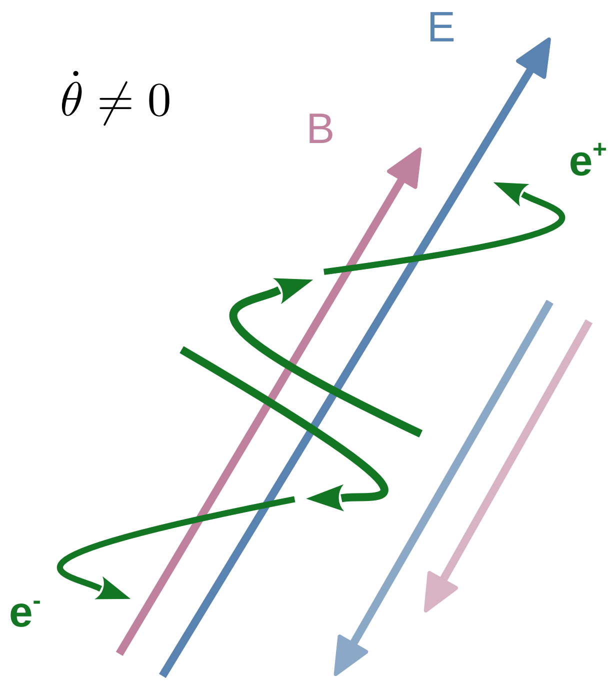
Axion assisted Schwinger effect



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based on
2101.05192 and 2204.10842
w Yohei Ema and Kyohei Mukaida

Axion assisted Schwinger effect

Schwinger particle production in electric field:

$$\sim \quad \exp\left(\frac{-m^2 - p_T^2}{E}\right)$$

Axion assisted Schwinger particle production in electric field

$$\sim \quad \exp\left(-\frac{m^2}{E}\right) \quad \text{for} \quad \dot{\theta}^2 \gtrsim \frac{m^2 p_T^2}{E}$$

→ high momentum part of distribution exponentially enhanced

in presence of large axion velocity

Outline

- Solving the Dirac equation with axion background field
- Axion assisted Schwinger effect and interpretation
- Basis independence and transient phenomena

Dirac equation

$$S = \int d^4x \left[\frac{1}{2} (\partial\phi)^2 - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left(i\not{D} - m e^{2 i c_m \phi / f_a \gamma_5} \right) \psi + c_A \frac{\alpha}{4\pi f_a} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} + c_5 \frac{\partial_\mu \phi}{f_a} \bar{\psi} \gamma^\mu \gamma_5 \psi \right]$$

axion — gauge field — fermion — shift symmetric axion couplings

→ Dirac equation

$$\left[i\not{D} - m e^{2 i \theta_m \gamma_5} + \partial_\mu \theta_5 \gamma^\mu \gamma_5 \right] \psi = 0 \,, \qquad \theta_i = c_i \phi / f_a$$

(note symmetry $\psi \to e^{i c \gamma_5 \phi / f_a}$: $c_5 \to c_5 - c \,,\; c_m \to c_m + c$)

→ Solve for constant axion velocity $\dot{\theta}$ and E-field $E \hat{e}_z$ (switched on over finite time T)

Solutions to the Dirac equation

formal solution:

solutions of Dirac equation for constant background fields, $\dot{\theta} = 0\,, \quad E = -\dot{A}_z = 0$

$$\psi = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} e^{i\theta_5\gamma_5} \sum_{\lambda=1,2} \left[B_\lambda u_\lambda e^{-i\int dt\Omega} + D_\lambda^\dagger v_\lambda e^{+i\int dt\Omega} \right]$$

creation/annihilation operators

positive/negative frequency modes

$$\Omega = \sqrt{(p_z - gQA_z)^2 + (m^2 + p_T^2)}$$

$$B_\lambda = \sum_{\lambda'=1,2} \left[\alpha_\lambda^{(\lambda')} b_{\lambda'} - (-)^{\lambda+\lambda'} \beta_\lambda^{(\lambda')*} d_{\lambda'}^\dagger \right]$$

$$D_\lambda^\dagger = \sum_{\lambda'=1,2} \left[\beta_\lambda^{(\lambda')} b_{\lambda'} + (-)^{\lambda+\lambda'} \alpha_\lambda^{(\lambda')*} d_{\lambda'}^\dagger \right]$$

creation/annihilation operators at infinite past

Bogoliubov coefficients

→ analytical solutions for u_λ, v_λ (fixes basis)

numerically solve Dirac equation for Bogoliubov coefficients, IC $\alpha = 1, \beta = 0$

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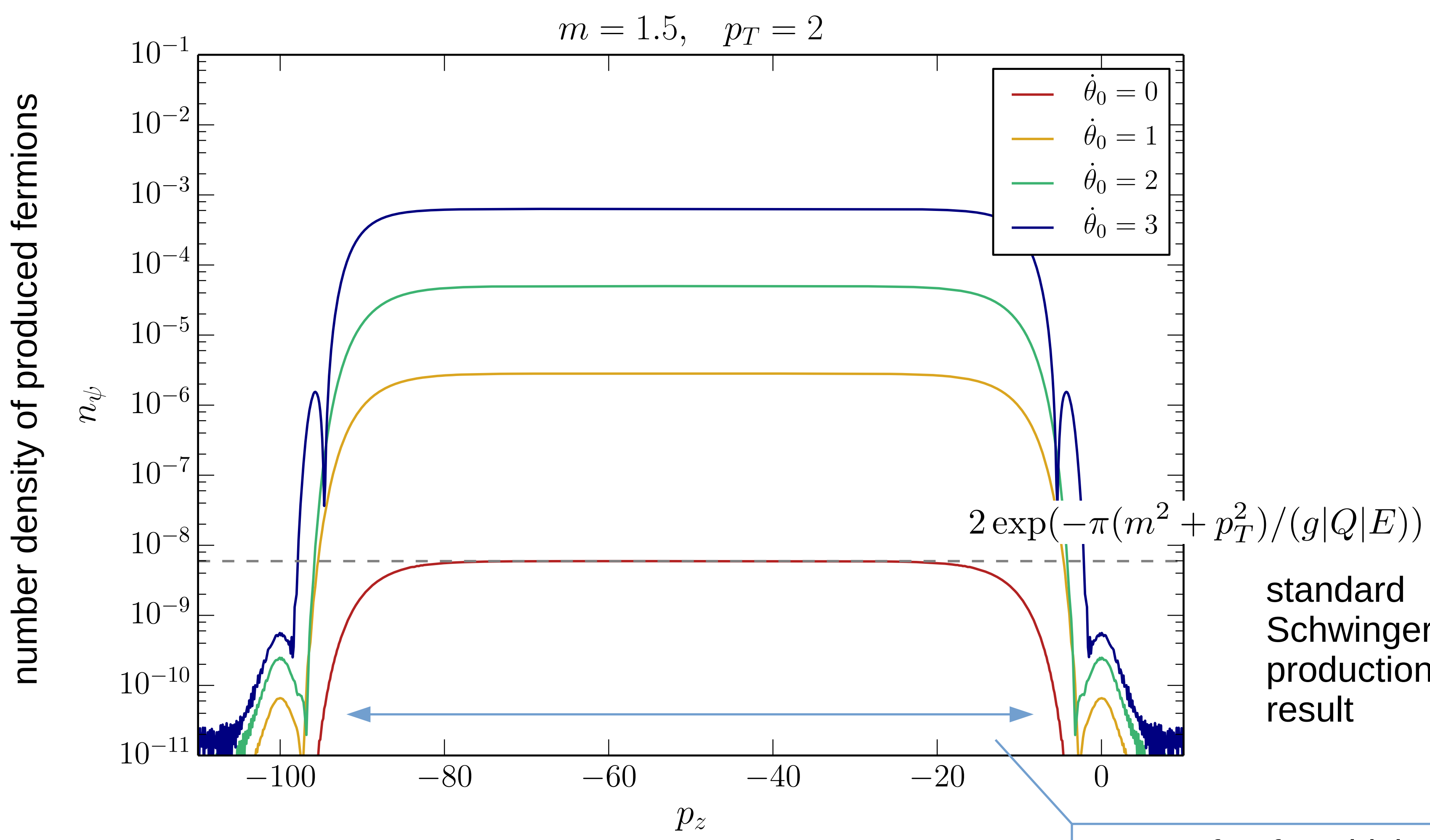
Numerical solutions



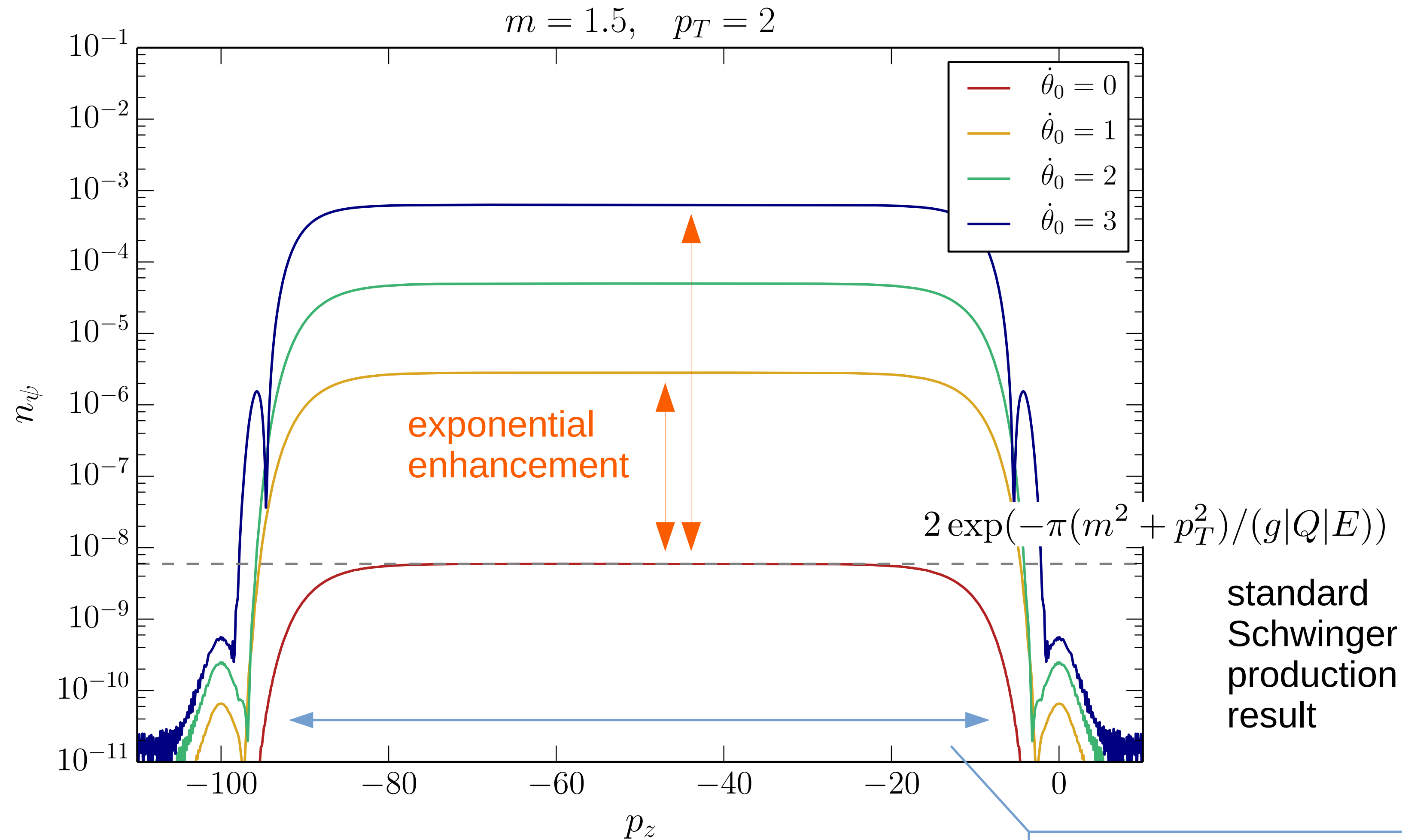
standard Schwinger production result

range of p_z for which $p_z + gQEt = 0$

units:
$g|Q|E = 1$

fermion momentum (in E-field direction)

Numerical solutions



number density of produced fermions

fermion momentum (in E-field direction)

standard Schwinger production result

range of p_z for which $p_z + gQEt = 0$

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Non-relativistic limit

$$\mathcal{L}_\eta = \eta^\dagger i\partial_0 \eta + \frac{1}{2m}\eta^\dagger \left(\Pi^2 - 2\dot{\theta}_{5+m}\Pi\cdot\sigma + \dot{\theta}_{5+m}^2\right)\eta + \mathcal{O}\left(\frac{1}{m^2}\right)\,, \quad \Pi = (p_x, p_y, p_z - gQA_z)$$

axion induced spin-momentum coupling

$$\rightarrow \quad \tilde{\Omega}_{\rm NR}^{\pm} = \frac{1}{2m}\left(\sqrt{(p_z - gQA_z)^2 + p_T^2} \pm \dot{\theta}_{5+m}\right)^2$$

minimized for non-zero axion velocity

spin-momentum coupling can compensate p_T suppression

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Analytical solution in FCB

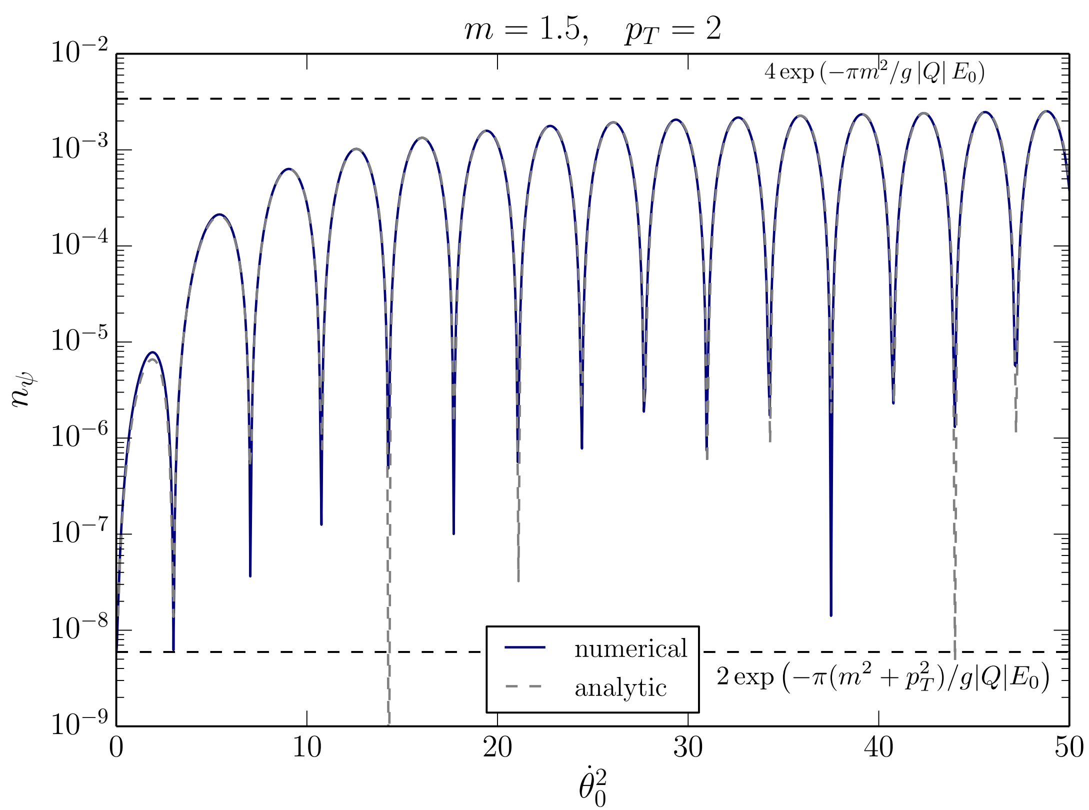
see also [Kitamoto, Yamada `21]

Choose $c_m = 0$ (fermion current basis, FCB).

relevant energy levels:

$$\tilde{\Omega}^{\pm}(t) = \sqrt{\left(\sqrt{(p_z - gQA_z)^2 + p_T^2} \pm \dot{\theta}_{5+m}\right)^2 + m^2}$$

solve using
WKB approximation



Basis dependence

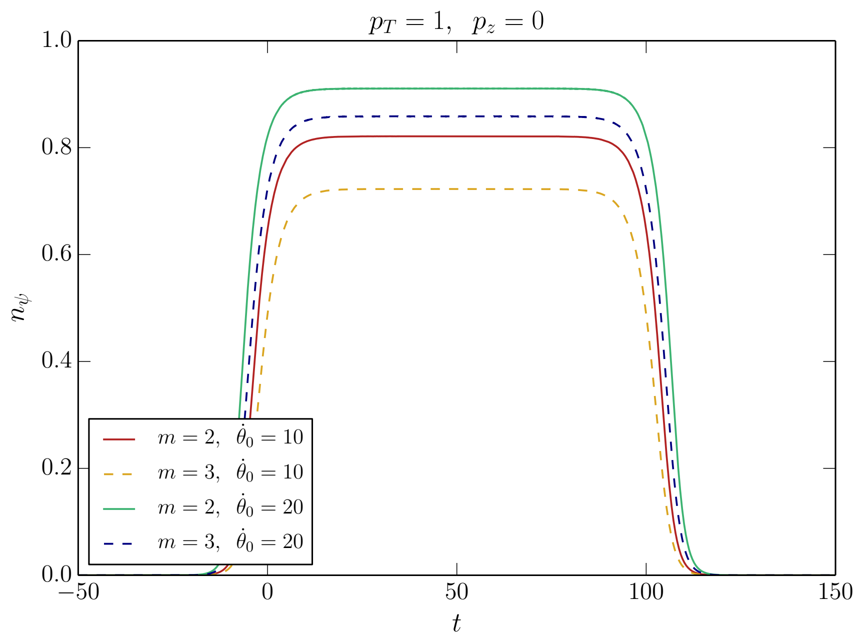
- for time-independent background, solutions u_λ, v_λ are uniquely fixed*
- for time-dependent background, any complete set of u_λ, v_λ is a priori equally good
- for evaluating physical observables in the asymptotic future (= time-independent background) all choices trivially give same result
- when evaluating physical observables at intermediate times (transient phenomena) we need to take care of divergences, regularization and renormalization
- Obviously, if we do everything correctly, physical observables, even at intermediate times, will not depend on the basis choice.

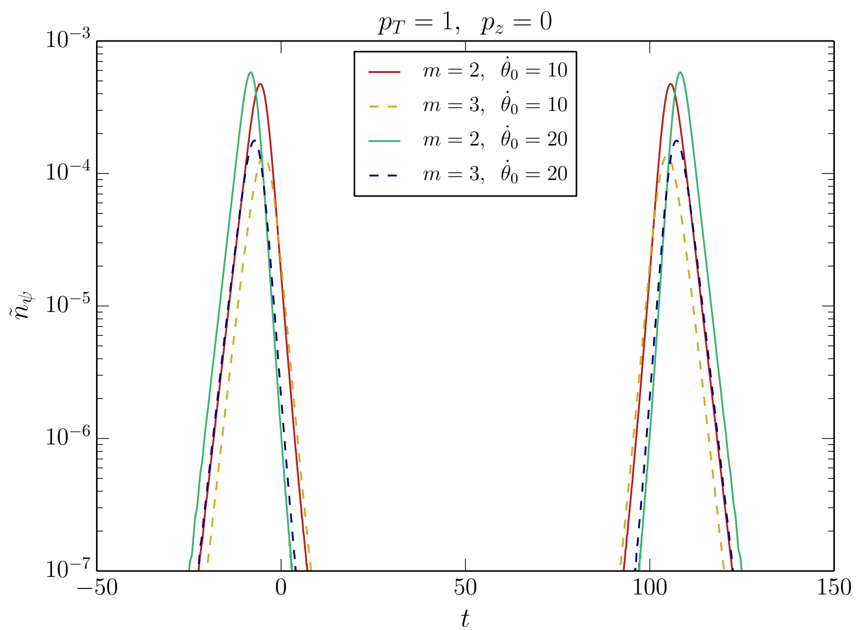
see also [Adshead, Lozanov `21]

* up to normalization, symmetries, degenerate eigenvalues

Pitfalls and resolutions (1)

number density $n_\psi \sim |\beta|^2$ no E-field, $\dot\theta \neq 0$ for 0 < t < 100





Hamiltonian basis $c_5 = 0$

Fermion current basis $c_m = 0$

units:
$p_T = 1$

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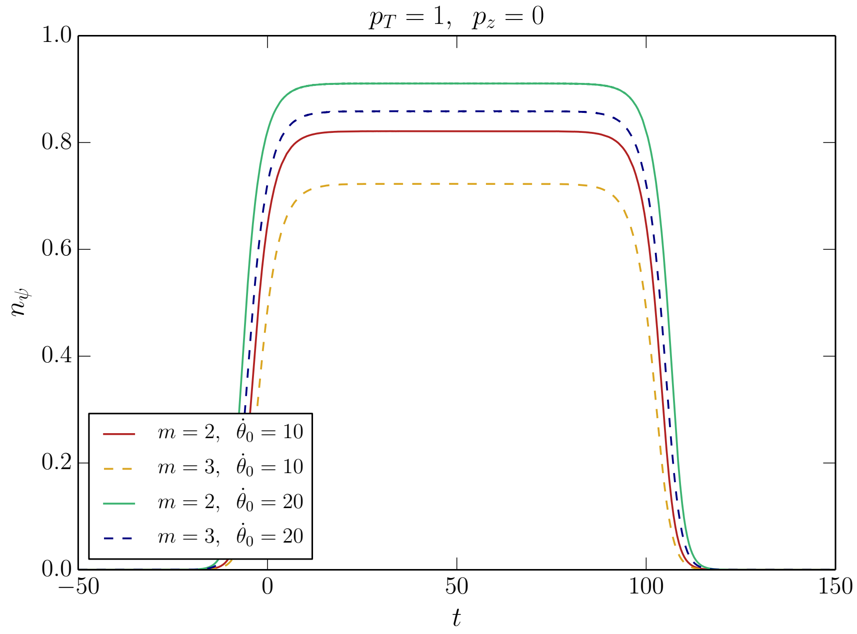

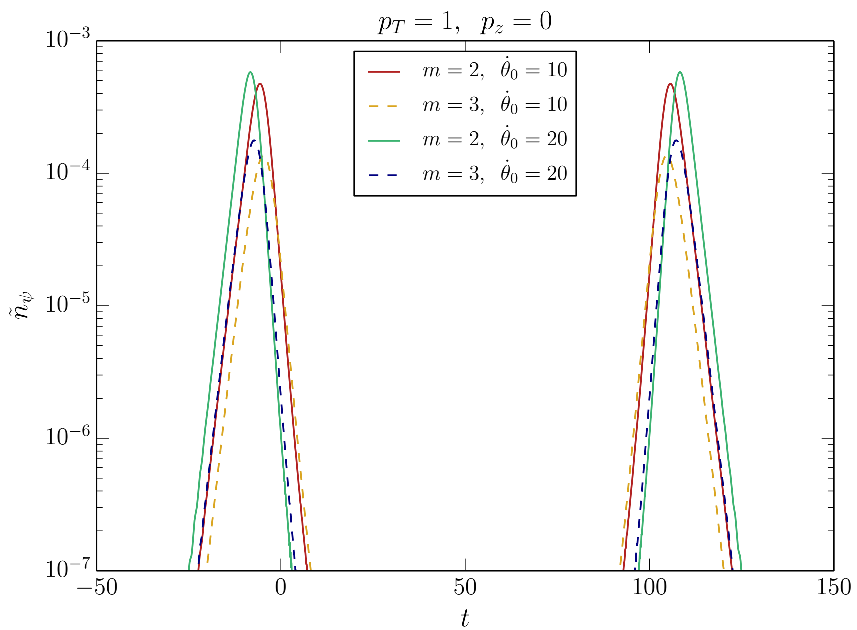

Hamiltonian basis $c_5 = 0$ Fermion current basis $c_m = 0$

→ $|\beta|^2$ is not an observable

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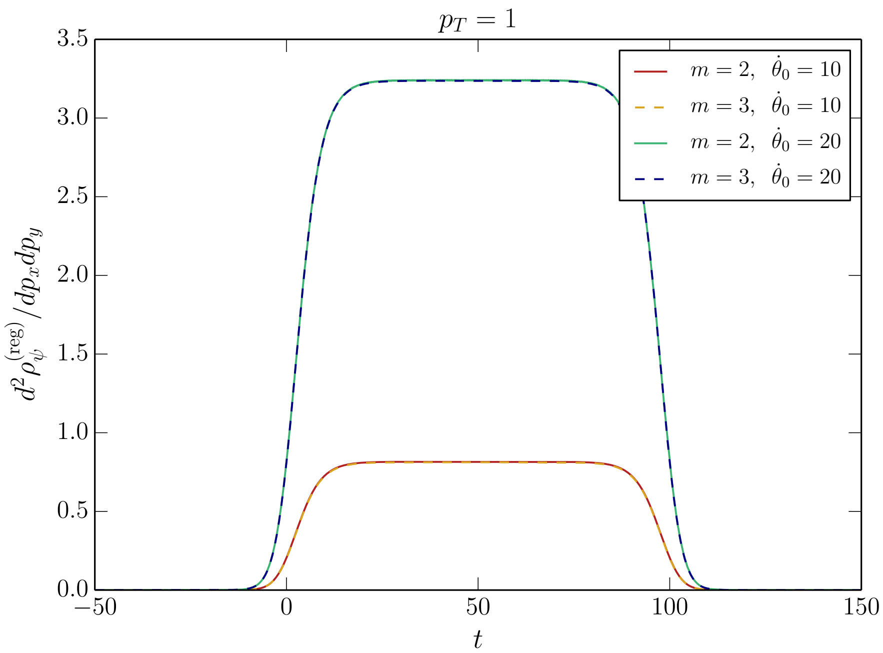
Pitfalls and resolutions (2)

energy density $$\rho_\psi = \frac{\langle H_\psi \rangle}{\text{Vol}} = \rho_\psi^{(\text{reg})} + \rho_\psi^{(\text{vac})}$$



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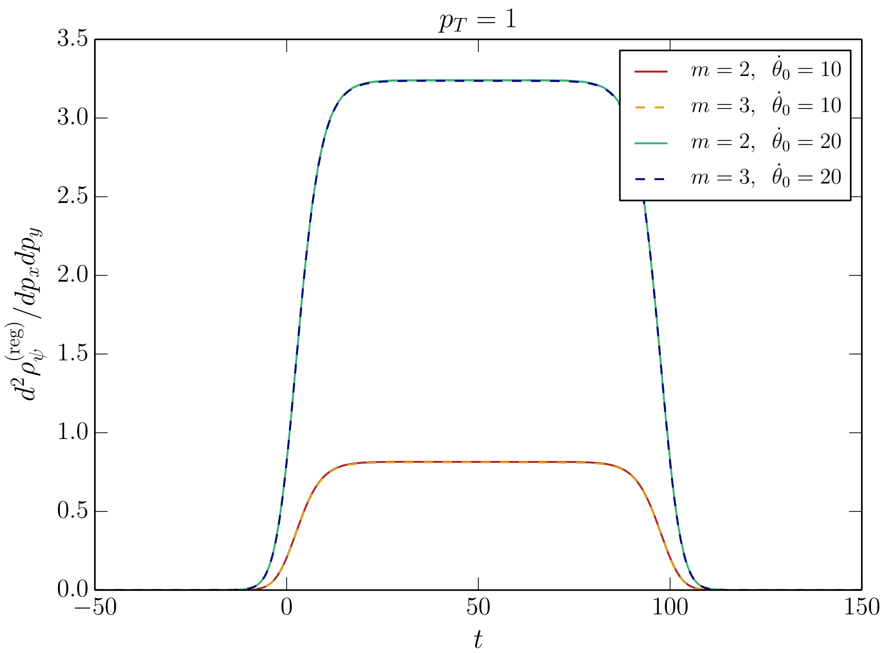
Pitfalls and resolutions (2)

energy density $\rho_\psi = \frac{\langle H_\psi \rangle}{\text{Vol}} = \rho_\psi^{(\text{reg})} + \rho_\psi^{(\text{vac})}$ $\rho^{(\text{diff})} \equiv : \rho_\psi^{(\text{reg})} : |_{\text{HB}} - : \rho_\psi^{(\text{reg})} : |_{\text{FCB}}$



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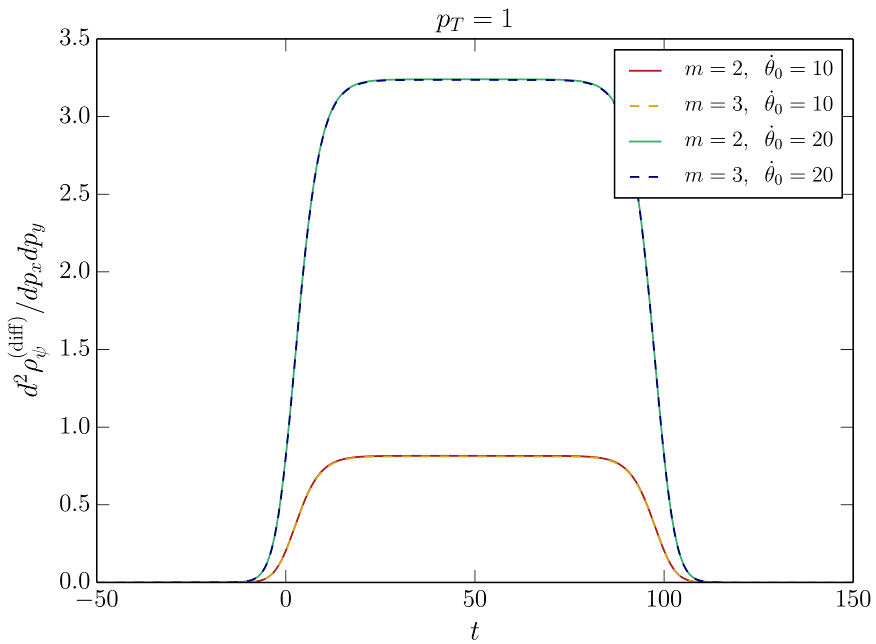

Fermion current basis $c_m = 0$

- normal ordering assumes that the argument of the regulator function is an eigenvalue of the energy density operator (btw, dimensional regularization also fails in FCB due to γ_5)

Final thoughts

- Everything is frame independent, massless limit and anomaly equation are reproduced (as it should!)

- exponential enhancement of large momentum particles in axion assisted Schwinger effect

  → requires large axion velocity

  → pheno consequences ?

- O(1) transient energy density (even no exp($-m^2/E$) supression!)

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Thank you!

UV sensitivity

- very large transient effect found in physical observable ρ_ψ , frame independent
- arises even without any EM background fields
- can be calculated analytically:

$$\rho_\psi^{(\mathrm{diff})} = \frac{m^2 \dot{\theta}_{5+m}^2}{4\pi^2} \log\left(\frac{\Lambda^2}{m^2}\right) + \frac{\dot{\theta}_{5+m}^4}{4\pi^2}$$

($\rho_\psi \to 0 \text{ for } m \to 0$ as expected)

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dim-8 operator

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→ low energy axion EFT is non-renormalizable

discussing this dim-8 operator requires specification of UV theory