Phenomenology of particle production during axion inflation

Marco Peloso, University of Padua

- Axion inflation
- Gauge field amplification → Density perturbations; → Gravitational waves

  Density perturbations → Gravitational waves; Density perturbations → Primordial Black Holes

- Strong backreaction on background dynamics

- Density perturbations $\zeta \sim \frac{\delta\rho}{\rho}$ observed through ΔT_{CMB}
- Gravitational waves from inflation not yet observed $r \equiv \frac{P_h}{P_\zeta} = \frac{\langle hh \rangle}{\langle \zeta\zeta \rangle}$
- $r < 0.032$ (Planck+Bicep '21); $r \lesssim \mathrm{O}\left(10^{-3}\right)$ from CMB-S4

Slow roll inflation + standard assumption of $\delta\rho$ and GW from amplification of vacuum modes (due to the inflationary expansion)

$$V^{1/4} \simeq 10^{16}\,\text{GeV}\left(\frac{r}{0.01}\right)^{1/4}, \quad \Delta\phi \gtrsim M_p \left(\frac{r}{0.01}\right)^{1/2}$$

Lyth '96

If observed:

- Inflaton scans a Planckian range during inflation
- Scale of inflation $\cong$ GUT scale
- Parametrically : $m_\phi^2 \sim \frac{V}{\Delta\phi^2} \sim \mathrm{O}\left(10^{13}\,\text{GeV}\right)^2$

Motivations for axion inflation

$\Delta\phi > M_p$ not expected in a generic low-energy effective QFT

$$V = V_{\text{renormalizable}}(\phi) + \phi^4 \sum_{n=1}^{\infty} c_n \frac{\phi^n}{M^n}$$

Hard to achieve flatness $\dfrac{M_p V'}{V}, \dfrac{M_p^2 V''}{V} \ll 1$ unless $M \gg M_p$ (e.g., η problem in supergravity)

Shift symmetry $\phi \rightarrow \phi + C$. E.g. axion (natural) inflation

Freese, Frieman, Olinto '90

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + V_{\cancel{\text{shift}}}(\phi) + \frac{C}{f}\partial_\mu \phi \, \bar{\psi}\gamma^\mu \gamma_5 \psi + \frac{\alpha}{f}\phi \, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

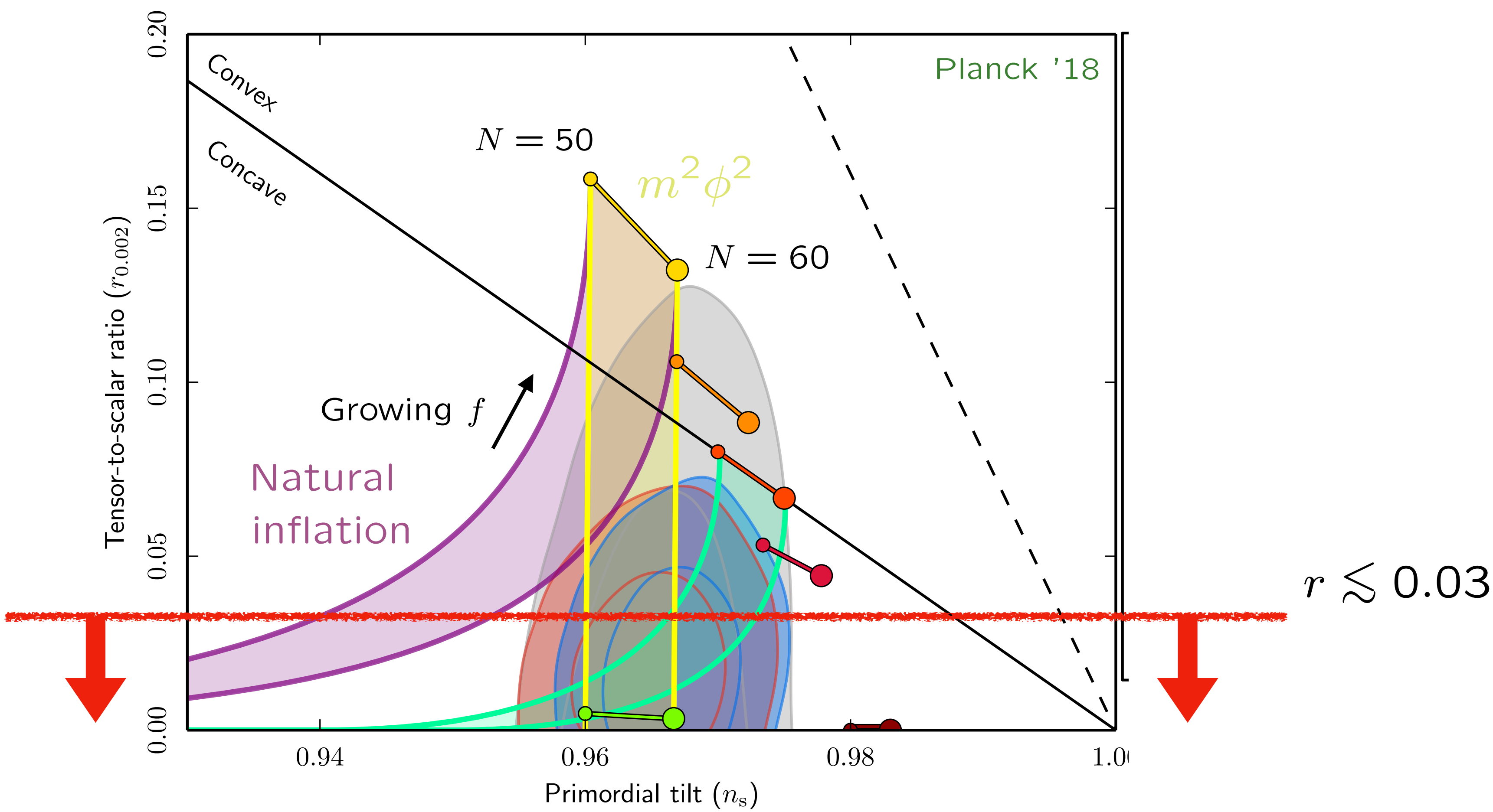
- Smallness of $V_{\cancel{\text{shift}}}$ technically natural. $\Delta V \propto V_{\cancel{\text{shift}}}$
- Constrained couplings to matter (predictivity)

Are we speaking about a null set? (Is axion inflation still viable?)

Common to identify axion inflation with "natural inflation"

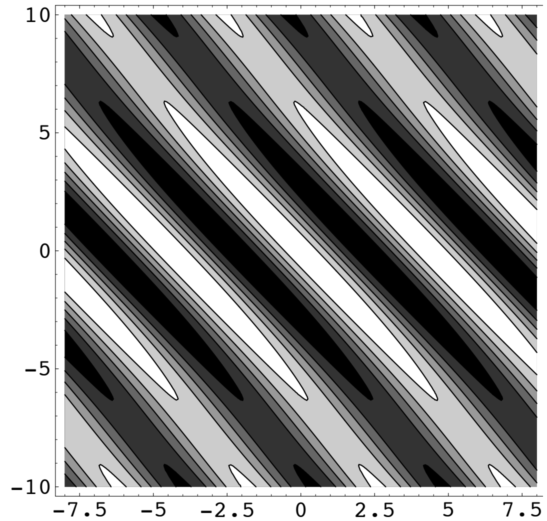
Freese, Frieman, Olinto '90

$$V = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$



Aligned natural inflation

Kim, Nilles, MP '05



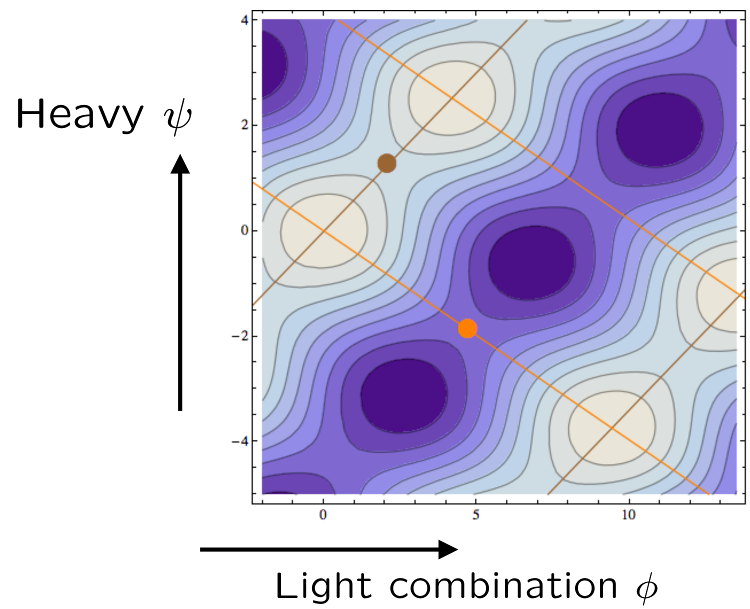
$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{\theta}{f_1} + \frac{\rho}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\theta}{f_2} + \frac{\rho}{g_2}\right)\right]$$

$f_{\text{eff}} \gg f_i,\, g_i$ if $\dfrac{f_1}{g_1} \simeq \dfrac{f_2}{g_2}$

- Proposed to produce $f_{\text{eff}} > M_p$ from sub-Planckian $f_i,\, g_i$

  (gravitational instanton corrections may still be a problem)

- In agreement with CMB



denote by ϕ (ψ) the light (heavy) eigenstate

Fields rescaled $\simeq$ curvature in 2 directions

Vertical direction ψ much heavier; inflation on valleys with $\dfrac{\partial V}{\partial \psi} = 0\,,\ \dfrac{\partial^2 V}{\partial \psi^2} > 0$

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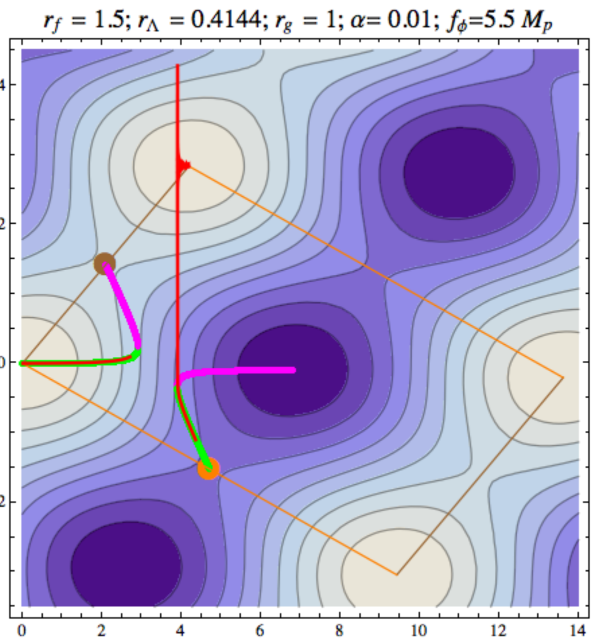
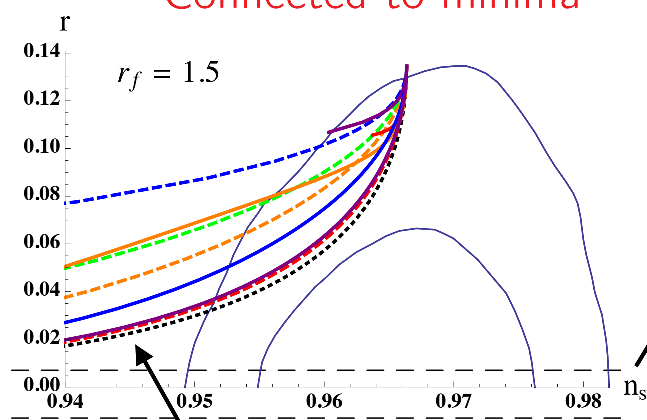


MP, Unal '15

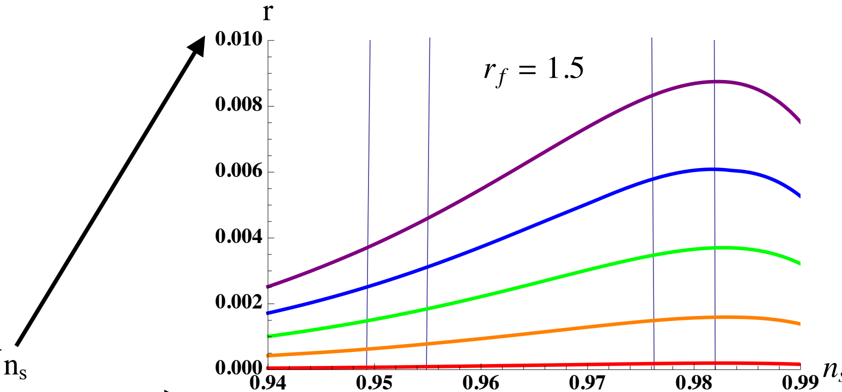
For some parameters inflationary trajectories ending because

(1) reach a minimum or (2) become unstable in heavy direction

Connected to minima



Disconnected



All above natural inflation !

Recall $r = 16\epsilon = 8M_p^2\left(\frac{V'}{V}\right)^2$

For solutions (1), flattening V also increases the duration of inflation.

Limited possibility to decrease ϵ at $N \sim 60$ e-folds before end of inflation.

Vector production from $\frac{\alpha\,\phi}{f}\,F\,\tilde{F}$

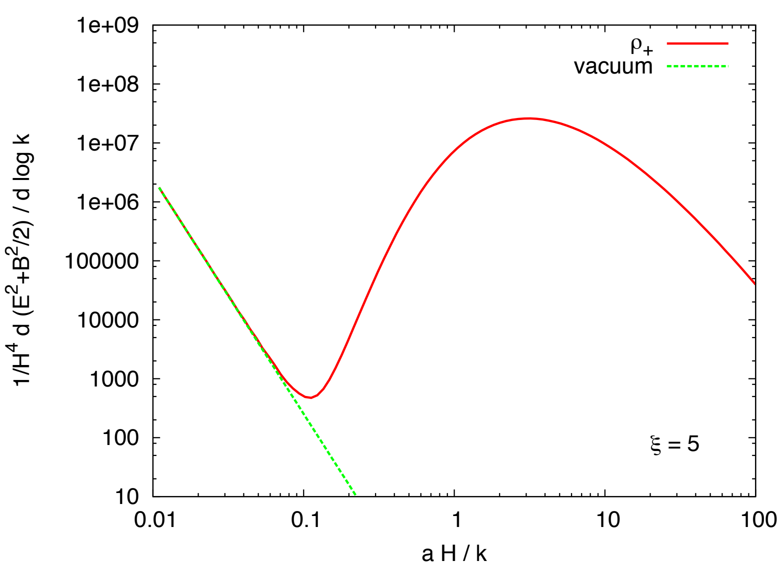
Turner, Widrow '88

Garretson, Field, Carroll '92

Anber, Sorbo '06

$$\Rightarrow \quad \left(\frac{\partial^2}{\partial\tau^2}+k^2\mp 2\,a\,H\,k\,\xi\right)A_\pm(\tau,\,k)=0 \qquad \xi\equiv\frac{\alpha\,\dot{\phi}^{(0)}}{2\,f\,H}$$

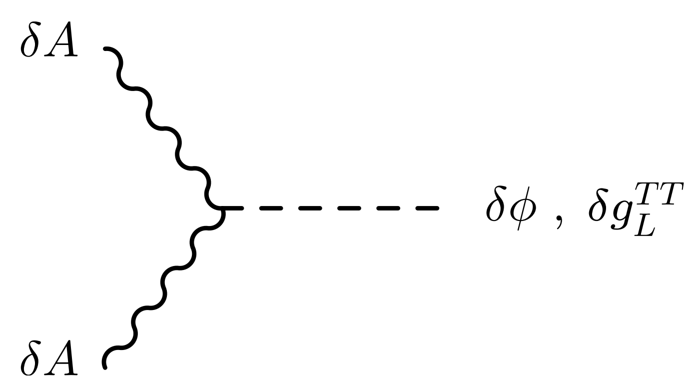
Physical ρ in one mode



- One tachyonic helicity at horizon crossing
- Then diluted by expansion
- Max amplitude $A_+ \propto \mathrm{e}^{\pi\xi}$

Amplified gauge fields source scalar and tensor perturbations

Barnaby, MP '10



- Scalar non-G @ CMB scales $\rightarrow$ $f/\alpha \gtrsim 10^{16}$ GeV
- GW less produced ($1/M_p$ vs α/f)

Barnaby, MP '10

In this regime, negligible backreaction on background dynamics

Under perturbative control; confirmed by lattice

MP, Sorbo, Unal '16
Caravano, Komatsu, Lozanov, Weller '22

General lesson: Several mechanisms for additional GW, result in a decrease of r once also extra density perturbations are accounted for

Observation of GW through the CMB $\Longleftrightarrow$	$V^{1/4} \simeq 10^{16}\,\text{GeV} \left(\frac{r}{0.01}\right)^{1/4}$ $\Delta\phi \gtrsim M_p \left(\frac{r}{0.01}\right)^{1/2}$

Robust!

How robust ? Cost for evading it ?

- No direct coupling with inflaton (Source gravitationally coupled to both GW and inflaton)
- Relativistic source (GW are produced by quadrupole moment; ζ by energy density)
- Source active only for limited time (GW observed only on a small window; ζ provides constrains on many more scales)

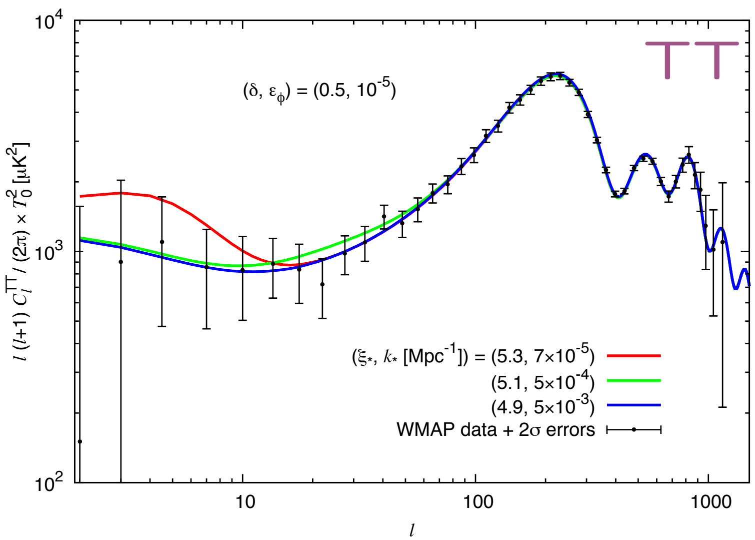
These 3 ingredients present in Namba, MP, Shiraishi, Sorbo, Unal '15

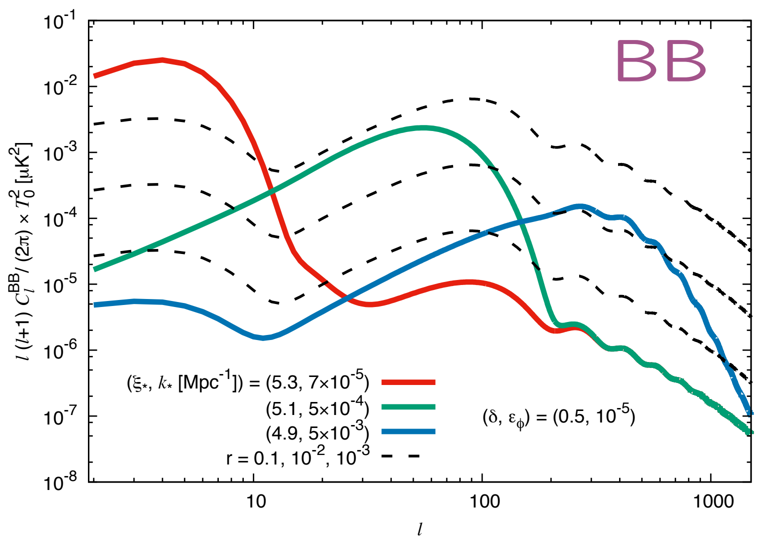
$$\mathcal{L} = \underbrace{-\frac{1}{2}(\partial\varphi)^2 - U(\varphi)}_{\text{inflaton sector}} \underbrace{- \frac{1}{2}(\partial\sigma)^2 - V(\sigma) - \frac{1}{4}F^2 - \frac{\alpha\,\sigma}{4f}F\tilde{F}}_{\text{extra sector}}$$

$m_\sigma = \mathrm{O}(H)$ so to roll for few e-folds

- Gives visible r at arbitrarily small r_{vacuum} / scale of inflation

Three examples with $\epsilon_\phi = 10^{-5}$ (so that $r_{\text{vacuum}} = 16\,\epsilon$ is unobservable):





Shown for comparison: scale invariant

$r = 0.1$

$r = 0.01$

$r = 0.001$

- Distinguishable from vacuum GW by tensor running

Moral: Hard, but not impossible, to violate $V \leftrightarrow r$ relation. Limits from scalar non-G $\rightarrow$ specific conditions $\rightarrow$ distinguish from vacuum GW

Naturally blue signals in axion inflation

Back to production from inflaton Recall $A_+ \propto \mathrm{e}^{\pi\xi}$, $\xi = \frac{\dot\phi}{2fH} \propto \sqrt{\epsilon}$

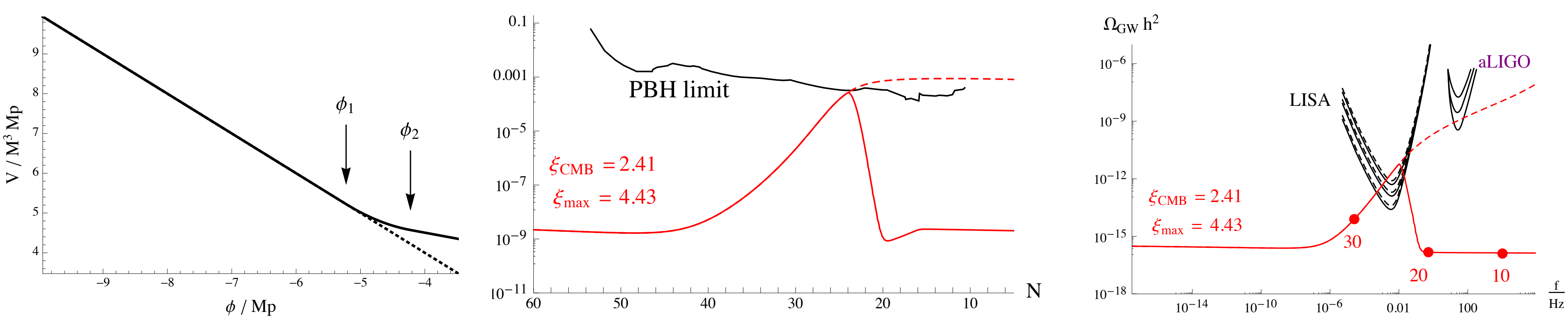
Inflaton speeds up $\rightarrow$ signal naturally grows at small scales

- GW at interferometers Cook, Sorbo '12; Barnaby, Pajer, MP '12 Domcke, Pieroni, Binétruy '16
- PBH Linde, Mooij, Pajer '12; Bugaev, Klimai '13 Garcia-Bellido, MP, Unal '16, '17

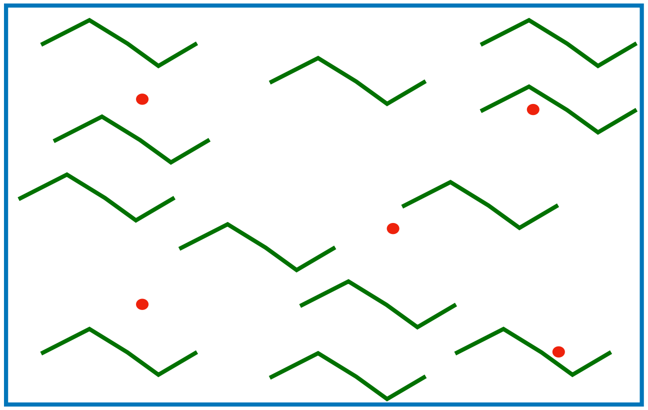
GW in CMB limited by scalar non-G

GW at interferometers limited by $\zeta \rightarrow$ PBH

highly model dependent, due to $\propto \mathrm{e}^{\dot\phi}$

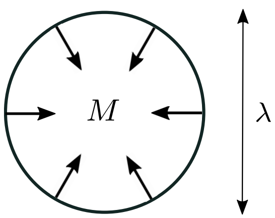


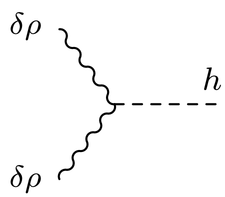
PBH $\leftarrow$ enhanched $\delta\rho$ $\rightarrow$ GW (after inflation)



Increase $\delta\rho$ everywhere, so to have (rare) regions above threshold for collapsing to PBH. Enhanced $\delta\rho$ unavoidably produces GW at horizon re-entry (standard GR; independent of the inflationary mechanism of $\delta\rho$ production)

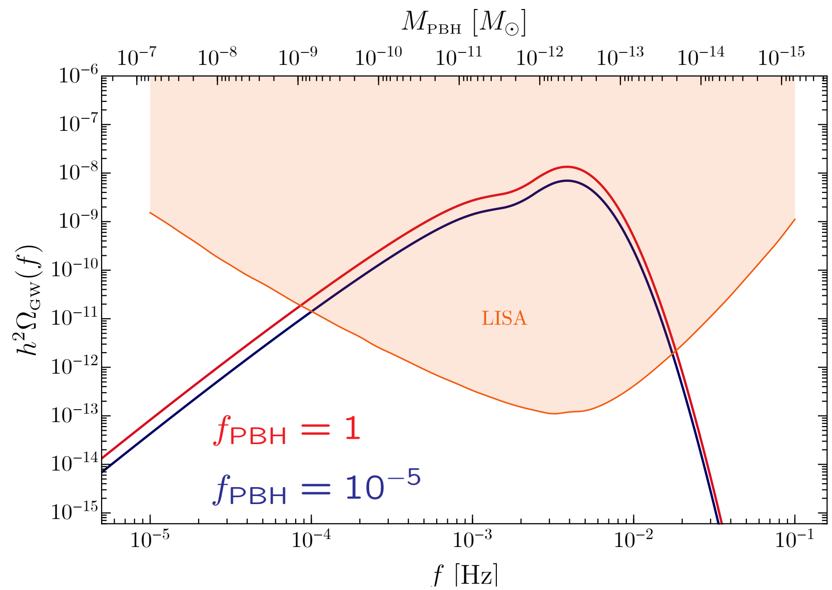
PBH mass $\rightarrow$ wavelength enhanced $\delta\rho$ modes $\rightarrow$ GW frequency





$$f_{\rm GW} \sim \frac{1}{\lambda} \sim 3\,{\rm mHz}\sqrt{\frac{10^{-12}\,M_\odot}{M}}$$

Sub-lunar $M_{\rm PBH}$ allowed window for PBH dark-matter results in GW in LISA band. In fact, GW allow to also probe existence of PBH of insignificant abundance



Cosmo vs. Astro Stochastic Gravitational Wave Background

Once SGWB observed, how can we extract a cosmological component from the astrophysical one ?

- Frequency dependence
- Arrival direction
- Chirality
- Statistics

The NG that we cannot see ...

Great amount of information in $\left\langle h_{\lambda_1}\left(\vec{k}_1\right)\, h_{\lambda_2}\left(\vec{k}_2\right)\, h_{\lambda_3}\left(\vec{k}_3\right)\right\rangle$

Any measure of NG requires phase coherency.

Phases $\quad h_{\vec{k}}\left(t,\,\vec{k}\right) = \mathrm{e}^{-i\,k\,t}\, h_{\vec{k}} + \mathrm{e}^{i\,k\,t}\, h^*_{\vec{k}}$

No pbm. in PS $\left\langle h\left(\vec{k}_1\right)\, h\left(\vec{k}_2\right)\right\rangle = \delta^{(3)}\left(\vec{k}_1 + \vec{k}_2\right)\left[\underbrace{\mathrm{e}^{-i\,(k_1 - k_2)\,t}}_{1}\, h_{\vec{k}_1}\, h^*_{\vec{k}_2} + \dots\right]$

Phases in BS $\left\langle h\left(\vec{k}_1\right) h\left(\vec{k}_2\right) h\left(\vec{k}_3\right)\right\rangle \propto \delta^{(3)}\left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3\right)\left[\mathrm{e}^{-i\,(k_1 + k_2 - k_3)\,t}\, h_{\vec{k}_1}\, h_{\vec{k}_2}\, h^*_{\vec{k}_3} + \dots\right]$

$$\left\langle \mathrm{signal}^3 \right\rangle = \int d^3k_1 d^3k_2 d^3k_3\, \mathcal{R}^{(3)}\left(\vec{k}_1,\, \vec{k}_2,\, \vec{k}_3\right)\, \left\langle h^3\left(\vec{k}_i\right)\right\rangle$$

Modes with same k_i but different $\hat{k}_i$ should have the same phase or

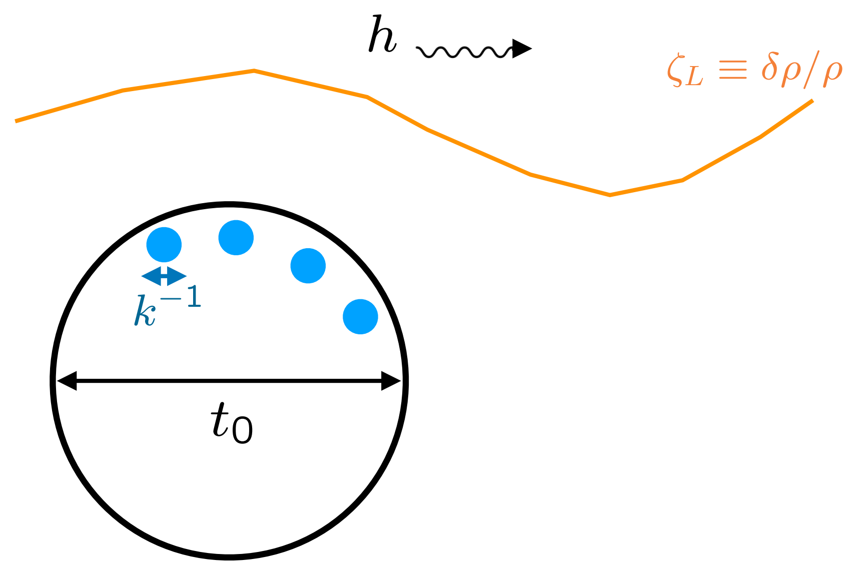
signal $\rightarrow 0$ in angular integral

$$h_{\vec{k}}\left(t,\,\vec{k}\right)=\mathrm{e}^{-i\,k\,t}\,h_{\vec{k}}+\mathrm{e}^{i\,k\,t}\,h^*_{\vec{k}}\;\;\Rightarrow\;\;\left\langle h^3\right\rangle\propto\mathrm{e}^{-i\,(k_1+k_1+k_2)\,t}\,h_{\vec{k}_1}\,h_{\vec{k}_2}\,h_{\vec{k}_3}+\ldots$$

Even if coherent at the origin (e.g., from inflation), two $\neq$ physical effects lead to decoherence

Bartolo, De Luca, Franciolini, Lewis, MP, Racco, Riotto '18

(1) Propagation in perturbed universe



Shapiro time delay: GW from $\neq$ directions

accumulate $\neq$ time shifts $\mathrm{e}^{-ik\left(t+\int^t dt'\zeta_L\right)}$

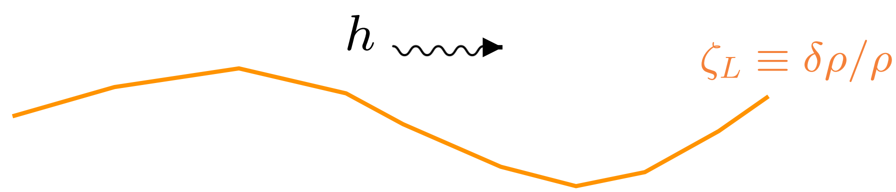
(2) Finite observation time

$k_{\text{resolved}}=k\pm\frac{1}{T_{\text{obs}}}$ $\;\Rightarrow\;$ Superposition of $\mathrm{e}^{-i\left(k-\frac{1}{T_{\text{obs}}}\right)t_0}+\ldots+\mathrm{e}^{-i\left(k+\frac{1}{T_{\text{obs}}}\right)t_0}$

This also generates $\Delta\phi\sim\frac{t_0}{T_{\text{obs}}}\gg1$, destroying coherency

... the NG that we might see ...

Non-G, angular anisotropies, and a probe of the large scale structure of the Universe

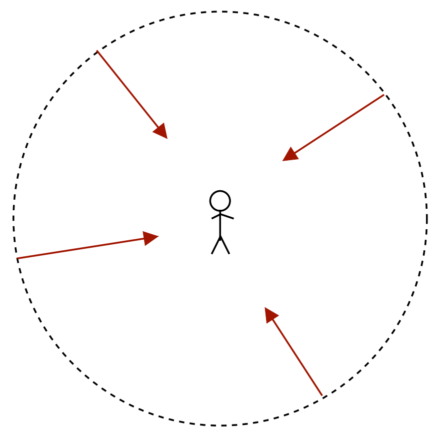


Production mechanism & propagation imprint anisotropies, $\rho_{\rm GW}\,(\vec{x}) \propto \dot{h}_{ij}\dot{h}_{ij}$

- Treatment as CMB

Alba, Maldacena '15; Contaldi '16; Cusin, Pitrou, Uzan '17; Jenkins, Sakellariadou '18; Bartolo, Bertacca, Matarrese, MP, Ricciardone, Riotto, Tasinato '19

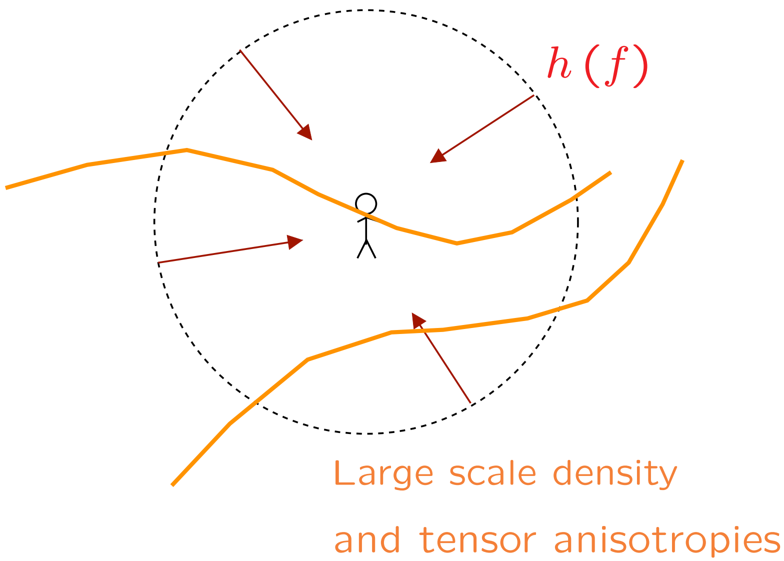
$$\rho_{\rm GW} = \sum_{\ell m} a_{\ell m}^{\rm GW}\, Y_{\ell m}$$



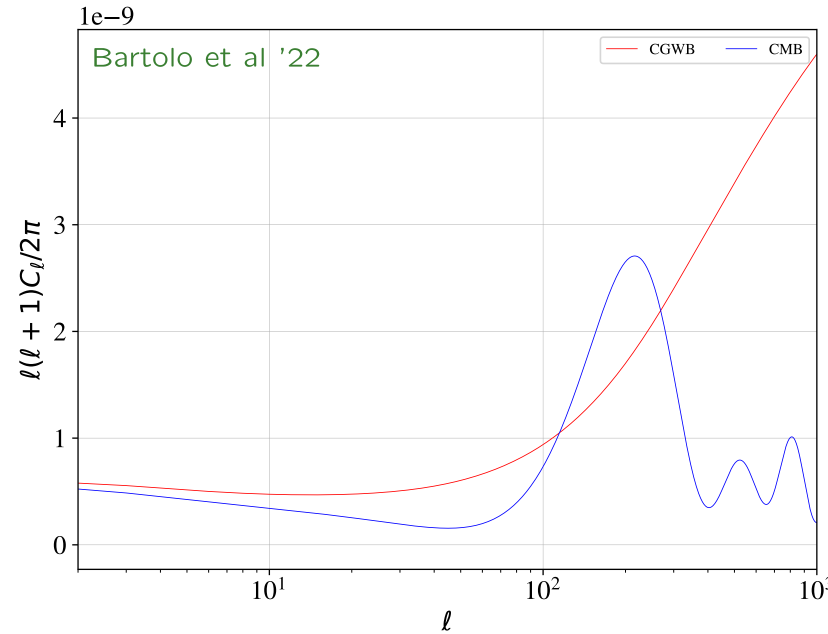
$$\langle a_{\ell m}\, a^*_{\ell' m'} \rangle = C_\ell\, \delta_{\ell\ell'}\, \delta_{mm'}$$ Angular power spectrum

$$\langle a_{\ell_1 m_1}\, a_{\ell_2 m_2}\, a_{\ell_3 m_3} \rangle \propto b_{\ell_1 \ell_2 \ell_3}$$ Bispectrum (non-G)

Anisotopies from the propagation



Sachs-Wolfe



Bispectrum from higher order interactions. Already at linear order, due to propagation, induced by the non-Gaussianity of $\delta\rho$. At large scales

$$b_{\ell_1,\ell_2\ell_3} \simeq 2\, f_{\rm NL}\left[C_{\ell_1}\,C_{\ell_2} + C_{\ell_1}\,C_{\ell_3} + C_{\ell_2}\,C_{\ell_3}\right]$$

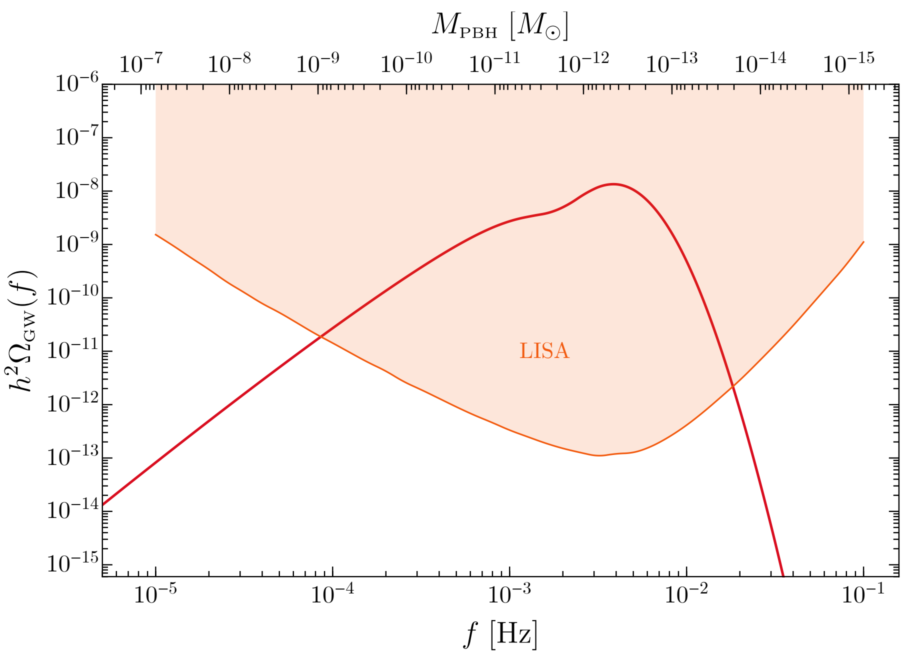
"local" scalar NG,

$$\delta\rho \sim \delta\rho_{\rm g} + f_{\rm NL}\,\delta\rho_g^2$$

New probe of large scale anisotropies (like CMB photons)

Anisotropies & non-G at the production - GW in models with PBH

Bartolo et al '19



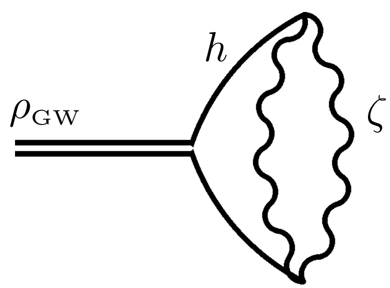
Very large GW signal @LISA

in models of PBH-DM.

Is is isotropic ? Is it Gaussian?

$$\zeta + \zeta \to h$$

$$\rho_{\rm GW} \sim \left\langle \dot{h}^2 \right\rangle \sim \left\langle \zeta^4 \right\rangle$$



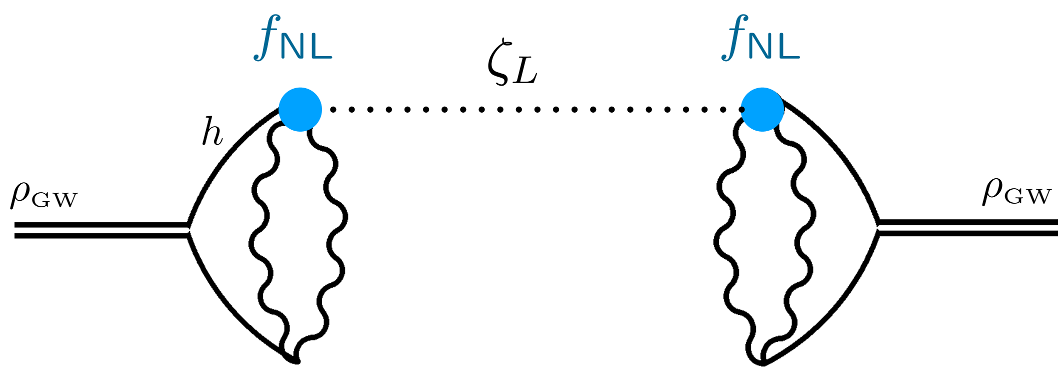
ζ is a short-scale mode, that

generates GW of $f \sim$ mHz today

In presence of scalar non-G,

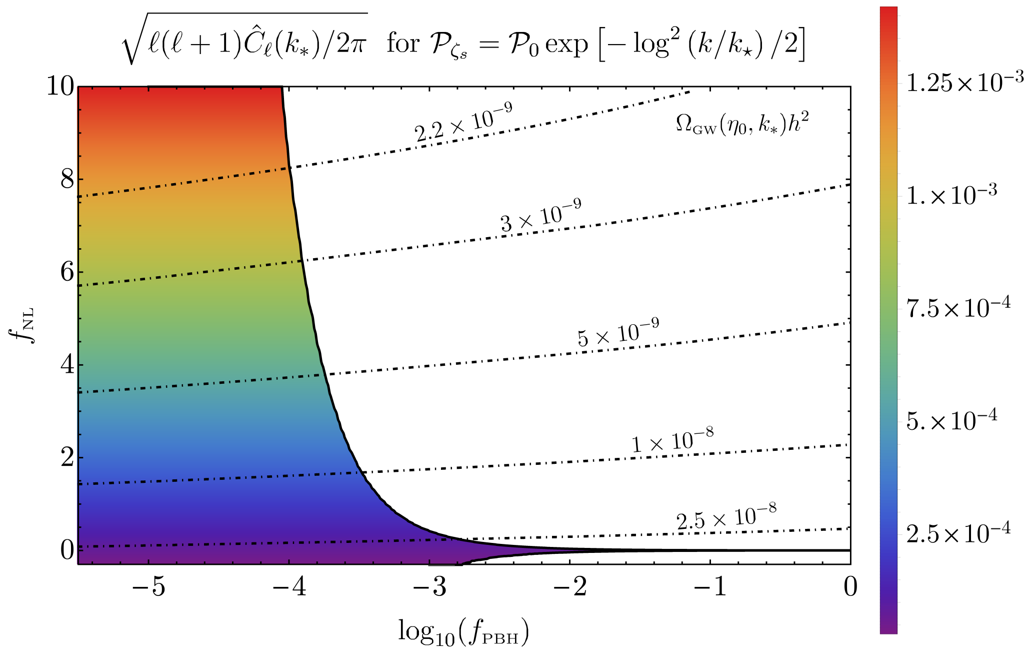
a long mode ζ_L modulates

the power of ζ on small scales,

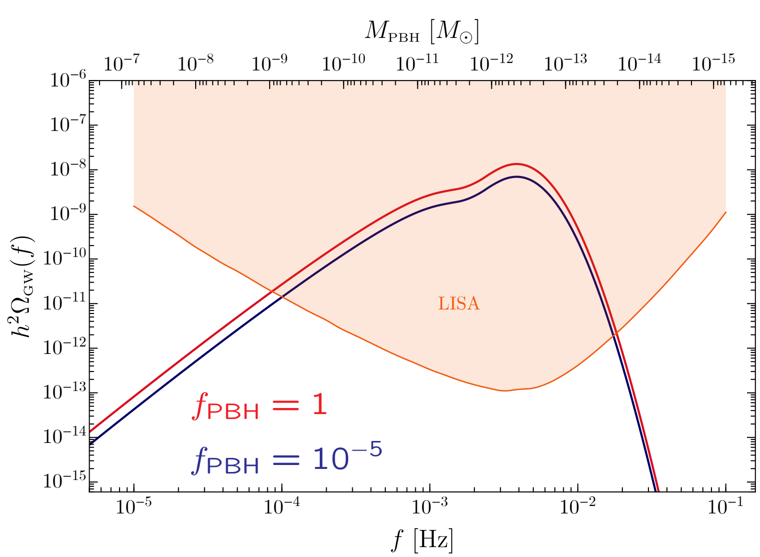


- Greater scalar $f_{\rm NL}$ leads to greater anisotropies and non-G of GW.
- $-11.1 \le f_{\rm NL} \le 9.3$, at 95% C.L. Planck '19
- Isocurvature constraints impose a tighter limit on $f_{\rm NL}$ for PBH-DM

Young, Byrnes '15



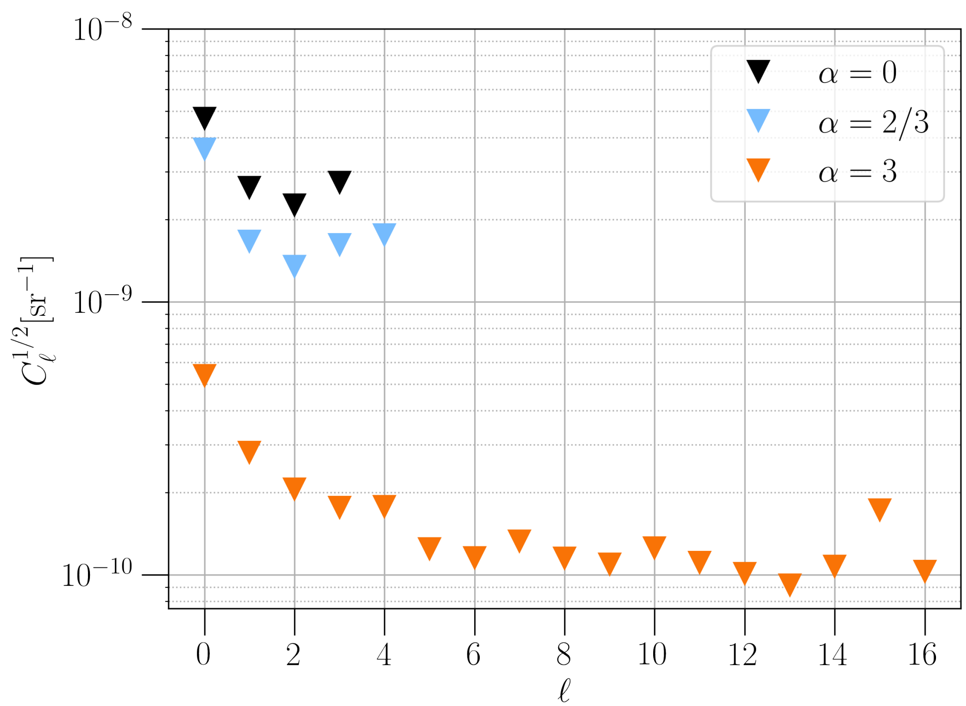
Observing a bump at LISA, with significant anisotropy and non-G indicates that the PBH constitute only a small fraction of the DM



Slight change of P_ζ $\rightarrow$ large change of $f_{\rm PBH}$ / slight change in $\Omega_{\rm GW}$.

Anisotropies can differentiate between these two cases.

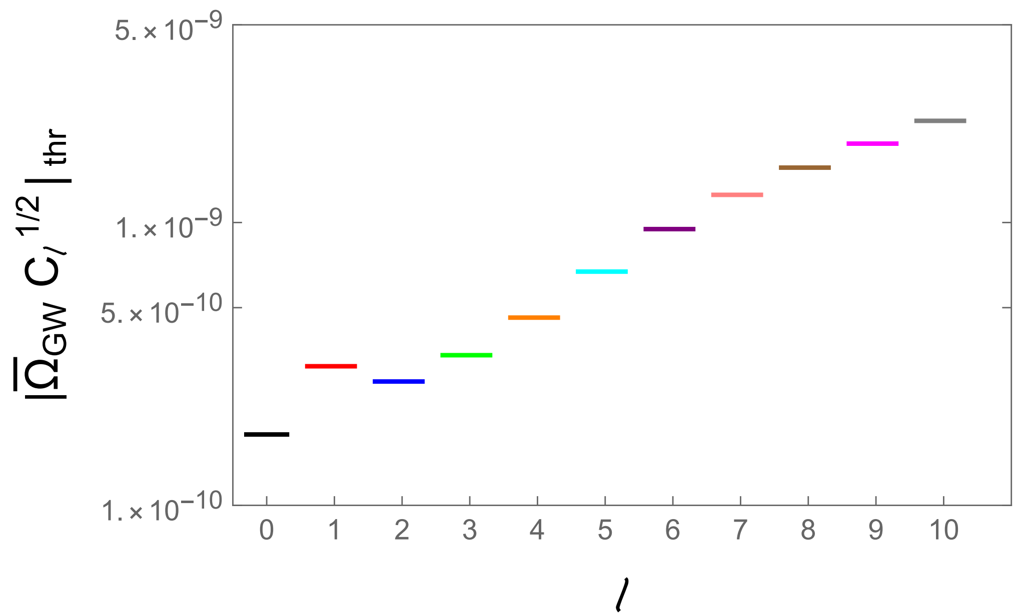
. . . can we see it ?



LIGO + Virgo '22

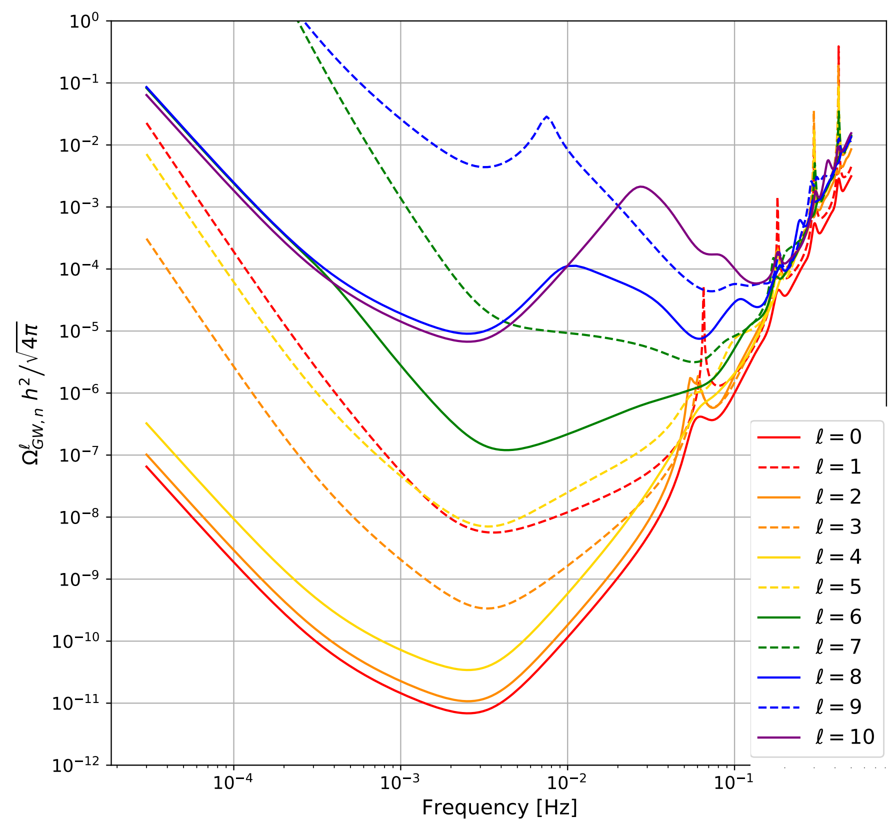
$$\Omega_{\rm GW} \propto \left(\frac{f}{25\,\mathrm{Hz}}\right)^{\alpha}$$

Adding ET ($\alpha = 0$)



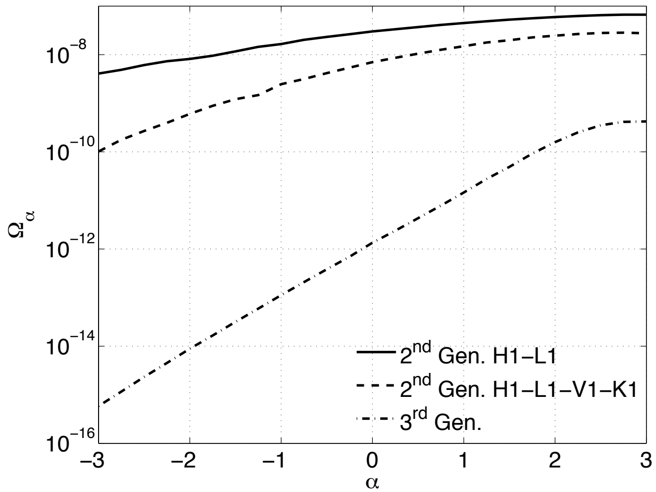
Mentasti, Contaldi, MP, to appear

LISA sensitivity



Bartolo et al '22

Measurement of GW polarization



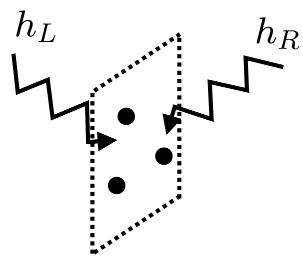
Crowder, Namba, Mandic, Mukohyama, MP '12

Assume $\Omega_{\rm GW,L} = \Omega_\alpha \left(\frac{f}{100\,{\rm Hz}}\right)^\alpha$ and $\Omega_{\rm GW,R} = 0$

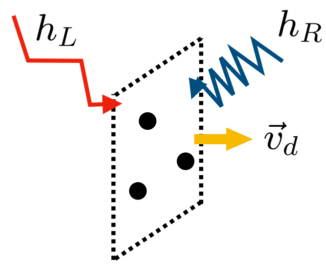
Amplitude needed to detect $\Omega_{\rm GW}$

and exclude $\Omega_{\rm GW,R} = \Omega_{\rm GW,L}$ at 2σ

Two GWs related by a mirror symmetry produce the same response in a planar detector. Cannot detect net circular polarization of an isotropic SGWB



Isotropy in any case broken by peculiar motion of the solar system. Assumption, $v_d \simeq 10^{-3}$ as CMB



$$\mathrm{SNR}_{\rm LISA} \simeq \frac{v_d}{10^{-3}} \frac{\Omega_{\rm GW,R} - \Omega_{\rm GW,L}}{1.2 \cdot 10^{-11}} \sqrt{\frac{T}{3\,{\rm years}}}$$

Domcke, García-Bellido, MP, Pieroni Ricciardone, Sorbo, Tasinato '19

(one order of magnitude greater than estimate in Seto '06)

Backreaction on the background dynamics

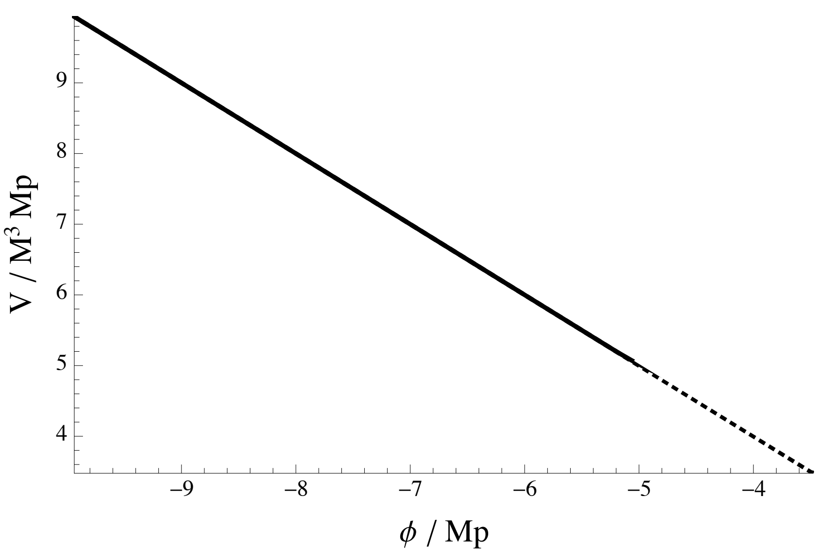
Recall $A_+ \propto e^{\pi\xi}$, $\xi = \dfrac{\dot{\phi}}{2fH} \propto \sqrt{\epsilon}$

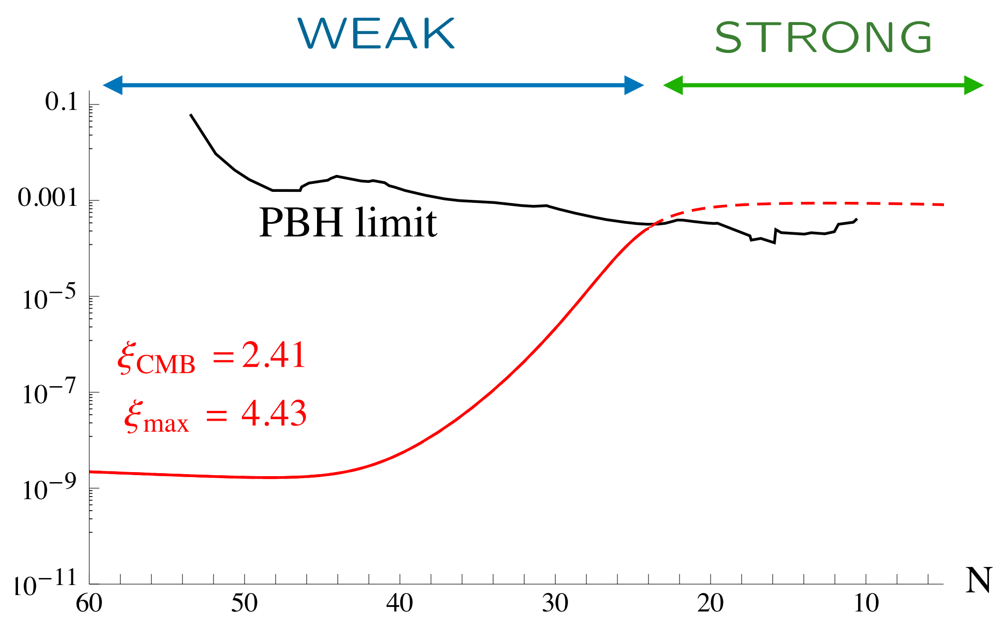
- Initial stage of weak bacreaction of the poduced $\vec{A}$ on ϕ dynamics.

  Standard increase of $\dot{\phi}(t)$ and marked growth of sourced signals

- Later stage of strong backreaction. $\dot{\phi}(t)$ increases much more slowly

  and sourced signals flatten out.





The instability of the Anber and Sorbo solution

MP, Sorbo '22

- With dissipation $\ddot{\phi} + 3H\dot{\phi} + \boxed{V' = -\Gamma\,\dot{\phi}}$ can inflate also if V is steep

  Berera '95

  Could help with string th. arguments for $\Delta\phi < \ldots$ and $V' > \ldots$

- Anber-Sorbo mechanism simple & well defined QFT realization

$$\ddot{\phi} + 3H\dot{\phi} + V' = \frac{\alpha}{f}\,F\,\tilde{F}\left[\dot{\phi}\right]$$

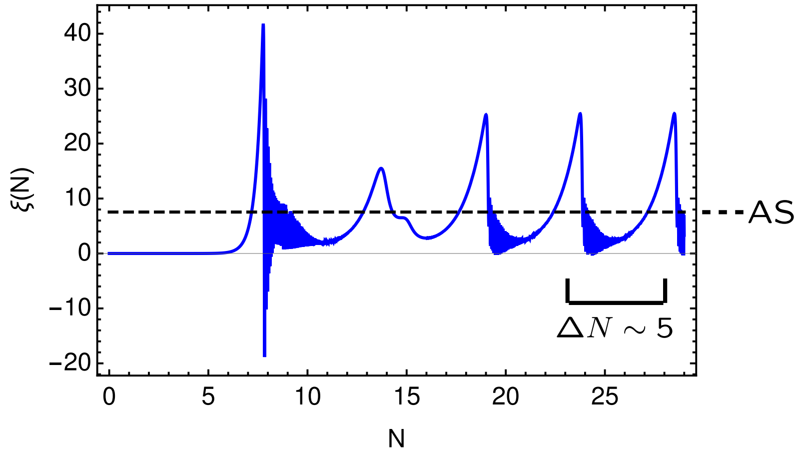
- Assuming a steady-state solution, $\dot{\phi} = \text{constant}$,

$$V' \simeq -2.4 \cdot 10^{-4}\,\frac{\alpha\,H^4}{f}\,\frac{\mathrm{e}^{2\pi\xi}}{\xi^4} \qquad , \qquad \xi \equiv \frac{\alpha\,\dot{\phi}}{2fH}$$

  In original work, $\xi \simeq 20$ to have enough inflation

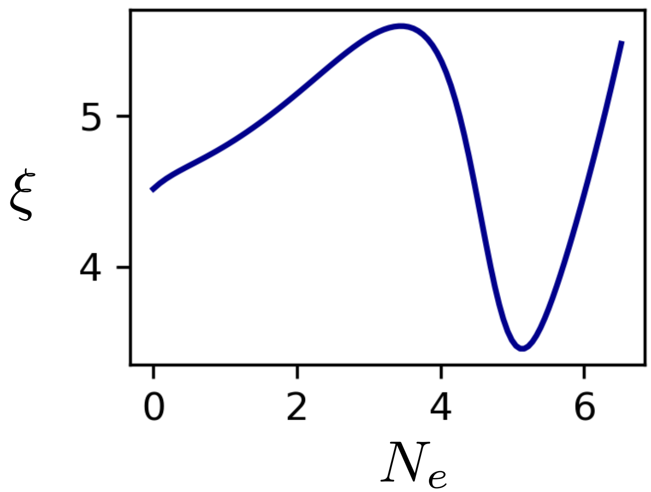
- Oscillatory behaviour from simplified numerical solutions of the system (set of gauge modes + homogeneous inflaton)

Cheng, Lee, Ng '15; Notari, Tywoniuk '16; Dall'Agata, González-Martín, Papageorgiou, MP '19



- Confirmed by full lattice simulation $\phi\,(t,\,\vec{x})$, $A^{\mu}\,(t,\,\vec{x})$

Caravano, Komatsu, Lozanov, Weller '22



- Interpreted as delayed effect between the moment the gauge quanta are produced and the moment they backreact on $\phi\,(t)$

Domcke, Guidetti, Welling, Westphal '20
Gorbar, Schmitz, Sobol, Vilchinskii '21

Analytical study: $\phi(t) = \bar{\phi}(t) + \delta\phi(t)$, $A^\mu\left(t, \vec{k}\right) = \bar{A}^\mu\left(t, \vec{k}\right) + \delta A^\mu\left(t, \vec{k}\right)$

of the homogeneous inflaton & gauge modes around the AS solution

MP, Sorbo '22

$$\delta\phi'' + 2aH\delta\phi' + a^2 V''\delta\phi = -\frac{\alpha}{fa^2}\int \frac{d^3k}{(2\pi)^3}\frac{k}{2}\frac{\partial}{\partial\tau}\left[\bar{A}\,\delta A^* + \bar{A}^*\,\delta A\right]$$

$$\delta A'' + \left(k^2 - \frac{k\,\bar{\phi}'}{f}\right)\delta A = \frac{\alpha\,\bar{A}}{f}\,\delta\phi'$$

- Formally solve 2nd eq for δA as a function of $\delta\phi'$

$$\delta A(\tau,\, k) = \frac{\alpha\, k}{f}\int^\tau d\tau' G_k\left(\tau,\,\tau'\right)\,\bar{A}\left(\tau',\, k\right)\delta\phi'\left(\tau'\right)$$

- Insert solution in 1st eq $\;\rightarrow\;$ integro-differential eq for $\delta\phi$

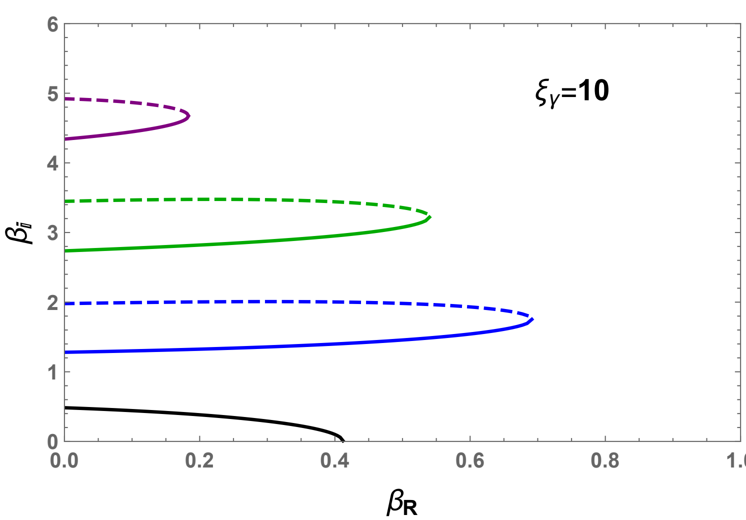
$\xi_\gamma \equiv \xi\,\gamma$

$$\delta\phi''(\tau) + 2aH\delta\phi'(\tau) + a^2 V''\delta\phi(\tau) \simeq \frac{\alpha^2}{f^2 a^2}\frac{e^{2\pi\xi}}{2^8\pi^2\xi^5}\int^{\tau}\frac{d\tau'}{(-\tau')^4}\delta\phi'(\tau')\,\frac{\partial}{\partial\tau}\int_0^{4\xi_\gamma^2} dy\, y^3\sqrt{\tau\tau'}\left[e^{-4\sqrt{y}} - e^{-4\sqrt{y}\sqrt{\frac{-\tau}{-\tau'}}}\right]$$

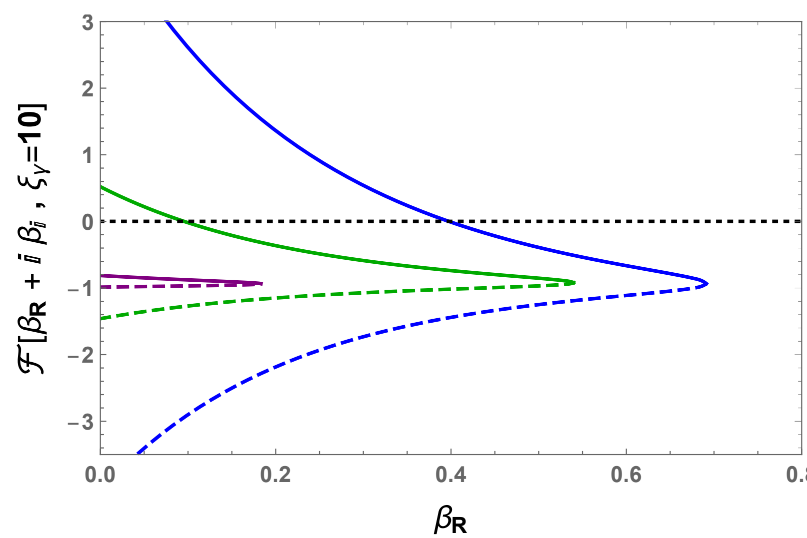
- Look for $\delta\phi \propto (-\tau)^{-\beta} \equiv a^{\mathrm{Re}\,\beta}\cos\left(\mathrm{Im}\,\beta \times N + \text{phase}\right)$

Inserting this and doing the integrals → homogeneous eq in time (all terms scale as $\tau^{-\beta-2}$). Therefore left wih an algebraic equation for complex β.

$$\frac{\xi\, f\, V''}{\alpha\,(-V')} \simeq \frac{4\beta(\beta+3)}{(2\beta-1)(2\beta+7)}\left[\frac{1}{(8\,\xi_\gamma)^{2\beta-1}}\frac{\Gamma(2\beta+8)}{\Gamma(9)} - 1\right] \equiv \mathcal{F}[\zeta,\,\xi_\gamma]$$

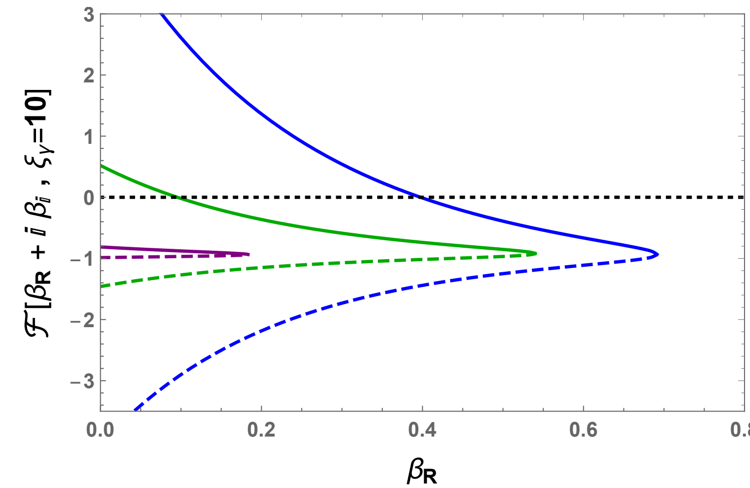


Curves of real $\mathcal{F}$



$$\delta\phi \propto (-\tau)^{-\beta} \equiv a^{\mathrm{Re}\,\beta} \cos(\mathrm{Im}\,\beta \times N + \text{phase})$$

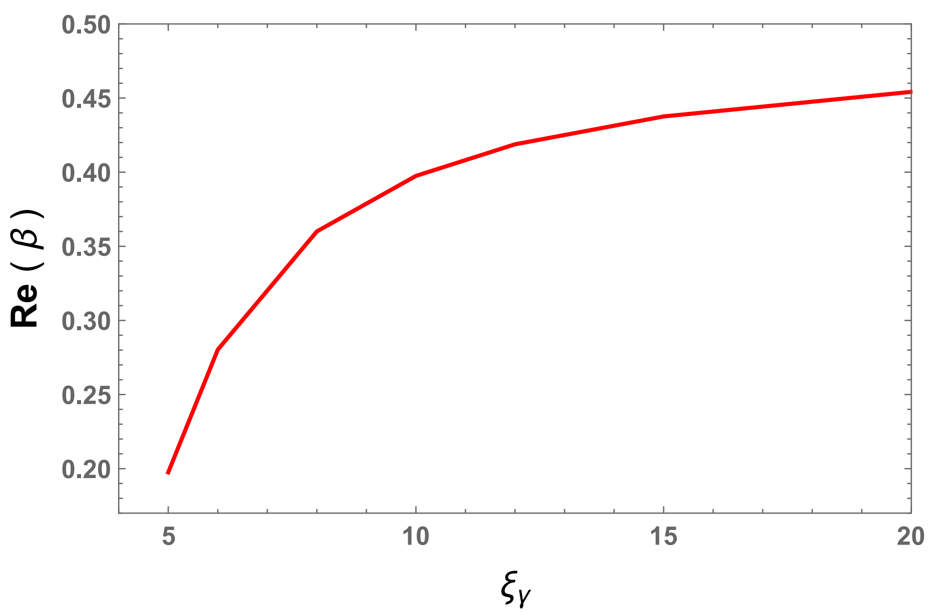
$$\frac{\xi\, f\, V''}{\alpha\,(-V')} \simeq \frac{4\beta\,(\beta+3)}{(2\beta-1)\,(2\beta+7)} \left[\frac{1}{(8\,\xi_\gamma)^{2\beta-1}} \frac{\Gamma(2\beta+8)}{\Gamma(9)} - 1\right] \equiv \mathcal{F}[\zeta,\, \xi_\gamma]$$

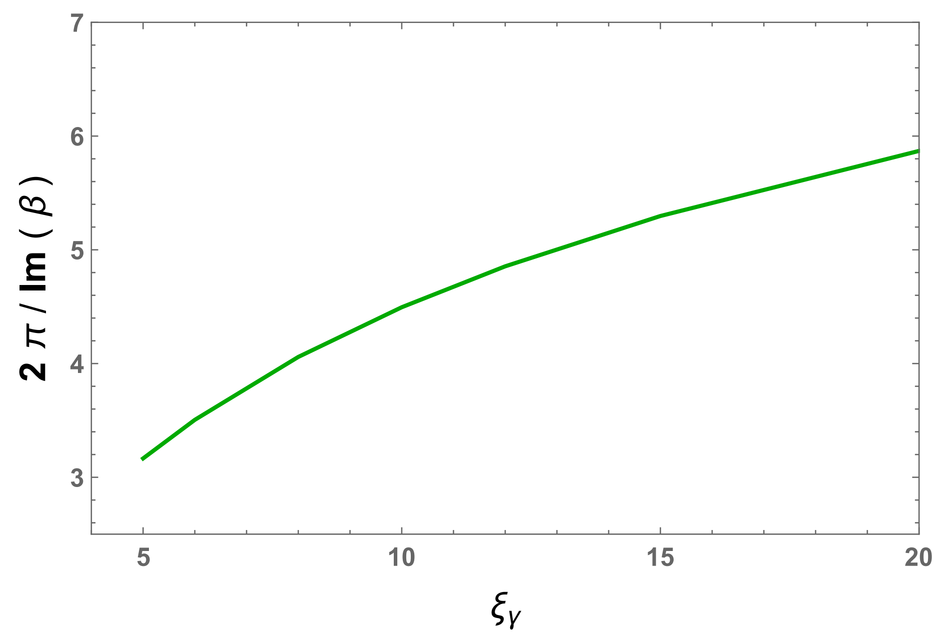


$\delta\phi\,(t)$ linear combination of these modes

$$\frac{\rho_A}{\rho_\phi} \sim \left|\frac{\xi\, f\, V''}{\alpha\,(-V')}\right| \ll 1 \text{ in AS}$$

Look for most unstable solution (greatest β_R) with $\mathcal{F} = 0$





Summary

- Theoretical motivations for an axion inflaton
- Predictivity on couplings: reviewed some phenomenology of $\phi F\tilde{F}$
- Scalar NG, GW at CMB and interferometer scales, seeds for PBH
- SGWB as probe of primordial non-G. Angular dependence is key
- Regime of weak backreaction of produced gauge modes on

  background dynamics fully understood
- Oscillatory behavior of $\dot{\phi}$ in the regime of strong backreaction