Hadronic Axions with Parametrically Enhanced Electromagnetic Couplings

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[Anton Sokolov, AR, JHEP 06 (2021) 123, arXiv:2104.02574]
[Anton Sokolov, AR, PoS EPS-HEP2021 (2022) 178, arXiv:2109.08503]
[Anton Sokolov, AR, arXiv:2205.02605]

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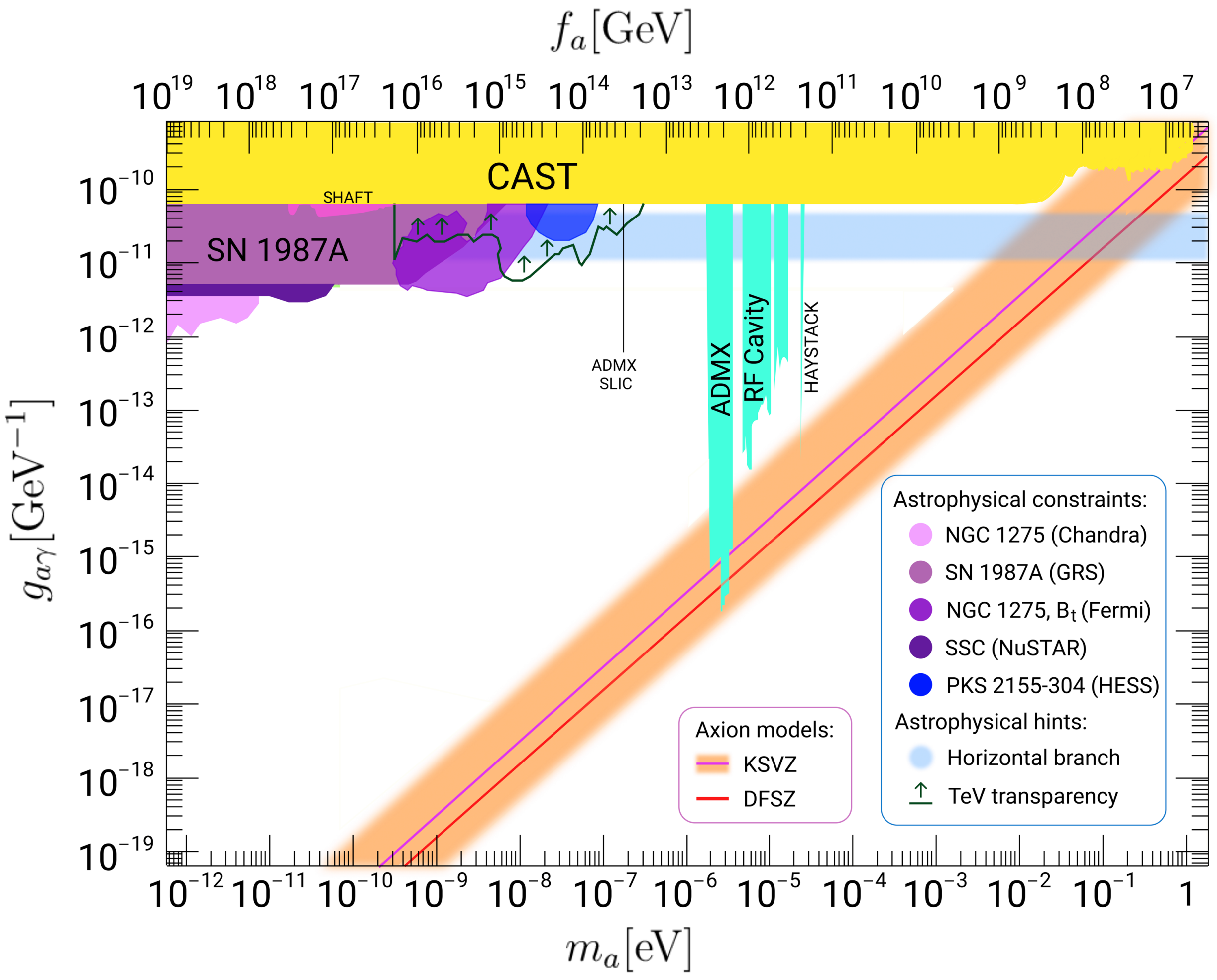
DESY.

Introduction

The Quest for the Axion

- There are many experiments hunting for the axion
- Most of them based on the coupling to the photon

$$\mathcal{L} \supset \frac{g_{a\gamma\gamma}}{4}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv g_{a\gamma\gamma}\, a\, \mathbf{E}\cdot\mathbf{B}$$ [Sikivie `83]



adapted from [Sokolov,AR 21]

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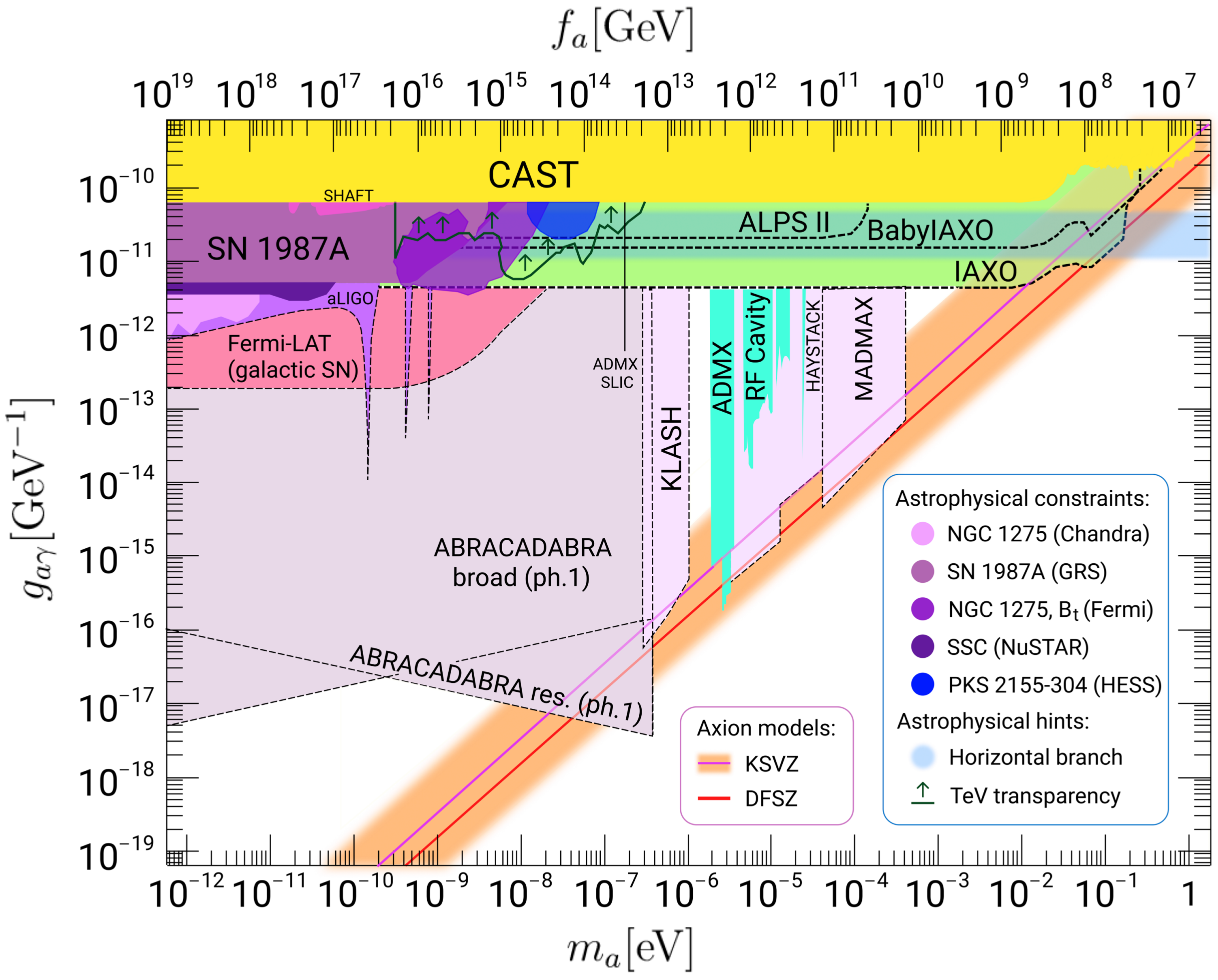
- The aim of the next generation of such experiments is to reach the “band” of photon couplings predicted by vanilla axion models (KSVZ, DFSZ)



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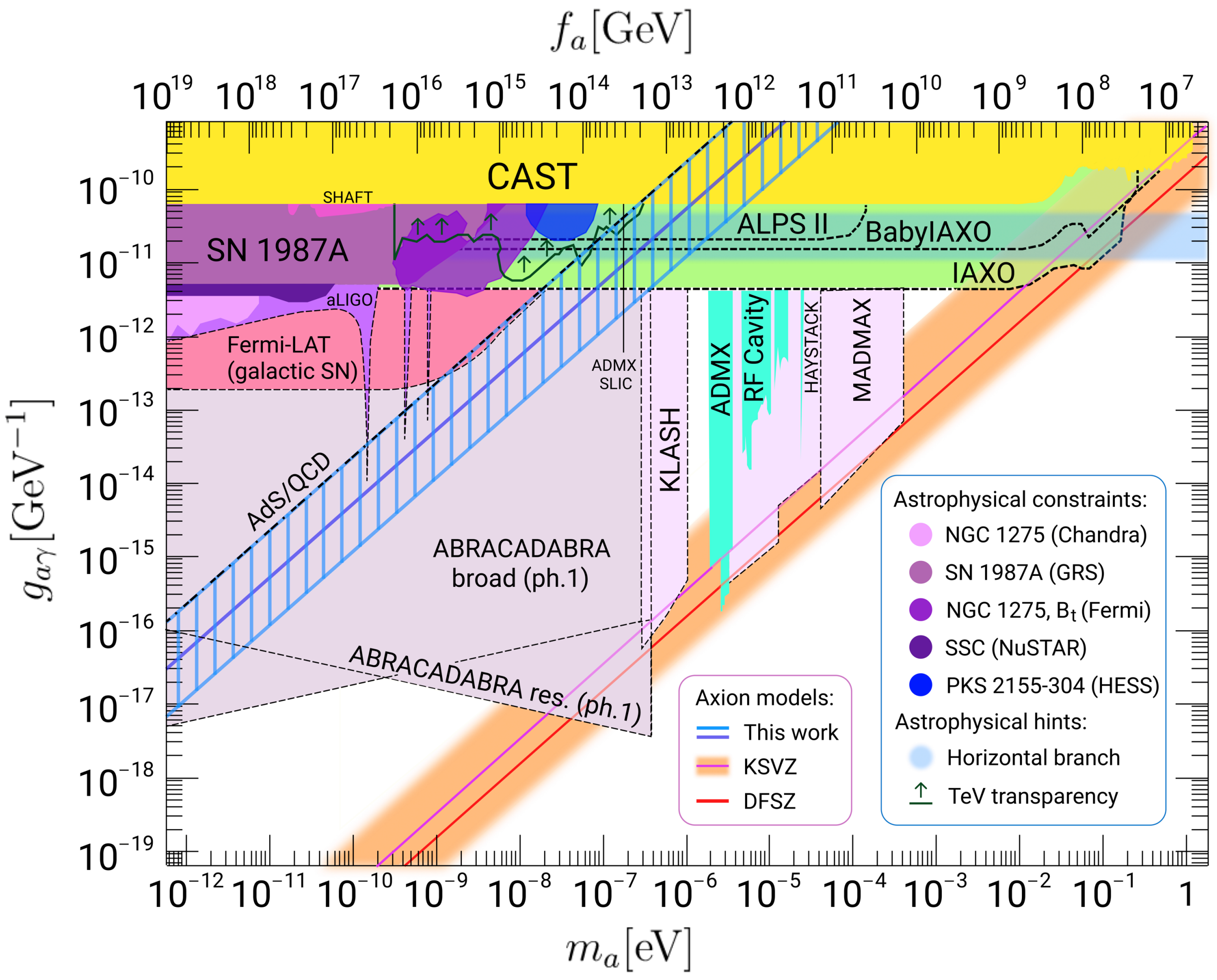
- **Report here on the construction of variants of the KSVZ model with enhanced photon couplings which can be probed very soon**



adapted from [Sokolov,AR 21]

KSVZ Axion Model

[Kim 79;Shifman,Vainshtein,Zakharov 80]

Recap

- Add to SM a singlet complex scalar field σ, featuring a spontaneously broken global $U(1)_{\rm PQ}$ symmetry, and a vector-like fermion $\mathcal{Q} = \mathcal{Q}_L + \mathcal{Q}_R$ in the fundamental of colour, singlet under $SU(2)_L$ and neutral under hypercharge.
- Assuming that under $U(1)_{\rm PQ}$ the fields transform as

$$\sigma \to e^{i\alpha}\sigma\,, \qquad \mathcal{Q}_L \to e^{i\alpha/2}\mathcal{Q}_L\,, \qquad \mathcal{Q}_R \to e^{-i\alpha/2}\mathcal{Q}_R$$

the most general renormalizable Lagrangian can be written as

$$\mathcal{L}_{\rm KSVZ} = |\partial_\mu \sigma|^2 - \lambda_\sigma \left(|\sigma|^2 - \frac{v_\sigma^2}{2}\right)^2 + \overline{\mathcal{Q}}\, i\gamma_\mu D^\mu \mathcal{Q} - \left(y_{\mathcal{Q}} \overline{\mathcal{Q}}_L \mathcal{Q}_R \sigma + \text{h.c.}\right)$$

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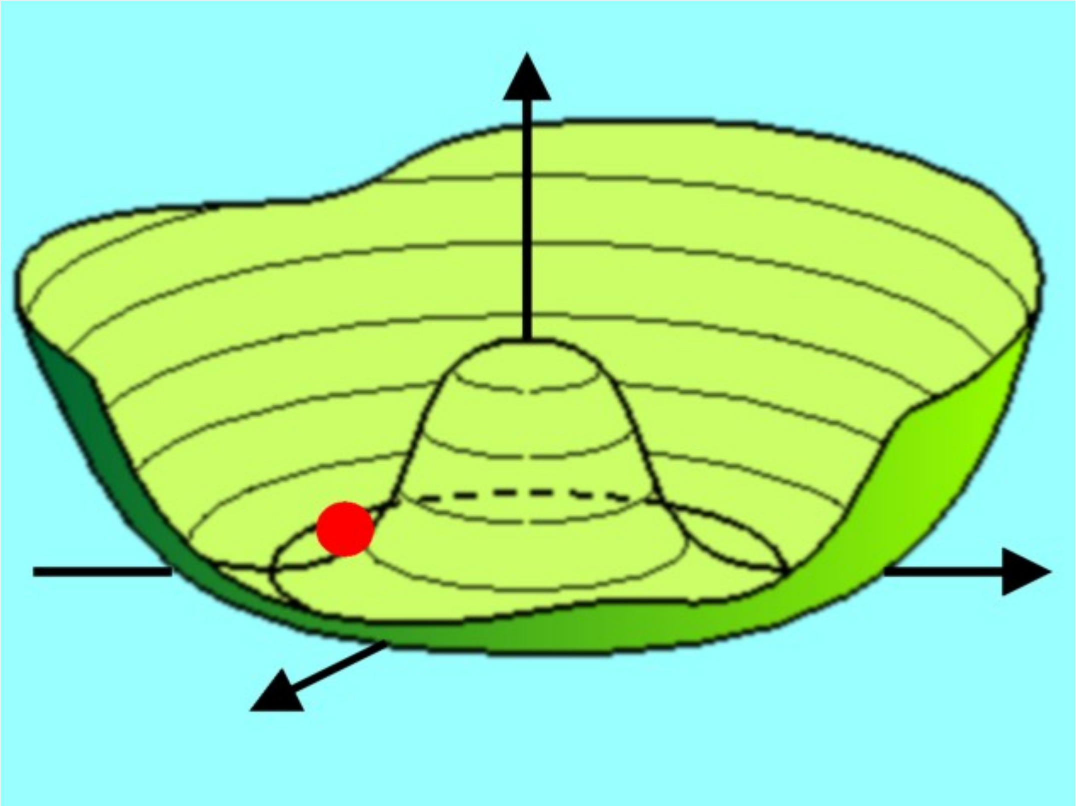
- Decomposing the scalar field in polar coordinates,

$$\sigma(x) = \frac{1}{\sqrt{2}} \left(v_\sigma + \rho(x) \right) \mathrm{e}^{ia(x)/v_\sigma}$$

  we see that this model features three BSM particles

1. Excitation of Goldstone field $a(x)$: massless at tree level
2. Excitation of radial field $\rho(x)$: $m_\rho = \sqrt{2\lambda_\sigma} v_\sigma$
3. New fermion: $m_{\mathcal{Q}} = \frac{y_{\mathcal{Q}}}{\sqrt{2}} v_\sigma$



[Raffelt]

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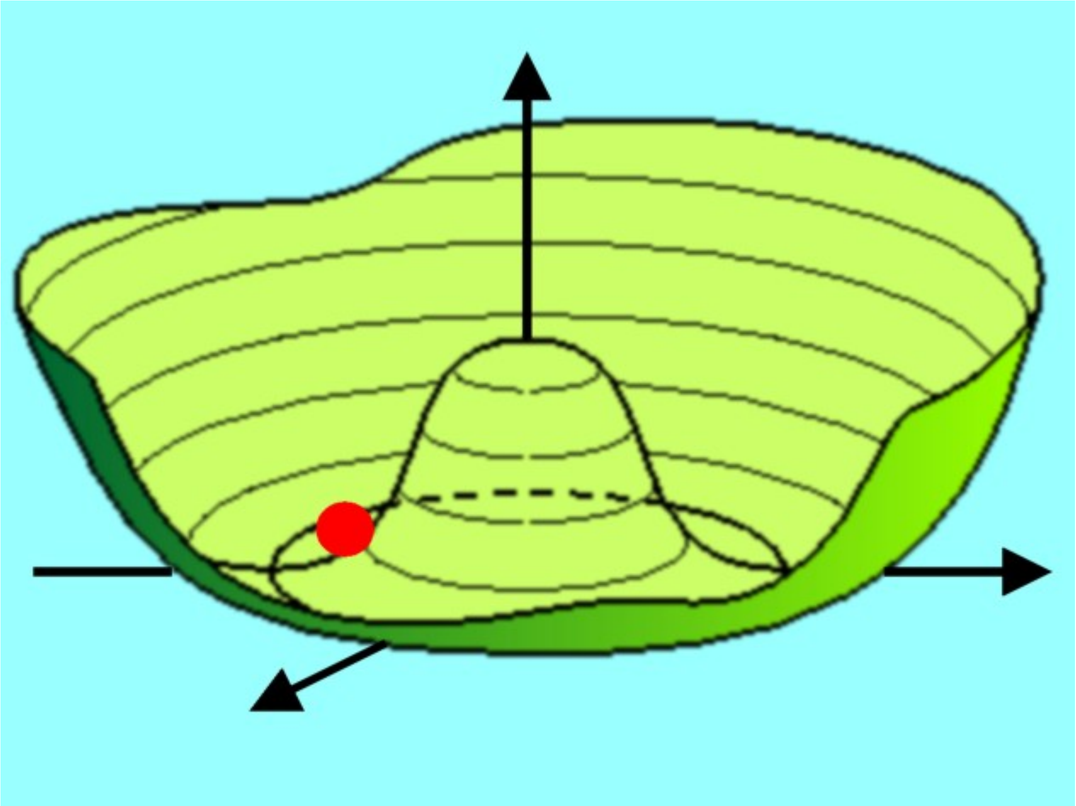
- For large PQ breaking scale v_σ, the latter two are heavy and may be integrated out, if we are only interested at the effective theory at energies much less than the breaking scale



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- Integrate out $\rho(x)$:

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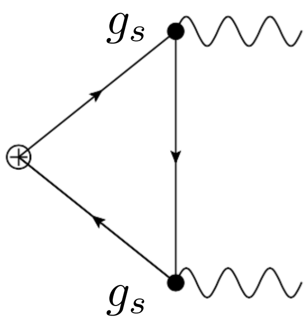
[Fujikawa 79]

$U(1)_{PQ} \times SU(3)_c \times SU(3)_c$ anomaly



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- Now we can also safely integrate out the heavy exotic quark:

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KSVZ Axion Model

[Kim 79;Shifman,Vainshtein,Zakharov 80]

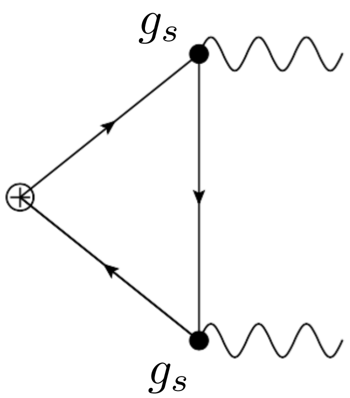
Recap

- Allowing for more general representations of the exotic coloured fermion under $SU(3)_c$ x $U(1)_E$, generalized KSVZ axion described by

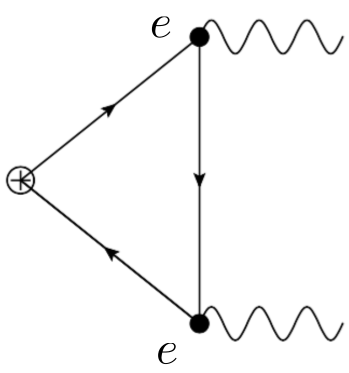
$$\mathcal{L}_{\rm KSVZ} \simeq \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G} + \frac{e^2}{32\pi^2}\frac{E}{N}\frac{a}{f_a}F\tilde{F}$$

  - Axion decay constant: $f_a = v_\sigma/2N$
  - Anomaly coefficients N and E:

$U(1)_{PQ}$ x $SU(3)_c$ x $SU(3)_c$



$U(1)_{PQ}$ x $U(1)_E$ x $U(1)_E$



- Exotic fermion in fundamental representation of $SU(3)_c$ with electric charge -1/3: $N=\frac{1}{2}, E=3\left(\frac{1}{3}\right)^2, \frac{E}{N}=\frac{2}{3}$
- Exotic fermion in fundamental representation of $SU(3)_c$ with electric charge 2/3: $N=\frac{1}{2}, E=3\left(\frac{2}{3}\right)^2, \frac{E}{N}=\frac{8}{3}$

KSVZ Axion Model

[Kim 79;Shifman,Vainshtein,Zakharov 80]

Recap

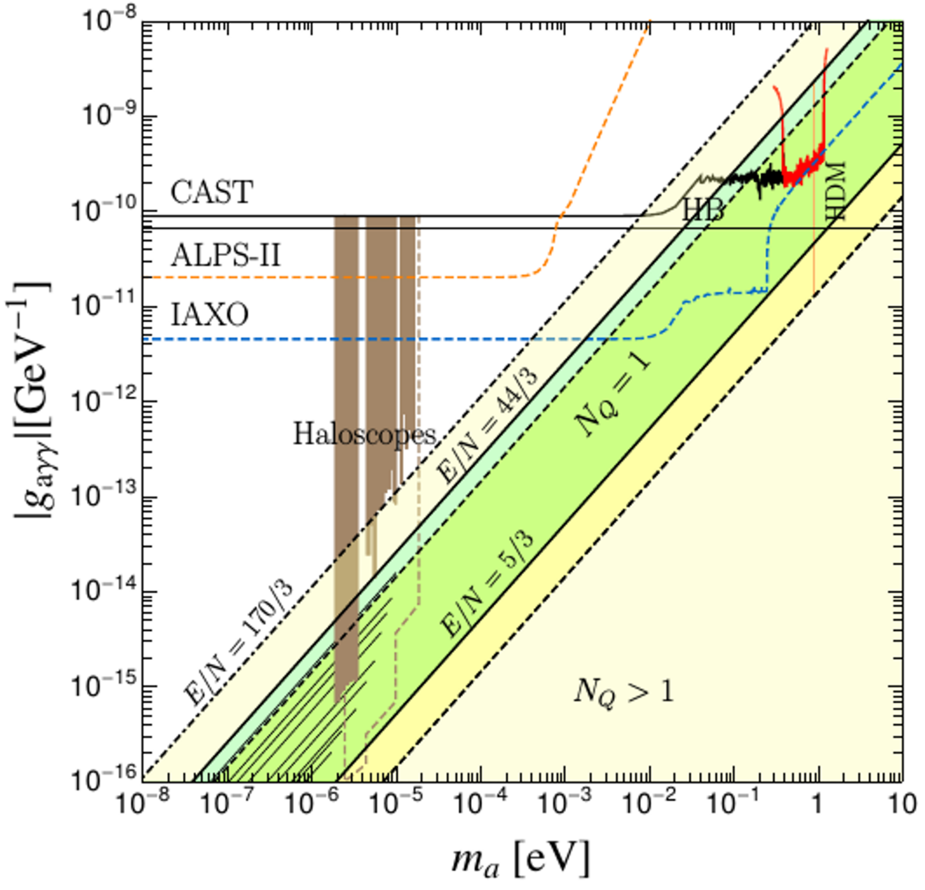
- Using chiral perturbation theory, one can then determine the low energy effective Lagrangian below the confinement scale:

$$\mathcal{L}_{\rm eff} \supset \frac{1}{2}\partial^\mu a\,\partial_\mu a - \frac{1}{2}m_a^2 a^2 + \frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots$$

- Axion mass: $$m_a \approx \frac{m_\pi f_\pi}{f_a}\frac{\sqrt{z}}{1+z} \qquad z = m_u/m_d$$
- Coupling to photon: $$g_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}\right)$$

- Requirement that model should be weakly coupled up to Planck scale limits boost factor, [Di Luzio,Mescia,Nardi 16, 18]

$$\frac{E}{N} \leq 170/3$$



[Di Luzio,Mescia,Nardi 18]

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Recap

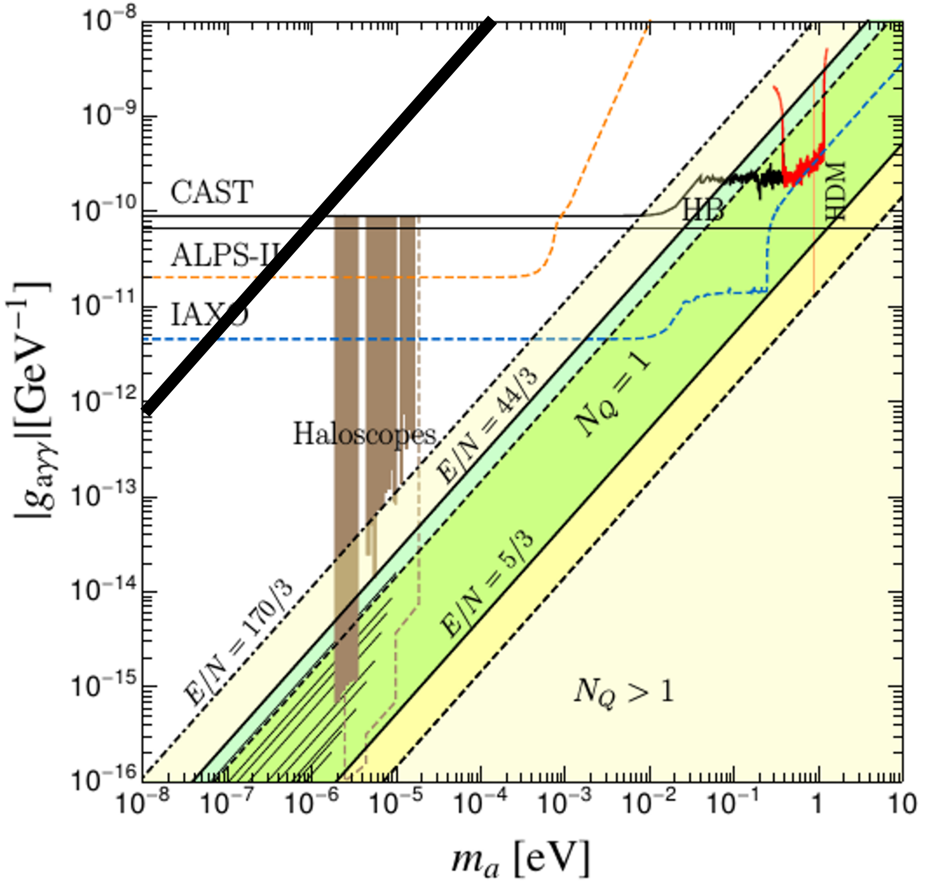
- Using chiral perturbation theory, one can then determine the low energy effective Lagrangian below the confinement scale:

$$\mathcal{L}_{\rm eff} \supset \frac{1}{2}\partial^\mu a\,\partial_\mu a - \frac{1}{2}m_a^2 a^2 + \frac{1}{4}g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots$$

- Axion mass: $m_a \approx \frac{m_\pi f_\pi}{f_a}\frac{\sqrt{z}}{1+z}$ $\qquad z = m_u/m_d$
- Coupling to photon: $g_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}\right)$

- Requirement that model should be weakly coupled up to Planck scale limits boost factor, $\frac{E}{N} \leq 170/3$ [Di Luzio,Mescia,Nardi 16, 18]

- **Further boost possible if exotic colored fermion carries magnetic charge in addition to electric charge** [Sokolov,AR 21, 22]
  - Boost due to charge quantization: $\alpha \to \alpha_{\rm M} \sim 1/\alpha$



[Di Luzio,Mescia,Nardi 18]

Variant KSVZ Axion Model

What if the exotic quark carries also a magnetic charge?

- Have to extend Quantum Electrodynamics (QED) to Quantum Electromagnetodynamics (QEMD):

$$\mathcal{L} = \sum_k \overline{\psi}_k \left(i\partial\!\!\!/ - m_k - e_k A\!\!\!/^{(\mathrm{E})} - g_k A\!\!\!/^{(\mathrm{M})} \right) \psi_k$$

[Zwanziger `71]

$$+ \frac{1}{8}\mathrm{tr}\left[(\partial \wedge A^{(\mathrm{E})}) \cdot (\partial \wedge A^{(\mathrm{E})}) \right] + \frac{1}{8}\mathrm{tr}\left[(\partial \wedge A^{(\mathrm{M})}) \cdot (\partial \wedge A^{(\mathrm{M})}) \right]$$

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$$\partial_\mu F^{\mu\nu} = \sum_i e_i \overline{\psi}_i \gamma^\nu \psi_i\,,$$

$$\partial_\mu \tilde{F}^{\mu\nu} = \sum_i g_i \overline{\psi}_i \gamma^\nu \psi_i\,,$$

where

$$F = \frac{1}{n^2}\left\{ n \wedge [n\cdot(\partial \wedge A^{(\mathrm{E})})] - (n \wedge [n\cdot(\partial \wedge A^{(\mathrm{M})})])^d \right\}$$

$$\tilde{F} = \frac{1}{n^2}\left\{ (n \wedge [n\cdot(\partial \wedge A^{(\mathrm{E})})])^d + n \wedge [n\cdot(\partial \wedge A^{(\mathrm{M})})] \right\}$$

$$\partial^2 (n \cdot A^{(\mathrm{E})})^2 = \partial^2 (n \cdot A^{(\mathrm{M})})^2 = 0$$

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  - Only four independent phase space variables, corresponding to the two physical degrees of the photon
  - Has been shown by path integral techniques that time-ordered Green's functions of gauge-invariant local operators are independent of n^μ if the **Dirac-Schwinger-Zwanziger charge quantization condition** holds,

$$e_i g_j - e_j g_i = 4\pi n_{ij}, \quad n_{ij} \in \mathbb{Z}$$

[Brandt, Neri, Zwanziger `78]

Variant KSVZ Axion Model

What if the exotic quark carries also a magnetic charge?

- Integrate out $\rho(x)$:

$$\mathcal{L}'_{\rm KSVZ} = \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{\mathcal{Q}}\, i\gamma_\mu D^\mu \mathcal{Q} - \left(m_{\mathcal{Q}} \overline{\mathcal{Q}}_L \mathcal{Q}_R e^{ia/v_\sigma} + \text{h.c.} \right)$$

- Last term can be brought to the form of a standard mass term by performing the field-dependent axial transformation, $\mathcal{Q} \to e^{-\frac{i}{2}\gamma_5 \frac{a}{v_\sigma}} \mathcal{Q}$, that is

$$\mathcal{Q}_L \to e^{\frac{i}{2}\frac{a}{v_\sigma}} \mathcal{Q}_L , \qquad \mathcal{Q}_R \to e^{-\frac{i}{2}\frac{a}{v_\sigma}} \mathcal{Q}_R$$

- However, fermionic measure in path integral is not invariant under axial transformations, cf.

$$\mathcal{D}\mathcal{Q}\mathcal{D}\bar{\mathcal{Q}} \to = \mathcal{D}\mathcal{Q}\mathcal{D}\bar{\mathcal{Q}}\; e^{i\int d^4x \mathcal{L}_F(x)}$$ where $$\mathcal{L}_{\rm F} = \frac{\alpha_s}{8\pi}\frac{a}{2Nf_a} G\tilde{G} + \mathcal{L}_{\rm F}^{\rm QEMD}$$ [Anton Sokolov, AR, arXiv:2205.02605]

$$\mathcal{L}_{\rm F}^{\rm QEMD} = \frac{a}{v_\sigma} \cdot \lim_{\substack{\Lambda\to\infty \\ x\to y}} \text{tr}\left\{\gamma_5 \exp\left(\not{\mathcal{D}}^2/\Lambda^2\right)\delta^4(x-y)\right\}$$ with $$\mathcal{D}_\mu = \partial_\mu - ie\, q_{\mathcal{Q}} A_\mu^{(\rm E)} - ig_0\, g_{\mathcal{Q}} A_\mu^{(\rm M)}$$

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[Anton Sokolov, AR, arXiv:2205.02605]

$$\mathcal{L}_{\rm F}^{\rm QEMD} = \frac{a}{f_a}\left(\frac{\alpha}{8\pi}\frac{E}{N}\text{tr}\left\{ \left(\partial \wedge A^{(\rm E)}\right) \left(\partial \wedge A^{(\rm E)}\right)^d \right\} + \frac{\alpha_M}{8\pi}\frac{M}{N}\,\text{tr}\left\{ \left(\partial \wedge A^{(\rm M)}\right) \left(\partial \wedge A^{(\rm M)}\right)^d \right\} + \frac{\sqrt{\alpha\alpha_M}}{4\pi}\frac{D}{N}\,\text{tr}\left\{ \left(\partial \wedge A^{(\rm E)}\right) \left(\partial \wedge A^{(\rm M)}\right)^d \right\} \right)$$

  - If Q in fundamental of $SU(3)_{\rm color}$ then N=1/2 and $E = 3\, q_{\mathcal{Q}}^2\,,\ M = 3\, g_{\mathcal{Q}}^2\,,\ D = 3\, q_{\mathcal{Q}} g_{\mathcal{Q}}$
  - All three terms respect shift symmetry

Variant KSVZ Axion Model

Phenomenological implications

- Axion Maxwell equations after integrating out exotic quark: [Anton Sokolov, AR, arXiv:2205.02605]

$$\left(\partial^2 - m_a^2\right) a \;=\; -\frac{1}{4}\left(g_{a\mathrm{EE}} + g_{a\mathrm{MM}}\right) F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{2}\, g_{a\mathrm{EM}} F_{\mu\nu} F^{\mu\nu}$$

$$\begin{aligned} \partial_\mu F^{\mu\nu} - g_{a\mathrm{EE}}\,\partial_\mu a\,\tilde{F}^{\mu\nu} + g_{a\mathrm{EM}}\,\partial_\mu a\,F^{\mu\nu} &= j_e^{\nu}\,; \\ \partial_\mu \tilde{F}^{\mu\nu} + g_{a\mathrm{MM}}\,\partial_\mu a\,F^{\mu\nu} - g_{a\mathrm{EM}}\,\partial_\mu a\,\tilde{F}^{\mu\nu} &= 0 \end{aligned}$$

$$g_{a\mathrm{MM}} = \frac{\alpha_M}{2\pi f_a}\frac{M}{N} \qquad g_{a\mathrm{EM}} = \frac{\sqrt{\alpha\alpha_M}}{2\pi f_a}\frac{D}{N} \qquad g_{a\mathrm{EE}} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92\right)$$

- Because of DSZ charge quantization condition, magnetic coupling huge, cf.

$$\alpha \equiv e^2/4\pi \approx 1/137, \qquad \alpha_M \equiv g_0^2/4\pi = 9\pi/\alpha \approx 3.87 \times 10^3$$

- Correspondingly, $g_{a\mathrm{MM}} \gg g_{a\mathrm{EM}} \gg g_{a\mathrm{EE}}$

Variant KSVZ Axion Model

Phenomenological implications

- Axion Maxwell equations in terms of field strengths: [Anton Sokolov, AR, arXiv:2205.02605]

$$\left(\partial^2 - m_a^2\right) a \;=\; \left(g_{a\mathrm{EE}} + g_{a\mathrm{MM}}\right) \mathbf{E}_0 \cdot \mathbf{B}_0 + g_{a\mathrm{EM}} \left(\mathbf{E}_0^2 - \mathbf{B}_0^2\right) \;,$$

$$\boldsymbol{\nabla} \times \mathbf{B}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{EE}} \left(\mathbf{E}_0 \times \boldsymbol{\nabla} a - \dot{a}\mathbf{B}_0\right) + g_{a\mathrm{EM}} \left(\mathbf{B}_0 \times \boldsymbol{\nabla} a + \dot{a}\mathbf{E}_0\right) \;,$$

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$$g_{a\mathrm{MM}} = \frac{\alpha_M}{2\pi f_a} \frac{M}{N} \quad \gg \quad g_{a\mathrm{EM}} = \frac{\sqrt{\alpha\alpha_M}}{2\pi f_a} \frac{D}{N} \quad \gg \quad g_{a\mathrm{EE}} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - 1.92\right)$$

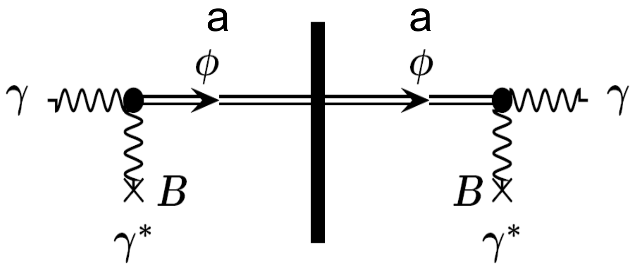
Variant KSVZ Axion Model

Phenomenological implications

- Axion-photon conversion in external field described by

$$\left(\partial^2 - m_a^2\right) a \; = \; \left(g_{a\mathrm{EE}} + g_{a\mathrm{MM}}\right) \mathbf{E}_0 \cdot \mathbf{B}_0 + g_{a\mathrm{EM}} \left(\mathbf{E}_0^2 - \mathbf{B}_0^2\right)$$

- Constraints from axion-photon conversion stay approximately the same, with the identification $g_{a\gamma\gamma} \to g_{aMM}$
- LSW:



$$P(\gamma_{\|} \to a \to \gamma) \simeq 16\, \frac{(g_{a\mathrm{MM}}\omega B_0)^4}{m_a^8}\, \sin^4\left(\frac{m_a^2\, L_{B_0}}{4\omega}\right),$$

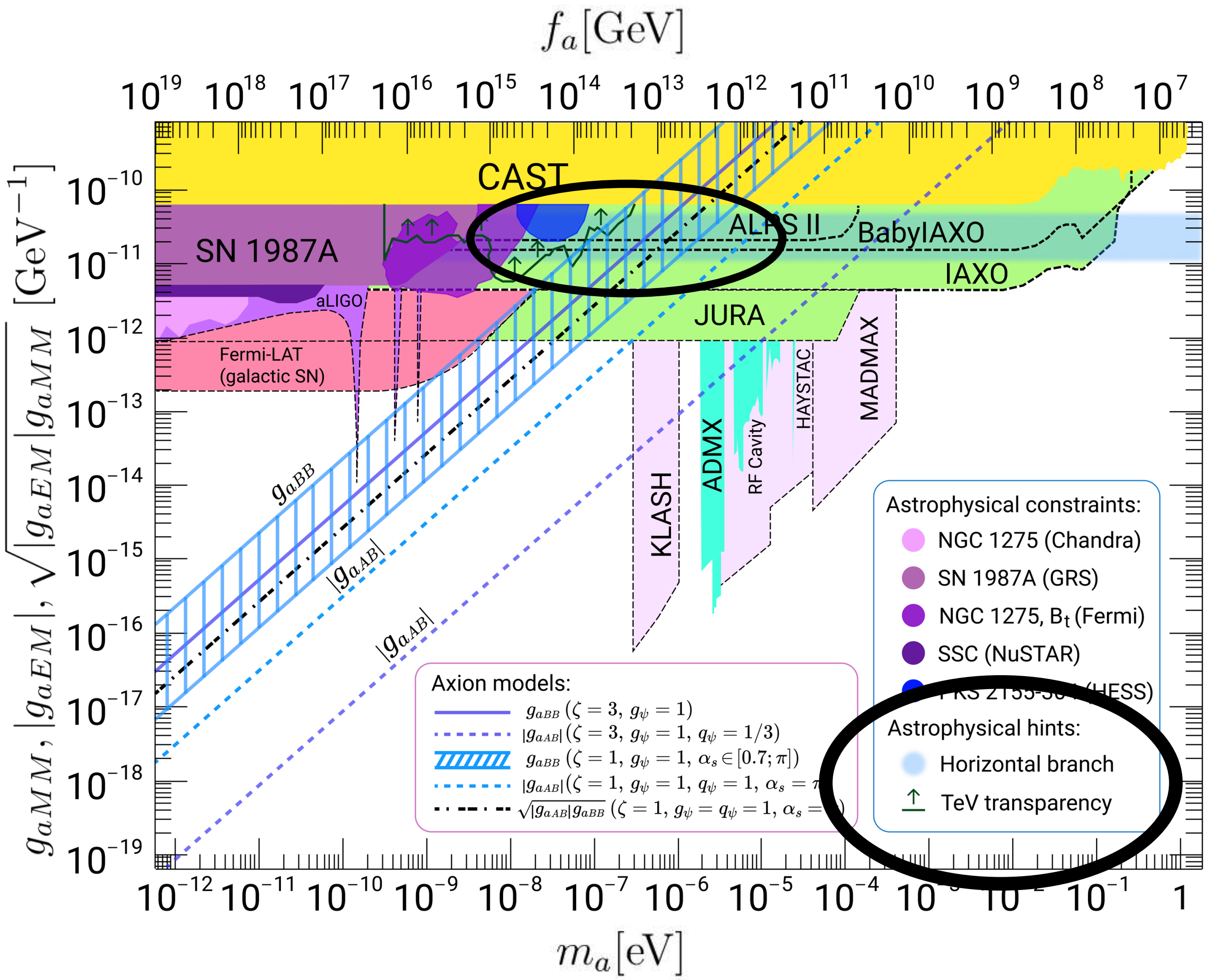
$$P(\gamma_{\perp} \to a \to \gamma) \simeq 16\, \frac{(g_{a\mathrm{EM}}\omega B_0)^2 (g_{a\mathrm{MM}}\omega B_0)^2}{m_a^8}\, \sin^4\left(\frac{m_a^2\, L_{B_0}}{4\omega}\right)$$

  - If signal detected in both channels, one can compare the theoretically derived ratio of CP-violating and CP-conserving couplings in a given model with the experiment

Variant KSVZ Axion Model

Phenomenological implications

- This axion solves strong CP problem and simultaneously may explain astrophysical hints
  - Excessive energy losses of horizontal branch stars in globular clusters
  - Transparency of universe for TeV gamma rays



[Sokolov,AR 22]

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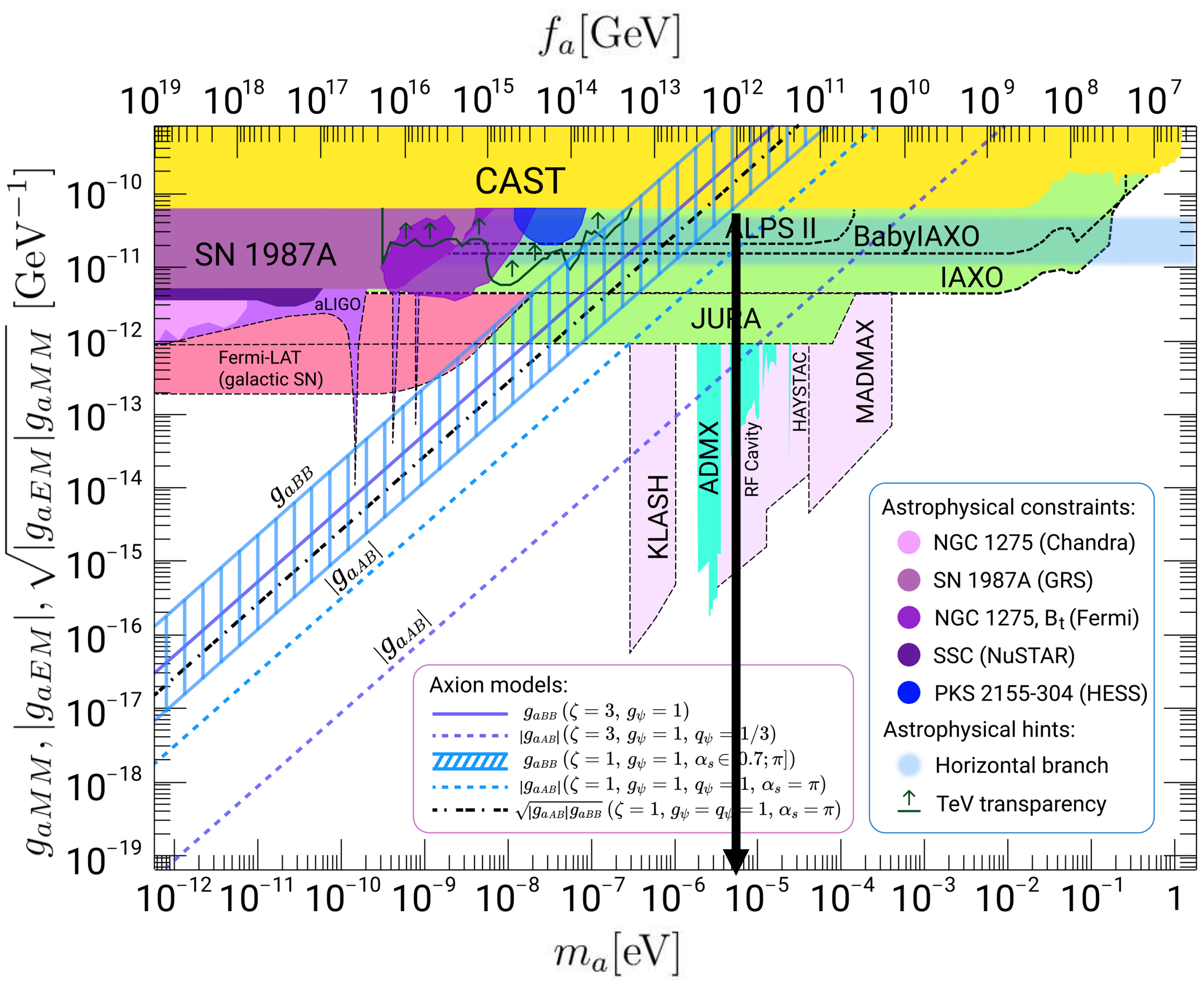

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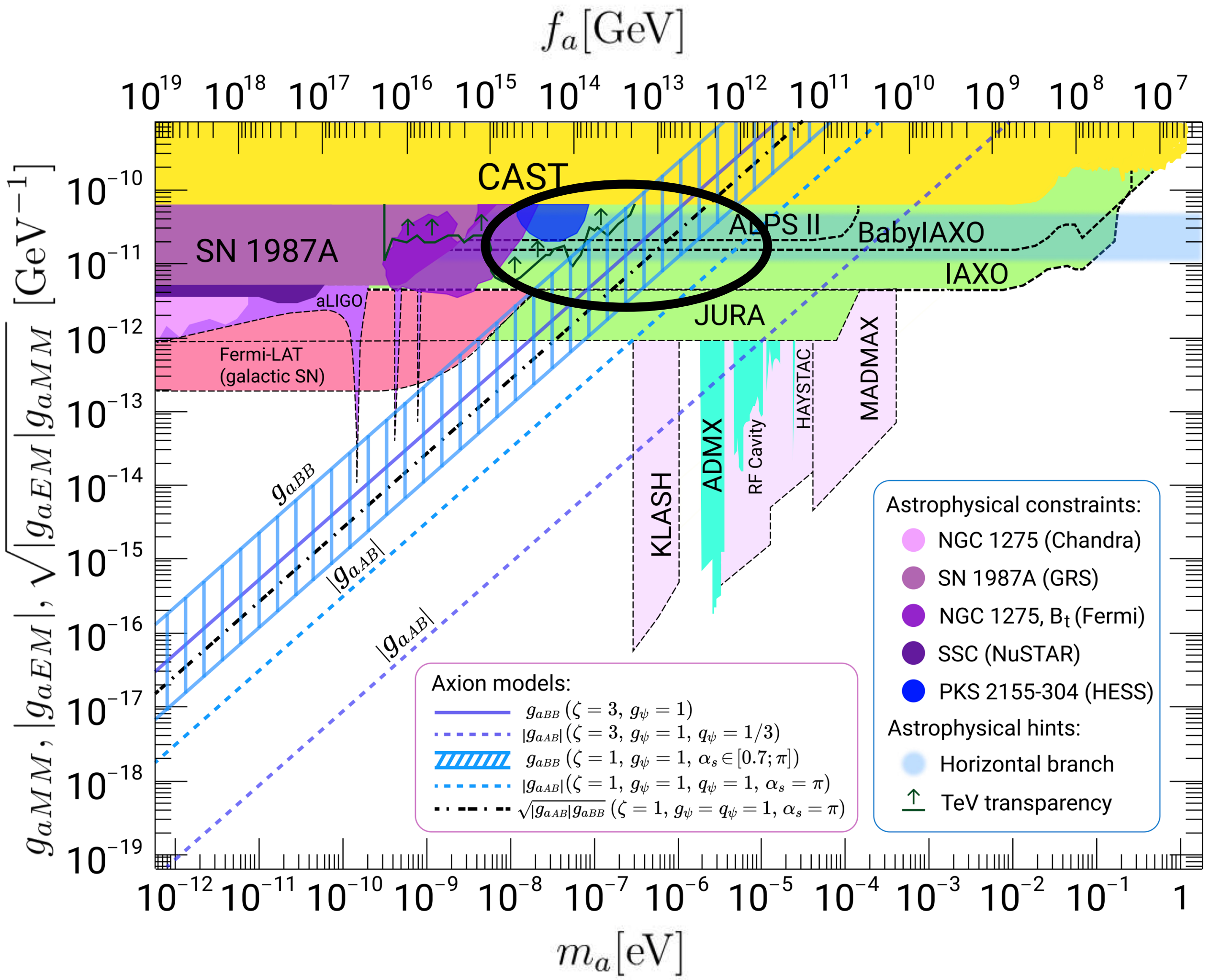

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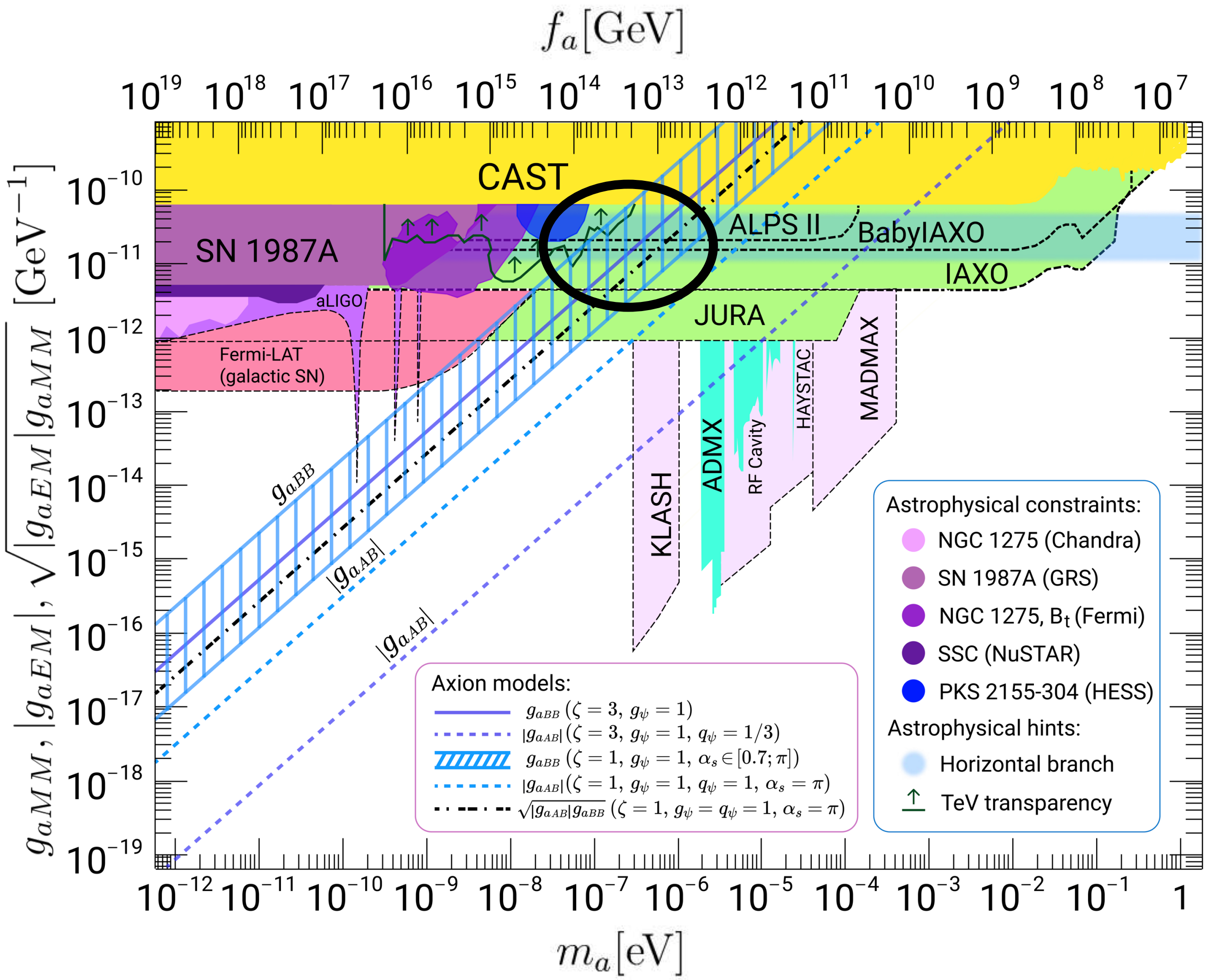
- An axion in the parameter range by ALPS II and (Baby)IAXO naturally 100% DM, cf.

$$\frac{\Omega_{\text{axion DM}}}{\Omega_{\text{DM}}} \approx \left(\frac{6\ \mu\text{eV}}{m_a}\right)^{1.165} \theta^2_{\text{initial}}$$



[Sokolov,AR 22]

Variant KSVZ Axion Model

Phenomenological implications

- At low mass, m_a<< L, where L is the size of a detector exploiting a laboratory magnetic field, the interactions of the latter with DM axions
  - Can not be described by conventional axion-photon conversion
  - However, they will lead to oscillating axion-DM induced electric and magnetic induced fields, as can be inferred from the axion Maxwell equations

$$\nabla \times \mathbf{B}_a - \dot{\mathbf{E}}_a = g_{a\mathrm{EE}} \left(\mathbf{E}_0 \times \nabla a - \dot{a}\mathbf{B}_0\right) + g_{a\mathrm{EM}} \left(\mathbf{B}_0 \times \nabla a + \dot{a}\mathbf{E}_0\right) \,,$$

$$\nabla \times \mathbf{E}_a + \dot{\mathbf{B}}_a = -g_{a\mathrm{MM}} \left(\mathbf{B}_0 \times \nabla a + \dot{a}\mathbf{E}_0\right) - g_{a\mathrm{EM}} \left(\mathbf{E}_0 \times \nabla a - \dot{a}\mathbf{B}_0\right) \,,$$

$$\nabla \cdot \mathbf{B}_a = -g_{a\mathrm{MM}}\, \mathbf{E}_0 \cdot \nabla a + g_{a\mathrm{EM}}\, \mathbf{B}_0 \cdot \nabla a \,,$$

$$\nabla \cdot \mathbf{E}_a = g_{a\mathrm{EE}}\, \mathbf{B}_0 \cdot \nabla a - g_{a\mathrm{EM}}\, \mathbf{E}_0 \cdot \nabla a$$

- The presently lumped element axion DM searches, such as ABRACADABRA, ADMX SLIC, DM Radio, SHAFT, WISPLC, only search for oscillating axion-DM-induced magnetic fields
- However, to probe our variant KSVZ axion, one should aim for measuring induced electric fields

Conclusions

- Proposed a new family of KSVZ-type axion models where the exotic quark carries magnetic charge
- These models have parametrically enhanced axion-photon coupling
- These models can explain various “hints” with one stroke
  - Strong CP conservation
  - Quantisation of charge
  - Observed dark matter abundance
  - Anomalous TeV-transparency of the Universe
  - Cooling of horizontal branch stars in globular clusters
- For masses above 10^{-8} eV, these models can be probed decisively with ALPS II and (Baby)IAXO