Evolution of Peccei-Quinn fields for axions with relatively low decay constants

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based on work in progress in collaboration with P. Kozów

Outline

- Introduction
- Evolution of PQ field during inflation
  - radiative corrections
  - corrections from space-time curvature
- Evolution of PQ field after inflation
  - thermal corrections
- Axions as cold and warm dark matter
- Summary

Introduction

- **Axions (QCD axions and ALPs) are interesting candidates for Dark Matter**
  - **pseudo Goldstone bosons of spontaneously broken Peccei-Quinn global $U(1)_{\mathrm{PQ}}$ symmetry**
  - **phase component of complex PQ scalar**

$$\Phi = \frac{1}{\sqrt{2}} S e^{ia/f_a} = \frac{1}{\sqrt{2}} S e^{i\theta}$$

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- **Different mechanisms leading to relic abundance of axions**
  - **misalignment** — Dine . . . Sikivie . . . Wilczek 1983
  - **parametric resonance** — Kofman et al 1994
  - **rotating axions** — Co Hall Harigaya 2019
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  - ...
- Two basic scenarios for misalignment
  - $U(1)_{\mathrm{PQ}}$ broken (during inflation)

$$f_a \gg H_I$$

  - $U(1)_{\mathrm{PQ}}$ unbroken (during inflation)

$$f_a \ll H_I$$

Simple potential for PQ field

$$V\left(\Phi\right)=\lambda_{\Phi}\left(|\Phi|^{2}-\frac{f_a}{2}\right)^{2}=\frac{1}{4}\lambda_{\Phi}\left(S^{2}-f_a^{2}\right)^{2}$$

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- $U(1)_{\mathrm{PQ}}$ **broken** during and after inflation because $f_a \gg H_I$
  - $S = f_a$
  - $\langle\theta\rangle$ determined by some stochastic process during the phase transition from unbroken to broken $U(1)_{\mathrm{PQ}}$
  - isocurvature perturbations are generated during inflation $\langle\delta\theta^2\rangle \propto H_I/f_a \ll 1$

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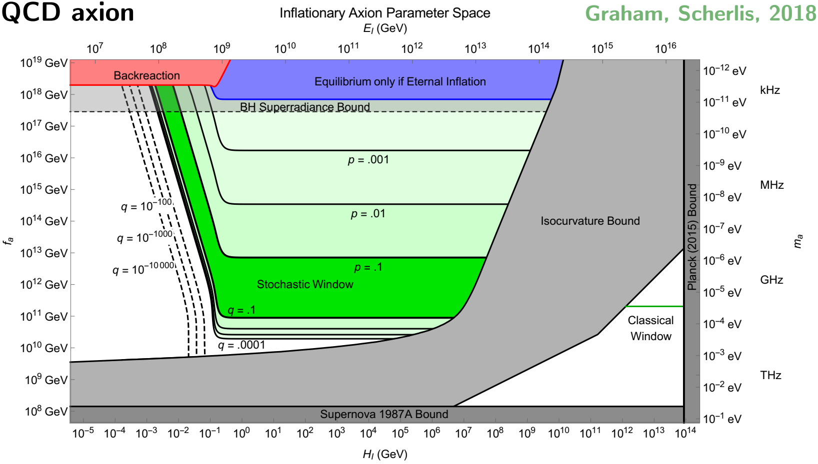
- $U(1)_{\rm PQ}$ unbroken until T drops below $T_c = \mathcal{O}(f_a) \ll H_I$
  - before the phase transition: $S = 0$, θ undefined
  - just after phase transition:
    $S = f_a$
    flat stochastic distribution of θ with $\left\langle\delta\theta^2\right\rangle = \frac{\pi^2}{3}$
  - isocurvature white noise
    but only on scales smaller than the Hubble radius when the axion field starts to oscillate

Feix et al 2019

QCD axion

Graham, Scherlis, 2018



- **Broken $U(1)_{\mathrm{PQ}}$:**
  wide range of acceptable values of $f_a \gg H_I$
- **Unbroken $U(1)_{\mathrm{PQ}}$:**
  one possible value of f_a ("classical window")

What does "unbroken $U(1)_{\mathrm{PQ}}$" mean?

- Potential with (global) minimum at $\Phi_{\mathrm{PQ}} = 0$
  does not guarantee that Φ_{PQ} vanishes
- Light enough fields fluctuate during inflation
- Heavy enough fields displaced from a minimum of their potential oscillate

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In this talk I will concentrate on models in which PQ-like field has nontrivial dynamics during and after inflation

PQ field during inflation

Axion field a is massless (or at least very light) during inflation
$\Rightarrow$ it fluctuates on average by $H_I/2\pi$ during each Hubble time

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- have average values, $\langle S_i \rangle$ and $\langle \theta_i \rangle$, determined by stochastic processes
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Dynamics after inflation may lead to production of axions as

- cold dark matter (CDM)
  - e.g. misalignment
- warm dark matter (WDM)
  - e.g. parametric resonance production

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Bounds on isocurvature perturbations lead to very strong upper bounds on some couplings

e.g. kinetic misalignment: $10^{-36} \lesssim \lambda_\Phi \lesssim 10^{-22} \lll 1$

PQ field during inflation

If $\lambda_\Phi \lll 1$ one should consider corrections

- **radiative**
- **thermal**
- **geometric (curvature of space-time)**

Radiative corrections

We use Coleman-Weinberg (CW) potential adopting Gildener-Weinberg approach

The PQ scalar couples to some scalars ϕ_i and some fermions ψ_j

$$\mathcal{L} \supset \sum_i \left(\frac{1}{2} m_i^2 \phi_i^2 + \frac{1}{2} \lambda_i |\Phi|^2 \phi_i^2 \right) + \sum_j y_j \Phi \overline{\psi}_j \psi_j$$

which gives the CW potential

$$V = \frac{1}{64\pi^2} \left\{ \sum_i M_{\phi_i}^4 \left[\ln\left(\frac{M_{\phi_i}^2}{\mu^2} \right) - \frac{3}{2} \right] - 4 \sum_j M_{\psi_j}^4 \left[\ln\left(\frac{M_{\psi_j}^2}{\mu^2} \right) - \frac{3}{2} \right] \right\}$$

$$M_{\phi_i}^2 = m_i^2 + \frac{1}{2} \lambda_i S^2 \qquad M_{\psi_j}^2 = \frac{1}{2} y_j^2 S^2$$

μ – scale at which running PQ self-coupling vanishes: $\lambda_\Phi(\mu) = 0$

Bosonic contribution must dominate for large values of S

Radiative corrections

Two simple limiting cases:

- $V_{\rm CW}(S)$ **dominated by scalar(s)** **(SD)**
  - **simple realization:** N_s **scalars with** $m_i = m$ **and** $\lambda_i = \lambda$

$$V_{\rm CW}(S) \approx \frac{N_s}{64\pi^2}\left(m^2 + \frac{1}{2}\lambda S^2\right)^2 \left[\ln\left(\frac{m^2 + \frac{1}{2}\lambda S^2}{\mu^2}\right) - \frac{3}{2}\right]$$

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- $V_{\rm CW}(S)$ **with "quasi-SUSY" spectrum (qS)**
  - simple realization: $4N_f$ scalars and N_f fermions with $y_j = y$, $m_i = m$, $\lambda_i = \lambda$ and $y^2 = (1-\epsilon)\lambda$ with $0 < \epsilon \ll 1$

$$V_{\rm CW}(S) \approx \frac{N_f}{16\pi^2}\left\{\left(m^2 + \frac{1}{2}\lambda S^2\right)^2 \left[\ln\left(\frac{m^2 + \frac{1}{2}\lambda S^2}{\mu^2}\right) - \frac{3}{2}\right] - \frac{1}{4}(1-\epsilon)^2\lambda^2 S^4 \left[\ln\left(\frac{(1-\epsilon)\lambda S^2}{2\mu^2}\right) - \frac{3}{2}\right]\right\}$$

  - $V_{\rm CW} \to 0$ for $m \to 0$ and $\epsilon \to 0$

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  - $V_{\rm CW} \to 0$ for $m \to 0$ and $\epsilon \to 0$
- Minimum of $V_{\rm CW}(S)$ at $S^2 \approx \frac{2\mu^2}{\lambda}$ $\left(f_a \to \sqrt{\frac{2}{\lambda}}\,\mu\right)$

CW potential in curved space-time

Markkanen et al 2018

$$V = \frac{1}{64\pi^2}\sum_i \left\{ M_{\phi_i}^4 \left[\ln\left(\frac{M_{\phi_i}^2}{\mu^2}\right) - \frac{3}{2}\right] + \frac{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}}{90}\ln\left(\frac{M_{\phi_i}^2}{\mu^2}\right)\right\}$$
$$-\frac{4}{64\pi^2}\sum_j \left\{ M_{\psi_j}^4 \left[\ln\left(\frac{M_{\psi_j}^2}{\mu^2}\right) - \frac{3}{2}\right] + \frac{\frac{7}{8}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + R_{\mu\nu}R^{\mu\nu}}{90}\ln\left(\frac{M_{\psi_j}^2}{\mu^2}\right)\right\}$$

$$M_{\phi_i}^2 = m_i^2 + \lambda_i |\Phi|^2 + \left(\xi_i - \frac{1}{6}\right) R$$

$$M_{\psi_j}^2 = y_j^2 |\Phi|^2 + \frac{1}{12} R$$

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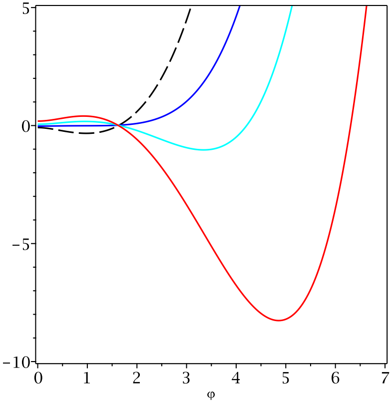
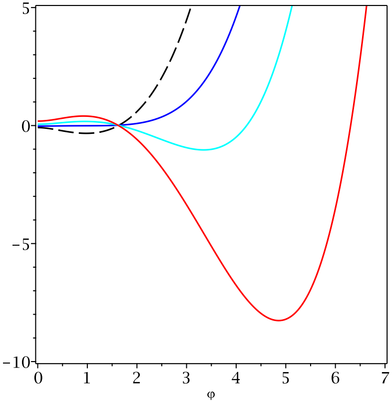
	inflation	MD	RD
R	$12H^2$	$3H^2$	0
$R_{\mu\nu}R^{\mu\nu}$	$36H^4$	$9H^4$	$12H^4$
$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$	$24H^4$	$15H^4$	$24H^4$



Potential during inflation is more complicated (as compared to $R = 0$ case) and usually have second (much) deeper minimum for (much) bigger value of S

$U(1)_{\mathrm{PQ}}$ may be restored (for some values of H_I even global minimum at $S = 0$)



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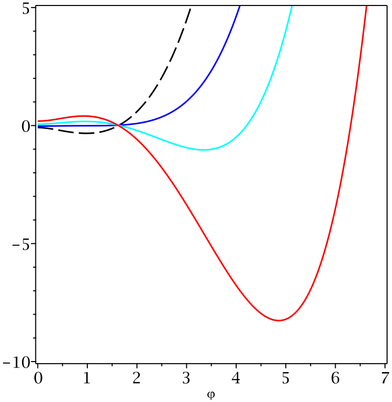
During inflation stochastic fluctuations of a light field "compete" with classical evolution caused by its potential

After long enough time the system approaches the Fokker-Planck probability distribution: $P(\Phi) \propto \exp\left[-\frac{8\pi^2}{3}\,\frac{V(\Phi)}{H_I^4}\right]$

Starobinsky, Yokoyama, 1994



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Stochastic processes during inflation have somewhat different character than in the case of standard $\lambda_\Phi (S^2 - f_a^2)^2$ potential

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PQ field after inflation

Just after inflation fields S and θ are almost homogeneous

model	SD	qS	$\lambda_\Phi \vert\Phi\vert^4$
$S_i^2 \sim$	$2\frac{(2-12\xi)H_I^2-m^2}{\lambda}$	$\frac{(3-12\xi)H_I^2-m^2}{\lambda\epsilon}$	$\frac{H_I^2}{\sqrt{\lambda_\Phi}}$

θ_i - **"accidental"**

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θ_i - "accidental"

Saxion field S starts to oscillate when the Hubble parameter drops to approximately $H_i \approx \frac{1}{3} m_S^{\rm eff}$

Axion field θ_i stays unchanged

- until saxion oscillations are completely dumped at $S=0$ and θ becomes undefined

or

- until axion potential is generated by non-perturbartive effects at (much) lower temperatures

PQ field after inflation

Proposed picture in $\lambda_\Phi(S^2 - f_a^2)^2$ theory

Shtanov et al, Kofman et al 1994

- saxion field S starts to oscillate when H drops below about one third of the effective saxion mass
- energy stored in saxion oscillations
  - redshifts due to Hubble expansion
  - transfers to particles via perturbative decays
  - transfers to particles via non-perturbative parametric resonance
- parametric resonance simplification:
  whole initial energy is used to produce approximately equal number of saxion and axion particles via resonance
- axions contribute to WDM
- saxions (and surviving saxion oscillations) must be thermalized in order not to dominate the energy density of the Universe
  - e.g. via Higgs portal $\lambda_{H\Phi}|\Phi|^2 H^\dagger H$ Co et al 2020

Thermal corrections

Our model

- **Thermal effects depend on the same fields and couplings as the CW potential does**
  - **it is not necessary to consider other sources like e.g. the Higgs portal (but it can be added)**

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Our model

- Thermal effects depend on the same fields and couplings as the CW potential does
  - it is not necessary to consider other sources like e.g. the Higgs portal (but it can be added)
- We concentrate on two kinds of thermal effects
  - thermal mass
  - thermalization of oscillations

Thermal mass of the saxion in our model

$$m_T^2 = \left({\sum_i}' \lambda_i + \frac{1}{2} {\sum_j}' g_j^2 \right) \frac{T^2}{24}$$

Primes denote that the sums are over particles which

- **are not (much) heavier than T**
- **have number densities not much smaller than in the equilibrium**

Let us define the effective number of such particles $n_{\rm eff}$:

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For QCD axion n_{eff} is not very small

Saxion field starts to oscillate when H drops below $H_i \approx \frac{1}{3} m_S^{\text{eff}}$

- If saxion effective mass is dominated by the zero-temperature V_{CW}

$$H_i^{(0)} \approx \begin{cases} \sqrt{\frac{\lambda N_s}{48\pi^2}\left[(1-6\xi)-\frac{m^2}{4H_I^2}\right]\ln\left(\frac{(2-12\xi)H_I^2-m^2}{\mu^2}\right)}\, H_I & \text{(SD)} \\ \sqrt{\frac{\lambda N_f}{8\pi^2}\left[(1-4\xi)-\frac{2m^2}{9H_I^2}\right]\ln\left(\frac{(3-12\xi)H_I^2-m^2}{2\epsilon\mu^2}\right)}\, H_I & \text{(qS)} \end{cases}$$

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- If saxion effective mass is dominated by the thermal contribution

$$H_i^{(T)} \approx \begin{cases} \frac{\sqrt{5}}{144\sqrt{\pi^3 g_*}}\, n_{\rm eff}\lambda M_{\rm Pl} < H_{RH} & \text{if} \quad T_{RH} > \widetilde{T}_{RH} \\ \sqrt[6]{\frac{5}{g_*}}\,\frac{\sqrt[3]{12}}{72\sqrt{\pi}}\,(n_{\rm eff}\lambda)^{2/3}\, T_{RH}^{2/3} M_{\rm Pl}^{1/3} > H_{RH} & \text{if} \quad T_{RH} < \widetilde{T}_{RH} \end{cases}$$

where $\widetilde{T}_{RH} = \sqrt{\frac{5 n_{\rm eff}\lambda}{96\pi^3 g_*}}\, M_{\rm Pl}$

Saxion oscillations

For $T_{RH} > \widetilde{T}_{RH}$: $$\frac{H_i^{(0)}}{H_i^{(T)}} \propto \frac{N_s^{1/2}}{n_{\rm eff}} \frac{H_I}{\sqrt{\lambda} M_{\rm Pl}}$$

$H_i^{(T)} > H_i^{(0)}$ unless $n_{\rm eff} \ll 1$ or $\lambda \lesssim \mathcal{O}\left(10^{-10}\right)\left(\frac{H_I}{10^{14}\text{GeV}}\right)^2$

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- For substantial part of the parameter space thermal mass strongly dominates over the zero-temperature CW potential for quite long time

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- For substantial part of the parameter space thermal mass strongly dominates over the zero-temperature CW potential for quite long time
- $U(1)_{\rm PQ}$ is unbroken after inflation

Saxion oscillations in potential dominated by thermal mass

- No parametric resonance production of (s)axions in quadratic potential (with constant mass)
- Thermal mass does change with time (decreasing T)

$$\ddot{S} + 3H\dot{S} + \frac{n_{\rm eff}\lambda}{24}T^2 S = 0$$

Resonant production of particles only for unacceptably high T_{RH}

Saxion oscillations in potential dominated by thermal mass

- No parametric resonance production of (s)axions in quadratic potential (with constant mass)
- Thermal mass does change with time (decreasing T)

$$\ddot{S} + 3H\dot{S} + \frac{n_{\rm eff}\lambda}{24}T^2 S = 0$$

Resonant production of particles only for unacceptably high T_{RH}

- Amplitude of saxion oscillations decreases in expanding universe
  - $A_S \sim \mathcal{R}^{-21/16}$ during reheating process (MD)
  - $A_S \sim \mathcal{R}^{-1}$ after reheating is completed (RD)
- Number of particles which could be produced via parametric resonance is bounded roughly by V/m
  - it decreases as $\mathcal{R}^{-3}$ in pure $\lambda_\Phi S^4$ potential
  - with thermal mass it decreases longer
    and during (MD) much faster, almost as $\mathcal{R}^{-4}$

Saxion oscillations in potential dominated by thermal mass

- The maximal energy of thermal mass potential decreases slightly slower than the maximal energy of zero-temperature CW potential
- $\Rightarrow$ If thermal mass dominates at the beginning of saxion oscillations it dominates also at later times
- $\Rightarrow$ No (or very few) particles produced via parametric resonance

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The above conclusions are valid as long as the thermal mass is much bigger than the curvature of the zero-temperature potential (for small S)

Thermal mass domination fades away when temperature decreases to about

$$\widetilde{T}^2 \approx \mathcal{O}(0.1)\frac{N_s}{n_{\rm eff}}m^2 \ln\left(\frac{e\mu^2}{m^2}\right)$$

How much has the amplitude of saxion oscillations decreased till such time?

Amplitude of saxion oscillations at $\widetilde{T}$
(for "quasi-SUSY" spectrum if oscillations started after reheating):

$$\frac{A_S^2(\widetilde{T})}{S_{\min}^2} \approx \mathcal{O}\left(10^3\right)\frac{m^2}{\mu^2}\ln\left(\frac{e\mu^2}{m^2}\right)\frac{N_s}{n_{\text{eff}}^2}\frac{(3-12\xi)H_I^2-m^2}{\epsilon\lambda M_{\text{Pl}}^2}$$

- **For spectrum dominated by scalar(s)**

$$(3-13\xi)\to(2-12\xi) \qquad \epsilon\to 1$$

- **If oscillations started before the completion of reheating**

$$\text{extra factor:}\quad \mathcal{O}\left(10\right)\left(\frac{T_{RH}}{\sqrt{n_{\text{eff}}\lambda}M_{\text{Pl}}}\right)^{1/3}$$

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Production of cold and warm axions depends strongly on $\dfrac{A_S(\widetilde{T})}{S_{\rm min}}$

- $A_S(\widetilde{T}) \gg S_{\min}$:
  Warm axions produced via parametric resonance but
  - Available energy decreased during thermal mass domination
  - Less effective (stochastic) resonance Greene et al 1997
  - (Much) less axions and saxions produced
  - Amounts of produced axions and saxions may be very different

Relic axions

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  - Relic density depends on stochastic processes before inflation
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- $A_S(\widetilde{T}) \ll S_{\rm min}$:
  Cold axions produced via misalignment mechanism
  - Tachyonic instability important — Felder et al 2000
  - Dynamics no longer "remembers" the initial value θ_i
  - Results similar to "classical window":
    $\langle \delta\theta^2 \rangle = \frac{\pi^2}{3}$, white noise isocurvature perturbations at low scales

Relic axions

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- $\frac{m^2}{\mu^2} \ln\left(\frac{e\mu^2}{m^2}\right) \leq 1 \qquad \xrightarrow[m\to 0]{} 0 \qquad \xrightarrow[m\to\sqrt{e}\mu]{} 0$
- $\frac{N_s}{n_{\text{eff}}^2}$ may be $\mathcal{O}(1)$ but may be $\gg 1$ if n_{eff} is small
- constraints from bounds on isocurvature perturbations

$$\epsilon\lambda \lesssim 10^{-8} \quad \Rightarrow \quad \frac{(3-12\xi)H_I^2 - m^2}{\epsilon\lambda M_{\text{Pl}}^2} \gtrsim \mathcal{O}\left(\frac{H_I}{10^{15}\,\text{GeV}}\right)^2$$

- extra factor if $T_i > T_R$: $\mathcal{O}(10)\sqrt[3]{\frac{T_{RH}}{\sqrt{n_{\text{eff}}\lambda}M_{\text{Pl}}}} \gtrsim \mathcal{O}\left(\sqrt[3]{\frac{T_{RH}}{\sqrt{n_{\text{eff}}}\,10^{10}\,\text{GeV}}}\right)$

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Relic density of cold and warm axions depends quite strongly on many details of a model

Thermalization

In models with substantial production of warm axions

- saxions are also produced
- residual saxion oscillations may survive

Saxion particles and saxion condensate could dominate energy density of the universe

$\Rightarrow$ Thermalization of saxions is necessary

- Thermalization mediated by the same particles which contribute to the thermal mass of PQ field
- Contribution from scalars $\Gamma_{\rm th} \sim \frac{\lambda^2 S^2}{\alpha_{\rm th} T}$ Mukaida Nakayama 2012
  less effective (than usually assumed)
- Contribution from fermions $\Gamma_{\rm th} \sim y^4 \alpha_{\rm th} T$
  - when oscillations start: $\Gamma_{\rm th} \sim y^3 m_T \lll m_T \sim H$
  - during reheating: $\Gamma_{\rm th}/H$ scales as $\mathcal{R}^{9/8} \sim T^{-3}$
  - after reheating: $\Gamma_{\rm th}/H$ scales as $\mathcal{R} \sim T^{-1}$
- If $T_i < T_{RH}$ thermalization occurs very late: $T_{\rm th} < y^3 T_{RH}$
- Thermalization temperature is higher if saxion oscillations start before reheating: $T_{\rm th} \propto (T_i/T_{RH})^3$

Summary

- Peccei-Quinn field with non-trivial dynamics during and after inflation must have extremely small self-coupling
- Corrections to its potential may be crucial
  - radiative, thermal, geometric
- During inflation
  - saxion potential has (second) minimum at $S \gg f_a$
  - $\langle S_i \rangle$, $\langle \theta_i \rangle$, $\langle \delta S_i^2 \rangle$, $\langle \delta \theta_i^2 \rangle$ determined by stochastic processes
  - but $\langle S_i \rangle$ close to the position of minimum at $S \gg f_a$
- After inflation
  - thermal corrections very important for the evolution of saxion field
  - evolution of S and production of particles via parametric resonance depend quite strongly on details of a model
- Relic abundance of axions (also model-dependent)
  - contribution to WDM may be (strongly) suppressed
  - contribution to CDM may be stochastic or not
  - isocurvature perturbations may be standard (generated during inflation) or may have the form of white noise at small scales
- Numerical simulations necessary to get precise predictions