Distinguishing string-theoretic axions from field-theoretic ones with a precision study of axion couplings

Kiwoon Choi

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KC, Im, Seong and Kim, arXiv:2106.05816

기초과학연구원
Institute for Basic Science

Outline

- Introduction
- Axion couplings:

  KSVZ-type, DFSZ-type, String-theoretic axions
- Distinguishing different type of axions with experimentally measurable low energy axion couplings
- Conclusion

Axions or axion-like particles (ALPs) are light pseudo-scalar bosons postulated in many well-motivated models for BSM physics:

* Strong CP problem: QCD axion
* Weak scale hierarchy problem: Relaxion
* Dark matter
* Inflaton for natural inflation
* axion-driven baryogenesis, magnetogenesis, ...
* Generic prediction of string theory

Axions introduced for different reasons generically have quite different masses and different pattern of couplings.

Axions have a finite field range (periodic):

$$a(x) \cong a(x) + 2\pi f_a \quad (f_a = \text{axion decay constant})$$

Axions can be naturally light as their interactions are constrained by an approximate PQ-symmetry non-linearly realized in low energy effective theory, involving a constant shift of the axion field:

$$U(1)_{\rm PQ}: \quad a(x) \;\to\; a(x) + \text{constant}$$

Terminology:

"axions or ALP" = Pseudo-Nambu-Goldstone bosons associated with generic $U(1)_{PQ}$

"QCD axion" = Specific type of axion with $U(1)_{PQ}$ broken dominantly by the QCD anomaly to solve the strong CP problem

Classification of axions by the origin of the field variable

* Field-theoretic axions from the phase of $U(1)_{\rm PQ}$-charged complex scalar fields

PQWW, KSVZ, DFSZ, …

$$\sigma(x) = \rho(x) e^{ia(x)/f_a} \quad \left(\frac{f_a}{\sqrt{2}} = \langle \rho \rangle\right)$$

* String-theoretic axions from p-form gauge fields in string theory (or in any theory with extra-dim)

Witten '84, Svrcek & Witten '06, …

$$a(x) = \int_{\Sigma_p} C^{(p)}_{[m_1,..,m_p]}(x,y)\, dy^{m_1}...dy^{m_p} \quad (\Sigma_p = p\text{-cycle in extra-dim})$$

or $\quad \partial_\mu a = \epsilon_{\mu\nu\rho\sigma}(\partial^\nu B^{\rho\sigma} + ...) \quad (B_{\mu\nu} = \text{2-form gauge field in 4D spacetime})$

Field-theoretic axions have a UV completion with **linearly realized** $U(1)_{\rm PQ}$ ($f_a = 0$), while string-theoretic axions do not have such a limit within 4-dim effective theory.

Classification by low energy couplings

For model-independent discussion of axion couplings, it is convenient to use

* a field basis for which all fields other than the axion are PQ-invariant:

Georgi, Kaplan, Randall '85

$$U(1)_{\rm PQ}: \quad a(x) \ \rightarrow \ a(x) + \text{constant},$$

$$\Phi = (\psi, H, ...) \ \rightarrow \ \Phi$$

* angular field variable for axion:

$$\theta(x) = \frac{a(x)}{f_a} \cong \theta(x) + 2\pi$$

➔ Axion couplings $\propto \dfrac{1}{f_a}$

Axion couplings to the SM at high scales around f_a :

PQ-invariant derivative couplings to matter fields described by real-valued c_ψ & c_H

$$\mathcal{L}_{\rm eff} = \frac{1}{2} f_a^2 (\partial_\mu \theta)^2 + \partial_\mu \theta \Big(\sum_\psi c_\psi \bar{\psi} \sigma^\mu \psi + i c_H (H^\dagger D^\mu H - D^\mu H^\dagger H) \Big) \quad \text{(PQ-invariant)}$$

$$+ \sum_{F^A = G,W,B} c_A \frac{g_A^2}{32\pi^2} \theta(x) F^{A\mu\nu} \tilde{F}^A_{\mu\nu} - V_0(\theta(x)) - m_H^2(\theta(x)) |H|^2 + \ldots \quad \text{(PQ-breaking)}$$

Bare axion potential

Couplings to the SM gauge fields normalized in such a way that c_G , c_W , c_B are rational numbers or integers.

$\frac{g_A^2}{8\pi^2}$ = either perturbative loop-factor (field-theoretic axions)

or $1/S_{\rm instanton}$ (string-theoretic axion).

Non-derivative coupling to $|H|^2$ (axion-dependent Higgs mass)

Each coupling in this effective lagrangian is not an observable by itself, while their reparametrization-invariant combinations are observable quantities.

QCD axion postulated to solve the strong CP problem

$U(1)_{\rm PQ}$ is broken dominantly by the coupling to the gluon anomaly:

Peccei & Quinn

$$\frac{\mathbf{c}_G}{32\pi^2}\frac{a(x)}{f_a}G^{\mu\nu}\tilde{G}_{\mu\nu}$$

PQ-breaking couplings other than the anomalous couplings to the SM gauge fields are highly suppressed as

$$\frac{\partial}{\partial\theta}\Big(V_0(\theta), \frac{1}{16\pi^2}m_H^2(\theta)\Lambda^2, ...\Big) \lesssim 10^{-10} f_\pi^2 m_\pi^2$$

➔ $$V_{\rm axion}(\theta) = -\frac{f_\pi^2 m_\pi^2}{m_u + m_d}\sqrt{m_u^2 + m_d^2 + 2m_u m_d\cos(c_G\theta)} + \mathcal{O}(10^{-10} f_\pi^2 m_\pi^2)$$

P- and CP-invariant QCD: $|\theta_{\rm QCD}| \equiv |c_G\Big\langle\frac{a}{f_a}\Big\rangle| \lesssim 10^{-10}$

$$m_a \simeq 5.7\,\mathbf{c}_G\left(\frac{10^{12}{\rm GeV}}{f_a}\right)\mu{\rm eV}$$

Ultra-light ALP which may constitute (part of) dark matter

(e.g. fuzzy dark matter with $m_a \sim 10^{-20} - 10^{-21}\,\mathrm{eV}$)

No coupling to the gluon anomaly (**c_G** = 0) , but may have non-zero coupling to the EW anomalies

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_B B^{\mu\nu}\tilde{B}_{\mu\nu} + c_W W^{\mu\nu}\tilde{W}_{\mu\nu}\right)$$

All other PQ-breakings are so tiny, yielding $m_a \ll m_{a_{\rm QCD}} \sim \frac{m_\pi f_\pi}{f_a}$

Relaxion postulated to solve the weak scale hierarchy problem:

Higgs mass is relaxed by the cosmological excursion of the relaxion field:

Graham, Kaplan & Rajendran '15

$$m_H^2(\theta)|H|^2$$

Low energy axion couplings:

At $\mu = \mathcal{O}(1)$ GeV

photon gluons light quarks and electron

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_\gamma F^{\mu\nu}\tilde{F}_{\mu\nu} + c_G G^{\alpha\mu\nu}G^{\alpha}_{\mu\nu}\right) + \sum_{\Psi=u,d,e}\frac{\partial_\mu a}{2f_a}C_\Psi\bar{\Psi}\gamma^\mu\gamma_5\Psi$$

$$c_\gamma = c_W + c_B, \quad C_\Psi = C^0_\Psi + \Delta C_\Psi$$

Rational numbers which are typically of O(1).

Tree level values

Radiative corrections

$$C^0_u = c_{q_1}(f_a) + c_{u^c_1}(f_a) + c_H(f_a),$$
$$C^0_d = c_{q_1}(f_a) + c_{d^c_1}(f_a) - c_H(f_a),$$
$$C^0_e = c_{\ell_1}(f_a) + c_{e^c_1}(f_a) - c_H(f_a).$$

Measurable axion couplings in low energy experiments:

$$\frac{1}{2}g_{a\gamma}a(x)\vec{E}\cdot\vec{B}+\partial_\mu a(x)\left(\frac{g_{ap}}{2m_p}\bar{p}\gamma^\mu\gamma_5 p+\frac{g_{an}}{2m_n}\bar{n}\gamma^\mu\gamma_5 n+\frac{g_{ae}}{2m_e}\bar{e}\gamma^\mu\gamma_5 e\right)$$

$$g_{a\gamma} \simeq \frac{\alpha_{\rm em}}{2\pi}\frac{1}{f_a}\left(c_\gamma-\frac{2}{3}\left(\frac{m_u+4m_d}{m_u+m_d}\right)c_G\right)$$

$$g_{ap} \simeq \frac{m_p}{f_a}\left(C_u\Delta u+C_d\Delta d-\left(\frac{m_d}{m_u+m_d}\Delta u+\frac{m_u}{m_u+m_d}\Delta d\right)c_G\right)$$

$$\simeq \frac{m_p}{f_a}\left(0.88C_u-0.39C_d-0.47c_G\right)$$

$$g_{an} \simeq \frac{m_n}{f_a}\left(C_d\Delta u+C_u\Delta d-\left(\frac{m_u}{m_u+m_d}\Delta u+\frac{m_d}{m_u+m_d}\Delta d\right)c_G\right)$$

$$\simeq \frac{m_n}{f_a}\left(-0.39C_u+0.88C_d-0.02c_G\right)$$

accidental cancellation

The parts depending on $\mathbf{c_G}$ represent the contribution from the axion-meson mixing.

$$g_{ae} \simeq \frac{m_e}{f_a}C_e$$

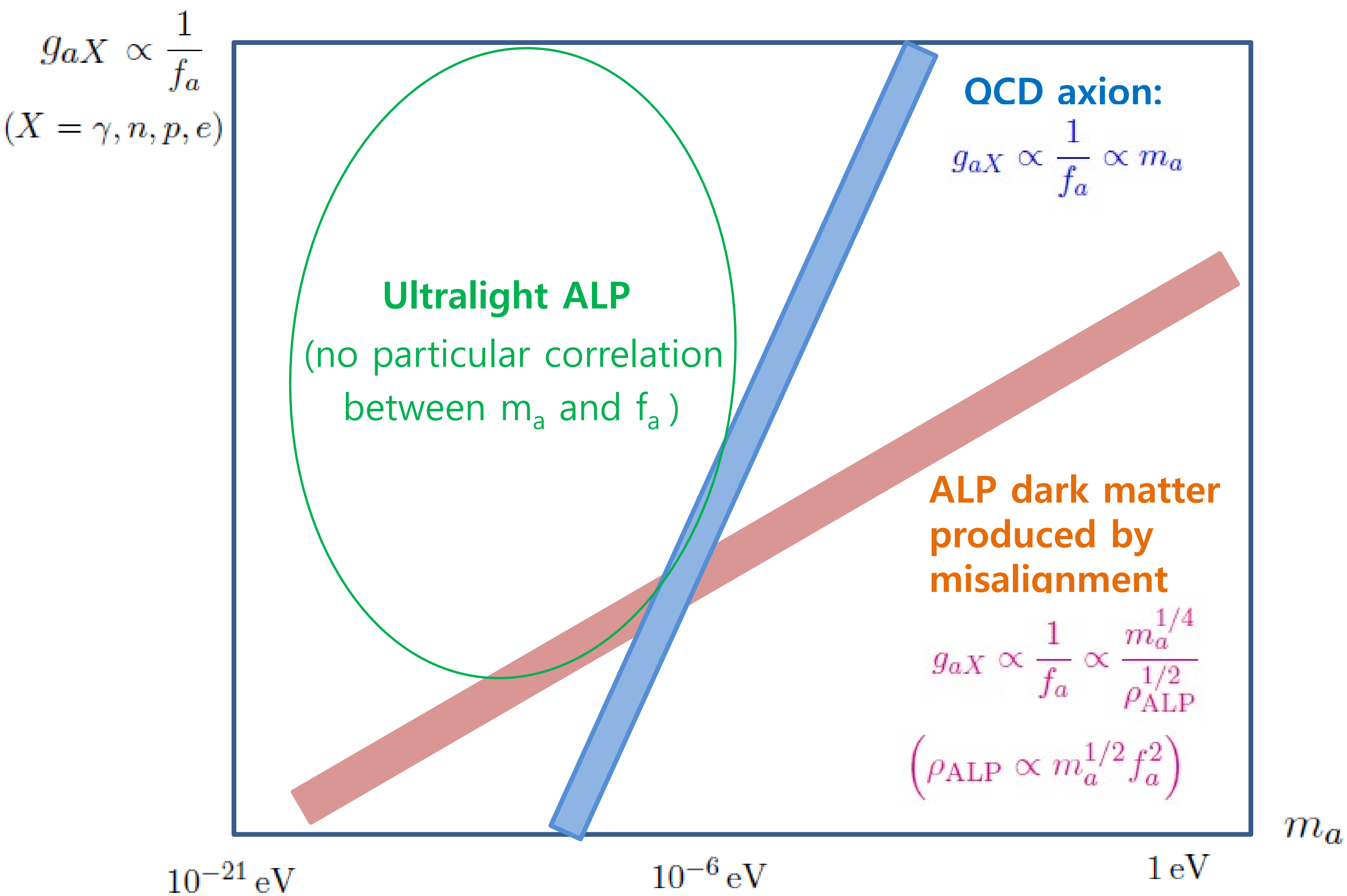
$\Delta u = 0.897(27),\ \Delta d = -0.376(27)$ for $C_{u,d}$ at $\mu = 2$ GeV

$\left(s^\mu\Delta q = \langle p|\bar{q}\gamma^\mu\gamma_5 q|p\rangle\ \ (q=u,d)\right)$

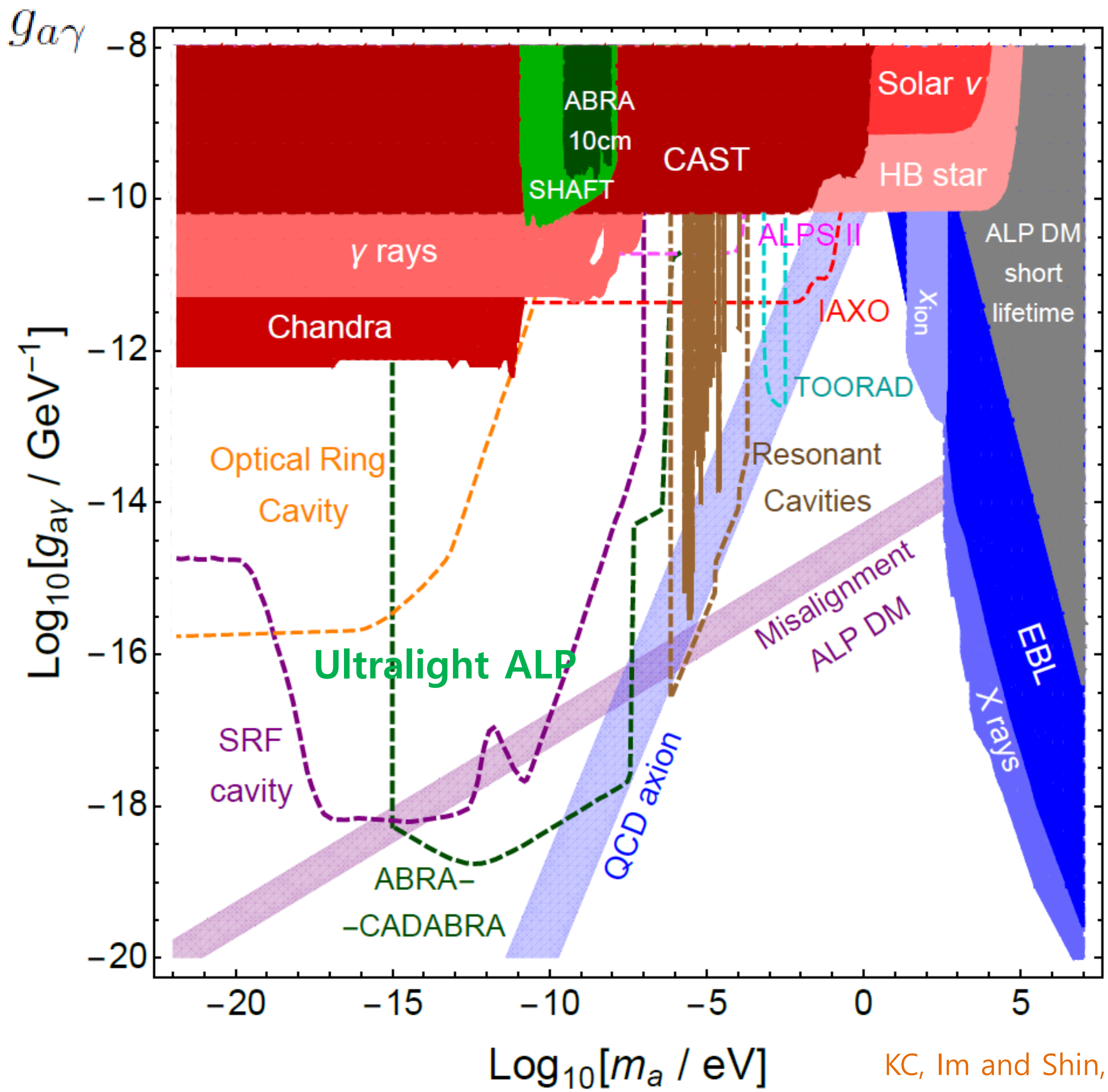
Cortona et al, arXiv:1511.02867

Typical parameter space for theoretically well-motivated light axions which may have good potential to be experimentally detected:

(Assume there is no big hierarchy among $\{c_A, c_\psi, \ldots\}$)



Observational bounds and the sensitivities of planned experiments



KC, Im and Shin, arXiv:2012.02029

Resonant cavity:
Sikivie '83,
Semertzidis et al '09

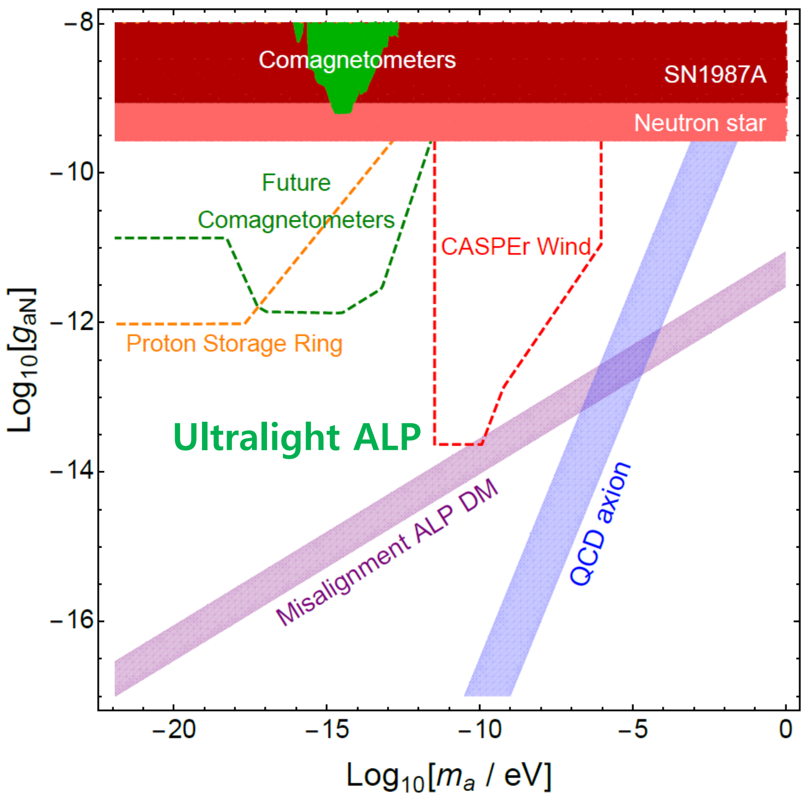
ABRACADABRA:
Kahn et al '16

Optical ring cavity:
Obata et al '18

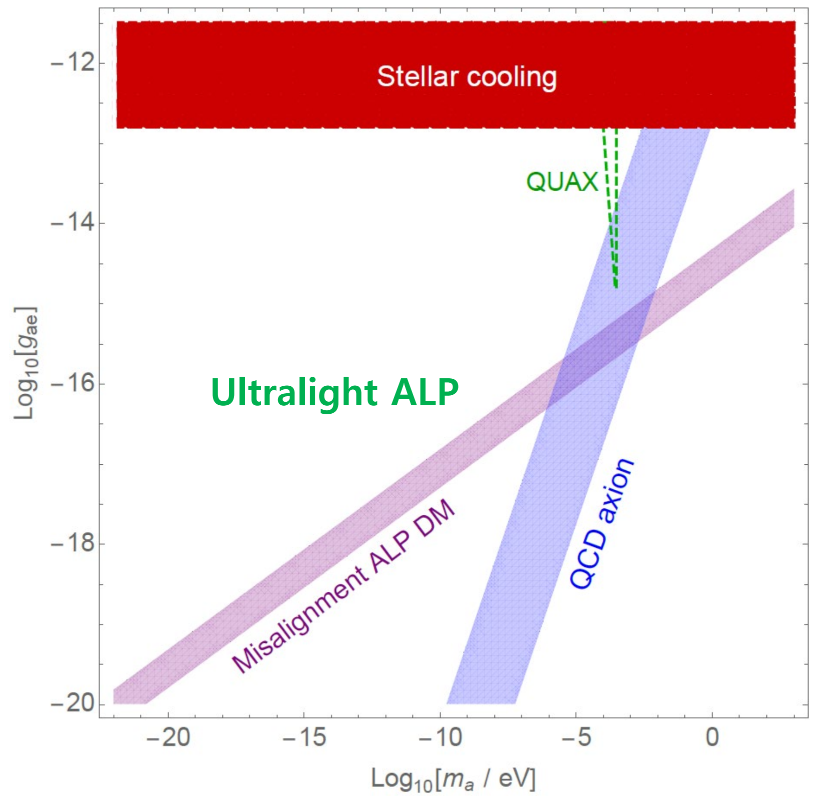
TOORAD:
Marsh et al '19

SRF cavity:
Berlin et al '20

g_{aN}



g_{ae}



Once an axion is discovered by any means, so its mass is known, it will be much easier to experimentally measure the axion couplings, which may make it possible to determine all of g_{aX} $(X = \gamma, n, p, e)$ in some future.

Couplings of field-theoretic axions

UV completion with linearly realized

$$U(1)_{\rm PQ}:\ \ \Phi\ \rightarrow\ e^{iX_\Phi\alpha}\Phi \quad (X_\Phi = \text{quantized PQ-charges})$$

Couplings of field-theoretic axions (in the unit of $1/f_a$) are determined mostly by the quantized PQ-charges which are either of order unity or precisely zero.

* Couplings to the photon and gluons:

PQWW, KSVZ, DFSZ, ..

$$c_\gamma = \sum_\psi X_\psi Q^2_{\rm em}(\psi), \quad c_G = \sum_\psi X_\psi Q^2_{\rm color}(\psi) \quad (\psi = \text{chiral fermions})$$

* Tree-level couplings to the light quarks and electron:

$$\begin{aligned} C_u^0 &= X_Q + X_{u^c} + ... \\ C_d^0 &= X_Q + X_{d^c} + ..., \\ C_e^0 &= X_L + X_{e^c} + ... \end{aligned}$$

Minimal **DFSZ:** SM fields have flavor-universal non-zero $U(1)_{\rm PQ}$ charges as

Dine-Fischler-Srednicki-Zhitnitsky

$$c_{H_u} = c_{H_d} = -\frac{1}{2}, \quad c_\psi = \frac{1}{4} \ (\psi \in {\rm SM}), \ \ c_G = c_W = \frac{3}{5} c_B = 3$$

➔ $$C_u^0 = \cos^2\beta, \quad C_d^0 = C_e^0 = \sin^2\beta \quad \left(\tan\beta = H_u/H_d\right)$$

Minimal **KSVZ:** All SM fields are neutral under $U(1)_{\rm PQ}$, but there exist exotic PQ-charged ($SU(2)_W$-singlet) quark Q & Q^c with $\mathcal{L}_{\rm Yukawa} = y_Q \sigma Q Q^c$

Kim-Shifman-Vainshtein-Zakharov

$$c_Q = c_{Q^c} = -\frac{1}{2}, \quad c_\psi = 0 \ \ (\psi \in {\rm SM}), \quad c_G = 1, \quad c_W = 0, \quad c_B = 6Y_Q^2$$

➔ $$C_\Psi^0 = 0 \quad \left(\sim \frac{f_a^2}{M_P^2} \lesssim \mathcal{O}\Big(\frac{1}{S_{\rm ins}^2}\Big) = \mathcal{O}\Big(\Big(\frac{g_{\rm GUT}^2}{8\pi^2}\Big)^2\Big)\right) \qquad (\Psi = u, d, e)$$

(from Planck-scale-suppressed higher-dim operator $\frac{1}{M_P^2}(\sigma^* \partial_\mu \sigma)\bar\psi\sigma^\mu\psi$

with the weak gravity conjecture bound $f_a \lesssim M_P/S_{\rm ins}$ $(S_{\rm ins} \sim g_{\rm GUT}^2/8\pi^2)$)

Arkani-Hamed et al '07

For our discussion, we can categorize generic field-theoretic axions into two classes:

DFSZ-type : SM fields are PQ-charged

$$\rightarrow \quad c_G \text{ and/or } c_{W,B} = \mathcal{O}(1)$$

$$C_\Psi^0 = \mathcal{O}(1) \quad (\Psi = u, d, e)$$

KSVZ-type: SM fields are all PQ-neutral, but there exist some exotic PQ-charged fermions charged under the SM gauge group

$$\rightarrow \quad c_G \text{ and/or } c_{W,B} = \mathcal{O}(1)$$

$$C_\Psi^0 = \mathcal{O}\Big(\frac{f_a^2}{M_P^2}\Big) \lesssim \mathcal{O}\Big(\Big(\frac{g_{\rm GUT}^2}{8\pi^2}\Big)^2\Big)$$

Couplings of string-theoretic axions

$$a(x) = \int_{\Sigma_p} C^{(p)}_{[m_1,..,m_p]}(x,y)\, dy^{m_1} ... dy^{m_p}$$

or $\quad \partial_\mu a = \epsilon_{\mu\nu\rho\sigma}(\partial^\nu B^{\rho\sigma} + ...)$

For each string-theoretic axion, there exists a modulus partner which forms an N=1 chiral superfield together with the axion:

$$T = \tau(x) + i\frac{a(x)}{f_a}$$

τ = Modulus partner of an angular axion field $\frac{a(x)}{f_a} \cong \frac{a(x)}{f_a} + 2\pi$

= Euclidean action (**S**$_{\text{ins}}$) of the brane instanton which is a low energy consequence of the brane which couples to the underlying p-form gauge field

$$\propto \text{Vol}(\Sigma_p) \;\; \text{and/or} \;\; \frac{1}{g_{\text{st}}^n} \quad (g_{\text{st}} = \text{string coupling})$$

4D effective SUGRA

Kahler potential: $K = K_0(T+T^*) + Z_I(T+T^*)\Phi_I^*\Phi_I + ...,$

Gauge kinetic function: $\mathcal{F}_A = \frac{c_A}{8\pi^2}T + ...$ $\left(c_A = \text{rational numbers of order unity}\right)$

$$\left(-\frac{1}{4}\text{Re}(\mathcal{F}_A)F^{A\mu\nu}F^A_{\mu\nu} + \frac{1}{4}\text{Im}(\mathcal{F}_A)F^{A\mu\nu}\tilde{F}^A_{\mu\nu}\right)$$

Axion couplings at $\mu \sim f_a \sim M_{\rm st}$:

$$\frac{1}{2}\partial_\mu a\partial^\mu a + \frac{\partial_\mu a}{f_a}\left[ic_\phi(\phi^* D^\mu\phi - \phi D^\mu\phi^*) + c_\psi\bar{\psi}\bar{\sigma}^\mu\psi\right] + \frac{c_A}{32\pi^2}\frac{a(x)}{f_a}F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$\frac{1}{2}f_a^2 = M_P^2\frac{\partial^2 K_0}{\partial T\partial T^*}$$

$$c_{\phi_I} = \frac{\partial \ln Z_I}{\partial T}, \quad c_{\psi_I} = \frac{\partial \ln(e^{-K_0/2}Z_I)}{\partial T}, \quad c_A = 8\pi^2\frac{\partial}{\partial T}\mathcal{F}_A,$$

Moduli vacuum value:

* Axion weak gravity conjecture: $\tau = S_{\rm ins} \lesssim \mathcal{O}\left(\frac{M_P}{f_a}\right)$ Arkani-Hamed et al '07

* Axion potential induced by the brane-instanton: Dine et al '85; Blumenhagen et al '09; ...

$$\delta V = e^{-\tau}\Lambda^2 M_P^2 \cos\left(\frac{a}{f_a}\right) \quad \text{with} \quad \Lambda^2 \sim m_{3/2}M_P \ \text{or}\ m_{3/2}^2$$

For QCD axion, $\delta V \lesssim 10^{-10} m_\pi^2 f_\pi^2$ ➔ $\tau \gtrsim \ln\left(10^{10}\Lambda^2/f_\pi m_\pi\right)$

For ultra-light ALP, $\delta V \lesssim m_a^2 f_a^2$ ➔ $\tau \gtrsim 2\ln\left(\Lambda/m_a\right)$

(Note that this lower bound is meaningful only for $m_a \ll m_{3/2}$.)

* To avoid a too large $\frac{1}{g_{\rm GUT}^2} = {\rm Re}(\mathcal{F}_A) = c_A \frac{\tau}{8\pi^2} + \ldots$ ➔ $\tau \lesssim \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right)$

➔ $\tau = \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right)$

For $\tau = S_{\rm ins} = \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right) \gg 1$,

$$\frac{\partial K_0}{\partial \tau} \sim \tau \frac{\partial^2 K_0}{\partial \tau^2} \sim \tau \frac{f_a^2}{M_P^2} \lesssim \mathcal{O}\Big(\frac{g_{\rm GUT}^2}{8\pi^2}\Big),$$

$$\frac{\partial \ln Z_I}{\partial \tau} \sim \frac{1}{\tau} = \mathcal{O}\Big(\frac{g_{\rm GUT}^2}{8\pi^2}\Big),$$

➔ $$c_\psi \sim c_\phi = \mathcal{O}\left(\frac{1}{\tau}\right) = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right)$$

➔ $$C_\Psi^0 = \mathcal{O}\left(\frac{1}{\tau}\right) = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right) \quad (\Psi = u, d, e)$$

for string-theoretic axions

Overall strength of axion couplings is determined by f_a .

Is there any range of f_a favored by string-theoretic axions?

In string compactifications without a big hierarchy between the string scale and the 4D Planck scale, KC & Kim, '85 Svrcek & Witten '06

$$f_a = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2} M_P\right) \sim 10^{16} - 10^{17}\,{\rm GeV}$$

However, in string compactifications with a large compactification volume and/or a large warp factor which would generate a big scale hierarchy, f_a of string-theoretic axions can be anywhere in the range

$$10^7 - 10^8\ {\rm GeV} \lesssim f_a \lesssim \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2} M_P\right)$$

Burgess, Ibanez, Quevedo '99,
Conclon '06,
Cicoli et al '12;
...

Astrophysics

Weak Gravity Conjecture

Axions in heterotic string or M-theory

Witten '84; KC & Kim '85; Banks & Dine '96; KC '97

$$K_0 = -\ln(S+S^*) - 3\ln(T+T^*),$$
$$Z_I = \frac{1}{T+T^*} - \frac{\beta}{3}\frac{1}{S+S^*},$$
$$\mathcal{F}_{E6} = \frac{1}{8\pi^2}(S+\beta T), \quad \mathcal{F}_{E8} = \frac{1}{8\pi^2}(S-\beta T)$$

Model-independent axion & Kahler axion from NS 2-form

$$S = \tau_1 + i\frac{a_1}{f_1}, \quad T = \tau_2 + i\frac{a_2}{f_2}$$

Ibanez & Nilles '86; Lukas, Ovrut, Waldram '97

$$\beta = \frac{1}{4\pi}\int J\wedge\left[\mathrm{tr}(F\wedge F) - \frac{1}{2}\mathrm{tr}(R\wedge R)\right] \qquad \mathrm{Re}(\mathcal{F}_{E_6}) = \frac{1}{g^2_{\rm GUT}}, \quad \mathrm{Re}(\mathcal{F}_{E_8}) = \frac{1}{g^2_{\rm hid}}$$

For the QCD axion combination which is decoupled from the hidden E8-sector,

$$f_a = \sqrt{2}M_P\frac{g^2_{\rm GUT}}{8\pi^2}\frac{g^2_{\rm hid}\sqrt{(g^2_{\rm hid}+g^2_{\rm GUT})^2 - g^2_{\rm hid}g^2_{\rm GUT}}}{|g^4_{\rm hid} - g^4_{\rm GUT}|} = \mathcal{O}(10^{16})\,\mathrm{GeV}$$

$$c_A = 1, \quad c_{\phi_I}(f_a) = \omega_\phi\frac{g^2_{\rm GUT}}{16\pi^2}, \quad c_{\psi_I} = \omega_\psi\frac{g^2_{\rm GUT}}{16\pi^2}$$

$$\omega_\phi = \frac{g^2_{\rm hid}\left[(g^2_{\rm hid}+g^2_{\rm GUT})^2 + 2g^2_{\rm hid}g^2_{\rm GUT}\right]}{(g^2_{\rm hid}+2g^2_{\rm GUT})\,(g^4_{\rm hid}-g^4_{\rm GUT})} \sim \omega_\psi = \frac{g^2_{\rm hid}\left[(g^2_{\rm hid}+g^2_{\rm GUT})^2 - g^2_{\rm hid}g^2_{\rm GUT}\right]}{(g^2_{\rm hid}+2g^2_{\rm GUT})\,(g^4_{\rm hid}-g^4_{\rm GUT})} = \mathcal{O}(1)$$

Axions in Large Volume Scenario of Type IIB with D7 supporting the SM:

Conlon '06; Cicoli, Goodsell, Ringwald '12

$$K = -3\ln(T_b + T_b^*) + \frac{(T+T^*)^{3/2}}{(T_b+T_b^*)^{3/2}} + \frac{(T+T^*)^{\omega_I}}{(T_b+T_b^*)}\Phi_I^*\Phi_I, \quad \mathcal{F}_A = \frac{1}{8\pi^2}T,$$

Axions from RR 4-form

$$T_b = \tau_b + i\theta_b, \quad T = \tau + i\frac{a}{f_a}$$

$$\tau_b \sim \mathcal{V}_{\rm CY}^{2/3} \sim \left(\frac{M_P}{M_{\rm st}}\right)^{4/3} \sim e^{\tau_s} \gg \frac{8\pi^2}{g_{\rm GUT}^2}, \qquad \tau = \frac{8\pi^2}{g_{\rm GUT}^2}$$

$f_a \sim \dfrac{M_P}{\tau_b^{3/4}} \sim M_{\rm st}$: Axion decay constant exponentially lower than M_P

$\omega_I = \dfrac{1}{2}$ for charged matter fields on the intersections of D7

Conlon, Cremades, Quevedo '06

$$c_A = 1, \quad c_{\phi_I}(f_a) = \omega_I\frac{g_{\rm GUT}^2}{16\pi^2} + \mathcal{O}\left(\frac{1}{\tau_b}\right), \quad c_{\psi_I} = \omega_I\frac{g_{\rm GUT}^2}{16\pi^2} + \mathcal{O}\left(\frac{1}{\tau_b}\right).$$

Three types of axions with different couplings ratios

$$\frac{C_\Psi^0}{c_A} = \frac{\text{tree-level axion-matter}}{\text{quantized axion-gauge}} \quad (\Psi = u, d, e; \;\; A = \gamma, G)$$

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_\gamma F^{\mu\nu}\tilde{F}_{\mu\nu} + c_G G^{\alpha\mu\nu}G^{\alpha}_{\mu\nu}\right) + \sum_{\Psi=u,d,e}\frac{\partial_\mu a}{2f_a}C_\Psi\bar{\Psi}\gamma^\mu\gamma_5\Psi$$

$$c_\gamma = c_W + c_B, \quad C_\Psi = C_\Psi^0 + \Delta C_\Psi$$

Tree level values

Radiative corrections

c_A =rational numbers
($A=\gamma$, G)

DFSZ-type: $\frac{C_\Psi^0}{c_A} = \mathcal{O}(1)$

KSVZ-type: $\frac{C_\Psi^0}{c_A} \sim \frac{f_a^2}{M_P^2} \lesssim \mathcal{O}\left(\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right)^2\right)$

String-theoretic: $\frac{C_\Psi^0}{c_A} \sim \frac{1}{\tau} = \frac{1}{S_{\rm ins}} = \mathcal{O}\left(\frac{g_{\rm GUT}^2}{8\pi^2}\right)$

Would it be possible to distinguish these three types of axions by experimentally measurable $\mathrm{g_{aX}}$ $(\mathbf{X} = \gamma, \mathbf{p}, \mathbf{n}, \mathbf{e})$?

$$\frac{1}{2} g_{a\gamma} a(x) \vec{E} \cdot \vec{B} + \partial_\mu a(x) \left(\frac{g_{ap}}{2m_p} \bar{p} \gamma^\mu \gamma_5 p + \frac{g_{an}}{2m_n} \bar{n} \gamma^\mu \gamma_5 n + \frac{g_{ae}}{2m_e} \bar{e} \gamma^\mu \gamma_5 e \right)$$

Both KSVZ-type and string-theoretic axions have $C^0_\Psi / c_A \ll 1$, therefore one needs to examine radiative corrections ΔC_Ψ $(\Psi = u, d, e)$ to see if they have a distinguishable pattern of $\mathrm{g_{aX}}$ $(\mathbf{X} = \gamma, \mathbf{p}, \mathbf{n}, \mathbf{e})$.

RG evolution of axion couplings

High scale axion couplings at $\mu \sim f_a$:

$$\frac{\partial_\mu a}{f_a}\left[ic_\phi(\phi^* D^\mu \phi - \phi D^\mu \phi^*) + c_\psi \bar{\psi}\bar{\sigma}^\mu \psi\right] + c_A \frac{g_A^2}{32\pi^2}\frac{a(x)}{f_a} F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$\phi = \{H_u, H_d, ...\} \quad \psi = \{Q_i, u_i^c, d_i^c, L_i, e_i^c\} \quad F^A_{\mu\nu} = (G^\alpha_{\mu\nu}, W^i_{\mu\nu}, B_{\mu\nu})$$

$$c_A = \{c_G, c_W, c_B\} : \text{ rational numbers}$$

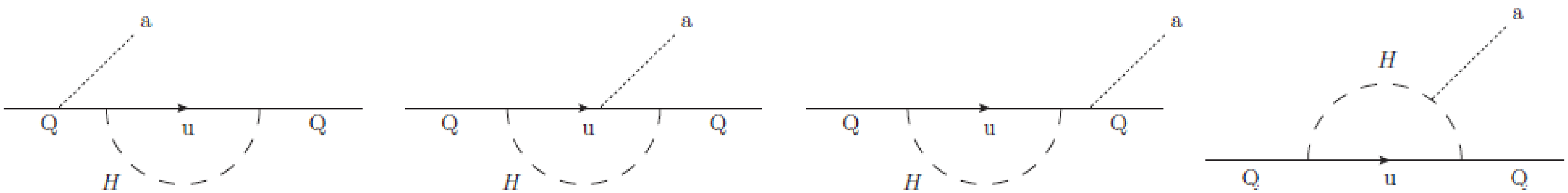
$$c_\phi(f_a) = c_\phi^0, \quad c_\psi(f_a) = c_\psi^0$$

RG evolution of axion couplings

(reparameterization covariance, diagrammatic computation, spurion analysis on N=1 superspace for SUSY models)

Yukawa-induced 1-loop running:

KC, Im, Park, Yun, 1708.00021



$$\left.\frac{d\mathbf{c}_F}{d\ln\mu}\right|_{1-\text{loop}} = \frac{\xi_y}{16\pi^2}\sum_{f,\alpha}\left(\frac{1}{2}\{\mathbf{c}_F, \mathbf{y}^\dagger_{fF\alpha}\mathbf{y}_{fF\alpha}\} + \mathbf{y}^\dagger_{fF\alpha}\mathbf{c}_f^T\mathbf{y}_{fF\alpha} + c_{H_\alpha}\mathbf{y}^\dagger_{fF\alpha}\mathbf{y}_{fF\alpha}\right),$$

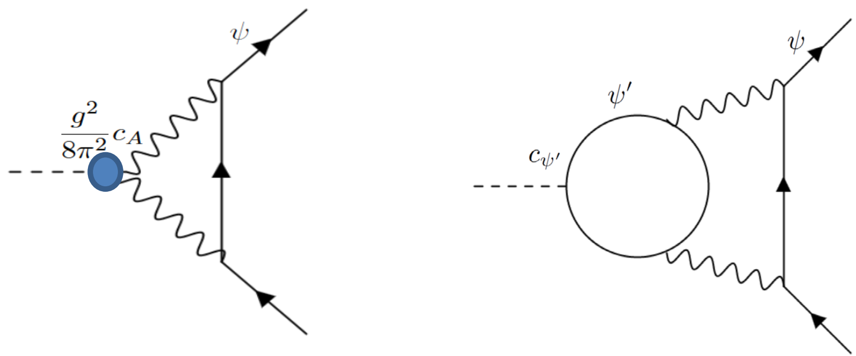
$$\left.\frac{d\mathbf{c}_f^T}{d\ln\mu}\right|_{1-\text{loop}} = \frac{\xi_y}{16\pi^2}\sum_{F,\alpha}\left(\frac{1}{2}\{\mathbf{c}_f^T, \mathbf{y}_{fF\alpha}\mathbf{y}^\dagger_{fF\alpha}\} + \mathbf{y}_{fF\alpha}\mathbf{c}_F\mathbf{y}^\dagger_{fF\alpha} + c_{H_\alpha}\mathbf{y}_{fF\alpha}\mathbf{y}^\dagger_{fF\alpha}\right),$$

$$\left.\frac{dc_{H_\alpha}}{d\ln\mu}\right|_{1-\text{loop}} = \frac{1}{8\pi^2}\sum_{f,F}\left(c_{H_\alpha}\text{tr}(\mathbf{y}^\dagger_{fF\alpha}\mathbf{y}_{fF\alpha}) + \text{tr}(\mathbf{y}_{fF\alpha}\mathbf{c}_F\mathbf{y}^\dagger_{fF\alpha}) + \text{tr}(\mathbf{y}^\dagger_{fF\alpha}\mathbf{c}_f^T\mathbf{y}_{fF\alpha})\right),$$

$$F_i = \{q_i, \ell_i\} \qquad f_i = \{u_i^c, d_i^c, e_i^c\}$$

$$\text{non-SUSY}: \quad \xi_y = 1 \qquad \text{SUSY}: \quad \xi_y = 2$$

Gauge-induced 2-loop (computationally 1-loop) running:



KC, Im, Shin, 2012.05029
Baur et al, 2012.12272
KC, Im, Seong, Kim, arXiv:2106.05816

$$\left.\frac{d\mathbf{c}_\psi}{d\ln\mu}\right|_{2-\text{loop}} = -\xi_g \sum_A \frac{3}{2}\left(\frac{g_A^2}{8\pi^2}\right)^2 \mathbb{C}_A(\psi)\Big(c_A - 2\sum_{\psi'}\text{tr}(\mathbf{c}_{\psi'})\mathbb{T}_A(\psi')\Big)\mathbb{1},$$

$$\left.\frac{dc_{H_\alpha}}{d\ln\mu}\right|_{2-\text{loop}} = -\xi_H \sum_A \frac{3}{2}\left(\frac{g_A^2}{8\pi^2}\right)^2 \mathbb{C}_A(H_\alpha)\Big(c_A - 2\sum_{\psi'}\text{tr}(\mathbf{c}_{\psi'})\mathbb{T}_A(\psi')\Big),$$

$\mathbb{C}_A(\Phi)$ = quadratic Casimir $\qquad$ $\mathbb{T}_A(\Phi)$ = Dynkin index

$$\text{non-SUSY}: \quad \xi_g = 1, \quad \xi_H = 0 \qquad\qquad \text{SUSY}: \quad \xi_g = \xi_H = \frac{2}{3}$$

MSSM: $$\frac{d\mathbf{c}_Q}{d\ln\mu} = \frac{\xi_y}{16\pi^2}\left(\frac{1}{2}\{\mathbf{c}_Q,\tilde{\mathbf{y}}_u^\dagger\tilde{\mathbf{y}}_u+\tilde{\mathbf{y}}_d^\dagger\tilde{\mathbf{y}}_d\}+\tilde{\mathbf{y}}_u^\dagger\mathbf{c}_{u^c}^T\tilde{\mathbf{y}}_u+\tilde{\mathbf{y}}_d^\dagger\mathbf{c}_{d^c}^T\tilde{\mathbf{y}}_d+c_{H_u}\tilde{\mathbf{y}}_u^\dagger\tilde{\mathbf{y}}_u+c_{H_d}\tilde{\mathbf{y}}_d^\dagger\tilde{\mathbf{y}}_d\right)$$
$$-\xi_g\left(\frac{\alpha_s^2}{2\pi^2}\tilde{c}_G+\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W+\frac{\alpha_1^2}{96\pi^2}\tilde{c}_B\right)\mathbb{1}\,,$$
$$\frac{d\mathbf{c}_{u^c}^T}{d\ln\mu} = \frac{\xi_y}{8\pi^2}\left(\frac{1}{2}\{\mathbf{c}_{u^c}^T,\tilde{\mathbf{y}}_u\tilde{\mathbf{y}}_u^\dagger\}+\tilde{\mathbf{y}}_u\mathbf{c}_Q\tilde{\mathbf{y}}_u^\dagger+c_{H_u}\tilde{\mathbf{y}}_u\tilde{\mathbf{y}}_u^\dagger\right)-\xi_g\left(\frac{\alpha_s^2}{2\pi^2}\tilde{c}_G+\frac{\alpha_1^2}{6\pi^2}\tilde{c}_B\right)\mathbb{1}\,,$$
$$\frac{d\mathbf{c}_{d^c}^T}{d\ln\mu} = \frac{\xi_y}{8\pi^2}\left(\frac{1}{2}\{\mathbf{c}_{d^c}^T,\tilde{\mathbf{y}}_d\tilde{\mathbf{y}}_d^\dagger\}+\tilde{\mathbf{y}}_d\mathbf{c}_Q\tilde{\mathbf{y}}_d^\dagger+c_{H_d}\tilde{\mathbf{y}}_d\tilde{\mathbf{y}}_d^\dagger\right)-\xi_g\left(\frac{\alpha_s^2}{2\pi^2}\tilde{c}_G+\frac{\alpha_1^2}{24\pi^2}\tilde{c}_B\right)\mathbb{1}\,,$$
$$\frac{d\mathbf{c}_L}{d\ln\mu} = \frac{\xi_y}{16\pi^2}\left(\frac{1}{2}\{\mathbf{c}_L,\tilde{\mathbf{y}}_e^\dagger\tilde{\mathbf{y}}_e\}+\tilde{\mathbf{y}}_e^\dagger\mathbf{c}_{e^c}^T\tilde{\mathbf{y}}_e+c_{H_e}\tilde{\mathbf{y}}_e^\dagger\tilde{\mathbf{y}}_e\right)-\xi_g\left(\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W+\frac{3\alpha_1^2}{32\pi^2}\tilde{c}_B\right)\mathbb{1}\,,$$
$$\frac{d\mathbf{c}_{e^c}^T}{d\ln\mu} = \frac{\xi_y}{8\pi^2}\left(\frac{1}{2}\{\mathbf{c}_{e^c}^T,\tilde{\mathbf{y}}_e\tilde{\mathbf{y}}_e^\dagger\}+\tilde{\mathbf{y}}_e\mathbf{c}_L\tilde{\mathbf{y}}_e^\dagger+c_{H_e}\tilde{\mathbf{y}}_e\tilde{\mathbf{y}}_e^\dagger\right)-\xi_g\frac{3\alpha_1^2}{8\pi^2}\tilde{c}_B\mathbb{1}\,,$$
$$\frac{dc_{H_u}}{d\ln\mu} = \frac{3}{8\pi^2}\left(c_{H_u}\mathrm{tr}(\tilde{\mathbf{y}}_u^\dagger\tilde{\mathbf{y}}_u)+\mathrm{tr}(\tilde{\mathbf{y}}_u\mathbf{c}_Q\tilde{\mathbf{y}}_u^\dagger)+\mathrm{tr}(\tilde{\mathbf{y}}_u^\dagger\mathbf{c}_{u^c}^T\tilde{\mathbf{y}}_u)\right)-\xi_H\left(\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W+\frac{3\alpha_1^2}{32\pi^2}\tilde{c}_B\right),$$
$$\frac{dc_{H_d}}{d\ln\mu} = \frac{3}{8\pi^2}\left(c_{H_d}\mathrm{tr}(\tilde{\mathbf{y}}_d^\dagger\tilde{\mathbf{y}}_d)+\mathrm{tr}(\tilde{\mathbf{y}}_d\mathbf{c}_Q\tilde{\mathbf{y}}_d^\dagger)+\mathrm{tr}(\tilde{\mathbf{y}}_d^\dagger\mathbf{c}_{d^c}^T\tilde{\mathbf{y}}_d)\right)$$
$$+\frac{1}{8\pi^2}\left(c_{H_d}\mathrm{tr}(\tilde{\mathbf{y}}_e^\dagger\tilde{\mathbf{y}}_e)+\mathrm{tr}(\tilde{\mathbf{y}}_e\mathbf{c}_L\tilde{\mathbf{y}}_e^\dagger)+\mathrm{tr}(\tilde{\mathbf{y}}_e^\dagger\mathbf{c}_{e^c}^T\tilde{\mathbf{y}}_e)\right)-\xi_H\left(\frac{9\alpha_2^2}{32\pi^2}\tilde{c}_W+\frac{3\alpha_1^2}{32\pi^2}\tilde{c}_B\right),$$

$$\tilde{c}_G = c_G-\mathrm{tr}(2\mathbf{c}_\mathrm{Q}+\mathbf{c}_{\mathrm{u^c}}+\mathbf{c}_{\mathrm{d^c}})\,,$$
$$\tilde{c}_W = c_W-\mathrm{tr}(3\mathbf{c}_\mathrm{Q}+\mathbf{c}_\mathrm{L})-\frac{3}{2}\xi_\mathrm{H}(\mathrm{c}_{\mathrm{H_u}}+\mathrm{c}_{\mathrm{H_d}})\,,$$
$$\tilde{c}_B = c_B-\mathrm{tr}\left(\frac{1}{3}(\mathbf{c}_\mathrm{Q}+8\,\mathbf{c}_{\mathrm{u^c}}+2\,\mathbf{c}_{\mathrm{d^c}})+\mathbf{c}_\mathrm{L}+2\mathbf{c}_{\mathrm{e^c}}\right)-\frac{3}{2}\xi_\mathrm{H}(\mathrm{c}_{\mathrm{H_u}}+\mathrm{c}_{\mathrm{H_d}})$$

One can systematically integrate the RG evolution, while including the relevant threshold corrections to compute $\Delta C_\Psi \ (\Psi = u, d, e)$ at $\mu = 2 \text{ GeV}$, and then use

$$\frac{1}{2} g_{a\gamma} a(x) \vec{E} \cdot \vec{B} + \partial_\mu a(x) \left(\frac{g_{ap}}{2m_p} \bar{p} \gamma^\mu \gamma_5 p + \frac{g_{an}}{2m_n} \bar{n} \gamma^\mu \gamma_5 n + \frac{g_{ae}}{2m_e} \bar{e} \gamma^\mu \gamma_5 e \right)$$

$$g_{a\gamma} \simeq \frac{\alpha_{\rm em}}{2\pi} \frac{1}{f_a} \left(c_\gamma - \frac{2}{3} \left(\frac{m_u + 4m_d}{m_u + m_d} \right) c_G \right)$$

$$g_{ap} \simeq \frac{m_p}{f_a} \left(C_u \Delta u + C_d \Delta d - \left(\frac{m_d}{m_u + m_d} \Delta u + \frac{m_u}{m_u + m_d} \Delta d \right) c_G \right)$$

$$\simeq \frac{m_p}{f_a} \left(0.88 C_u - 0.39 C_d - 0.47 c_G \right)$$

$$g_{an} \simeq \frac{m_n}{f_a} \left(C_d \Delta u + C_u \Delta d - \left(\frac{m_u}{m_u + m_d} \Delta u + \frac{m_d}{m_u + m_d} \Delta d \right) c_G \right)$$

$$\simeq \frac{m_n}{f_a} \left(-0.39 C_u + 0.88 C_d - 0.02 c_G \right)$$

$$g_{ae} \simeq \frac{m_e}{f_a} C_e$$

accidental cancellation for the contribution from the axion-meson mixing

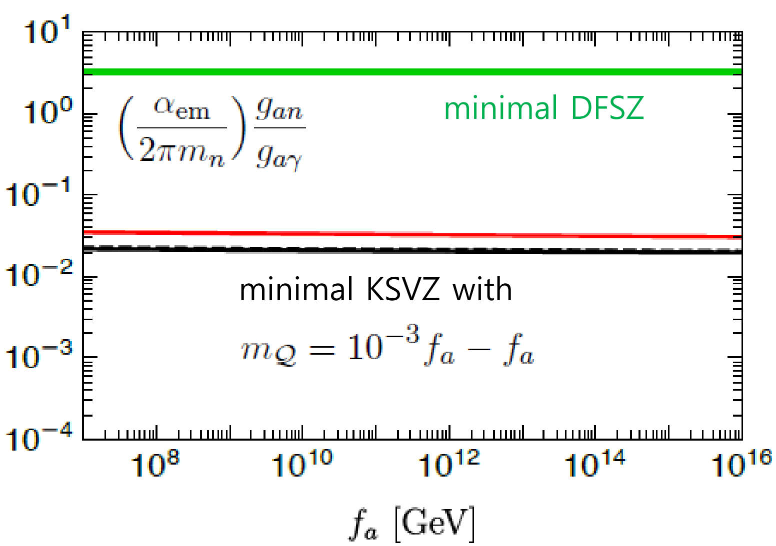
$g_{aX}/g_{a\gamma}\ \left(X = p, n, e\right)$ including radiative corrections

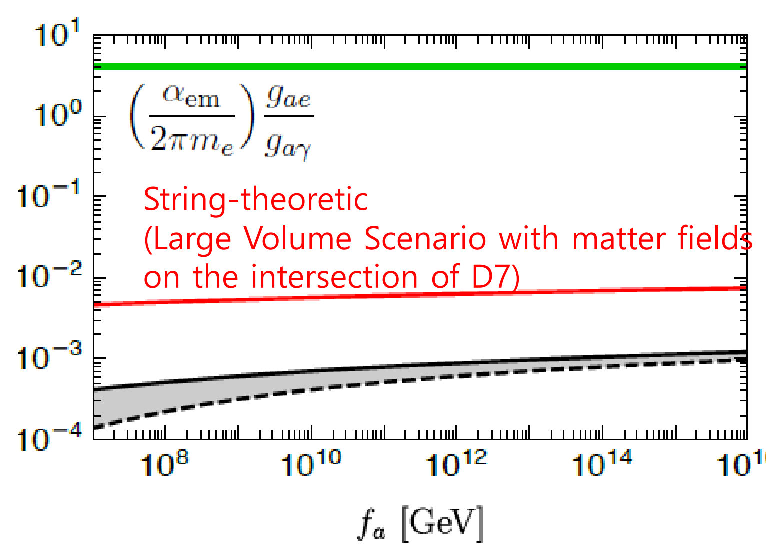
KC, Im, Seong, Kim, arXiv:2106.05816

QCD axion $(c_G \neq 0)$

* All three-types of QCD axions have a similar value of $g_{ap}/g_{a\gamma}$ because of the contribution from the axion-meson mixing induced by $\mathbf{c_G}$.

* As the axion-meson mixing contribution to g_{an} & g_{ae} are small, these three-type of QCD axions have distinguishable pattern of $g_{an}/g_{a\gamma}$ and $g_{ae}/g_{a\gamma}$.



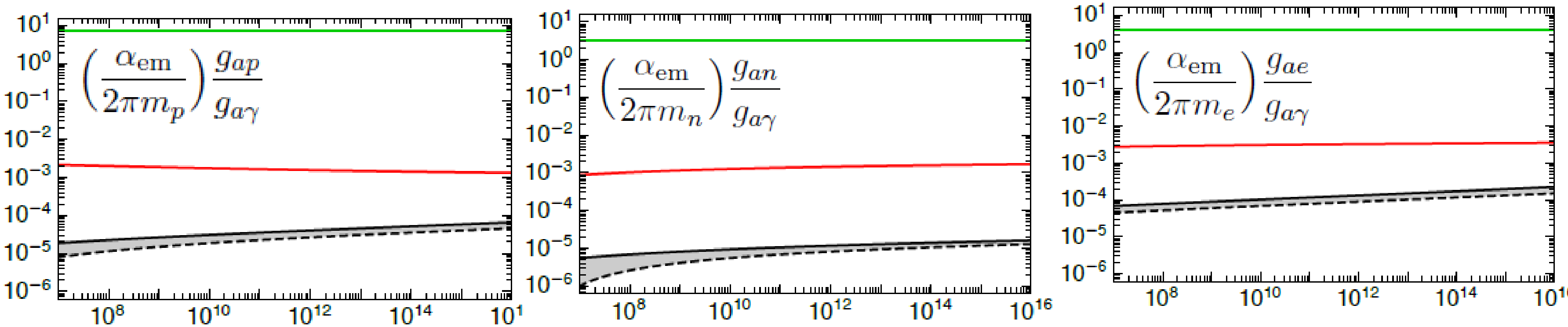


$$\left(\frac{g_{an}}{g_{a\gamma}}\right)_{\rm DFSZ} \gg \left(\frac{g_{an}}{g_{a\gamma}}\right)_{\rm string} \simeq \left(\frac{g_{an}}{g_{a\gamma}}\right)_{\rm KSVZ} \qquad \left(\frac{g_{ae}}{g_{a\gamma}}\right)_{\rm DFSZ} \gg \left(\frac{g_{ae}}{g_{a\gamma}}\right)_{\rm string} \gg \left(\frac{g_{ae}}{g_{a\gamma}}\right)_{\rm KSVZ}$$

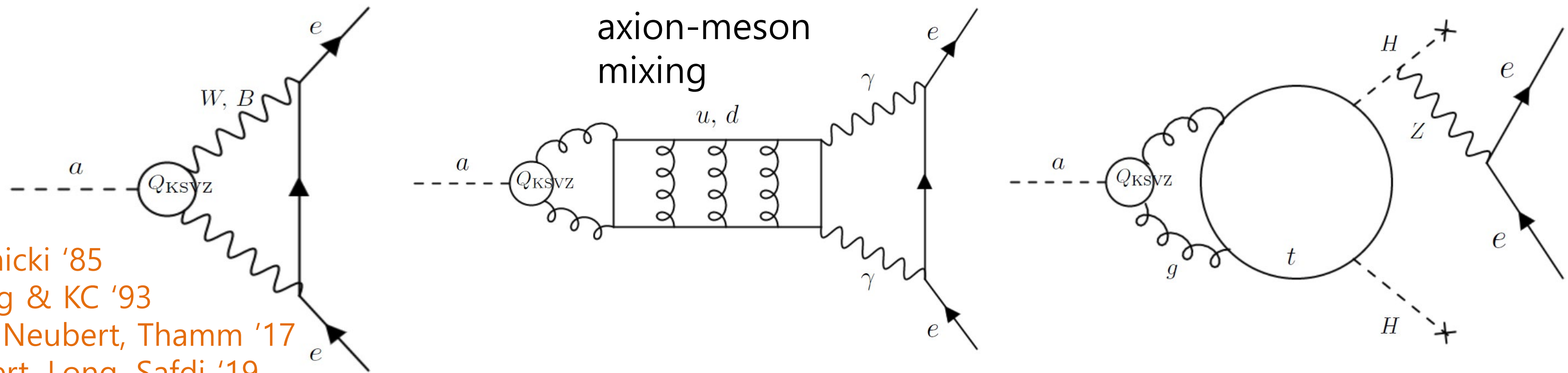
Ultra-light ALP $(c_G = 0,\ c_B \text{ or } c_W = 1)$

Different type of ALPs have clearly distinguishable pattern for all three coupling ratios:

$$\left(\frac{g_{aX}}{g_{a\gamma}}\right)_{\rm DFSZ} \gg \left(\frac{g_{aX}}{g_{a\gamma}}\right)_{\rm string} \gg \left(\frac{g_{aX}}{g_{a\gamma}}\right)_{\rm KSVZ} \qquad \left(X = p, n, e\right)$$



Coupling of KSVZ-type QCD axion to the electron



Srednicki '85
Chang & KC '93
Baur, Neubert, Thamm '17
Dessert, Long, Safdi '19

Two-loop RG-running from the exotic heavy quark mass to m_e

$$\left(g_{ae}\right)_{\rm KSVZ} = \frac{m_e}{f_a}\left[\frac{3\alpha_{\rm em}^2}{4\pi^2}\left(\frac{3}{8}\frac{c_W}{s_W^4}+\frac{5}{8}\frac{c_B}{c_W^4}\right)\ln\frac{M_{Q_{\rm KSVZ}}}{m_e} - \frac{\alpha_{\rm em}^2}{2\pi^2}\left(\frac{4m_d+m_u}{m_u+m_d}\right)c_G \ln\frac{4\pi f_\pi}{m_e}\right.$$

$$\left. + \,\mathcal{O}\left(\frac{y_t^2\alpha_s^2}{8\pi^4}\right)c_G \ln\frac{M_{Q_{\rm KSVZ}}}{m_t}\right]$$

axion-meson-mixing

higher-loop

$$= 10^{-4}\frac{m_e}{f_a}\left[1.3\, c_B + 5.4\, c_W - 0.3\, c_G + 8.3\, c_G\right] \quad \text{for} \quad f_a \gtrsim m_{Q_{\rm KSVZ}} = 10^{10}\,\text{GeV}$$

The higher loop contribution which was not considered before gives the dominant contribution.

Conclusion

- Axions can be categorized into three types by the origin of the axion field variable and the pattern of low energy couplings:

  Field-theoretic: DFSZ-type, KSVZ-type & String-theoretic

  These three type of axions have parametrically different ratios between the couplings to gauge fields and the couplings to matter fields at the UV scale.

- If an axion is discovered, so its mass is identified, it might be possible to measure multiple number of low energy axion couplings and determine the coupling ratios $\mathbf{g_{aX} / g_{a\gamma}}$ **(X=p,n,e)** which can give us information on the underlying axion model.

  DFSZ-type axions have a clearly distinct pattern of low energy couplings, while KSVZ-type and string-theoretic axions have a numerically similar pattern.

  Therefore it requires a precision analysis taking into account the radiative corrections to axion couplings to see if string-theoretic axions can be discriminated from KSVZ-type axions by the measurable $\mathbf{g_{aX} / g_{a\gamma}}$ **(X=p,n,e)**.

- For **QCD axions,** $g_{ae}/g_{a\gamma}$ of KSVZ-type and string-theoretic axions differ by about an order of magnitude, while $g_{aN}/g_{a\gamma}$ are similar to each other.

  For **ultra-light ALPs** without the coupling to gluons, different types of ALPs have clearly distinguishable pattern for all three coupling ratios
  $g_{aX}/g_{a\gamma}$ (X=p,n,e).

- The coupling of KSVZ-type QCD axion to the electron is dominated by the higher-loop contribution involving the top quark and Higgs boson, which was ignored in the previous studies.

Thank you for your attention.