Axion quality and flavour from compositeness / 5D

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Outline

- Axion quality problem
- Solution in composite axion & 5D models
- ‘Warped’ axion model
- 5D DFSZ model & flavour
- Neutrino masses in 5D
- Conclusion

Axion quality problem

- To solve the strong CP problem, non-QCD sources of explicit PQ breaking must be highly suppressed
- *But* global symmetries are expected to be broken by gravity

e.g. consider $\cancel{PQ}$ operators of the form:

$$\mathcal{L}_{\cancel{PQ}} = \frac{c_n}{M_{Pl}^{n-4}} \Phi^n + \text{c.c} \qquad \Phi = F_a e^{ia/F_a}$$

These give contributions to the axion potential that shift its minimum:

$$V(a) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a}{F_a} + \bar{\theta}\right) - c_n F_a^4 \left(\frac{F_a}{M_{Pl}}\right)^{n-4} \cos\left(\frac{a}{F_a} + \delta\right)$$

Axion quality problem

$$V(a) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a}{F_a} + \bar{\theta}\right) - c_n F_a^4 \left(\frac{F_a}{M_{Pl}}\right)^{n-4} \cos\left(\frac{a}{F_a} + \delta\right)$$

For O(1) phase difference, minimum of the potential is roughly

$$\langle \tfrac{a}{F_a} + \bar{\theta} \rangle \sim 10^{67} \times c_n \left(\frac{F_a}{M_{Pl}}\right)^n$$

But to preserve solution to the strong CP problem, we require $\left|\langle \tfrac{a}{F_a} + \bar{\theta} \rangle\right| \lesssim 10^{-10}$

Unless $c_n \lll 1$, any explicit PQ breaking must be restricted to high dimension operators:

e.g. for $F_a \sim 10^9$ GeV require $n \gtrsim 9$

Some possible solutions

Many ideas to produce “high quality” axions that are protected from non-QCD sources of PQ breaking:

- (Gauged) discrete symmetries
- Gauged PQ symmetries
- Heavy axion models
- Composite axions
- Axions in extra dimensions
- …

Composite axion

- Axion could be a pseudo-NGB of a new strongly interacting sector (analogous to pions in QCD)

  [Kim '85, Choi, Kim '85]

- $U(1)_{PQ}$ is a global symmetry of the strong sector that is spontaneously broken upon confinement

- Axion is insensitive to explicit PQ breaking in the UV if all PQ-charged operators have sufficiently large mass dimension

Composite axion: example

Gavela, Ibe, Quilez, Yanagida '18

- Chiral $SU(5)$ confining gauge theory

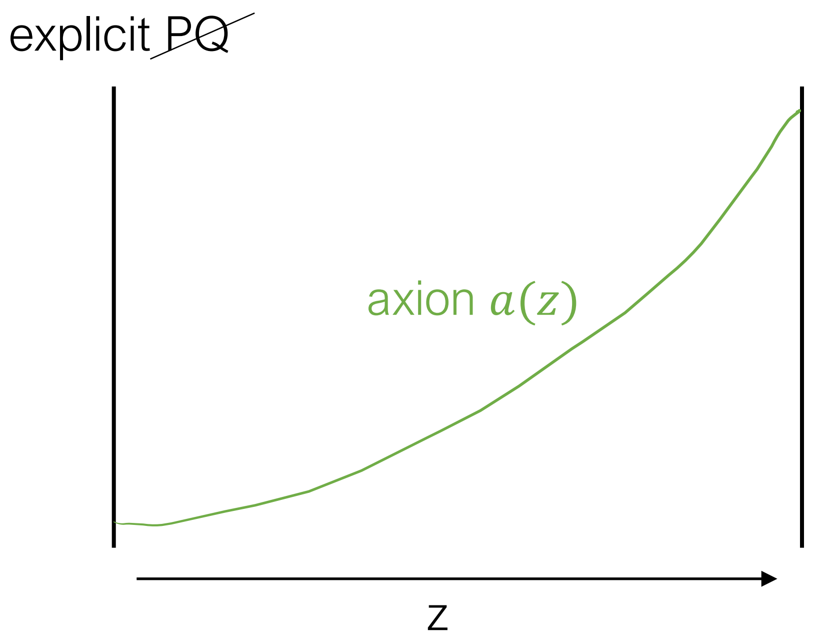
	$SU(5)$	$SU(3)_c$	$U(1)_{PQ}$
$\psi_{\bar{5}}$	$\bar{\mathbf{5}}$	$\mathbf{R}$	-3
ψ_{10}	$\mathbf{10}$	$\mathbf{R}$	1

- Assume that confinement spontaneously breaks $U(1)_{PQ}$ → composite axion
- Lowest dimension $SU(5)$-invariant, PQ breaking operator is dimension-9: $\langle \bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10} \rangle \sim \Lambda_5^9$

$$\mathcal{L}_{\cancel{PQ}} = c\,\frac{1}{4\pi}\frac{1}{M_{\text{Pl}}^5}\frac{1}{2!\,4!}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10} \longrightarrow \mathcal{L}_{\cancel{PQ}} = c\,\frac{(4\pi)^2}{2!\,4!}\left(\frac{N}{5}\right)^9 \underline{\frac{f_a^9}{M_{\text{Pl}}^5}}\, e^{-i\frac{10}{N}a/f_a} + \text{h.c.}$$

Axion quality in 5D

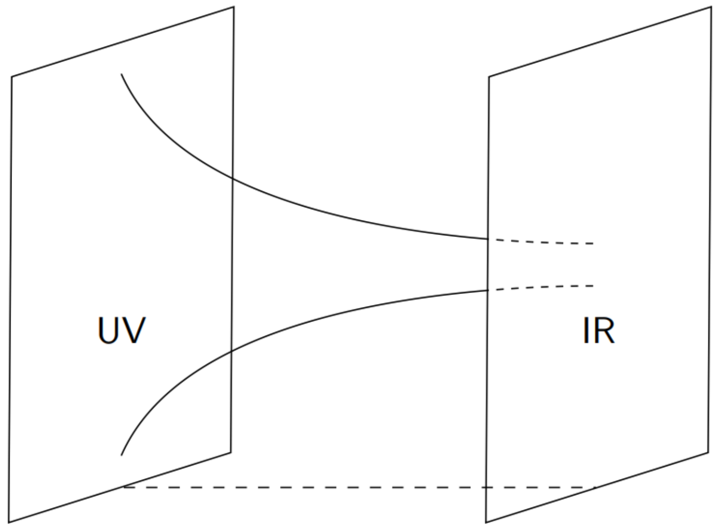
- In an extra dimension, axion can be sequestered from explicit sources of PQ breaking
- Identify axion with 5th component of bulk gauge field $V_M = (V_\mu, \underline{V_z})$ [Choi '03]
- Axion protected by combination of 5D gauge symmetry and locality



Warped extra dimensions

Randall, Sundrum '99

- Compact extra dimension with 4D branes located at the endpoints (S_1/Z_2 orbifold)
- Fundamental mass scales ~M_{pl}
- 'Warped' metric can naturally generate hierarchically smaller effective 4D PQ scale



$z_{UV} \sim (M_{Pl})^{-1}$

$$A(z) = \frac{1}{kz_{UV}} \sim 1$$

$z_{IR} \sim (\Lambda_{PQ})^{-1}$

$$A(z) = \frac{1}{kz_{IR}} \ll 1$$

$$ds^2 = \underline{A(z)^2} \left(dx^2 + dz^2\right)$$

'warp factor'

$$A(z) = \frac{1}{kz} \quad (AdS_5)$$

AdS/CFT

Maldacena '98; Witten '98; Klebavnov, Witten '99
Arkani-Hamed .et .al. '00; Rattazzi, Zaffaroni '01
Perez-Victoria '01; Contino, Pomarol '04

- 5D AdS model is dual to a 4D conformal field theory (AdS/CFT correspondence)
- IR brane corresponds to spontaneous breaking of conformal symmetry in the CFT (mass gapped theory)

Axion in 5D warped spacetime has a dual 4D composite axion model

AdS/CFT dictionary

Slice of AdS_5	4D CFT + elementary sector
Bulk field, $\phi(x^\mu, y)$	Operator, $\mathcal{O}$
5D bulk mass	Operator dimension, $\Delta_{\mathcal{O}}$
UV localised zero mode	Elementary state
IR localised zero mode	Massless CFT bound state
KK modes $(m_n \neq 0)$	CFT bound states
Gauge symmetry $\mathcal{G}$,	Global symmetry $\mathcal{G}$,

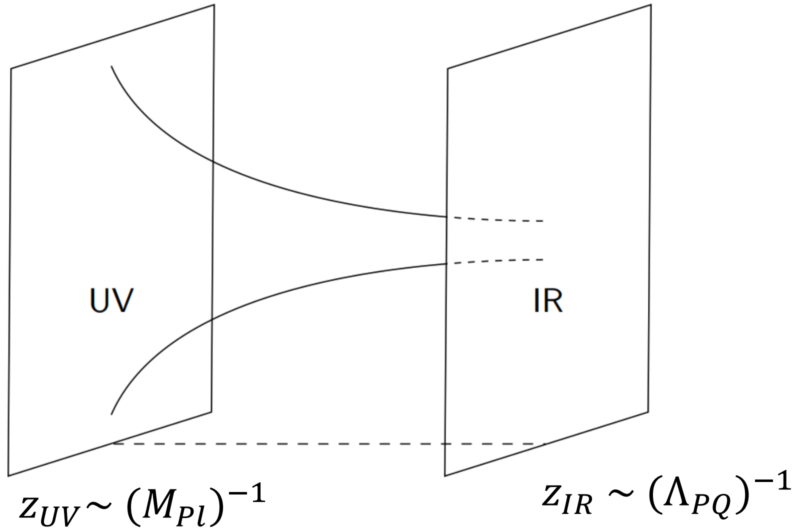
Warped axion model

5D $U(1)_{PQ}$ gauge theory in a slice of AdS_5

$$S = 2\int_{z_{UV}}^{z_{IR}} d^5x\,\sqrt{-g}\left(-\frac{1}{4g_5^2}F^{MN}F_{MN} - \frac{1}{2}(D^M\Phi^\dagger)(D_M\Phi) - \frac{1}{2}m_\Phi^2\Phi^\dagger\Phi + \text{G.F.}\right)$$
$$-\int d^4x\,\sqrt{-g_4}\,U(\Phi)$$

PQ charge +1

Boundary potentials



$$ds^2 = \frac{1}{(kz)^2}\left(dx^2 + dz^2\right)$$

Scalar condensate

- Scalar obtains z-dependent VEV $\Phi = \eta(z) e^{ia(x,z)}$
- Spontaneously breaks $U(1)_{PQ}$ gauge symmetry

operator dimension

$$\Delta = 2 + \sqrt{4 + m_\Phi^2/k^2}$$

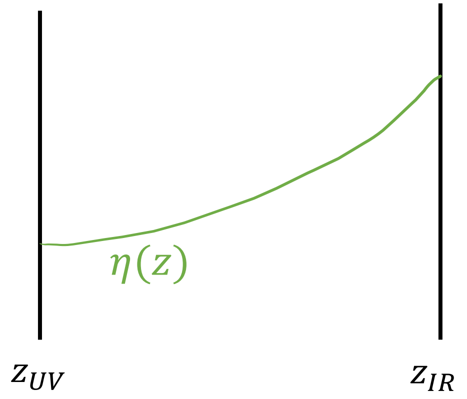
$$\eta(z) = k^{3/2}\left(\lambda\,(kz)^{4-\Delta} + \sigma\,(kz)^{\Delta}\right)$$

source term for operator $\mathcal{O}$

$$\lambda = \frac{\ell_{UV}}{\Delta - 4 + b_{UV}}$$

scalar condensate $\langle \mathcal{O} \rangle$

$$\sigma = \sigma_0\,(kz_{IR})^{-\Delta} \sim (\Lambda_{PQ})^{\Delta}$$



Pseudo-scalar spectrum

- Two coupled 4D pseudoscalar degrees of freedom

$$\Phi = \eta(z) e^{\underline{ia(x^\mu, z)}} \qquad\qquad V_M = (V_\mu, \underline{V_z})$$

- Compactification leads to a tower of Kaluza-Klein modes

$$a(x^\mu, z) = \sum_{n=0}^{\infty} f_a^{(n)}(z) a^{(n)}(x^\mu) \qquad\qquad V_z(x^\mu, z) = \sum_{n=0}^{\infty} f_{V_z}^{(n)}(z) a^{(n)}(x^\mu)$$

z-dependent profile

4D scalar mode

$$\Box a^{(n)} = m_n^2 a^{(n)}$$

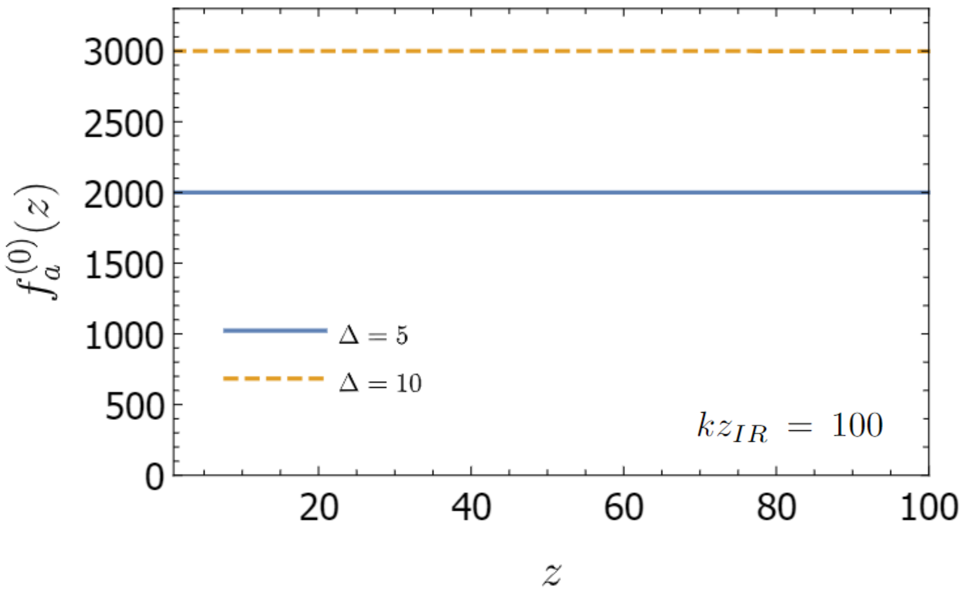
- a, V_z expanded in same set of 4D modes

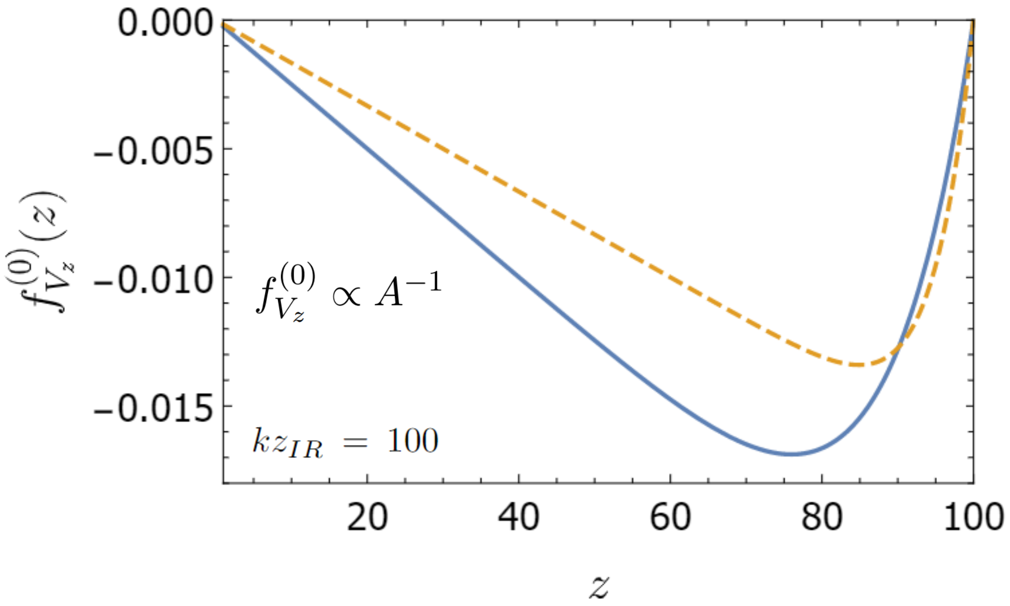
Massless axion ($\ell_{UV}=0$)

- In absence of explicit PQ breaking, spectrum contains a single massless zero mode axion, $a^{(0)}(x^\mu)$

$$f_a^{(0)}(z) \simeq \frac{z_{IR}}{\sigma_0}\sqrt{\Delta - 1} + \mathcal{O}\left(\sigma_0^2\right)$$

$$f_{V_z}^{(0)}(z) = \partial_z f_a^{(0)}$$





4D PQ symmetry

- The 5D $U(1)_{PQ}$ gauge symmetry acts as

$$a \to a + \alpha(x^\mu, z) \qquad V_z \to V_z + \partial_z \alpha(x^\mu, z) \qquad \alpha = \text{gauge parameter}$$

- Consider the specific gauge transformation

$$\alpha(x^\mu, z) = \alpha_0 \, \underbrace{f_a^{(0)}}_{\text{axion profile}}(z)$$

- This acts on 4D axion mode as a shift symmetry! (recall $f_{V_z}^{(0)}(z) = \partial_z f_a^{(0)}$)

$$a^{(0)}(x^\mu) \to a^{(0)}(x^\mu) + \alpha_0$$

Explicit PQ breaking

How to introduce PQ breaking when $U(1)_{PQ}$ is a *gauge* symmetry? → boundary conditions

- Boundary conditions for vector modes: $V_\mu\Big|_{z_{UV}} = 0\,, \qquad \partial_z V_\mu\Big|_{z_{IR}} = 0$
- Restricts gauge parameter at the boundaries $\boxed{\partial_\mu \alpha\Big|_{z_{UV}} = 0} \qquad \partial_z \alpha\Big|_{z_{IR}} = 0$

$U(1)_{PQ}$ *gauge* symmetry is effectively reduced to a *global* symmetry on the UV boundary!

All sources of explicit PQ breaking MUST reside on the UV boundary

UV PQ breaking

- Introduce the global PQ breaking UV boundary term

$$
\begin{aligned}
U_{UV}(\Phi) &\supset (-\ell_{UV} k^{5/2}\Phi + \mathrm{h.c.}) \\
&= -2\ell_{UV} k^{5/2} \eta \cos(a)
\end{aligned}
$$

- Models UV sources of PQ breaking that could lead to an axion quality problem

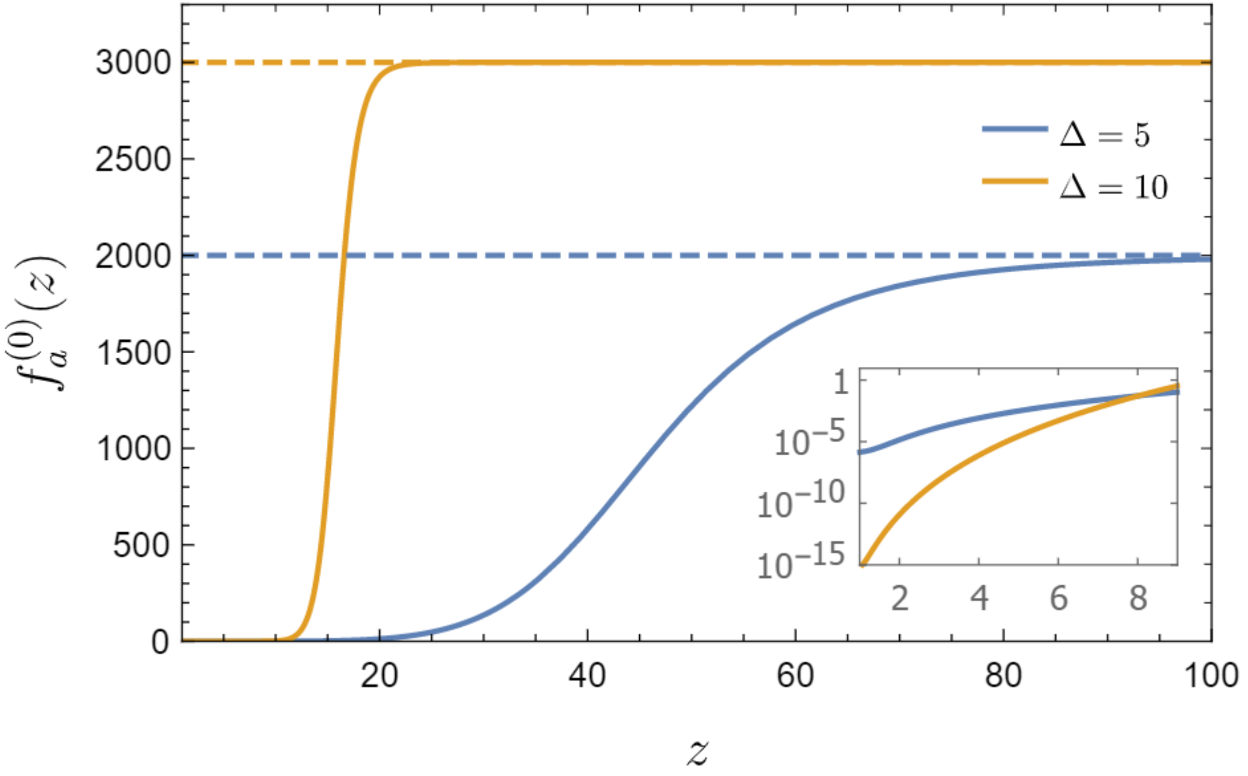
  (Could also add higher order Φ^n terms, but these give sub-leading effects)

Still have an exact 5D gauge symmetry that satisfies $\alpha(z_{UV}) = 0$

Axion solution with UV breaking

- For large Δ axion profile is suppressed near the UV brane

  → Protects axion potential from UV localised sources of explicit breaking



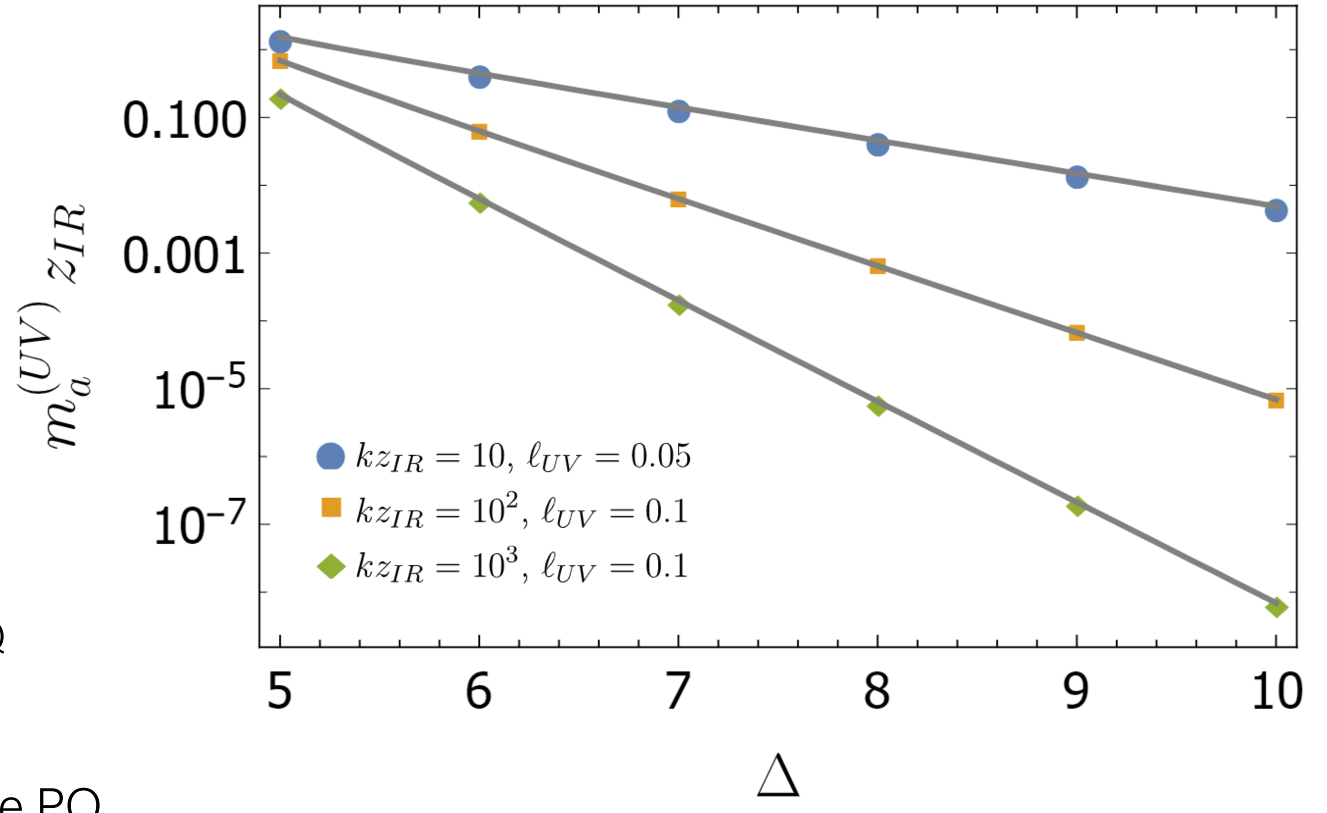
Axion mass from UV

Contribution to axion mass from explicit PQ breaking on UV brane $U_{UV}(\Phi) \supset -2\ell_{UV} k^{5/2} \eta \cos(a)$

$$(m_a^{(UV)})^2 \simeq \frac{4\ell_{UV}}{\sigma_0} \frac{(\Delta-1)(\Delta-2)}{\Delta - 4 + b_{UV}} \left(\frac{z_{IR}}{z_{UV}}\right)^{4-\Delta} z_{IR}^{-2}$$

$$\sim l_{UV} \left(\frac{\Lambda_{PQ}}{M_{Pl}}\right)^{\Delta-4} \Lambda_{PQ}^2$$

suppressed relative to PQ scale for large Δ

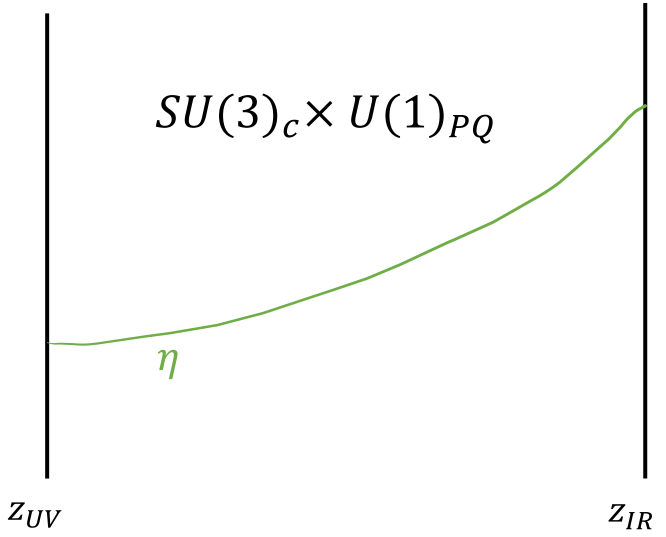


Scaling consistent with a 4D composite axion model where PQ is broken by an operator of dimension Δ

What about PQ breaking from QCD?

If QCD is localised on the UV brane its contribution to the axion potential is also suppressed

Solution: Allow QCD to propagate in the bulk



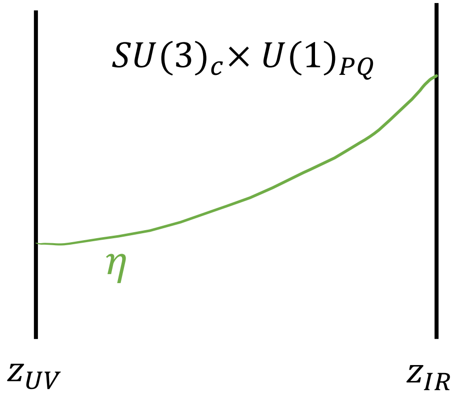
4D dual is a composite axion model with constituents charged under QCD

What about PQ breaking from QCD?

Origin of PQ breaking is a 5D Chern-Simons term* ($U(1)_{PQ}$ invariant up to a total derivative)

$$S_{CS} = -\frac{\kappa}{128\pi^2}\int_{z_{UV}}^{z_{IR}} d^5x\, \epsilon^{MNPQR} V_M G^c_{NP} G^c_{QR} + \underbrace{\frac{\kappa}{64\pi^2}\int d^4x\, a\, G^c \widetilde{G}^c \Big|_{z_{IR}}}_{}$$

cancels gauge anomaly
(IR only!)



Integrating out extra dimension, effective 4D action is

$$S_{eff} = \int d^4x \left(\frac{1}{2} a^{(0)} \left(\Box - m_a^2\right) a^{(0)} + \frac{g_s^2}{32\pi^2 F_a} a^{(0)} G\tilde{G} \right)$$

$$\boxed{F_a \simeq \frac{1}{\kappa}\frac{\sigma_0}{\sqrt{\Delta - 1}} z_{IR}^{-1} \sim \Lambda_{PQ}}$$

unsuppressed!

*such a term can also be obtained by integrating out bulk fermions

Solution to quality problem

- Combing the QCD and UV contributions to the axion potential gives

$$V(a^{(0)}) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \bar{\theta}\right) - (m_a^{(UV)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \delta\right)$$

$$(m_a^{(QCD)})^2 \simeq \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 F_\pi^2}{F_a^2}$$

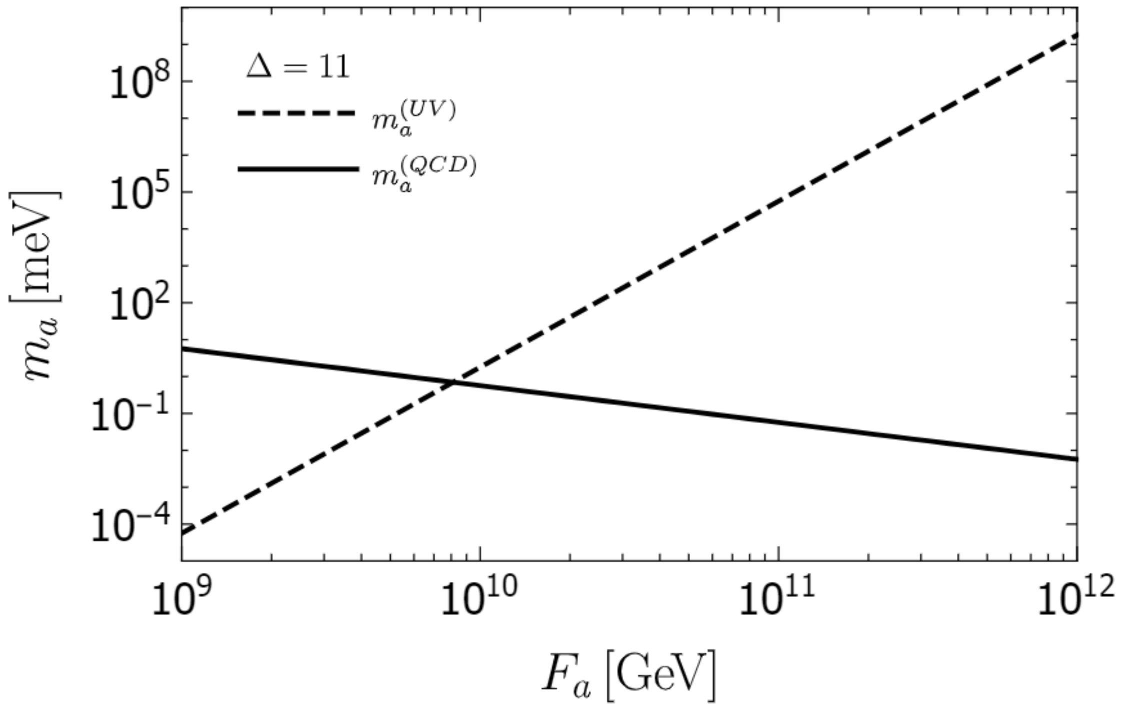
$$(m_a^{(UV)})^2 \sim \left(\frac{F_a}{M_{Pl}}\right)^{\Delta-4} F_a^2$$

- Requiring the effective theta angle $|\bar{\theta}_{eff}| \lesssim 10^{-10}$ leads to

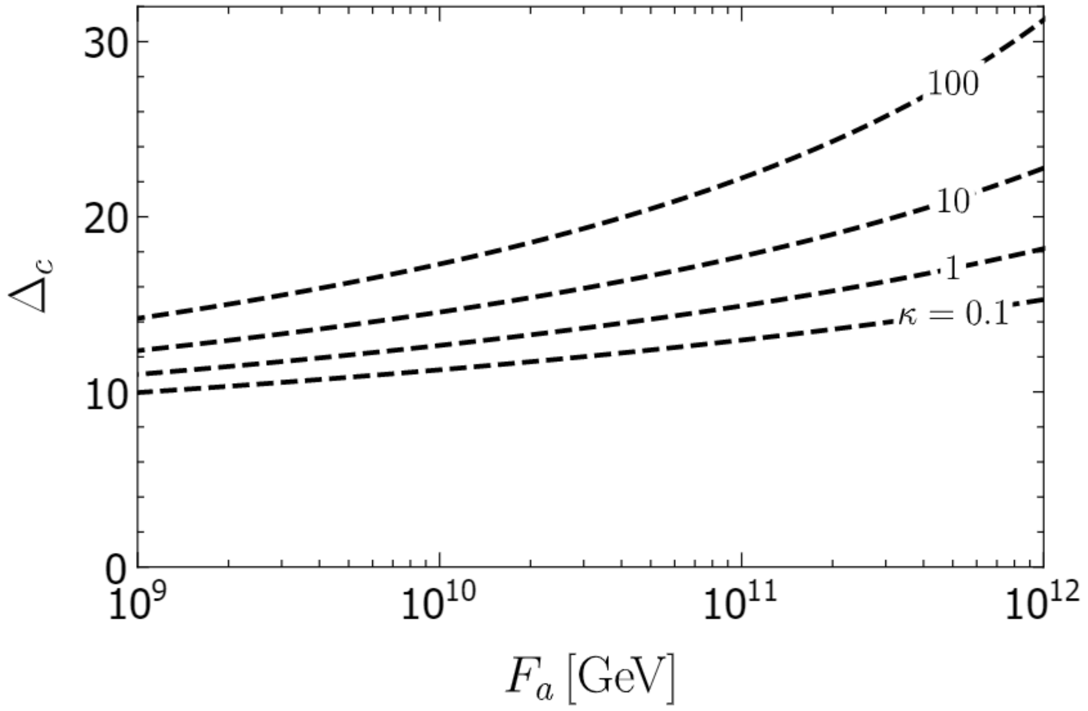
$$m_a^{(UV)} \lesssim 10^{-5}\, m_a^{(QCD)}$$

Solution to quality problem

$$(m_a^{(UV)})^2 \sim \left(\frac{F_a}{M_{Pl}}\right)^{\Delta-4} F_a^2$$

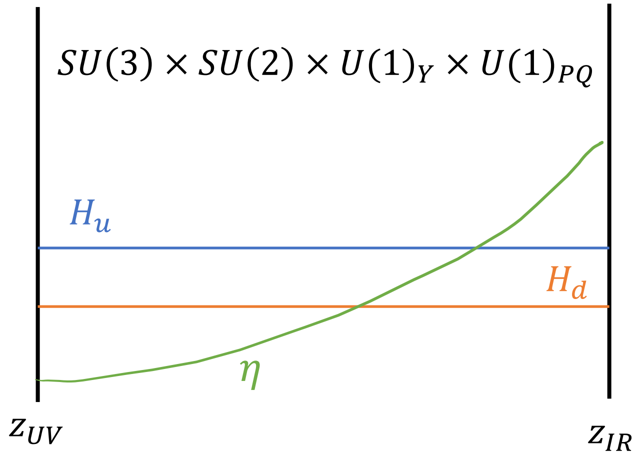


To solve quality problem $\Delta > \Delta_c$



5D DFSZ model

- Extend model with SM fermions, gauge bosons promoted to bulk 5D fields
- Two Higgs doublets $H_{u,d} \sim (\mathbf{2}, \mp\frac{1}{2})$ with PQ charges $X_{H_{u,d}}$
- $SU(2)$, $U(1)_{PQ}$ spontaneously broken by 5D (z-dependent) Higgs VEVs



$$H_u = \frac{v_u}{\sqrt{2}} e^{i\frac{a_u(x,z)}{v_u}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad H_d = \frac{v_d}{\sqrt{2}} e^{i\frac{a_d(x,z)}{v_d}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \Phi = \eta(z) e^{ia(x,z)}$$

- Effective 4D electroweak VEV $v^2 \approx (v_u^2 + v_d^2)/k$

Fermions in 5D

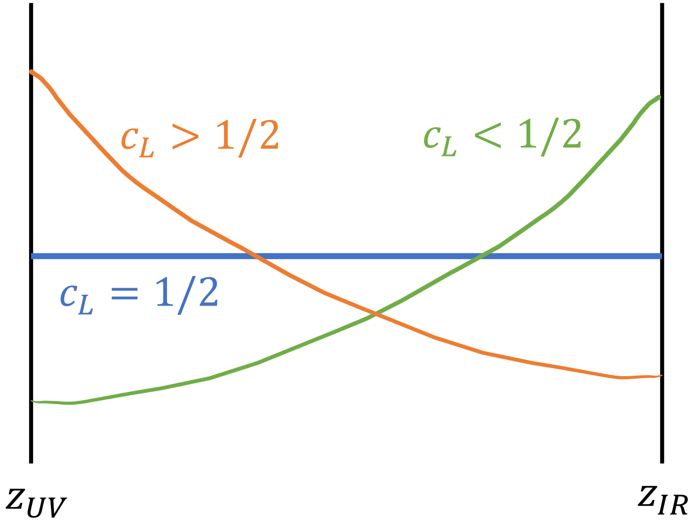
Gherghetta, Pomarol '03
Contino, Pomarol '04

$$S_\Psi = -\int d^5x\,\sqrt{-g}\left(\frac{i}{2}\left(\bar\Psi\Gamma^M D_M\Psi - D_M\bar\Psi\Gamma^M\Psi\right) + \underline{m_\Psi\bar\Psi\Psi}\right)$$

- Massless chiral zero-mode (SM fermion) + vector-like KK modes

- 5D mass determines the localisation of the zero mode

$$\underbrace{}_{}m_\Psi = \underset{O(1)}{c_L}\,\underset{\sim M_{Pl}}{k}$$



$$\tilde f_L^0 \sim (kz)^{\frac{1}{2}-c_L}$$

$$\tilde f_R^0 \sim (kz)^{\frac{1}{2}+c_R}$$

- In dual 4D theory, UV (IR) localised modes are elementary (composite)

Flavour hierarchies

5D DSFZ model is also a model of flavour

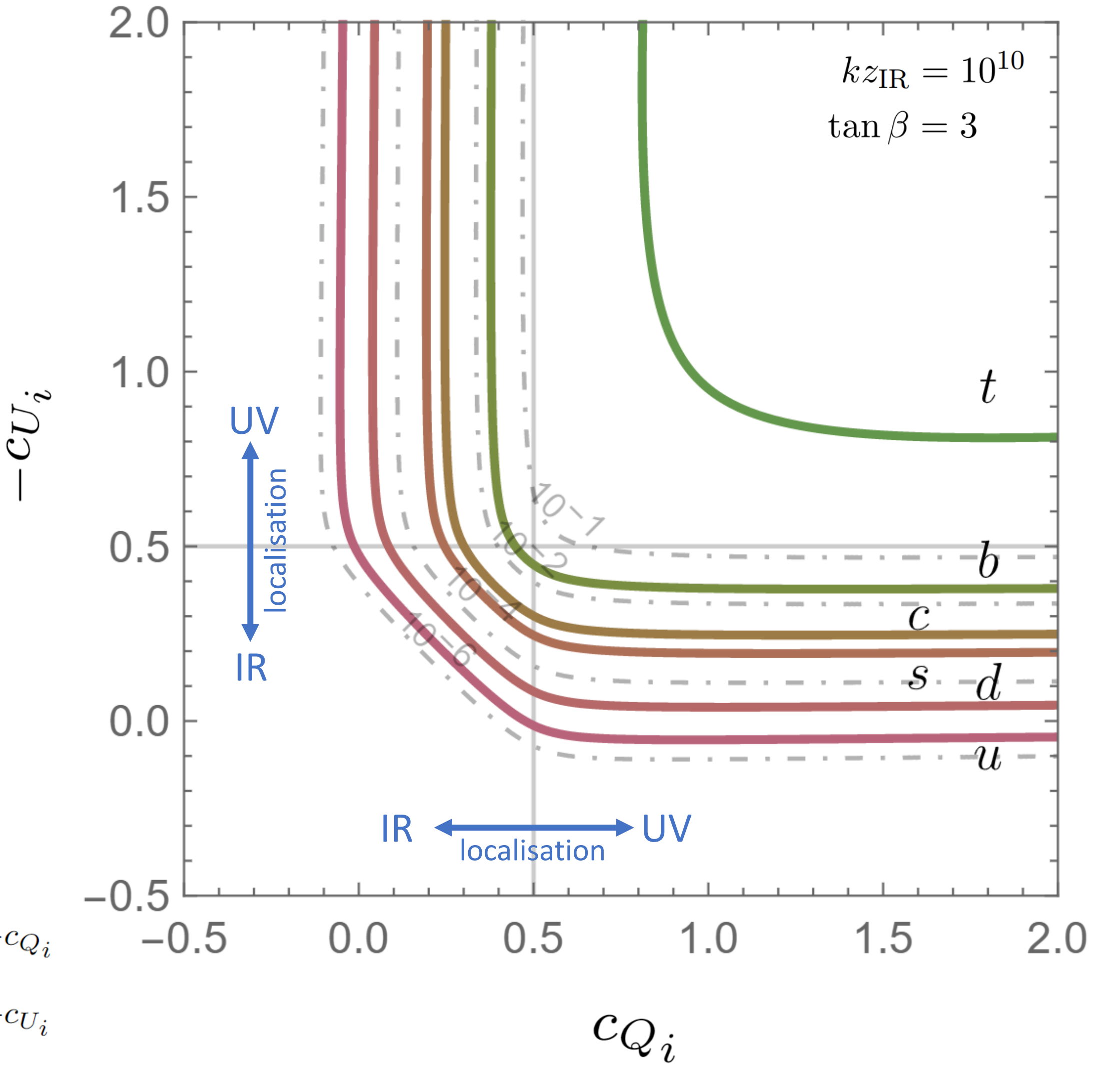
- Assume $\mathcal{O}(1)$ 5D Yukawa couplings

$$y^{(5)}_{u,ij}\bar{Q}_i U_j H_u$$

- Hierarchies in masses & CKM/PMNS angles due to profile overlaps

$$m_u^{ij} = y^{(5)}_{u,ij}\frac{\sqrt{2}v_u}{\sqrt{k}}\int_{z_{UV}}^{z_{IR}}\frac{dz}{(kz)^5}\; f^0_{Q_{iL}}(z) f^0_{U_{jR}}(z)$$

$$\tilde{f}^0_{Q_{iL}} \sim (kz)^{\frac{1}{2}-c_{Q_i}}$$
$$\tilde{f}^0_{U_{iR}} \sim (kz)^{\frac{1}{2}+c_{U_i}}$$



Scalar solutions

- Now have 4 coupled pseudo-scalar fields H_u, H_d, V_z, a

$$H_u = \frac{v_u}{\sqrt{2}} e^{i\frac{a_u(x,z)}{v_u}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad H_d = \frac{v_d}{\sqrt{2}} e^{i\frac{a_d(x,z)}{v_d}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \Phi = \eta(z) e^{ia(x,z)} \qquad V_M = (V_\mu, V_z)$$

- a_u, a_d Kaluza-Klein spectrum also contain the axion mode

$$\frac{a_{u,d}}{v_{u,d}} = X_{H_{u,d}} \underline{f_a^0(z)\, a^0(x^\mu)} + \text{ massive pseudoscalar modes}$$

- Leads to axion-fermion couplings from 5D Yukawa terms

$$S_{\text{Yukawa}} = -2 \int_{z_{UV}}^{z_{IR}} d^5x \sqrt{-g} \frac{1}{\sqrt{k}} \left(y^{(5)}_{u,ij} \bar{Q}_i U_j H_u + y^{(5)}_{d,ij} \bar{Q}_i D_j H_d + y^{(5)}_{e,ij} \bar{L}_i E_j H_d + \text{h.c.} \right)$$

Flavour-violating couplings

- After integrating over z, effective 4D couplings are $S_{4D} \supset i \int d^4x \, \frac{\partial_\mu a^0}{2F_a} \left(\bar{u}_i \gamma^\mu \left((c_u^V)_{ij} - (c_u^A)_{ij} \gamma^5 \right) u_j \right)$

$$\frac{(c_u^{V,A})_{ij}}{F_a} = X_{H_u} \int_{z_{UV}}^{z_{IR}} \frac{dz}{(kz)^4} \, \hat{f}^0_{a_X} \left((A_R^u)_{ik} \underline{(f^0_{U_{kR}})^2} (A_R^{u\dagger})_{kj} \mp (A_L^u)_{ik} \underline{(f^0_{Q_{kL}})^2} (A_L^{u\dagger})_{kj} \right)$$

5D profiles

mixing matrices

- Different 5D fermion profiles for each generation → off-diagonal axion couplings

- Similar to 4D models with generation-dependent PQ charges:

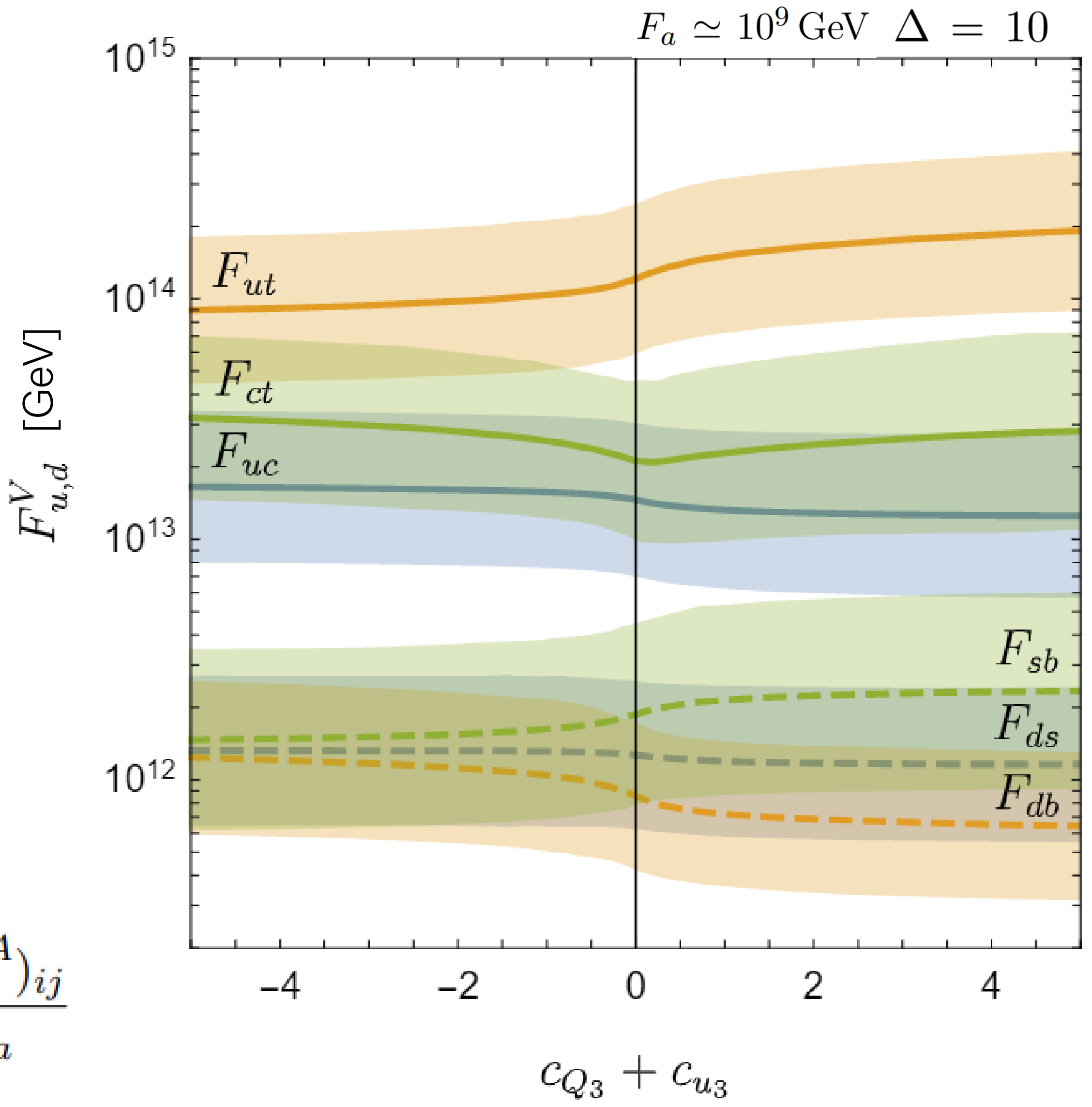
$$-i\frac{\partial_\mu a}{f} \left(\bar{u}_L (A_L^u)^\dagger \underline{X_Q} A_L^u \gamma^\mu u_L + \bar{u}_R (A_R^u)^\dagger \underline{X_u} A_R^u \gamma^\mu u_R + \cdots \right)$$

$$(X_Q)_{ij} = X_{Q_i} \delta_{ij}$$

Axion-fermion couplings

- Assume anarchic $\mathcal{O}(1)$ 5D Yukawa couplings
- Fit the 5D mass parameters, $c_{Q_i}, c_{U_i}, c_{D_i}$ to the quark masses and CKM matrix
- Flavour diagonal couplings are generally larger by a few orders of magnitude
- Similar results for the charged lepton sector

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$



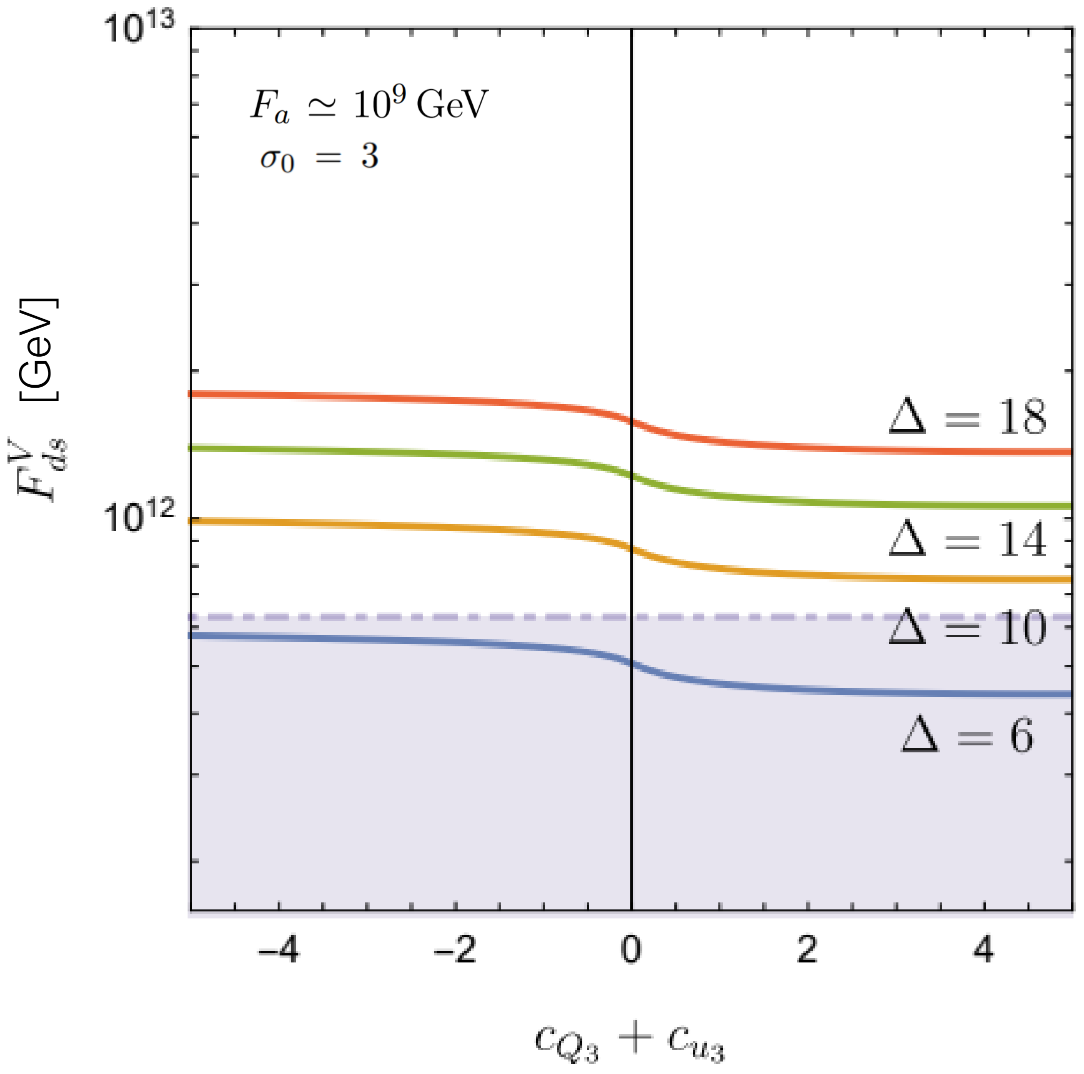
Axion-fermion couplings

- Flavour-violating couplings can be tested in rare meson decays
- Strongest constraint is from $K^+ \to \pi^+ a$

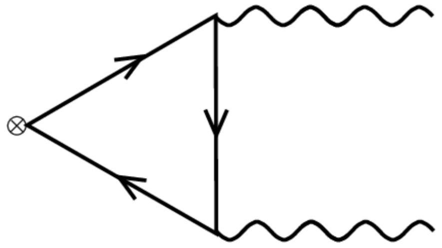
$$(F_d^V)_{12} \gtrsim 6.8 \times 10^{11}\,\mathrm{GeV}$$

[E949 experiment]

- Improved bound from KOTO, NA62



Axion-gauge boson couplings



- Triangle diagrams with bulk fermions produce a $U(1)_{PQ}$ anomaly and contribute to the axion-gluon and axion-photon couplings

- Anomalies (associated with the presence of chiral zero modes) are localised on the boundaries:

[Arkani-Hamed, Cohen, Georgi '01]

$$\eta^{MN}\partial_M J_N^a = \frac{1}{64\pi^2} A(R) d^{abc}\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^b F_{\rho\sigma}^c \underline{\left(\delta(z-z_{UV})+\delta(z-z_{IR})\right)}$$

- Anomaly coefficients are as in 4D

e.g. $U(1)_{PQ}U(1)_{EM}{}^2$: $A_{EM} = 3\left(-\frac{5}{3}X_Q + \frac{4}{3}X_U + \frac{1}{3}X_D - X_L + X_E\right) = -4\left(X_{H_u}+X_{H_d}\right)$

X_i = PQ charge

Axion-gauge boson couplings

- Integrating over extra dimension leads to the effective action

$$S_{4D} \supset \int d^4x\, a^0 \underbrace{f_a^0(z_{IR})}_{\sim (\Lambda_{PQ})^{-1}} \left(\frac{e^2}{32\pi^2} A_{EM} F^0 \widetilde{F}^0 + \frac{g_s^2}{32\pi^2} A_{QCD} G^{0c} \widetilde{G}^{0c} \right)$$

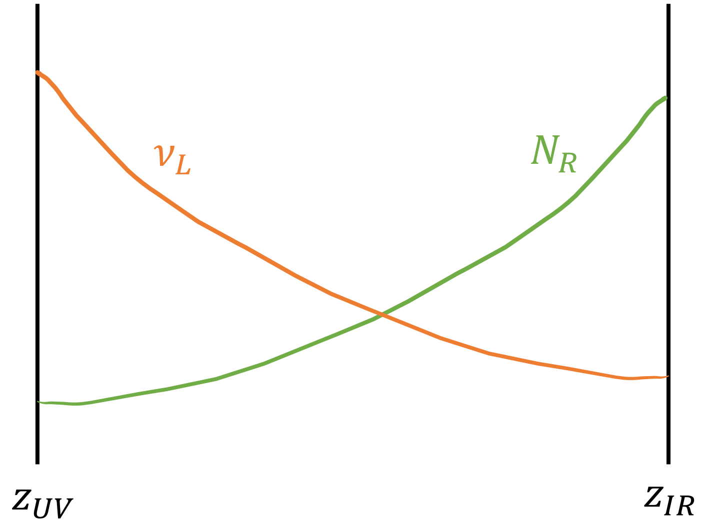
This contribution gives $\dfrac{E}{N} = \dfrac{A_{EM}}{A_{QCD}} = \dfrac{8}{3}$ $\longrightarrow$ same as 4D DFSZ model

- But can also be contribution from 5D Chern-Simons terms $\cancel{\dfrac{E}{N} = \dfrac{A_{EM}}{A_{QCD}} = \dfrac{8}{3}}$

Neutrino mass

- Can generate neutrino masses by adding bulk singlet fermions, N_i (X_N =2)
- Majorana mass terms break $U(1)_{PQ}$ and are localised on the UV boundary

$$\frac{1}{2}\left(b_{N,ij}\overline{N_i^c}N_j + \frac{y_{N,ij}^{(5)}}{k^{3/2}}\Phi\overline{N_i^c}N_j + \text{h.c.}\right)\delta(z - z_{UV})$$



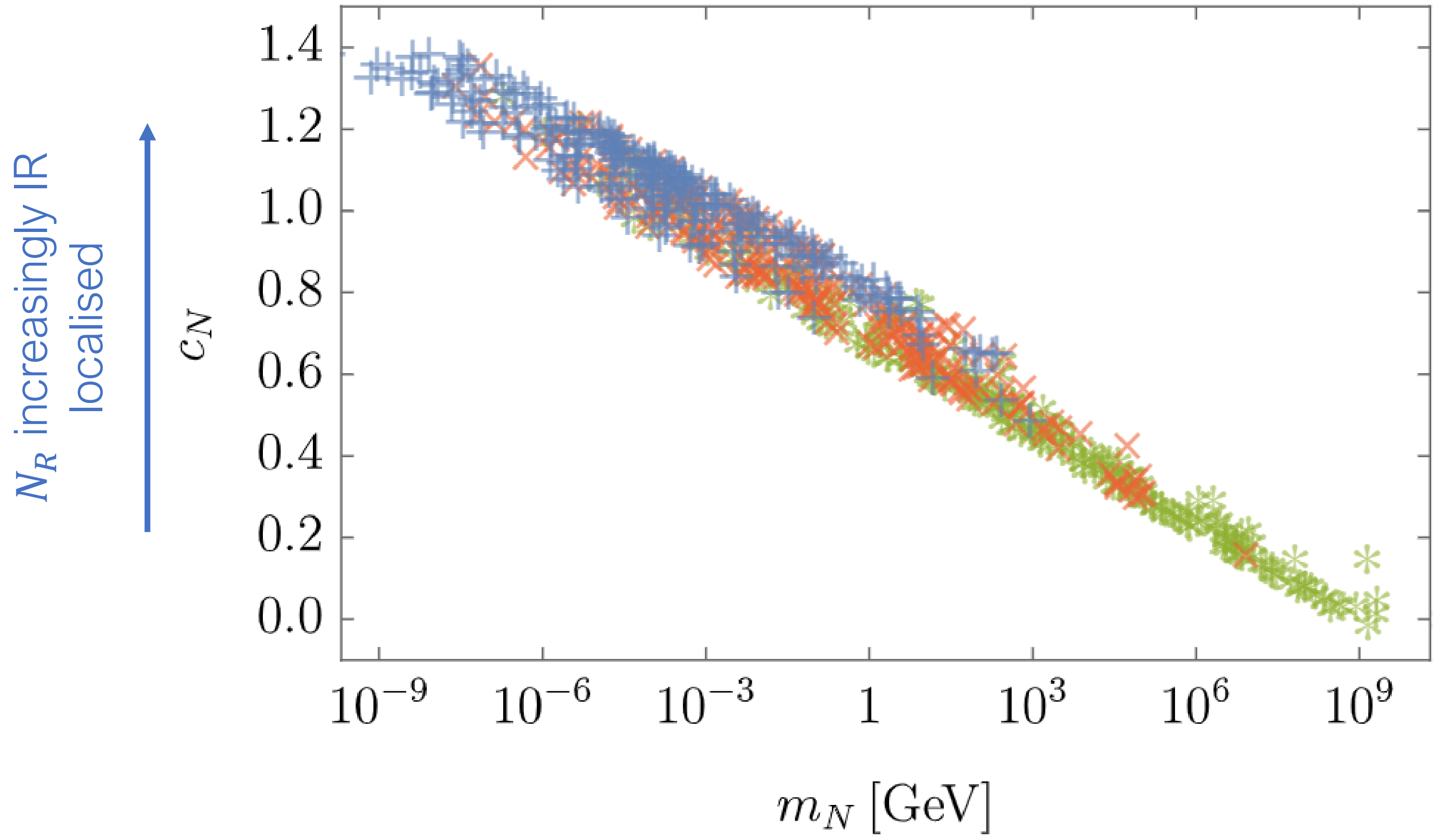
- If N_i are IR localised ($C_N > 1/2$), have hierarchically small effective 4D Majorana mass

$$M_M \sim \underline{\left(\frac{\Lambda_{PQ}}{M_{Pl}}\right)}^{1-2c_N} M_{Pl}$$

- Can obtain naturally light active neutrino masses by localising υ_L in the UV

Naturally light sterile neutrinos

- Naturally light sterile neutrinos, while producing active neutrino masses and PMNS angles with $O(1)$ 5D Yukawas



Summary

- Axion solution to the strong CP problem requires $U(1)_{PQ}$ to be an excellent global symmetry
  → axion quality problem

- Composite axions can be protected from explicit PQ breaking in UV

- In dual 5D warped model axion quality is ensured via locality → axion is localised towards the IR

- 5D DFSZ model also provides explanation of flavour hierarchies

- Can be probed via flavour-violating axion couplings