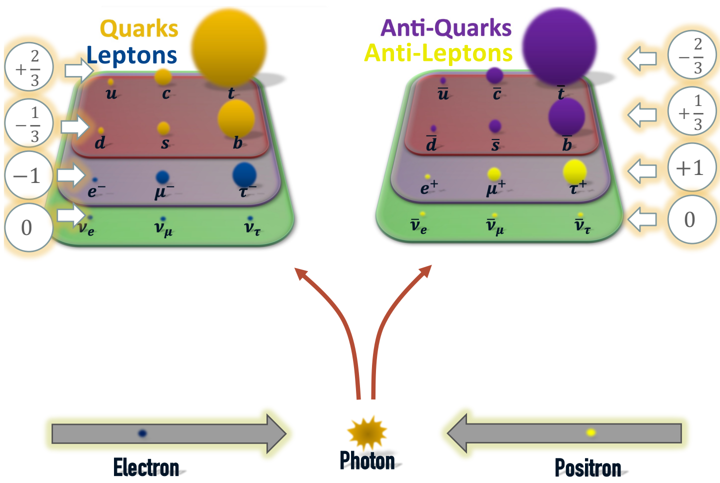
What happens in e^+e^- collisions?
Quark colors and the R value

Belle II Masterclass

In this masterclass we want to investigate how many different quark color charges exist. For this purpose, it is worth to take some time to understand what exactly happens during an e^+e^- collision. Especially important for us is the type of particles that can be created and how often this happens.
If a particle and antiparticle collide (like in this example an electron e^- and a positron e^+), they „annihilate“ to a state of pure energy: a photon. Subsequently, this photon can use the energy to create an arbitrary particle P as well as an antiparticle $\bar{P}$ as long as the energy is high enough.



Frequency of occurrence of different particles

The photon is the exchange particle of the electromagnetic force. Accordingly, it couples to every particle proportional to their electric charge Q_P, or even more accurately to its square:

$$\gamma \to P\bar{P} \quad \text{is proportional to} \quad (Q_P)^2$$

Hence, we can calculate the frequency of occurrence N of e.g. a b quark / $\bar{b}$ antiquark pair:

$$N\left(e^+e^- \to \gamma \to b\bar{b}\right) = \left(-\frac{1}{3}\right)^2 \cdot XY = \frac{1}{9} \cdot XY$$

XY is a constant factor, which is equal for all particles and can be ignored at the moment.

Exercise a)

According the calculation of above's example, try to answer the following: What is the frequency of occurrence for the following particle-antiparticle pairs:

$$N\left(e^+e^- \to \gamma \to u\bar{u}\right) = \qquad \cdot XY$$

$$N\left(e^+e^- \to \gamma \to s\bar{s}\right) = \qquad \cdot XY$$

$$N\left(e^+e^- \to \gamma \to \tau^+\tau^-\right) = \qquad \cdot XY$$

Exercise b)

Maybe you already spotted some regularities. Let's turn this around: For which particle-antiparticle pairs is the frequency of occurrence given by the following?

$$N\left(e^+e^- \to \gamma \to P\bar{P}\right) = \left(\frac{2}{3}\right)^2 \cdot XY = \frac{4}{9} \cdot XY \qquad P = \qquad , \qquad ,$$

Exercise c)

To be able to create a particle-antiparticle pair, the energy needs to be sufficiently high. Let's assume that we build a particle accelerator that produces enough energy in the collision point to (only) produce u-, d- and s-quarks (+ anti quarks). What is the total frequency of occurrence of the overall quark production in this case? **Hint:** You might want to calculate this in a couple of steps on a note pad.

$$N\left(e^+e^- \to \gamma \to u\bar{u}/d\bar{d}/s\bar{s}\right) = \qquad \cdot XY$$

How does this change when we increase the collision energy sufficiently such that the production of c-quarks becomes possible as well?

$$N\left(e^+e^- \to \gamma \to u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}\right) = \qquad \cdot XY$$

Putting the particles into perspective

As the factor XY is constant for all particles in good approximation, it fortunately cancels when the ratio of occurrences of different particles is calculated.
Hence, simply counting how often a process occurs (like e.g. $e^+e^- \to \gamma \to \mu^+\mu^-$) and comparing this to the frequency of occurrence of an other process already tells you a lot about the particle's nature.

Exercise d)

What do you expect for the following frequency of occurrence ratios?

$$\frac{N\left(e^+e^- \to \gamma \to u\bar{u}\right)}{N\left(e^+e^- \to \gamma \to s\bar{s}\right)} = \frac{\qquad \cdot XY}{\qquad \cdot XY} =$$

$$\frac{N\left(e^+e^- \to \gamma \to \tau^+\tau^-\right)}{N\left(e^+e^- \to \gamma \to \mu^+\mu^-\right)} = \frac{\qquad \cdot XY}{\qquad \cdot XY} =$$

Exercise e) The R value

We are nearly at our destination. In particle detectors it is usually relatively easy to distinguish different leptons (namely electrons $e^\pm$, muons $\mu^\pm$ and tau-leptons $\tau^\pm$). Quarks, however, are way harder to distinguish. Therefore, one typically simply measures the total amount of quarks that were produced (u, d, s, c, b or t).

Let's assume that the energy is sufficiently high to create u, d, s and c quarks (as well as leptons): What ratio do you expect by comparing quarks to tau-leptons?
Hint: Your solutions of exercise a) and c) might be helpful.

$$\frac{N\left(e^+e^- \to \gamma \to u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}\right)}{N\left(e^+e^- \to \gamma \to \tau^+\tau^-\right)} = \frac{\qquad \cdot XY}{\qquad \cdot XY} =$$

In order to reduce missmeasurements of leptons, the ratio of quarks to leptons in general is even more interesting. For this purpose, one can simply average over the amount of different lepton types (as shown in exercise d), muons and tauons are created equally often). Correspondingly, we get the same result as above by calculating the following ratio:

$$R = \frac{N\left(e^+e^- \to \gamma \to u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}\right)}{\frac{1}{2}\cdot\left[N\left(e^+e^- \to \gamma \to \mu^+\mu^-\right) + N\left(e^+e^- \to \gamma \to \tau^+\tau^-\right)\right]} = \boxed{\text{The same as above}}$$

This ratio is called the R **value**.

It becomes COLORFUL!

You might have asked yourself: *"What the hell! What does this have to do with quark colors?"* Well, it is time to disclose the mystery.
But first a small setback: You were fooled shamelessly! Actually, the factor XY in the frequency of occurrence of different particles is not always equal! The following was kept secret until now:
In contrast to leptons, quarks and antiquarks are influenced by the **strong force**. This is exactly the reason that makes quarks group together to form protons and neutrons. Furthermore, this force is responsible for the fact that atomic nuclei are glued together despite of the positive charges of all the protons.
Particles that couple to the strong force need to be „strongly charged“. Let's remember, previously we learned that couplings to the electromagnetic force are governed by the known electric charge. For the strong force, it's very similar just that everything is much stronger.
Due to reasons that we will learn later, these „strong“ charges are called **colors**. While it's commonly known that there are two types of electromagnetic charges (positive and negative ones that can be differently strong), for the strong charge this is not necessarily true. Exactly this is the topic for this masterclass:
How many strong charges / colors do exist or differently said:

Exercise f) How many colors does a quark come in?

The electromagnetic force cannot see any colors. Therefore she is „color-blind“. This means that quarks can be created in all possible colors and every color is equally common. Equivalent, the frequency of occurrence of one quark type increases with the total numbers of different quark colors N_F that exist:

$$N\left(e^+e^- \to \gamma \to q\bar{q}\right) = N_\mathrm{F} \cdot \left(Q_q\right)^2 \cdot XY$$

Accordingly, how would the ratio of exercise e) (i.e. the R value) change, if there are five different quark colors?

$$R = \frac{N\left(e^+e^- \to \gamma \to u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}\right)}{\frac{1}{2}\cdot\left[N\left(e^+e^- \to \gamma \to \mu^+\mu^-\right) + N\left(e^+e^- \to \gamma \to \tau^+\tau^-\right)\right]} = \frac{\qquad\qquad \cdot XY}{\qquad\qquad \cdot XY} =$$

And last but not least: What would be the R value, if there are only three different quark colors?

$$R = \frac{N\left(e^+e^- \to \gamma \to u\bar{u}/d\bar{d}/s\bar{s}/c\bar{c}\right)}{\frac{1}{2}\cdot\left[N\left(e^+e^- \to \gamma \to \mu^+\mu^-\right) + N\left(e^+e^- \to \gamma \to \tau^+\tau^-\right)\right]} = \frac{\qquad\qquad \cdot XY}{\qquad\qquad \cdot XY} =$$