

NLO unitarity and stability bounds in extended Higgs sectors

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Outline

- Perturbative Unitarity at Next-to-Leading Order
- Vaccum Stability
- Models with Higgs Triplets
- Constraints on the Parameter Space

Introductions

Standard Model (SM) of particle physics is a framework
 → good agreement with the collider data

Open questions : Neutrino masses , Dark Matter , Baryon asymmetry of the Universe (BAU) , Electroweak Phase Transitions (EWPT)

- SM might be a simplified version of a more complicated model
- Do the LHC data preclude the existence of additional multiplets in the scalar sector of the SM ?

Prior to the Higgs Discovery

Standard Model

$$V(\phi) = -\mu'^2 |\phi|^2 + \lambda |\phi|^4$$

Higgs mass : $m_h = \sqrt{2\lambda} v$



 $m_h \rightarrow$ free parameter This is a potential at tree-level Theoretical considerations were used to place bound on the Higgs mass

... such as perturbative unitarity bounds

Perturbative Unitarity

• Unitarity of the S-matrix, $S^{\dagger}S = I$,

$$\left|a_{\ell}^{2\to 2} - \frac{1}{2}i\right|^2 + \sum_{k>2} \left|a_{\ell}^{2\to k}\right|^2 = \frac{1}{4}.$$
 (1)

 $\rightarrow a_{\ell}$'s are the ℓ -th partial-wave amplitudes

• For a given $2 \rightarrow 2$ process, the unitarity bounds :

$$\left|a_{\ell} - \frac{1}{2}i\right|^2 \le \frac{1}{4} \tag{2}$$

- At tree-level, $a_{\ell} \in \mathbb{R}$, leads to a strong bound, $|\operatorname{Re}(a_{\ell})| \leq \frac{1}{2}$.
- Beyond tree-level, $a_{\ell} \notin \mathbb{R}$, hence above bound gets weaker.

Partial-Wave Amplitudes

- Let's consider high energy limit to compute scattering amplitudes
 - $SU(2)_L \otimes U(1)_Y$ symmetry is intact at high energies, leads to block diagonal form of scattering matrix.
 - Most dominant contribution comes from $\ell = 0$ partial-wave.
- For a given process, $i \rightarrow f$,

$$(a_0)_{i,f}(s) = \frac{1}{16\pi s} \int_{-s}^{0} dt \mathcal{M}_{i\to f}(s,t),$$
 (3)

 $\rightarrow \mathcal{M}_{i \rightarrow f}$ represents the sum of scattering amplitudes.

$2 \rightarrow 2$ Scattering Amplitudes

- Let's consider an extended Higgs sector with quartic couplings λ_i (i = 1,..,j)
 - In the limit, $s \gg |\lambda_i| v^2 \gg M_W^2$, only 1PI diagrams survive at one-loop.



$$\lambda_i^0 = \lambda_i + \delta \lambda_i , \quad \delta \Lambda = \frac{1}{16\pi^2 \epsilon} \beta_\Lambda \tag{4}$$

- Scattering amplitudes at one-loop,

$$\mathcal{M}_{1\mathrm{PI}}^{2\to2} = \frac{\lambda_i \lambda_j}{16\pi^2} \left[\frac{1}{\epsilon} + 2 - \ln\left(\frac{-p^2 - i0_+}{\mu^2}\right) \right] , \quad (5)$$

One-loop corrections to *S*-matrix

• For a given process, $i \rightarrow f$, arXiv: 1512.04567, 1702.08511

$$(a_0)_{i,f}(s) = -\frac{1}{16\pi}b_0 + \frac{1}{256\pi^3}\left(b_1^R + ib_1'\right), \qquad (6)$$

or,

$$256\pi^{3}(\mathbf{a_{0}^{Q,Y}})_{\rm NLO} = -16\pi^{2}\mathbf{b_{0}} + (i\pi - 1)\mathbf{b_{0}} \cdot \mathbf{b_{0}} + 3\beta_{\mathbf{b_{0}}}.$$

– ${\bf b_0}$ is the tree-level S-matrix and $\beta_{{\bf b_0}}$ represents the matrix involving beta functions.

- For example, if
$$b_0 = a\lambda_i + b\lambda_j$$
, then
 $\beta_{b_0} = a\beta_{\lambda_i} + b\beta_{\lambda_j}, \ \forall a, b \in \mathbb{R}.$

Vaccum Stability

• Potential with bi-quadratic form,

$$V_{2.0}^{(4)} = \lambda_{\chi} |\chi^{0}|^{4} + \lambda_{\chi\xi} |\chi^{0}|^{2} (\xi^{0})^{2} + \lambda_{\xi} (\xi^{0})^{4}$$
(7)
$$V_{2.0}^{(4)} > 0 \implies \lambda_{\chi} > 0 \land \lambda_{\xi} > 0 \land \lambda_{\chi\xi} + 2\sqrt{\lambda_{\chi}\lambda_{\xi}} > 0$$

• Potential with non bi-quadratic form,

$$V_{3.0}^{(4)} = \lambda_{\phi} |\phi^{0}|^{4} + \lambda_{\xi} (\xi^{0})^{4} + \lambda_{\chi} |\chi^{0}|^{4} + \lambda_{\phi\xi} |\phi^{0}|^{2} (\xi^{0})^{2} + \lambda_{\chi\xi} |\chi^{0}|^{2} (\xi^{0})^{2} + \left(\lambda_{\phi\chi} + \frac{\kappa_{2}}{2}\right) |\phi^{0}|^{2} |\chi^{0}|^{2} + \frac{\kappa_{3}}{\sqrt{2}} \left[(\phi^{0})^{2} \xi^{0} \chi^{0*} + \text{h.c.} \right]$$
(8)

 \rightarrow Introduce a dimensionless gauge-invariant parameter,

$$\zeta = \frac{1}{2} \frac{\left[(\phi^0)^2 \xi^0 \chi^{0*} + \text{h.c.} \right]}{|\phi^0|^2 \sqrt{|\chi^0|^2 (\xi^0)^2}} \,, \quad \text{with} \quad \zeta \in [-1, 1]$$

Positivity conditions will depend on the parameter ζ .

Theoretical Bounds in Weakly Interacting Theories

• Standard Model : Prior to the Higgs discovery

Positivity conditions : $\lambda > 0$

• Two-Higgs Doublet Model :

arXiv: 1512.04567, 1609.01290

In principle, unitarity can hold up to $\lambda(s) pprox 15$

$$R_1 = rac{\left|a_0^{
m NLO}
ight|}{\left|a_0^{
m LO} + a_0^{
m NLO}
ight|}, \quad R_1' = rac{\left|a_0^{
m NLO}
ight|}{\left|a_0^{
m LO}
ight|},$$

 $R_1 = 1$, or $R'_1 = 1$, perturbativity is violated when $\lambda(s) \sim 4.3$ Positivity conditions : $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$, ...

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LHC era



ATLAS Collaboration, Nature 607 (2022)

CMS Collaboration, Nature 607 (2022)

$$\mu_i^f = \frac{\sigma_i \times B^f}{(\sigma_i \times B^f)_{\rm SM}}, \qquad \kappa_f = \frac{g_{hff}}{g_{hff}^{\rm SM}}, \quad \kappa_V = \frac{g_{hVV}}{g_{hVV}^{\rm SM}}$$

LHC era



ATLAS Collaboration, Nature 607 (2022)

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$$\mu_i^f = \frac{\sigma_i \times B^f}{(\sigma_i \times B^f)_{\rm SM}}, \qquad \kappa_f = \frac{g_{hff}}{g_{hff}^{\rm SM}}, \quad \kappa_V = \frac{g_{hVV}}{g_{hVV}^{\rm SM}}$$

Any deviation from unity suggesting that the SM needs to be extended



Search for BSM @ forefront of particle physics research



- S It can also explain neutrino oscillation , EWBG , Dark-Matter puzzle
- It offers a much richer prospect for collider experiments

Can be probed in particle colliders and in cosmological observatories



Triplet extended Higgs sector with Custodial Symmetry :

SM doublet ϕ (T = 1/2, Y = 1/2) + Real triplet ξ (T = 1, Y = 0) + Complex triplet χ (T = 1, Y = 1)

$$\langle \phi \rangle = v_{\phi}, \langle \xi \rangle = v_{\xi}, \langle \chi \rangle = v_{\chi} \longrightarrow \rho = 1$$
 in tree-level if $v_{\chi} = v_{\xi}$

* Georgi-Machacek (GM) model :

Equality of triplet VEVs is preserved by the Higgs potential [Georgi, Machacek '85; Chanowitz, Golden '85] Interactions among the Higgs fields maintained $SU(2)_I \times SU(2)_P$ symmetry

* extended Georgi-Machacek (eGM) model :

Equality of triplet VEVs is obtained by tuning the potential parameters @ tree-level [Kundu, Mondal, Pal '21] Higgs interactions does not maintained $SU(2)_L \times SU(2)_R$ symmetry ...Similar interactions were considered in 2HDM with softly broken Z₂ symmetry

GM and eGM models are classified based on their symmetry, with identical field contents



In eGM model, both H^+ and F^+ couple to fermions

 \rightarrow Much richer flavor physics phenomenology

• Positivity of the Higgs potential

$$\begin{split} V^{(4)} &= \lambda_{\phi} \left(\phi^{\dagger} \phi \right)^{2} + \lambda_{\zeta} \left(\xi^{\dagger} \xi \right)^{2} + \lambda_{\chi} \left(\chi^{\dagger} \chi \right)^{2} + \tilde{\lambda}_{\chi} \left| \tilde{\chi}^{\dagger} \chi \right|^{2} + \lambda_{\phi \xi} \left(\phi^{\dagger} \phi \right) \left(\xi^{\dagger} \xi \right) \\ &+ \lambda_{\phi \chi} \left(\phi^{\dagger} \phi \right) \left(\chi^{\dagger} \chi \right) + \lambda_{\chi \xi} \left(\chi^{\dagger} \chi \right) \left(\xi^{\dagger} \xi \right) + \kappa_{1} \left| \xi^{\dagger} \chi \right|^{2} + \kappa_{2} \left(\phi^{\dagger} \tau_{\alpha} \phi \right) \left(\chi^{\dagger} \tau_{\alpha} \chi \right) + \kappa_{3} \left[\left(\phi^{T} \varepsilon \tau_{\alpha} \phi \right) \left(\chi^{\dagger} \tau_{\alpha} \xi \right) + \text{h.c.} \right] > 0 \end{split}$$

• Yukawa and quartic couplings of the theory need to be in perturbative regime

 $y_i < \sqrt{4\pi}$ and $\lambda_i < 4\pi$

9 Quartic couplings should satisfy the unitarity conditions @ one-loop

$$\left|a_{\ell}^{2\to 2} - \frac{1}{2}i\right|^2 + \sum_{k>2} \left|a_{\ell}^{2\to k}\right|^2 = \frac{1}{4}.$$

NLO corrections to the LO amplitudes should be smaller in magnitude

$$|a_\ell^{NLO}| < |a_\ell^{LO}|$$

It is not possible to recast the necessary and sufficient positivity conditions into a fully analytical, compact form

Ensure that boundedness of the potential in any directions of field space

Numerically, 3-field direction BFB conditions (neither necessary nor sufficient) are a very good approximation of all 13-field direction BFB conditions

[Moultaka, Peyranère '21, D. Chowdhury, P. Mondal, S.S. 2404.18996]

These 3-field direction BFB conditions are faster numerically



Perturbative Unitarity

$\mathbf{Q}^{\mathbf{Y}}$	0			1/2		1		3/2	2
0	$\phi^{0*}\phi^{0}$	$\frac{\xi^0\xi^0}{\sqrt{2}}$	$\chi^{0*}\chi^{0}$	$\phi^0 \xi^0$	$\phi^{0*}\chi^{0}$	$\frac{\phi^0 \phi^0}{\sqrt{2}}$	$\chi^0 \xi^0$	$\phi^0\chi^0$	$\frac{\chi^0 \chi^0}{\sqrt{2}}$
U	$\phi^+\phi^-$	ξ+ξ-	$\frac{\chi^+\chi^-}{\chi^{++}\chi^{}}$	$\phi^{+}\xi^{-}$	$\chi^+ \phi^-$		χ+ξ-		
	.0× .4	a0.a.t.	0* +	10 e.t.	(0× ±	(0.4	+0 ±	(0 ±	+ 0
1	$\phi^{\circ-}\phi'$	ξ°ξ΄	$\chi^{\circ}\chi^{\circ}$	φ°ξ'	$\phi^{\circ}\chi^{\circ}$	φ°φ'	ξ°χ'	$\phi^{\circ}\chi'$	x' x°
1			$x^{-}x^{++}$	$\phi^+ \xi^0$	$\chi^{++}\phi^{-}$	$\chi^0 \xi^+$	$\chi^{++}\xi^{-}$	$\phi^+\chi^0$	
		$\frac{\xi^+\xi^+}{\sqrt{2}}$	$\chi^{0*}\chi^{++}$	$\phi^+\xi^+$	$\phi^{0*}\chi^{++}$	$\frac{\phi^+\phi^+}{\sqrt{2}}$	$\xi^+\chi^+$	$\phi^+\chi^+$	$\chi^0 \chi^{++}$
2							$\xi^0 \chi^{++}$	$\phi^0 \chi^{++}$	$\frac{\chi^+\chi^+}{\sqrt{2}}$
3	×		X			$\xi^+\chi^{++}$	$\phi^+\chi^{++}$	$\chi^{+}\chi^{++}$	
4	×			Х		X		Х	$\tfrac{\chi^{++}\chi^{++}}{\sqrt{2}}$

Two-particle basis states broken down by their total charge Q and total hypercharge Y.

 \rightarrow 16, 15, 11, 3, 1 unique tree-level eigenvalues for the blocks with Q=0,1,2,3,4, respectively. Out of these, total 19 eigenvalues are appeared to be independent.

Perturbative Unitarity

NLO unitarity :

 $256\pi^3 (\mathbf{a_0^{Q,Y}})_{\rm NLO} = -16\pi^2 \mathbf{b_0} + (i\pi - 1)\mathbf{b_0} \cdot \mathbf{b_0} + 3\beta_{\mathbf{b_0}} \,.$



NLO unitarity significantly refine the parameter space

arXiv: 2404.18996

Higgs Signal strengths



arXiv: 2404.18996

Combined global fits



arXiv: 2404.18996

Combined global fits



@ 95.4% CL limit on mass differences and quartic couplings

-> Flavor or electroweak precision data could be used to constrain the model further. (Work in progress ...)

arXiv: 2404 18996

Thank You !

Questions?

Backup slides

Higgs potential with triplets

SM Higgs doublet (ϕ , Y = 1/2) + real triplet (ξ , Y = 0) + complex triplet (χ , Y = 1)

$$V = -m_{\phi}^{2}(\phi^{\dagger}\phi) - m_{\xi}^{2}(\xi^{\dagger}\xi) - m_{\chi}^{2}(\chi^{\dagger}\chi) + \mu_{1}(\chi^{\dagger}t_{a}\chi)\xi_{a} + \mu_{2}(\phi^{\dagger}\tau_{a}\phi)\xi_{a} + \mu_{3}\left[(\phi^{T}\epsilon\tau_{a}\phi)\tilde{\chi}_{a} + \text{h.c.}\right] + \lambda_{\phi}(\phi^{\dagger}\phi)^{2} + \lambda_{\xi}(\xi^{\dagger}\xi)^{2} + \lambda_{\chi}(\chi^{\dagger}\chi)^{2} + \tilde{\lambda}_{\chi}|\tilde{\chi}^{\dagger}\chi|^{2} + \lambda_{\phi\xi}(\phi^{\dagger}\phi)(\xi^{\dagger}\xi) + \lambda_{\phi\chi}(\phi^{\dagger}\phi)(\chi^{\dagger}\chi) + \lambda_{\chi\xi}(\chi^{\dagger}\chi)(\xi^{\dagger}\xi) + \kappa_{1}|\xi^{\dagger}\chi|^{2} + \kappa_{2}(\phi^{\dagger}\tau_{a}\phi)(\chi^{\dagger}t_{a}\chi) + \kappa_{3}\left[(\phi^{T}\epsilon\tau_{a}\phi)(\chi^{\dagger}t_{a}\xi) + \text{h.c.}\right]$$

A. Kundu, P. Mondal, P.B. Pal, PRD 105 (2022)

$$\begin{split} \langle \phi \rangle &= \mathbf{v}_{\phi} \,, \ \langle \xi \rangle = \mathbf{v}_{\xi} \,, \ \langle \chi \rangle = \mathbf{v}_{\chi} \\ \rho &= \frac{\mathbf{v}_{\phi}^2 + 4\left(\mathbf{v}_{\xi}^2 + \mathbf{v}_{\chi}^2\right)}{\mathbf{v}_{\phi}^2 + 8\mathbf{v}_{\chi}^2} \quad \Rightarrow \quad \rho = 1 \quad \text{requires} \quad \boxed{\mathbf{v}_{\chi} = \mathbf{v}_{\xi}} \end{split}$$

Collider Phenomenology

🌻 Yukawa sector :

Only the doublet couples to fermions

triplet VEV (v_{χ}) : $v_{\phi}^2 + 8v_{\chi}^2 = v^2$ and $\tan \beta = \frac{v_{\phi}}{2\sqrt{2}v_{\chi}}$ $v_{\chi} \downarrow \qquad \tan \beta \uparrow$

......Similar phenomenology as in type-I 2HDM

Additional features:

The addition of a singly charged scalar (F^+) coupled to fermions, along with the presence of a doubly charged scalar, makes these models highly interesting for collider studies.



Higgs Signal Strengths : HEPfit Implementation



$$\mu_i^f = \frac{\sigma B \ (i \to H \to f)}{\sigma B_{SM} \ (i \to H \to f)}$$

- $f \in \{ZZ, WW, \gamma\gamma, Z\gamma, \mu\mu, bb, \tau\tau\}$
- Make all possible observables μ^f_i for different production and decay modes
- Fit to the ALTAS and CMS data on (correlated) observables μ_i^f for a BSM model
- Present the results on the (new) observables from the combined fit

$$\kappa_V = c_lpha c_eta - \sqrt{rac{8}{3}} s_lpha s_eta \,, \quad ext{and} \quad \kappa_f = rac{c_lpha}{c_eta} \,,$$





Higgs Signal Strengths : LHC data

ATLAS Run 2

	Signal strength	Value		Ca	rrelatio	on mat	rix		\mathcal{L} [fb ⁻¹]	Source
Ì	HER.MA	1.04 ± 0.10	1	-0.13	0	0	0	0		
	$\mu_{VBF}^{\gamma\gamma}$	1.20 ± 0.26	-0.13	1	0	0	0	0		
	Pin	1.5 ± 0.55	0	0	1	-0.37	0	-0.11		
	$\mu_{2h}^{\gamma\gamma}$	-0.2 ± 0.55	0	0	-0.37	1	0	0	139	[18]
	$\mu_{\rm tth}^{\gamma\gamma}$	0.89 ± 0.31	0	0	0	0	1	-0.44		
	$\mu_{th}^{\gamma\gamma}$	3 ± 3.5	0	0	-0.11	0	-0.44	1		
	μ_{eeT}^{ZZ}	0.95 ± 0.1	1	-0.22	-0.27	0				
	μ_{VBF}^{ZZ}	1.19 ± 0.45	-0.22	1	0	0				
	μ_{Vb}^{ZZ}	1.43 ± 1.0	-0.27	0	1	-0.18			139	1.0
	μ_{uh}^{ZZ}	1.69 ± 1.45	0	0	-0.18	1				
	$\mu_{incl.}^{ZZ}$	1.0 ± 0.1							139	[4]
Ē	$\mu_{ggF,bbh}^{WW}$	1.15 ± 0.135								
	μ_{VBF}^{WW}	0.93 ± 0.21							139	[17]
	$\mu^{WW}_{ggF,bbh,VBF}$	1.09 ± 0.11								
Г	μ_{VDT}^{U}	0.90 ± 0.18	1	-0.24	0	0				
	$\mu_{ggF,Mh}^{\tau\tau}$	0.96 ± 0.31	-0.24	1	-0.29	0				1100
	μ_{Vh}	0.98 ± 0.60	0	-0.29	1	0			139	[10]
	$\mu_{\rm mb, th}^{rr}$	1.06 ± 1.18	0	0	0	1				
Г	µ ^{bb} yyr	0.95 ± 0.37							126	(9)
	μ_{Wh}^{bb}	0.95 ± 0.26							139	[6]
	$\mu_{22_1}^{bb}$	1.08 ± 0.24							139	[6]
	μ_{Vh}^{bb}	1.02 ± 0.17							139	[6]
	$\mu_{\rm tth,th}^{bb}$	0.35 ± 0.35							139	[12]
	$\mu_{pp}^{s\mu}$	1.2 ± 0.6							139	[7]
	$\mu_{to}^{Z\gamma}$	2.0 ± 0.95							139	[5]

CMS Run 2

Signal strength	Value	Correlation matrix	\mathcal{L} [fb ⁻¹]	Source
$\begin{array}{c} \mu_{\rm ggh,bbh}^{\gamma\gamma} \\ \mu_{\rm VBF}^{\gamma\gamma} \\ \mu_{\rm Vh}^{\gamma\gamma} \\ \mu_{\rm Vh}^{\gamma\gamma} \end{array}$	$\begin{array}{c} 1.07\pm 0.11\\ 1.04\pm 0.32\\ 1.34\pm 0.34\\ 1.35\pm 0.31\end{array}$		137	[11]
$\mu_{\text{ggh,bbh,tth,t}}^{ZZ}$ $\mu_{\text{VBF,Vh}}^{ZZ}$	$\begin{array}{c c} 0.95 \pm 0.13 \\ 0.82 \pm 0.34 \end{array}$	1 -0.11 -0.11 1	137	[10]
$\begin{array}{c} \mu_{\rm ggh}^{WW} \\ \mu_{\rm VBF}^{WW} \\ \mu_{\rm Zh}^{WW} \\ \mu_{\rm Wh}^{WW} \end{array}$	$\begin{array}{c} 0.92 \pm 0.11 \\ 0.71 \pm 0.26 \\ 2.0 \pm 0.7 \\ 2.2 \pm 0.6 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	138	[16]
$\mu_{\text{incl.}}^{\tau\tau}$ $\mu_{\text{ggh}}^{\tau\tau}$ $\mu_{\text{qqh}}^{\tau\tau}$ $\mu_{\text{Vh}}^{\tau\tau}$	$\begin{array}{c} 0.93 \pm 0.12 \\ 0.97 \pm 0.19 \\ 0.68 \pm 0.23 \\ 1.80 \pm 0.44 \end{array}$		138	[15]
$\mu_{\rm qqh}^{bb}$ $\mu_{\rm ggh}^{bb}$	1.59 ± 0.60 -2.7 ± 3.89	1 -0.75 -0.75 1	90.8	[19]
$\mu^{\mu\mu}_{\text{ggh,tth}}$ $\mu^{\mu\mu}_{\text{VBF,Vh}}$	$\begin{array}{c} 0.66 \pm 0.67 \\ 1.85 \pm 0.86 \end{array}$	1 -0.24 -0.24 1	137	[8]
$\mu_{pp}^{Z\gamma}$	2.4 ± 0.9		138	[14]