

NLO unitarity and stability bounds in extended Higgs sectors

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Outline

- Perturbative Unitarity at Next-to-Leading Order
- Vaccum Stability
- Models with Higgs Triplets
- Constraints on the Parameter Space

Introductions

• Standard Model (SM) of particle physics is a framework \longrightarrow good agreement with the collider data

Open questions : Neutrino masses , Dark Matter , Baryon asymmetry of the Universe (BAU) , Electroweak Phase Transitions (EWPT)

- SM might be a simplified version of a more complicated model
- Do the LHC data preclude the existence of additional multiplets in the scalar sector of the SM ?

Prior to the Higgs Discovery

Standard Model

$$
V(\phi) = -\mu'^2 |\phi|^2 + \lambda |\phi|^4
$$

Higgs mass : $m_h =$ √ 2λ v

 $\mu'^2>0,~~\lambda>0$

 $m_h \rightarrow$ free parameter This is a potential at tree-level Theoretical considerations were used to place bound on the Higgs mass

... such as perturbative unitarity bounds

Perturbative Unitarity

• Unitarity of the S-matrix, $S^{\dagger}S = I$,

$$
\left| a_{\ell}^{2 \to 2} - \frac{1}{2} i \right|^2 + \sum_{k > 2} \left| a_{\ell}^{2 \to k} \right|^2 = \frac{1}{4} \,. \tag{1}
$$

 \rightarrow a_{ℓ} 's are the ℓ -th partial-wave amplitudes

• For a given $2 \rightarrow 2$ process, the unitarity bounds :

$$
\left|a_{\ell}-\frac{1}{2}i\right|^2 \leq \frac{1}{4} \tag{2}
$$

- At tree-level, $a_\ell \in \mathbb{R}$, leads to a strong bound, $|\mathsf{Re}(a_\ell)| \leq \frac{1}{2}$.
- Beyond tree-level, $a_{\ell} \notin \mathbb{R}$, hence above bound gets weaker.

Partial-Wave Amplitudes

- Let's consider high energy limit to compute scattering amplitudes
	- $SU(2)_L \otimes U(1)_Y$ symmetry is intact at high energies, leads to block diagonal form of scattering matrix.
	- Most dominant contribution comes from $\ell = 0$ partial-wave.
- For a given process, $i \rightarrow f$,

$$
(a_0)_{i,f}(s) = \frac{1}{16\pi s} \int_{-s}^0 dt \mathcal{M}_{i\to f}(s,t) , \qquad (3)
$$

 \rightarrow $\mathcal{M}_{i\rightarrow f}$ represents the sum of scattering amplitudes.

$2 \rightarrow 2$ Scattering Amplitudes

- Let's consider an extended Higgs sector with quartic couplings λ_i $(i = 1, ..., i)$
	- $-$ In the limit, $s \gg |\lambda_i| v^2 \gg M_W^2$, only 1PI diagrams survive at one-loop.

$$
\lambda_i^0 = \lambda_i + \delta \lambda_i, \quad \delta \Lambda = \frac{1}{16\pi^2 \epsilon} \beta_\Lambda \tag{4}
$$

– Scattering amplitudes at one-loop,

$$
\mathcal{M}_{1\text{PI}}^{2\to 2} = \frac{\lambda_i \lambda_j}{16\pi^2} \left[\frac{1}{\epsilon} + 2 - \ln\left(\frac{-p^2 - i0_+}{\mu^2}\right) \right], \quad (5)
$$

One-loop corrections to S-matrix

• For a given process, $i \rightarrow f$, arXiv: 1512.04567, 1702.08511

$$
(a_0)_{i,f}(s) = -\frac{1}{16\pi}b_0 + \frac{1}{256\pi^3}\left(b_1^R + ib_1^I\right), \qquad (6)
$$

or,

$$
256\pi^3(\mathbf{a_0^{Q,Y}})_{\text{NLO}} = -16\pi^2\mathbf{b_0} + (i\pi - 1)\mathbf{b_0}\cdot\mathbf{b_0} + 3\beta_{\mathbf{b_0}}.
$$

 \mathbf{b}_0 is the tree-level S -matrix and $\beta_{\mathbf{b}_0}$ represents the matrix involving beta functions.

- For example, if
$$
b_0 = a\lambda_i + b\lambda_j
$$
, then
 $\beta_{b_0} = a\beta_{\lambda_i} + b\beta_{\lambda_j}$, $\forall a, b \in \mathbb{R}$.

Vaccum Stability

• Potential with bi-quadratic form,

$$
V_{2.0}^{(4)} = \lambda_{\chi} |\chi^0|^4 + \lambda_{\chi\xi} |\chi^0|^2 (\xi^0)^2 + \lambda_{\xi} (\xi^0)^4 \tag{7}
$$

$$
V_{2.0}^{(4)} > 0 \implies \lambda_{\chi} > 0 \land \lambda_{\xi} > 0 \land \lambda_{\chi\xi} + 2\sqrt{\lambda_{\chi}\lambda_{\xi}} > 0
$$

• Potential with non bi-quadratic form,

$$
V_{3.0}^{(4)} = \lambda_{\phi} |\phi^0|^4 + \lambda_{\xi} (\xi^0)^4 + \lambda_{\chi} |\chi^0|^4 + \lambda_{\phi\xi} |\phi^0|^2 (\xi^0)^2 + \lambda_{\chi\xi} |\chi^0|^2 (\xi^0)^2
$$

+ $\left(\lambda_{\phi\chi} + \frac{\kappa_2}{2} \right) |\phi^0|^2 |\chi^0|^2 + \frac{\kappa_3}{\sqrt{2}} \left[(\phi^0)^2 \xi^0 \chi^{0*} + \text{h.c.} \right] (8)$

 \rightarrow Introduce a dimensionless gauge-invariant parameter,

$$
\zeta = \frac{1}{2} \frac{\left[(\phi^0)^2 \xi^0 \chi^{0*} + \text{h.c.} \right]}{|\phi^0|^2 \sqrt{|\chi^0|^2 (\xi^0)^2}}, \quad \text{with} \quad \zeta \in [-1, 1]
$$

Positivity conditions will depend on the parameter ζ.

Theoretical Bounds in Weakly Interacting Theories

• Standard Model : Prior to the Higgs discovery

LO unitarity : $\lambda \leq \frac{8\pi}{3}$	[Lee, Quigg, Thacker '77]
NLO unitarity : $\lambda \leq 2 - 2.5$ [Dawson, Eillenbrock '89; Durand, Johnson, Lopez'92]	SM Higgs scenario
No revised limit @2-loop [Durand, Maher, Rieszellmann, 92]	SM Higgs scenario

Positivity conditions : $\lambda > 0$

• Two-Higgs Doublet Model : arXiv: 1512.04567, 1609.01290

In principle, unitarity can hold up to $\lambda(s) \approx 15$

$$
R_1 = \frac{|a_0^{\text{NLO}}|}{|a_0^{\text{LO}} + a_0^{\text{NLO}}|}, \quad R_1' = \frac{|a_0^{\text{NLO}}|}{|a_0^{\text{LO}}|},
$$

 $R_1=1,~~$ or $~R'_1=1,$ perturbativity is violated when $\lambda(s)\sim4.3$ Positivity conditions : $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0$, ...

LHC era

ATLAS Collaboration, Nature 607 (2022) CMS Collaboration, Nature 607 (2022)

$$
\mu_i^f = \frac{\sigma_i \times B^f}{(\sigma_i \times B^f)_{\text{SM}}}, \qquad \kappa_f = \frac{g_{hff}}{g_{hff}^{\text{SM}}}, \quad \kappa_V = \frac{g_{hVV}}{g_{hVV}^{\text{SM}}}
$$

LHC era

ATLAS Collaboration, Nature 607 (2022) CMS Collaboration, Nature 607 (2022)

$$
\mu_i^f = \frac{\sigma_i \times B^f}{(\sigma_i \times B^f)_{\text{SM}}}, \qquad \kappa_f = \frac{g_{hff}}{g_{hff}^{\text{SM}}}, \quad \kappa_V = \frac{g_{hVV}}{g_{hVV}^{\text{SM}}}
$$

Any deviation from unity suggesting that the SM needs to be extended

Search for BSM @ forefront of particle physics research

- **S** It can also explain neutrino oscillation, EWBG, Dark-Matter puzzle
- \bullet It offers a much richer prospect for collider experiments

Can be probed in particle colliders and in cosmological observatories

Triplet extended Higgs sector with Custodial Symmetry:

SM doublet ϕ (T = 1/2, Y = 1/2) + Real triplet ξ (T = 1, Y = 0) + Complex triplet γ (T = 1, Y = 1)

$$
\langle \phi \rangle = v_{\phi}, \langle \xi \rangle = v_{\xi}, \langle \chi \rangle = v_{\chi} \longrightarrow \rho = 1 \text{ in tree-level if } \left(v_{\chi} = v_{\xi} \right)
$$

* Georgi-Machacek (GM) model:

Equality of triplet VEVs is preserved by the Higgs potential [Georgi, Machacek '85; Chanowitz, Golden '85] Interactions among the Higgs fields maintained $SU(2)_I \times SU(2)_R$ symmetry

* extended Georgi-Machacek (eGM) model:

Equality of triplet VEVs is obtained by tuning the potential parameters @ tree-level [Kundu, Mondal, Pal '21] Higgs interactions does not maintained $SU(2)_I \times SU(2)_R$ symmetry ... Similar interactions were considered in 2HDM with softly broken Z_2 symmetry

GM and eGM models are classified based on their symmetry, with identical field contents

In eGM model, both H^+ and F^+ couple to fermions

 \rightarrow Much richer flavor physics phenomenology

Positivity of the Higgs potential \bullet

$$
\begin{split} V^{(4)}&=\lambda_{\phi}\big(\phi^{\dagger}\phi\big)^{2}+\lambda_{\xi}\big(\xi^{\dagger}\xi\big)^{2}+\lambda_{\chi}\big(\chi^{\dagger}\chi\big)^{2}+\tilde{\lambda}_{\chi}\Big|\tilde{\chi}^{\dagger}\chi\Big|^{2}+\lambda_{\phi\xi}\big(\phi^{\dagger}\phi\big)\big(\xi^{\dagger}\xi\big)\\ &+\lambda_{\phi\chi}\big(\phi^{\dagger}\phi\big)\big(\chi^{\dagger}\chi\big)+\lambda_{\chi\xi}\big(\chi^{\dagger}\chi\big)\big(\xi^{\dagger}\xi\big)+\kappa_{1}\Big|\xi^{\dagger}\chi\Big|^{2}+\kappa_{2}\big(\phi^{\dagger}\tau_{\alpha}\phi\big)\big(\chi^{\dagger}\tau_{\alpha}\chi\big)+\kappa_{3}\Big[\big(\phi^{T}c\tau_{\alpha}\phi\big)\big(\chi^{\dagger}\tau_{\alpha}\xi\big)+\text{h.c.}\Big]>0 \end{split}
$$

. Yukawa and quartic couplings of the theory need to be in perturbative regime

 $y_i < \sqrt{4\pi}$ and $\lambda_i < 4\pi$

Quartic couplings should satisfy the unitarity conditions @ one-loop \bullet

$$
\left| a_{\ell}^{2 \to 2} - \frac{1}{2}i \right|^2 + \sum_{k > 2} \left| a_{\ell}^{2 \to k} \right|^2 = \frac{1}{4}.
$$

NLO corrections to the LO amplitudes should be smaller in magnitude \bullet

$$
|\,a_\ell^{NLO}|<|\,a_\ell^{LO}|
$$

It is not possible to recast the necessary and sufficient positivity conditions into a fully analytical, compact form

Ensure that boundedness of the potential in any directions of field space

Numerically, 3-field direction BFB conditions (neither necessary nor sufficient) are a very good approximation of all 13-field direction BFB conditions

[Moultoka, Peyronère '21, D. Chowdhury, P. Mondal, S.S. 2404-18996]

These 3-field direction BFB conditions are faster numerically

Perturbative Unitarity

Two-particle basis states broken down by their total charge Q and total hypercharge Y.

 \rightarrow 16, 15, 11, 3, 1 unique tree-level eigenvalues for the blocks with $Q = 0, 1, 2, 3, 4$, respectively. Out of these, total 19 eigenvalues are appeared to be independent.

Perturbative Unitarity

NLO unitarity :

 $256\pi^3(\mathbf{a_0^{Q,Y}})$ $\mathbf{Q}_{0}^{(Q,Y})_{\text{NLO}} = -16\pi^2 \mathbf{b}_0 + (i\pi - 1)\mathbf{b}_0 \cdot \mathbf{b}_0 + 3\beta_{\mathbf{b}_0}$.

NLO unitarity significantly refine the parameter space

arXiv: 2404.18996

Higgs Signal strengths

arXiv: 2404.18996

Combined global fits

arXiv: 2404.18996

Combined global fits

@ 95.4% CL limit on mass differences and quartic couplings

Flavor or electroweak precision data could be used to constrain the model further. (Work in progress ...)

arXiv: 2404.18996

Thank You !

Questions?

Backup slides

Higgs potential with triplets

SM Higgs doublet $(\phi, Y = 1/2)$ + real triplet $(\xi, Y = 0)$ + complex triplet $(\chi, Y = 1)$

$$
V = -m_{\phi}^{2}(\phi^{\dagger}\phi) - m_{\xi}^{2}(\xi^{\dagger}\xi) - m_{\chi}^{2}(\chi^{\dagger}\chi) + \mu_{1}(\chi^{\dagger}t_{a}\chi)\xi_{a} + \mu_{2}(\phi^{\dagger}\tau_{a}\phi)\xi_{a}
$$

+
$$
\mu_{3}\left[(\phi^{\dagger}\epsilon\tau_{a}\phi)\tilde{\chi}_{a} + \text{h.c.}\right] + \lambda_{\phi}(\phi^{\dagger}\phi)^{2} + \lambda_{\xi}(\xi^{\dagger}\xi)^{2} + \lambda_{\chi}(\chi^{\dagger}\chi)^{2}
$$

+
$$
\tilde{\lambda}_{\chi}|\tilde{\chi}^{\dagger}\chi|^{2} + \lambda_{\phi\xi}(\phi^{\dagger}\phi)(\xi^{\dagger}\xi) + \lambda_{\phi\chi}(\phi^{\dagger}\phi)(\chi^{\dagger}\chi) + \lambda_{\chi\xi}(\chi^{\dagger}\chi)(\xi^{\dagger}\xi)
$$

+
$$
\kappa_{1}|\xi^{\dagger}\chi|^{2} + \kappa_{2}(\phi^{\dagger}\tau_{a}\phi)(\chi^{\dagger}t_{a}\chi) + \kappa_{3}\left[(\phi^{\dagger}\epsilon\tau_{a}\phi)(\chi^{\dagger}t_{a}\xi) + \text{h.c.}\right]
$$

A. Kundu, P. Mondal, P.B. Pal, PRD 105 (2022)

$$
\langle \phi \rangle = v_{\phi}, \ \langle \xi \rangle = v_{\xi}, \ \langle \chi \rangle = v_{\chi}
$$
\n
$$
\rho = \frac{v_{\phi}^2 + 4\left(v_{\xi}^2 + v_{\chi}^2\right)}{v_{\phi}^2 + 8v_{\chi}^2} \Rightarrow \rho = 1 \quad \text{requires} \quad \boxed{v_{\chi} = v_{\xi}}
$$

Collider Phenomenology

Yukawa sector: \bullet

Only the doublet couples to fermions

triplet VEV (v_x) : $v_{\phi}^2 + 8v_x^2 = v^2$ and $\tan \beta = \frac{v_{\phi}}{2\sqrt{2}v_x}$ v_x $\tan \beta$

......Similar phenomenology as in type-I 2HDM

Additional features:

The addition of a singly charged scalar (F^+) coupled to fermions, along with the presence of a doubly charged scalar, makes these models highly interesting for collider studies.

Higgs Signal Strengths: HEPfit Implementation

- \bullet Make all possible observables μ_i^f for different production and decay modes
- \bullet Fit to the ALTAS and CMS data on (correlated) observables μ^f_i for a BSM model
- Present the results on the (new) observables from the combined fit

$$
\kappa_V = c_\alpha c_\beta - \sqrt{\frac{8}{3}} s_\alpha s_\beta
$$
, and $\kappa_f = \frac{c_\alpha}{c_\beta}$,

Higgs Signal Strengths: LHC data

ATLAS Run 2

CMS Run 2

