Galaxy Clustering Beyond 2-Point Statistics

DAVID J. BAUMGART AND J. N. FRY THE ASTROPHYSICAL JOURNAL, 375:25–34, 1991 July 1

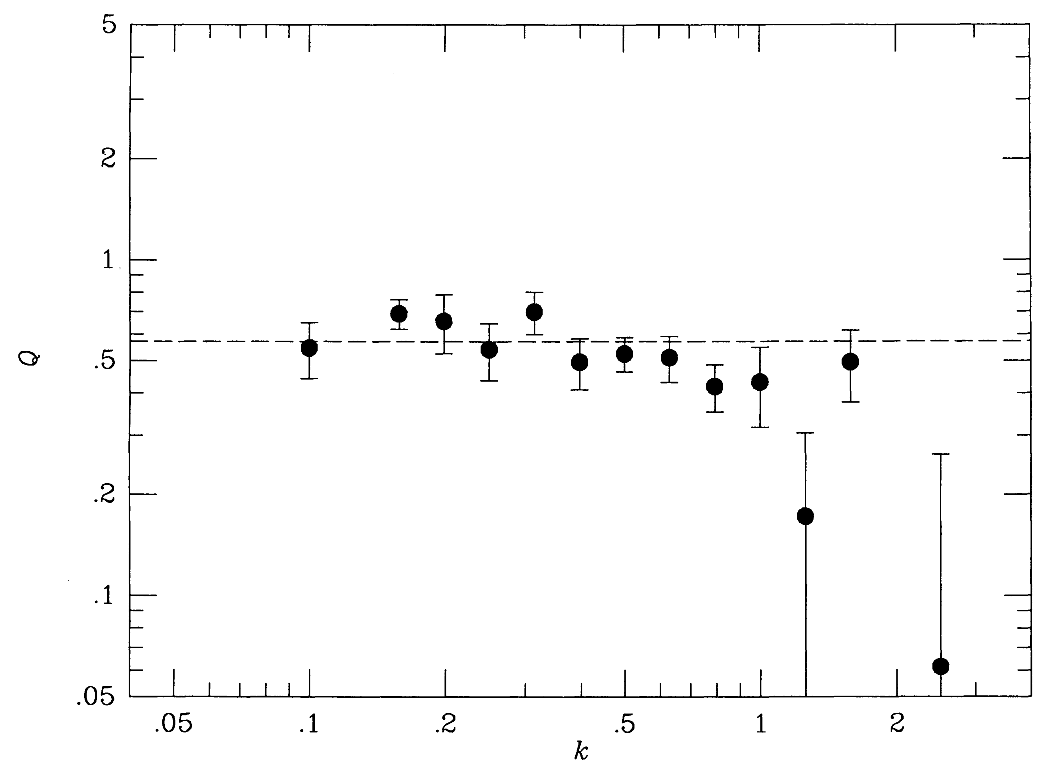

FIG. 8.—Reduced three-point amplitude $Q = B/3P^2$, averaged over all samples, plotted against k. The dashed line is $Q = \frac{4}{7}$.

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New Strategies for Extracting Cosmology from Galaxy Surveys
3rd edition

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What

I will discuss *exclusively* the plain, boring bispectrum

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, B(k_1, k_2, k_3)$$

and 3-point correlation function

$$\langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\delta(\mathbf{x}_3)\rangle = \zeta(x_{12}, x_{23}, x_{13})$$

Why

Our signal is non-Gaussian

Signal-to-noise

Bispectrum S/N is comparable to the power spectrum

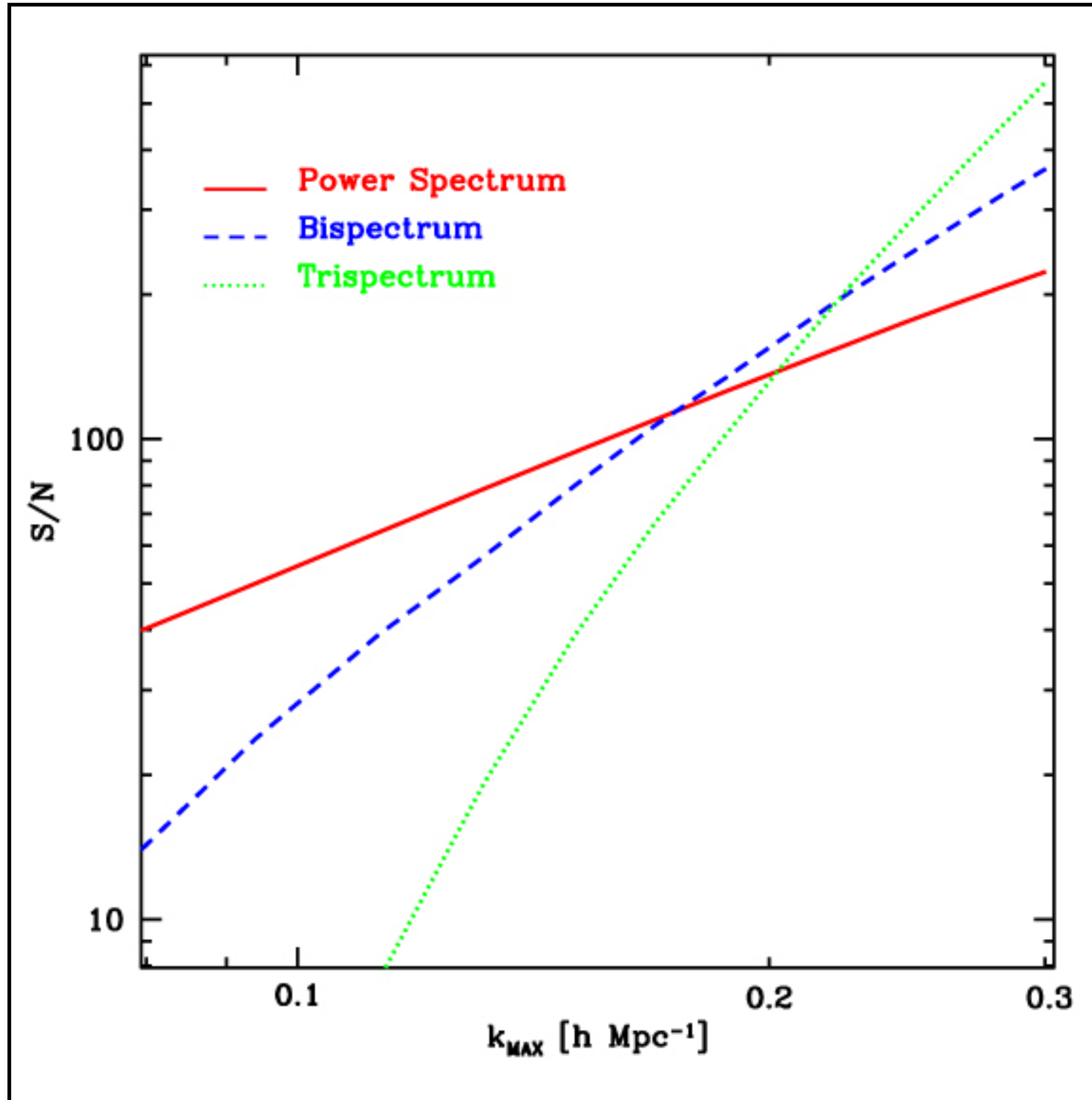
but the signal is distributed over a large number of triangular configurations

To extract enough information we must **get to small scales!**

$$\left(\frac{S}{N}\right)^2_P = \sum_{k}^{k_{\max}} \frac{P^2(k)}{\Delta P^2(k)}$$

$$\left(\frac{S}{N}\right)^2_B = \sum_{\text{triangles}}^{k_{\max}} \frac{B^2(k_1,k_2,k_3)}{\Delta B^2(k_1,k_2,k_3)}$$

ES & Scoccimarro (2005)



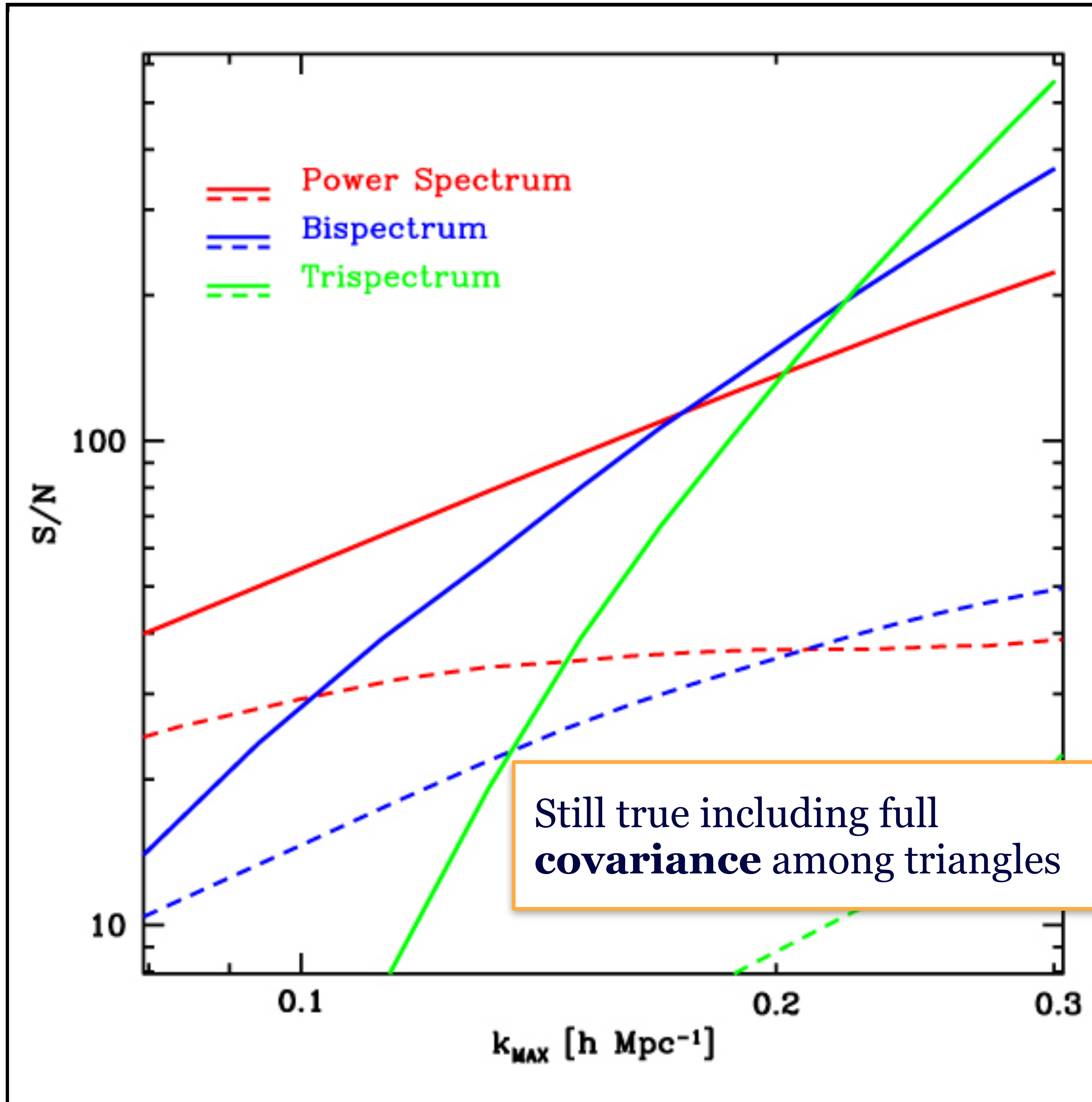
Signal-to-noise

Bispectrum S/N is comparable to the power spectrum

but the signal is distributed over a large number of triangular configurations

$$\left(\frac{S}{N}\right)^2_P = \sum_{k_i,k_j}^{k_{\max}} P(k_i)C_{ij}^{-1}P(k_j)$$

$$\left(\frac{S}{N}\right)^2_B = \sum_{\text{triangles } i,j}^{k_{\max}} B_i C_{ij}^{-1} B_j$$



Still true including full **covariance** among triangles

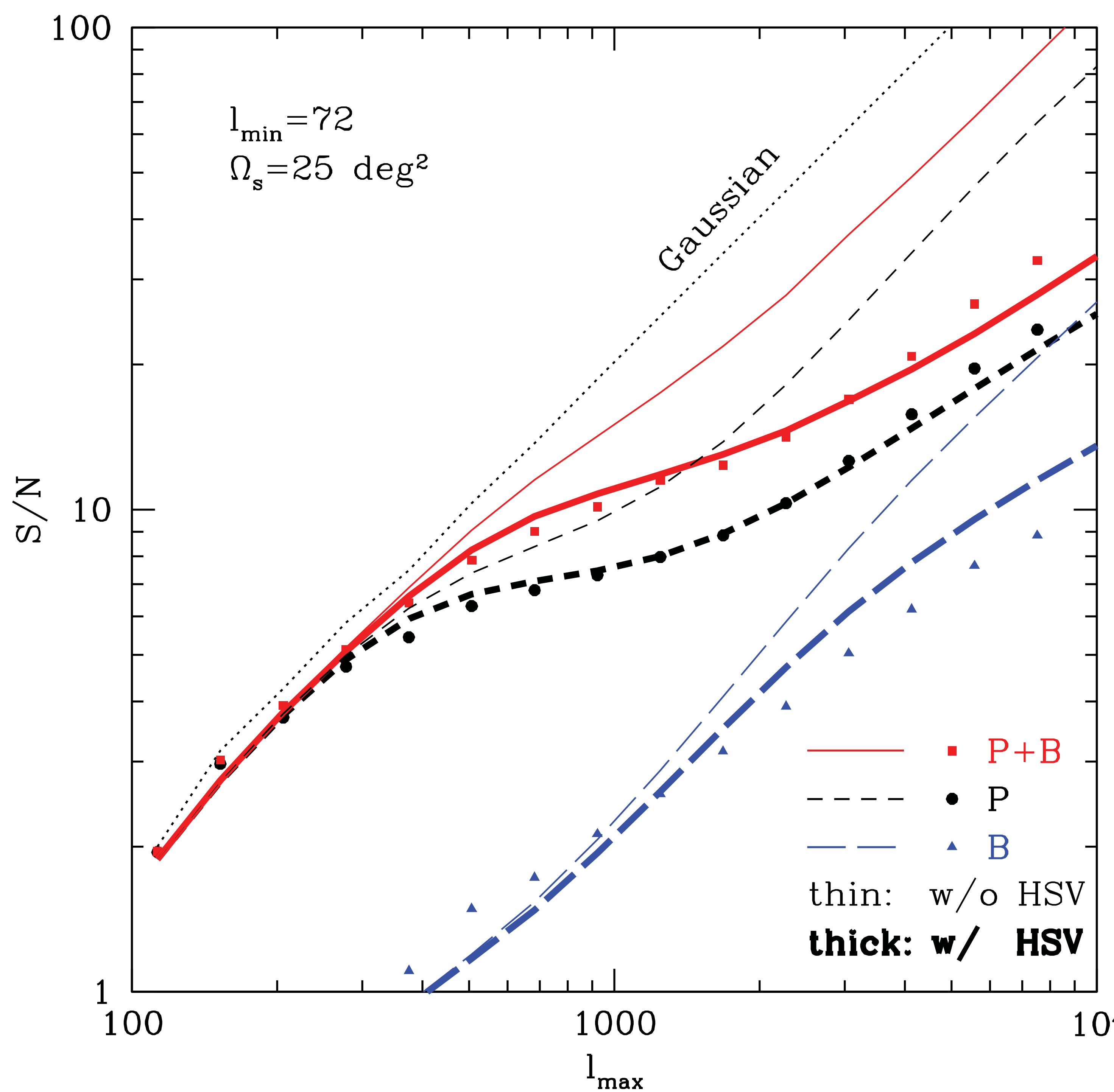
ES & Scoccimarro (2005)

Signal-to-noise

An example from
Weak Lensing

If we stop at mildly non-linear scales, the bispectrum and trispectrum contributions might be the most relevant

Kayo, Takada & Jain (2013)



Why

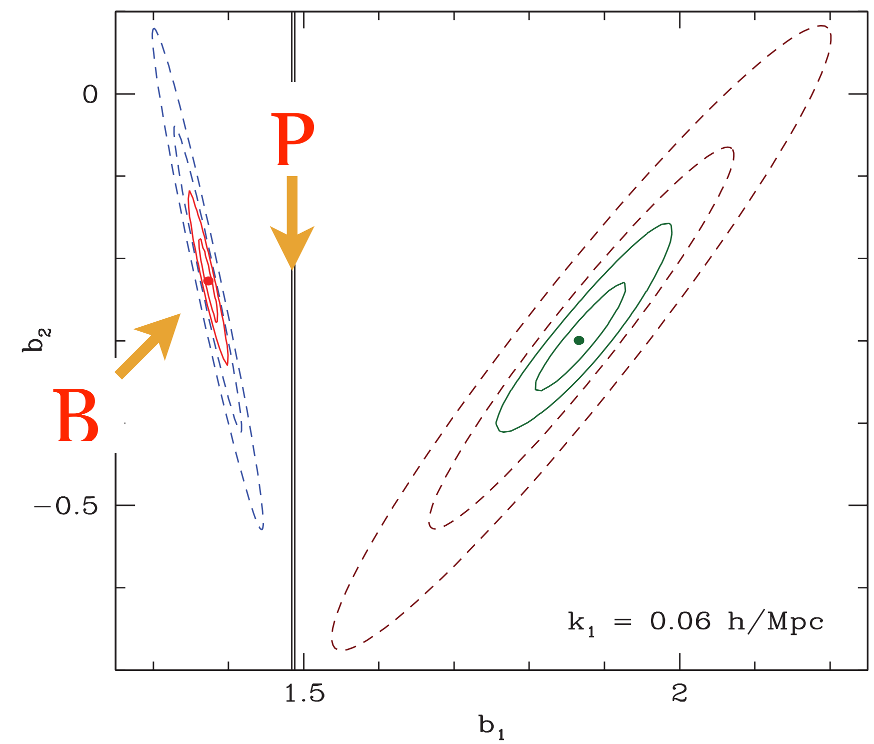
Our signal is non-Gaussian

Constraints on nonlinear bias

Bispectrum as diagnostics

Diagnostics: *an example*

Pollack *et al.* (2012)

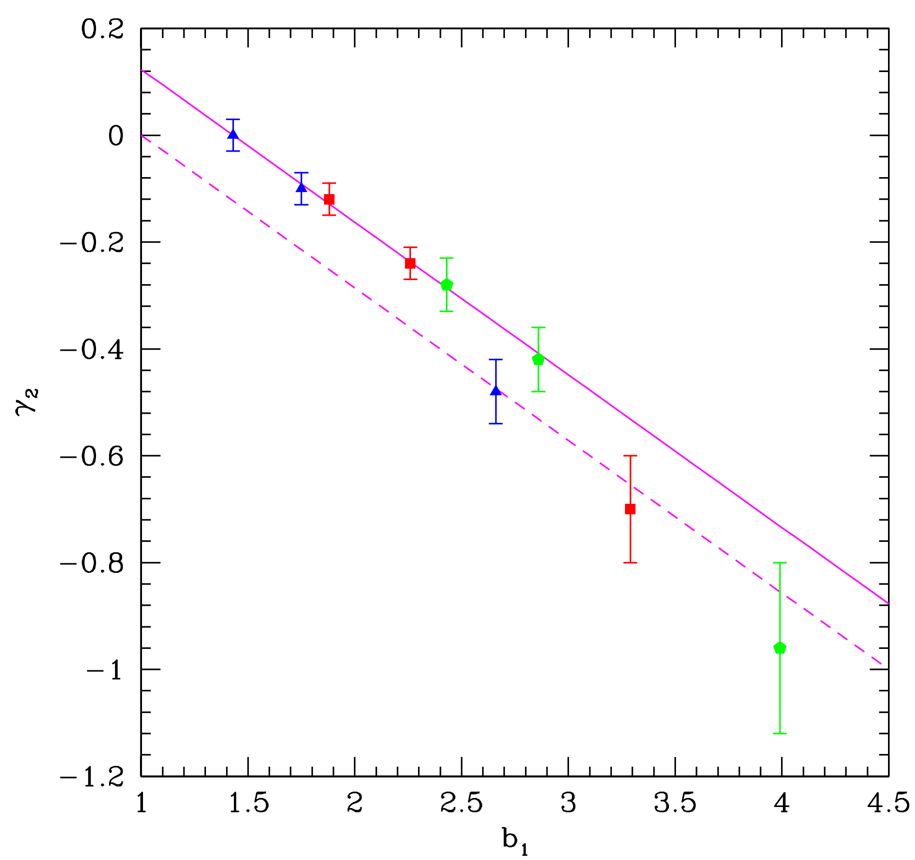


Measurements of the galaxy bispectrum in N-body simulations can identify a problem in our understanding of galaxy bias

In a **local bias** model, the linear b_1 bias determined from the power spectrum was inconsistent with the one determined from the bispectrum

The solution was a **nonlocal bias** contribution, now part of the model

$$\delta_g(\boldsymbol{k}) = b_1 \delta(\boldsymbol{k}) + \frac{b_2}{2} \delta^2(\boldsymbol{k}) + b_{\mathcal{G}_2} \mathcal{G}_2(\boldsymbol{k})$$



Chan *et al.* (2012), see also Baldauf *et al. (2012)*

Why

Our signal is non-Gaussian

Constraints on nonlinear bias

Bispectrum as diagnostics

Full-Shape analysis is now assumed to be a joint analysis of power spectrum and bispectrum (with some caveats), 2pcf+3pcf is coming soon

BAO are now detected in 3pcf

Primordial Non-Gaussianity (particularly nonlocal)

Parity violation

GR effects, etc …

Outlook

Modelling

Anisotropies

Window convolution

Covariance

Recent data analyses

(*For Fourier & configuration space*)

Bispectrum at tree-level

Galaxy density in redshift space: more nonlinearity

$$\delta_s(\mathbf{k}) = Z_1(\mathbf{k})\delta_L(\mathbf{k}) + \int d^3q\, Z_2(\mathbf{q}, \mathbf{k}-\mathbf{q})\delta_L(\mathbf{q})\delta_L(\mathbf{k}-\mathbf{q}) + \dots$$

$$\begin{aligned} Z_1(\mathbf{k}) &= b_1 + f\mu^2\,, \\ Z_2(\mathbf{k}_1,\mathbf{k}_2) &= \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1,\mathbf{k}_2) + b_{\mathcal{G}_2} S(\mathbf{k}_1,\mathbf{k}_2) + f\mu_{12}^2 G_2(\mathbf{k}_1,\mathbf{k}_2) + \\ &\quad + \frac{f\mu_{12}k_{12}}{2}\left[\frac{\mu_1}{k_1} Z_1(\mathbf{k}_2) + \frac{\mu_2}{k_2} Z_1(\mathbf{k}_1)\right] \end{aligned}$$

Redshift-space
PT kernels

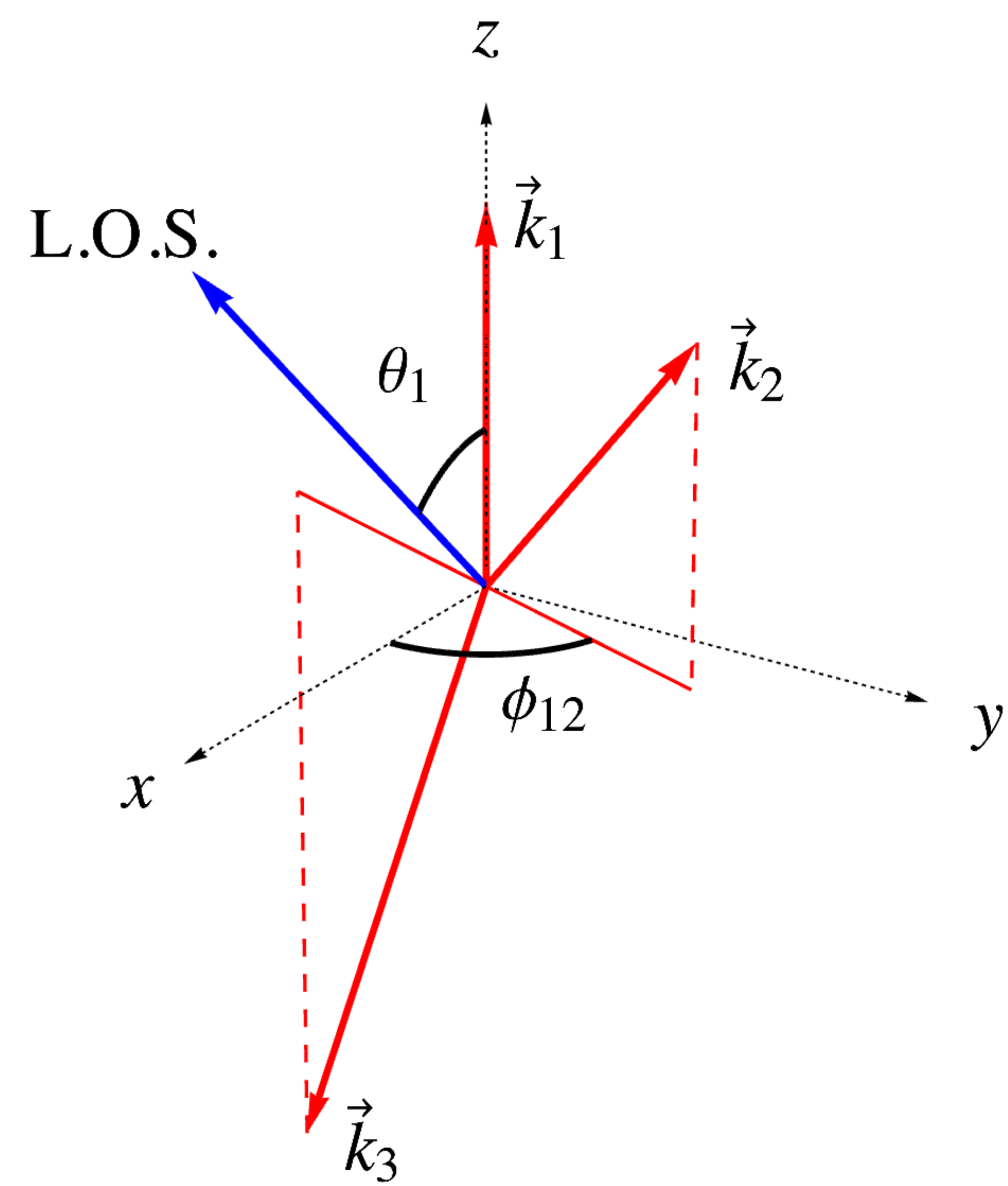
$$B_s(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = B_s^{(\mathrm{det})}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) + B_s^{(\mathrm{stoch})}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$B_s^{(\mathrm{det})}(\mathbf{k}_1,\mathbf{k}_2,\hat{n}) = 2\,Z_1(\mathbf{k}_1)\,Z_1(\mathbf{k}_2)Z_2(\mathbf{k}_1,\mathbf{k}_2)P_L(k_1)P_L(k_2) + 2\ \mathrm{perm.}$$

$$B_s^{(\mathrm{stoch})}(\mathbf{k}_1,\mathbf{k}_2,\hat{n}) = \frac{1}{\bar{n}}\left[(1+\alpha_1)\,b_1 + (1+\alpha_3)\,f\,\mu^2\right]\,Z_1(\mathbf{k}_1)\,P_L(k_1) + 2\ \mathrm{perm.} + \frac{1+\alpha_2}{\bar{n}^2}$$

Redshift-space anisotropies: bispectrum *multipoles*

$$B_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = B_s(k_1, k_2, k_3, \theta_1, \phi_{12})$$



The orientation of the triangle w.r.t. the line-of-sight now matters

Different choices are possible (see e.g. Hashimoto *et al*., 2017, Gualdi & Verde, 2020)

We follow Scoccimarro *et al*. (1999), with the FFT-based estimator of Scoccimarro (2015).

$$B_s(k_1, k_2, k_3, \theta_1, \phi_{12}) = \sum_{\ell,m} B_{\ell,m}(k_1, k_2, k_3) Y_{\ell,m}(\theta_1, \phi_{12})$$

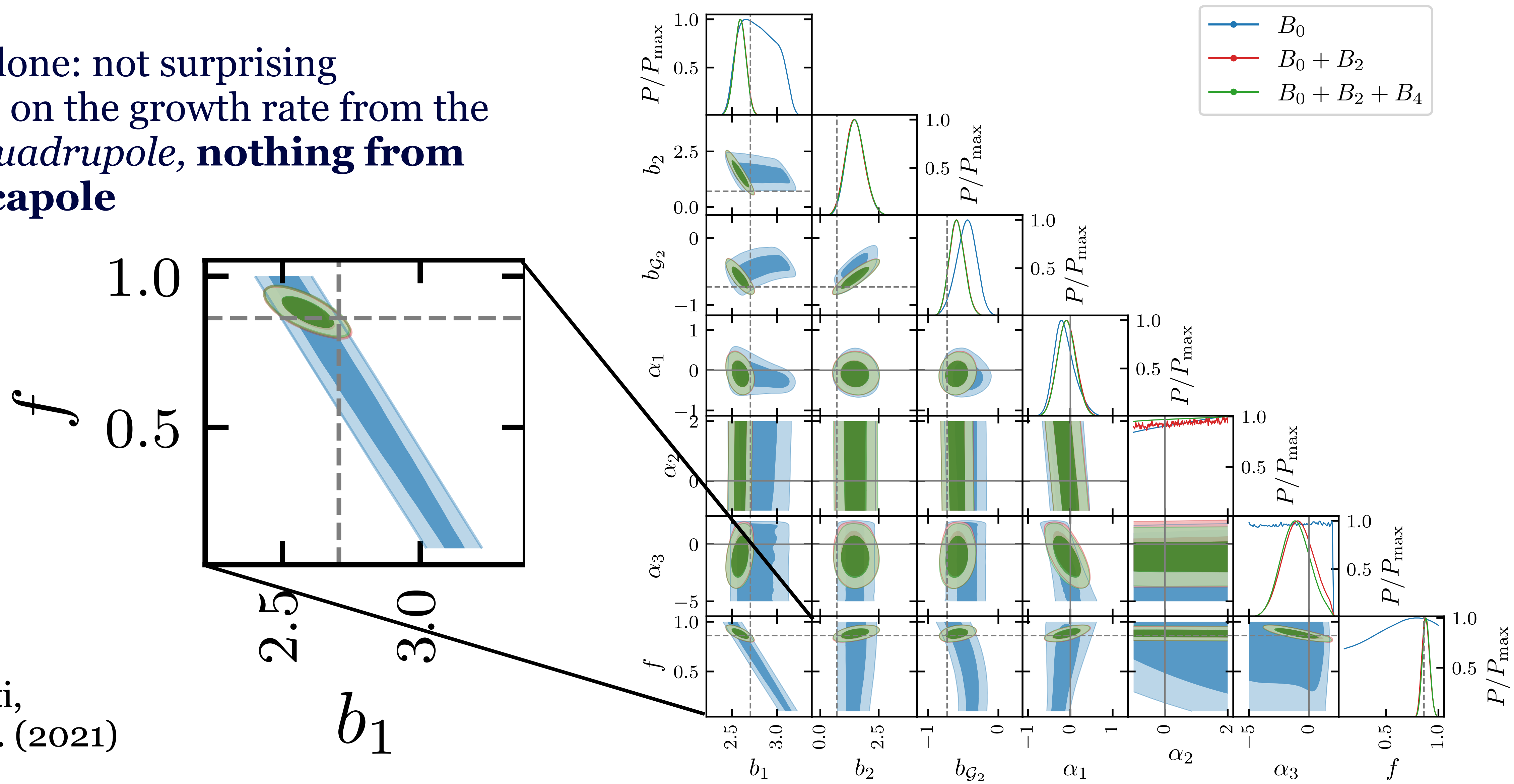
$$\mu_1 \equiv \mu \equiv \cos\theta_1$$

Redshift-space anisotropies: bispectrum *multipoles*

Test of bispectrum multipoles: halos on $1000\,h^{-3}\mathrm{Gpc}^3$ of cumulative volume

bias parameters + f

Bispectrum alone: not surprising improvement on the growth rate from the bispectrum *quadrupole*, **nothing from the hexadecapole**



Rizzo, Moretti,
Pardede *et al.* (2021)

See also Gualdi & Verde (2020), Gualdi *et al.* (2021), D'Amico et al. (2022)

Redshift-space modelling: tree-level, *monopole*

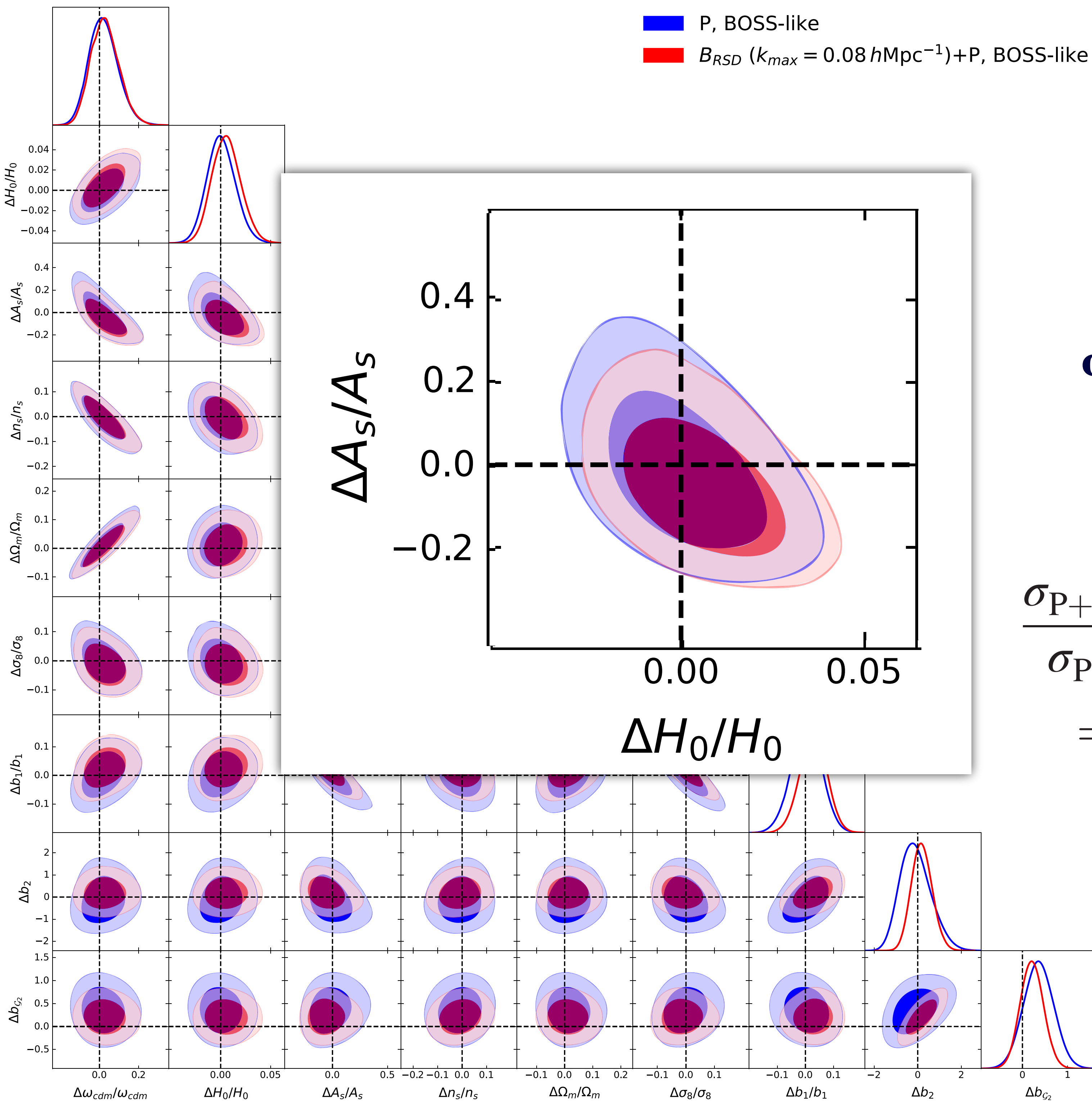


FIG. 7. Same as Fig. 5 but with the covariance rescaled by 100 to match the BOSS survey volume.

Test of **tree-level** bispectrum
in redshift space
EFTofLSS

BOSS-like HOD

Some (10%) improvement
on amplitude parameters (A_s, σ_8)
on BOSS-like volume

On full volume, 566 h^{-3}Gpc^3:

$$\frac{\sigma_{\rm P+B}}{\sigma_{\rm P}}\{\omega_{\rm cdm}, h, n_s, A_s, \Omega_m, \sigma_8\}$$
$$= \{0.88, 0.94, 0.86, 0.95, 0.89, 0.96\},$$

Ivanov *et al.* (2022)

Redshift-space modelling: tree-level, beyond EFT

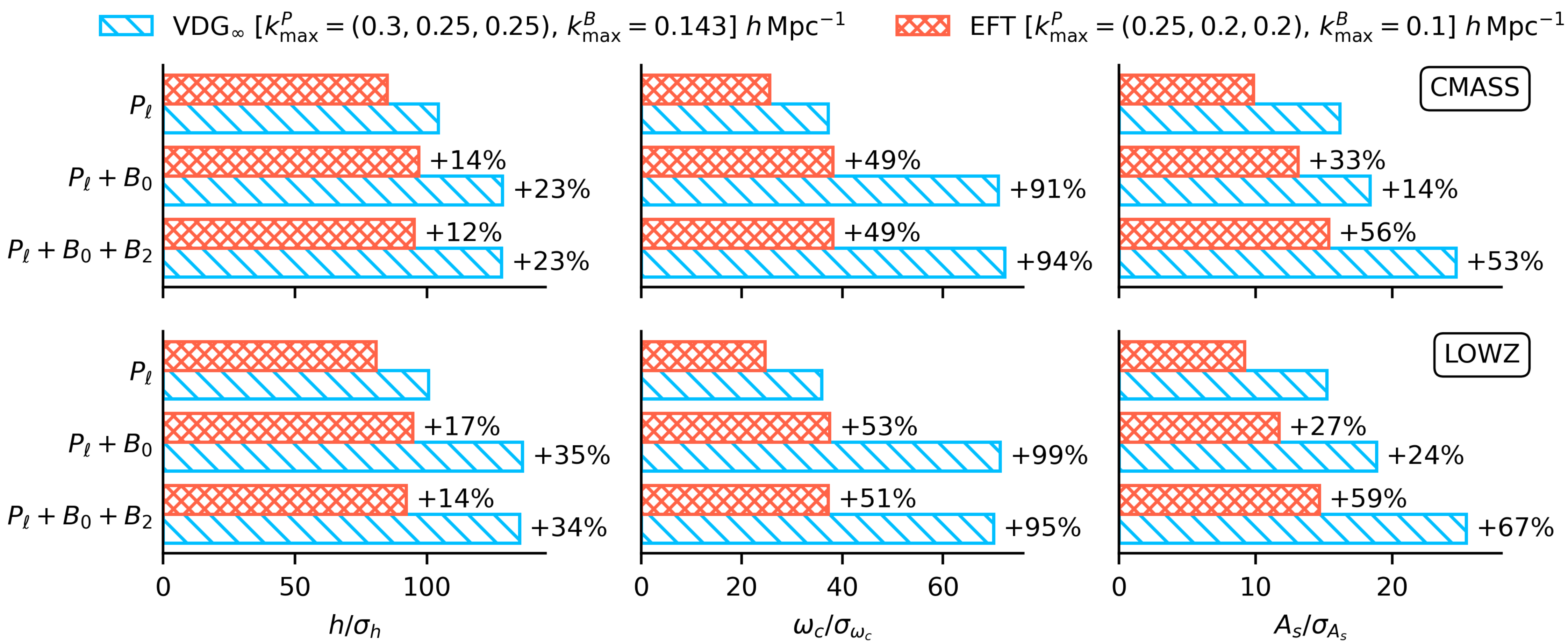


FIG. 17. Inverse relative uncertainties on the three cosmological parameters h, ω_c, and A_s obtained for the CMASS and LOWZ samples (top and bottom rows, respectively). Each panel depicts three cases corresponding to the power spectrum multipoles alone and in combination with either the bispectrum monopole, or bispectrum monopole and quadrupole. The percentages indicate the relative improvement over the power spectrum alone. Scale cuts for each model were chosen to maximise constraining power under the condition that FoB $< 1\sigma$.

Eggemeier *et al.* (2025)

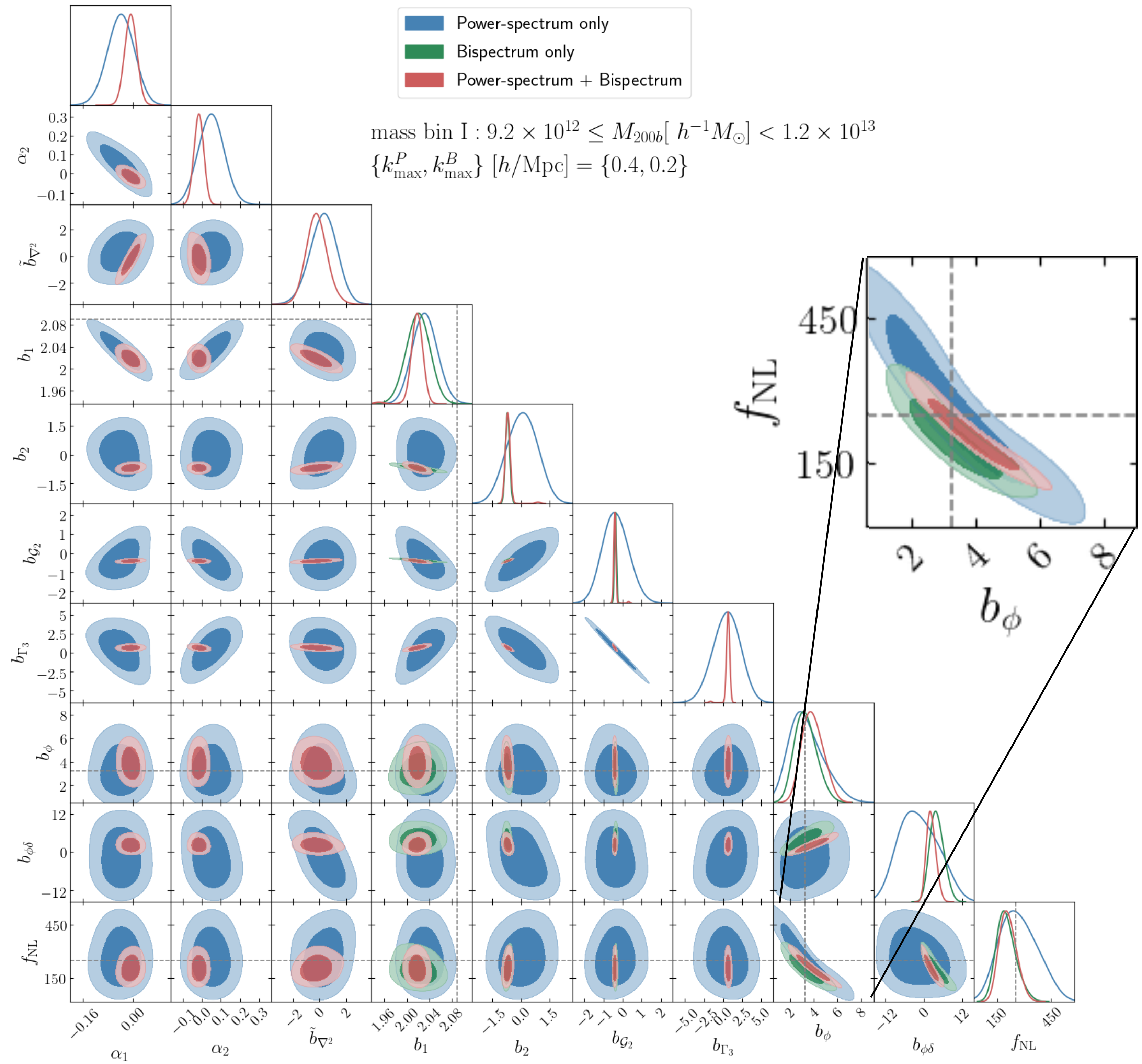
Beyond ΛCDM: Primordial non-Gaussianity

Test of the power spectrum & bispectrum model in real space

Eos simulations, $80\,h^{-3}\,\mathrm{Gpc}^3$
Halo catalogs

Significant improvement (factor of 5) over power spectrum only

Also from the reduction of the $f_{\rm NL} - b_\phi$ degeneracy



Moradinezhad *et al.* (2021)

Bispectrum at one-loop: *matter*

The reach of perturbative models
(as a function of survey volume)

Tree-level
(Fry, 1984)

1-loop SPT
(Scoccimarro, 1997; 1998)

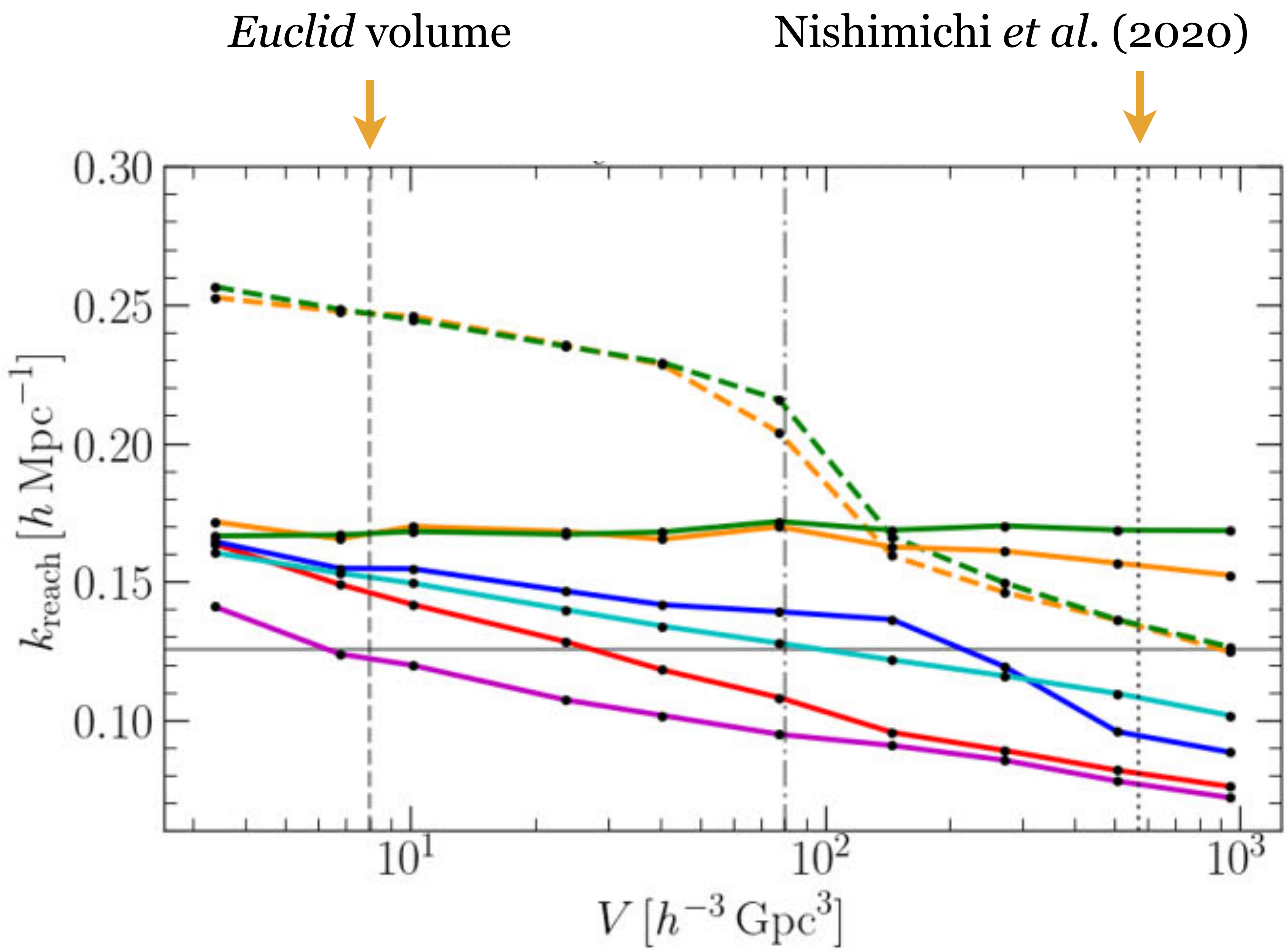
Renormalised PT
(Bernardeau, Crocce & Scoccimarro, 2008; 2012)

Lagrangian PT
(Matsubara, 2008)

EFTofLSS
EFTofLSS (IR-res)
(Angulo *et al.*, 2015;
Baldauf *et al.*, 2015)

But also more phenomenological models are available:
Scoccimarro & Couchmann (2001);
Gil-Marín *et al.* (2012)

Much to gain to go to one-loop
... but numerically demanding!



Alkhanishvili *et al.* (2019)

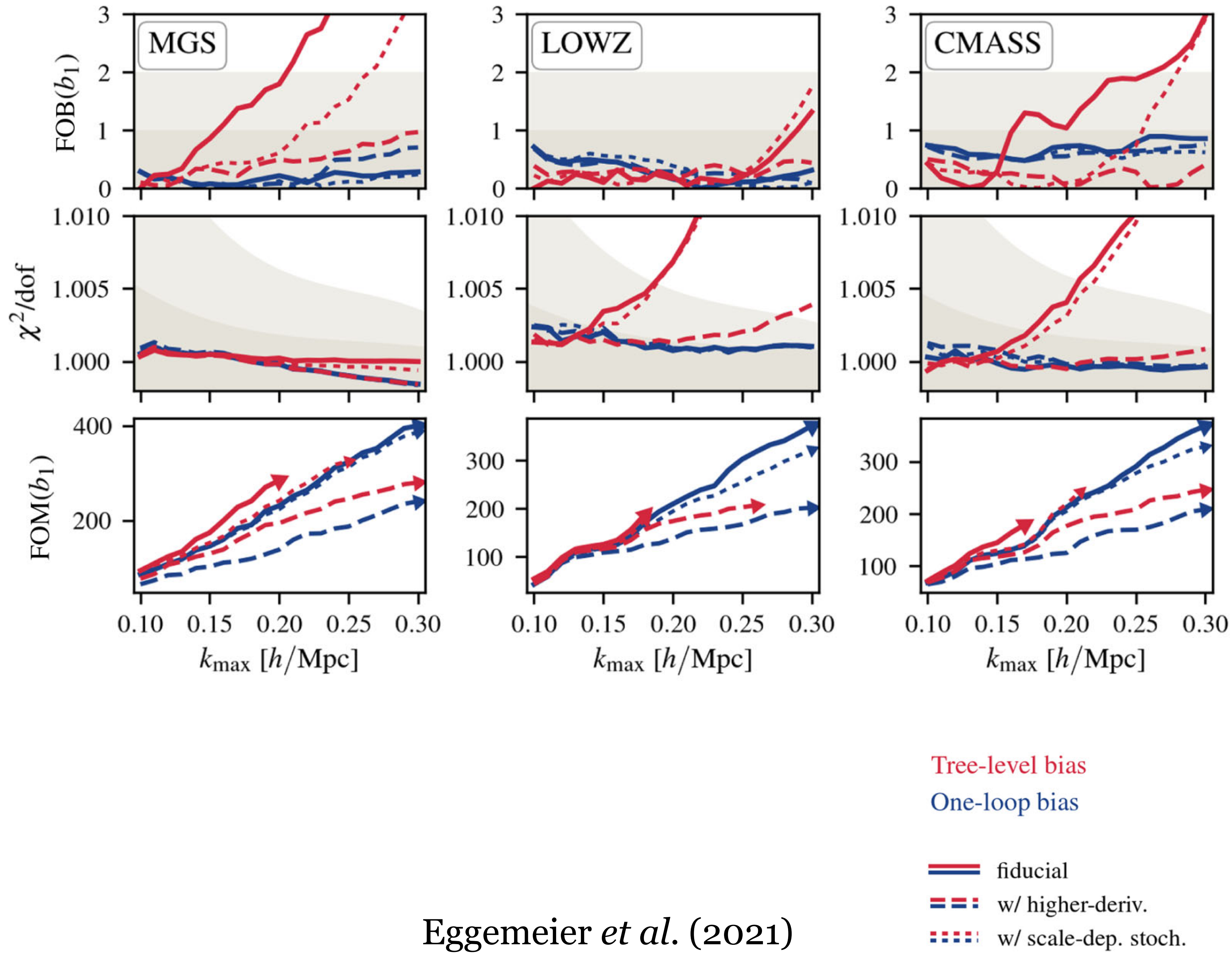
Bispectrum at one-loop: *galaxies in real-space*

Test of 1-loop bispectrum bias model in real space

HOD galaxies (CMASS, LOWZ)
& halos
$6\,\mathrm{Gpc}^3 h^{-3}$

8 parameters (tree-level B)
15 parameters (one-loop B)

One-loop corrections greatly extend the reach of the model and its potential to constrain its parameters
(despite their larger number)



Eggemeier *et al.* (2021)

Bispectrum at one-loop: *redshift-space monopole*

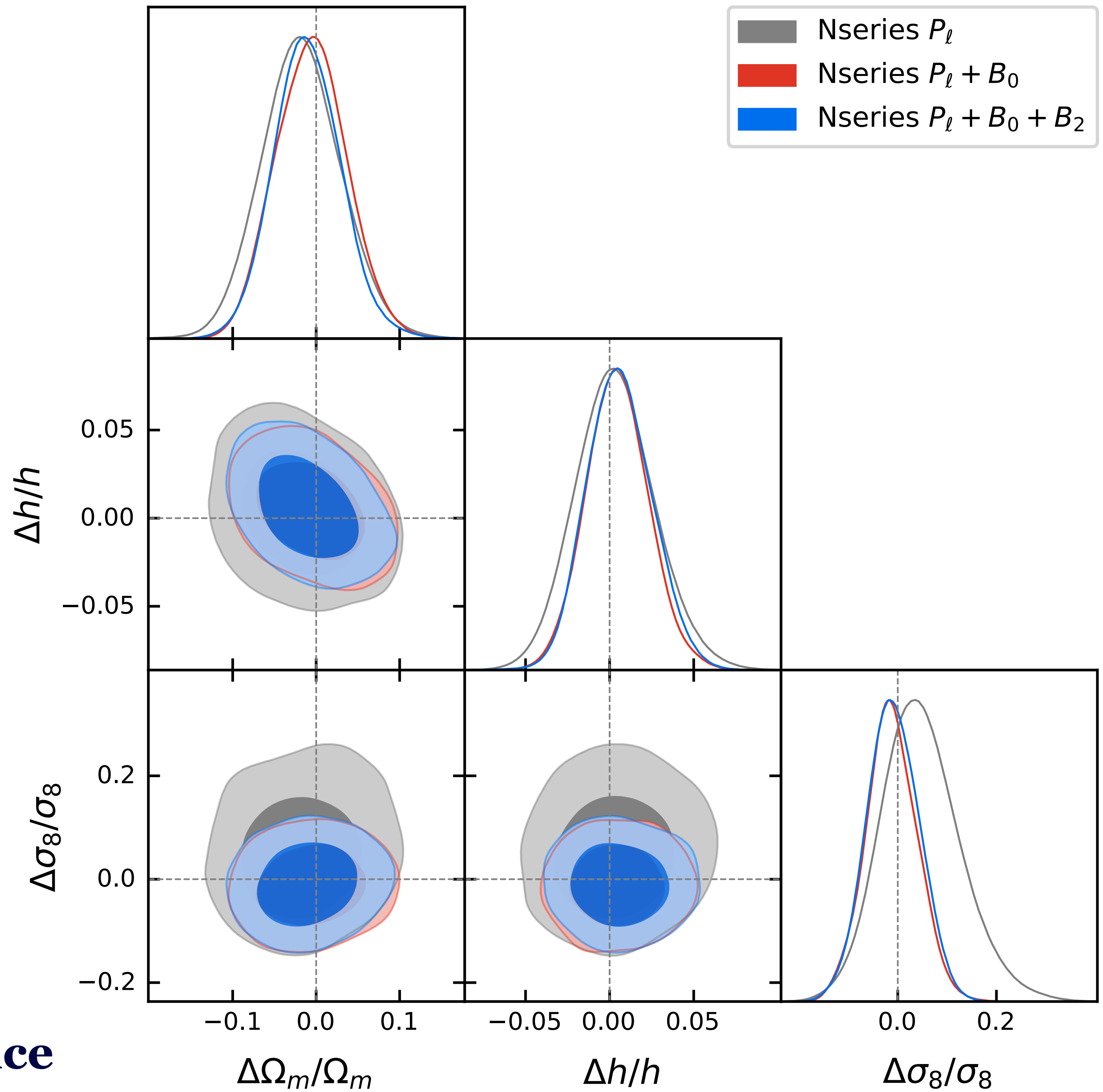
Test of the bispectrum model:
B_0 at 1-loop
B_2 tree-level

CMASS HOD mocks + window

Significant improvement adding B_0 at one-loop wrt to power spectrum only, much less adding B_2 tree-level (but very limited number triangles in this case ...)

See also Philcox *et al.* (2023): not much larger improvement on σ_8 at one-loop (10% over tree-level)

In both cases the **cosmology-dependence of loop corrections is fixed**.
See Anastasiou *et al.* (2024), Bakx *et al.* (2024) for effort to speed up evaluation



D'Amico *et al.* (2022)

Beyond ΛCDM: Tests with Primordial non-Gaussainity

Test of the power spectrum & bispectrum model in real space

Eos simulations, $80\,h^{-3}\,\mathrm{Gpc}^3$ Halo catalogs

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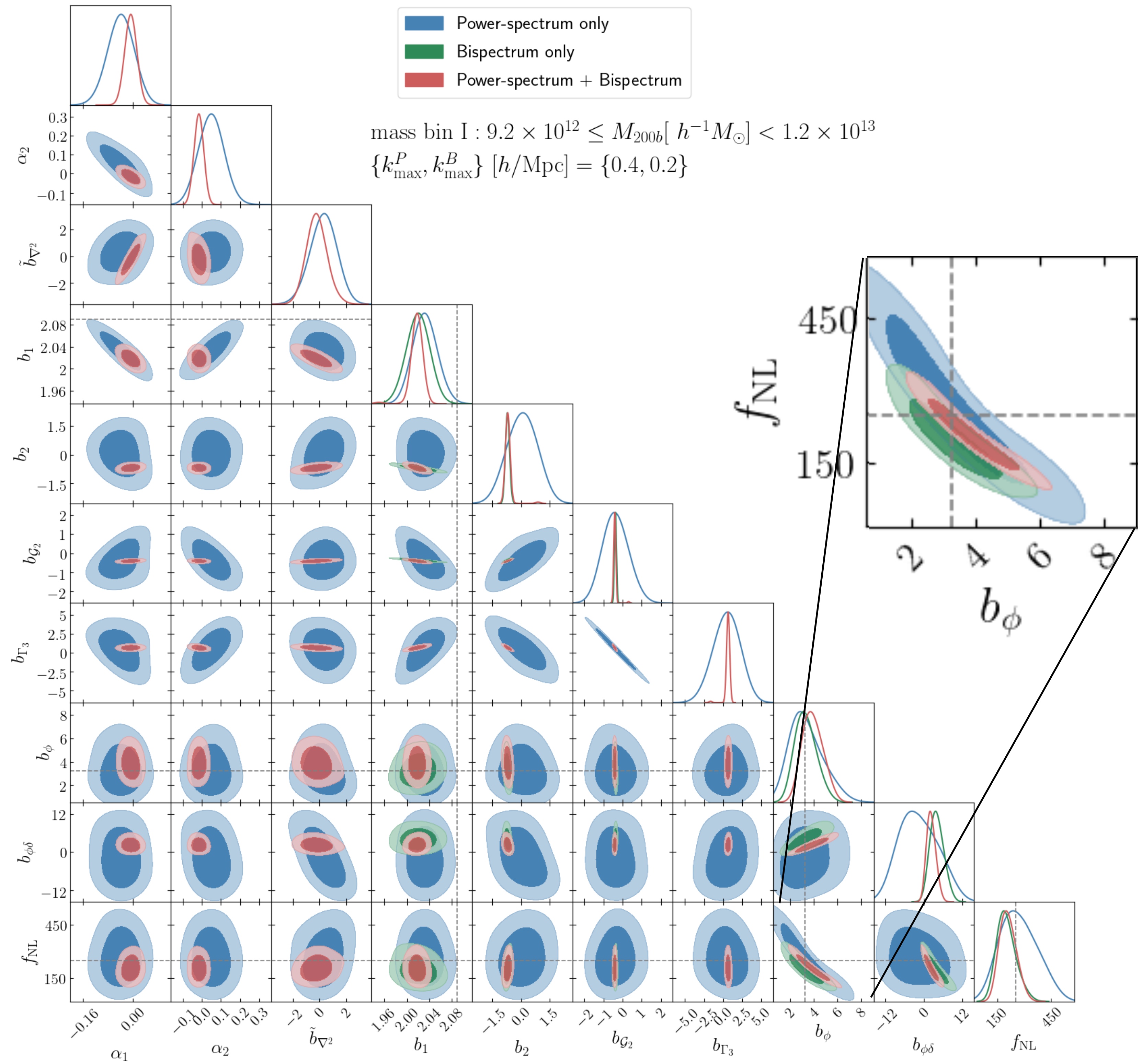


Moradinezhad *et al.* (2021)

Window convolution

The convolution of the bispectrum prediction with the window function is a problem ..

$$\tilde{B}(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} B_W(\mathbf{k}_1 - \mathbf{p}_1, \mathbf{k}_2 - \mathbf{p}_2) B(\mathbf{p}_1, \mathbf{p}_2)$$

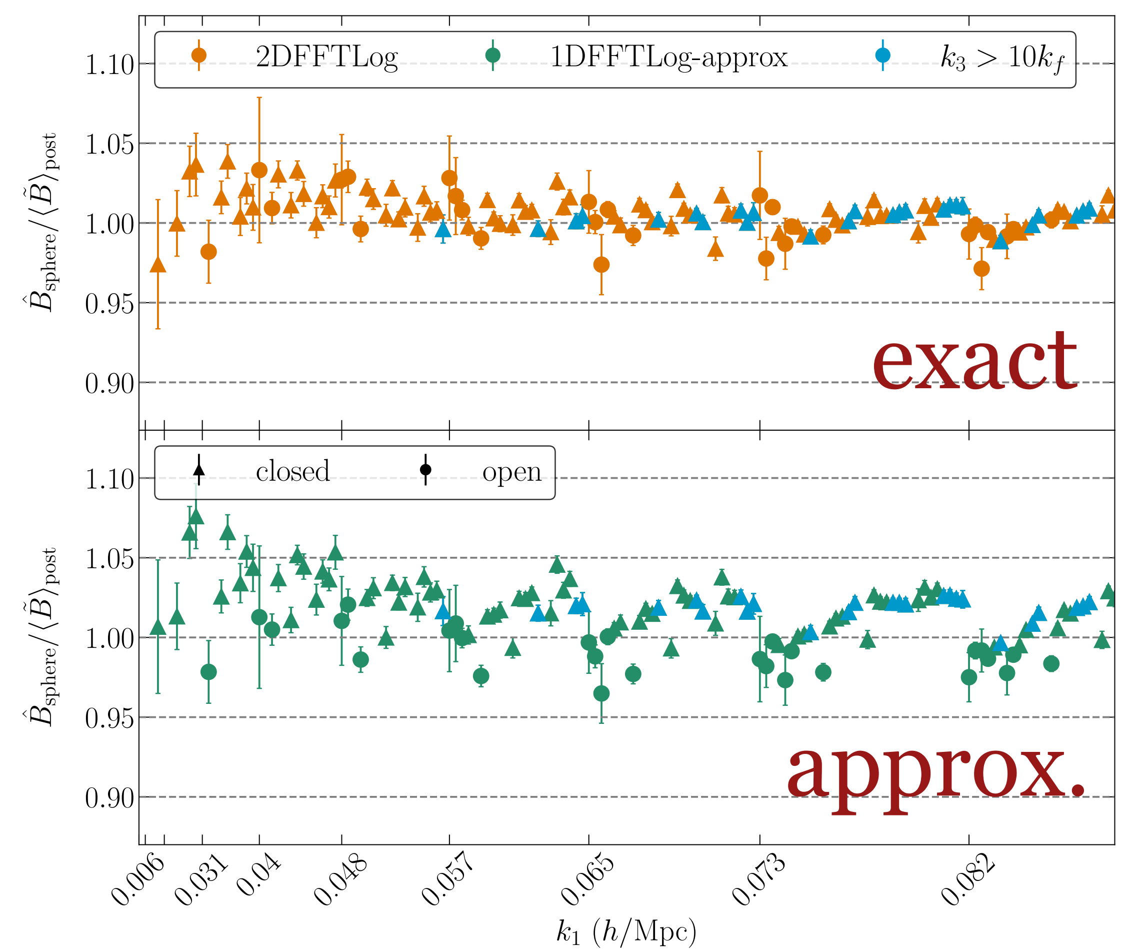
Three approaches so far:

- "Tree-level" approximation (Gil-Marín *et al.*, 2015)

  $\tilde{B} \simeq 2 Z_1(\mathbf{k}_1) Z_1(\mathbf{k}_2) Z_2(\mathbf{k}_1, \mathbf{k}_2) \tilde{P}(k_1) \tilde{P}(k_2) + \text{perm}\,.$

- Cubic ("windowless") estimator (Philcox, 2021)

- Exact convolution (Pardede *et al.*, 2022) →



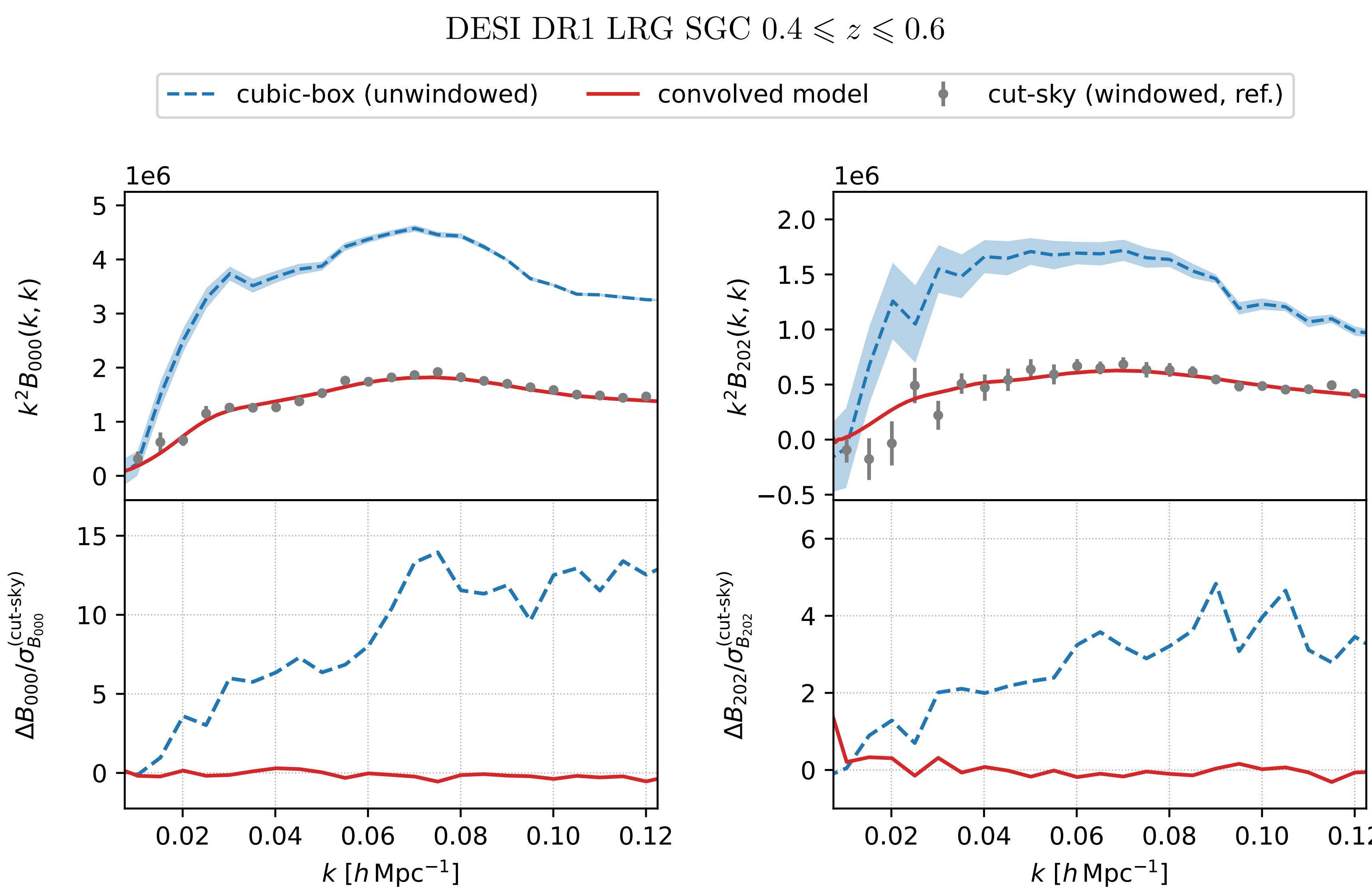
Pardede *et al.* (2022)

Window convolution: Tripolar Spherical Harmonics decomposition

$$B(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{n}}) = \sum_{\ell_1+\ell_2+L \in 2\mathbb{Z}} B_{\ell_1 \ell_2 L}(k_1, k_2)\, S_{\ell_1 \ell_2 L}(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \hat{\mathbf{n}})$$

Sugiyama *et al.* (2019)

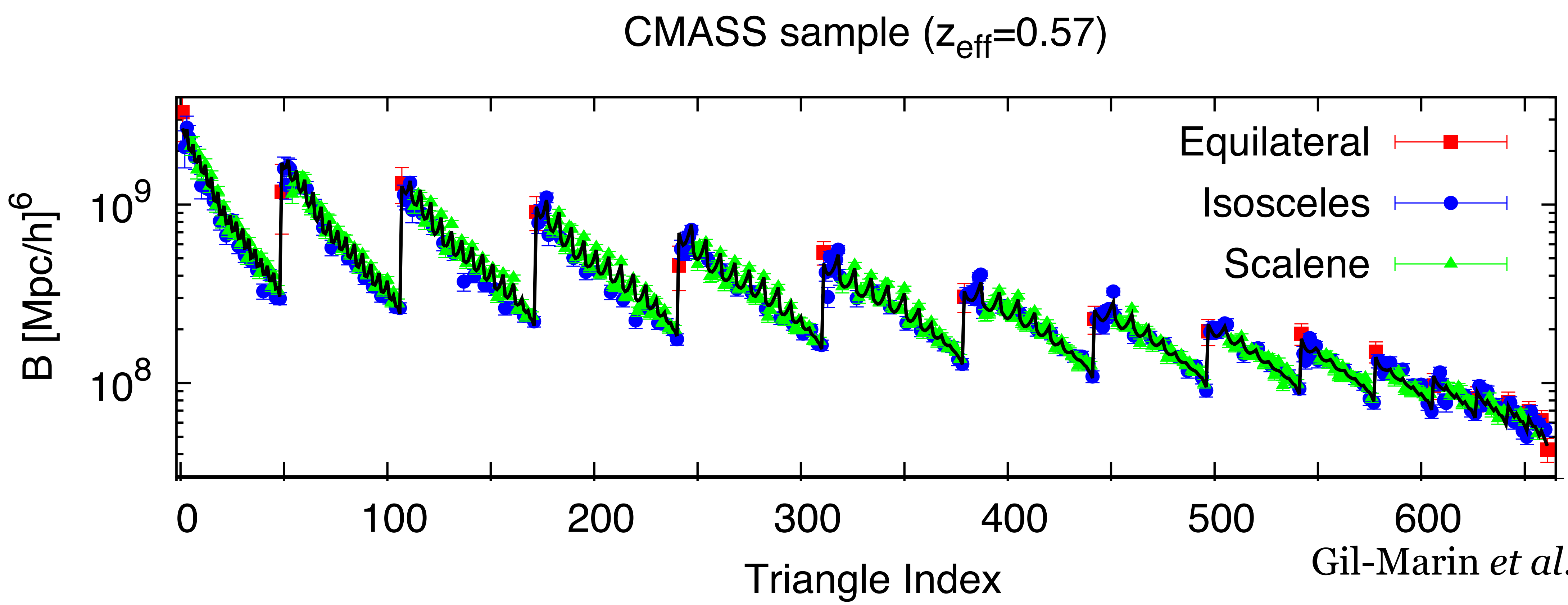
$$S_{\ell_1 \ell_2 L}(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \hat{\mathbf{n}}) = H^{-1}_{\ell_1 \ell_2 L} \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} y^{m_1}_{\ell_1}(\hat{\mathbf{k}}_1)\, y^{m_2}_{\ell_2}(\hat{\mathbf{k}}_2)\, y^{M}_{L}(\hat{\mathbf{n}})$$



Wang *et al.* (2025)

Bispectrum covariance

The bispectrum signal is **distributed over a large number of configurations**



Gil-Marin *et al.* (2017)

A robust, numerical estimates of such a **large covariance matrix** requires a **large number of mocks**

Bispectrum covariance

$$\mathbf{C}^{B}_{\ell_1\ell_2}(t_i,t_j) = \mathbf{C}^{B\,(PPP)}_{\ell_1\ell_2}(t_i,t_j) + \mathbf{C}^{B\,(BB)}_{\ell_1\ell_2}(t_i,t_j) + \mathbf{C}^{B\,(PT)}_{\ell_1\ell_2}(t_i,t_j) + \mathbf{C}^{B\,(P_6)}_{\ell_1\ell_2}(t_i,t_j)$$

$$\mathbf{C}^{B\,(PPP)}_{\ell_1\ell_2}(t_i,t_j) = \frac{(2\ell_1+1)(2\ell_2+1)}{N_{t_i}k_f^3}\delta^K_{ij}\sum_{\ell_3,\ell_4,\ell_5} P_{\mathrm{tot},\ell_3}(k_{1,i})P_{\mathrm{tot},\ell_4}(k_{2,i})P_{\mathrm{tot},\ell_5}(k_{3,i}) \times R_{\ell_1,\ell_2,\ell_3,\ell_4,\ell_5}(k_{1,i},k_{2,i},k_{3,i})\,,$$

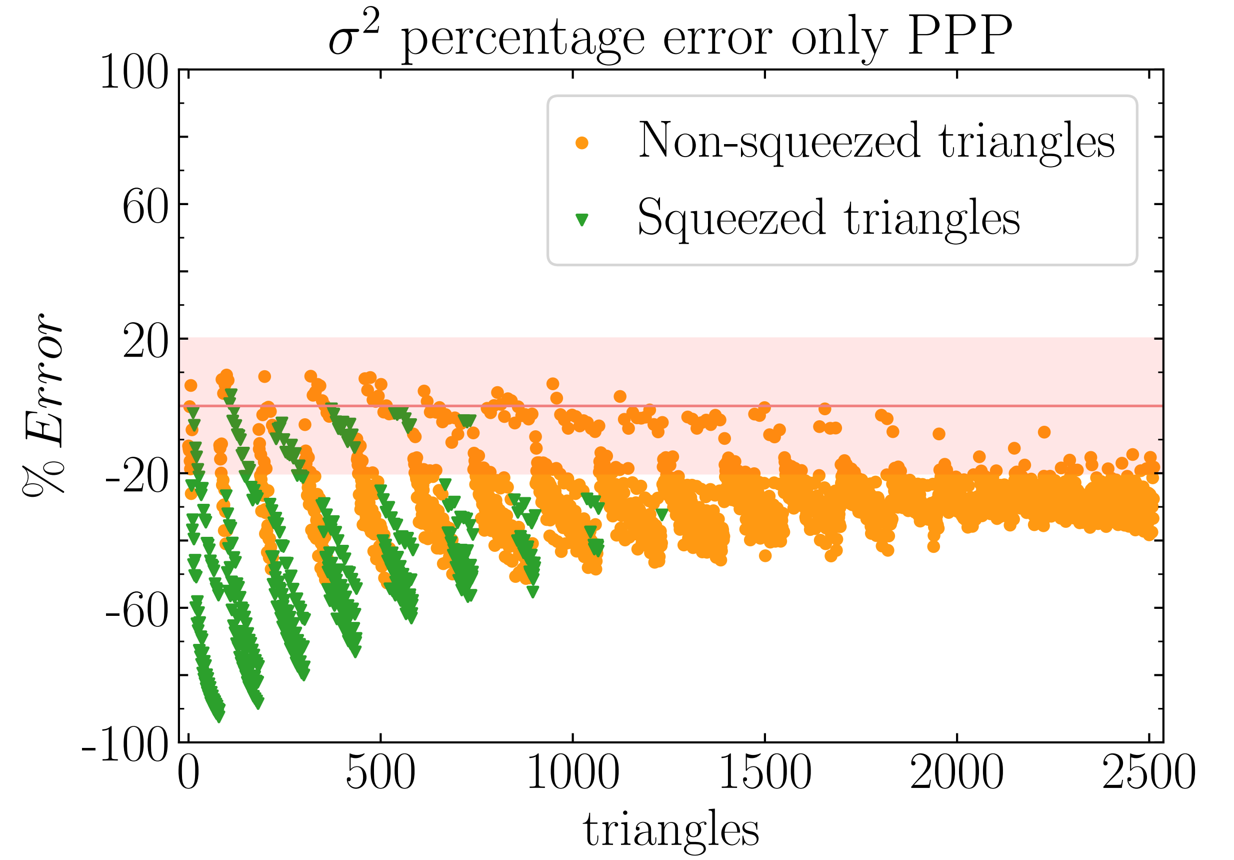
$$\mathbf{C}^{B\,(BB)}_{\ell_1\ell_2}(t_i,t_j) = (2\ell_1+1)(2\ell_2+1)\sum_{\ell_3,m_3}\sum_{\ell_4,m_4} B^{m_3}_{\mathrm{tot},\ell_3}(k_{1,i},k_{2,i},k_{3,j})\,B^{m_4}_{\mathrm{tot},\ell_4}(k_{1,j},k_{2,j},k_{3,i}) \times S^{(3,3)}_{\ell_1,\ell_2,\ell_3,\ell_4;m_3,m_4}(t_i,t_j) + 8\,\mathrm{perm.}\,,$$

$$\mathbf{C}^{B\,(PT)}_{\ell_1\ell_2}(t_i,t_j) = \frac{(2\ell_1+1)(2\ell_2+1)}{N_{t_i}N_{t_j}}\sum_{\mathbf{q}'s\in\mathbf{k}^i s}\sum_{\mathbf{p}'s\in\mathbf{k}^j s}\delta_K(\mathbf{q}_{123})\delta_K(\mathbf{p}_{123})\delta_K(\mathbf{q}_3+\mathbf{p}_{12})\delta_K(\mathbf{p}_3+\mathbf{q}_{12}) P_{\mathrm{tot}}(\mathbf{q}_3)T_{\mathrm{tot}}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{p}_1,\mathbf{p}_2)\mathcal{L}_{\ell_1}(\hat{q}_1\cdot\hat{n})\mathcal{L}_{\ell_2}(\hat{p}_1\cdot\hat{n}) + 8\ \mathrm{perm.}\,.$$

$$\mathbf{C}^{B\,(P_6)}_{\ell_1\ell_2}(t_i,t_j) = \ \ldots$$

(and no window effect ...)

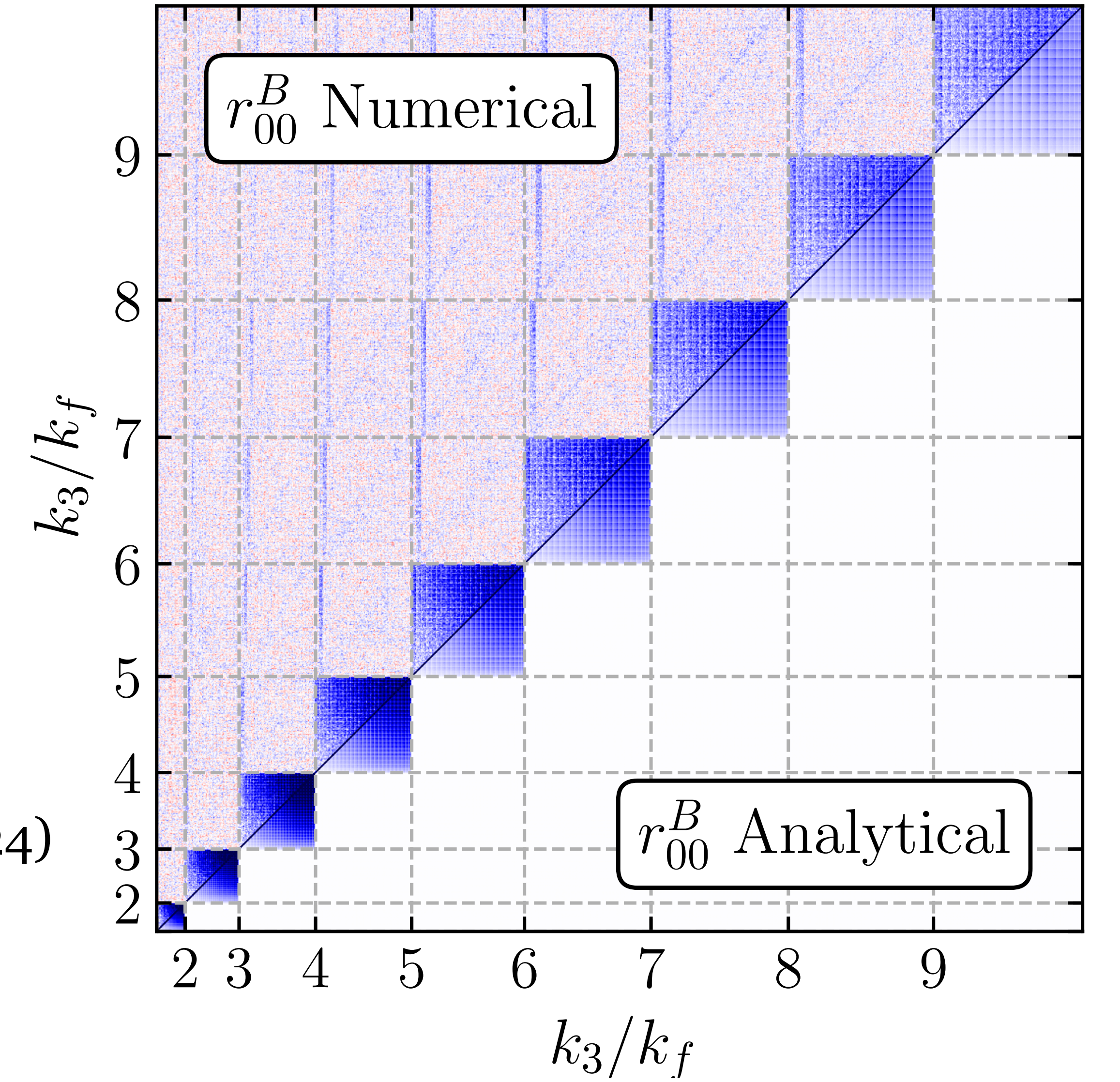
Bispectrum *non-Gaussian* covariance



Unlike the power spectrum, for squeezed bispectrum configurations the non-Gaussian contribution can be larger than the Gaussian one

Barreira (2019)
Biagetti, Castiblanco *et al.* (2022)
Floss *et al.* (2023)

An approximate, simple prediction can be found, also in redshift space (no window)

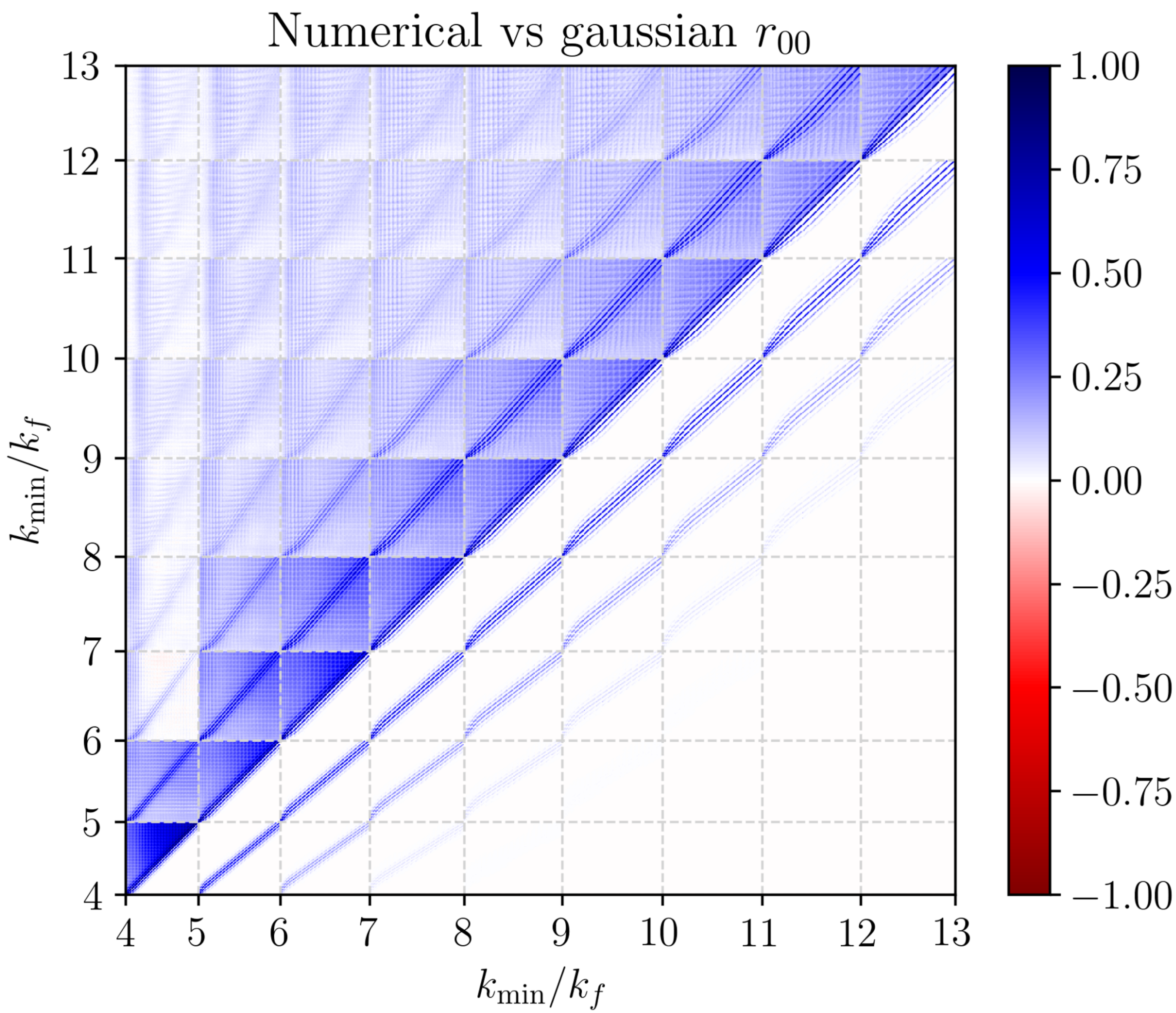


Salvalaggio, Castiblanco *et al.* (2024)

See also Sugiyama *et al.* (2022)

Bispectrum covariance: window function effects

Something can be done, also to include the window effects



Salvalaggio *et al.* (In prep.)

To sum up: bispectrum issues

- **The model**
  tree-level PT *vs* **one-loop PT** *vs* **phenomenological**
  we probably need to go beyond tree-level, but loop + AP integrations are challenging

- **Anisotropy**
  monopole *vs* **monopole + quadrupole**
  we already have multipoles estimators, so …

- **Window function**
  approximated *vs* **exact** *vs* **windowless**
  it would be very *nice to test both exact convolution and windowless*

- **Covariance**
  numerical *vs* theoretical
  I cannot see how we can limit ourselves to one of the two approaches … we must do everything we can!

- **Alternative estimators**
  Skew-spectra (Schmittfull *et al.*, 2015; Moradinezhad *et al.* 2020; …)
  Tri-polar Spherical Harmonic Decomposition (Sugiyama *et al.*, 2017)
  Modal estimator (Fergusson *et al.*, 2012; Byun *et al.*, 2021) … and more …

BOSS data: Gil-Marin *et al.* (2017)

- *data:* monopole (825 triangles, $\Delta k = 0.01 h\,\mathrm{Mpc}^{-1}$)

- *model*: fit to N-body + tree-level bias & RSD (+AP)
  $0.03\,h\,\mathrm{Mpc}^{-1} \le k \le 0.18\,h\,\mathrm{Mpc}^{-1}$
  $0.03\,h\,\mathrm{Mpc}^{-1} \le k \le 0.22\,h\,\mathrm{Mpc}^{-1}$

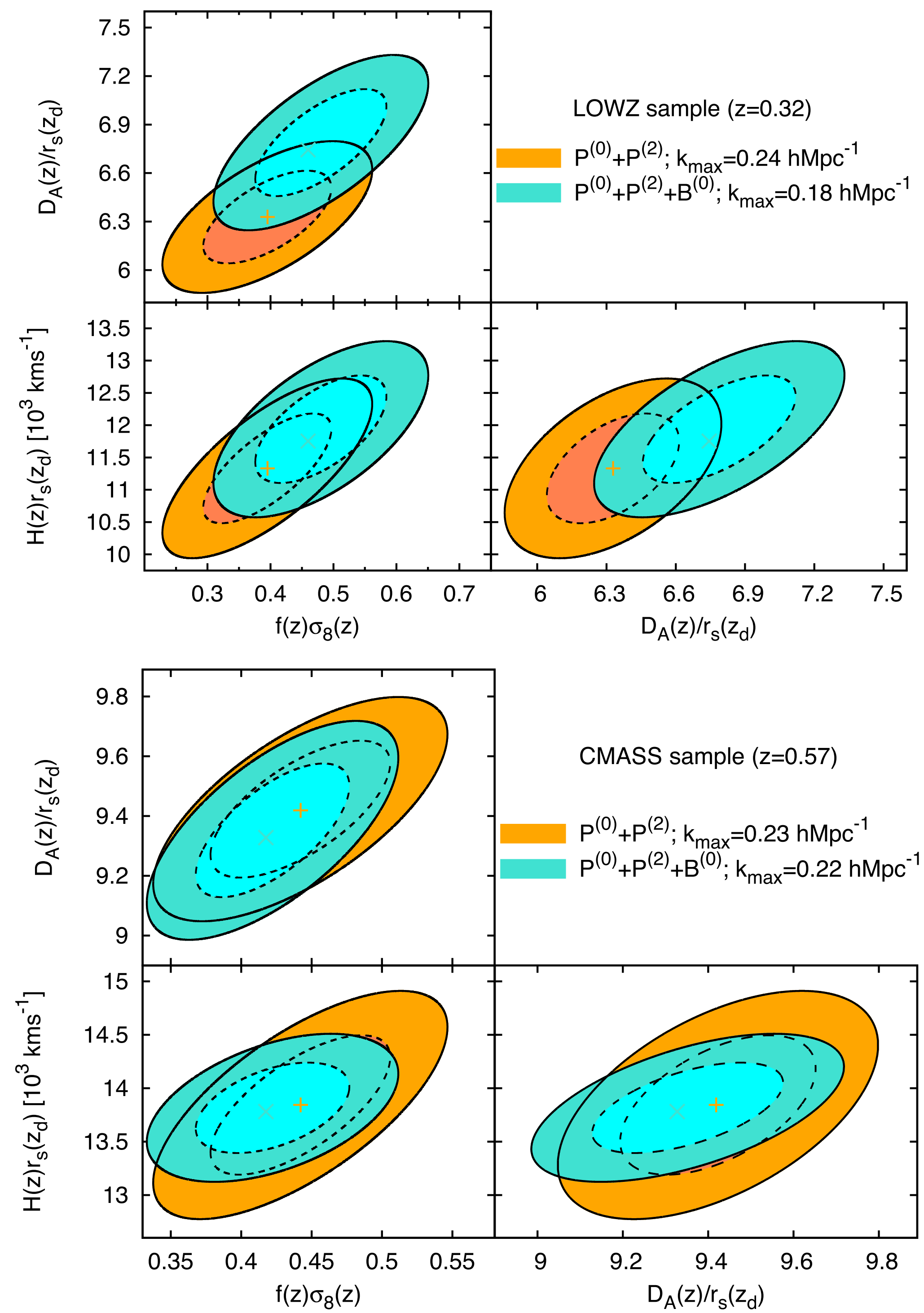
- *window*: approximation

$$\widetilde{B} \simeq Z_1(\mathbf{k}_1) Z_1(\mathbf{k}_2) Z_2(\mathbf{k}_1, \mathbf{k}_2) \widetilde{P}(k_1) \widetilde{P}(k_2)$$

- *covariance*: numerical (2048 Patchy mocks)

- *analysis*: template fitting

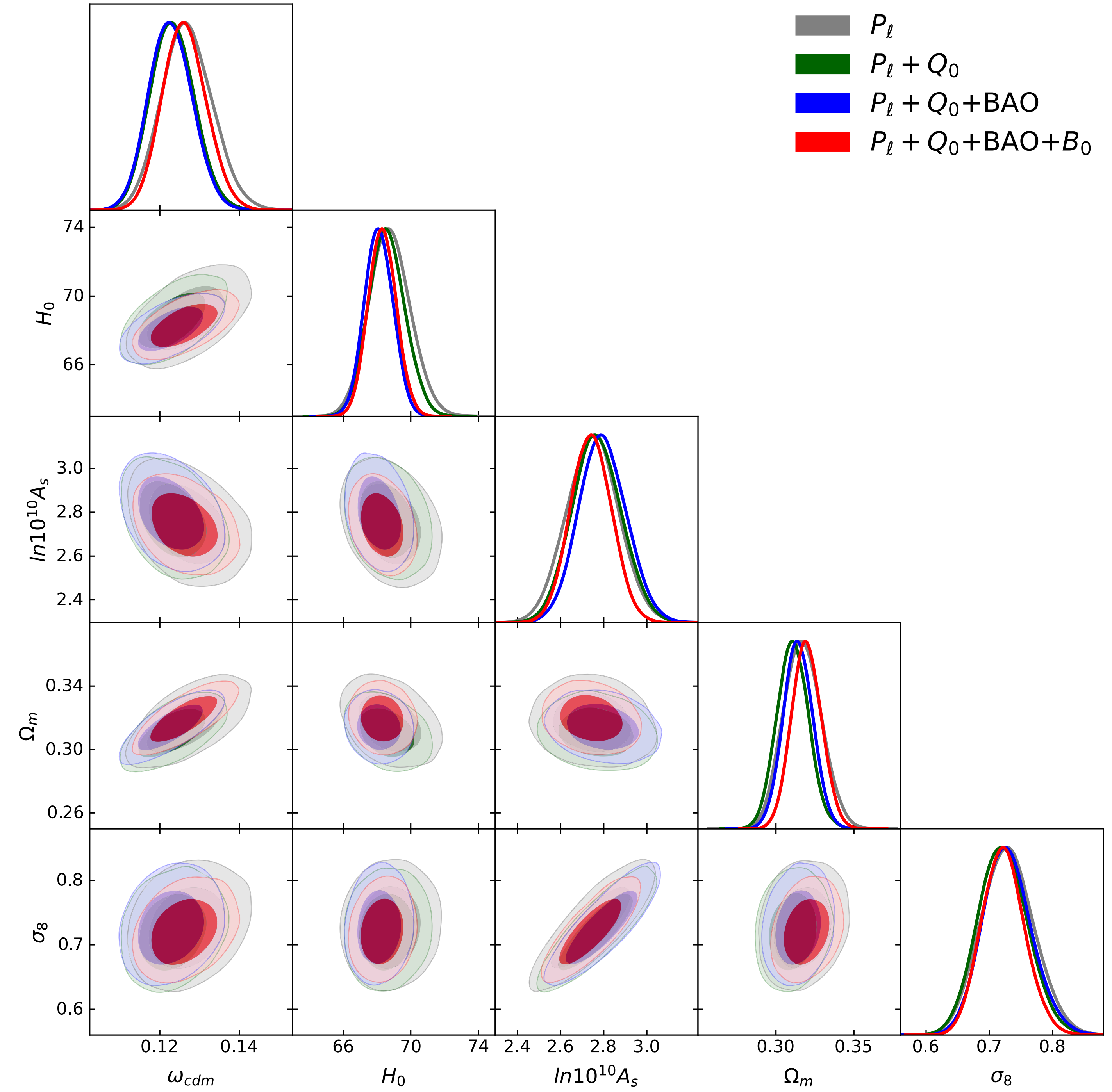
$$\{b_1, b_2, A_{\rm noise}, \sigma^P_{\rm FoG}, \sigma^B_{\rm FoG}, f, \sigma_8, \alpha_\parallel, \alpha_\perp\}.$$



Significant improvement, up to 50%
(for CMASS)

BOSS data: Philcox & Ivanov (2022)

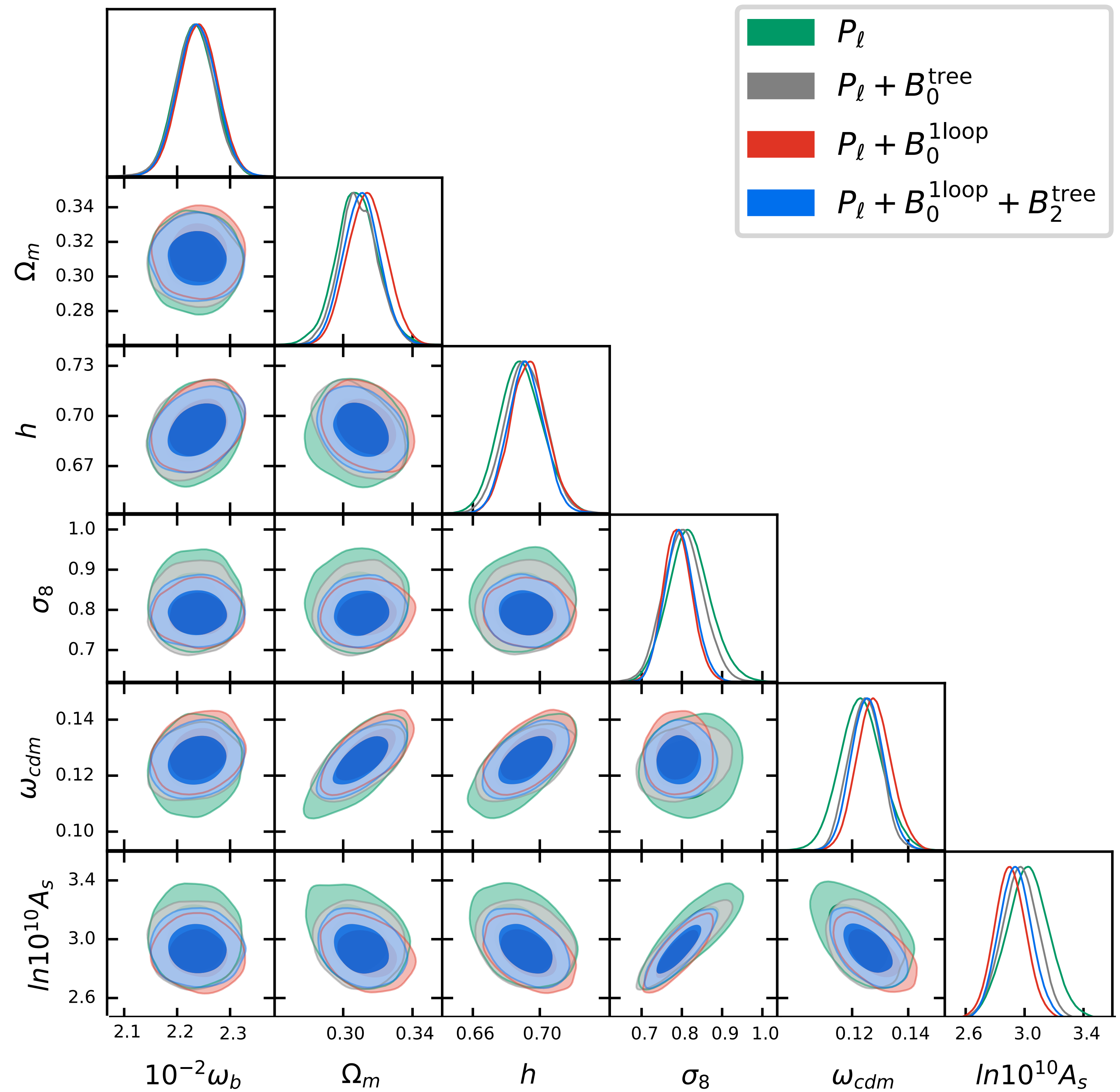
- *data:* monopole (62 triangles, $\Delta k = 0.01 h\mathrm{Mpc}^{-1}$, $0.01 \leq k \leq 0.08\, h\mathrm{Mpc}^{-1}$)
- *model*: tree-level
- *window*: windowless estimator
- *covariance*: numerical (2048 Patchy mocks)
- *analysis*: full-shape 3/4 cosmo + 13 bias/noise parameters



13% improvement on σ_8

BOSS data: D'Amico *et al.* (2022)

- *data:* monopole & quadrupole (150 triangles for B_0, 9 for B_2, $\Delta k = 0.02\,h\mathrm{Mpc}^{-1}$, $0.02 \leq k \leq 0.21\,h\mathrm{Mpc}^{-1}$ for CMASS)
- *model*: 1-loop for B_0, tree-level for B_2
- *window*: approximation
- *covariance*: numerical (2048 Patchy mocks)
- *analysis*: full-shape 3 cosmo + 12 bias/noise parameters



Significant improvement (30% for σ_8) from one-loop B_0, rather than B_2

BOSS data: Philcox *et al.* (2023)

- *data:* monopole, **quadrupole & hexadecapole** (62 triangles, $\Delta k = 0.01 h\mathrm{Mpc}^{-1}$, $0.01 \leq k \leq 0.08\, h\mathrm{Mpc}^{-1}$)
- *model*: tree-level
- *window*: cubic estimator
- *covariance*: numerical (2048 Patchy mocks)
- *analysis*: full-shape 3/4 cosmo + 13 bias/noise parameters

overall 30% improvements on σ_8 w.r.t. the power spectrum alone

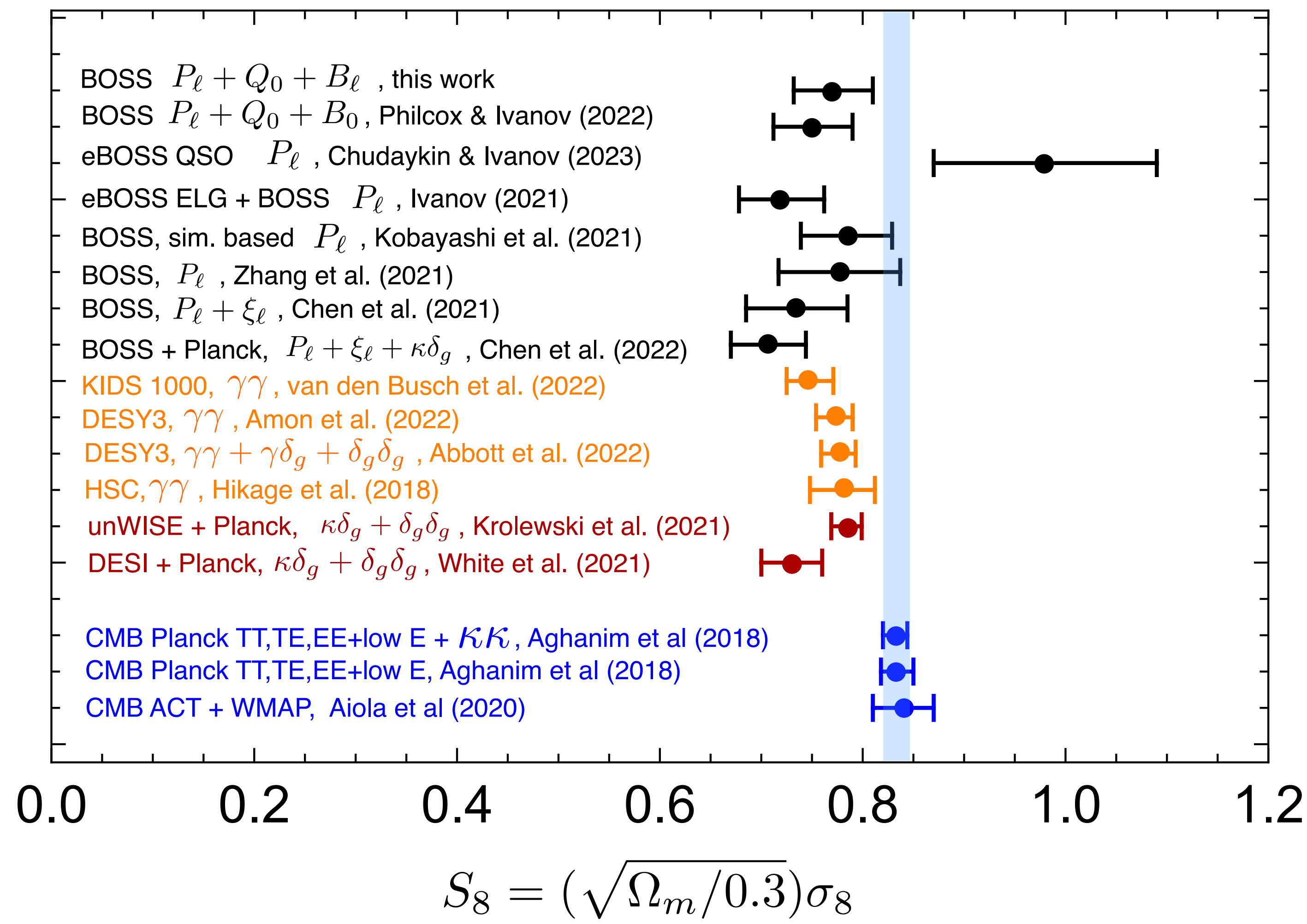


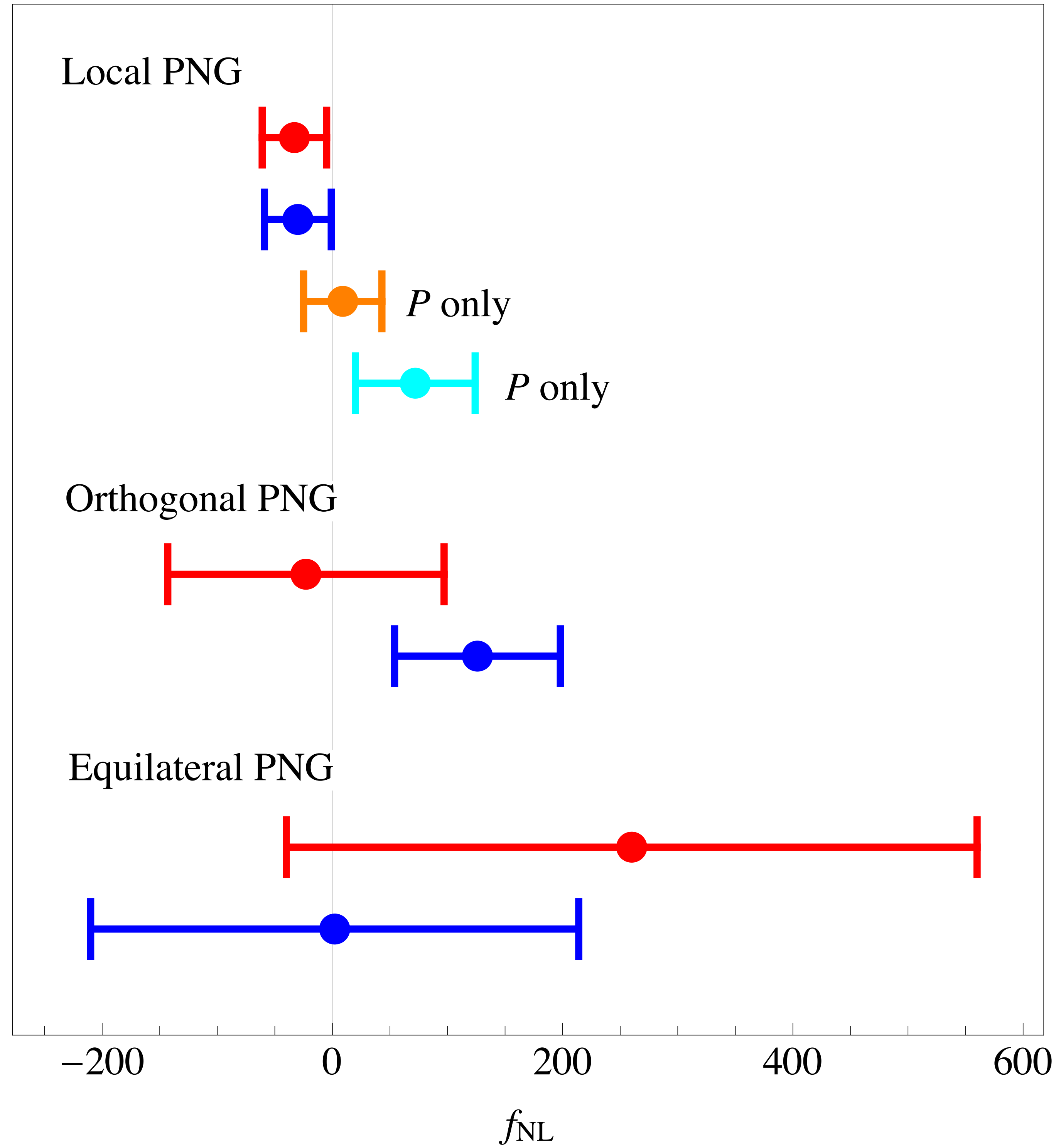
Figure 4. A compilation of some direct and indirect measurements of the growth parameter S_8, from spectroscopic surveys, weak lensing, and the CMB. Errorbars shown approximately correspond to the 68% CL, and our measurement is shown in the top row. Further detail is given in Ref. [70] and the main text.

BOSS analysis beyond ΛCDM: Primordial non-Gaussianity

1. The bispectrum greatly improves constraints on local PNG and …

2. … it *allows* those on single-field inflation models

D'Amico *et al.* (2022)
Cabass *et al.* (2022A, 2022B)



DESI DR1 data: Novel Mast *et al.* (2025)

- *data:* monopole only (100 triangles and some, $\Delta k = 0.01 h\mathrm{Mpc}^{-1}$, $0.01 \leq k \leq 0.12\, h\mathrm{Mpc}^{-1}$)
- *model*: fit to N-body + tree-level bias & phenomenological RSD
- *window*: approximation $\widetilde{B} \simeq Z_1(\mathbf{k}_1) Z_1(\mathbf{k}_2) Z_2(\mathbf{k}_1, \mathbf{k}_2) \widetilde{P}(k_1) \widetilde{P}(k_2)$
- *covariance*: numerical (1000 EZmocks rescaled)
- *analysis*: template fitting

9% improvements on $\alpha_{\rm iso}$ and $f\sigma_8$ w.r.t. the power spectrum alone

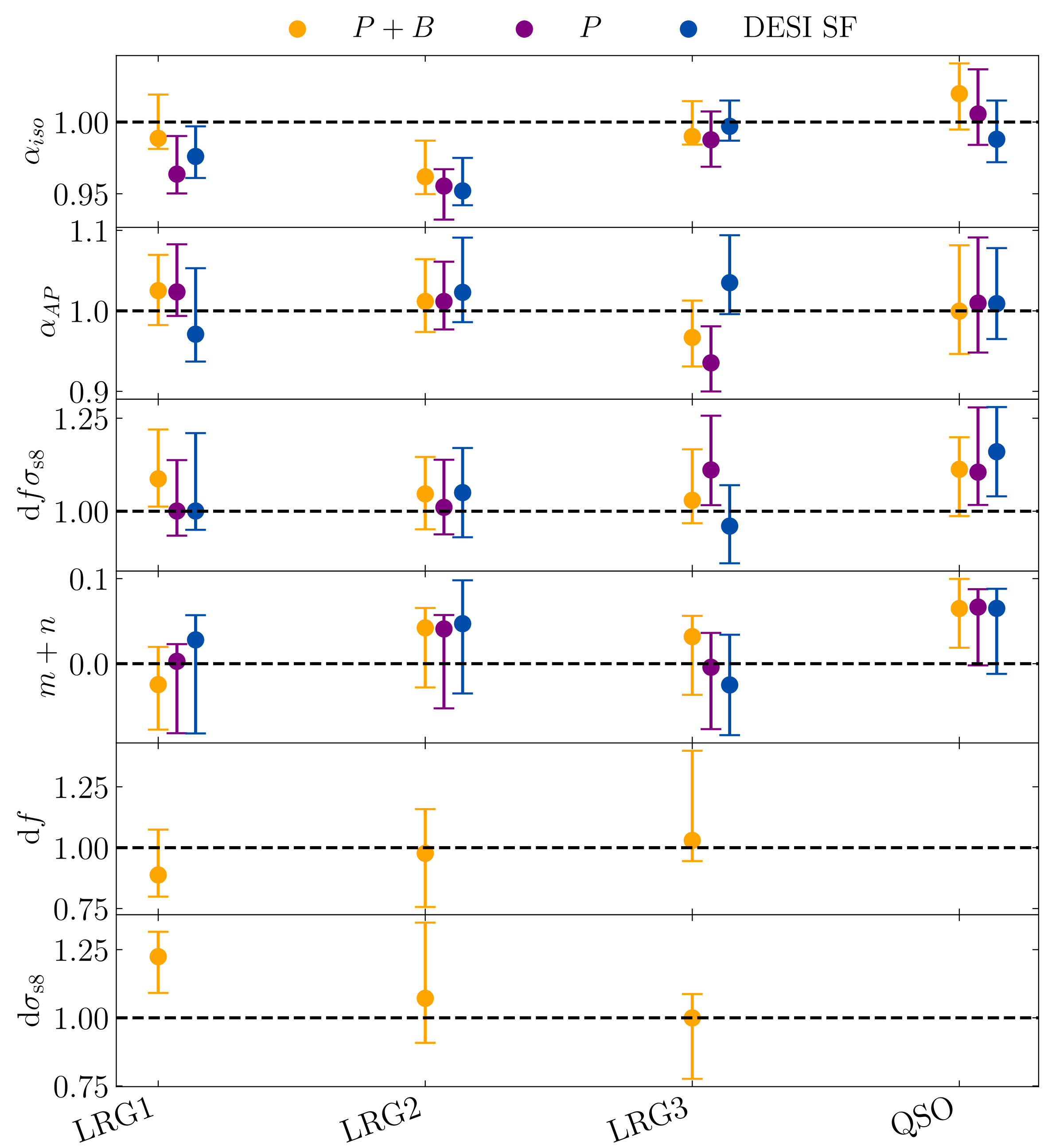
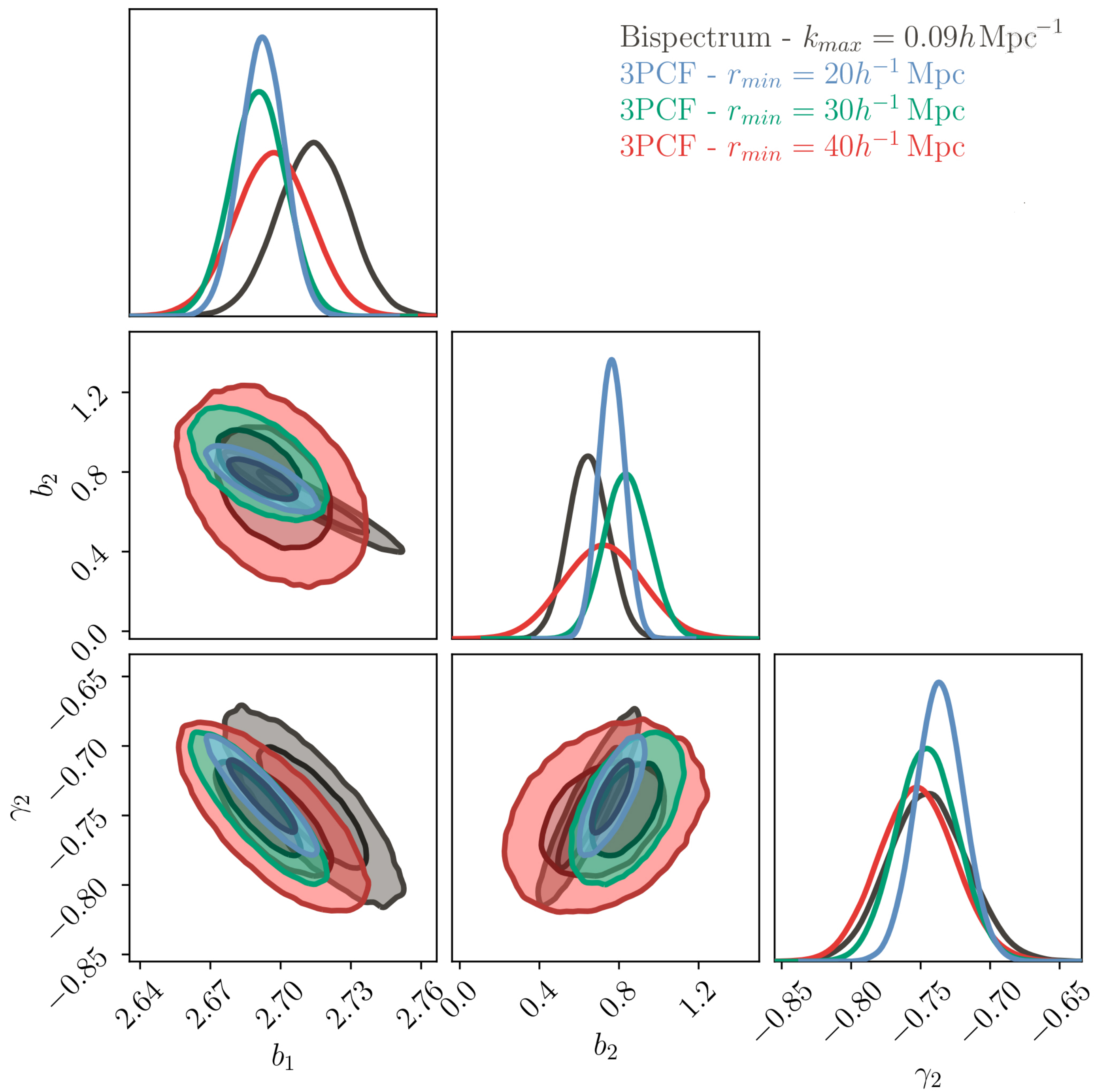


Figure 10. Constraints (68% C.L.) on the parameters $\{\alpha_{\rm iso}, \alpha_{\rm AP}, df\sigma_{s8}, m+n, df, d\sigma_{s8}\}$ for the unblinded DESI data in the four redshift bins (LRG1, LRG2, LRG3, QSO) as indicated in abscissa. The different colours correspond to the baseline $P+B$ (orange), P only (purple) and the official DESI SF (ShapeFit) analysis [64] results (blue). The dashed line marks the fiducial, `c000`, cosmology, and df, $d\sigma_{s8}$, and $df\sigma_{s8}$ are computed with respect to the fiducial model (i.e., $f/f^{\tt c000}$, $\sigma_{s8}/\sigma_{s8}^{\tt c000}$). Despite the analysis differences, the P results presented here are very consistent with the official DESI ones. The addition of the bispectrum breaks the $f\sigma_{s8}$ degeneracy, in addition it tightens the error bars especially on the $df\sigma_{s8}, \alpha_{\rm iso}$ and $m+n$ parameters. A rigorous interpretation of the ShapeFit constraints in terms of ΛCDM and extensions of it will be provided in [84].

3PCF

The fast estimator of Slepian & Eisenstein (2015) made it possible to efficiently measure (almost) all large-scale configurations

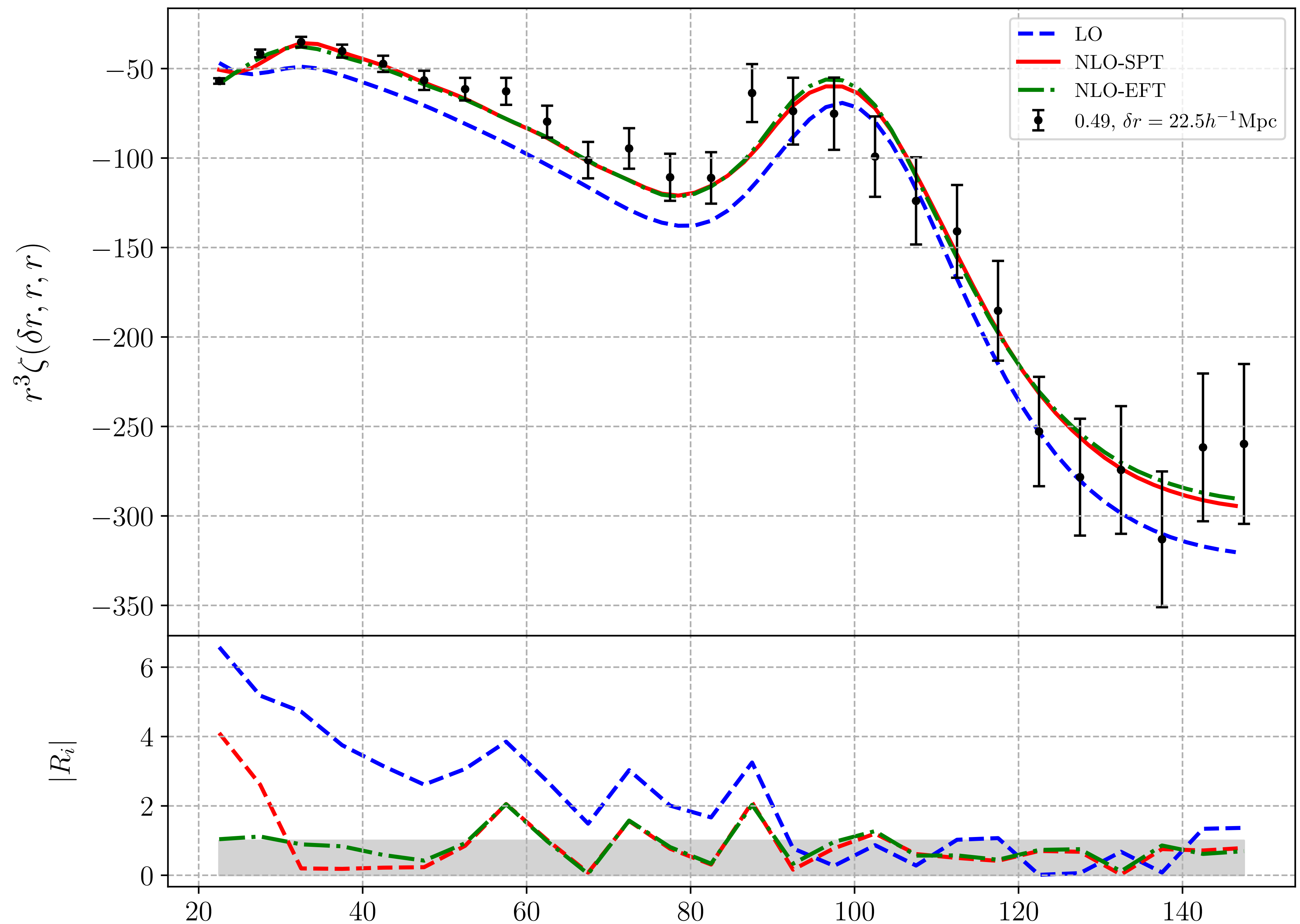


Veropalumbo *et al.* (2022)

3PCF: PT models

The model has been so far tree-level

Something is moving for the one-loop prediction with 2D-FFTlog: an application to the **matter 3PCF**

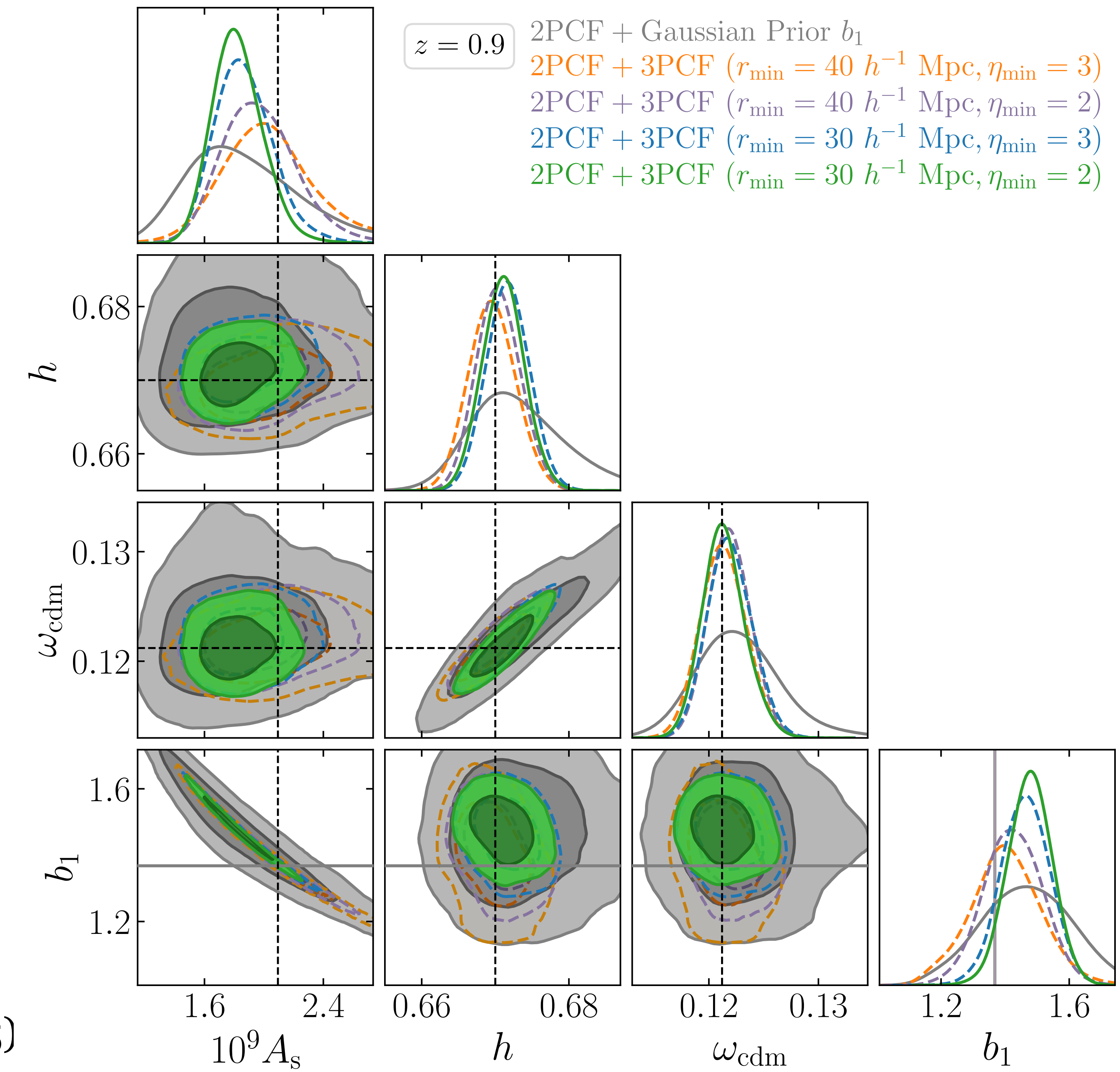


Guidi *et al.* (2023)

3PCF: fast model evaluation

First 2PCF+3PCF full-shape analysis based on an emulator (for tree-level model)

In real space (redshift space coming up)



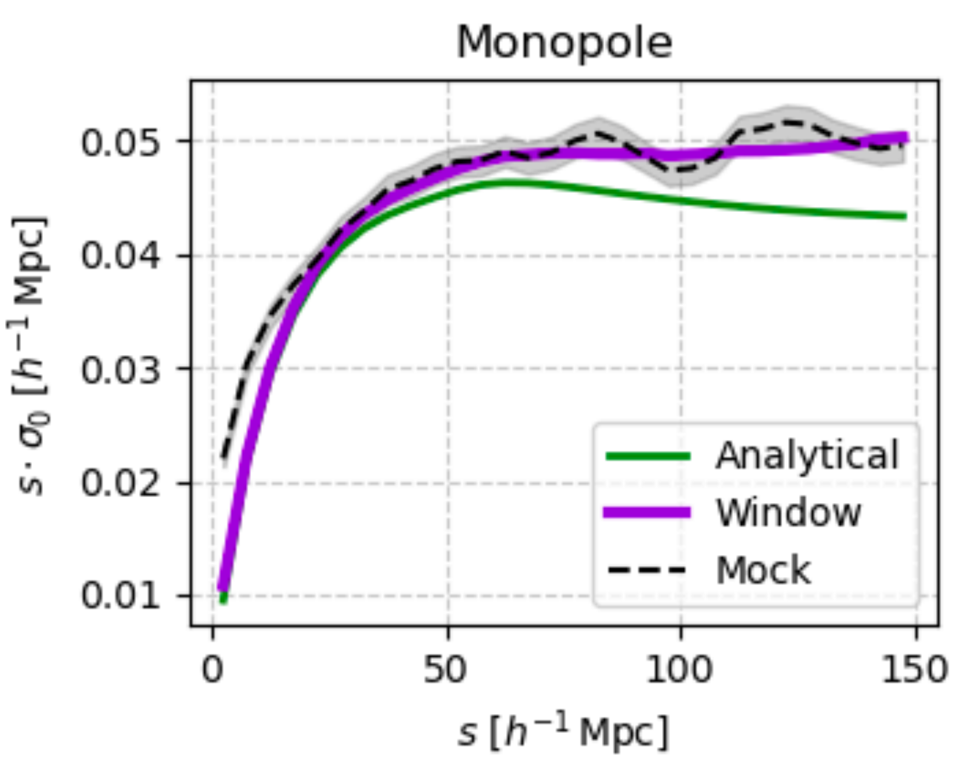
Euclid collaboration: Guidi *et al.* (2025)

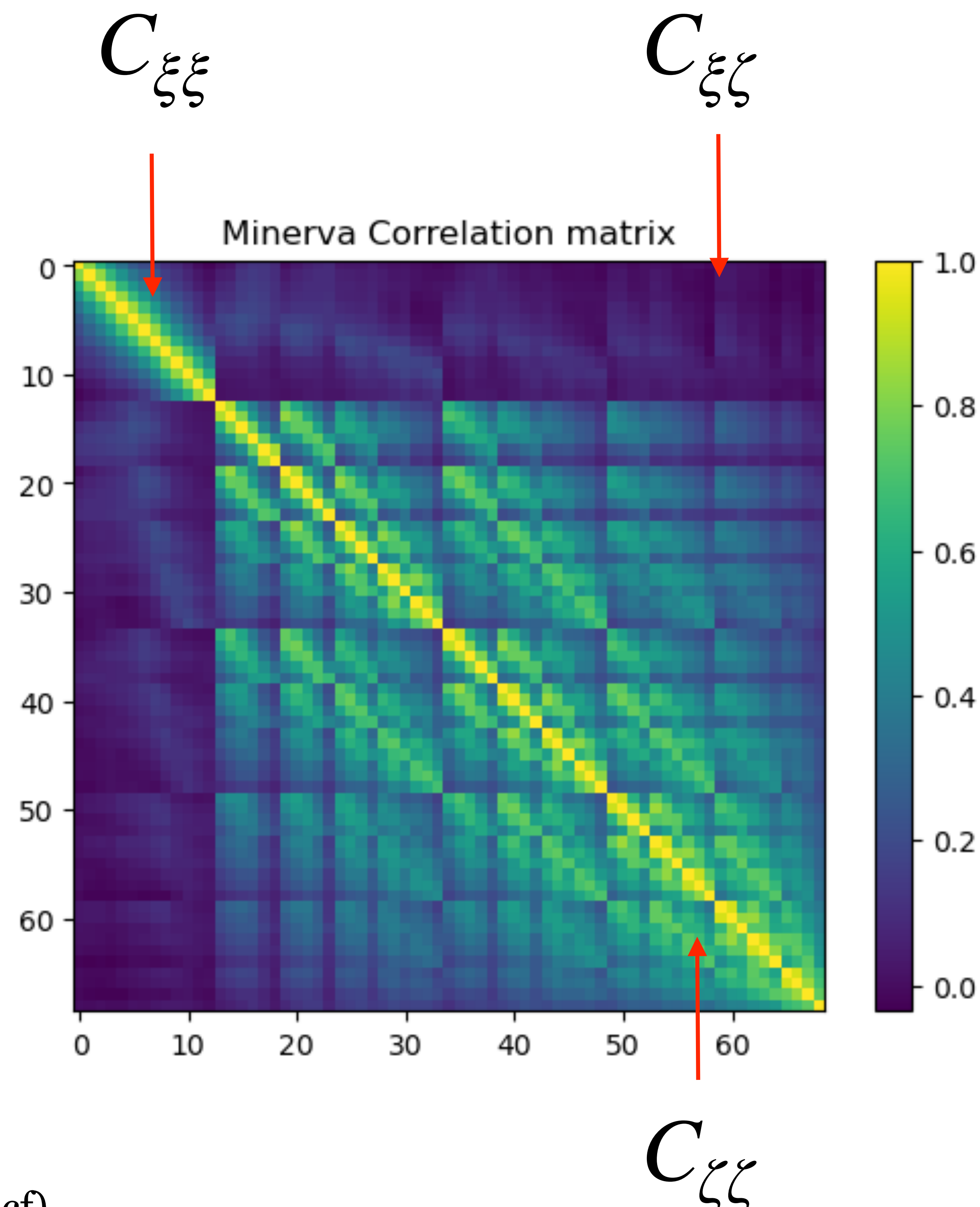
3PCF: covariance

We have two problems here:

- Volume effects
- Non-Gaussian contributions

RASCALC (Philcox *et al.*, 2020) and similar codes (WinCov) can account for both *in principle*



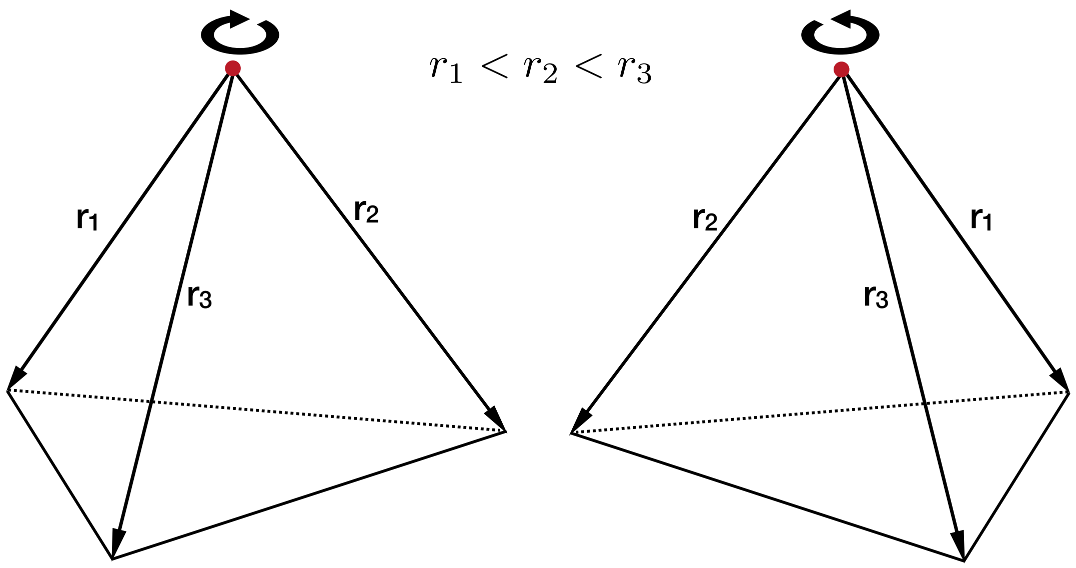


A. Veropalumbo for the Euclid collaboration (2pcf)

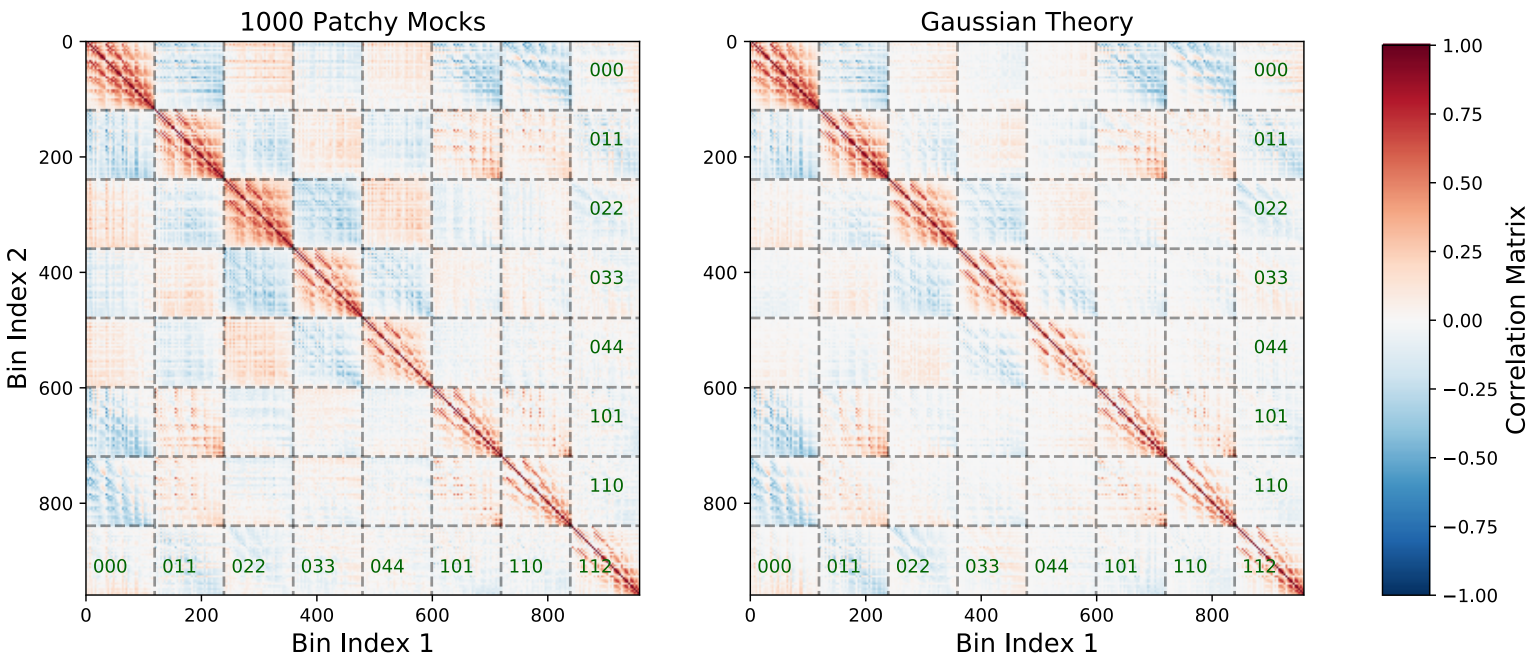
4PCF

The galaxy 4PCF can provide a test of parity of the galaxy distribution

We have hints/detections …
… but also doubts



The covariance could play a major role here, and it is hard to estimate, numerically *and* analytically



Philcox *et al.* (2021), Philcox (2022), Hou *et al.* (2023), DESI (*in prep.*)

To sum up: 3pcf issues

- **The model**
  tree-level PT
  this is already challenging given the 2D FT, emulation needed for a full-shape analysis

- **Anisotropy**
  monopole *vs* **monopole + quadrupole**
  they can be measured, so they will be used

- **Covariance**
  numerical *vs* theoretical
  It is important, but not obvious, to have a reliable covariance including all non-Gaussian contributions, a challenge both for numerical as for analytical estimates

What is left to be done

	Bispectrum	3pcf
Model	Efficient one-loop evaluation Do we agree on the model? Should we go beyond EFT?	Efficient tree-level evaluation (one loop?)
Window	More work to do but a solution is available for FKP estimator, Other options also available, but it would be nice to have a fair comparison	No need
Covariance	We must make sure all non-Gaussian and super-sample contributions are there for *both* numerical and analytical estimates	