SEXTEN CENTER FOR ASTROPHYSICS RICCARDO GIACCONI

New Strategies for Extracting Cosmology from Galaxy Surveys — 3rd edition

JAX-based one-loop EFT code for full-shape analysis of galaxy power spectrum

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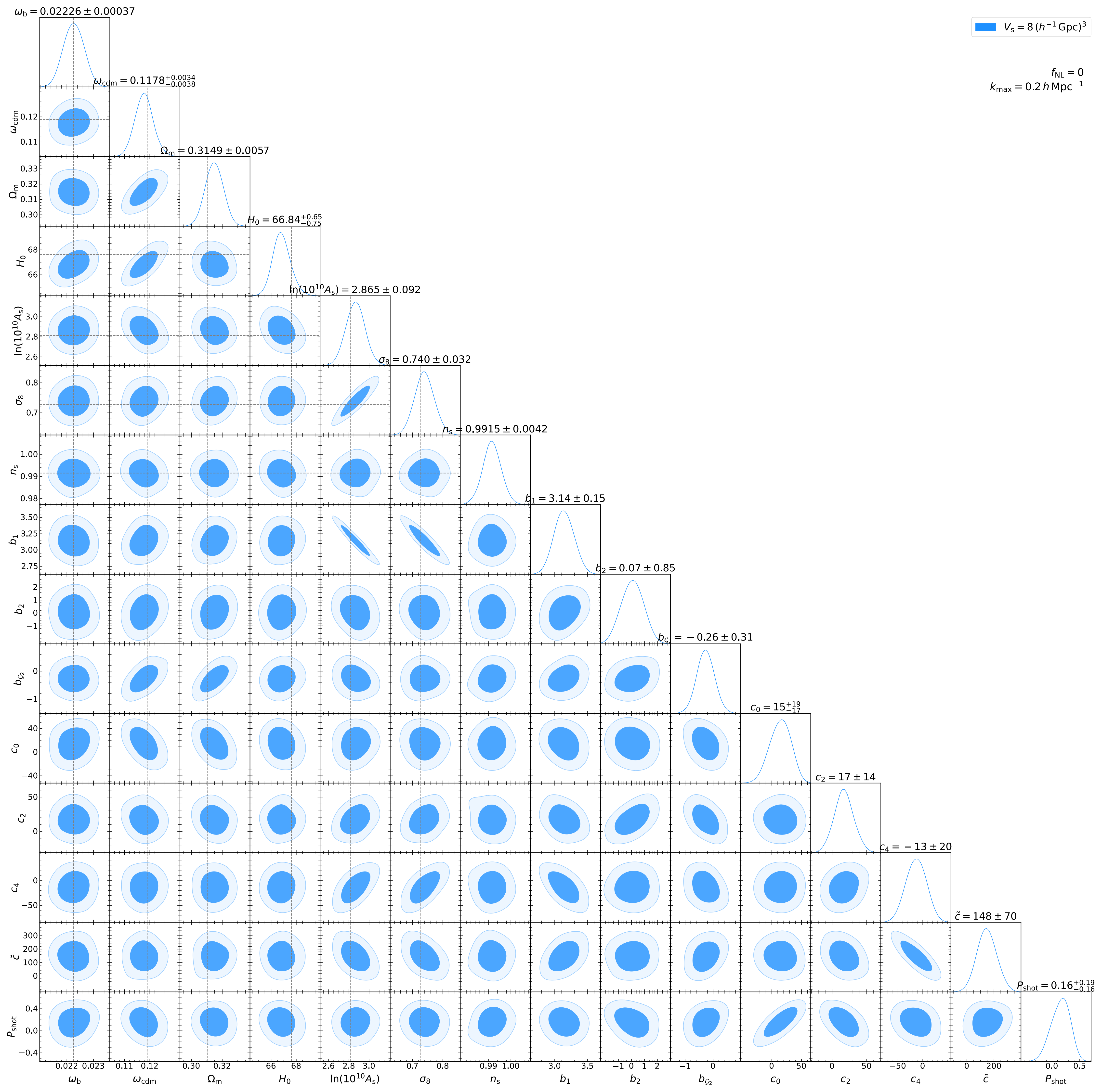
PT-based analysis of new-generation galaxy surveys?

In future galaxy survey analyses, we can have a lot of redshift bins and sub-samples.

EFT approach requires to include ~10 nuisance parameters (galaxy bias, other counterterms, stochasticity) for each redshift/sub-sample.

-> Parameter inference using e.g. MCMC becomes hard and hard when combining different redshifts/sub-samples....



MCMC using parameter derivatives: Hamiltonian Monte Carlo

For achieving scalable inference in high-dimensional parameter spaces, Hamiltonian Monte Carlo algorithms would be useful.

The leapfrog integrator in HMC utilizes the gradient of the target probability density with respect to the parameters, $\nabla_\theta \mathscr{L}(\theta)$

Algorithm 1 Hamiltonian Monte Carlo

Given $\theta^0, \epsilon, L, \mathcal{L}, M$:
for $m = 1$ to M **do**
  Sample $r^0 \sim \mathcal{N}(0, I)$.
  Set $\theta^m \leftarrow \theta^{m-1}, \tilde{\theta} \leftarrow \theta^{m-1}, \tilde{r} \leftarrow r^0$.
  for $i = 1$ to L **do**
    Set $\tilde{\theta}, \tilde{r} \leftarrow \text{Leapfrog}(\tilde{\theta}, \tilde{r}, \epsilon)$.
  end for
  With probability $\alpha = \min\left\{1, \frac{\exp\{\mathcal{L}(\tilde{\theta}) - \frac{1}{2}\tilde{r}\cdot\tilde{r}\}}{\exp\{\mathcal{L}(\theta^{m-1}) - \frac{1}{2}r^0 \cdot r^0\}}\right\}$, set $\theta^m \leftarrow \tilde{\theta}$, $r^m \leftarrow -\tilde{r}$.
end for

function Leapfrog(θ, r, ϵ)
Set $\tilde{r} \leftarrow r + (\epsilon/2)\boxed{\nabla_\theta \mathcal{L}(\theta)}$.
Set $\tilde{\theta} \leftarrow \theta + \epsilon \tilde{r}$.
Set $\tilde{r} \leftarrow \tilde{r} + (\epsilon/2)\boxed{\nabla_\theta \mathcal{L}(\tilde{\theta})}$.
return $\tilde{\theta}, \tilde{r}$.

Hoffman & Gelman 2011

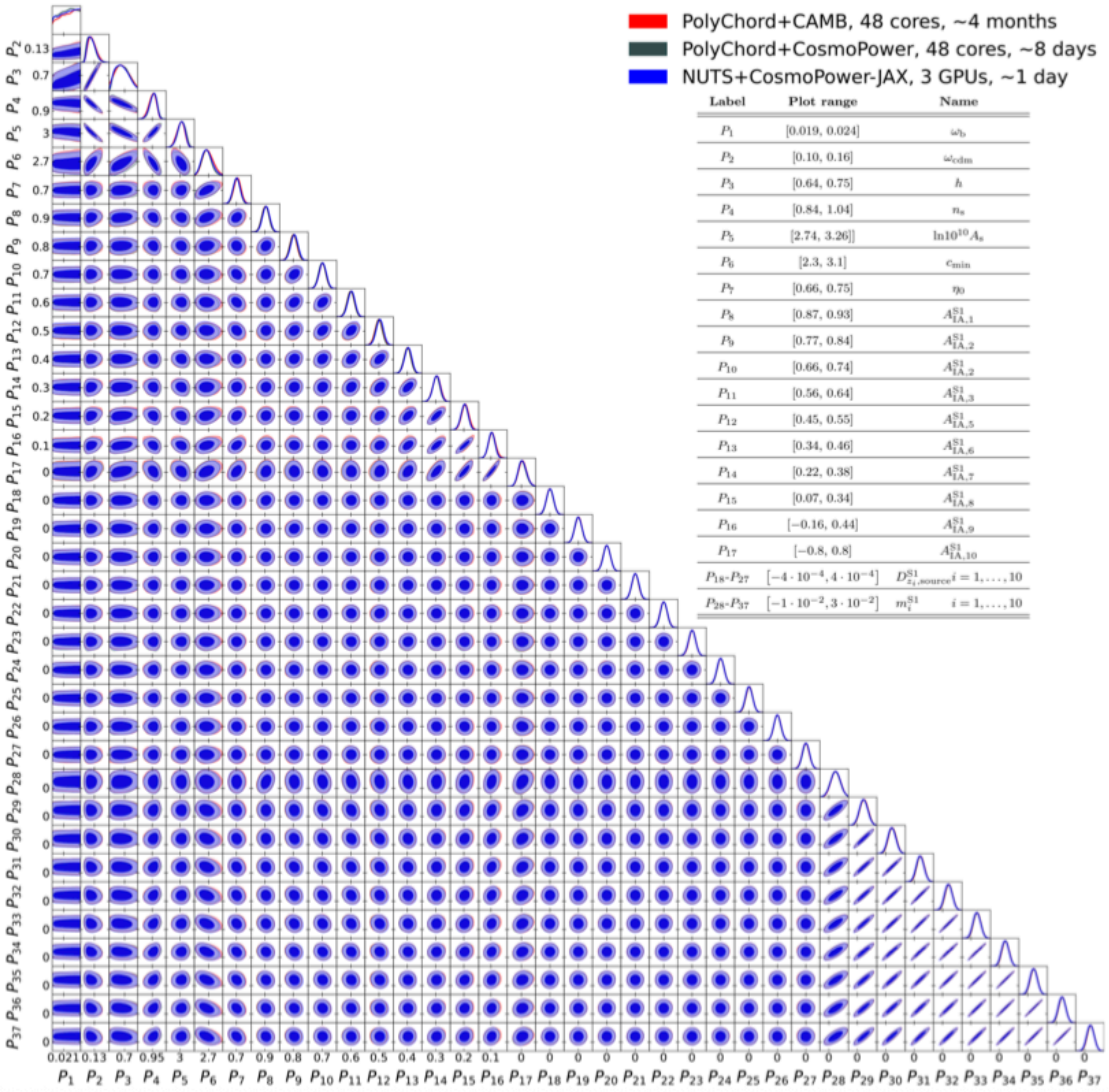
Differentiable code using JAX

We can resort to Google JAX, which enables development of differentiable codes that run not only on CPUs, but also on GPUs (or TPUs).

For example, Piras & Spurio Mancini 2023 developed Cosmopower-JAX, a differentiable emulator of the linear power spectrum.

Combining with HMC and 3 GPUs, it finishes the inference of 37 parameters in ~1 day.

Piras & Spurio Mancini 2023



Label	Plot range	Name
P_1	[0.019, 0.024]	ω_b
P_2	[0.10, 0.16]	ω_{cdm}
P_3	[0.64, 0.75]	h
P_4	[0.84, 1.04]	n_s
P_5	[2.74, 3.26]]	$\ln 10^{10} A_s$
P_6	[2.3, 3.1]	c_{min}
P_7	[0.66, 0.75]	η_0
P_8	[0.87, 0.93]	$A^{S1}_{IA,1}$
P_9	[0.77, 0.84]	$A^{S1}_{IA,2}$
P_{10}	[0.66, 0.74]	$A^{S1}_{IA,2}$
P_{11}	[0.56, 0.64]	$A^{S1}_{IA,3}$
P_{12}	[0.45, 0.55]	$A^{S1}_{IA,5}$
P_{13}	[0.34, 0.46]	$A^{S1}_{IA,6}$
P_{14}	[0.22, 0.38]	$A^{S1}_{IA,7}$
P_{15}	[0.07, 0.34]	$A^{S1}_{IA,8}$
P_{16}	[−0.16, 0.44]	$A^{S1}_{IA,9}$
P_{17}	[−0.8, 0.8]	$A^{S1}_{IA,10}$
P_{18}-P_{27}	$[-4 \cdot 10^{-4}, 4 \cdot 10^{-4}]$	$D^{S1}_{z_i,\mathrm{source}} i = 1, \ldots, 10$
P_{28}-P_{37}	$[-1 \cdot 10^{-2}, 3 \cdot 10^{-2}]$	m^{S1}_i $i = 1, \ldots, 10$

Eulerian PT of galaxy power spectrum

Expansion of galaxy density fluctuations in Eulerian PT, up to 3rd order (e.g. CLASS-PT)

$$\delta_{\rm g}(\mathbf{x}) = b_1\delta(\mathbf{x}) + \frac{b_2}{2}\delta^2(\mathbf{x}) + b_{\mathcal{G}_2}\mathcal{G}_2(\mathbf{x}) + \frac{b_3}{6}\delta^3(\mathbf{x}) + b_{\mathcal{G}_3}\mathcal{G}_3(\mathbf{x}) + b_{\delta\mathcal{G}_2}\delta(\mathbf{x})\mathcal{G}_2(\mathbf{x}) + b_\Gamma\Gamma(\mathbf{x}) + R_*^2\nabla_{\mathbf{x}}^2\delta(\mathbf{x})$$

Redshift-space distortions expanded to 3rd order (e.g. Senatore & Zaldarriaga 2014)

$$\delta_{\rm g}^{(\rm s)}(\mathbf{k}) = \delta_{\rm g}(\mathbf{k}) - ik_\parallel u_\parallel(\mathbf{k}) - ik_\parallel[u_\parallel\delta_{\rm g}](\mathbf{k}) - \frac{1}{2}k_\parallel^2[u_\parallel^2](\mathbf{k}) - \frac{1}{2}k_\parallel^2[u_\parallel^2\delta_{\rm g}](\mathbf{k}) - \frac{1}{6}ik_\parallel^3[u_\parallel^3](\mathbf{k}).$$

Galaxy density fluctuation is expressed with PT kernels Z_n that encode bias expansion and RSD, as well as gravitational evolution.

$$\delta_{\rm g}(\mathbf{k}) = Z_1(\mathbf{k})\delta_{\rm L}(\mathbf{k}) + \int_{\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}} Z_2(\mathbf{k}_1,\mathbf{k}_2)\delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2) + \int_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3=\mathbf{k}} Z_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_2)\delta_{\rm L}(\mathbf{k}_1)\delta_{\rm L}(\mathbf{k}_3)$$

Power spectrum up to one-loop order is expressed by

$$P_{\rm gg}(\mathbf{k}) = [Z_1(\mathbf{k})]^2P_{\rm lin}(k) + 2\int_{\mathbf{p}}[Z_2(\mathbf{p},\mathbf{k}-\mathbf{p})]^2P_{\rm lin}(p)P_{\rm lin}(|\mathbf{k}-\mathbf{p}|) + 6Z_1(\mathbf{k})P_{\rm lin}(k)\int_{\mathbf{p}}Z_3(\mathbf{k},\mathbf{p},-\mathbf{p})P_{\rm lin}(p)$$

+ counterterms, stochasticity, IR resummation, (and AP effect)

Lagrangian PT of galaxy power spectrum

LPT is based on the displacement that relates the Eulerian coordinates $\mathbf{x}$ and the Lagrangian coordinates $\mathbf{q}$,

$$\mathbf{x} = \mathbf{q} + \mathbf{\Psi}(\mathbf{q}) \qquad\qquad 1 + \delta_{\rm g}(\mathbf{x}) = \int_{\mathbf{q}} F(\mathbf{q})\delta_{\rm D}(\mathbf{x} - \mathbf{q} - \mathbf{\Psi}(\mathbf{q}))$$

Galaxy power spectrum in LPT is (e.g. Matsubara 2008, Carlson, Reid & White 2012)

$$P_{\rm gg}(\mathbf{k}) = \int_{\mathbf{q}} e^{i\mathbf{k}\cdot\mathbf{q}} \left\langle e^{i\mathbf{k}\cdot\mathbf{\Delta}} F(\mathbf{q}_1)F(\mathbf{q}_2)\right\rangle_{\mathbf{q}=\mathbf{q}_2-\mathbf{q}_1} \qquad \mathbf{\Delta} \equiv \mathbf{\Psi}(\mathbf{q}_2) - \mathbf{\Psi}(\mathbf{q}_1)$$

Galaxy bias in Lagrangian space is encoded in (e.g. Matsubara 2008, Vlah, Castorina & White 2016)

$$F(\mathbf{q}) = F[\delta(\mathbf{q}), s^2(\mathbf{q}), \nabla^2\delta(\mathbf{q}), \cdots] = \int_{\mathbf{J}} \tilde{F}(\mathbf{J}) e^{i\sum_j J_{O_j}(\mathbf{q})O_j(\mathbf{q})}$$

The power spectrum becomes

$$P_{\rm gg}(\mathbf{k}) = \int_{\mathbf{q}} e^{i\mathbf{k}\cdot\mathbf{q}} \int_{\mathbf{J}_1,\mathbf{J}_2} \tilde{F}(\mathbf{J}_1)\tilde{F}(\mathbf{J}_2) K(\mathbf{k}, \mathbf{q}, \mathbf{J}_1, \mathbf{J}_2)$$

where

$$K(\mathbf{k}, \mathbf{q}, \mathbf{J}_1, \mathbf{J}_2) \equiv \left\langle e^{i\{\mathbf{k}\cdot\mathbf{\Delta} + \sum_j [J_{O_j}(\mathbf{q}_1)O_j(\mathbf{q}_1) + J_{O_j}(\mathbf{q}_2)O_j(\mathbf{q}_2)]} \right\rangle_{\mathbf{q}=\mathbf{q}_2-\mathbf{q}_1}$$

Lagrangian PT of galaxy power spectrum

The moment generating function

$$K(\mathbf{k},\mathbf{q},\mathbf{J}_1,\mathbf{J}_2) \equiv \left\langle e^{i\{\mathbf{k}\cdot\boldsymbol{\Delta}+\sum_j [J_{O_j}(\mathbf{q}_1)O_j(\mathbf{q}_1)+J_{O_j}(\mathbf{q}_2)O_j(\mathbf{q}_2)]} \right\rangle_{\mathbf{q}=\mathbf{q}_2-\mathbf{q}_1}$$

By expanding it into cumulants (+ Taylor expansion), and defining the Lagrangian bias parameters as

$$b_n = \int_{\mathbf{J}} \tilde{F}(\mathbf{J}) e^{-\frac{1}{2}J_\delta^2\sigma^2(\Lambda)}(iJ_\delta)^n,\, b_O = \int_{\mathbf{J}} \tilde{F}(\mathbf{J}) e^{-\frac{1}{2}J_\delta^2\sigma^2(\Lambda)} iJ_O$$

LPT galaxy power spectrum up to one-loop order is (e.g. Chen et al. 2021)

$$\begin{aligned}
P_{\rm gg}(\mathbf{k}) = \int_{\mathbf{q}} e^{i\mathbf{k}\cdot\mathbf{q}} e^{-\frac{1}{2}k_ik_jA_{ij}^{<}} \Big\{ & 1 - \frac{1}{2}k_ik_jA_{ij}^{>} - \frac{1}{2}k_ik_jA_{ij}^{1-\rm loop} - \frac{i}{6}k_ik_jk_kW_{ijk} + \frac{1}{8}k_ik_jk_kk_lA_{ij}^{>}A_{kl}^{>} \\
& + 2ib_1\left(1-\frac{1}{2}k_ik_jA_{ij}^{>}\right)k_iU_i^{10} + b_1^2\left(1-\frac{1}{2}k_ik_jA_{ij}^{>}\right)\xi_{\rm lin} - b_1k_ik_jA_{ij}^{10} + ib_1^2k_iU_i^{11} + ib_2k_iU_i^{20} \\
& + (b_1^2+b_2)k_ik_jU_{{\rm lin},i}U_{{\rm lin},j} + b_2^2\xi_{\rm lin}^2 + 2ib_1b_2\xi_{\rm lin}k_iU_{{\rm lin},i} \\
& - b_sk_ik_j\Upsilon_{ij} + 2ib_sk_iV_i^{10} + 2ib_1b_sk_iV_i^{12} + b_2b_s\chi + b_s^2\zeta + \cdots \Big\}
\end{aligned}$$

+ counterterms, stochasticity (and AP effect)

Implementation of PT terms using FFTs ①

FFTLog-based method for fast evaluation of one-loop integrals (Simonovic et al. 2018, CLASS-PT, Lee 2024)

- Perform FFTLog on $P_{\rm lin}$ to decompose it into powers of k, $$P_{\rm lin}(k) = \sum_{m=-N/2}^{N/2} c_m k^{\nu+i\eta_m}$$

- Decompose the PT kernels into rational functions of $\mathbf{q}$, $\mathbf{k}-\mathbf{q}$ and $\hat{\mathbf{n}}\cdot\mathbf{q}$, and insert the analytic solutions of the form:

$$\int_{\mathbf{q}} \frac{(\hat{\mathbf{n}}\cdot\mathbf{q})^{n_z}}{q^{2\nu_1}|\mathbf{k}-\mathbf{q}|^{2\nu_2}} = k^{3-2(\nu_1+\nu_2)+n_z} \boxed{\mathsf{I}^{(n_z)}(\mu,\nu_1,\nu_2)}$$

analytic solution

- (If needed, take care of IR/UV parts that are not expressed by the above analytic solution.)

-> The one-loop integrals reduce to a linear combination of analytic solutions.

$$P_{22}(k,\mu) = k^3 \sum_{m_1,m_2} c_{m_1} c_{m_2} k^{-2(\nu_1+\nu_2)} \sum_{n_1,n_2,n_z} 2 f_{22}(n_1,n_2,n_z)\, \mathsf{I}^{(n_z)}(\mu,\nu_1+n_1,\nu_2+n_2).$$

Implementation of PT terms using FFTs ②

Another 1D FFT method (Schmittfull, Vlah & McDonald 2016) based on the generalized correlation function:

$$\xi_n^\ell(r) = \int_0^\infty \frac{k^2 \mathrm{d}k}{2\pi^2} j_\ell(kr) k^n P_{\mathrm{lin}}(k)$$

One-loop EPT terms consist of the Hankel transforms of different $\xi_n^\ell(r)$'s:

$$P_{22}(k) \supset \int_{\mathbf{p}} p^{n_1} |\mathbf{k}-\mathbf{p}|^{n_2} \mathcal{L}_\ell(\hat{\mathbf{p}} \cdot \widehat{\mathbf{k}-\mathbf{p}}) P_{\mathrm{lin}}(p) P_{\mathrm{lin}}(|\mathbf{k}-\mathbf{p}|) = (-1)^\ell \int_0^\infty 4\pi r^2 \mathrm{d}r\, j_0(kr) \xi_{n_1}^\ell(r) \xi_{n_2}^\ell(r)$$

$$P_{13}(k) \supset k^m P_{\mathrm{lin}}(k) \int_{\mathbf{p}} \frac{p^n \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \mathbf{p})}{|\mathbf{k}-\mathbf{p}|^2} P_{\mathrm{lin}}(p) = k^m P_{\mathrm{lin}}(k) \int_0^\infty \mathrm{d}r r\, j_\ell(kr) \xi_n^\ell(kr)$$

One-loop LPT terms are also constructed by the Hankel transforms of these functions, e.g.

$$A_{ij}^{(22)}(\mathbf{q}) = X_{22}(q)\delta_{ij} + Y_{22}(q)\hat{q}_i\hat{q}_j$$

$$X_{22}(q) = \int_0^\infty \frac{\mathrm{d}k}{2\pi^2} \frac{2}{3}[1 - j_0(kq) - j_2(kq)] \frac{9}{98} Q_1(k), Y_{22}(q) = \int_0^\infty \frac{\mathrm{d}k}{2\pi^2} 2 j_2(kq) \frac{9}{98} Q_1(k)$$

where

$$Q_1(k) = \int_{\mathbf{p}} \left[\frac{k^2 p^2 - (\mathbf{k}\cdot\mathbf{p})^2}{p^2 |\mathbf{k}-\mathbf{p}|^2} \right]^2 P_{\mathrm{lin}}(p) P_{\mathrm{lin}}(|\mathbf{k}-\mathbf{p}|)$$

Angular integrals of redshift-space LPT

Redshift-space LPT power spectrum consists of the angular internals of the form

$$I_{m,n}(A,B,C) = i^{m-n}\int_{-1}^{1}\frac{\mathrm{d}\mu_{\mathbf{q}}}{2}\int_{0}^{2\pi}\frac{\mathrm{d}\phi}{2\pi}e^{iA\mu_{\mathbf{q}}+B\mu_{\mathbf{q}}^2-iC\sqrt{1-\mu_{\mathbf{q}}^2}\cos\phi}\mu_{\mathbf{q}}^m\left(\sqrt{1-\mu_{\mathbf{q}}^2}\cos\phi\right)^n$$

This type of integrals has the analytic solution (Chen et al. 2021),

$$I_{m,n}(A,B,C) = e^B\sum_{\ell=0}^{\infty}\left(\frac{-2}{\rho}\right)^{\ell}G_{\ell}^{(m,n)}(A,B,C)j_{\ell}(\rho) \qquad \rho=\sqrt{A^2+C^2}$$

which is derived by using the recursion relation

$$G_{\ell}^{(m+1,n)} = \frac{\partial G_{\ell}^{(m,n)}}{\partial A}+\frac{A}{2}G_{\ell-1}^{(m,n)},\ \ G_{\ell}^{(m,n+1)} = \frac{\partial G_{\ell}^{(m,n)}}{\partial C}+\frac{C}{2}G_{\ell-1}^{(m,n)}$$

By iteratively applying the recursion relation, we can obtain the expression for arbitrary (m,n) in terms of $G_{\ell}^{(0,0)}$ and its derivatives, e.g.

$$G_{\ell}^{(1,1)} = \frac{\partial^2 G_{\ell}^{(0,0)}}{\partial A\partial C}+\frac{C}{2}\frac{\partial G_{\ell-1}^{(0,0)}}{\partial A}+\frac{A}{2}\frac{\partial G_{\ell-1}^{(0,0)}}{\partial C}+\frac{AC}{4}G_{\ell-2}^{(0,0)}$$

Angular integrals of redshift-space LPT

Vlah & White 2018 found the analytic expression using the hypergeometric functions,

$$G_\ell^{(0,0)}(A,B,C) = \sum_{n=\ell}^{\infty} \frac{\Gamma(\ell+n+\frac{1}{2})}{\Gamma(\ell+1)\Gamma(n+\frac{1}{2})\Gamma(1-\ell+n)} \left(\frac{BA^2}{\rho^2}\right)^n {}_2F_1\left(\frac{1}{2}-n, -n; \frac{1}{2}-\ell-n; \frac{\rho^2}{A^2}\right)$$

This integral arises even in the Zeldovich approximation in redshift space.

In this work, I've found that it reduces to a closed-form expression

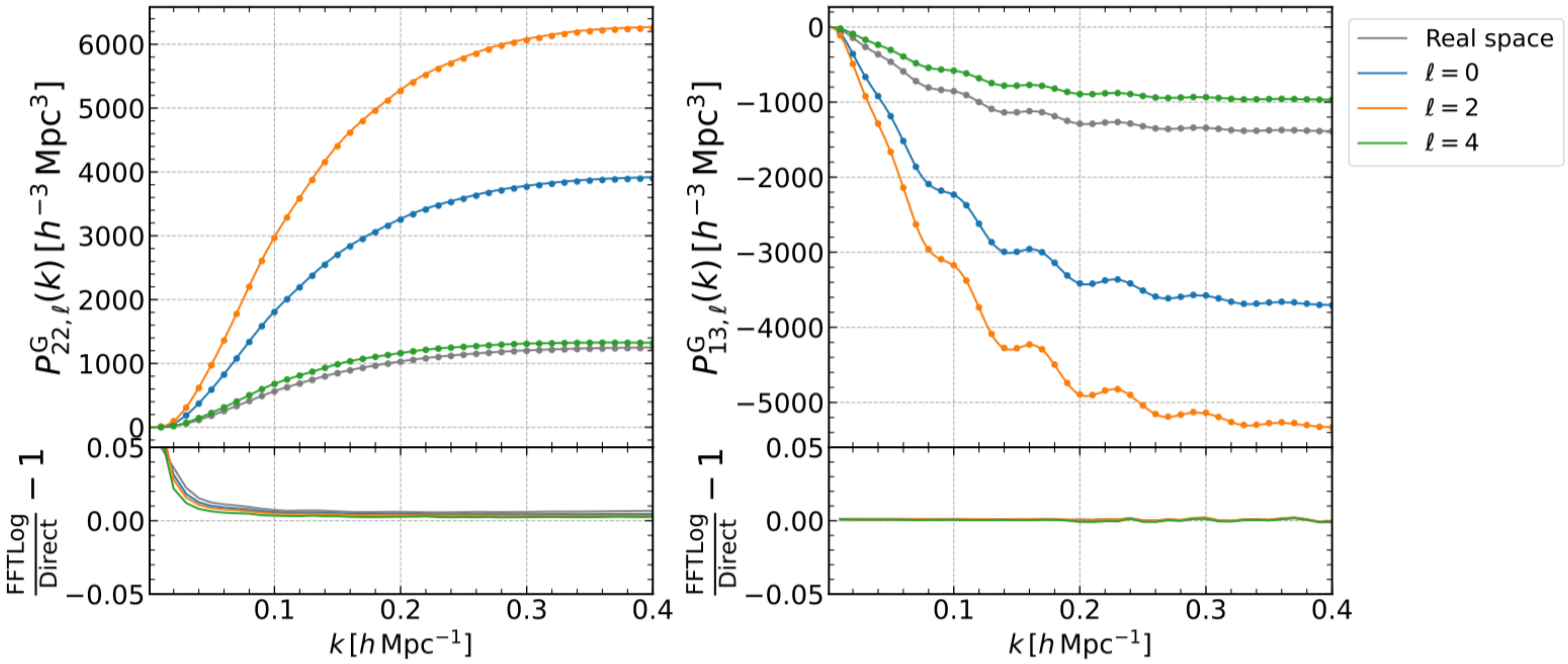
$$G_\ell^{(0,0)}(A,B,C) = e^{-\frac{BC^2}{\rho^2}} \sum_{i=0}^{\ell} \sum_{k=0}^{\ell} c_{(\ell,k,i)} (-B)^{\ell+i} \left(\frac{C^2}{\rho^2}\right)^{\ell+i-k}$$

with the coefficients $c_{(\ell,k,i)}$ are rational numbers that depend on (ℓ, k, i), expressed using generalized hypergeometric functions.

This expression enables JAX implementation of the angular integrals, without implementing the hypergeometric functions in JAX.

Demo: one-loop EPT with brute-force integration

Cross check of 1-loop contributions between FFTLog-based implementation (solid lines) and direct numerical integration (symbols).

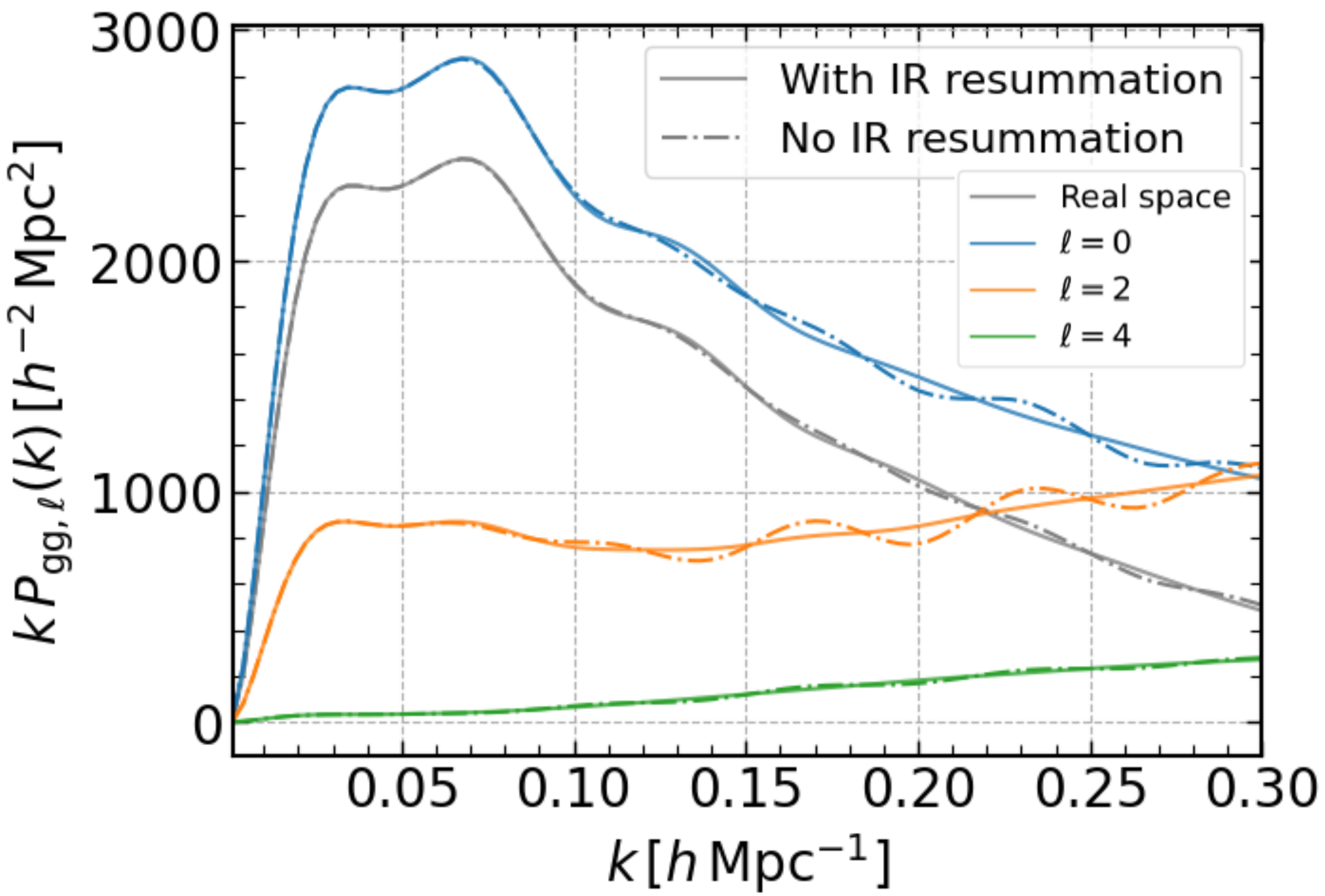


Implementation of IR resummation

Damp the BAO wiggles by the factor calculated with the no-wiggle part

$$P_{\rm gg}(k,\mu) = (b_1 + f\mu^2)^2 \left\{ P_{\rm nw}(k) + e^{-k^2\Sigma^2_{\rm tot}(\mu)} P_{\rm w}(k) \left[1 + k^2\Sigma^2_{\rm tot}(\mu)\right] \right\}$$
$$+ P^{\text{1-loop}}_{\rm gg}[P_{\rm nw}](k,\mu) + e^{-k^2\Sigma^2_{\rm tot}(\mu)} P^{\text{1-loop}}_{\rm gg}[P_{\rm w}](k,\mu)$$

Ivanov & Sibiryakov 2018



Damping factor in redshift space (Ivanov & Sibiryakov 2018)

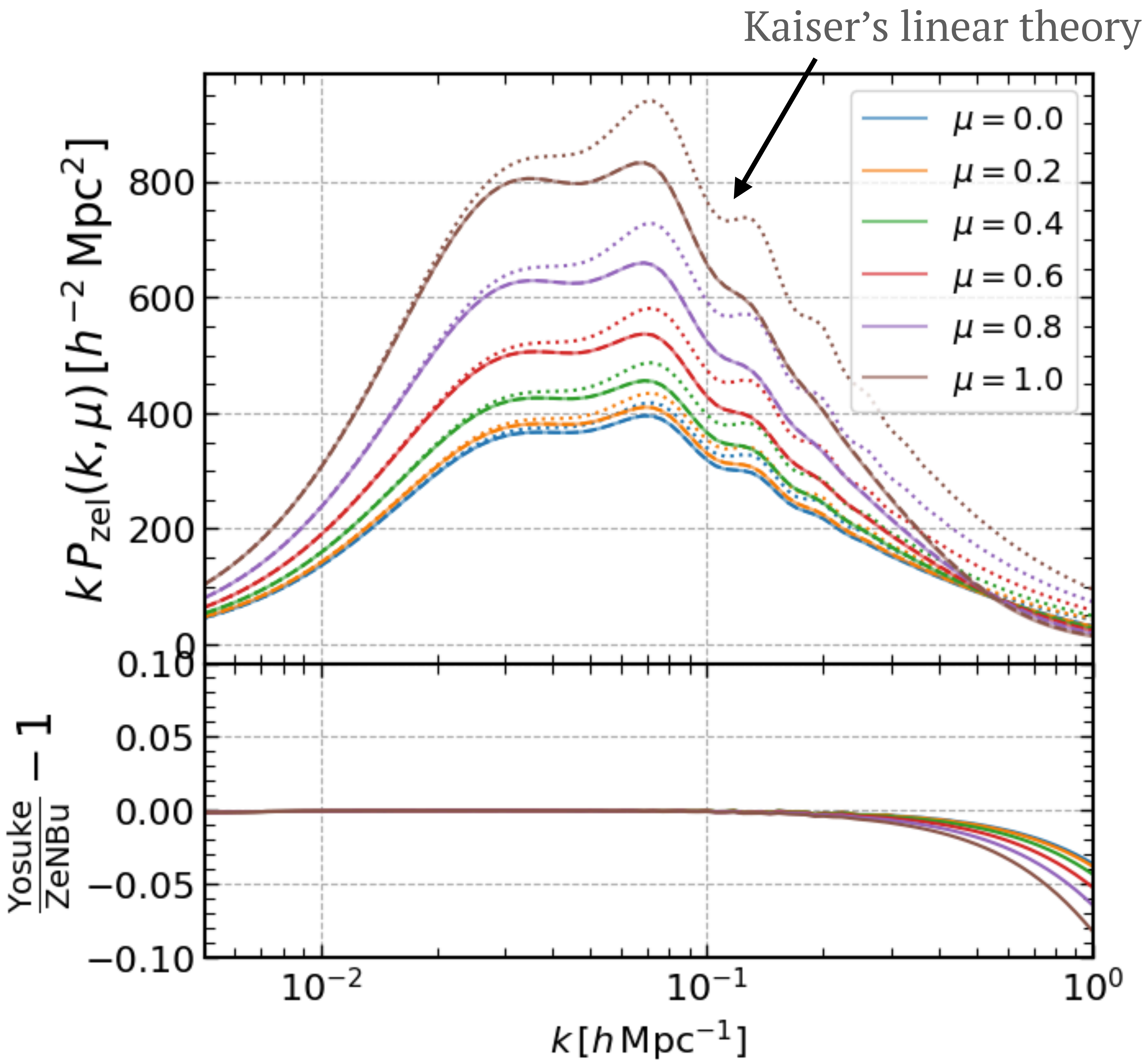
$$\Sigma^2_{\rm tot}(\mu) = [1 + f\mu^2(2+f)]\Sigma^2 + f^2\mu^2(\mu^2-1)\delta\Sigma^2$$

$$\Sigma^2 \equiv \int_0^{k_S} \frac{dq}{6\pi^2} P_{\rm nw}(q) \left[1 - j_0\left(\frac{q}{k_{\rm osc}}\right) + 2 j_2\left(\frac{q}{k_{\rm osc}}\right)\right]$$

$$\delta\Sigma^2 \equiv \int_0^{k_S} \frac{dq}{2\pi^2} P_{\rm nw}(q) j_2\left(\frac{q}{k_{\rm osc}}\right).$$

Demo: Zeldovich approximation in redshift space

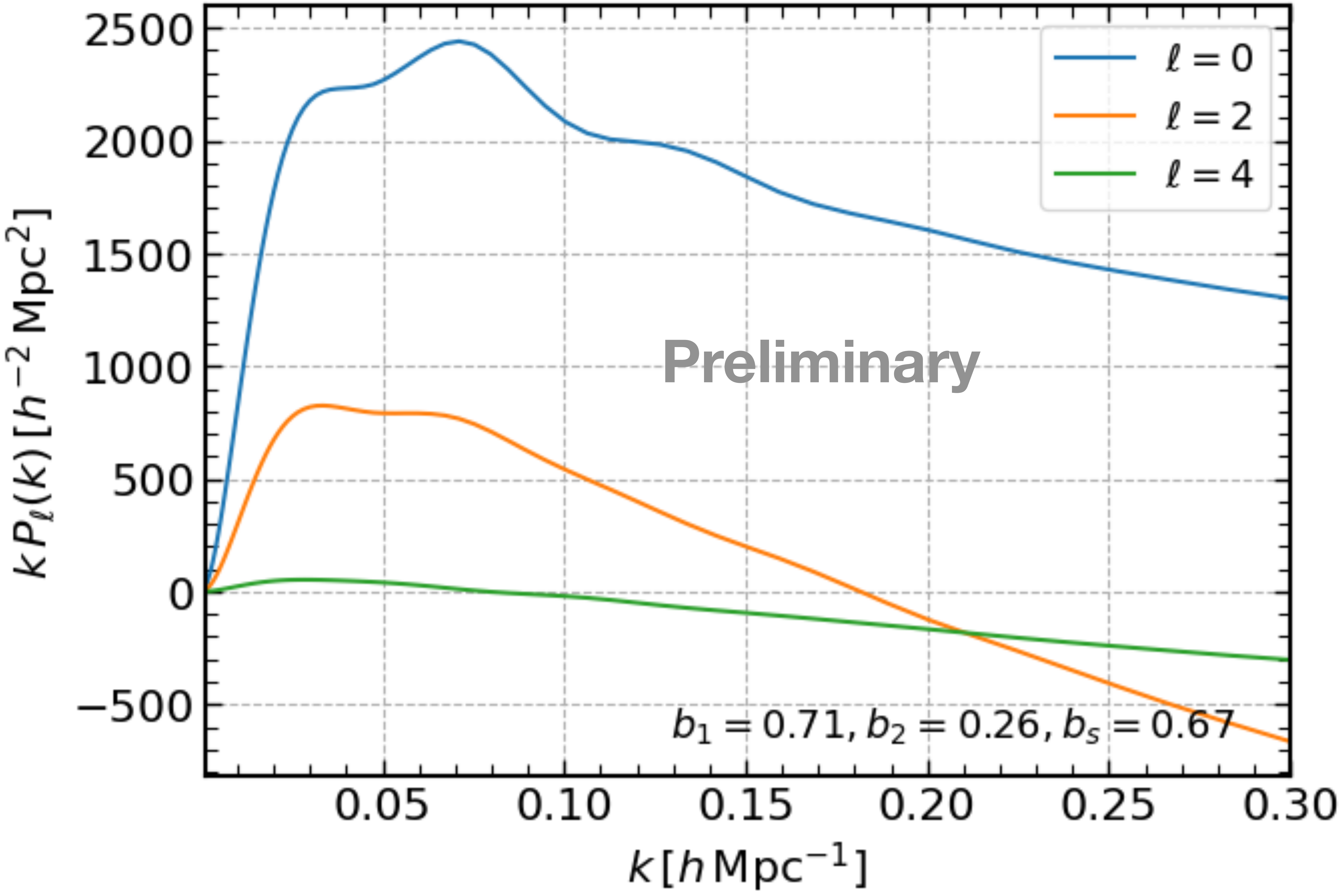
The JAX implementation of redshift-space Zeldovich power spectrum is compared to that calculated using ZeMBu (a code similar to velociletor)



Demo: redshift-space one-loop LPT power spectrum

The redshift-space multipole moments, including up to second-order bias terms (b_1, b_2, b_s)

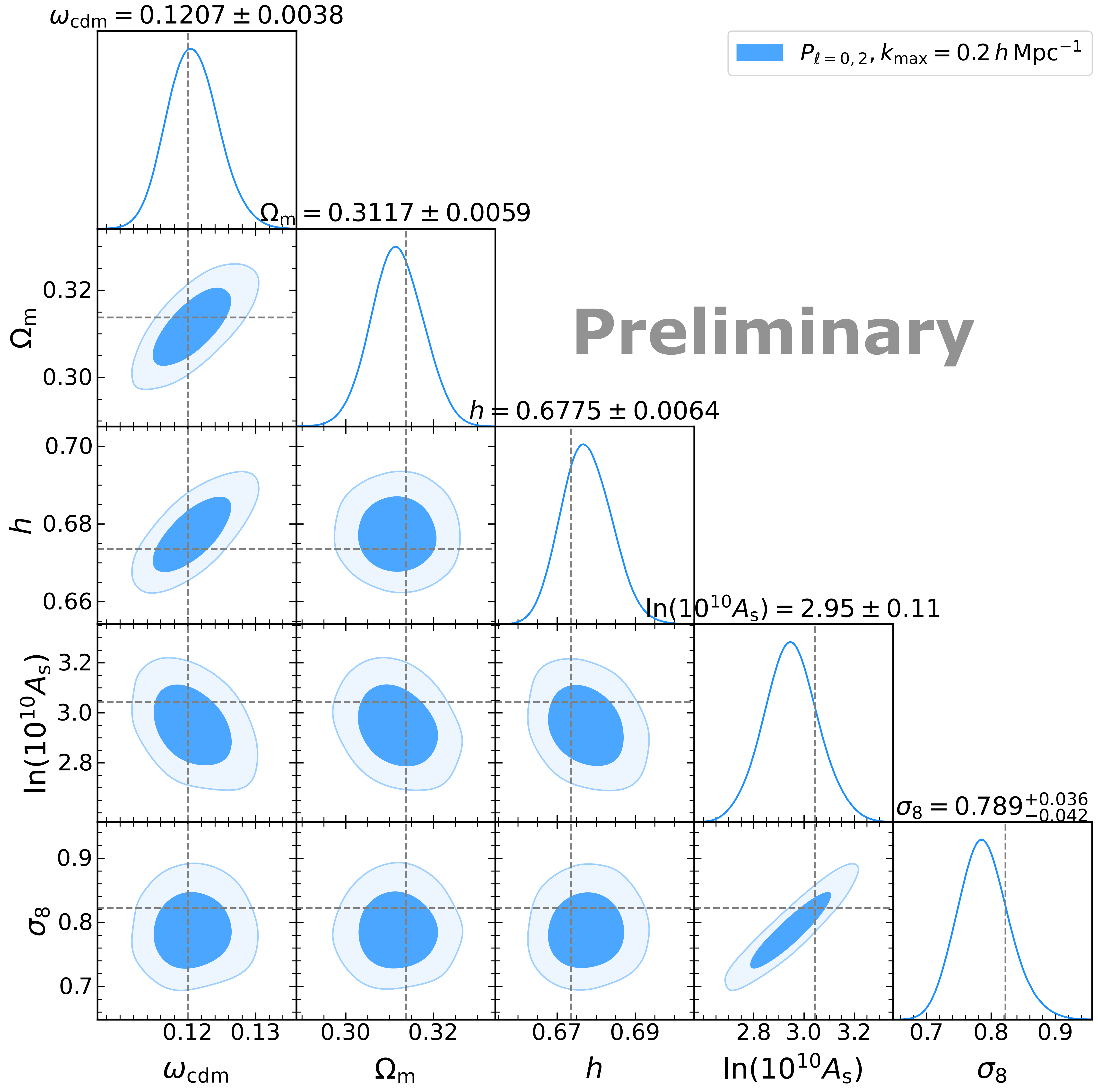
My code computes the LPT power spectrum multipoles in O(1) msec after JIT compilation, on a laptop CPUs.



Parameter inference using Hamiltonian Monte Carlo

Running parameter inference using Hamiltonian Monte Carlo (No-U-Turn Sampler) sampling in Numpyro.

Currently, O(10) mins on a GPU to converge the chains.



Summary

We are developing a pure JAX code of the galaxy power spectrum based on the Eulerian & Lagrangian one-loop EFT framework.

The code is differentiable with respect to cosmology, galaxy bias, and other EFT parameters.

We design the code to be compatible with JIT compilation provided by JAX. After JIT compilation, it computes the galaxy power spectrum in O(1) CPU milliseconds.

Detailed comparison with the existing codes and running Hamiltonian Monte Carlo inference are ongoing.

Backup: Galaxy power spectrum with PNG

JAX implementation of galaxy power spectrum with local PNG (Cabass et al. 2022) is now ongoing.
My Python implementation has been incorporated into the SPHEREx cosmology pipeline.