TB, Eggemeier, Kurita, Vlah, Chisari (2025)

TB, Kurita, Eggemeier, Vlah, Chisari (in prep.)

Bispectrum of Intrinsic Alignment

Thomas Bakx (PhD Candidate at Utrecht University)

With Alex Eggemeier, Toshiki Kurita, Elisa Chisari, Zvonimir Vlah

Sesto Galaxy Clustering Workshop - 11/07/2025

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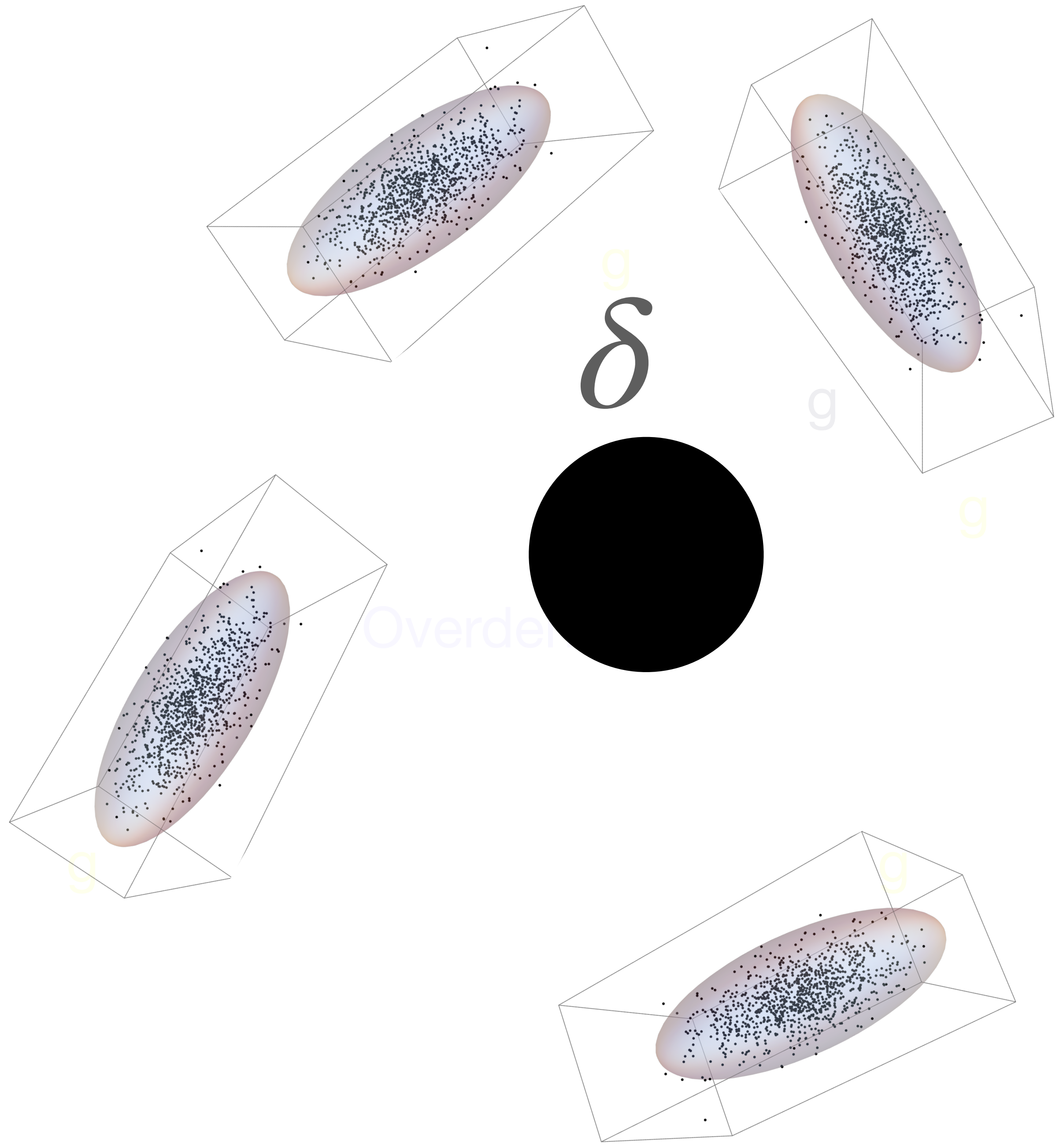
Intrinsic Alignments: a Primer (1)



- Shapes of galaxies are distorted by large-scale structure
- Therefore, shapes of galaxies are *correlated!*
- First considered in the context of *weak gravitational lensing*

Catelan, Bland, Kamionkowski (2000); Hirata, Seljak (2004); Bridle, King (2007)

- Detected in galaxy surveys many times by now

Mandelbaum, Hirata++ [SDSS] (2006)

Hirata, Mandelbaum++ [2SLAQ+SDSS] (2007)

Martens, Hirata, Ross, Fang [BOSS] (2018)

Johnston, Georgiou++ [KIDS+GAMA] (2020)

Fortuna, Hoekstra++ [KIDS] (2021)

Singh, Mandelbaum, More [BOSS] (2023)

Kurita, Takada [BOSS] (2023)

Lamman, Eisenstein++ [DESI] (2023)

Intrinsic Alignments: a Primer (2)

- An impact on many cosmological and astrophysical phenomena!
  - Redshift Space Distortions (selection effects)
  - Self-Interacting Dark Matter
  - Merger Histories of Galaxies
  - Baryon Acoustic Oscillations
  - Magnetic Fields
  - Parity-Violating Physics
  - Gravitational Waves
  - Anisotropic Primordial Non-Gaussianity
  - Cosmic Isotropy

Krause, Hirata (2010)

Hirata (2009)

Lamman, Eisenstein++ [DESI] (2023)

Harvey, Chisari, Robertson, McCarthy (2021)

Lee, Moon (2022)

Xu, Jing, Zhao, Cuesta (2023)

van Dompseler, Georgiou, Chisari (2023)

Saga, Shiraishi, Akitsu, Okumura (2023)

Philcox, Konig, Alexander, Spergel (2023)

Biagetti, Orlanda (2020)

Kurita, Takada (2023)

Shiraishi, Okumura, Akitsu (2023)

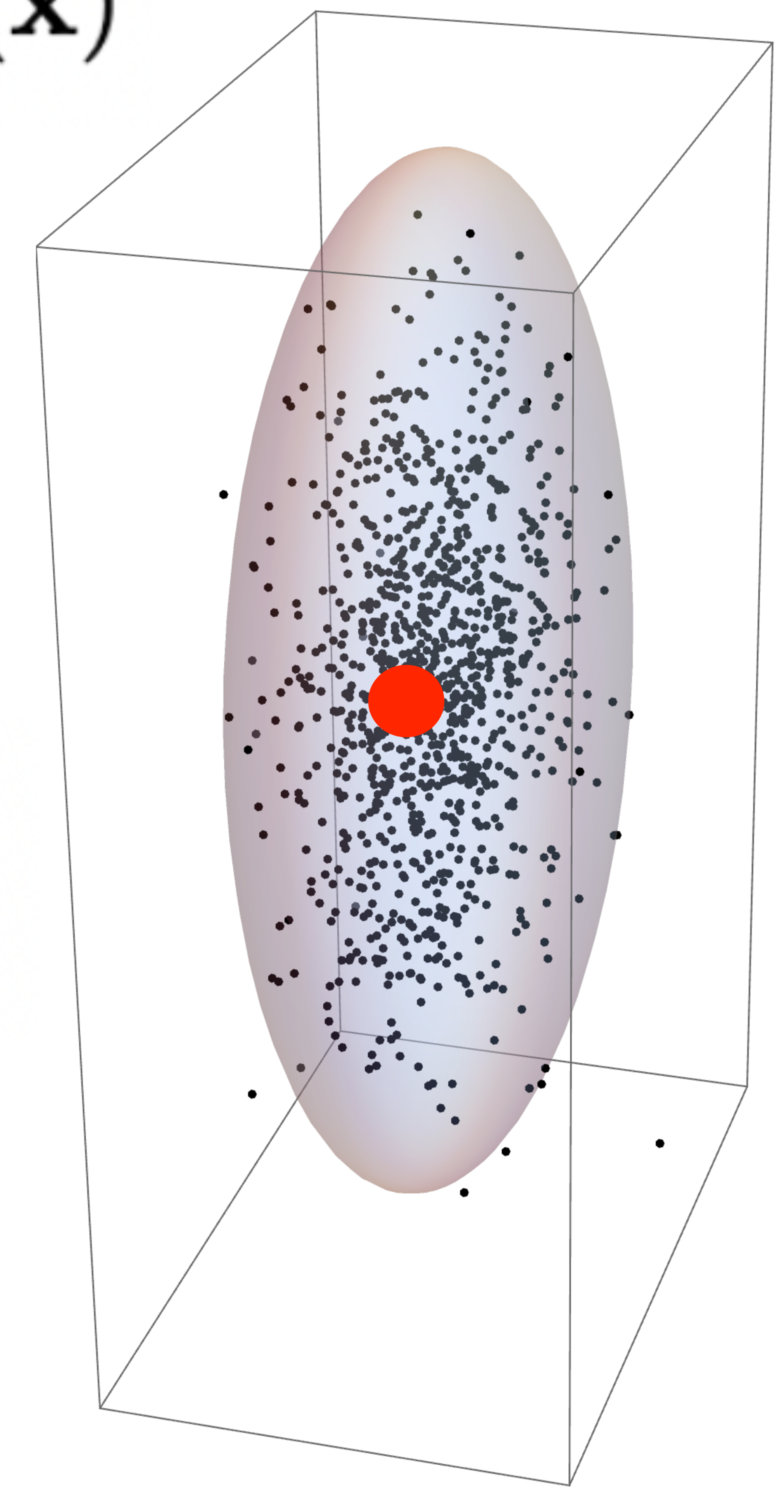
Intrinsic Alignments: a Primer (3)

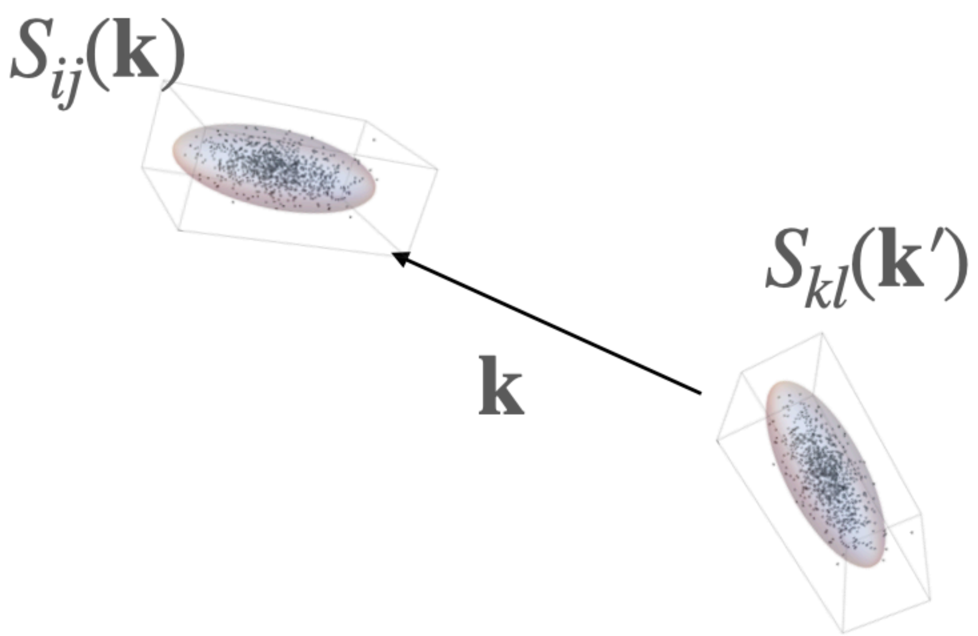
- Intrinsic alignment is a 3D phenomenon $I_{ij}(\mathbf{x})$

$$I_{ij} \propto \sum w_p \Delta \mathbf{x}_{p,i} \Delta \mathbf{x}_{p,j};$$

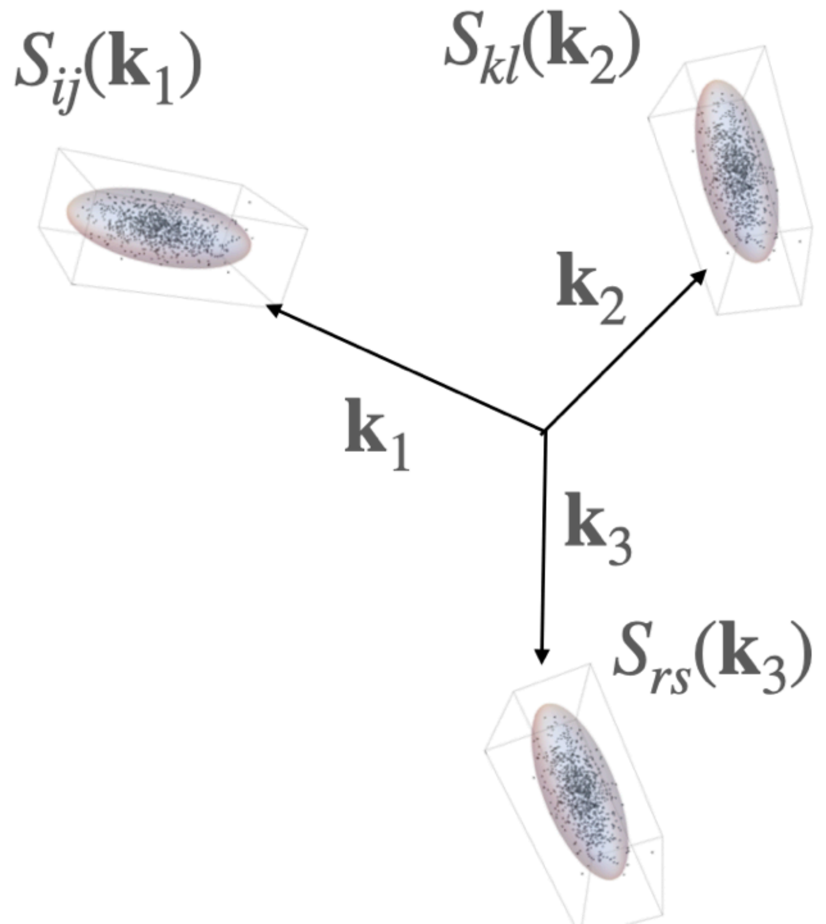
$$I_{ij}(\mathbf{x}) := \sum_\alpha I_{ij}(\mathbf{x}_\alpha)\delta(\mathbf{x} - \mathbf{x}_\alpha);$$

$$S_{ij}(\mathbf{x}) := \frac{I_{ij}(\mathbf{x}) - \langle I_{ij}\rangle}{\langle \mathrm{Tr}(I_{ij})\rangle} = \frac{1}{3}\delta_{ij}\delta_s(\mathbf{x}) + g_{ij}(\mathbf{x}).$$





$$\langle S_{ij}(\mathbf{k}_1) S_{kl}(\mathbf{k}_2)\rangle = (2\pi)^3 P_{ijkl}(\mathbf{k}_1)\delta^D(\mathbf{k}_1 + \mathbf{k}_2)$$



- IA is described by a 3D **traceless field** $g_{ij}(\mathbf{x})$

Vlah, Chisari, Schmidt (2019,2020)

Schmitz, Hirata, Blazek (2019), Pyne, Tenneti, Joachimi (2022)

$$\langle S_{ij}(\mathbf{k}_1) S_{kl}(\mathbf{k}_2) S_{rs}(\mathbf{k}_3)\rangle = (2\pi)^3 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_{ijklrs}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

Galaxy Surveys in 2025: Beyond Two-Point

- Much theoretical work in 2000s and on
- Galaxy bispectrum applied to BOSS data…

Gil-Marin++ (2015)

Philcox++ (2022)

d'Amico++ (2022)

Lu++ (2024)

Ivanov++ (2023)

- And recently also in DESI: Novell-Masot++ (2025)
- Makes sense to apply this methodology to intrinsic alignment as well
    - Access all information in three-point statistics of galaxy shapes
    - Can constrain IA directly if spectroscopic redshifts are available
- Spoiler: forecast for bispectrum B_{DDE} an SNR ≈ 30 for $k < 0.15\, h/\mathrm{Mpc}$ for DESI LRGs

Intrinsic Alignments: Observables

- What can we actually *observe?*
- Shape field is 3D, but can only observe *projection* on the sky

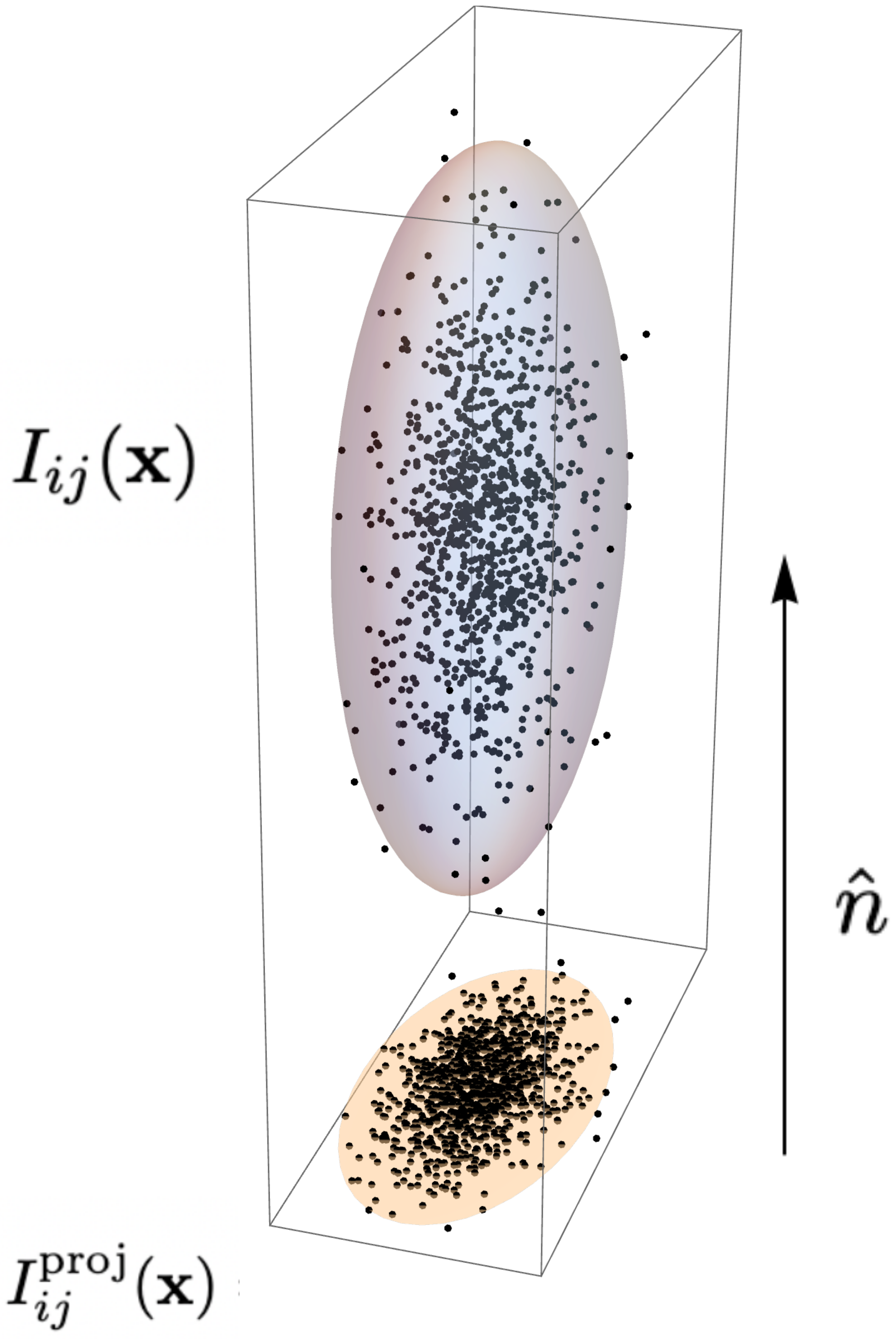
$$I_{ij}^{\mathrm{proj}}(\mathbf{x}) = \begin{pmatrix} I_{11} & I_{12} \\ I_{12} & I_{22} \end{pmatrix}; \quad \gamma_{ij} = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}; \quad \gamma_1 = \frac{I_{11} - I_{22}}{2}, \quad \gamma_2 = I_{12}.$$

- Decomposition into E- and B-modes in 3D Fourier Space

$$E(\mathbf{k}) + iB(\mathbf{k}) = (\gamma_1(\mathbf{k}) + i\gamma_2(\mathbf{k}))e^{-2i\phi_k}.$$

- Also have a contribution from weak gravitational lensing (which is not 3D)

$$\gamma^{\mathrm{obs}} = \gamma^{\mathrm{lens}} + \gamma^{\mathrm{IA}}$$

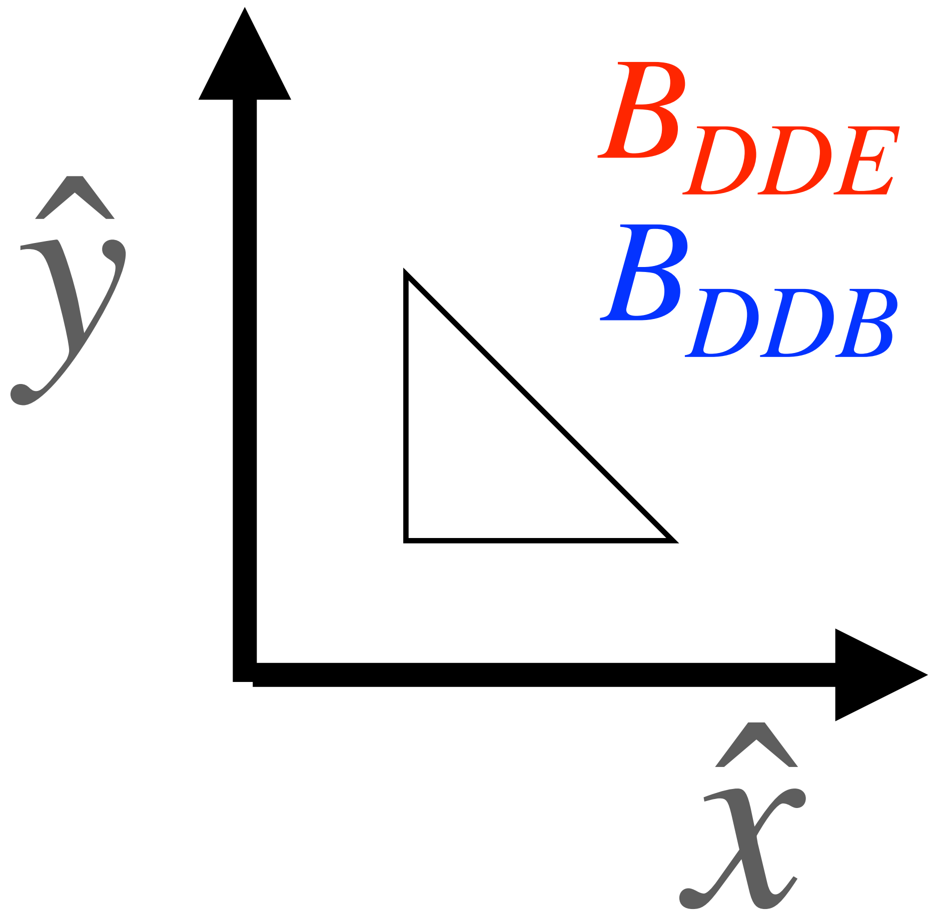


Bispectra on the sky

- All possible bispectra are nonzero, *even if parity conserved!*

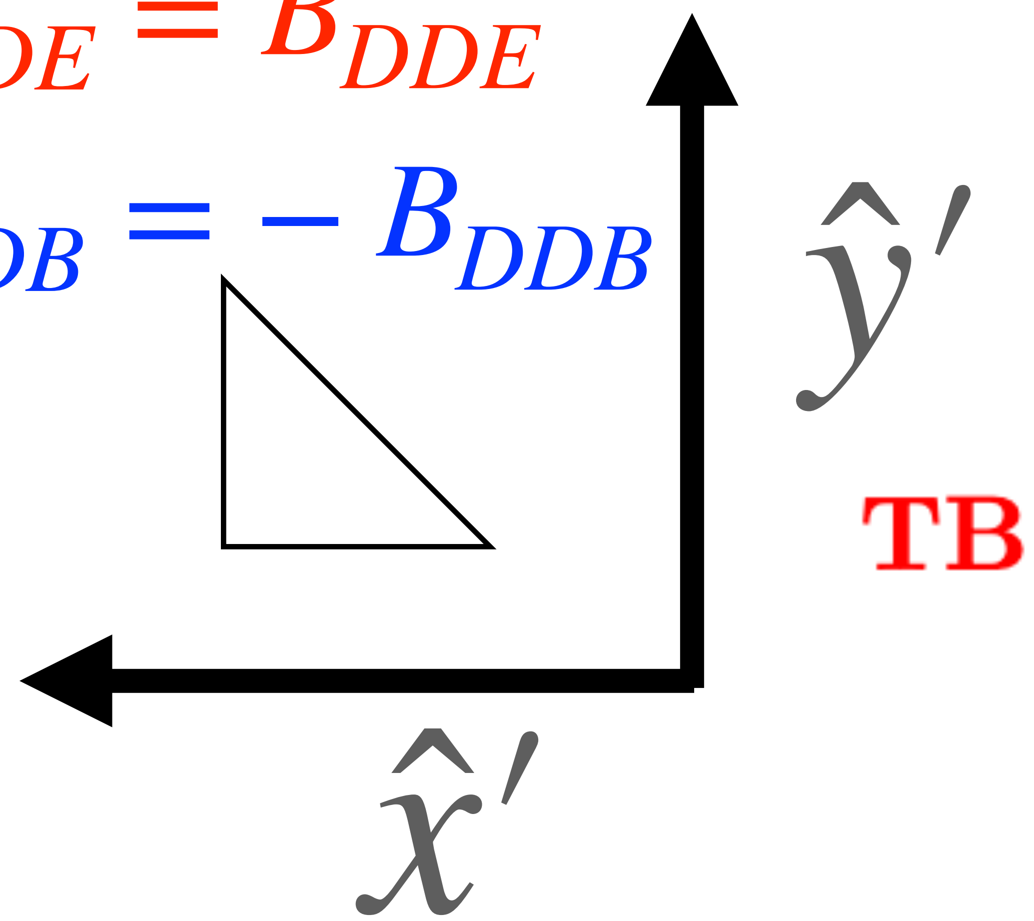
$B_{DDE}, B_{DDB} \quad B_{DEE}, B_{DBB}, B_{DEB} \quad B_{EEE}, B_{EBB}, B_{EEB}, B_{BBB}$

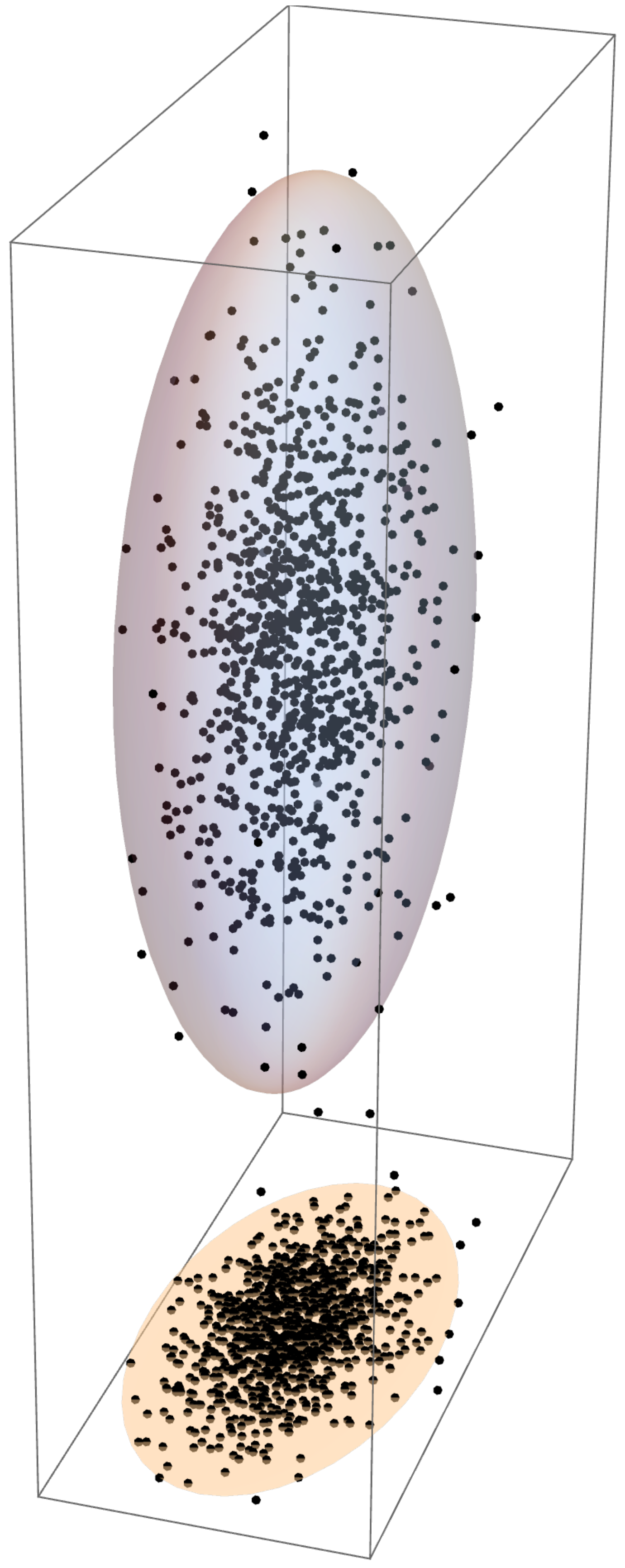
- Bispectra with an odd number of B-modes are parity-odd or ‘handed’:



$B'_{DDE} = B_{DDE}$

$B'_{DDB} = -B_{DDB}$





TB, Eggemeier, Kurita, Vlah, Chisari (2025)

- Analogy with CMB polarisation bispectra, e.g. B_{TTB} et cetera (but we work in flat sky)

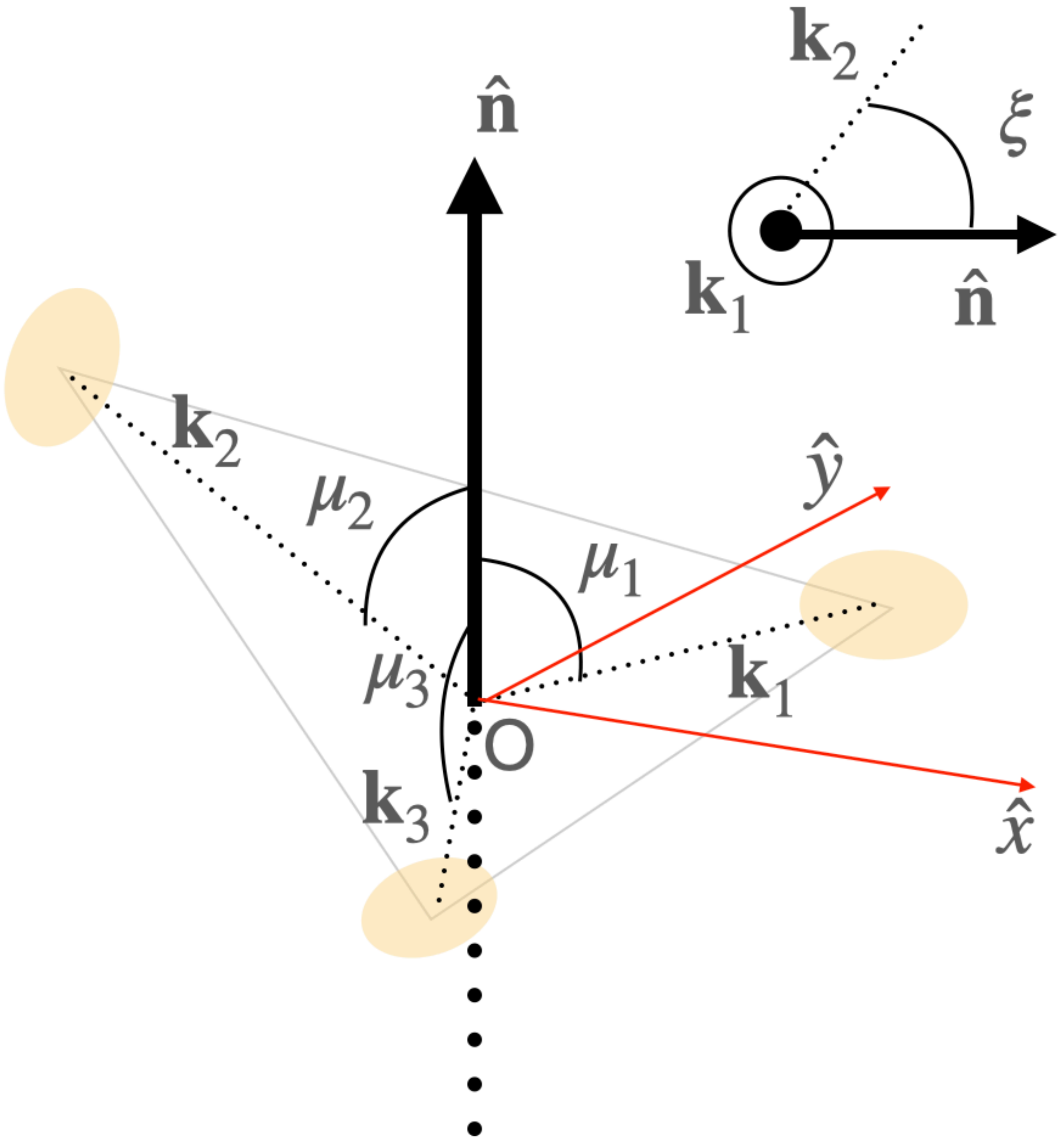
Multipoles

- Projected bispectrum is a function of *five* variables

$$B_{XYZ} = B_{XYZ}(k_1, k_2, k_3, \mu_1, \xi)$$

- Can write *multipole decomposition* (spherical averaging)

$$B^{\ell m}_{XYZ}(k_1, k_2, k_3) = \frac{2\ell+1}{4\pi}\int_{-1}^{1} \mathrm{d}\mu \int_0^{2\pi} \mathrm{d}\xi\, B_{XYZ}(k_1, k_2, k_3, \mu, \xi) Y_{\ell m}(\mu, \xi)$$



- Analogous to existing literature on galaxy bispectrum — Scoccimarro (2015)

- Parity-even (odd) $\Longleftrightarrow$ even (odd) in ξ — **TB**, Eggemeier, Kurita, Vlah, Chisari (2025)

$\propto \cos\xi$ $\propto \sin\xi$

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Separable estimators: use FFT

$$\hat{B}^{\ell_1\ell_2}_{XYZ}(k_1,k_2,k_3) = \frac{N_{\ell_1\ell_2}}{N_\Delta V}\sum_{\mathbf{q}_i\in\text{bin } i}\delta^K(\mathbf{q}_{123})\hat{X}(\mathbf{q}_1)\hat{Y}(\mathbf{q}_2)\hat{Z}(\mathbf{q}_3)\mathcal{P}^{\ell_1\ell_2}_{XYZ}(\{\hat{\mathbf{q}}_i\})$$

Scoccimarro (2015)

- Only two independent angles μ_1, μ_2, so two multipole indices (ℓ_1, ℓ_2)

$1-\mu_i^2$

- Use *associated* Legendre polynomials for angle weights for spin-2 fields (where $m=2$)

Kurita, Takada (2022)

$$\mathcal{P}^{\ell_1\ell_2}_{XYZ}(\{\hat{\mathbf{k}}_i\})_{\text{even}} = \mathcal{L}^{m_X}_{\ell_1}(\mu_1)\mathcal{L}^{m_Y}_{\ell_2}(\mu_2)\mathcal{L}^{m_Z}_{m_Z}(\mu_3)$$

Inoue++ (2024)

- Parity-odd: $\propto \sin\xi \propto \mathbf{k}_1 \times \mathbf{k}_2$

$$\mathcal{P}^{\ell_1\ell_2}_{XYZ}(\{\hat{\mathbf{k}}_i\})_{\text{odd}} = \mathcal{L}^{m_X}_{\ell_1}(\mu_1)\mathcal{L}^{m_Y}_{\ell_2}(\mu_2)\mathcal{L}^{m_Z}_{m_Z}(\mu_3)\big(\hat{\mathbf{n}}\cdot(\hat{\mathbf{k}}_1\times\hat{\mathbf{k}}_2)\big)$$

- Up to now, discussion was entirely *independent of dynamics*

Bispectrum Modelling

- 'TATT' model includes second order bias (3 pars) for shape field, $K_{ij} = \mathrm{TF}(\partial_i \partial_j \Phi)$

$$g_{ij}(\mathbf{x}, \tau) = c_1 K_{ij} + c_{K^2} \mathsf{TF}(K^2)_{ij} + c_{\delta K} \delta_m K_{ij} + c_t t_{ij} + \epsilon_{ij} + \epsilon_{ij}^{\delta} \delta + \epsilon_K K_{ij}$$

- EFT of IA provides framework for biasing at all orders

Blazek, MacCrann, Troxel, Fang (2017)

$$g_{ij}^{\mathrm{loc}}(\mathbf{x}, \tau) = \sum_{\mathcal{O}'} (c_{\mathcal{O}'} + \epsilon_{\mathcal{O}'}(\mathbf{x}, \tau)) \mathcal{O}'_{ij}(\mathbf{x}, \tau) + \epsilon_{ij}(\mathbf{x}, \tau)$$

Vlah, Chisari, Schmidt (2019,2020)

Desjacques, Jeong, Schmidt (2018)

- These models are robust on large scales, but not beyond say $k > 0.3\, h/\mathrm{Mpc}$
- Higher order IA biases have been detected for dark matter halos

Chen, Kokron (2023)

TB, Kurita, Chisari, Vlah, Schmidt (2023)

Akitsu, Li, Okumura (2023)

SNR for DESI LRG sample

TB, Eggemeier, Kurita, Vlah, Chisari (2025)

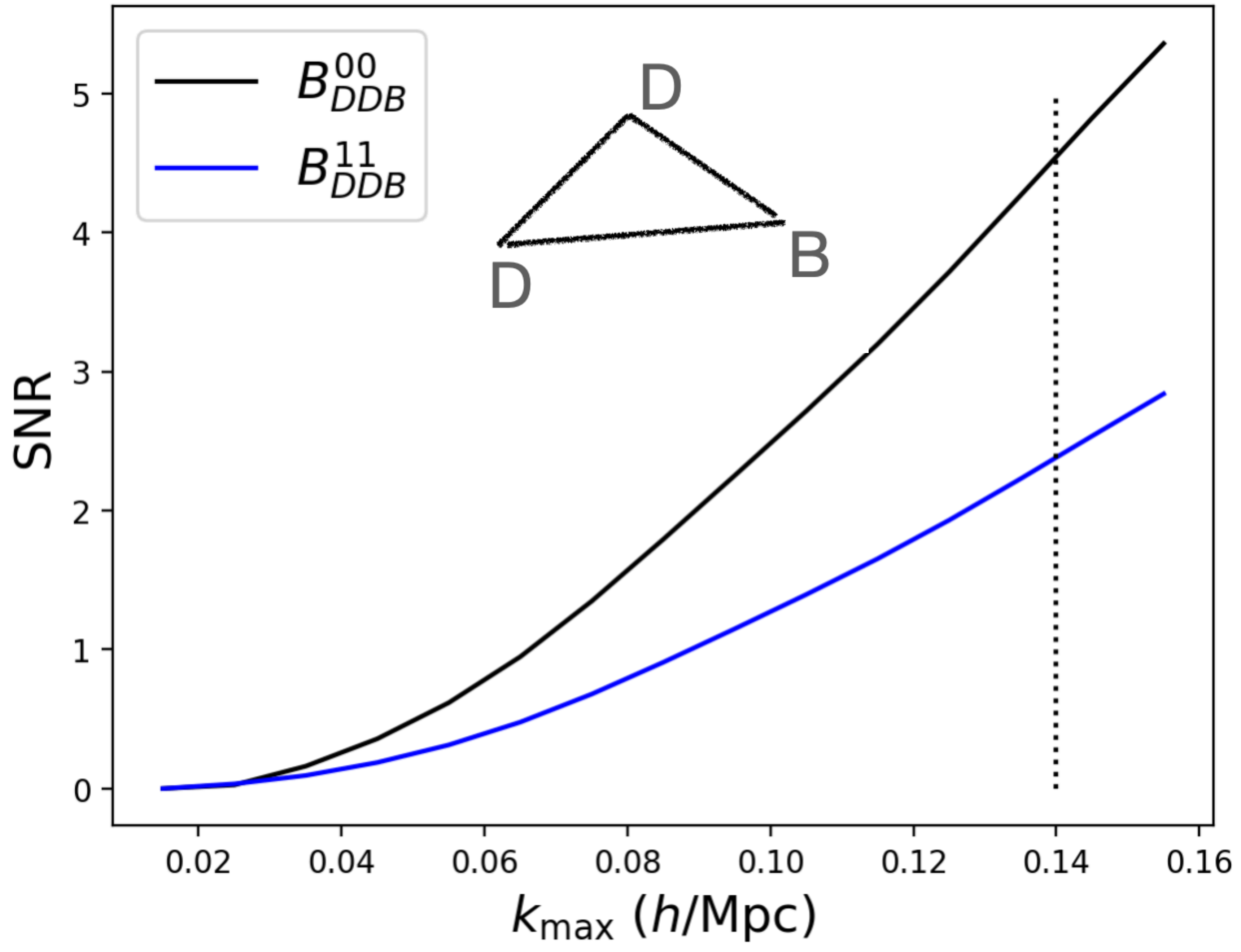
- Theoretical Gaussian covariance (should be a good approximation on large scales)
- Assume $f_{\text{sky}} = 0.1$ for overlap with LSST (high quality shape measurement - conservative)
- $0.4 < z < 0.8, \quad \bar{n} = 5 \cdot 10^{-4}\,(h/\text{Mpc})^3$ (volume-limited) Zhou++ (2022)
- This yields $V = 2.5\,(\text{Gpc}/h)^3$ and a total of $\sim 1.3 \cdot 10^6$ galaxies
- Use scales $k < 0.14\,h/\text{Mpc}$: tree-level PT applies and Gaussian covariance is reasonable
- Linear galaxy bias $b_1 \approx 2.1$ and linear alignment amplitude is $c_1 \approx -0.05$ ($A_{IA} = 4$)
- Second-order bias parameters fixed through co-evolution relations Chaussidon++ (2024)
- **All noise amplitudes $\alpha \propto \langle \epsilon\epsilon \rangle$ set to zero**

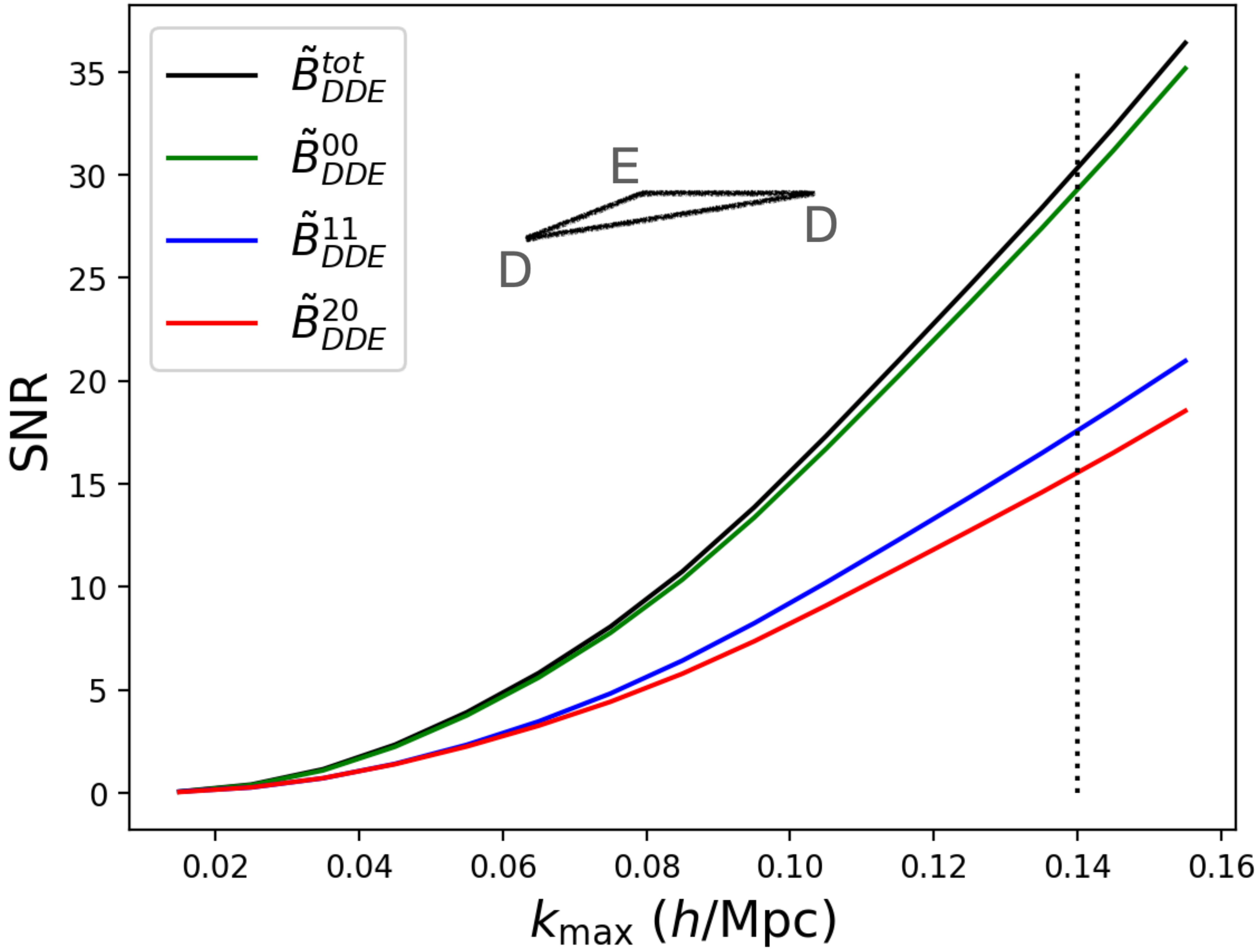
Kurita, Takada [BOSS] (2023)

Singh, Mandelbaum, More [BOSS] (2023)

One Shape Field: B_{DDE}, B_{DDB}

- Most SNR is in collinear configurations for B_{DDE}
- Parity-even multipoles look promising!
- Parity-odd signal is suppressed





More Shape Fields

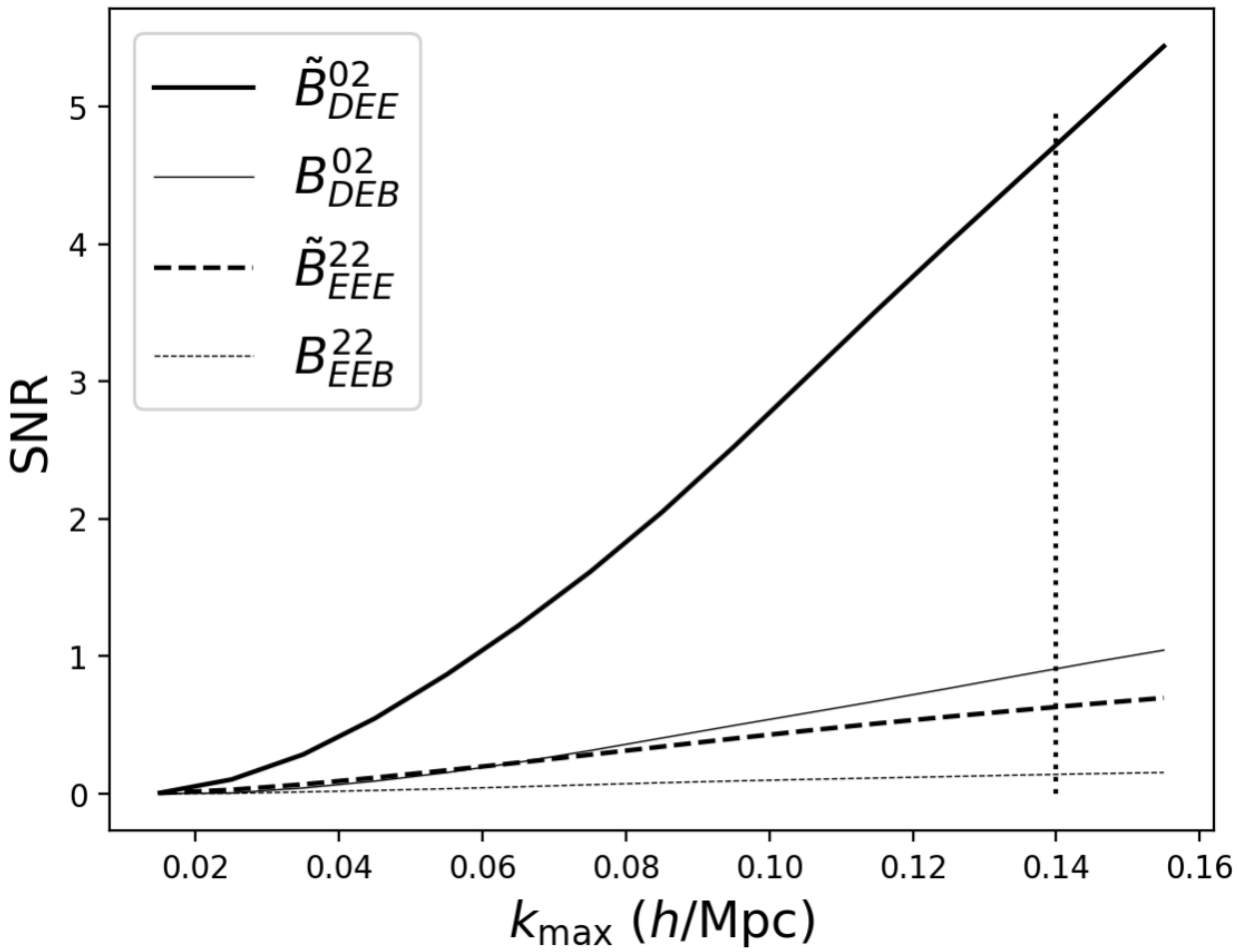
TB, Eggemeier, Kurita, Vlah, Chisari (2025)

- Difference w.r.t galaxy clustering: shape field is noisier

$P_{EE} \sim c_1^2 P_L(k) < \sigma_\gamma^2/\bar{n}$ for all scales!

- SNR plateaus quickly for most cases

- Extracting information is difficult with current samples

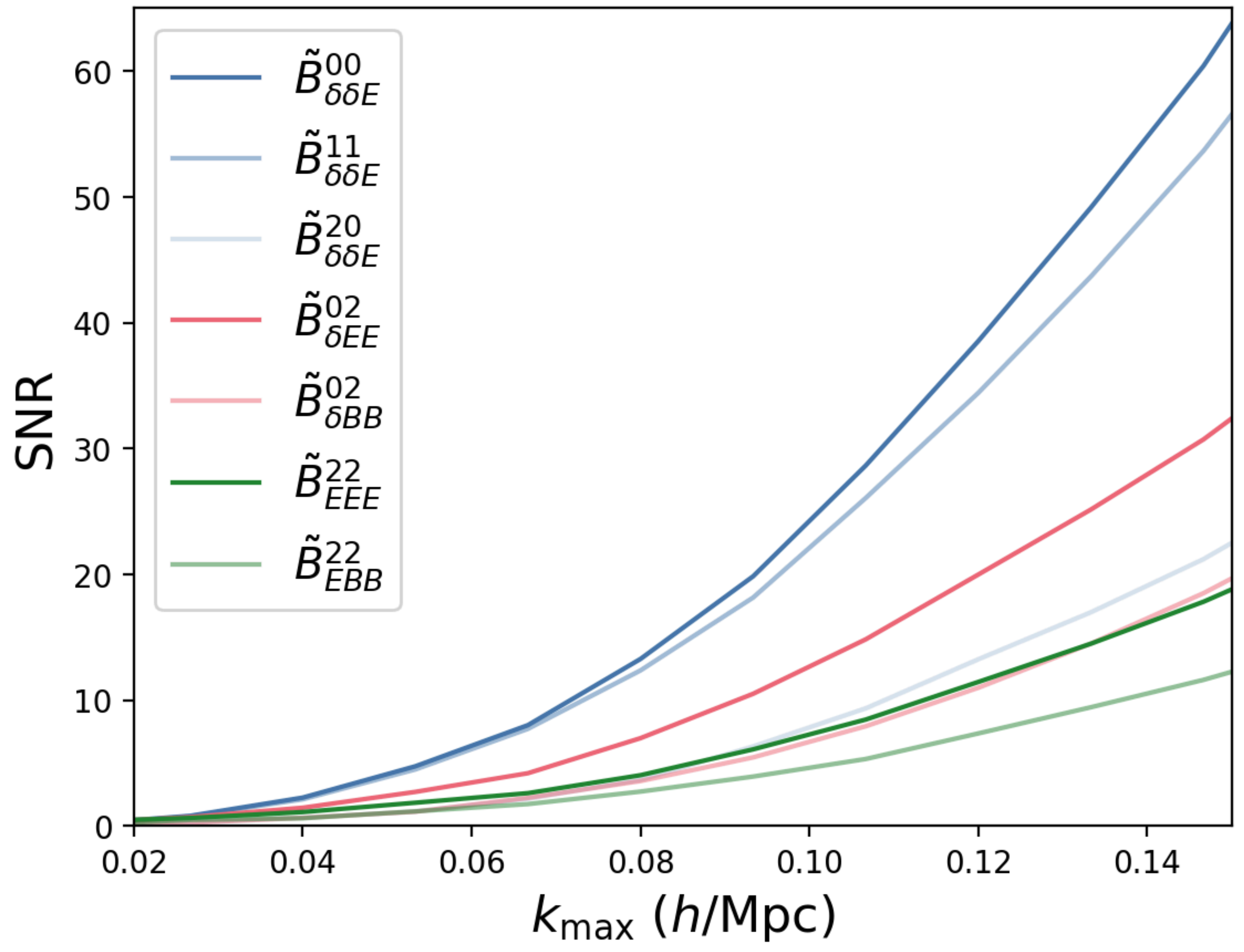


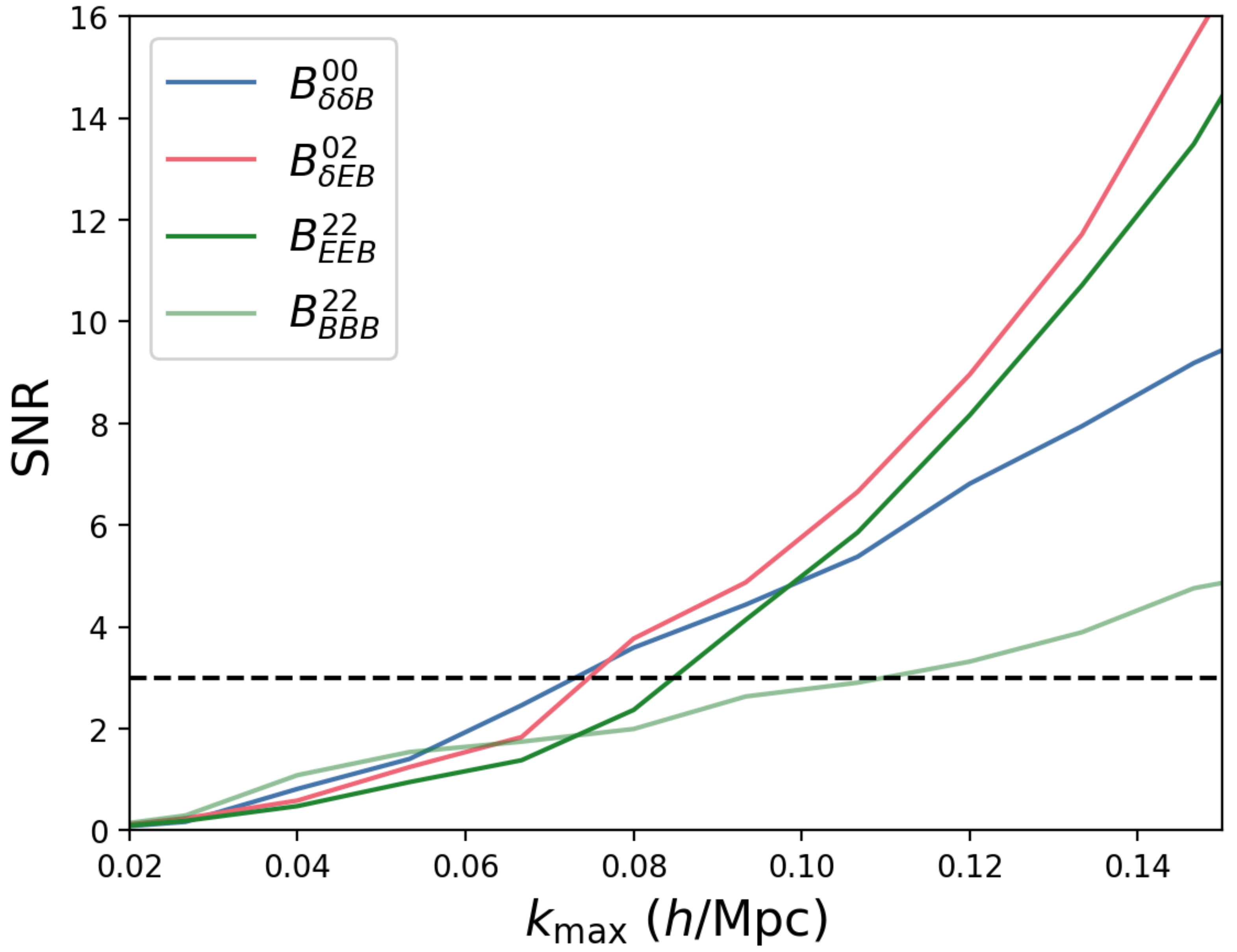
Model Tests (*preliminary results*)

- N-body simulations from the DarkQuest project, no RSD
- 20 realisations, each $V = 1\ (\mathrm{Gpc}/h)^3$ Kurita, Takada (2020)
- Use mass bin $\log[M/(h^{-1}M_\odot)] \in [12, 12.5]$, redshift zero
- Correlate *dark matter* with halo shapes E, B (not position)
- Five parameters in total: $c_1, c_{\delta K}, c_{KK}, c_t, \alpha^{1\delta}_{gg} \propto \langle \epsilon_{ij}\epsilon^{\delta}_{kl}\rangle$
- IA power spectrum analysis done before - fit *only bispectrum*

TB, Kurita, Chisari, Vlah, Schmidt (2023)

- All bispectrum combinations B_{XYZ} **detected in single volume**
- More than one shape field: mostly dominated by $\alpha^{1\delta}_{gg} \sim \sigma^2_\gamma/\bar{n}$





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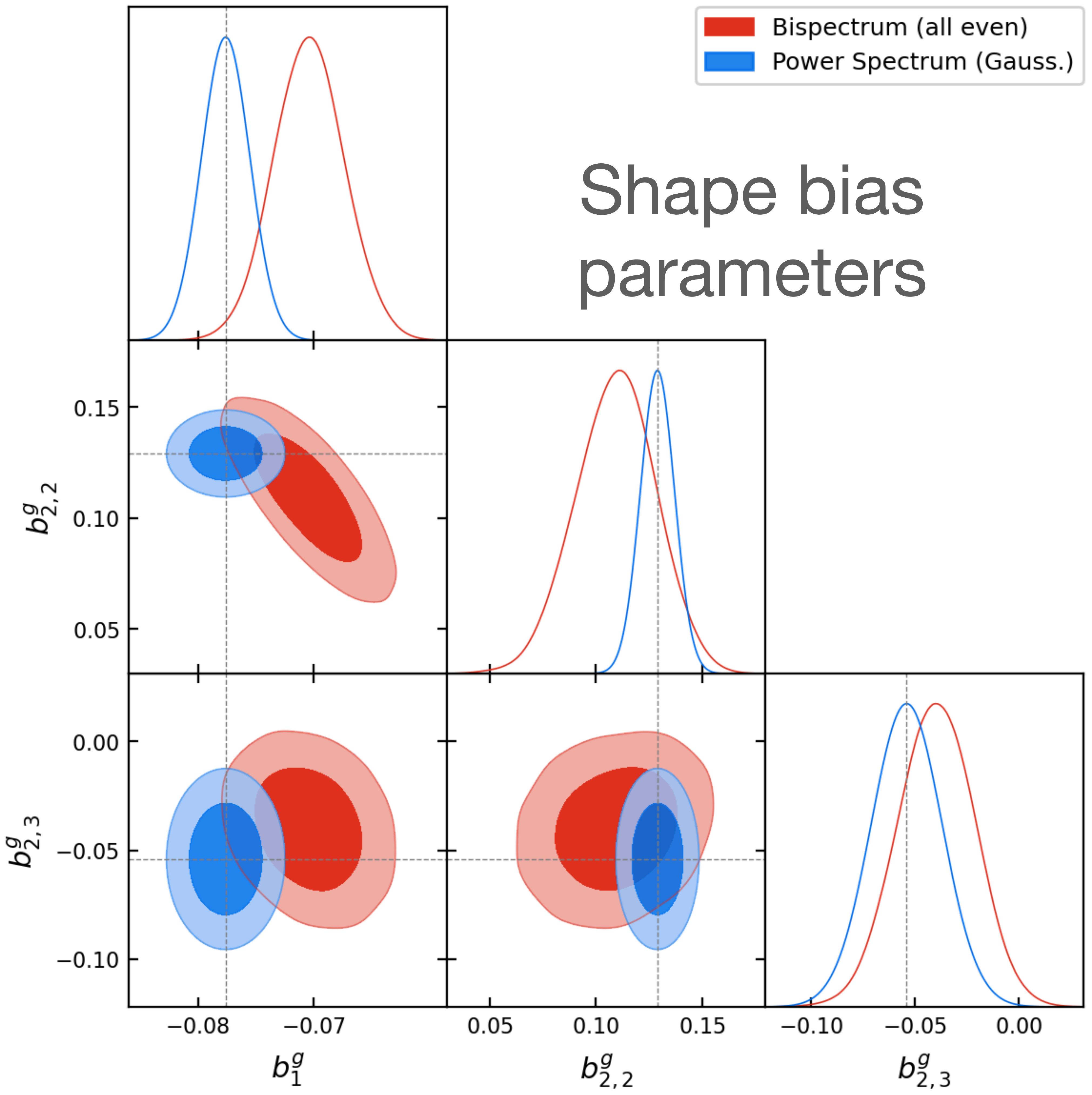
Model Tests (*preliminary results*)

TB, Kurita, Eggemeier, Vlah, Chisari (in prep.)

- Consistency between *all* multipoles!
- Consistency with IA power spectrum
- Model works up to $k \approx 0.11\, h/\mathrm{Mpc}$
- Degeneracies in IA power spectrum broken
- Very important to consider *higher multipoles:*

$$B^{00}_{DDE}, B^{11}_{DDE}, B^{20}_{DDE}$$

- Validate Gaussian covariance: broad agreement



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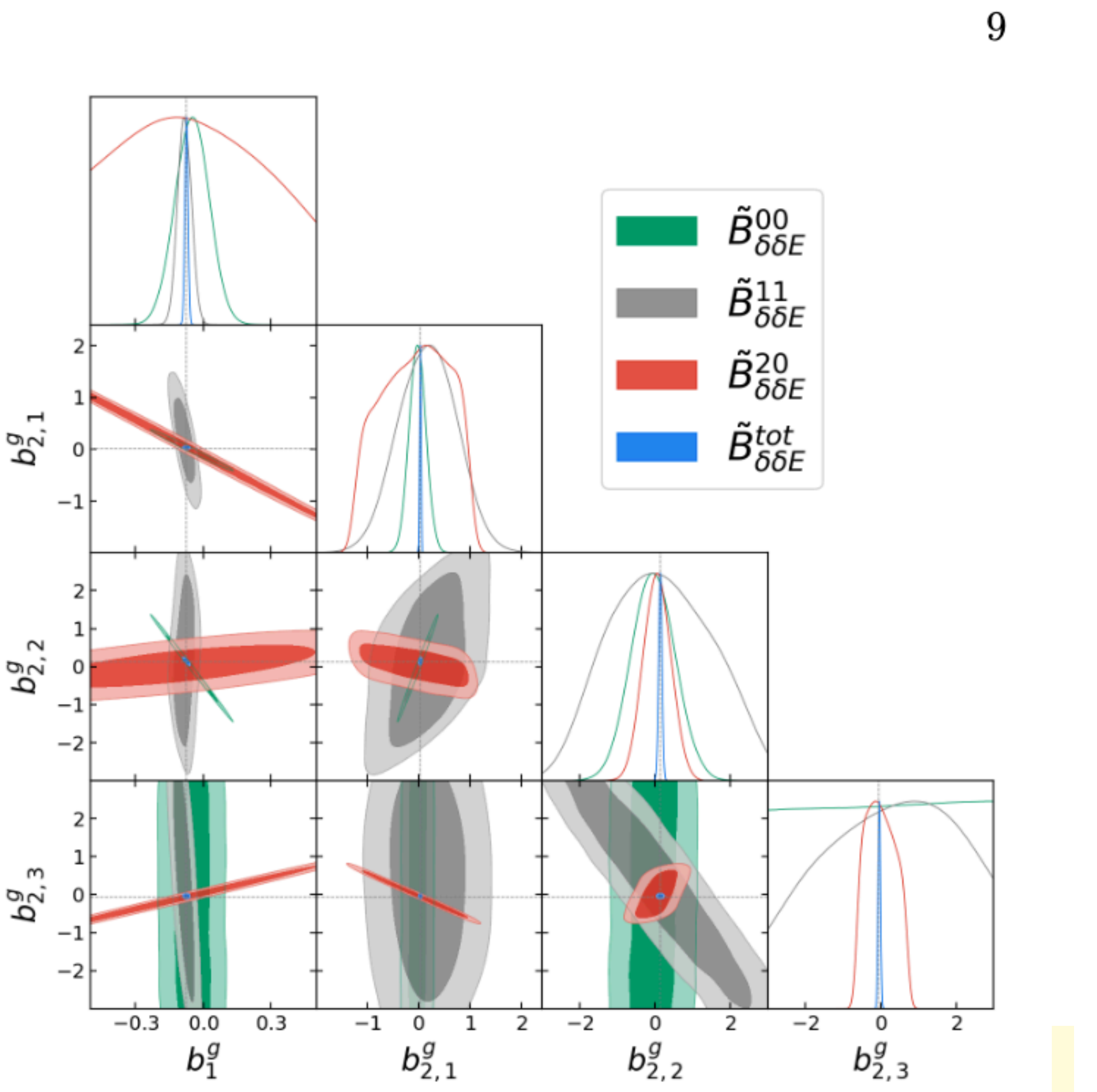
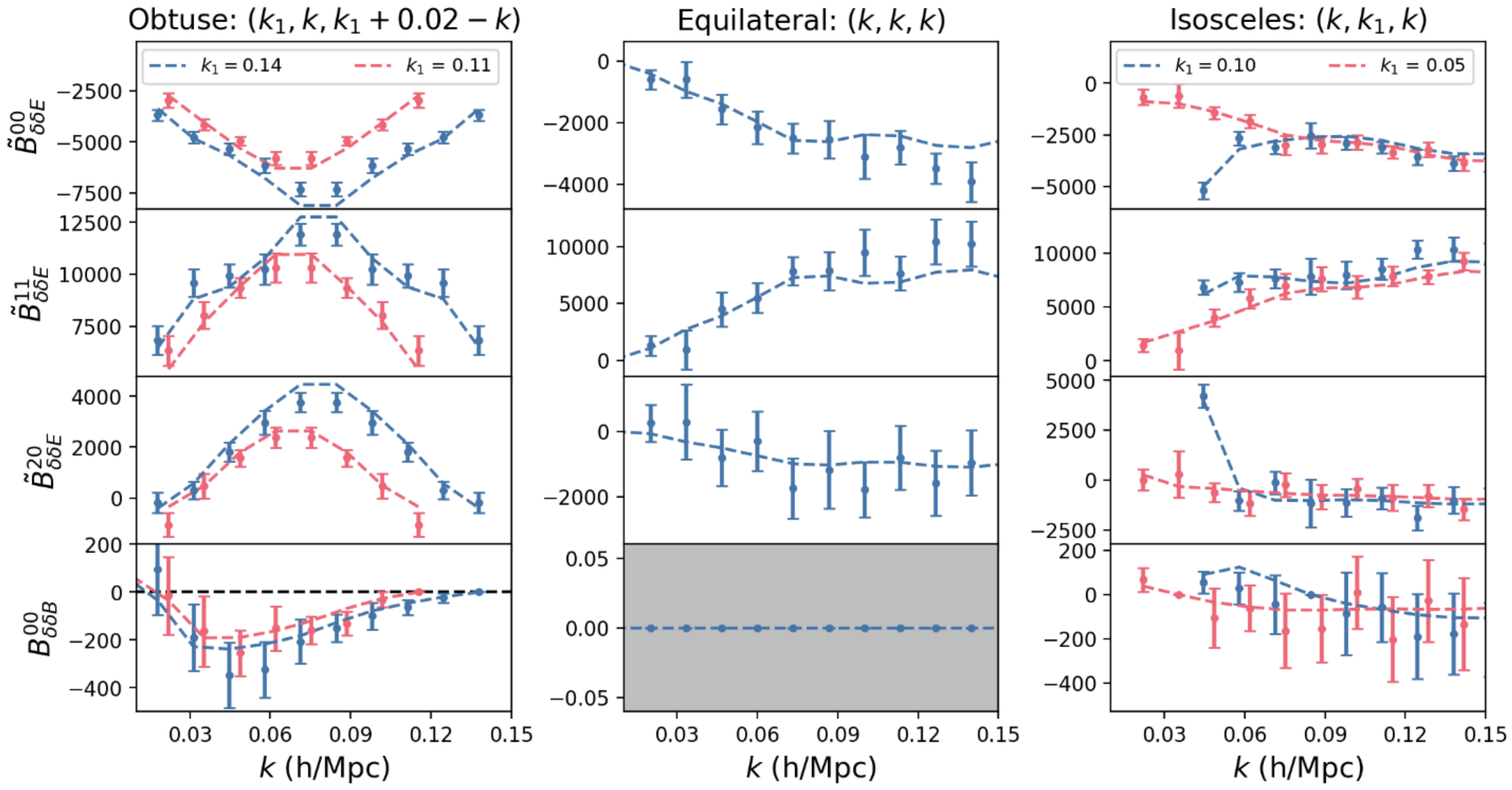
Open Questions / Future Avenues

- Use intrinsic alignment jointly with galaxy clustering (and other probes?)
- Consider compressed statistics, e.g. skew spectra
- More model building towards smaller scales, applications to mitigation for lensing
- Use IA bispectrum in direct measurements of IA
- Include redshift-space distortions consistently in the EFT
- Investigate stochastic contributions for realistic samples in e.g. hydrodynamical sims
- Thank you for your attention!

Bispectrum degeneracies





Obtuse: $(k_1, k, k_1 + 0.02 - k)$
Equilateral: (k, k, k)
Isosceles: (k, k_1, k)
$k_1 = 0.14$
$k_1 = 0.11$
$k_1 = 0.10$
$k_1 = 0.05$
$\tilde{B}^{00}_{\delta\delta E}$
$\tilde{B}^{11}_{\delta\delta E}$
$\tilde{B}^{20}_{\delta\delta E}$
$B^{00}_{\delta\delta B}$
k (h/Mpc)