NOISE IN (PARTICLE) DETECTOR SYSTEMS

LECTURE AT RADHARD SCHOOL (MINI WORKSHOP)

THESSALONIKI-BONN (DAAD PROGRAM)

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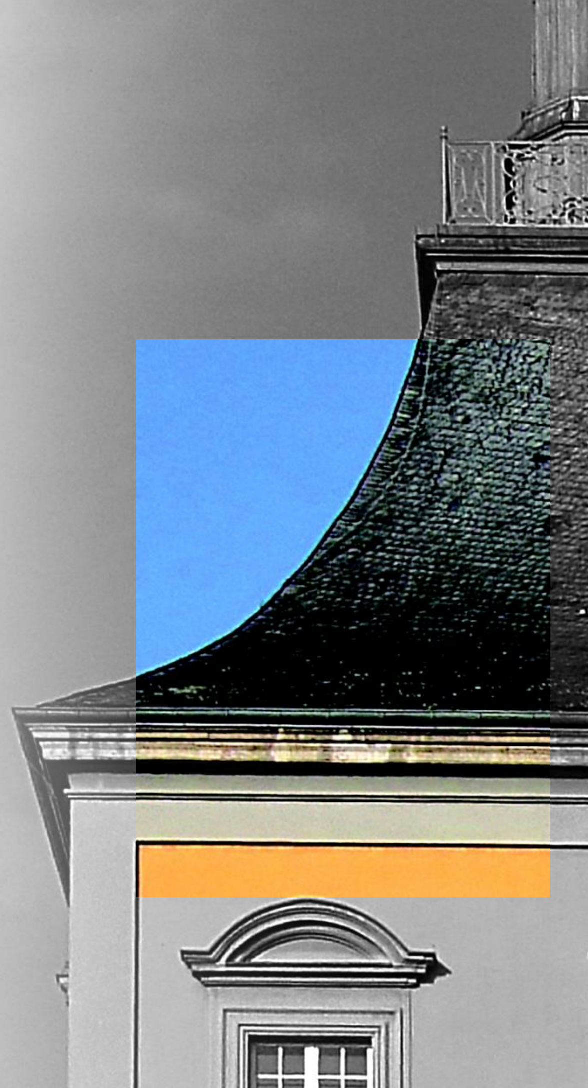

Disclaimer

Outline

- Signal fluctuations versus electronic noise
- Noise – what do you mean?
- Physical noise origins
- Noise in a typical detector readout system

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SI LAB Silizium Labor Bonn

Signal fluctuations and (electronic) noise

Distinguish

- Signal noise (better: signal fluctuations)
- Electronic noise

}
- inherent to a system

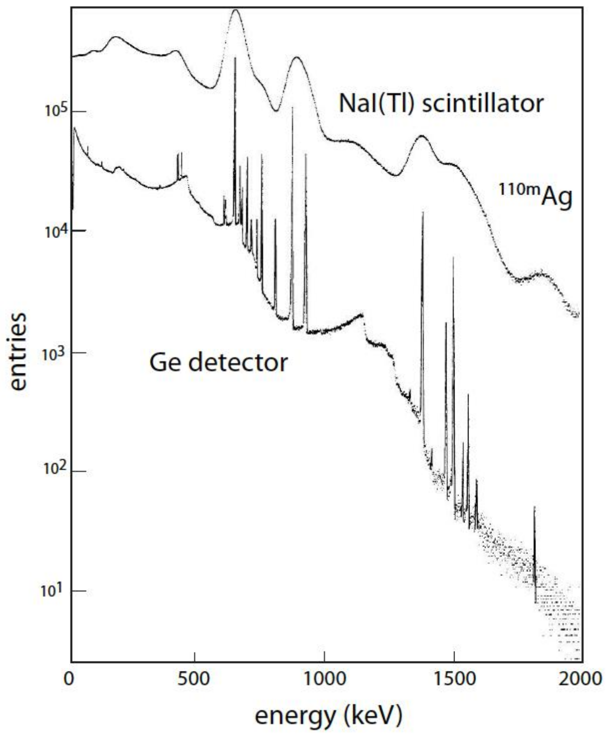
- EMI (electromagnetic interference)
  RFI (radio frequency interference)
  “pick-up” noise

often causing so-called “common-mode” noise

}
- introduced externally*
- different for every system
- => recognize and minimise

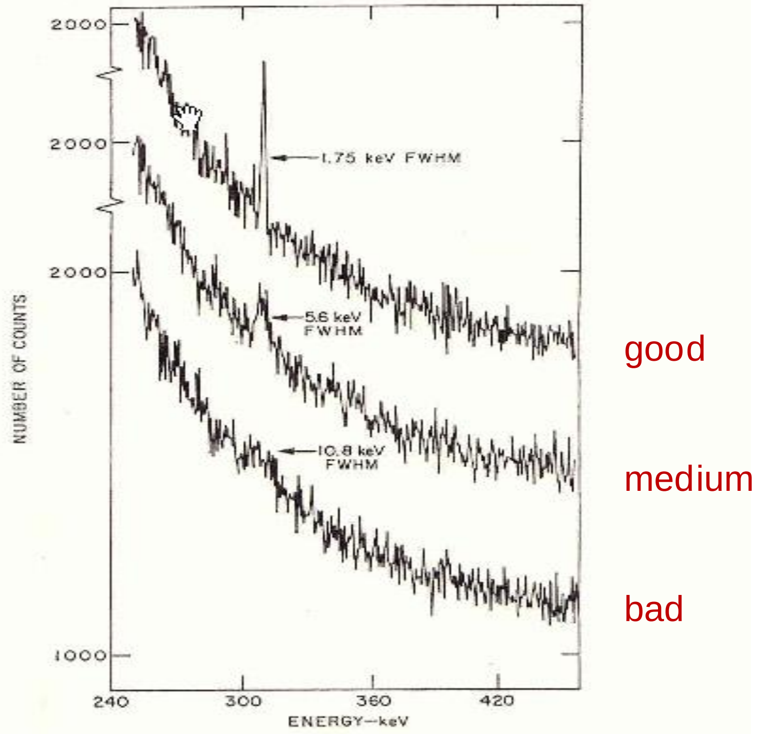
* e.g. from power supplies, digital switching, external RF signals, common grounding

Why bother?



(J.Cl. Philippot, IEEE Trans. Nucl. Sci. NS-17/3 (1970) 446)

Low noise improves the resolution and the ability to distinguish (signal) structures.



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. NS-19/1 (1972) 107

Low noise improves the signal-to-noise ratio
(narrow signal counts are in fewer bins and thus compete with fewer background counts).

examples from H. Spieler, 2005

Generic detector & R/O scheme: the dominant noise components

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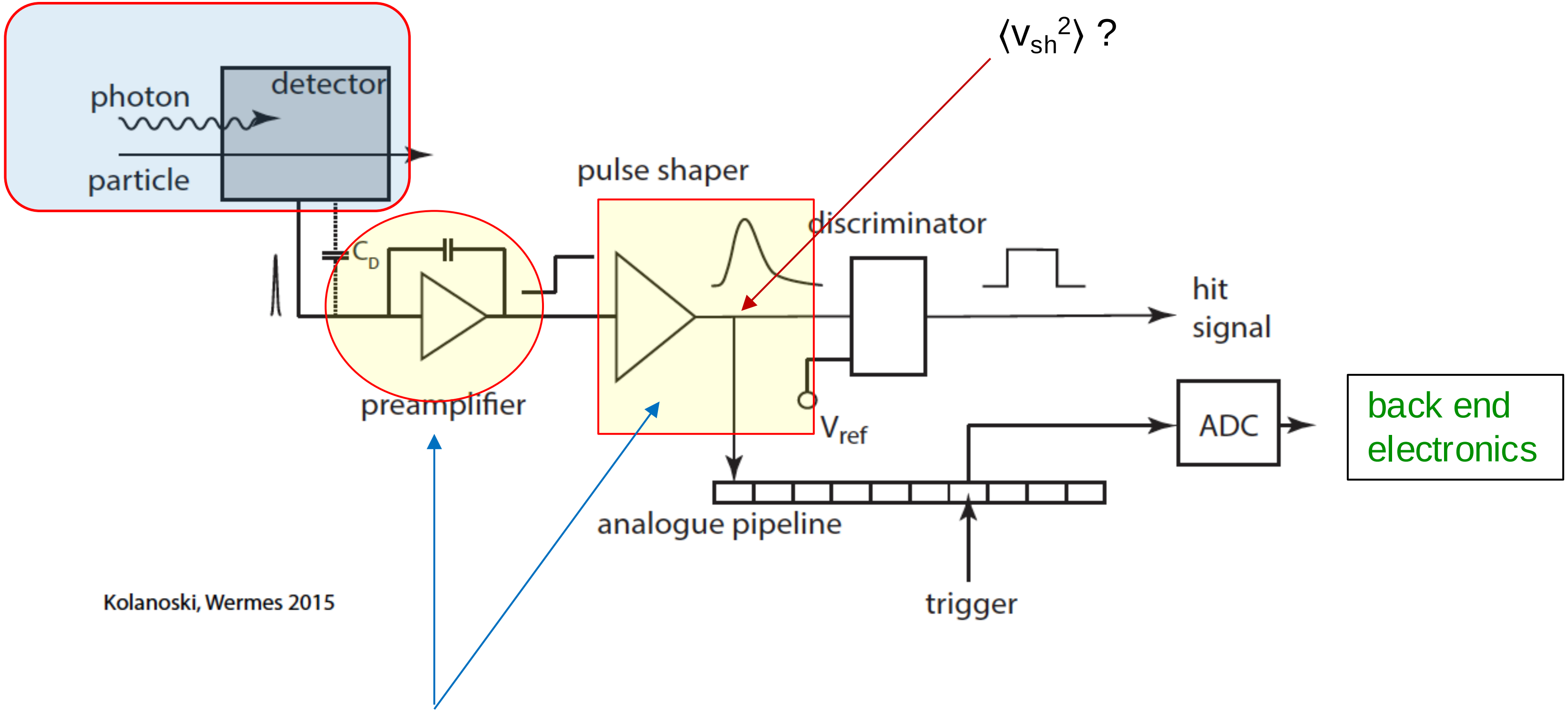
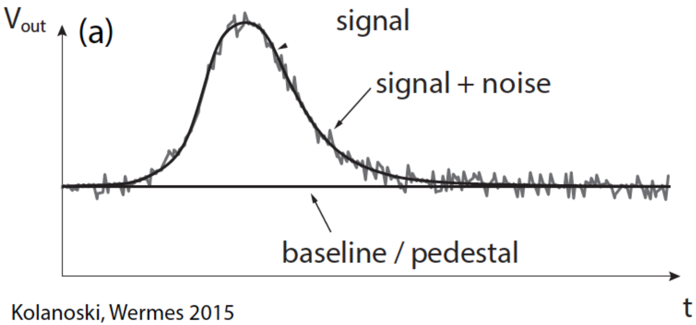
signal fluctuations

electronic noise



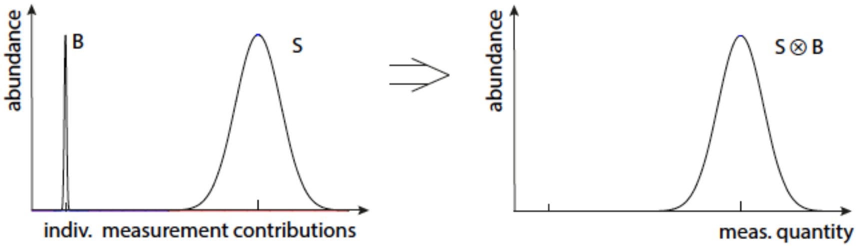
The dominant electronic noise of a system is hidden in these parts

Signal fluctuations and electronic noise



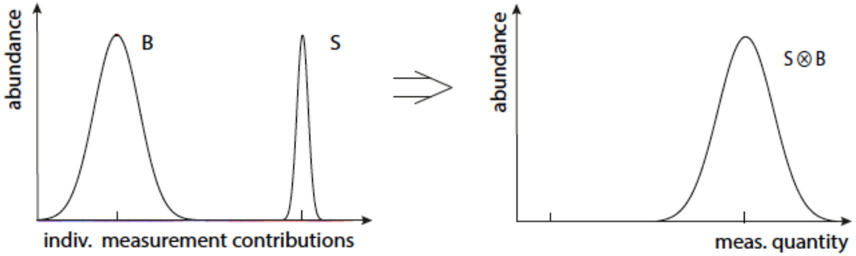
Kolanoski, Wermes 2015

case A: $\sigma_B \ll \sigma_S$



typical for NaI (Tl) crystal w/ PMT readout

case B: $\sigma_B \gg \sigma_S$



typical for silicon strip/pixel detectors

skip

EXAMPLE for **case A** ($\sigma_B \ll \sigma_S$): Positron Emission Tomography (PET)

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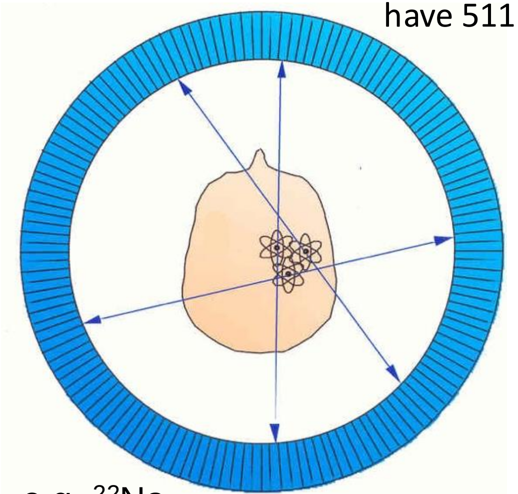
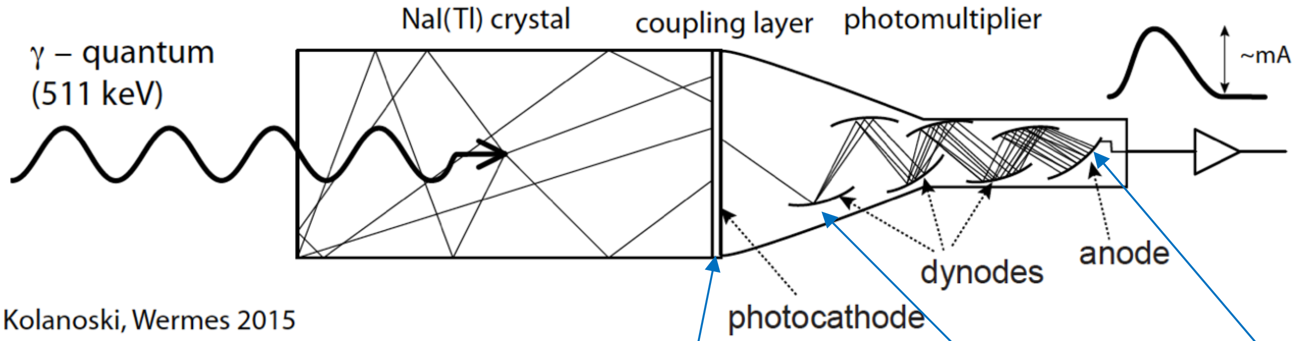
annihilation photons
have 511 keV



e.g. ^{22}Na
with PMT
readout

44 000 scintillation
photons per MeV



Kolanoski, Wermes 2015

22.000 photons -> 13.000 here -> 2.500 p.e. here -> 2×10^9 here

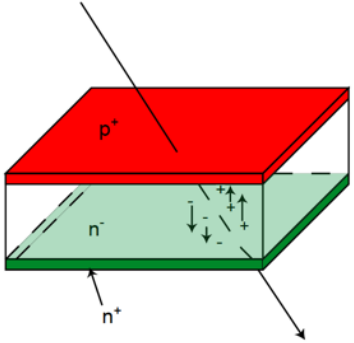
Where is the dominant noise source?

$$\sigma_{1.\,\mathrm{dyn}} = \sqrt{N_{1.\,\mathrm{dyn}}} = \sqrt{2500}\, e^- = 50\, e^- = 2\%\ N_{1.\,\mathrm{dyn}}$$

$$\Rightarrow \quad \sigma_{\mathrm{signal}} = \sigma_{1.\,\mathrm{dyn}} \times (\mathrm{amplification}) = 5 \times 10^7 e^- \,.$$

$\gg$ than PMT noise (typ. $10^{-(4\text{-}5)}$ × signal)

Example for **case B** ($\sigma_B \gg \sigma_S$): semiconductor detector



take e.g. 60 keV γ-ray from ^{241}Am

$$N_{e/h} = \frac{E_\gamma}{\omega_i} = \frac{60\,000\,\text{eV}}{3.65\,\text{eV}/(\text{e/h})} \approx 16\,500 \ \text{e/h pairs}\,.$$

fluctuations

$$\sigma_{\text{signal}} = \sqrt{N_{e/h} \cdot F} \qquad (F = \text{Fano factor,}$$
$$= \sqrt{N_{e/h} \cdot 0.1} = 40\, e^- \,.$$

Relative to signal $\frac{\sigma_{e/h}}{N_{e/h}} = 0.2\%$ (compare with 2% for NaI(Tl))

Noise in Si detectors ($\propto C_D$) is of order 100 e- (pixels) or 1000 e- (strips)
i.e. usually larger than the signal fluctuation.
=> electronic noise determines the (charge) resolution of the system

Signal Fluctuations

and

Fano Factor

Fano Factor I

The general computation of the Fano factor is complicated. We consider here as a simple system a silicon detector for which the discussion can essentially be reduced to two energy loss mechanisms (see also [903]): the generation of electron–hole pairs and lattice excitations (phonon excitations). For the creation of an e/h pair at least the band-gap energy (in silicon $E_G = 1.1\,\text{eV}$) is needed. For every event, however, the deposited energy is subdivided differently for the generation of e/h pairs or for lattice excitations such that on average the energy of $w_i = 3.65\,\text{eV}$ is needed to create one e/h pair.

Let us assume that in a process a fixed energy E_0 be deposited with every event in a detector, for example the energy of an X-ray or γ quantum from a radioactive source. This energy is available for the creation of N_p phonon excitations and of $N_{e/h}$ electron–hole pairs. We thus have

$$E_0 = E_i\, N_{e/h} + E_x\, N_p\,, \qquad (17.83)$$

where E_i and E_x are the (assumed fixed) energies necessary for one individual ionisation and one individual phonon excitation, respectively

simplified approach

only two energy loss mechanisms

1) e/h ionisation
2) phonon exc.

assume also Poissonian statistics

Fano Factor II

The energy E_0 can split arbitrarily between ionisations or lattice excitations. Since E_0 is fixed, however, and since therefore for every absorbed quantum the same energy is deposited, then in every absorption process a fluctuation of a larger E_0 portion into phonon excitation ($E_x\,\Delta N_p$) must be compensated by a correspondingly smaller E_0 portion for ionisation ($E_i\,(-\Delta N_{e/h})$), where ΔN_p and $\Delta N_{e/h}$ are the number fluctuations of phonons and e/h pairs for one individual event, respectively: !

$$E_x\,\Delta N_p - E_i\,\Delta N_{e/h} = 0\,.$$

Averaged over many absorption processes of the energy E_0 therefore yields:

$$E_x\,\sigma_p = E_i\,\sigma_{e/h}$$

$$\Rightarrow \quad \sigma_{e/h} = \sigma_p \frac{E_x}{E_i} = \sqrt{N_p}\frac{E_x}{E_i}\,,$$

with $N_p = \frac{E_0 - E_i N_{e/h}}{E_x}$ from slide 12

and with the average number of e/h pairs, $N_{e/h} = \frac{E_0}{\omega_i}$

$$\sigma_{e/h} = \frac{E_x}{E_i}\sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x}\frac{E_0}{\omega_i}} = \sqrt{\frac{E_0}{\omega_i}}\cdot\sqrt{\underbrace{\frac{E_x}{E_i}\left(\frac{\omega_i}{E_i} - 1\right)}_{=F \text{ (Fano factor)}}} = \sqrt{\frac{E_0}{\omega_i}\cdot F}$$

$$\Longrightarrow \sigma_{e/h} = \sqrt{N_{e/h}\cdot F}\,.$$

Note: E_0 is fixed

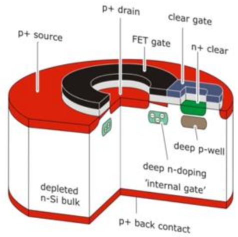
Result:
Energy resolution, i.e. $\sigma_{e/h}$ is in fact smaller than $\sqrt{N_{e/h}}$, since F is typically (e.g. in Si) << 1

Fano Factor III

Material	Si	Ge	GaAs	CdTe	diamond	Ar	liq. Ar
Fano factor	0.115	0.13	0.10	0.10	0.08	0.20	0.107–0.116

For detectors with more and also more complex signal generation processes, as for example scintillators, for which exciton processes also play a role (see section 13.3 on page 515), Fano factors larger than one ($F > 1$) can even occur [360].

Fano Factor IV: an example



high-resolution DEPFET sensor (for low-E X-ray imaging, slow shaping)

- measured electronic noise @ RT (!) $\sigma_n = 1.6\,e^-$

M. Porro et al., IEEE Trans. Nucl. Sci. 53 (2006), 401.

- signal fluctuation @ E_0 = 1 keV X-ray photons

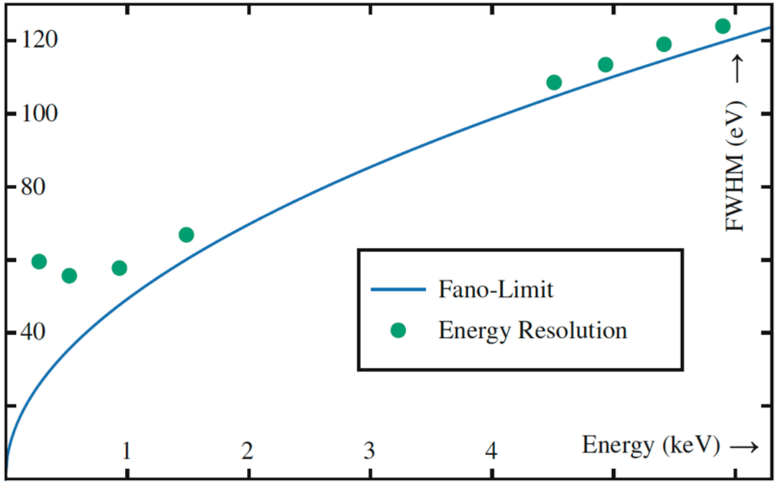
$$N_{e/h} = \frac{1000\,\text{eV}}{3.65\,\text{eV}} = 275$$

$$\sigma_{e/h} = \sqrt{N_{e/h} \cdot F} = \sqrt{275 \cdot 0.115} = 5.7$$

$$\Rightarrow \sigma_E = \frac{\sigma_{e/h}}{N_{e/h}} \cdot E_0 = 21\,\text{eV} \Rightarrow \text{FWHM} = 48\,\text{keV}$$

- noise of 1.6 eV

$$\Rightarrow \sigma_{E_n} = \frac{1.6}{275} \cdot 1\,\text{keV} = 6\,\text{eV} \Rightarrow \text{FWHM} = 14\,\text{eV}$$



useless to be better than the Fano limit

J. Schmidt et al., Microsc. Microanal. 24 (Suppl 1), 2018, https://doi.org/10.1017/S1431927618004130
N. Meidinger, J. Müller-Seidlitz, Handbook of X-ray and Gamma-ray Astrophysics, doi:10.1007/978-981-16-4544-0_20-1

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ELECTRONIC NOISE

Noise … what?

Noise terms

shot noise
Funkelrauschen
current noise

Widerstandsrauschen
burst noise
series noise

switching noise
popcorn noise
Schrotrauschen

pink noise
flicker noise
thermal noise

Nyquist noise
kT/C noise
Johnson noise

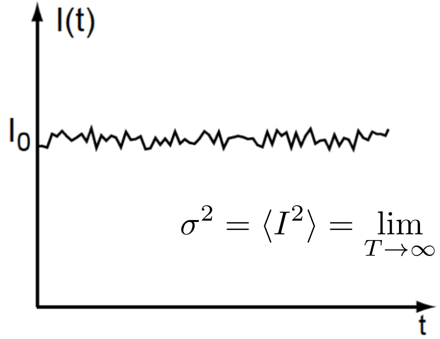
parallel noise

voltage noise
white noise
1/f noise ⟷ RTS noise

pick-up noise
common mode noise

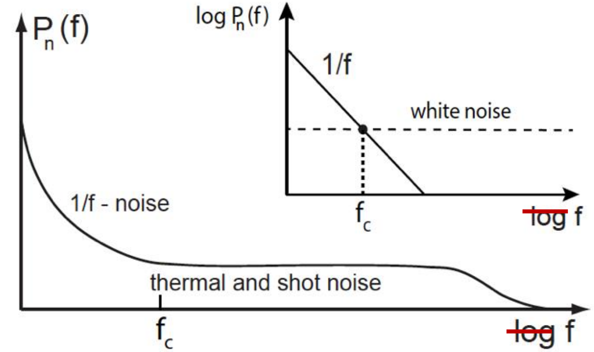
Quantifying Noise

"noise is a variation about a mean value" => quantified by the variance $\langle i^2 \rangle$ or $\langle v^2 \rangle$



$$\sigma^2 = \langle I^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T (I(t) - I_0)^2 \, dt$$

(a) Current noise as a function of time.



(b) Spectral noise density (schematic) as a function of frequency.

f = frequency

spectral noise (power) density

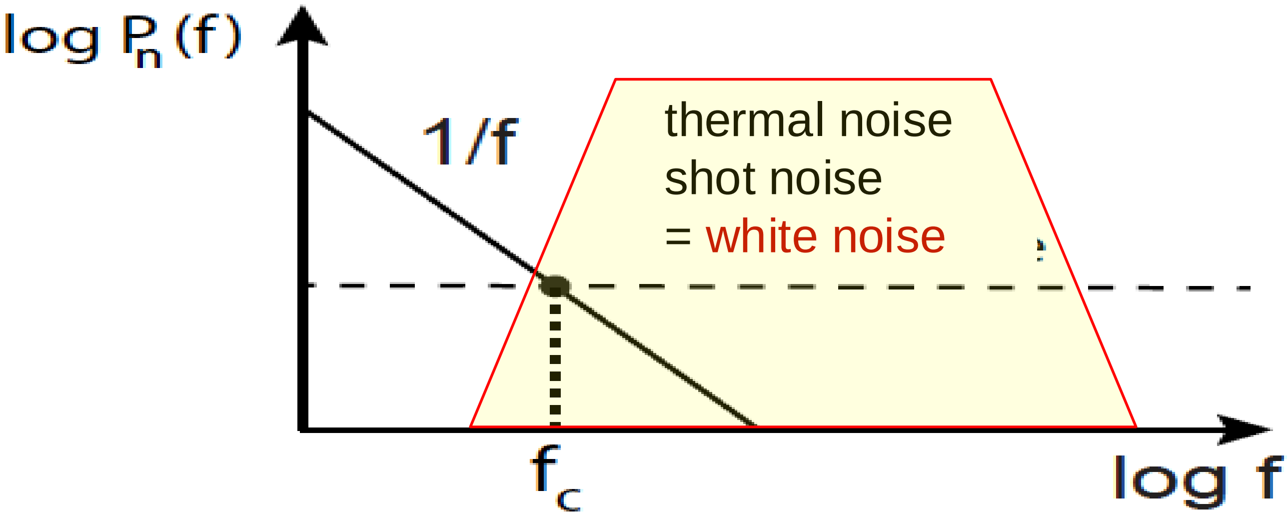
eg for thermal noise in R

$$\frac{dP_n}{df} = \frac{1}{R} \frac{d\left\langle v^2 \right\rangle}{df} = R \frac{d\left\langle i^2 \right\rangle}{df} \Rightarrow P_n = \int_0^\infty \frac{dP_n}{df} df$$

unit of "voltage noise power density" = $[\sqrt{v^2/df}] = V/\sqrt{Hz}$

unit of "current noise power density" = $[\sqrt{i^2/df}] = A/\sqrt{Hz}$

Notice …



filtering, i.e. limiting the Bandwidth by high- (CR) and low-pass (RC) filters

- reduces the noise
- but: yields a slower response

Noise origins
(… a bit tricky in parts)

Noise origins

A current $i = \frac{Nev}{d}$

can fluctuate in number

- fluctuations in carrier emission over a barrier
- fluctuations in trap/release processes

$$(di)^2 = \left(\frac{ev}{d} dN\right)^2 + \left(\frac{eN}{d} dv\right)^2$$

and in velocity

- Brownian motion (thermal)

Anticipating: noise physical origins

$$\langle i^2 \rangle = 2q\langle i \rangle df$$

shot noise

number fluctuation

$$\langle i^2 \rangle = \frac{4kT}{R} df$$

thermal noise

velocity fluctuation

$$\langle i^2 \rangle = \ \text{const.} \ 1/\mathrm{f}^{\alpha} df$$

1/f noise

number fluctuation

▷ skip derivations of noise origins?

The thermal noise formula I (somewhat simplified, full … see e.g. ref[1].)

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origin: thermal (Brownian motion) of charge carriers

Two ways to derive from first principles

1. Thermal velocity distribution of carriers
   => time (or frequency) dependence of induced current → difficult derivation

2. Application of Planck's law for thermal radiation
   ("hides" a bit the physics behind a general result of statistical mechanics)
   => yields the spectral density of the radiated power
   i.e. the power that can be extracted in thermal equilibrium

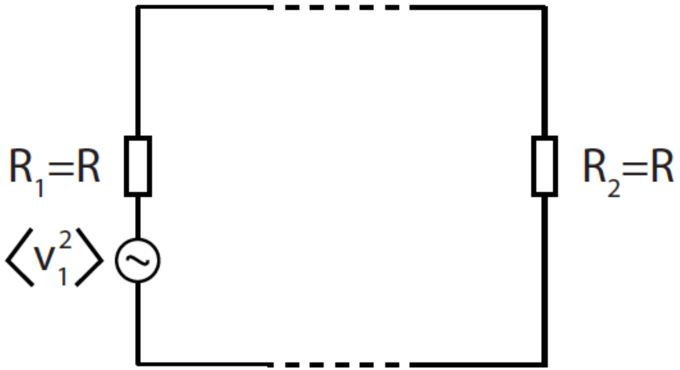
$$\frac{dP}{d\nu} = \frac{h\nu}{exp\left(\frac{h\nu}{kT}\right) - 1} \quad \rightarrow \; (\text{for}\, h\nu \ll kT) \quad = \frac{h\nu}{1 + \frac{h\nu}{kT} - 1} = kT$$

i.e. at sufficiently low frequencies (< THz) is P independent of ν
and is always the same amount in a bandwidth interval $\Delta\nu$

$P = kT\,\Delta\nu \;\rightarrow\; kT\,\Delta f$ (*)

The thermal noise formula II

To obtain the thermal noise voltage power spectrum of a resistor, consider an open resistor R_1 which generates a (quadratic) noise voltage $\langle v_1^2 \rangle$.



When both resistors are now short-circuited, the noise voltage $\langle v_1^2 \rangle$ over R_1 causes a voltage v^2 over R_2 yielding a noise power in R_2

$$P_{1\rightarrow 2} = \frac{v^2}{R_2} = \frac{\langle v_1^2 \rangle}{R_2} \left(\frac{R_2}{R_1 + R_2} \right)^2 = \frac{\langle v_1^2 \rangle}{4R}$$

with R_1 and R_2 having equal resistances ($R_1 = R_2 = R$).

In thermal equilibrium R_2 transfers the same noise power to R_1

$$P_{1\rightarrow 2} = P_{2\rightarrow 1}$$

for <u>every frequency portion</u> of the noise.

The factor 4 originates here from two resistors. The result, however, is general as shown in Nyquist's original paper.
Nyquist, H.: Phys. Rev. 32, p. 110 (1928)

The power <u>spectrum</u> (density) hence is a function of Δf, R, and of the temperature T.

The thermal noise formula III

with (*) P = kT df … we get

$$d\langle v_n^2\rangle = 4kTR\,df$$

and with Ohm's law relating $\langle i_n^2\rangle$ and $\langle v_n^2\rangle$

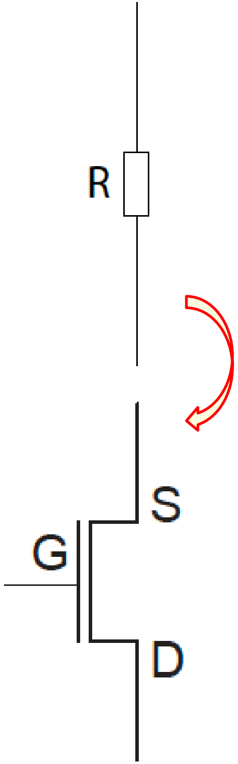
$$d\langle i_n^2\rangle = d\frac{\langle v_n^2\rangle}{R^2} = \frac{4kT}{R}df$$

Note: Thermal noise is always there (if T>0). It does not need power.

In a 1 kΩ resistor we find a current independent thermal current noise of or a voltage fluctuation over R of

$$\sqrt{\frac{d\langle i^2\rangle}{df}} = 4\frac{\mathrm{pA}}{\sqrt{\mathrm{Hz}}} \quad \text{or} \quad \sqrt{\frac{d\langle v^2\rangle}{df}} = 4\frac{\mathrm{nV}}{\sqrt{\mathrm{Hz}}}$$

Thermal noise in a MOSFET

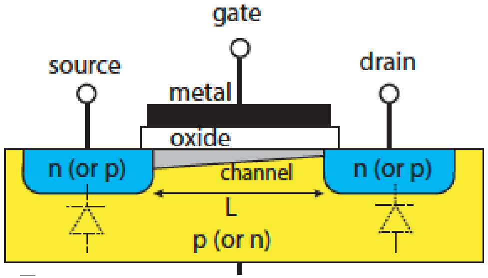


$$d\langle v_n^2\rangle = 4kTR\,df$$

$$d\left\langle i_n^2\right\rangle = \frac{4kT}{R}df$$

$$d\left\langle v_n^2\right\rangle = 4kT\gamma\frac{1}{g_m}df$$

$$d\left\langle i^2\right\rangle = 4kT\frac{g_m}{\gamma}df$$



$g_m = dI_D/dV_{GS}$

transconductance

γ = adjustment factor

$\frac{2}{3}$ in strong inversion

$\frac{1}{2}$ in weak inversion (subthreshold operation)

MOSFET operation characteristics

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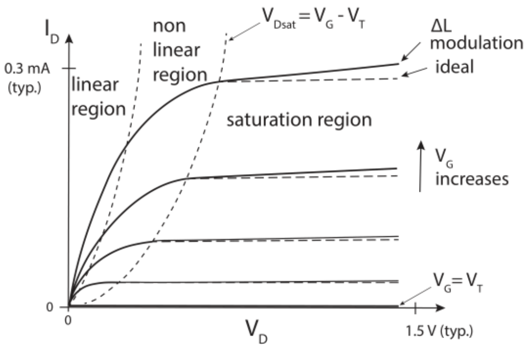
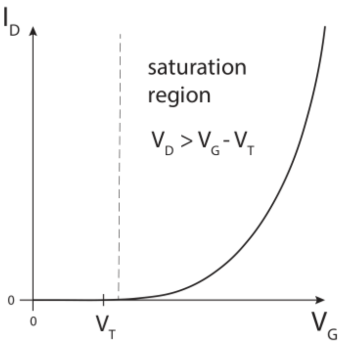
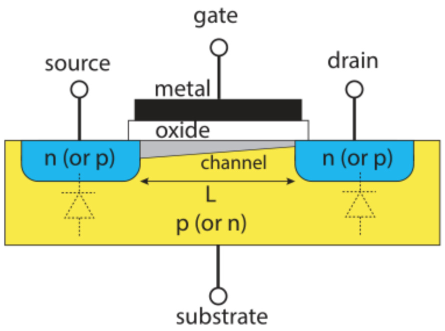


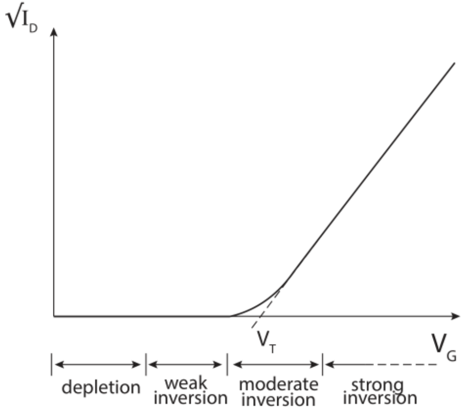
(a) Output characteristic.



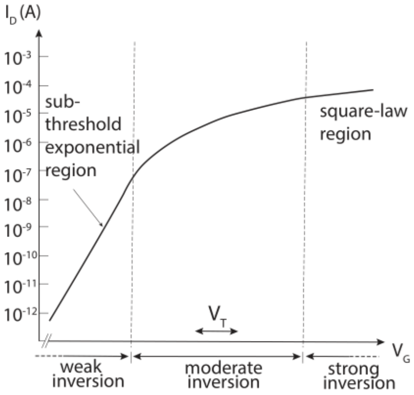
(b) Transfer characteristic (I_D vs. V_G).



(a) MOSFET.



(c) Transfer characteristic ($\sqrt{I_D}$ vs. V_G)



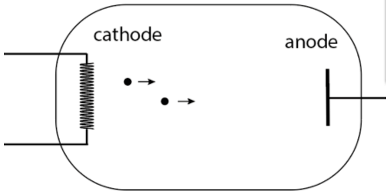
(d) Transfer characteristic near threshold.

modern low power
IC design usually operates
in weak or moderate inversion

Shot noise (somewhat simplified derivation, full … see e.g. ref [1])

origin: excess e⁻ injection into a device when quantisation plays a role (over a barrier, i.e. NOT in a resistor)

e.g.



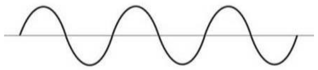
… but also in a pn boundary (detector diode)

e/h in depletion zone induce current pulses until recombination (short)

- the current pulses can be regarded as δ - functions,
  i.e. all frequencies contribute => white noise

$$\int_{-\infty}^{\infty} i_e(t)dt = e \quad \rightarrow \quad di_e/df = e \cdot 2 \qquad \text{because} \qquad \mathcal{L}[i(t)] = \mathcal{I}(s = \sigma + 2\pi i f) = \int_0^{\infty} i(t)e^{-st}\,dt$$

- for infinitely narrow df the spectral component *k* contributing is one sine wave
  with mean = 0 and rms = 1/√2



$$=> \sqrt{\frac{d\langle i_{e,k}^2\rangle}{df}} = \frac{2e}{\sqrt{2}} = \sqrt{2}e$$

- for N electrons of total average current ⟨i⟩ = Ne/t = Ne Δf we get

$$\langle i^2\rangle = \sum_{k=1}^{N}\left(\frac{di_{e,k}}{df}\right)^2 (df)^2 = 2Ne^2(df)^2 = 2e\underbrace{(Nedf)}_{\langle i\rangle = I_0}df = 2eI_0 df$$

needs a current

1mA (leakage) current yields

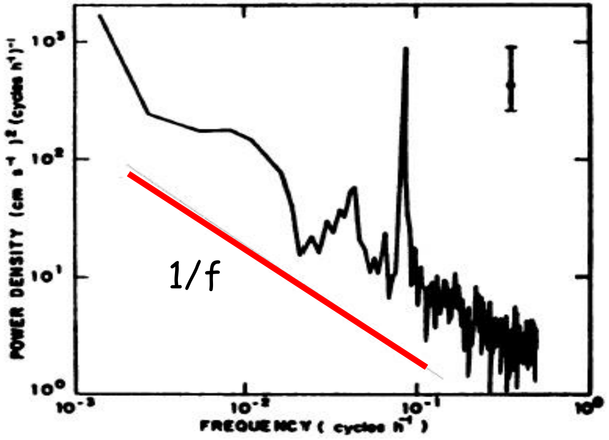
$$\sqrt{\frac{\Delta\langle i^2\rangle}{\Delta f}} = \sqrt{2eI_0} = 18\frac{\text{pA}}{\sqrt{\text{Hz}}}$$

of noise

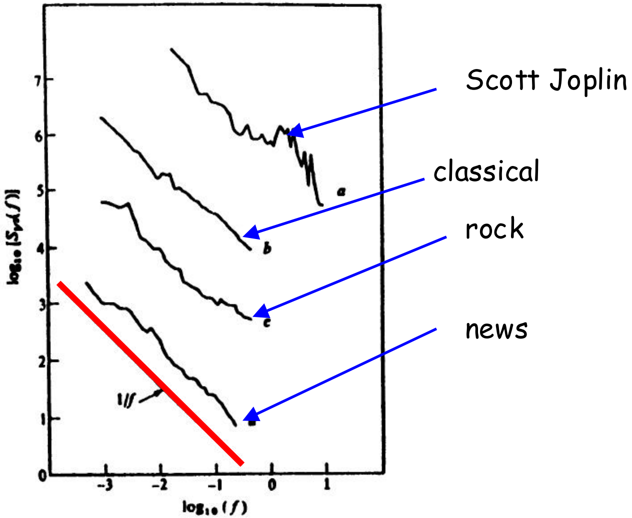
1/f noise - I

origin:

- superposition of relaxation processes with different time constants
- appears in many systems (ocean current velocity, music, broad casting, earthquake frequency spectra)
- many papers in literature (all you ever wanted to know) http://www.nslij-genetics.org/wli/1fnoise/



east-west component of ocean current velocity

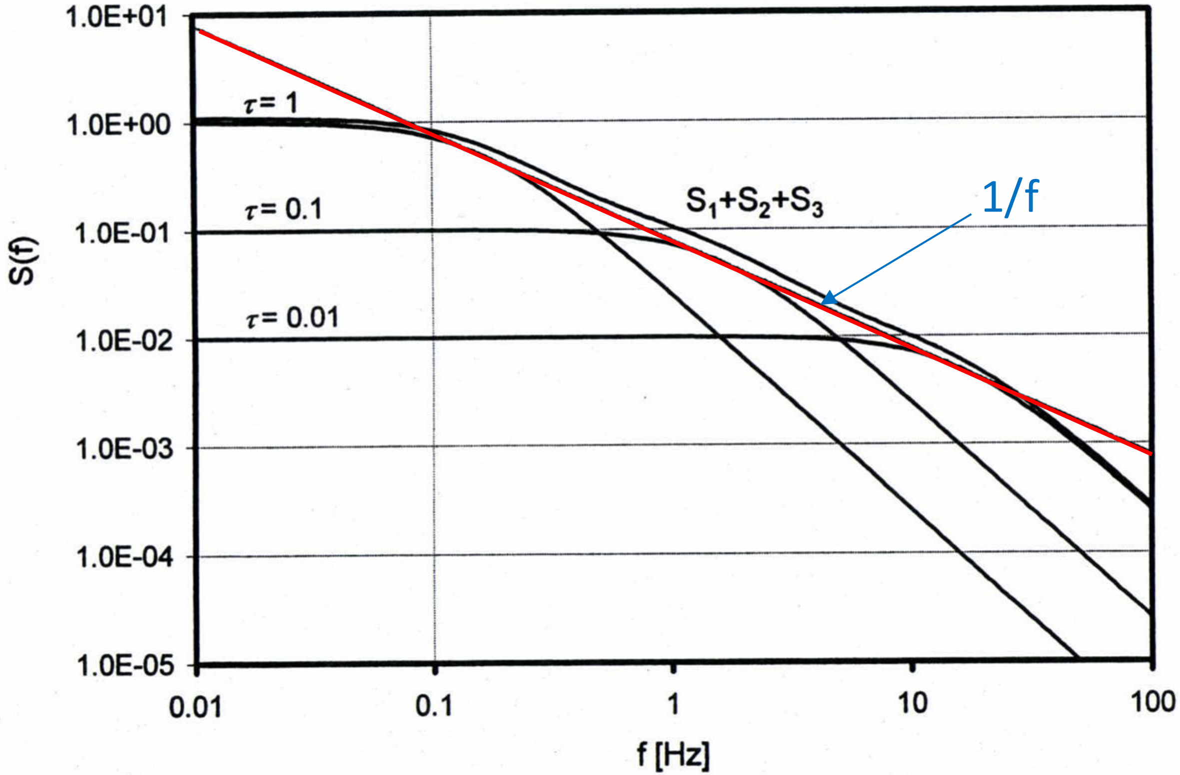


loudness fluctuations spectra of radio broadcasting

pictures from:
E. Milotti
U Udine

1/f noise - II

superposition of $1/f^2$ spectra with 3 time constants,
at least 1 order of magnitude differing from each other



1/f noise – III … in trap/release processes

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Assume a trapping site with relaxation time constant τ which releases electrons according to

$$N(t) = N_0\, \mathrm{e}^{-t/\tau} \quad \text{for } t \geq 0\,, \qquad N(t) = 0 \quad \text{else}$$

Fourier transforming this into the frequency domain yields

$$F(\omega) = \int_{-\infty}^{\infty} N(t) \mathrm{e}^{-i\omega t}\, dt = N_0 \int_0^{\infty} \mathrm{e}^{-(1/\tau + i\omega)t}\, dt = N_0 \frac{1}{1/\tau + i\omega}$$

For a whole sequence of such relaxation processes occurring at different times t_k

$$N(t, t_k) = N_0\, \mathrm{e}^{-\frac{t - t_k}{\tau}} \quad \text{for } t \geq t_k \;\;, \qquad N(t, t_k) = 0 \quad \text{else}$$

… but still with the same trapping time constant $\boldsymbol{\tau}$, one gets

$$F(\omega) = N_0 \sum_k \mathrm{e}^{i\omega t_k} \int_0^{\infty} \mathrm{e}^{-(1/\tau + i\omega)t} dt = \frac{N_0}{1/\tau + i\omega} \sum_k \mathrm{e}^{i\omega t_k}$$

The noise power spectrum then is obtained as

$$P(\omega) = \lim_{T \to \infty} \frac{1}{T} \left\langle |F(\omega)|^2 \right\rangle = \frac{N_0^2}{(1/\tau)^2 + \omega^2} \lim_{T \to \infty} \frac{1}{T} \left\langle \left| \sum_k \mathrm{e}^{i\omega t_k} \right|^2 \right\rangle = \frac{N_0^2}{(1/\tau)^2 + \omega^2} n$$

where n is the average rate of trapping/relaxation processes

1/f noise - IV

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If, in addition, one assumes that the relaxation time constants are different i.e. $\tau \rightarrow \tau_i$ and we integrate/sum over uniformly distributed $\tau_1 < \tau_i < \tau_2$, one finds

$$P(\omega) = \frac{1}{\frac{1}{\tau_1} - \frac{1}{\tau_2}} \int_{\frac{1}{\tau_2}}^{\frac{1}{\tau_1}} \frac{N_0^2\, n}{\left(\frac{1}{\tau}\right)^2 + \omega^2}\, d\left(\frac{1}{\tau}\right) = \frac{N_0^2\, n}{\omega\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)} \left[\arctan\frac{1}{\omega\tau_1} - \arctan\frac{1}{\omega\tau_2}\right]$$

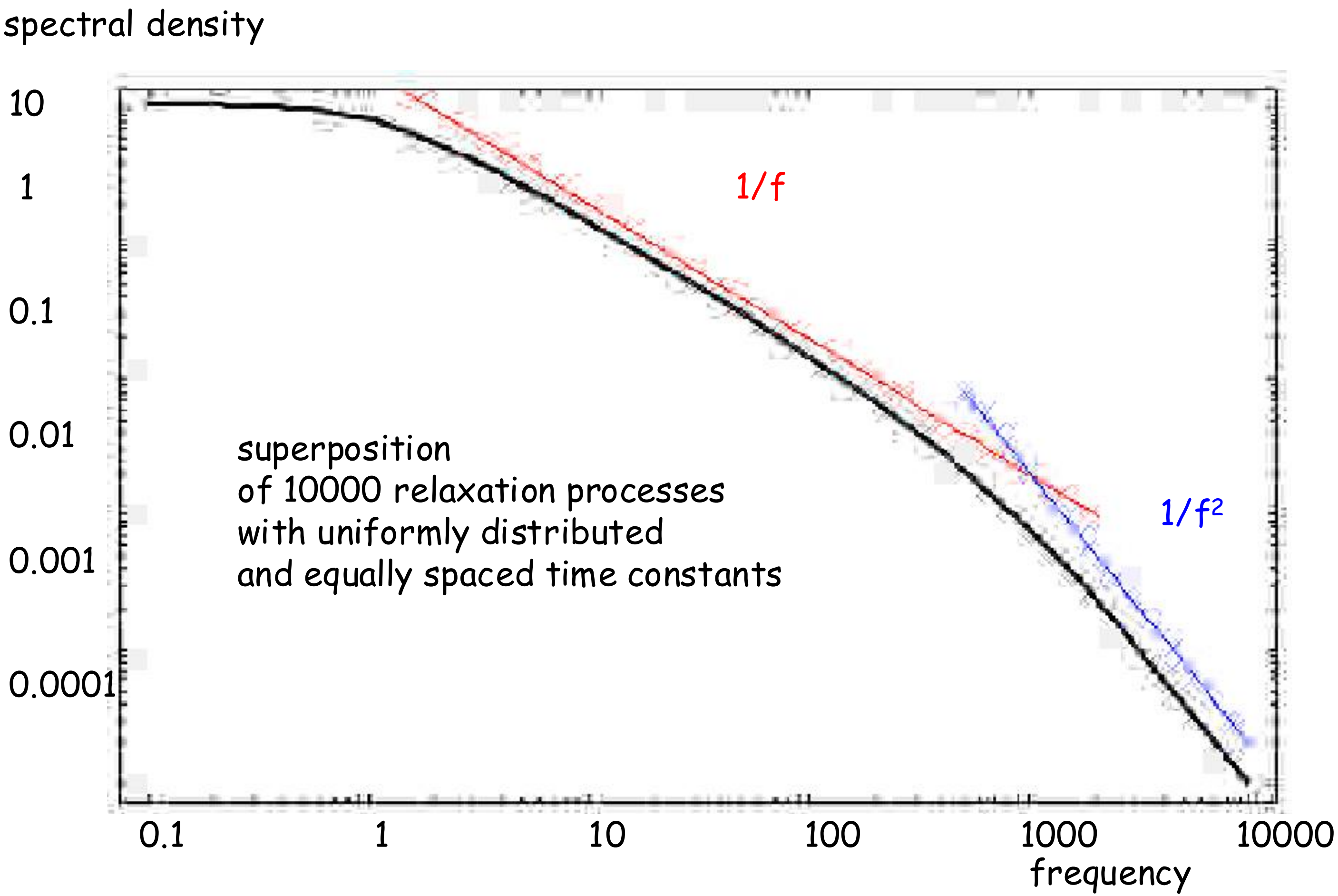
$$\approx \begin{cases} N_0^2\, n & \text{if } \; 0 < \omega \ll \frac{1}{\tau_1}, \frac{1}{\tau_2} & \rightarrow \quad \text{const.}\,, \\ \frac{N_0^2\, n\, \pi}{2\omega\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)} & \text{if } \; \frac{1}{\tau_2} \ll \omega \ll \frac{1}{\tau_1} & \rightarrow \quad \frac{1}{f}\,, \\ \frac{N_0^2\, n}{\omega^2} & \text{if } \; \frac{1}{\tau_1}, \frac{1}{\tau_2} \ll \omega & \rightarrow \quad \frac{1}{f^2}\,. \end{cases} \tag{I.23}$$

const

1/f

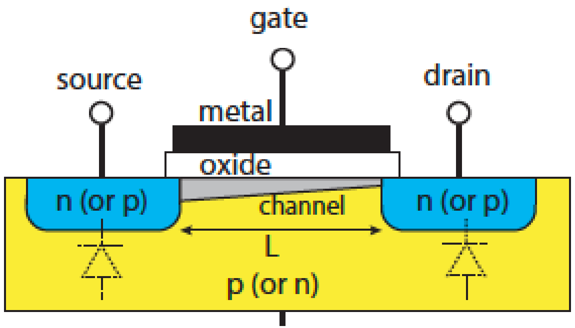
$1/f^2$

1/f noise - V



from:
E. Milotti
U Udine

1/f noise in a MOSFET



origin

- trapping and release of channel charges in gate oxide
- depends on gate area A = W × L

$$\frac{d\left\langle v_{1/f}^2\right\rangle}{df} = K_f \frac{1}{C'_{ox} W L} \frac{1}{f}$$

empirical parametrisation (e.g. PSPICE)

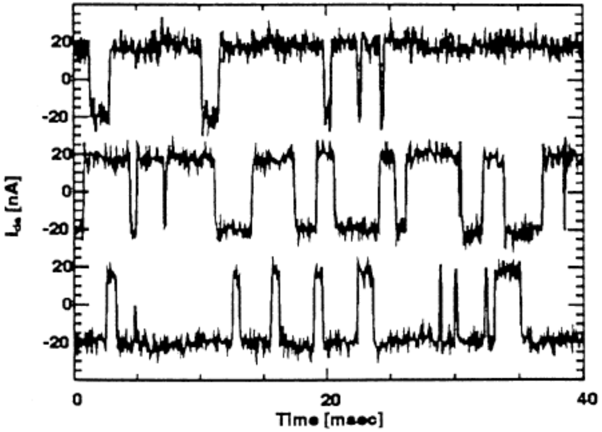
$$C'_{ox} = \frac{3}{2}\frac{C_{GS}}{WL} \approx \epsilon_0 \epsilon / d$$

$K_f^{NMOS} \approx 30 \times 10^{-25}$ J, $K_f^{PMOS} \approx 0.05\text{-}0.1 \times K_f^{NMOS}$

RTS noise

RTS noise = random telegraph signal noise

also called “burst noise” or “popcorn noise”



Occurs in electronics devices usually related to trapping/detrapping processes. The popping-up nature of individual RTS bursts eventually leads to the 1/f noise spectral density when noise of several traps with (very) different trapping times are superimposed.

Given the low frequency it is difficult to filter out and a nuisance for very low noise devices.

Fazit: Only three physical noise origins to consider …

- $\langle i^2 \rangle = \frac{4kT}{R} df$

thermal noise

thermal fluctuations (Brownian motion)
velocity fluctuation

(in resistors, transistor channels)

- $\langle i^2 \rangle = 2q \langle i \rangle df$

shot noise

fluctuations in hopping over
a barrier (shot)
number fluctuation

(where currents due to barrier crossings appear, e.g. in diodes, NOT in resistors)

- $\langle i^2 \rangle = \text{const.}\ 1/\text{f}^{\alpha} df$

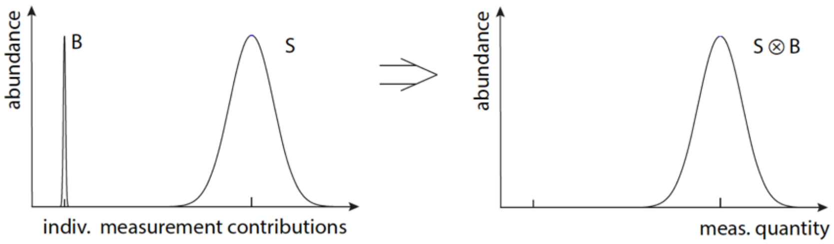
1/f noise

trap/release fluctuations of carriers
number fluctuation

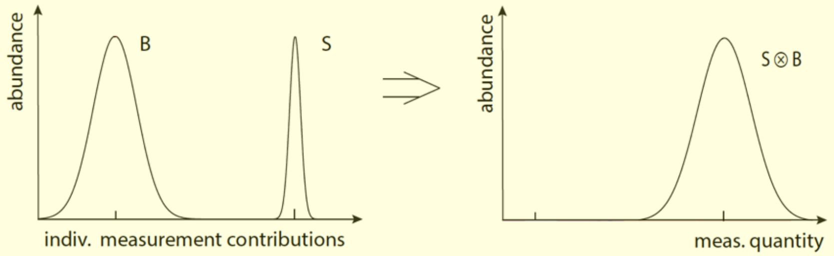
(whenever trapping occurs, e.g. in (MOS) transistor channels)

Remember when to care about noise …

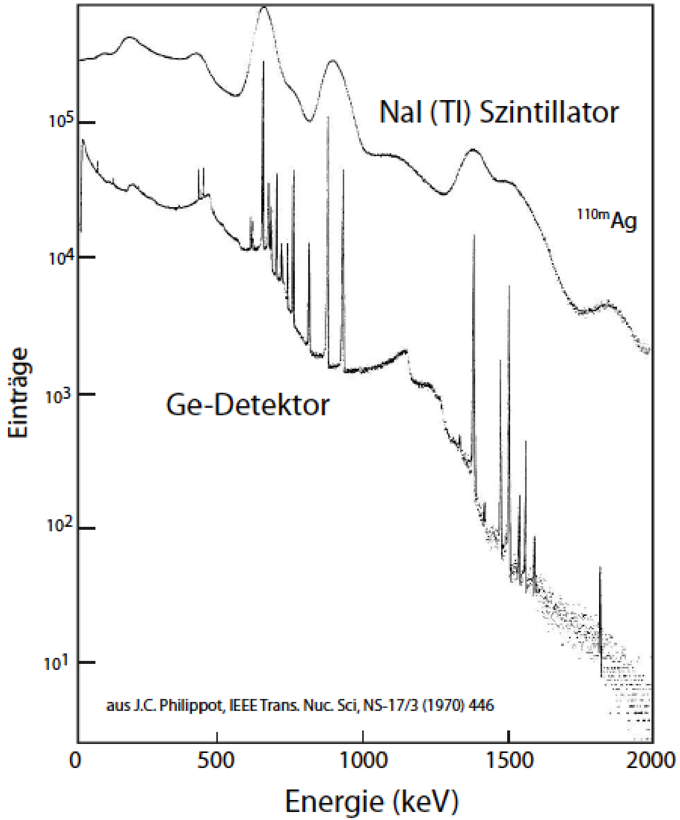
- always …
- but particularly, when the situation is like this



(a) $\sigma_S \gg \sigma_B$.

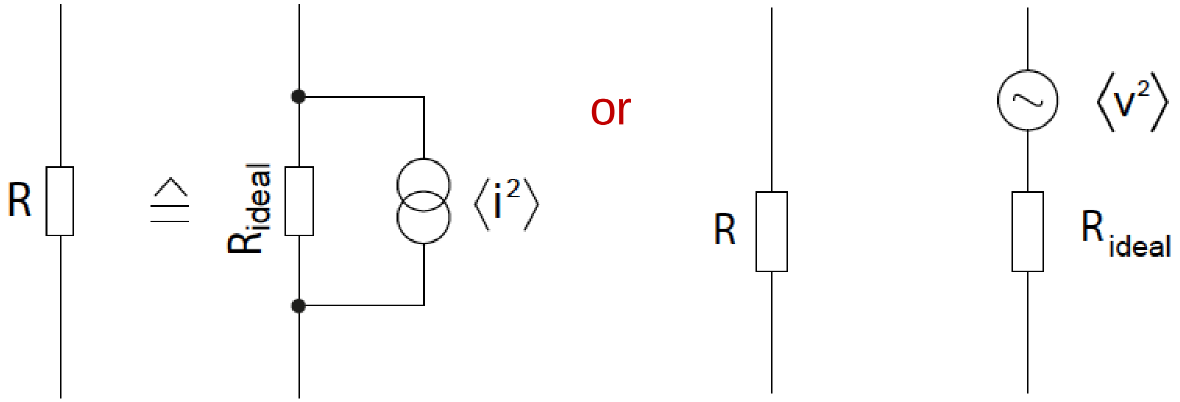


(b) $\sigma_S \ll \sigma_B$.



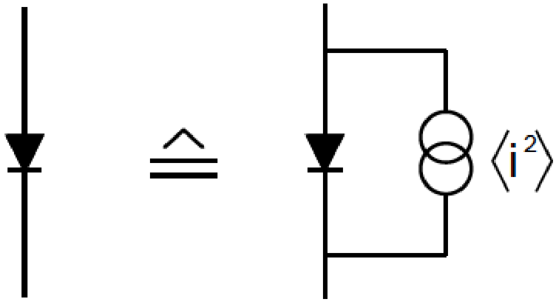
- Even if you are not interested in an energy measurement, remember … thresholds

Noisy circuit elements



(a) Replacement circuit with parallel current noise source.

(b) Replacement circuit with serial voltage noise source.



real (noisy) diode

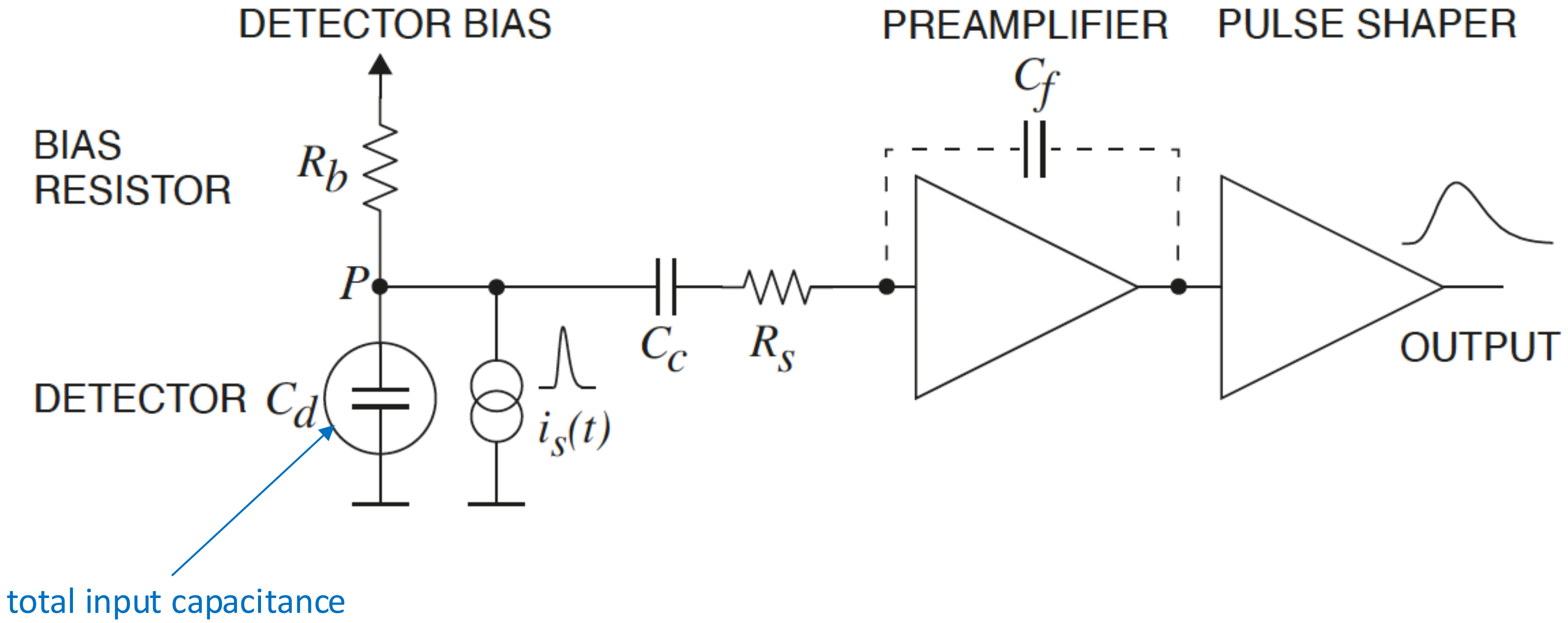
ideal diode with noise current source

Pop-up questions

- What is the difference between signal fluctuations and (electronic) noise and when do you have to worry particularly about the latter?
  - Signal fluctuations are fluctuating signal heights, whereas electronic noise comes from the amplification and readout electronics. Worry if B >> S.
- What is the Fano factor? When do you need to apply it and what is the resulting effect?
  - A factor to be applied to the simple Poissonian resolution to be expected, when a radiation signal always deposits its complete energy into a detector. For Si: e/h creations and phonon creation are correlated. F improves the resolution. For Si F is about 0.1.
- What does Fano limit mean?
  - Resolution limit due to signal fluctuations only.
- What are the most important electronic noise sources usually to consider in detector readout and what is their origin and dependence?
  - thermal noise i.e. Brownian motion e.g. in a resistor $\langle i^2 \rangle = 4kT/R$
    shot noise (current over barrier): $\langle i^2 \rangle = 2\, q \langle I \rangle\, df$
    1/f noise (trap-release processes): $\langle i^2 \rangle \propto 1/f\ \ df$
- Which noise sources do appear in a MOSFET?
  - thermal (R -> γ 1/gm) and 1/f noise

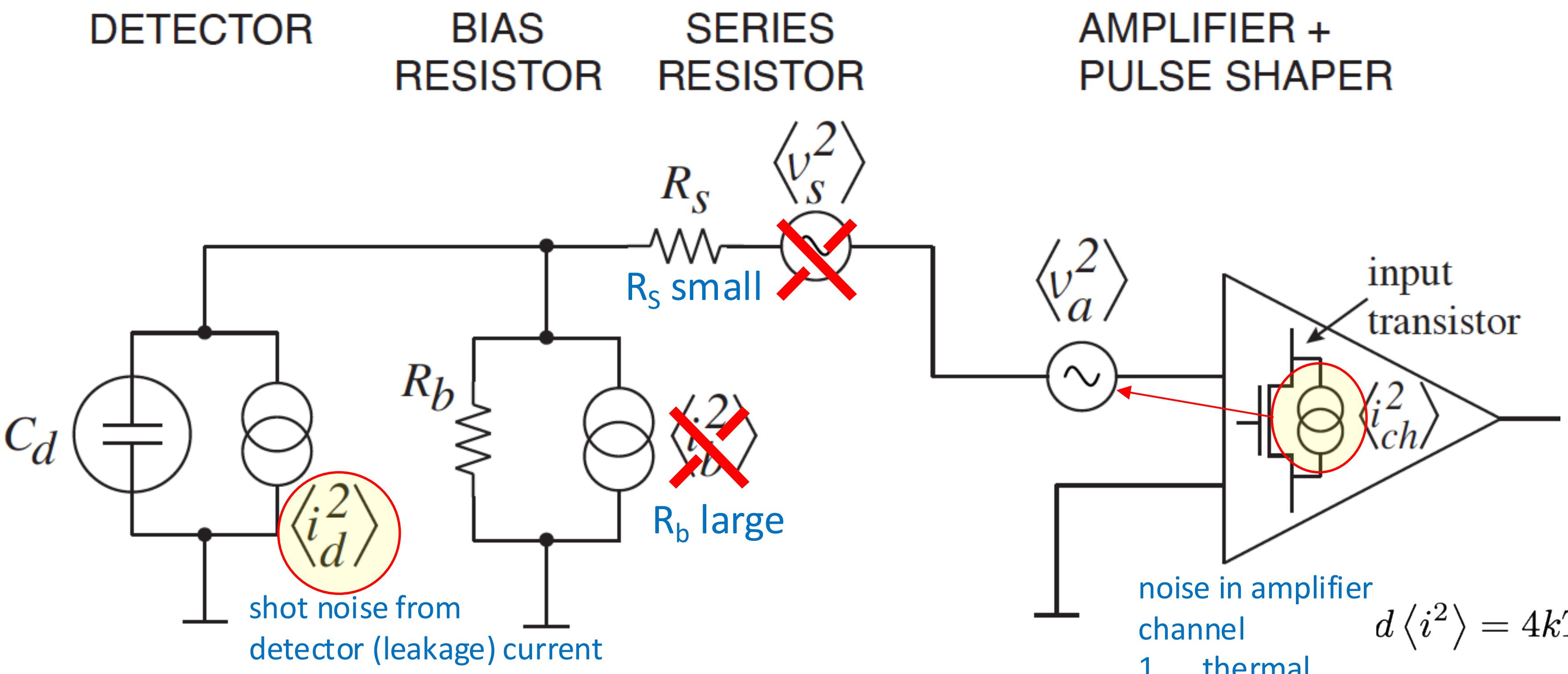
Noise in a typical detector readout system

Detector system



from: PDG-Review on Low-noise detector readout, N.Wermes (2022&2024), H. Spieler (2013)
in S. NAVAS *et al.* (Particle Data Group), Phys.Rev.D. **110**, 030001 (2024), doi 10.1103/PhysRevD.110.030001

Circuit diagram for equivalent noise analysis



$$d\langle i_d^2\rangle = 2eI_d\,df$$

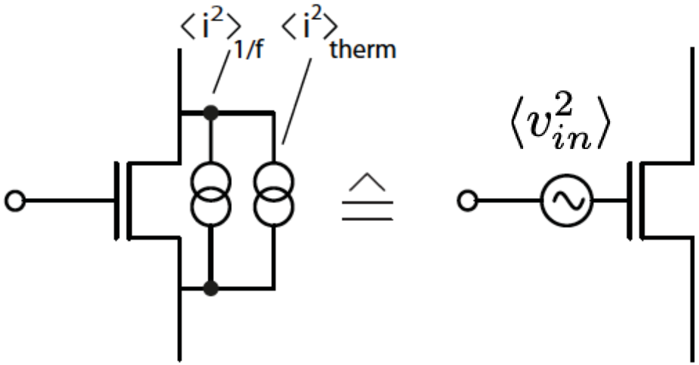
$$d\left\langle i^2\right\rangle = 4kT\frac{g_m}{\gamma}df$$

$$d\left\langle i^2\right\rangle = K_\alpha\frac{1}{f^\alpha}df$$

from: PDG-Review on Low-noise detector readout, N.Wermes (2022&2024), H. Spieler (2013)
in S. NAVAS *et al.* (Particle Data Group), Phys.Rev.D. **110**, 030001 (2024), doi 10.1103/PhysRevD.110.030001

Noise in a MOSFET (current or voltage noise?)

"parallel current noise can also be described by serial voltage noise"



via

$$\langle i_{\rm channel}^2 \rangle = \langle (g_m v_{\rm in})^2 \rangle$$

1/f noise
parametrized by C'_{ox} and K_f

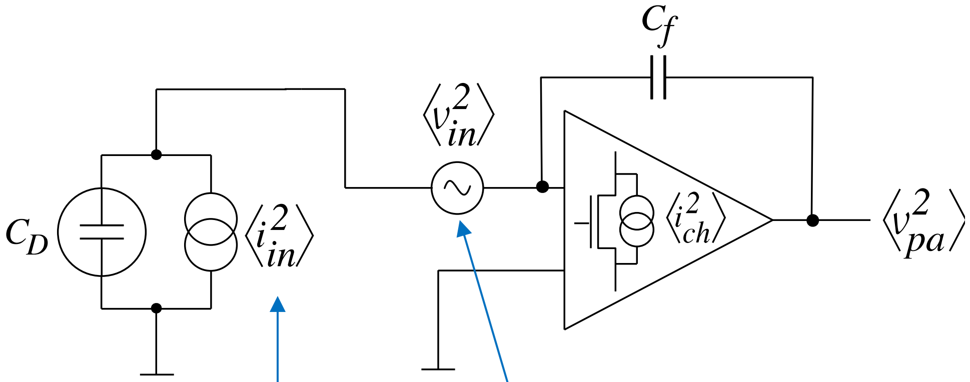
thermal noise

$$\frac{d\langle v_{in}^2 \rangle}{df} = 4kT\frac{1}{\gamma g_m} + K_f \frac{1}{C'_{ox} W L}\frac{1}{f}$$

* contributions assumed uncorrelated, adding in quadrature

Preamplifier output noise

let's now use a CSA and compute the noise output voltage …



shot noise
(det. leak.)

$$\frac{d\langle i^2 \rangle_{shot}}{df} = 2eI_d$$

thermal noise
1/f noise
in transistor channel

$$\frac{d\langle v_{in}^2 \rangle}{df} = 4kT\frac{1}{\gamma g_m} + K_f \frac{1}{C'_{ox}WL}\frac{1}{f}$$

now $<v_{pa}>^2$

The noise current, flowing through the feedback capacitance C_f , as well as the noise voltage at the preamplifier input, generate a noise voltage behind the preamplifier $\langle v_{pa}{}^2 \rangle$.

$$\langle v_{pa}^2 \rangle = \langle v_{in}^2 \rangle \left(\frac{\omega C_D}{\omega C_f} \right)^2$$

$$\langle v_{pa}^2 \rangle = \langle i_{in}^2 \rangle \left(\frac{1}{\omega C_f} \right)^2$$

$$\omega = 2\pi f$$

$$C_d \rightarrow C_D = C_{in}^{tot}$$

We then get …

current noise input

$$\frac{d\left\langle i^2\right\rangle_{shot}}{df} = 2eI_d$$

voltage noise input

$$\frac{d\left\langle v_{in}^2\right\rangle}{df} = 4kT\frac{1}{\gamma g_m} + K_f\frac{1}{C'_{ox}WL}\frac{1}{f}$$

$$\frac{d\left\langle v_{\mathrm{pa}}^2\right\rangle}{d\omega} = \frac{eI_0}{\pi\omega^2 C_f^2} + K_f\frac{1}{C'_{\mathrm{ox}}WL}\frac{C_D^2}{C_f^2}\frac{1}{\omega} + \frac{2kT}{\pi}\frac{1}{\gamma g_m}\frac{C_D^2}{C_f^2}$$

$$= \sum_{k=-2}^{0} c_K\omega^k$$

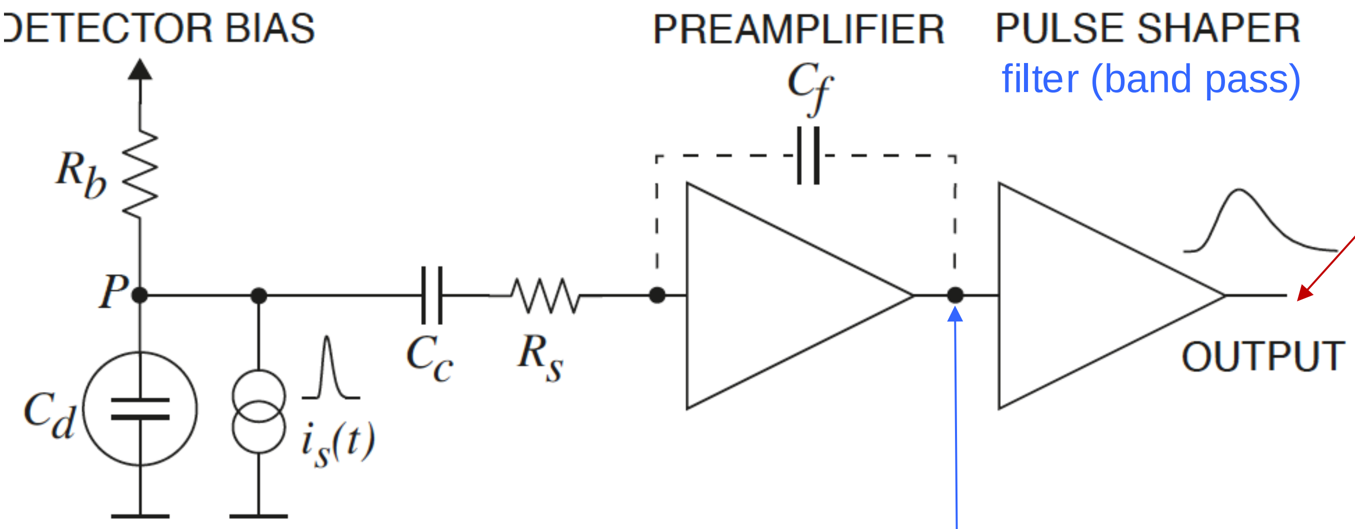
voltage noise output behind the CSA

with coefficients

$$c_{-2} = \frac{e}{\pi}I_0\frac{1}{C_f^2}, \qquad c_{-1} = K_f\frac{1}{C'_{\mathrm{ox}}WL}\frac{C_D^2}{C_f^2}, \qquad c_0 = \frac{2kT}{\pi}\frac{1}{\gamma g_m}\frac{C_D^2}{C_f^2}$$

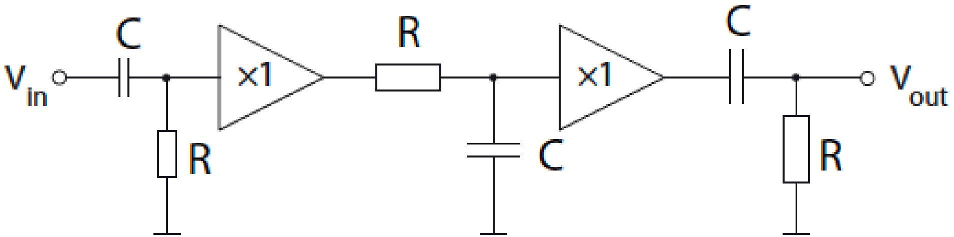
Noise in a preamp + shaper system

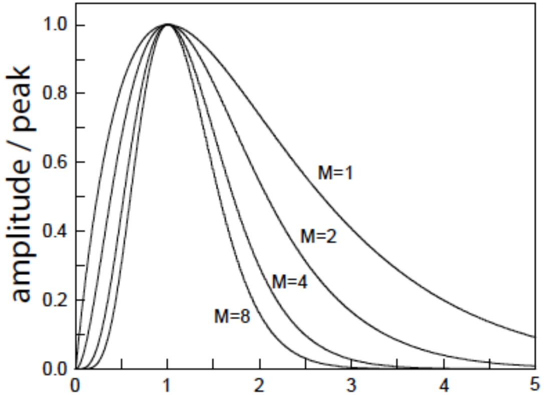
now $\langle v_{sh}^2 \rangle$?



so far: $\langle v_{pa}^2 \rangle$

shaper = high pass plus M low passes

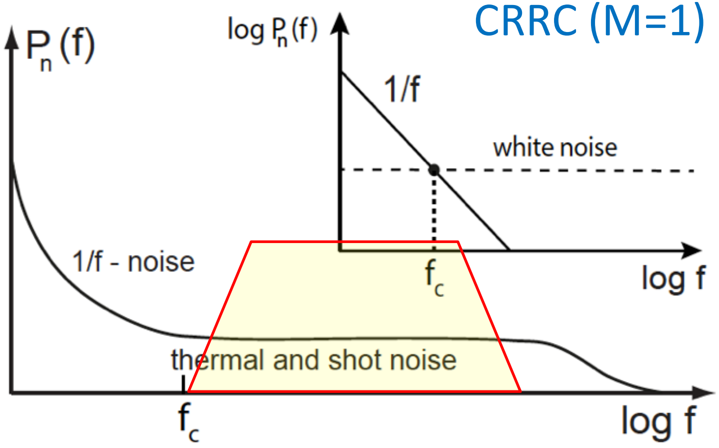




easiest and often realised

CRRC (M=1) shaper

$$f^{(1,M)}(t) = \frac{1}{M!}\left(\frac{t}{\tau}\right)^{M} e^{-t/\tau}$$



consequences for noise

BW limitation => lower noise

at the expense of loosing speed

Excursion: Laplace Transform I

very similar to Fourier transform (Laplace better for problems with initial value conditions
Fourier better for problems with boundary conditions)

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)\, \mathrm{e}^{-st} dt \qquad s = \sigma + i\omega$$

and the inverse transform

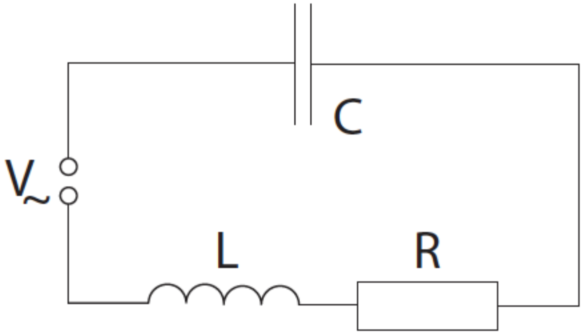
$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} F(s) \mathrm{e}^{st} ds = \begin{cases} f(t) \text{ for } & t \geq 0 \\ 0 \quad\ \text{for } & t < 0 \end{cases}$$

Excursion: Laplace Transform II

Operation or function	Time domain $f(t) = \mathcal{L}^{-1}[F(s)]$	Frequency domain $F(s) = \mathcal{L}[f(t)]$
linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
convolution	$\int_0^\infty f(t-t')g(t')dt'$	$F(s)G(s)$
nth derivative	$\frac{d^n}{dt^n} f(t)$	$s^n F(s)$
time integration	$\int_0^t f(t)dt$	$\frac{1}{s} F(s)$
scaling of t	$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
time shift	$f(t-t_0)$	$\mathrm{e}^{-st_0} F(s)$
damping	$\mathrm{e}^{-s_0 t} f(t)$	$F(s+s_0)$
multiplication	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
δ function	$\delta(t)$	1
derivative of the δ function	$\frac{d^n}{dt^n}\delta(t)$	s^n
step function	$\Theta(t)$	$\frac{1}{s}$
falling exponential	e^{-at}	$\frac{1}{s+a}$
rising exponential	$1 - \mathrm{e}^{-at}$	$\frac{a}{s(s+a)}$
power function	t^n	$\frac{n!}{s^{n+1}}$

Excursion: Laplace Transform III

very simple example



$$L\frac{d^2}{dt^2}i(t) + R\frac{d}{dt}i(t) + \frac{1}{C}i(t) = \frac{dV_\sim}{dt}$$

⇕

$$sLI(s) + RI(s) + \frac{1}{sC}I(s) = V(s)\,.$$

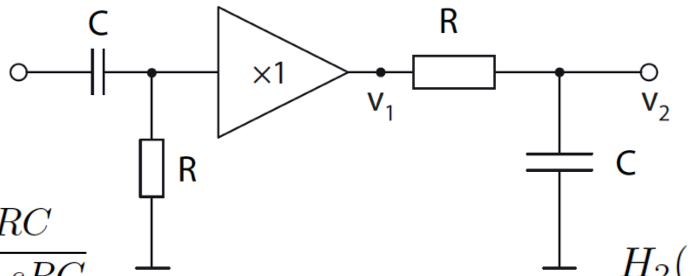
⇓

$$Z(s) = R + \frac{1}{sC} + sL$$

no differential equation to be solved !

Excursion: Laplace Transform IV

CR-RC shaper



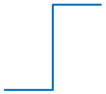
$$H_1(s) = \frac{sRC}{1 + sRC}$$

$$H_2(s) = \frac{1}{1 + sRC}$$

$$v_1(s) = H_1(s)\, v(s) = \frac{sRC}{1 + sRC}\, v(s) = \frac{s\tau}{1 + s\tau} v(s)\,,$$

$$v_2(s) = H_2(s)\, v_1(s) = \frac{1}{1 + sRC}\, v_1(s) = \frac{s\tau}{(1 + s\tau)^2} v(s)$$

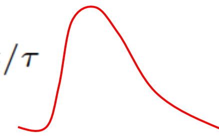
step function



$$v(t) = V_0\, \Theta(t) = \begin{cases} 0 & ,\ t \leq 0\,, \\ V_0 & ,\ t > 0\,, \end{cases}$$

$$v(s) = V_0 \frac{1}{s} \quad \Rightarrow \quad v_2(s) = \frac{V_0 \tau}{(1 + s\tau)^2}$$

$$\mathcal{L}^{-1} \quad \Rightarrow \quad v_2(t) = V_0 \frac{t}{\tau} \mathrm{e}^{-t/\tau}$$



Shaper transfer function (in → out)

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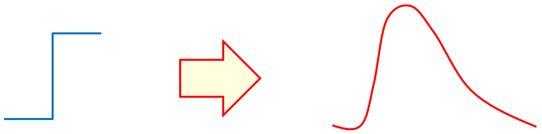
in the time domain



$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

A = amplitude

peaks at τ with peak height A / 2.71

in the frequency domain $1/s$

$$H(s) = \frac{s\tau}{(1+s\tau)^2} \rightarrow |H(\omega)|^2 = A^2\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right)^2$$

(with $s \rightarrow i\omega$)

for noise
need square

$$\langle v_{\mathrm{sh}}^2\rangle = \int_0^\infty \frac{d\langle v_{\mathrm{pa}}^2\rangle}{d\omega}\,|H(\omega)|^2\, d\omega$$

$$\langle v_{\mathrm{sh}}^2\rangle = \sum_{k=-2}^{0}\int_0^\infty c_k\omega^k\,|H(\omega)|^2\, d\omega$$

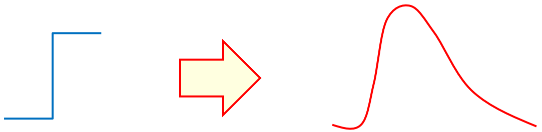
$$= A^2\frac{1}{2}\sum_{k=-2}^{0} c_k\tau^{-k-1}\,\Gamma\left(1+\frac{k+1}{2}\right)\Gamma\left(1-\frac{k+1}{2}\right)$$

c_k as before

$$\Gamma(x+1) = x\Gamma(x), \qquad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}, \qquad \Gamma(1) = 1$$

Shaper transfer function (in → out)

in the time domain



$$v_{sh}(t) = A\frac{t}{\tau}\,\mathrm{e}^{-\frac{t}{\tau}}$$

A = amplitude

peaks at τ with peak height A / 2.71

in the frequency domain $1/s$

$$H(s) = \frac{s\tau}{(1+s\tau)^2} \rightarrow |H(\omega)|^2 = A^2\left(\frac{\omega\tau}{1+\omega^2\tau^2}\right)^2$$

$$(\text{with } s \rightarrow i\omega)$$

executing the sum yields

$$\langle v_{\mathrm{sh}}^2\rangle = \frac{\pi}{4}A^2\left(c_{-2}\,\tau + \frac{2}{\pi}c_{-1} + c_0\frac{1}{\tau}\right)$$

with

$$c_{-2} = \frac{e}{\pi}I_0\frac{1}{C_f^2}\,, \qquad c_{-1} = K_f\frac{1}{C'_{\mathrm{ox}}WL}\frac{C_D^2}{C_f^2}\,, \qquad c_0 = \frac{2kT}{\pi}\frac{1}{\gamma g_m}\frac{C_D^2}{C_f^2}$$

Equivalent Noise Charge (ENC)

… want to express the noise in units of the signal at the input, i.e. "how many electrons would produce the noise voltage output behind the shaper that I see?"

$$\mathrm{ENC} = \frac{\text{noise output voltage (V)}}{\text{output voltage of a signal of } 1\,\mathrm{e}^- \ (\mathrm{V/e}^-)}$$

$$\mathrm{ENC}^2 = \frac{\langle v_{\mathrm{sh}}^2 \rangle}{v_{\mathrm{sig}}^2}$$

Equivalent noise charge

for 1e at the input we get $v_{\text{sig}} = \frac{A}{2.71}\frac{e}{C_f}$

peak of shaper pulse

$$\Rightarrow \text{ENC}^2\,(e^{-2}) = \frac{\langle v_{\text{sh}}^2 \rangle}{v_{\text{signal}}^2\,(1e^-)} = \frac{(2.71)^2}{4e^2}\left(eI_d\tau + 2C_D^2 K_f \frac{1}{C'_{ox}WL} + \frac{2kT}{\gamma g_m}\frac{C_D^2}{\tau} \right)$$

$$= a_{\text{shot}}\,\tau + a_{1/\text{f}} C_D^2 + a_{\text{therm}}\,\frac{C_D^2}{\tau}$$

using $\gamma = 2/3$ and $C'_{ox} = 6\text{fF}/\mu\text{m}^2, K_f = 33 \times 10^{-25} J$

$$\text{ENC}^2\,(e^{-2}) = 11\,\frac{I_0}{\text{nA}}\frac{\tau}{\text{ns}} + 800\,\frac{1}{WL/(\mu\text{m}^2)}\frac{C_D^2}{(100\,\text{fF})^2} + 8600\,\frac{1}{g_m/\text{mS}}\frac{C_D^2/(100\,\text{fF})^2}{\tau/\text{ns}}$$

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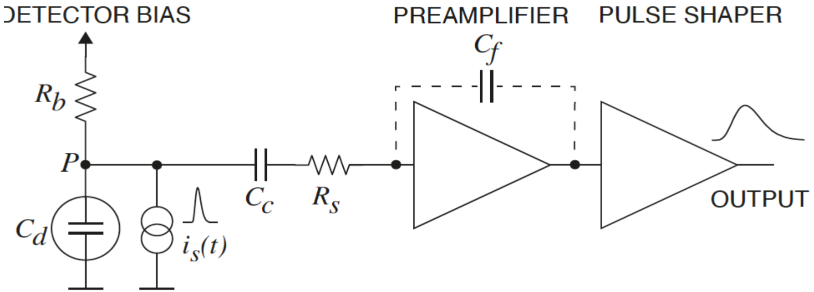
$$\langle i^2 \rangle = 2q\langle i \rangle df$$
$$\langle i^2 \rangle = \frac{4kT}{R} df$$
$$\langle i^2 \rangle = \text{const.}\ 1/\mathrm{f}^{\alpha} df$$



$$\mathrm{ENC}^2 = a_{\mathrm{shot}}\,\tau + a_{1/\mathrm{f}}\,C_D^2 + a_{\mathrm{therm}}\,\frac{C_D^2}{\tau}$$

– Shot noise, which is parallel current noise to the input, is proportional to the detector leakage current I_0 and increases with the filter time τ, since I_0 is effectively integrated over τ by the CSA–shaper system. While still being frequency independent (white) at the CSA input, $\langle v_{pa}^2 \rangle_{\mathrm{shot}}$ develops a $1/f^2$ dependence behind the preamplifier as described by (17.100), and a 1/f dependence after the shaper corresponding to a linear dependence on τ.

– Thermal noise in the transistor channel, while still being 'white' behind the preamplifier, is strongly reduced by the bandwidth limitation through the filter, leading to a decrease with $1/\tau$ after the shaper.

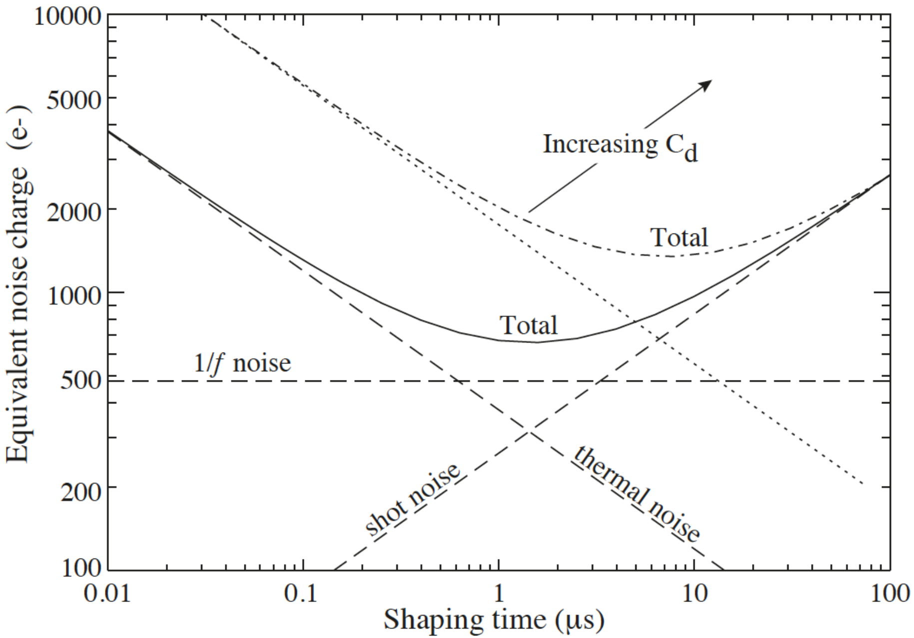
– For the 1/f noise part in the input transistor channel one would naively expect a larger contribution for large τ values (corresponding to small frequencies). This contribution, however, is cancelled by the bandwidth reduction by about the same factor, such that at the shaper output any τ dependence is no longer present.

Optimal filter time

$$\mathrm{ENC}^2 = a_{\mathrm{shot}}\,\tau + a_{1/\mathrm{f}}\,C_D^2 + a_{\mathrm{therm}}\,\frac{C_D^2}{\tau}$$

there is an optimal shaping time

$$\tau_{\mathrm{opt}} = \left(\frac{a_{\mathrm{therm}}}{a_{\mathrm{shot}}}\,C_D^2\right)^{1/2} = \left(\frac{4kT}{3\,eI_0 g_m}\,C_D^2\right)^{1/2}$$



see also: PDG-Review on Low-noise detector readout, N.Wermes (2022&2024), H. Spieler (2013)
in S. NAVAS *et al.* (Particle Data Group), Phys.Rev.D. **110**, 030001 (2024), doi 10.1103/PhysRevD.110.030001

Examples

Pixel detector. As an example featuring small electrodes and correspondingly small input capacitances we choose a silicon pixel detector (section 8.7) with parameters $C_D = 200\,\text{fF}$, $I_0 = 1\,\text{nA}$, $\tau = 50\,\text{ns}$, $W = 20\,\mu\text{m}$, $L = 0.5\,\mu\text{m}$, $g_m = 0.5\,\text{mS}$, where we assumed a typical leakage current before the detector received substantial radiation damage. With (17.110) an equivalent noise charge of

small changes on 26.9.24 (γ, C_{ox} , (K_f))

$$\text{ENC}^2 \approx (24\,e^-)^2(\text{shot}) + (17\,e^-)^2(1/\text{f}) + (25\,e^-)^2(\text{therm}) \approx (40\,e^-)^2 \longrightarrow (47\ \text{e-})^2$$

Strip detector. For a typical silicon microstrip detector (see section 8.6.2) after radiation damage one obtains with $C_D = 20\,\text{pF}$, $I_0 = 1\,\mu\text{A}$, $\tau = 50\,\text{ns}$, $W = 2000\,\mu\text{m}$, $L = 0.4\,\mu\text{m}$, $g_m = 5\,\text{mS}$:

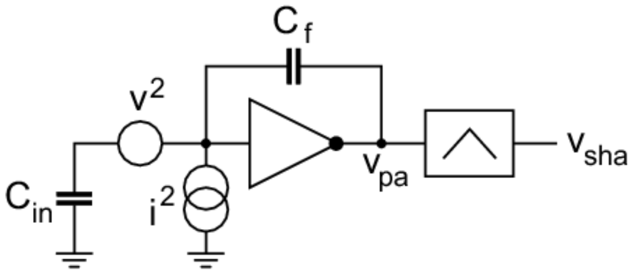
$$\text{ENC}^2 \approx (750\,e^-)^2(\text{shot}) + (200\,e^-)^2(1/\text{f}) + (800\,e^-)^2(\text{therm}) = (1100\,e^-)^2. \longrightarrow (1400\ \text{e-})^2$$

Liquid argon calorimeter. As an example of a detector with a large electrode capacitance we take a liquid argon calorimeter cell with typical values as given by the ATLAS electromagnetic calorimeter (see section 15.5.3.2 on page 597) in the central region. With the parameters $C_D = 1.5\,\text{nF}$, $I_0 = <2\,\mu\text{A}$, $\tau = 50\,\text{ns}$, $W = 3000\,\mu\text{m}$, $L = 0.25\,\mu\text{m}$, $g_m = 100\,\text{mS}$, i.e. assuming only a small (negligible) parallel shot noise (leakage current), one obtains:

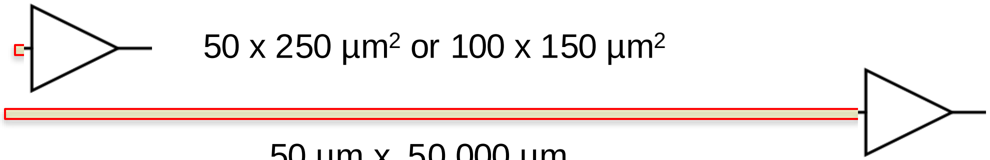
$$\text{ENC}^2 \approx (1000\,e^-)^2(\text{shot}) + (15000\,e^-)^2(1/\text{f}) + (13500\,e^-)^2(\text{therm}) \approx (20200\,e^-)^2. \longrightarrow (25000\ \text{e-})^2$$

Examples: Noise in pixel/strip/liq.Ar detector (ionisation detector)

… with CSA preamplifier & shaper



comparing pixels and strips



different I_{leak}

	C_D	I_0	τ	W	L	g_m	ENC therm	ENC 1/f	ENC shot	ENC tot
pixel	200 fF	1 nA	50 ns	20 µm	0.5 µm	0.5 mS	25 e⁻	17 e⁻	24 e⁻	**40 e⁻**
strip	20 pF	1 µA	50 ns	2000 µm	0.4 µm	5 mS	800 e⁻	200 e⁻	750 e⁻	**1100 e⁻**
liq. Ar	1.5 nF	2 µA	50 ns	3000 µm	0.25 µm	100 mS	13 500 e⁻	15 000 e⁻	1000 e⁻	**20 200 e⁻**

Pop-up questions

- What are the most important electronic noise sources in a typical ionisation detector readout and **what do they depend on** in a system with CSA and shaper?
  - shot noise from detector leakage current, thermal & 1/f noise in amplifying transistor channel: shot noise $\propto$ leakage current depends on τ; thermal noise ~ 1/gm × C_D × 1/τ; 1/f noise ~C_D
- What is ENC?
  - equivalent noise charge: refers the obtained noise to a signal of 1e at the input.
- The original f dependencies of thermal, shot and 1/f noise become completely different after CSA and shaper. Why?
  - Because the transfer functions of CSA and shaper are in parts frequency dependent.
- Why is there an “optimal shaping time”?
  - Because thermal noise falls with τ , shot noise rises with τ and 1/f noise is constant.

Thank you very much for your attention

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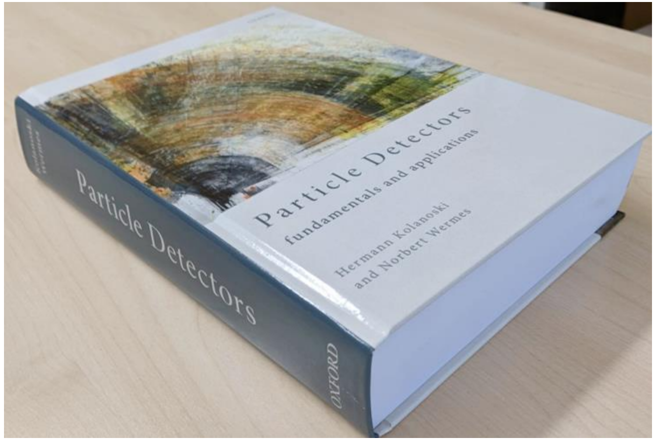
Content of lecture based on …

- Particle Data Group Review (2024)

35.9 Low-noise detector readout

Revised November 2021 by N. Wermes (Bonn U.), revised November 2013 by H. Spieler (LBNL).

new edition
to appear 2025



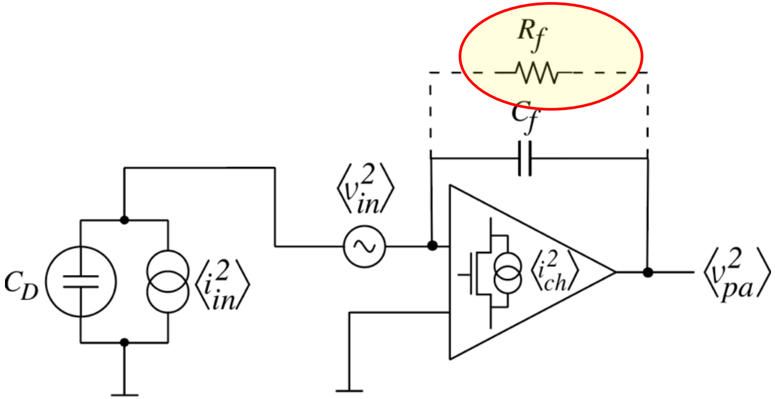
- Kolanoski, H. und Wermes, N.
  Teilchendetektoren – Grundlagen und Anwendungen (Springer/Spektrum 2016)

- Kolanoski, H. and Wermes, N.
  Particle Detectors – fundamentals and applications (Oxford University Press 2020)

  = ref [1]

BACKUP

… what about thermal noise in R-feedback ?



It acts on the preamplifier input in a very similar way as the leakage current shot noise contribution, i.e.

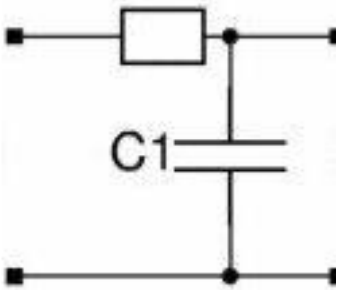
$$\frac{d\langle v_{\rm pa}^2\rangle}{d\omega} = \frac{eI_0}{\pi\omega^2 C_f^2}$$

$$2eI_0 \rightarrow \frac{4kT}{R_f}$$

$$\frac{d\langle v_{\rm pa}^2\rangle_{R_f}}{d\omega} = \frac{2kT}{R_f}\frac{1}{\pi\omega^2 C_f^2}$$

Its magnitude is usually small in comparison to the other contributions, in particular to the leakage-current-induced shot noise.

kT/C noise



- not a fundamental noise, but is thermal noise in the presence of a filtering capacitor (RC)
- the thermal white noise of an RC circuit has a band width of

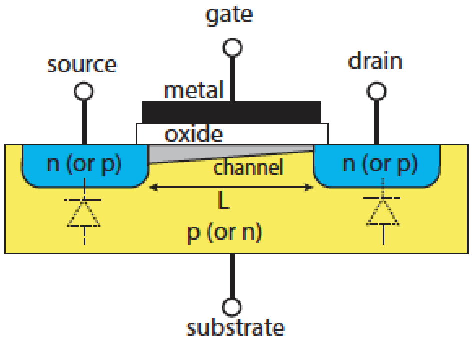
$$\Delta f = \frac{1}{2\pi}\int_0^\infty \frac{d\omega}{1+(\omega RC)^2} = \frac{1}{2\pi}\frac{\pi}{2RC} = \frac{1}{4RC}$$

- Hence

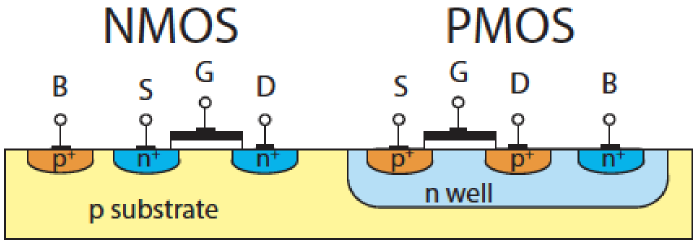
$$\langle v^2 \rangle = 4kTR\,\Delta f = \frac{4kTR}{4RC} = \frac{kT}{C}$$

- becomes independent of R

fresh-up: MOSFET -> NMOS, PMOS, CMOS



(a) MOSFET.

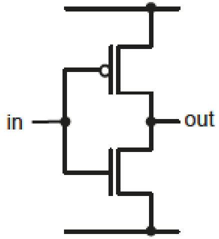


(b) CMOS.

- reminder: transistors operate in "inversion"

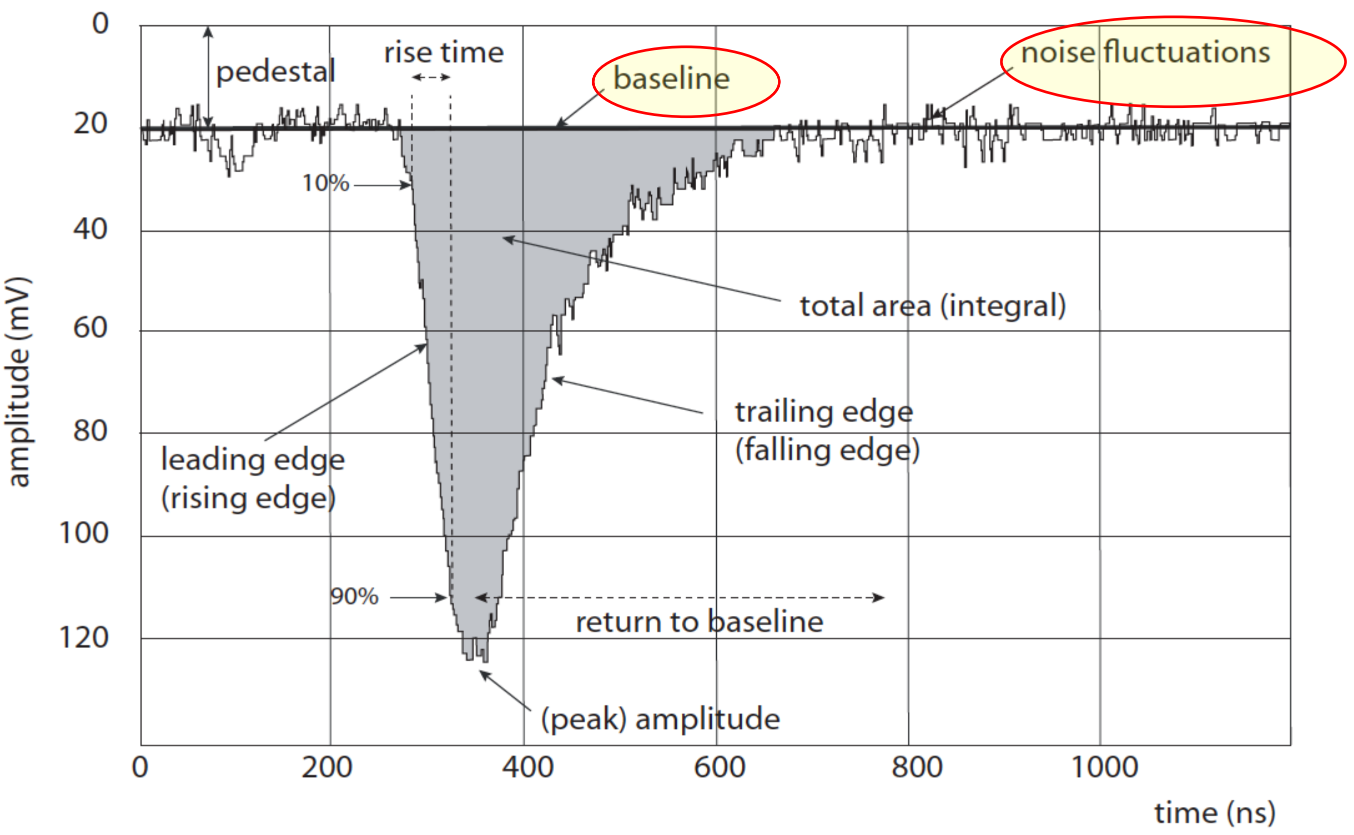
- NMOS: transistor channel current are electrons
- PMOS: transistor channel current are holes

- CMOS: both transistor types are realised in the same substrate.
  IMPORTANT for electronic circuits



(a) CMOS inverter.

Nomenclature, a typical detector pulse



Kolanoski, Wermes 2017