B05 – A novel approach to the baryon spectrum based on stochastic methods

NuMeriQS Retreat, Bonn

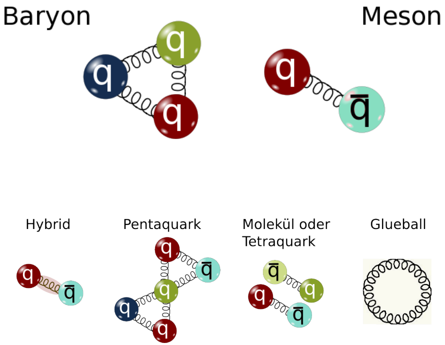
October 1, 2024 | Deborah Rönchen | Institute for Advanced Simulation, Forschungszentrum Jülich

Project members: Ulf-G. Meißner (PL), Deborah Rönchen (PL), Oleh Luniachek (PhD student)

Supported by DFG, NSFC, MKW NRW

HPC support by Jülich Supercomputing Centre

Subject of B05: Hadron physics



(Picture by NRW-FAIR Network)

Strong force:

- Matter fields: quarks (q)
  (almost free at high energies)

- Observed particles: hadrons
  (low and medium energies)

  - Mesons ($q\bar{q}$ states)
  - **Baryons** (qqq, $\bar{q}\bar{q}\bar{q}$ states)

    ↳ protons, neutrons, ...

  (+ exotic states ...)

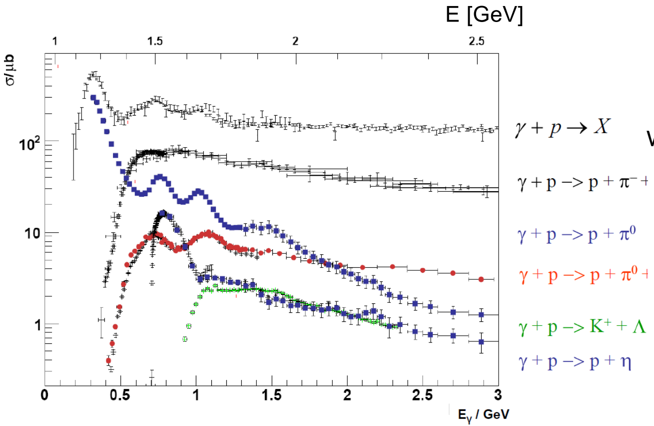
- gauge theory: Quantum Chromodynamics (QCD)
- no perturbative QCD at low & medium energies

Big question:

How do quarks and gluons form hadrons?

Experimental tests of strong force at medium energies

$\rightarrow$ measurements of hadronic cross sections and asymmetries



source: ELSA; data: ELSA, JLab, MAMI

What are those bumps?

- energy & angular momentum excitations of baryons **(resonances)**?
- background processes?
- something else?

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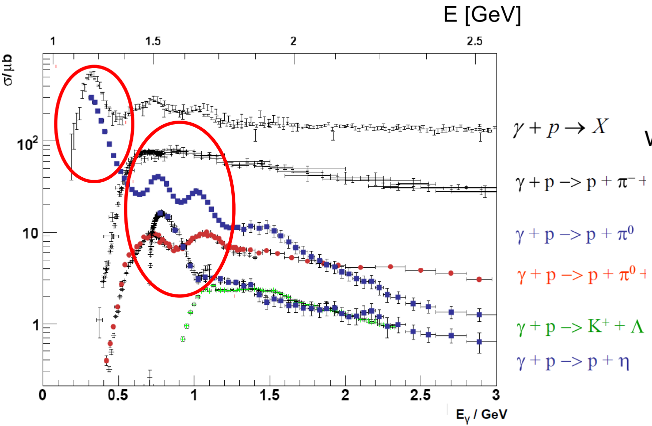
→ measurements of hadronic cross sections and asymmetries



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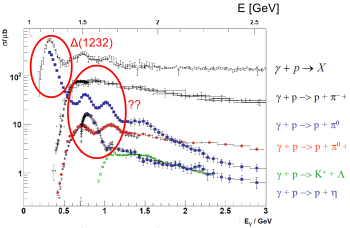
The excited baryon spectrum:

Connect experiment & QCD in the non-perturbative regime

How do quarks get confined in hadrons?

Experimental study of hadronic reactions



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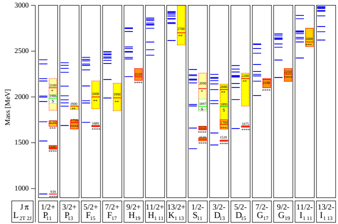
Theoretical predictions of excited hadrons
e.g. from relativistic quark models:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

⇒ Partial wave decomposition:
decompose data with respect to a conserved quantum number:

total angular momentum and parity J^P

⇒ search for resonances/excited states in those partial waves:
poles on the unphysical Riemann sheet

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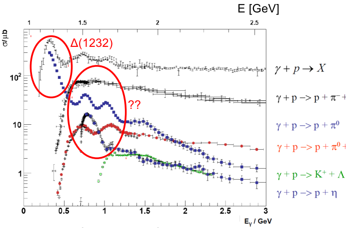
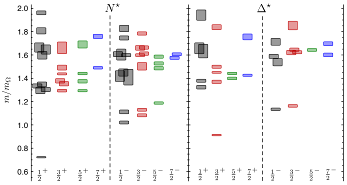


source: ELSA; data: ELSA, JLab, MAMI

$\Longleftrightarrow$

Theoretical predictions of excited hadrons
... or lattice calculations (with some limitations):



$m_\pi = 396$ MeV [Edwards *et al.*, Phys.Rev. D84 (2011)]

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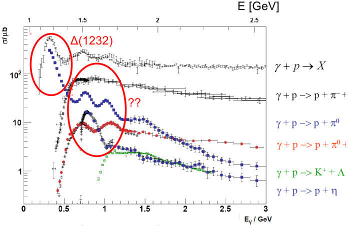

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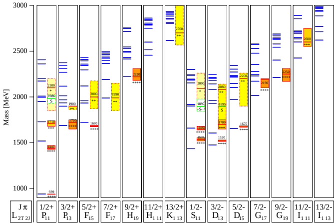

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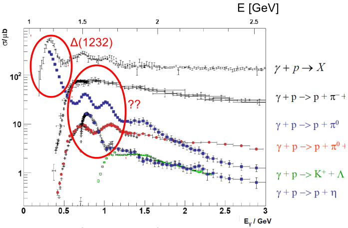

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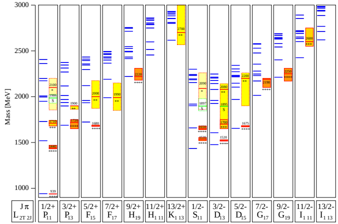

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In the past: **elastic or charge exchange πN scattering**

- "missing resonance problem"

In recent years: **photoproduction reactions**

- large data base, high quality polarization observables Prog.Part.Nucl.Phys. 125 (2022), Prog.Part.Nucl.Phys. 111 (2020)

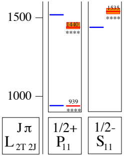
In the future: **electroproduction reactions**

- 10^5 data points for πN, ηN, KY, $\pi\pi N$ Review: e.g. Prog.Part.Nucl.Phys. 67 (2012)

The light baryon spectrum:

Many open questions

- Missing resonances?
- Different analyses often not agree on parameters or even existence of a state



E.g., the Roper resonance $N(1440)1/2^+$: discussed since $>$ 50 years

(Review: e.g. Burket, Roberts Rev.Mod.Phys. 91 (2019). Also: Mai, Meißner, Urbach Phys.Rept. 1001 (2023))

- q^3 quark models: first $1/2^-$ state lower than first $1/2^+$ state
- lattice QCD: e.g. Lang 2017 Phys. Rev. D 95, 014510

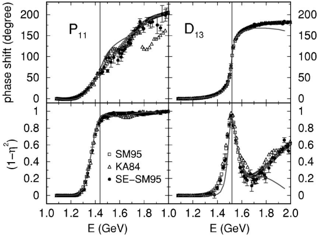


Fig. from PRC 62 025207 (2000)

- not a standard Breit-Wigner shape
- influence by meson-baryon background interaction?
- effects from nearby thresholds?

→ not a simple radial excitation of the nucleon?

→ information from photo- and electroproduction!
(Q^2 dependence of helicity amplitudes)

(Review: Ramalho & Pena Prog.Part.Nucl.Phys. 136 (2024))

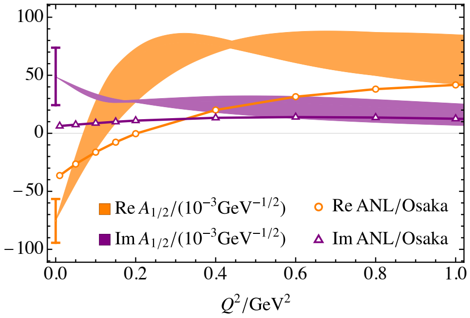
Baryon Transition Form Factors

Y.-F. Wang et al. PRL 133 (2024)

from the Jülich-Bonn-Washington model

Mai et al. EPJ A 59 (2023)

The Roper resonance $N(1440)1/2^+$:



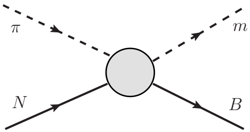
Prerequisite:

well-defined resonance parameters & uncertainties!

→ Jülich-Bonn model

- Zero crossing in Re$A_{1/2}$ at smaller Q^2 than in Breit-Wigner determinations or in ANL/OSAKA [Kamano, Few Body Syst. 59, 24 (2018)]
- important for quark models, DSE: meson cloud contributions or radial excitation of the nucleon?

Jülich-Bonn DCC approach for hadronic reactions



The Jülich-Bonn DCC approach for N^* and Δ resonances

pion-induced reactions

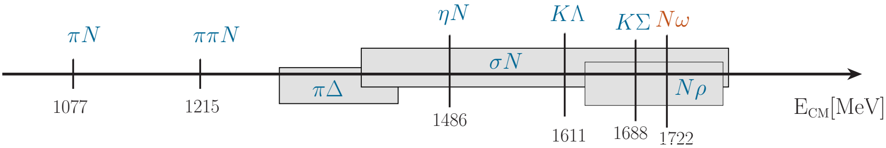
EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq\quad q^2\quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle\,\frac{1}{E-E_\gamma(q)+i\epsilon}\,\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

- channels ν, μ, γ:



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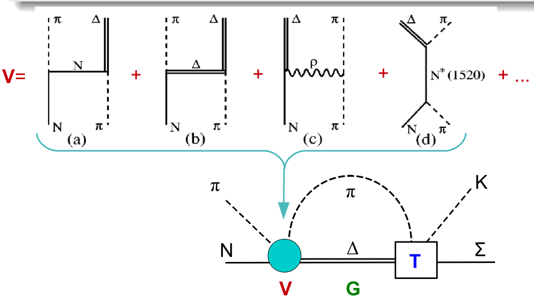
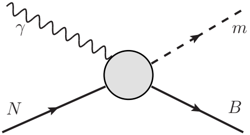


- potentials V constructed from effective $\mathcal{L}$
- t- and u-channel: T^{NP}

  dynamical generation of poles
- s-channel diagrams: T^{P}

  genuine resonance states
- contact terms

Photoproduction



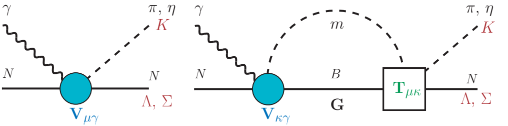
Photoproduction in a semi-phenomenological approach

EPJ A 50, 101 (2015)

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_{\kappa} V^{IJ}_{\kappa\gamma}$$

(partial wave basis)

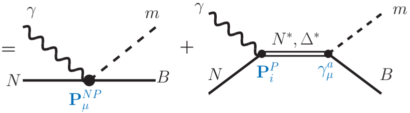


$m = \pi, \eta, K\ ,\ B = N, \Delta, \Lambda$

$T_{\mu\kappa}$: full hadronic T-matrix as in pion-induced reactions

Photoproduction potential: approximated by energy-dependent polynomials (field-theoretical description numerically too expensive)

$\mathbf{V}_{\mu\gamma}(E,q) =$



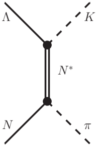
$$= \frac{\tilde{\gamma}^a_\mu(q)}{m_N} P^{\mathrm{NP}}_\mu(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^{\mathrm{P}}_i(E)}{E - m^b_i}$$

Simultaneous fit of pion- & photon-induced reactions

- calculate observables from T-matrix, Multipole amplitude M
- fit **free parameters** of $T/\,M$ to data

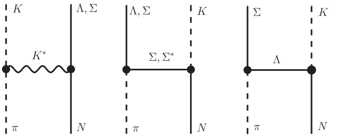
$\sigma = \frac{1}{2}\frac{4\pi}{p^2}\sum_{JLS,L'S'}|\tau^{JL'S'}_{LS}|^2$
with $\tau_{fi} = -\pi\sqrt{\rho_f\rho_i}\,T_{fi}$

s-channel: resonances (T^P)



m_{bare} + $f_{\pi NN^*}$

t- and u-channel exchange: “background“ (T^{NP})

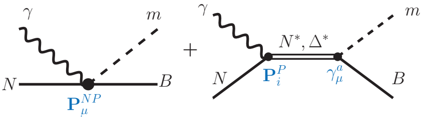


+ ...

cut offs Λ in form factors $\left(\frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 + \vec{q}^2}\right)^n$

(couplings fixed from SU(3))

- $\gamma p \to \pi N, \eta N, KY$: couplings of the polynomials



- couplings in contact terms: one per partial wave, couplings to πN, ηN, ($\pi\Delta$,) $K\Lambda$, $K\Sigma$

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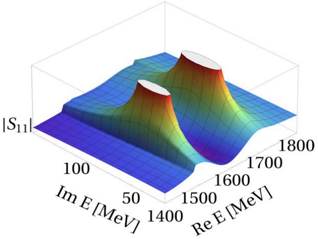
Fit parameters vs. resonance parameters

Quantities of interest (resonance properties) cannot be controlled directly

Resonances: poles in the full T-matrix

- on the unphysical Riemann sheet
- Re(E_0) = "mass", -2Im(E_0) = "width"
  residues$\rightarrow$ branching ratios
- NOT fit parameters!



Workflow:

- fit free model parameters to data $\rightarrow$ Amplitude (T-matrix)
- search for poles in T $\rightarrow$ resonance properties as in PDG listings

Resonance uncertainties

- from statistical & systematic uncertainties of exp. data
- from statistical & systematic uncertainties of the model

$\rightarrow$ extract uncertainties from **re-fits**

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Simultaneous fit of pion- & photon-induced reactions

Fitting procedure

JüBo Model:

- numerically expensive but theoretically well-founded formulation
- $\sim$ 900 fit parameters in total, $\sim$ 75,000 data points

  ↳ χ^2 minimization with MINUIT, parallelization in energy ($\sim$ 200 - 400 processes)

[JURECA, Jülich Supercomputing Centre, Journal of large-scale research facilities, 2, A**62** (2016)]

Disadvantages of using MINUIT:

- inefficient sampling of the parameter space
- cannot fit all parameters simultaneously
- cannot use parameter uncertainties as given by MINUIT
- re-fits: can obtain a few parameter sets $\rightarrow$ not enough to determine uncertainties of resonance parameters!

Goal of B05: Bayesian parameter estimation with HMC

HMC numerically challenging but rewarding:

- efficiently explore the high-dimensional parameter space
  (fit all parameters at once)
- determine resonance uncertainties from samples of parameter space
  (large enough number of samples)
- has never been applied in a complex coupled-channel framework $\rightarrow$ next level of precision for baryon spectroscopy

Advantages in JüBo:

- No sign problem: free parameters and χ^2 are real
- problem is ergodic: no singularities in χ^2

Work plan:

- Connect JüBo fortran code with HMC libraries (hand-tuned standard HMC)
- First application in a single-channel study, reduced parameter space
- coupled-channel fit, extension to $\eta' N$
- (explore more sophisticated HMC methods)

Summary

N^* and Δ resonance spectrum

- large amount of new data & many open questions (not only the Roper!)
- Prerequisite for a reliable spectrum:
  - well defined resonance parameters and extraction procedures
  - well defined uncertainty quantification!

Jülich-Bonn DCC analysis:

- Extraction of the N^* and Δ spectrum in a simultaneous analysis of pion- and photon-induced reactions [Eur.Phys.J.A 58 (2022) 229, PRC 109 (2024)]
- Electroproduction: *Jülich-Bonn-Washington* approach [Mai *et al.* PRC 103 (2021), PRC 106 (2022), EPJ A 59 (2023)]
  - Baryon transition form factors [Wang et al. PRL 133 (2024)]
- Λ^* and Σ^* resonance spectrum: in progress

Goals of B05:

- Bayesian parameter estimation with HMC
- well defined resonance uncertainties

Thank you for you attention!

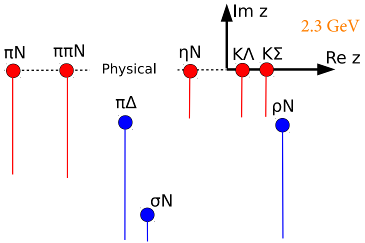
Appendix

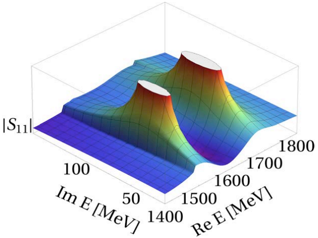
Resonance states

- (2 body) unitarity and analyticity respected (no on-shell factorization, dispersive parts included)
- opening of **inelastic channels** ⇒ **branch point** and new **Riemann sheet**

Resonances: poles in the full T-matrix

- on the unphysical Riemann sheet
- Pole position E_0 is the same in all channels
- Re(E_0) = "mass", -2Im(E_0) = "width"
  residues → branching ratios





3-body $\pi\pi N$ channel:

- parameterized effectively as $\pi\Delta$, σN, ρN
- $\pi N/\pi\pi$ subsystems fit the respective phase shifts

↳ branch points move into complex plane

Jülich-Bonn-Washington (JBW) parametrization

M. Mai *et al.* PRC 103 (2021), PRC 106 (2022), EPJ A 59 (2023)

$$\mathcal{M}_{\mu\gamma^*}(k,W,Q^2) = R_{\ell'}(\lambda, q/q_\gamma)\left(V_{\mu\gamma^*}(k,W,Q^2) + \sum_\kappa \int_0^\infty dp\, p^2\, T_{\mu\kappa}(k,p,W) G_\kappa(p,W) V_{\kappa\gamma^*}(p,W,Q^2)\right)$$

(Pseudo)-threshold behavior with meson/photon momenta

$$lim_{k\to 0} E_{\ell+} = k^\ell$$
$$lim_{q\to 0} L_{\ell+} = q^\ell$$
$$\dots$$

For $Q^2=0$ (real photons) identical to Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k,W,Q^2) = V^{\text{JUBO}}_{\mu\gamma}(k,W)\cdot \tilde{F}_D(Q^2)\cdot e^{-\beta^0_\mu Q^2/m_p^2}\left(1 + Q^2/m_p^2\beta^1_\mu + (Q^2/m_p^2)^2\beta^2_\mu\right)$$

←

Siegerts's theorem Siegert(1973) Amaldi et al.(1979) Tiator(2016)

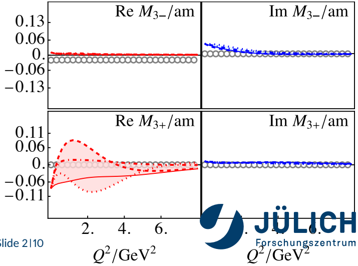
$$V^{L_{\ell\pm}} = (\text{const.})\cdot V^{E_{\ell\pm}}$$

...at pseudo-threshold

- simultaneous fit to πN, ηN, $K\Lambda$ electroproduction off proton $(W < 1.8$ GeV, $Q^2 < 8$ GeV$^2)$
- 533 fit parameters, 110.281 data points
- Input from JüBo: $V_{\mu\gamma}(k,W,Q^2=0)$, $T_{\mu\kappa}(k,p,W)$, $G_\kappa(p,W)$

  → universal pole positions and residues (fixed in this study)
- long-term goal: fit pion-, photo- and electron-induced reactions simultaneously

$\gamma^* p \to K\Lambda$ at $W = 1.7$ GeV



Baryon Transition Form Factors (TFFs)

Q^2 dependence of transition form factors (TFFs):
$\rightarrow$ conclusions on the nature of resonances

(e.g. 3 valence quark state, meson cloud contributions, ...)
Reviews: e.g. Rev.Mod.Phys. 91 (2019), Prog.Part.Nucl.Phys. 136 (2024)

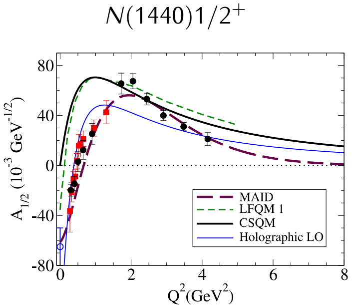


Figure from Prog.Part.Nucl.Phys. 136 104097 (2024)

TFFs from JBW:

- for the first time determined from a coupled-channel study of πN, ηN, and $K\Lambda$ electroproduction (+ constraints from photon & pion-induced reactions!)
- first estimation of TFFs for higher excited states
- from poles, not Breit-Wigner states

TFFs defined independently of the hadronic final state as Workman et al. PRC 87 (2013) :

$$H_h^{l\pm,I}(Q^2) = C_I \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \widetilde{R}^{l\pm,I}}} \widetilde{\mathcal{H}}_h^{l\pm,I}(Q^2) \,,$$

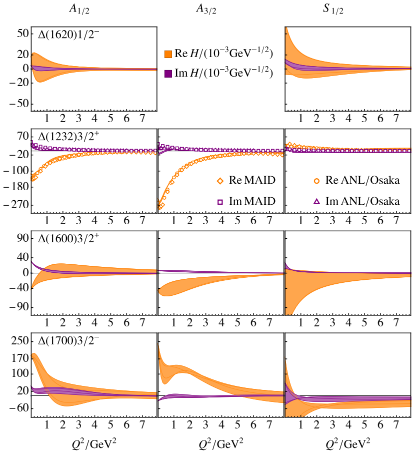
$h = 1/2, 3/2$ helicity, $\mathcal{H}$ ($=\mathcal{A}$ or $\mathcal{S}$) helicity amplitudes, $\widetilde{\mathcal{H}}$, $\widetilde{R}$ residues, z_p pole position

Baryon Transition Form Factors

Y.-F. Wang et al. PRL 133 (2024)

based on most recent JBW, pole parameters from JüBo2017

Δ states:



[ANL/OSAKA: Kamano Few Body Syst. 59, 24 (2018), MAID: Tiator et al. PRC94 (2016)]

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Baryon Transition Form Factors

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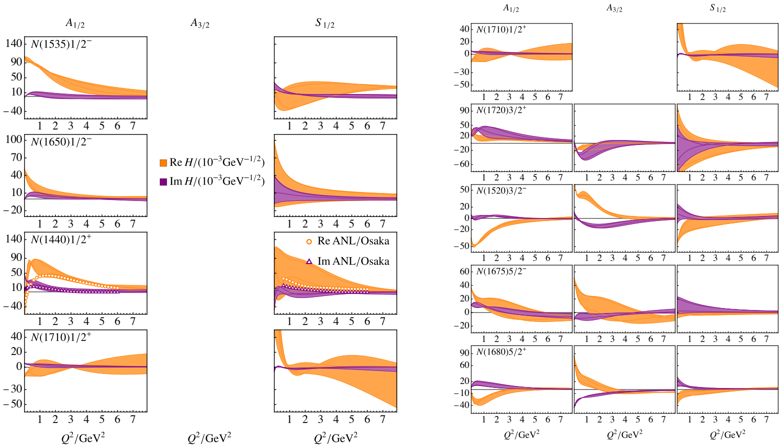
N^* states:



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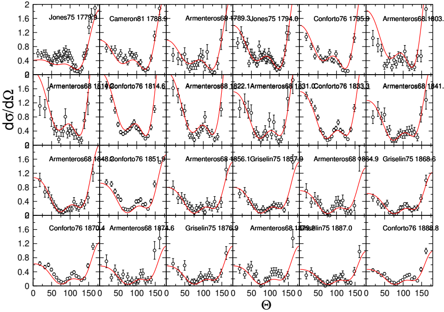
The Hyperon Spectrum: Λ^* and Σ^* resonances

Extension of JüBo to $\bar{K}N$ scattering: in progress

S. Rawat (preliminary)

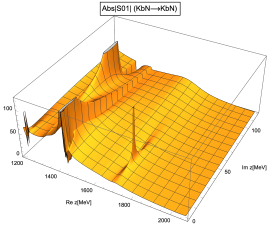
- use $SU(3)$ to adapt $\pi N \to X$ model to $\bar{K}N \to X$
- apply the same analysis tools (coupled-channel fits, pole search, ...) as for N^*s
- almost finished: coupled-channel fit to $\bar{K}N \to \bar{K}N$, $\pi\Lambda$, $\pi\Sigma$

Selected preliminary fit results $K^- p \to K^0 n$



"W-structure" due to pole in F_{05}

- Preliminary results $\Lambda(1405)$:
  $1430. - i11.19$ MeV & $1338. - i85.20$ MeV



- compares well with UChPT
  e.g. Mai 2015 (sol 4): $1429^{+8}_{-7} - i12^{+2}_{-3}$ MeV &
  $1325^{+15}_{-15} - i90^{+12}_{-18}$ MeV

JÜLICH Forschungszentrum

s-, t- and u-channel exchanges

- 21 s-channel states (resonances) coupling to πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\Delta$, ρN.
- t- and u-channel exchanges ("*background*", coupling constants fixed from SU(3)):

	πN	ρN	ηN	πΔ	σN	KΛ	KΣ
πN	N,Δ,$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$	N, Δ, Ct., π, ω, a_1	N, a_0	N, Δ, ρ	N, π	Σ, Σ*, K*	Λ, Σ, Σ*, K*
ρN		N, Δ, Ct., ρ	-	N, π	-	-	-
ηN			N, f_0	-	-	K*, Λ	Σ, Σ*, K*
πΔ				N, Δ, ρ	π	-	-
σN					N, σ	-	-
KΛ						Ξ, Ξ*, f_0, ω, ϕ	Ξ, Ξ*, ρ
KΣ							Ξ, Ξ*, f_0, ω, ϕ, ρ

Details of the formalism

Polynomials:

$$P_i^{\mathrm{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathrm{P}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathrm{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathrm{NP}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{\mu,n+1}^{\mathrm{NP}}(E-E_0)}$$

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathrm{P}}$, $g_{\mu,j}^{\mathrm{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

back

The scattering potential: s-channel resonances

$$V^{\mathrm{P}} = \sum_{i=0}^{n} \frac{\gamma^{a}_{\mu;i}\,\gamma^{c}_{\nu;i}}{z - m^{b}_{i}}$$

- i: resonance number per PW
- $\gamma^{c}_{\nu;i}$ ($\gamma^{a}_{\mu;i}$): creation (annihilation) vertex function with **bare coupling** f (free parameter)
- z: center-of-mass energy
- m^{b}_{i}: **bare mass** (free parameter)

- $J \leq 3/2$:

  $\gamma^{c}_{\nu;i}$ ($\gamma^{a}_{\mu;i}$) from effective $\mathcal{L}$

Vertex	$\mathcal{L}_{int}$
$N^*(S_{11})N\pi$	$\frac{f}{m_\pi}\bar{\Psi}_{N*}\gamma^{\mu}\vec{\tau}\,\partial_{\mu}\vec{\pi}\,\Psi \;+\;$ h.c.
$N^*(S_{11})N\eta$	$\frac{f}{m_\pi}\bar{\Psi}_{N*}\gamma^{\mu}\partial_{\mu}\eta\,\Psi \;+\;$ h.c.
$N^*(S_{11})N\rho$	$f\,\bar{\Psi}_{N*}\gamma^{5}\gamma^{\mu}\vec{\tau}\,\vec{\rho}_{\mu}\,\Psi \;+\;$ h.c.
$N^*(S_{11})\Delta\pi$	$\frac{f}{m_\pi}\bar{\Psi}_{N*}\gamma^{5}\vec{S}\partial_{\mu}\vec{\pi}\,\Delta^{\mu} \;+\;$ h.c.

- $5/2 \leq J \leq 9/2$:
  correct dependence on L (centrifugal barrier)

$$(\gamma^{a,c})_{\frac{5}{2}-} = \frac{k}{M}\,(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{5}{2}+} = \frac{k}{M}\,(\gamma^{a,c})_{\frac{3}{2}-}$$

$$(\gamma^{a,c})_{\frac{7}{2}-} = \frac{k^2}{M^2}\,(\gamma^{a,c})_{\frac{3}{2}-} \qquad (\gamma^{a,c})_{\frac{7}{2}+} = \frac{k^2}{M^2}\,(\gamma^{a,c})_{\frac{3}{2}+}$$

$$(\gamma^{a,c})_{\frac{9}{2}-} = \frac{k^3}{M^3}\,(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{9}{2}+} = \frac{k^3}{M^3}\,(\gamma^{a,c})_{\frac{3}{2}-}$$

Interaction potential from effective Lagrangian

J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); U.-G. Meißner, Phys. Rept. **161**, 213 (1988); B. Borasoy and U.-G. Meißner, Int. J. Mod. Phys. A **11**, 5183 (1996).

- consistent with the approximate (broken) chiral $SU(2) \times SU(2)$ symmetry of QCD

Vertex	$\mathcal{L}_{int}$	Vertex	$\mathcal{L}_{int}$
$NN\pi$	$-\frac{g_{NN\pi}}{m_\pi}\Psi\gamma^5\gamma^\mu\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi$	$NN\omega$	$-g_{NN\omega}\bar{\Psi}[\gamma^\mu - \frac{\kappa_\omega}{2m_N}\sigma^{\mu\nu}\partial_\nu]\omega_\mu\Psi$
$N\Delta\pi$	$\frac{g_{N\Delta\pi}}{m_\pi}\bar{\Delta}^\mu\vec{S}^\dagger\cdot\partial_\mu\vec{\pi}\Psi$ + h.c.	$\omega\pi\rho$	$\frac{g_{\omega\pi\rho}}{m_\omega}\epsilon_{\alpha\beta\mu\nu}\partial^\alpha\vec{\rho}^\beta\cdot\partial^\mu\vec{\pi}\omega^\nu$
$\rho\pi\pi$	$-g_{\rho\pi\pi}(\vec{\pi}\times\partial_\mu\vec{\pi})\cdot\vec{\rho}^\mu$	$N\Delta\rho$	$-i\frac{g_{N\Delta\rho}}{m_\rho}\bar{\Delta}^\mu\gamma^5\gamma^\mu\vec{S}^\dagger\cdot\vec{\rho}_{\mu\nu}\Psi$ + h.c.
$NN\rho$	$-g_{NN\rho}\Psi[\gamma^\mu - \frac{\kappa_\rho}{2m_N}\sigma^{\mu\nu}\partial_\nu]\vec{\tau}\cdot\vec{\rho}_\mu\Psi$	$\rho\rho\rho$	$g_{NN\rho}(\vec{\rho}_\mu\times\vec{\rho}_\nu)\cdot\vec{\rho}^{\mu\nu}$
$NN\sigma$	$-g_{NN\sigma}\bar{\Psi}\Psi\sigma$	$NN\rho\rho$	$\frac{\kappa_\rho g^2_{NN\rho}}{2m_N}\bar{\Psi}\sigma^{\mu\nu}\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\rho}_\nu)$
$\sigma\pi\pi$	$\frac{g_{\sigma\pi\pi}}{2m_\pi}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\sigma$	$\Delta\Delta\pi$	$\frac{g_{\Delta\Delta\pi}}{m_\pi}\bar{\Delta}_\mu\gamma^5\gamma^\nu\vec{T}\Delta^\mu\partial_\nu\vec{\pi}$
$\sigma\sigma\sigma$	$-g_{\sigma\sigma\sigma}m_\sigma\sigma\sigma\sigma$	$\Delta\Delta\rho$	$-g_{\Delta\Delta\rho}\bar{\Delta}_\tau(\gamma^\mu - i\frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta}\sigma^{\mu\nu}\partial_\nu)$
			$\cdot\vec{\rho}_\mu\cdot\vec{T}\Delta^\tau$
$NN\rho\pi$	$\frac{g_{NN\pi}}{m_\pi}2g_{NN\rho}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\pi})$	$NN\eta$	$-\frac{g_{NN\eta}}{m_\pi}\bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\eta\Psi$
NNa_1	$-\frac{g_{NN\pi}}{m_\pi}m_{a_1}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi\vec{a}_\mu$	NNa_0	$g_{NNa_0}m_\pi\bar{\Psi}\vec{\tau}\Psi\vec{a}_0$
$a_1\pi\rho$	$-\frac{2g_{\pi a_1\rho}}{m_{a_1}}[\partial_\mu\vec{\pi}\times\vec{a}_\nu - \partial_\nu\vec{\pi}\times\vec{a}_\mu]\cdot[\partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu]$	$\pi\eta a_0$	$g_{\pi\eta a_0}m_\pi\eta\vec{\pi}\cdot\vec{a}_0$
	$+\frac{2g_{\pi a_1\rho}}{2m_{a_1}}[\vec{\pi}\times(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)]\cdot[\partial^\mu\vec{a}^\nu - \partial^\nu\vec{a}^\mu]$		

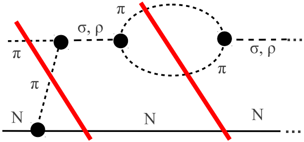
Theoretical constraints of the S-matrix

Unitarity: probability conservation

- 2-body unitarity
- 3-body unitarity:

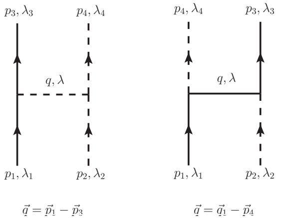
  discontinuities from t-channel exchanges

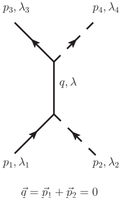
  → Meson exchange from requirements of the S-matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]



Analyticity: from unitarity and causality

- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut → Baryon u-channel exchange





→ Resonances