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B01 Quasi-Particle Dynamics in Low-Dimensional Topological Systems

Thomas Luu & Ulf Meißner

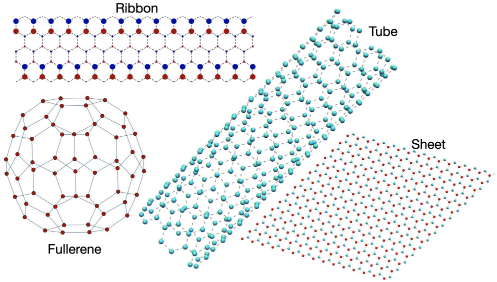
IAS-4, Forschungszentrum Jülich & HISKP, University of Bonn

October 1, 2024

In a nutshell...

Interested in dynamics of low-dimensional electronic systems

- At least one of the dimensions of the material is small (∼nanoscale)
- Quantum effects and strong correlations induce novel phenomena $\Longrightarrow$ non-perturbative
- Topology and symmetry play significant roles
- Perfect marriage of EFT and Monte Carlo methods



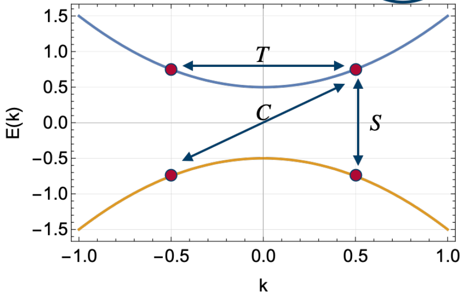
Our team. . .

There's me (TL), there's him (UM), and there's

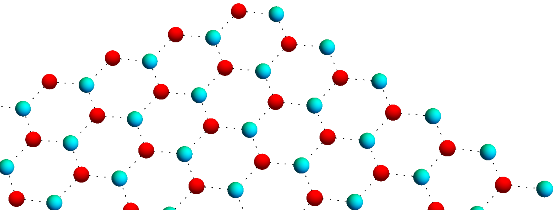
- Lin Wang (PD)
  - Comes from University of Konstanz
  - Degree from University of Science and Technology of China (USTC), Hefei, China
  - Research in
    - graphene/silicon quantum dots
    - topological superconductors/insulators
    - spintronics in low-dimensional systems including semiconductors and 2D materials
    - magnon topology
    - quantum anomalous Hall insulators
    - mesoscopic physics
- Starts on Oct. 1, 2024
- Collaborates with condensed matter experimentalists at RWTH

Symmetries relevant for low-D systems

- Time-reversal symmetry T: $T^2 = \pm 1$
  - $t = -t \implies E(k) = E(-k)$
- Charge conjugation symmetry (or particle-hole symmetry) C: $C^2 = \pm 1$
  - spectrum symmetric about zero: $E_+(k) = -E_-(-k)$
- Chiral symmetry (or sublattice symmetry) S: $S^2 = S$
  - $E_+(k) = -E_-(k)$

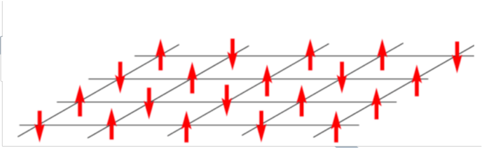


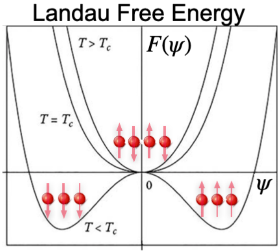
Normally, different phases of matter are distinguished by their ground-state symmetries (and lack thereof)



Phase transitions occur when symmetries get broken

- Classic example: Ising model





Another example studied by NuMeriQS scientists:

- Quantum phase transition of Hubbard model on a honeycomb lattice
  - J. Ostmeyer, **T.L.**, C. Urbach et al. [arXiv:2005.11112] Phys.Rev.B 102 (2020) 245105
  - J. Ostmeyer, **T.L.**, C. Urbach et al. [arXiv:2105.06936] Phys.Rev.B 104 (2021) 155142

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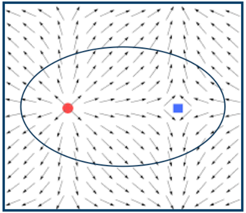
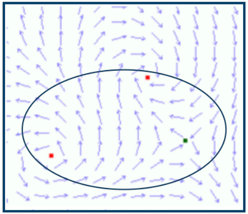
Phase transitions can also be classified by topology

- Another classic example: BKT transition (XY model)
  - Phases classified by topological invariant $\pi_1(S^1) = \mathbb{Z}$ (ie winding number)
  - Phases are distinct, but the ground states in each phase do not break the symmetry of the system



$J < J_c$
$|\nu| = 0 \in \mathbb{Z}$



$J > J_c$
$|\nu| = 1 \in \mathbb{Z}$

Intimately related to the Mermin-Wagner theorem

- continuous symmetries cannot be spontaneously broken at finite temperature in systems with sufficiently short-range interactions in dimensions d ≤ 2

A little bit more about "topology"

Topology 101

Topological Geometry

$\sim$

$\nsim$

$\sim$

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Topology 101

Topological Geometry

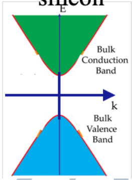
$\sim$

$\nsim$

$\sim$

Topological Matter

$H^{\text{solid}}_{\text{silicon}}$



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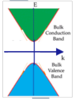
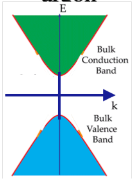


$H^{\text{solid}}_{\text{argon}}$



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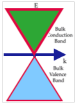
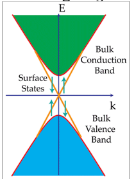


$H_{\text{Be}_2\text{Te}_3}$



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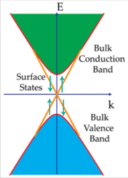
$H_{\text{Sb}_2\text{Te}_3}$



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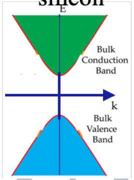

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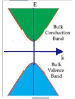
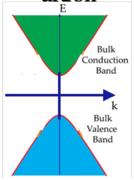


$H^{\text{solid}}_{\text{argon}}$



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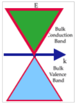

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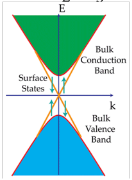

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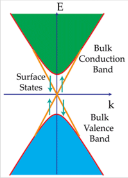
$H_{\text{Sb}_2\text{Te}_3}$



Invariants classified typically by $\mathbb{Z}$, $\mathbb{Z}_2$

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Classification of matter: the *ten-fold* way

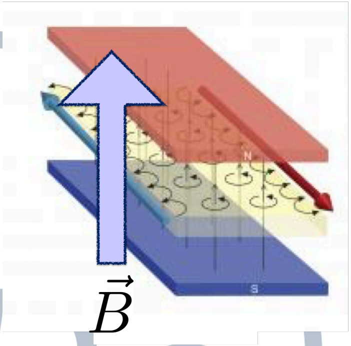
Symmetry				Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Atland & Zirnbauer, arXiv:cond-mat/9602137

Dyson, J.Math.Phys. 3 (1962) 1199

Novel forms of phenomena

fractional quantum hall effect

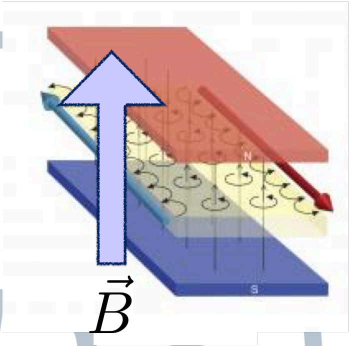


anyons as degrees of freedom in 2-D

$\mathbb{Z}$ invariants

Novel forms of phenomena

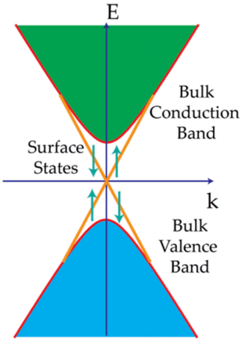
fractional quantum hall effect

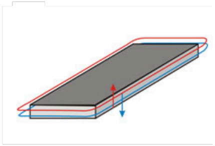


anyons as degrees of freedom in 2-D

$\mathbb{Z}$ invariants

topological insulator/superconductor





localized edge states due to boundary/bulk correspondence

$\mathbb{Z}_2$ invariants

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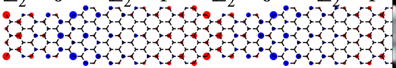
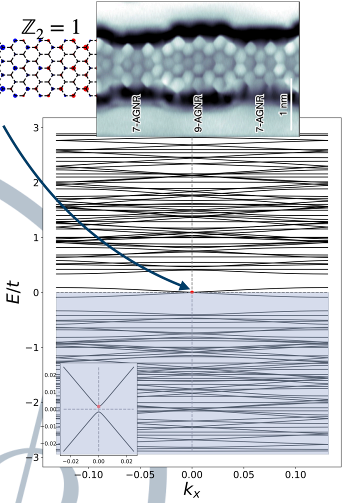
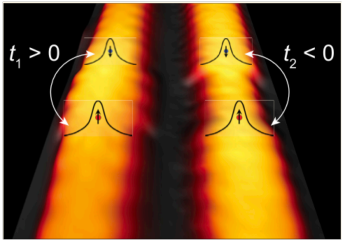
Another example: hybrid nanoribbons



Cao *et al*., Phys. Rev. Lett. **119**, 076401



All theoretical analysis based off *non-interacting* dynamics!



Rizzo *et al.,* ACS Nano 2021, 15, 12, 20633–20642

Potential application: Topological Quantum Dots

. . . and fault-tolerant quantum computing (one day)

Localization in the presence of interactions: **TL**, U.-G. Meißner, L. Razmadze Phys.Rev.B 106 (2022) 195422

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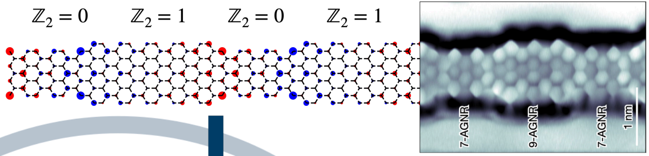
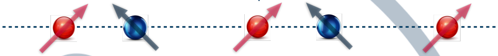
Simplifying the the theory *with interactions*

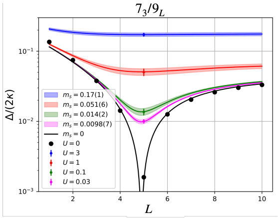




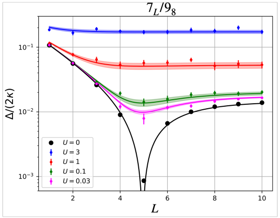
$$H_{1D} = -\sum_k a_k^\dagger \begin{pmatrix} m_s & t_A e^{ik} + t_B e^{-ik} \\ t_A e^{-ik} + t_B e^{ik} & -m_s \end{pmatrix} a_k$$

- fit $t_{A/B}$ to free theory

- fit m_s to underlying *interacting* theory



- predict spectrum of new geometries

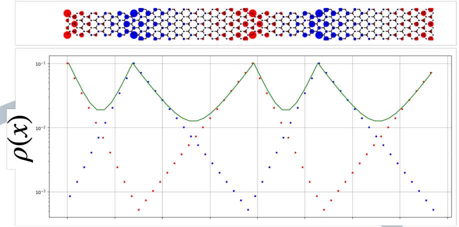


J. Ostmeyer, L. Razmadze, E. Berkowitz, **TL**, U.-G. Meißner Phys.Rev.B 109 (2024) 195135

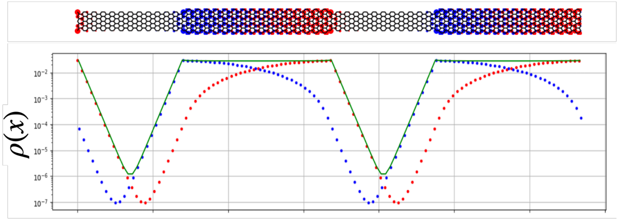
A new type of localization

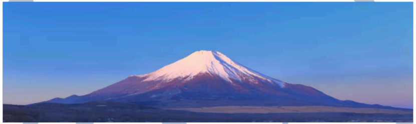
Localization due to $\mathbb{Z}_2$ invariants not the complete story

7/9 hybrid



9/11 hybrid





'Fuji' localization

'Kilimanjaro' localization

The extent of 'Kilimanjaro' localization can be controlled

see B01 poster

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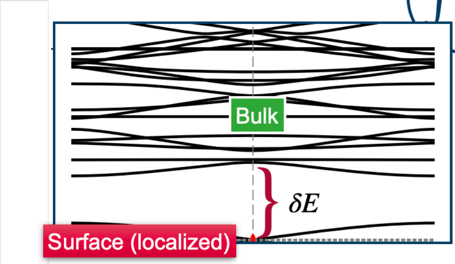
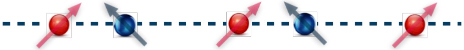
Ingredients for an effective (field) theory

- Separation of scales (ie energy gap to bulk states)



- Identification of relevant low-energy degrees of freedom



- Interaction terms constrained by symmetries

$$\delta H^i_{T,C,S} + \mathcal{O}\left(\left(\frac{q}{\delta E}\right)^{i+1}\right)$$

- Systematic power counting of terms

Takeaways

In case you weren't paying attention…

- Low-D materials offer fascinating novel phenomena, but require non-perturbative techniques due to strong correlation effects
- EFT methods applicable
  - symmetries are well established
  - identification of low-energy degrees of freedom
  - separation of scales (energy gap to bulk states)
  - systematic power counting
- Also great testbed for algorithmic testing and development, which already is leading to calculations in novel phase spaces