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The Classical Double Copy: A Duality Between Exact solutions in Gauge and Gravity theories

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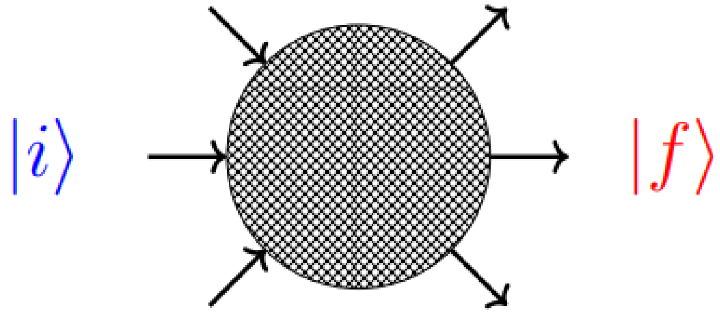
Introduction



- Theories of particle physics (such as the standard model) are best described by a special type of quantum field theory called *non-abelian gauge theories*.
- Recently, a new relationship between scattering amplitudes for *non-abelian gauge theories* and *gravity* has been discovered, known as the **Double Copy** *(Bern, Carrasco, and Johansson).*
- The Double Copy has allowed us to calculate previously unobtainable scattering amplitudes results in gravity, by "building" them out of analogous results in non-abelian gauge theories. (used as a tool in *gravitational scattering problems and gravitational waveform corrections*)

The Classical Double Copy

- It was soon realized that this duality at the level of scattering amplitudes, existed at the level of *classical physics* for **certain exact** solutions between **General Relativity** and **Classical** Non-abelian Gauge theories (e.g *Yang-Mills Theory*).
- This became known as the **Classical Double Copy** *(Luna, Monteiro, Nicholson, O'Connell, White)*.
- The best-known case of the Classical Double Copy is the **Weyl Double Copy**, which relates ***certain vacuum*** solutions in Classical Electromagnetism and General Relativity. *(Nicholson, O'Connell, Godazgar, Godazgar, Peinador Veiga, Pope)*
- This relationship relies on rewriting our solutions in the language of **two-component spinors**.

Weyl Double Copy in Practice

ELECTROMAGNETISM: We can write the electromagnetic field strength $\boldsymbol{F_{\mu\nu}}$ tensor in terms of **spinors**:

2-D Levi Civita Symbol

$$F_{\mu\nu} \rightarrow F_{AA'BB'} = \phi_{AB}\varepsilon_{A'B'} + \tilde{\phi}_{A'B'}\varepsilon_{AB}$$

Maxwell Spinor

Conjugate Maxwell Spinor

GRAVITY: For vacuum solutions, the *Riemann Curvature Tensor* $\boldsymbol{R_{\mu\nu\rho\lambda}}$ is reduced to the **Weyl Tensor** $\boldsymbol{W_{\mu\nu\rho\lambda}}$

The Weyl Tensor translated in terms of spinors is given as:

$$W_{\mu\nu\rho\lambda} \rightarrow W_{AA'BB'CC'DD'} = \Psi_{ABCD}\varepsilon_{A'B'}\varepsilon_{C'D'} + \tilde{\Psi}_{A'B'C'D'}\varepsilon_{AB}\varepsilon_{CD}$$

Weyl Spinor

Conjugate Weyl Spinor

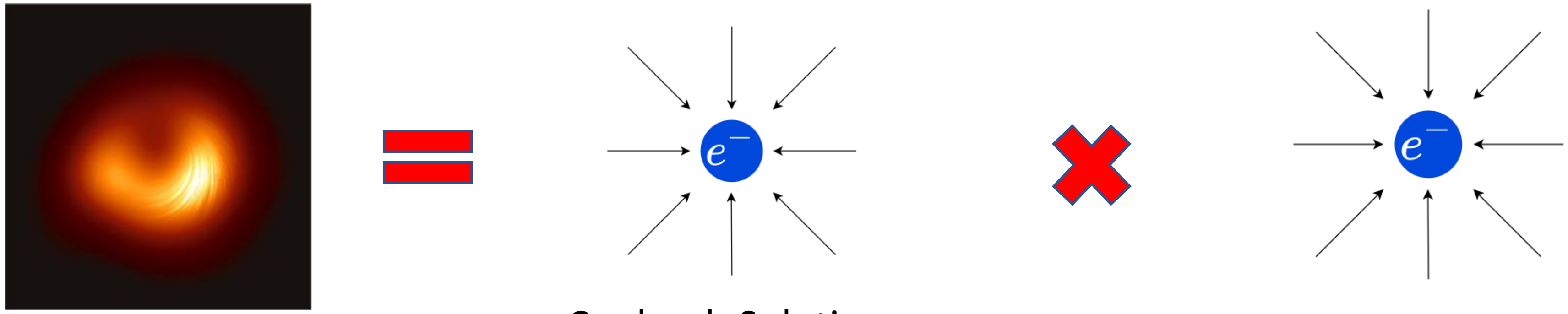
DOI: 10.3847/2041-8213/ab0ec7

Weyl Double Copy in Practice

For vacuum solutions in General Relativity that are of Petrov type ***D*** or ***N***, we can express them in terms of analogous solutions in electromagnetism:

Gravity $$\Psi_{ABCD} = \frac{\phi_{(AB}\phi_{CD)}}{S}$$ **Electromagnetism**

S is some scalar which is a harmonic function.



Schwarzschild Solution = Coulomb Solution × Coulomb Solution

Weyl Double Copy in *N = 0* Supergravity

The Weyl Double Copy has been extended to work for more *Exotic* theories of gravity such as *N = 0* Supergravity. *(KAW, White)*

N = 0 Supergravity = General Relativity (Graviton) + Dilaton + Axion

For *N=0* Supergravity, the Riemann Curvature tensor in the language of 2-components is *no longer* given by just the Weyl Tensor:

$$R_{\mu\nu\rho\lambda} \rightarrow \mathbb{X}_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} + \tilde{\mathbb{X}}_{A'B'C'D'}\epsilon_{AB}\epsilon_{CD} + \Phi_{ABC'D'}\epsilon_{A'B'}\epsilon_{CD} + \tilde{\Phi}_{A'B'CD}\epsilon_{AB}\epsilon_{C'D'}$$

$$\Psi_{ABCD} = \mathbb{X}_{(ABCD)}$$

Weyl Double Copy in *N = 0* Supergravity

For *N=0* Supergravity, in addition to the usual Weyl Double Copy statements, we can write down a Double Copy relationship for the so-called *mixed indexed fields* from the Riemann curvature spinor:

$$\Phi_{ABC'D'} = U^{C}{}_{C'}U^{D}{}_{D'}\left(\frac{\phi_{(AB}\phi_{CD)}}{S}\right)$$

$$U_{A}{}^{A'} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad U^{A}{}_{A'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\tilde{\Phi}_{A'B'CD} = U_{C}{}^{C'}U_{D}{}^{D'}\left(\frac{\tilde{\phi}_{(A'B'}\tilde{\phi}_{C'D')}}{\tilde{S}}\right)$$

Weyl Double Copy with Sources

The Weyl Double Copy has now been extended to work with non-vacuum solutions for Einstein-Maxwell Gravity. (*KAW, Moynihan, White, Manton, Easson, Svesko*)

Einstein-Maxwell Gravity = **General Relativity** + **Classical Electromagnetism**

These results were derived using methods inspired by methods from Twistor Theory and Quantum Field Theory. *(KAW, Moynihan, White)*

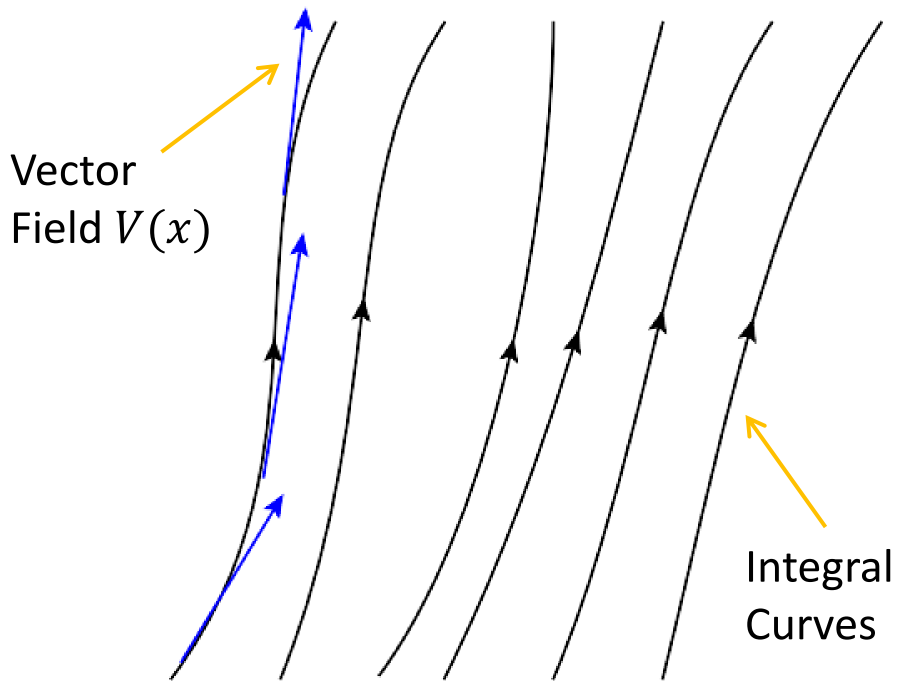
Kinematic Algebras

- One consequence of the Double Copy, was that gauge theories had a much *richer algebraic structure* than previously thought.
- Gauge theories are now known to possess so-called **Kinematic Algebras**.
- We currently believe that kinematics algebras are in general **not** Lie algebras, but some more general mathematical structures such as **homotopy algebras**. *(Reiterer; then Borsten, Jurco, Kim, Macrelli, Saemann, Wolf; Bonezzi, Chiaffrino, Diaz-Jaramilo, Hohm, Plefka)*
- Kinematic algebras were formerly only associated with Quantum Field Theories, but it has been shown that they exist for classical physics as well. *(KAW, Nagy, Wikeley, White)*

Diffeomorphisms and Kinematic Algebras

The Kinematic Algebra of a linear theory (e.g electromagnetism) is associated with special transformations called ***diffeomorphisms*** *(Fu, Krasnov).*

Diffeomorphism = simultaneous translation along all integral curves (field lines) of the vector field.



Recalling that $F_{\mu\nu}$ can be written in terms of so-called gauge (vector) fields $A_\mu(x)$:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu$$

We can think about this field as being “valued” in terms of diffeomorphism and so-called gauge (local symmetries) symmetries:

Infinitesimal diffeomorphism

Infinitesimal gauge transformation

$$\mathbf{A} = A^{\mu a}\partial_\mu \mathbf{T}^a \quad \longrightarrow \quad A^\mu = A^{\mu a}\mathbf{T}^a$$

Electromagnetism in Light Cone Gauge

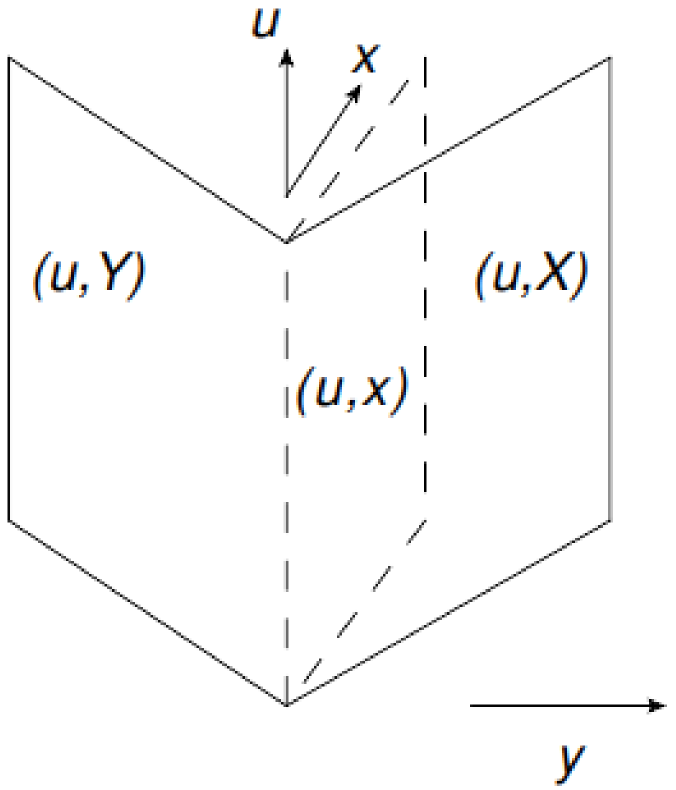
We can **fix** the *"local symmetries"* that arise in our gauge field, via something called a ***gauge transformation***.

For electromagnetism, we make a gauge choice that *restricts* our gauge field to a special gauge choice called **Light Cone gauge**.

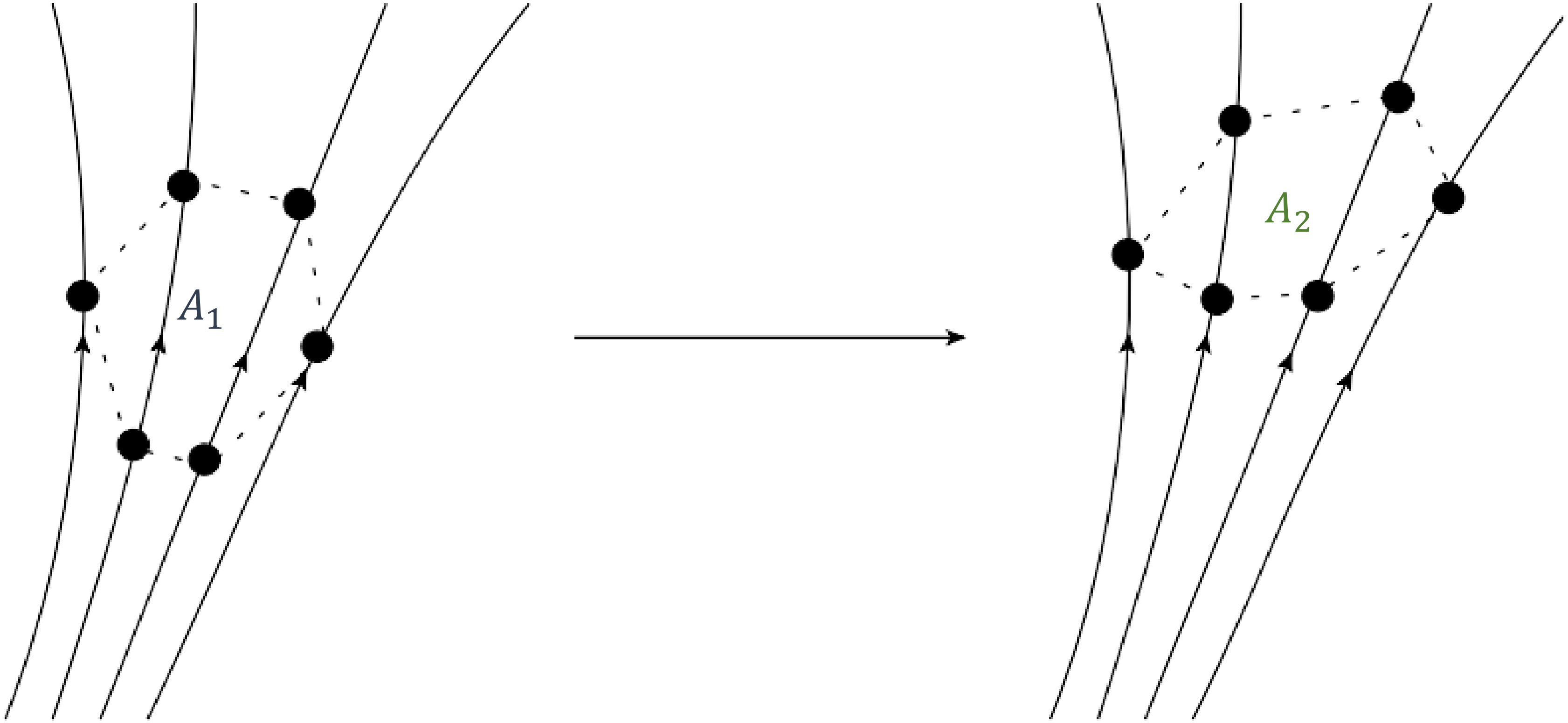
We describe Light cone gauge using a *special* set of coordinates (u, v, x, y).

In Light Cone gauge, we can directly **geometrically** *visualise* the kinematic algebra for the theory.

The kinematic algebra we see is a special type of diffeomorphism known as ***area-preserving diffeomorphisms*** (in particular *symplectomorphism*), which act **in 2-D planes** in either (u, x) or (u, y) planes.



Area Preserving Diffeomorphisms



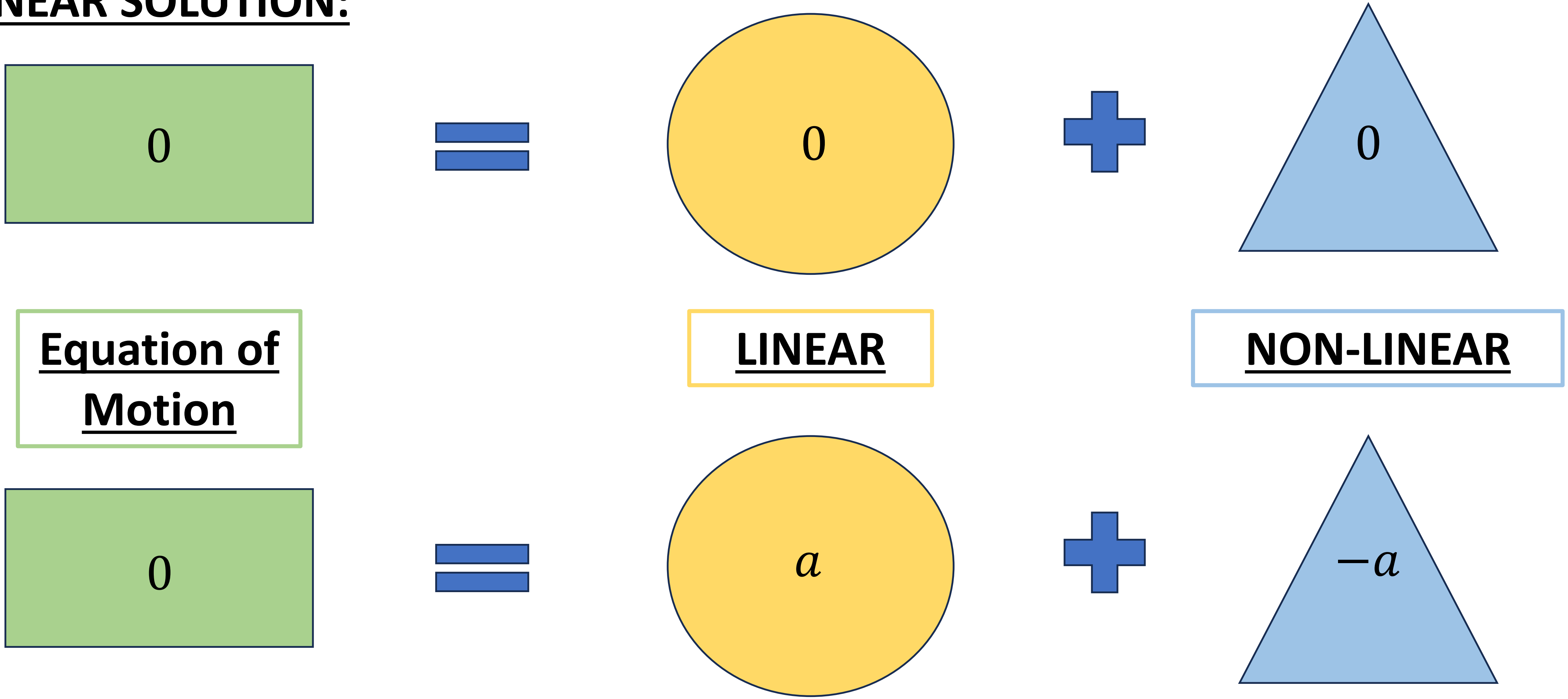
$$A_1 = A_2$$

Open Questions

- It turns out we can use our understanding of kinematic algebras in linear theories, to **construct** the *Lagrangians* of **non-linear** gauge theories (*QED coupled to scalar matter*) *(KAW, Nagy, Wikeley, White)* . Can we make this process more general and how is it related to the Double Copy?
- We can derive kinematic algebras for theories relevant to condensed matter physics (non-abelian Chern-Simons theory *(Ben-Shahar & Johansson)*) and beyond (fluid mechanics) *(KAW, Nagy, Wikeley, White)*. Can we gain any physical insight from the kinematic algebras ?
- Can we find (Classical) Double Copies for more exotic theories (CFT, AdS, Supergravity, de sitter, cosmology)?
- So far, every example of the Classical Double Copy has been **exact** solutions that **linearise** the *equations of motion* for both gravity and gauge theory. Can we find a Classical Double Copy for truly non-linear solutions?

Non-Linear Solutions vs Linear Solutions

LINEAR SOLUTION:



NON-LINEAR SOLUTION:

Conclusions

- The Double Copy is a duality between Non-abelian Gauge Theories and Gravity for both scattering amplitudes in quantum field theory and exact solutions in Classical Physics.
- The Classical Double Copy has been recently extended to work in non-vacuum solutions (Einstein-Maxwell Gravity) and exotic theories such as *N=0* supergravity.
- Kinematic algebras are new and exciting structures present in gauge theories.
- There is plenty more to explore on the Double Copy!

The Double Copy NEEDS you!