From Quantum Uncertainty to Classical Spinning Black Holes

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Based on 2112.07556, 2312.09960,...

with

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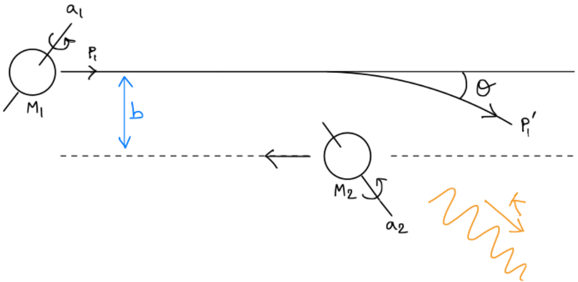
The Two-Body Problem

Gravitational wave physics requires **high precision**: need to solve

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu} \ .$$

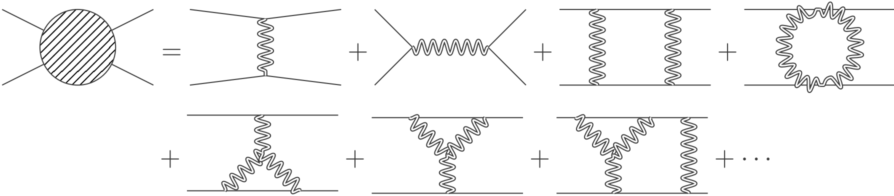
Complicated: spin, tidal effects, radiation, modified theories...



Ignores merger, but gets us quite far. Can be computed with **amplitudes**.

Amplitudes are Quantum

Solving the EFE's is hard. Can **independently** compute amplitudes — but they are **quantum** objects



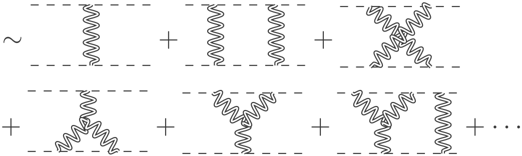
Classical physics: $\hbar \to 0$ implies **exponentiation**.

We could compute all the diagrams, but that's a lot of work! Can we do better?

The Eikonal

One known example of a better way: the **eikonal**

$$e^{\frac{i}{\hbar}\chi(b,s)} - 1 \sim$$



$$= i \int \hat{d}^4 q \hat{\delta}(2p_1 \cdot q)\hat{\delta}(2p_2 \cdot q) e^{-iq\cdot b}\mathcal{M}_4[q^2, s]$$

Cleverly packaged all the *classical* diagrams into a phase:

$$\chi = \chi_0 + \chi_1 + \chi_2 + \cdots, \qquad \text{where } \chi_i \sim \mathcal{O}(G^{1+i})$$

Only well established for scalars, conservative.

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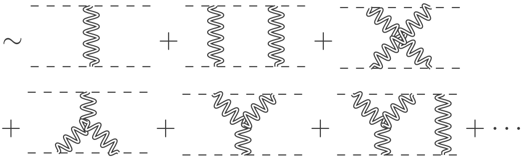

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Let's expand amplitudes into **fragments**

$$\mathcal{M}_{n,L} = G^{1+L}\hbar^{-k}(\mathcal{M}^{(0)}_{n,L} + \hbar\mathcal{M}^{(1)}_{n,L} + \hbar^2\mathcal{M}^{(2)}_{n,L} + \cdots)$$

$\hbar \to 0$ limit looks bad for $k > 1$... Cancellations must occur! We get the relationship

$$\tilde{\mathcal{M}}_{4,0} = \frac{\chi_0}{\hbar}, \quad \tilde{\mathcal{M}}_{4,1} = i\frac{\chi_0^2}{2\hbar^2} + \frac{\chi_1}{\hbar}, \quad \tilde{\mathcal{M}}_{4,2} = \frac{\chi_0^3}{6\hbar^3} + i\frac{\chi_0\chi_1}{\hbar^2} + \frac{\chi_2}{\hbar}, \quad \cdots$$

Infinite set of relations among fragments in the classical limit

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Where do these come from?

Quantum to Classical

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Minimal Uncertainty

Amplitudes are quantum – **uncertainty** built in:

$$(\Delta_\psi A)^2(\Delta_\psi B)^2 \geq \frac{1}{4}|\langle\psi|[A, B]|\psi\rangle|^2,$$

where $(\Delta_\psi A)^2 = \langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2$ is the variance.

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$$(\Delta_\psi A)^2 \simeq 0.$$

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Zero Variance Relations

Suppose $A = S^\dagger \mathcal{O}_A S$. Vanishing variance implies

$$\langle S^\dagger \mathcal{O}_A \mathcal{O}_A S \rangle \sim \langle S^\dagger \mathcal{O}_A S \rangle \langle S^\dagger \mathcal{O}_A S \rangle$$

Expanding $S = 1 + iT$, we get at leading order

$$\langle \mathcal{O}_A \mathcal{O}_A T \rangle - \langle T^\dagger \mathcal{O}_A \mathcal{O}_A \rangle \sim \langle T^\dagger \mathcal{O}_A \rangle \langle \mathcal{O}_A T \rangle - \langle \mathcal{O}_A T \rangle \langle \mathcal{O}_A T \rangle + \mathit{similar}$$

Amplitudes are defined via $\langle T \rangle \sim \mathcal{M} \delta^{(4)} \left(\sum_i p_i \right)$ and so we get

$$\langle \mathcal{O}_A \mathcal{O}_A \rangle \cdot \mathcal{M}_{q,L+1} \sim \langle \mathcal{O}_A \rangle \cdot \mathcal{M}_{r,L} \langle \mathcal{O}_A \rangle \cdot \mathcal{M}_{s,L}$$

Valid with spin, any number of legs/loops, conservative ($q = r = s$) and radiative ($r \neq s$).

Motivation beyond the eikonal.

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Motivation beyond the eikonal.

Eikonal state

Start with a two-particle state

$$|\Psi\rangle = \int_{p_1, p_2, a_1, a_2} |p_1, a_1; p_2, a_2\rangle$$

We **time-evolve** with the S-matrix (spin implicit)

$$S|\Psi\rangle = |\Psi\rangle + \int_{p_1', p_2', q} |p_1', p_2'\rangle \times \hat{\delta}(2p_1' \cdot q - q^2)\hat{\delta}(2p_2' \cdot q + q^2) i\mathcal{A}_4(s, t, m_i^2)$$

We can invert the eikonal

$$\hat{\delta}(2\tilde{p}_1 \cdot q)\hat{\delta}(2\tilde{p}_2 \cdot q) i\mathcal{A}_4(s, t, m_i^2) = \int \mathrm{d}^4x \, e^{iq\cdot x} \Big\{ (\exp(i\chi(x_\perp)) - 1) \Big\}$$

where $x_\perp^\mu = \Pi^\mu{}_\nu(\tilde{p}_1, \tilde{p}_2) x^\nu$ and

$$\tilde{X} = e^{q\cdot Y} X e^{-q\cdot Y}, \qquad Y_\mu = \frac{1}{2}\frac{\partial}{\partial p_2'^\mu} - \frac{1}{2}\frac{\partial}{\partial p_1'^\mu}.$$

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For $\hbar \to 0$, we find two stationary phase conditions

$$Q^\mu = -\frac{\partial\widetilde{\chi}}{\partial x_\mu}, \qquad X^\mu = b^\mu - \frac{\partial\widetilde{\chi}}{\partial Q_\mu}.$$

Q_μ is the **classical impulse** while X^μ picks up **iterations.**

The final state is then

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Final State With Radiation

Coherent states automatically minimise uncertainty:

$$a^\dagger \left|\alpha\right\rangle = \alpha \left|\alpha\right\rangle \qquad \Longrightarrow \qquad (\Delta_\alpha \hat{x})^2 = (\Delta_\alpha \hat{p})^2 = \frac{\hbar}{2}$$

in QFT, for us,

$$\left|p_1, p_2, \alpha\right\rangle = e^{-\frac{1}{2}\int_k |\alpha(k)|^2} \exp\left(\int_k \alpha(k) a^\dagger_\eta(k)\right) \left|p_1, p_2; 0\right\rangle$$

Compute lowest order to fix $\alpha \sim \mathcal{M}_5$, motivating the proposal

$$S\left|\psi\right\rangle = \int_{p_i, FT} \xi_{b_1'} \exp\left(\frac{i}{\hbar}\chi(x_\perp)\right)_{b_1}^{b_1'} \exp\left(\int_{\text{on-shell}} \mathcal{M}_5 a^\dagger\right)_{a_1'}^{b_1} \left|p_1', a_1', p_2'\right\rangle$$

This is our proposal for the final classical state including spin and radiation. Needs lots of testing!

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Observables in the Classical Limit

The change in some observable $\mathbb{O}_1 \in \mathcal{H}_1$ due to scattering is

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For large spin and zero radiation we have by BCH

$$[\mathbb{O}, e^{i\chi/\hbar}] = e^{i\chi/\hbar} \left(-\{\langle \mathbb{O} \rangle , \langle \tilde{\chi} \rangle\} - \frac{1}{2} \{\langle \tilde{\chi} \rangle , \{\langle \mathbb{O} \rangle , \langle \tilde{\chi} \rangle\}\} + \cdots \right)$$

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Find a nice formula to compute observables with spin

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Testing the Proposal

Test: compute non-trivial results. We find

- For $\mathbb{O}_1 = \mathbb{P}$, we find $\Delta p_1^\mu = Q^\mu$ up to $\mathcal{O}(G^2)$, including radiation reaction at $\mathcal{O}(G^3)$

$$\Delta p^\mu_{1,\mathrm{RR}} = i \left\langle\!\!\left\langle \tilde{\mathcal{M}}_5^*(x_1, x_2) \partial_1^\mu \tilde{\mathcal{M}}_5(x_1, x_2) \right\rangle\!\!\right\rangle .$$

- For $\mathbb{O}_1 = \mathbb{C}$ in Yang-Mills, we compute the change in colour-charge to one-loop

$$\Delta c^a = -\{c^a, \widetilde{\chi}\} - \frac{1}{2}\{\widetilde{\chi}, \{c^a, \widetilde{\chi}\}\} + \cdots$$

- For $\mathbb{O}_1 = \mathbb{W}$, we find the linear spin-kick up to one-loop $\mathcal{O}(G^2, s_1)$

$$\Delta s_1^\mu = s_1^\mu(\Delta p_1) - \{s_1^\mu, \widetilde{\chi}\} - \frac{1}{2}\{\widetilde{\chi}, \{s_1^\mu, \widetilde{\chi}\}\} + \cdots$$

The “direct” term can be traced to a boost, the rest rotation.

Where next

- More checks needed, refine proposal,
- Tidal effects, Higher dimensional operators
- Higher orders in G: tail effects, memory etc
- Bound states
- Combine spin + radiation
- Extract Radiation Reaction beyond $\mathcal{O}(G^3)$

Thank you for listening.

Any Questions?