Analytical Calculations for $Wt\bar{t}$ Production at NLO

Work in progess with Matteo Becchetti, Maximilian Delto, Philipp Kreer, Tiziano Peraro, Mattia Pozzoli and Lorenzo Tancredi

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SARA DITSCH

BONN FALL HEP MEETING 2024:

EMBRACING DIVERSITY IN HIGH ENERGY PHYSICS

Amplitudes and Wtt Production

SCATTERING AMPLITUDES

Theory
QFT

- Insight into underlying theory
- Connection to mathematics

$$d\sigma \propto |A|^2$$

Experiment
LHC

- Full Run-3 data and HL phase at LHC

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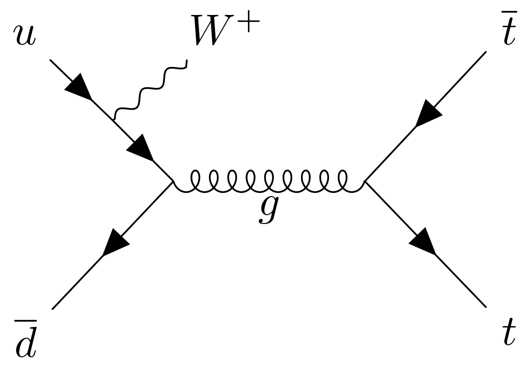

- Signature for BSM physics
- Background e.g. Ht$\bar{t}$ production
- Many scales, methodologically interesting

Representation of the Amplitude

FORM FACTORS IN THV [Peraro, Tancredi '20]

$$A(\{p_k\}) = \sum_{i=1}^{N} F_i(\{x_k\}) T_i$$

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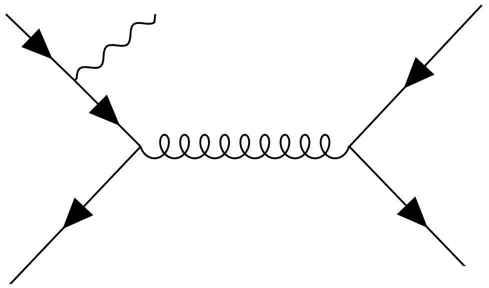

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⇨ One Basis for all loop orders

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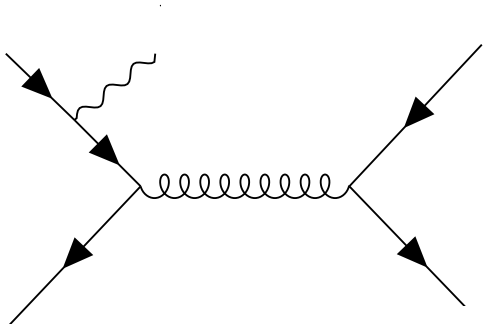

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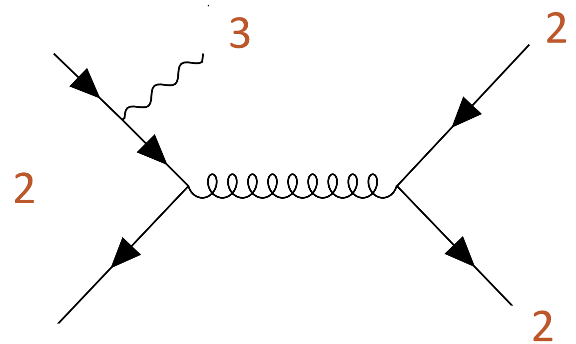

2×2×2×3 = 24

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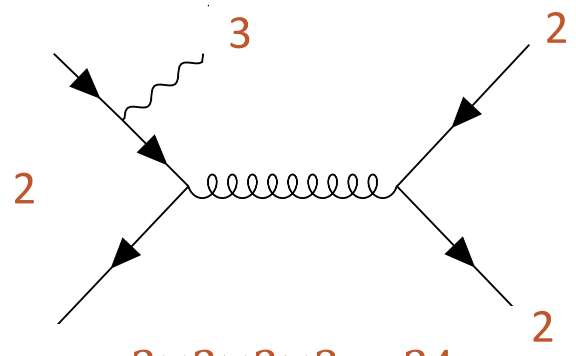

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$$T_i \equiv \left(\overline{V}_1 \{\not{p}_3, \not{p}_4\} U_2\right) \left(\overline{V}_3 \{\mathbb{1}, \not{p}_1, \not{p}_2, \not{p}_1 \not{p}_2\} U_4\right) \left(\varepsilon_5^* \cdot \{p_1, p_2, p_3\}\right)$$

Choice of Basis Tensors

WHAT IS A GOOD TENSOR BASIS?

- No spurious poles (e.g. gram determinant at tree-level)
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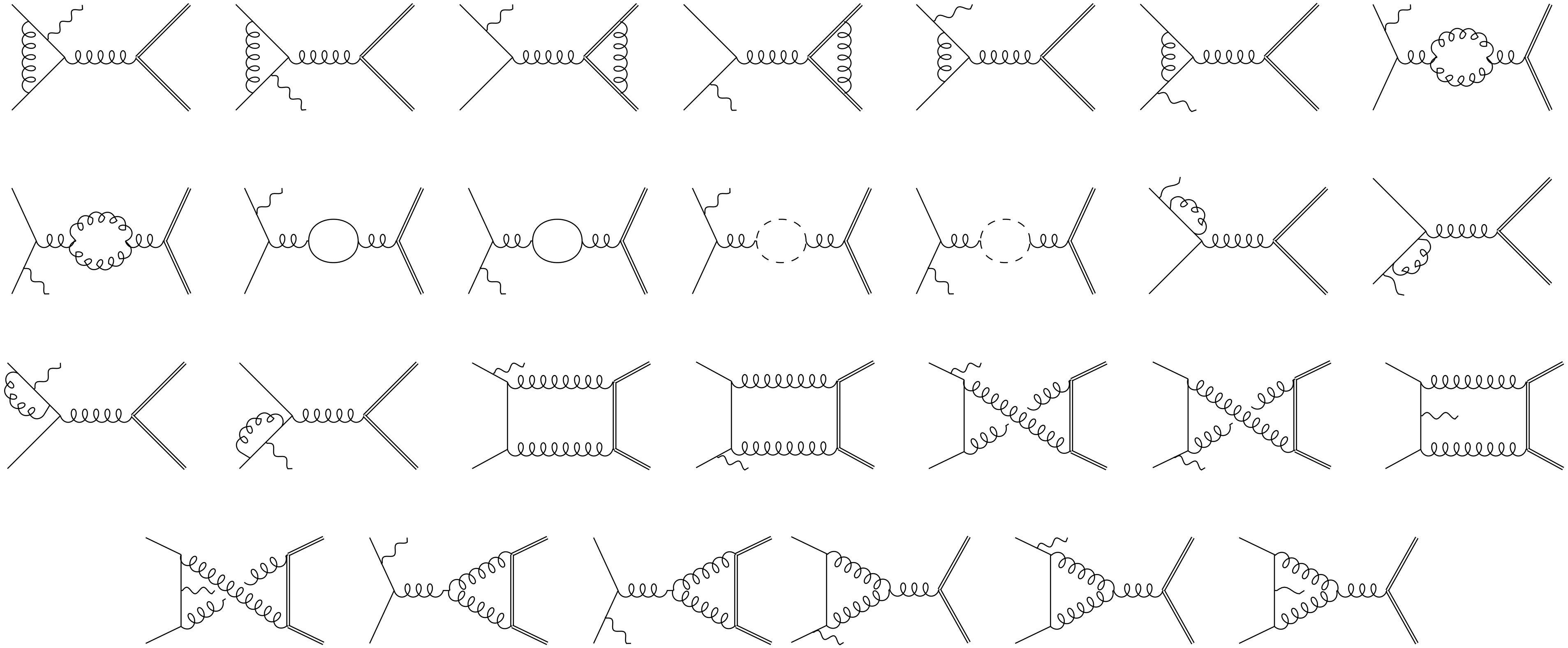
Not found for $Wt\bar{t}$ production

One-Loop Amplitude



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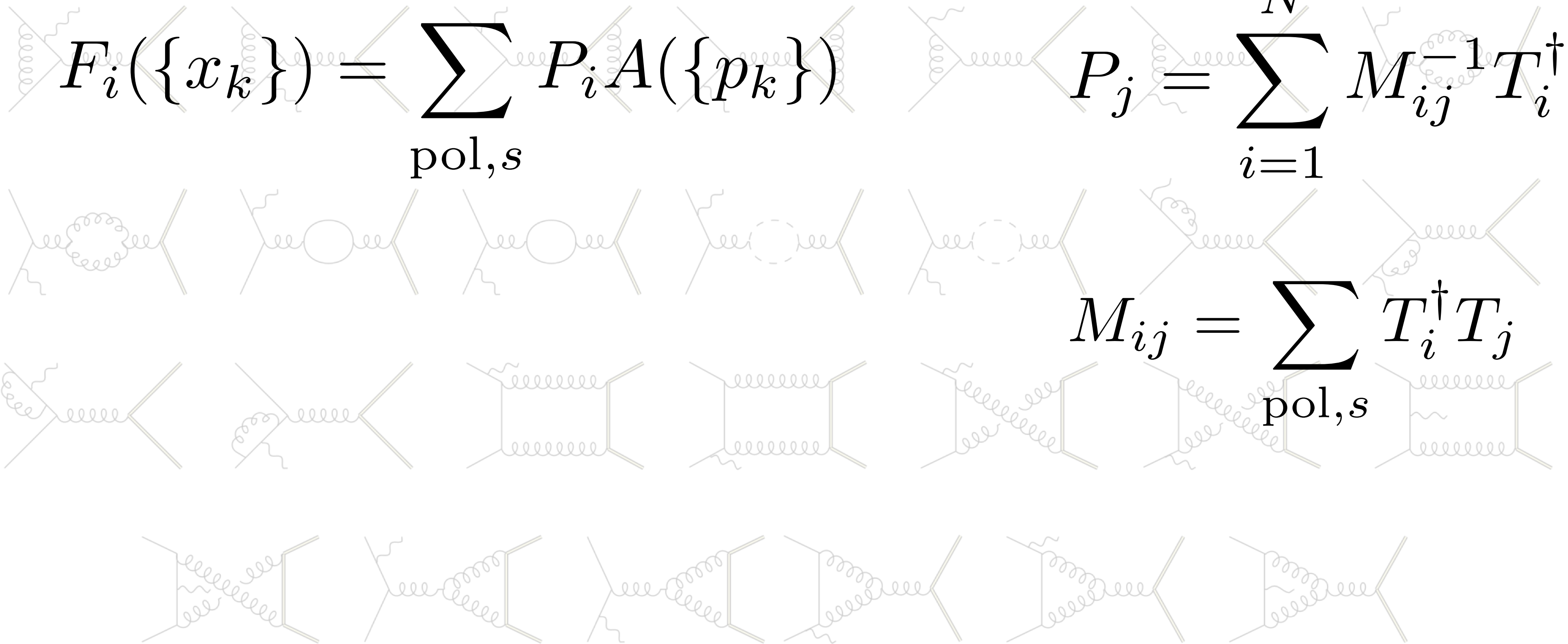

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$$I_{(a_1,..,a_n)} = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{1}{D_1^{a_1} D_2^{a_2} \dots D_n^{a_n}}$$

$$D_i = q_i(p_j, k)^2 - m_i^2$$

Reduction of Scalar Integrals

$$I = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{1}{D_1^{a_1} D_2^{a_2} \ldots D_n^{a_n}}$$

Integral families form a vector space with finite basis of integrals:

Master Integrals J [Smirnov, Petukhov ‘11]

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[Chetyrkin, Tkachov '81]

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[Chetyrkin, Tkachov '81]

$$I = \sum_i c_i(\{s_{ij}, d\}) J_i$$

$$F = \sum_k r_k(\{s_{ij}\}) I_k = \sum_{k,m} r_k(\{s_{ij}\}) c_m^{(k)}(\{s_{ij}\}, d) J_m$$

Computation of Master Integrals

METHOD OF DIFFERENTIAL EQUATIONS

$$\frac{\partial}{\partial x_i} J_j = \sum_k b_k I_k = \sum_n a_n J_n$$

$$\frac{\partial}{\partial x_i} \vec{J} = A_{x_i}(\{x_i\}, D)\vec{J}$$

[Kotikov '91] [Remiddi '97]

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$$\frac{\partial}{\partial x_i} \vec{J} = \varepsilon A_{x_i}(\{x_i\})\vec{J}$$

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[Henn '13]

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- Successive Integration in all Mandelstam variables
  → Not feasible

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Alternative:

- Find possible letters α_k from Baikov representation [Chen, Ma, Yang '22]
- Make ansatz and fix A_k from derivatives

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Solution:

- Combine terms with same denominators
- Find relations
- Partial fraction separately but with fixed relative order

→ Simplification of factor 10

Summary

- Separate tensor structure and kinematics: Form factor decomposition

  Is there a “better” basis? Why not?

- Group scalar integral into families and find canonical master integrals for families
- Reduce integrals to master integrals: IBP-reduction
- Compute master integrals: Method of (canonical) differential equations

  Compute DEQ matrix from Ansatz

- Simplify rational coefficients

  Find relations of expanded masters + partial fraction