Inclusive Semileptonic Decay of the D_s meson from Lattice QCD

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Extended Twisted Mass Collaboration

October 2024

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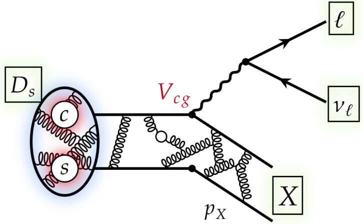
Introduction

Inclusive Semileptonic Decay $D_s \to X\ell\nu$ from Lattice QCD

- Final state is not necessary



[1] Gambino and Hashimoto 2020; Gambino, Hashimoto et al. 2022
[2] Asner et al. 2010
[3] Ablikim et al. 2021

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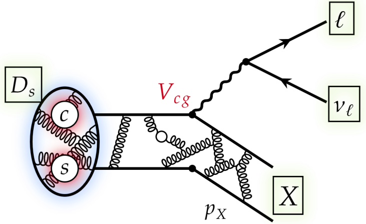

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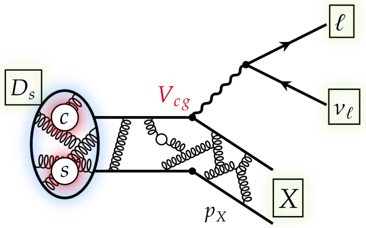
- Quark content $c\bar{s} \rightarrow s\bar{s}; d\bar{s}; c\bar{u}$



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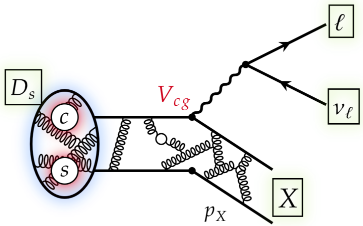
- non-perturbative; calculations from first principles



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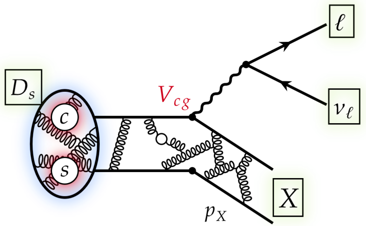
- Quark content $c\bar{s} \to s\bar{s}; d\bar{s}; c\bar{u}$
- non-perturbative; calculations from first principles
- preliminary studies[1]
- experimental results from CLEO-C[2] and BESIII[3]



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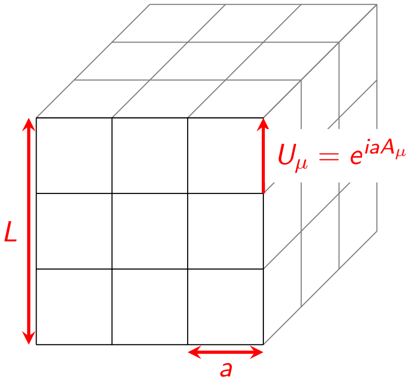
Lattice QCD

- Introduced in the 70s by Wilson[4], Kogut and Susskind[5]
- Non-perturbative
- Calculations from first principles
- Euclidean Time: ‘T=L’
- Fermion fields only defined at discrete points

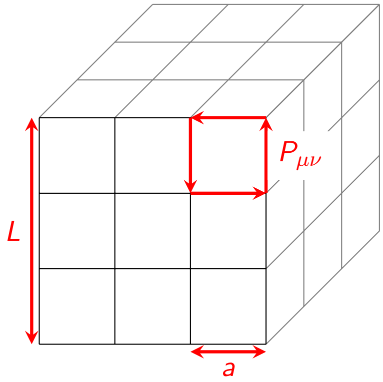


[4] Wilson 1974
[5] Kogut and Susskind 1975

Lattice QCD

- Introduced in the 70s by Wilson[4], Kogut and Susskind[5]
- Non-perturbative
- Calculations from first principles
- keep gauge invariance!
- $S[A] = \frac{1}{2g^2} \int \mathrm{d}^3 r \, \mathrm{Tr} \left[F_{\mu\nu}(\vec{r}) F_{\mu\nu}(\vec{r}) \right]$

  $\approx \beta \sum_n \sum_{\mu > \nu} \mathrm{Re}\,\mathrm{Tr}\big(1 - P_{\mu\nu}(\vec{n})\big)$



[4]Wilson 1974

[5]Kogut and Susskind 1975

Calculation of decay rates

- $\Gamma = G_F^2(|V_{cs}|^2\Gamma_{cs} + |V_{cd}|^2\Gamma_{cd} + \underbrace{|V_{us}|^2\Gamma_{su}}_{suppressed})$
- $\Gamma_{fg} = \int \frac{\mathrm{d}^3 p_\nu}{(2\pi)^3 2E_\nu} \frac{\mathrm{d}^3 p_\ell}{(2\pi)^3 2E_\ell} L_{\mu\nu}(p_\ell, p_\nu) H_{fg}^{\mu\nu}(p, p - p_\ell - p_\nu)\,,$

- change integration variables
- $\Gamma = \int \mathrm{d}e_l \mathrm{d}q_0 \mathrm{d}\boldsymbol{q}^2 \frac{\mathrm{d}\Gamma}{\mathrm{d}e_l \mathrm{d}q_0 \mathrm{d}\boldsymbol{q}^2}$

- lepton contribution: $e_l = \frac{p \cdot p_l}{m_{D_s}^2}$
- $(q_0, \boldsymbol{q}^2) = p - p_\ell - p_\nu$

Leptonic tensor from kinematics: What about hadronic tensor?

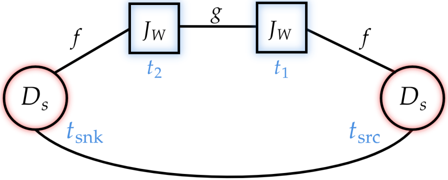
Γ_{fg} from lattice QCD

The hadronic tensor is the **spectral density** of the correlation function:

$$M_{fg}^{\mu\nu}(t, \boldsymbol{q}^2) = \int_0^\infty \mathrm{d}q_0 \, H_{fg}^{\mu\nu}(q_0, \boldsymbol{q}^2) e^{-q_0 t}$$

On the lattice:

$$M_{fg}^{\mu\nu}(t_2 - t_1, \boldsymbol{q}^2) = \lim_{\substack{t_{\mathrm{snk}} \mapsto +\infty \\ t_{\mathrm{src}} \mapsto -\infty}} \frac{C_{4\mathrm{pt}}^{\mu\nu}(t_{\mathrm{snk}}, t_2, t_1, t_{\mathrm{src}}; \boldsymbol{q})}{C_{2\mathrm{pt}}(t_{\mathrm{snk}} - t_2) C_{2\mathrm{pt}}(t_1 - t_{\mathrm{src}})}$$



- $t = t_2 - t_1 = a, 2a, \cdots$ **Euclidean time**
- $t_2 - t_{\mathrm{snk}},\ t_{\mathrm{src}} - t_1 \gg 0$ checked

Γ_{fg} from lattice QCD

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$$M_{fg}^{\mu\nu}(t, \boldsymbol{q}^2) = \int_0^\infty \mathrm{d}q_0 \, H_{fg}^{\mu\nu}(q_0, \boldsymbol{q}^2) e^{-q_0 t}$$

- Finite number of points for M(t): 30 – 50
- M has a statistical error
- H is continuous
- inverse Laplace-transform
- infeasible for several years: we are doing it now!

$$H^{\mu\nu} \underset{\text{very difficult}}{\overset{\text{easy}}{\rightleftarrows}} M^{\mu\nu}$$

Γ_{fg} from lattice QCD

$$24\pi^3 \frac{\mathrm{d}\Gamma_{fg}}{\mathrm{d}\boldsymbol{q}^2} = \sum_{n=0}^{2} |\boldsymbol{q}|^{3-n} \int_{q_0^{\min}}^{q_0^{\max}} \mathrm{d}q_0 (q_0^{\max} - q_0)^n Z_n$$

- Z_0, Z_1, Z_2 can be expressed as linear combinations of $H_{fg}^{\mu\nu}$
- allowed $q_0, \boldsymbol{q}^2$ range depends on flavour combination fg
- σ: smearing parameter
- calculate $\int H(q_0, \boldsymbol{q}^2)\mathrm{d}q_0$ directly with HLT-algorithm[6]
- numerical integration over $\boldsymbol{q}^2$

[6] Hansen, Lupo and Tantalo 2019

Γ_{fg} from lattice QCD

$$24\pi^3 \frac{\mathrm{d}\Gamma_{fg}}{\mathrm{d}\boldsymbol{q}^2} = \lim_{\sigma\mapsto 0} \sum_{n=0}^{2} |\boldsymbol{q}|^{3-n} \int_{q_0^{\min}}^{\infty} \mathrm{d}q_0 (q_0^{\max} - q_0)^n \theta_\sigma(q_0^{\max} - q_0) Z_n$$

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Configurations

name	L [fm]	a [fm]	M_π [MeV]
B48	3.82	0.080	≈ 135
B64	5.10	0.080	≈ 135
B96	7.64	0.080	≈ 135
C80	5.46	0.068	≈ 135
D96	5.46	0.057	≈ 135
E112	5.48	0.049	≈ 135

- ETMC-configurations
- $\mathcal{O}(a)$ and clover improved
- $N_f = 2 + 1 + 1$
- ten momenta per ensemble
- three decay channels
- two smearing kernels
- $\mathcal{O}(10)$ values of σ
- physical pion mass

Configurations

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B48	3.82	0.080	
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B96	7.64	0.080	
C80	5.46	0.068	
D96	5.46	0.057	
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Real world:

- $L \to \infty$
- $a \to 0$
- $\sigma \to 0$

- ETMC-configurations
- $\mathcal{O}(a)$ and clover improved
- $N_f = 2 + 1 + 1$
- ten momenta per ensemble
- three decay channels
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- $\mathcal{O}(10)$ values of σ
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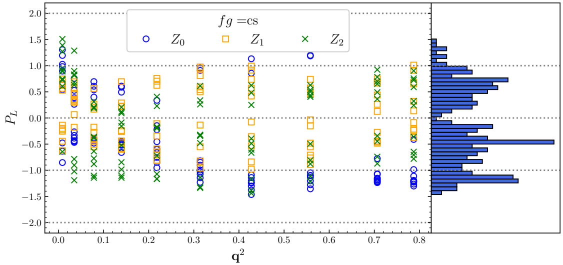
Finite-Volume-Effects

Quantify systematic effects of finite volume:

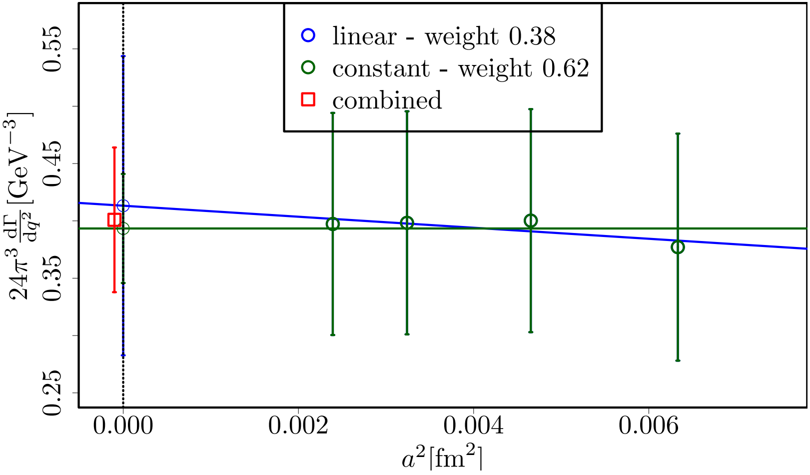
$$P_L(\sigma, q^2) = \frac{x(\sigma, q^2, L) - x\left(\sigma, q^2, \frac{3L}{2}\right)}{\sqrt{\Delta^2_{\text{stat}}(\sigma, q^2, L) + \Delta^2_{\text{stat}}\left(\sigma, q^2, \frac{3L}{2}\right)}}$$



Calculate systematic error:

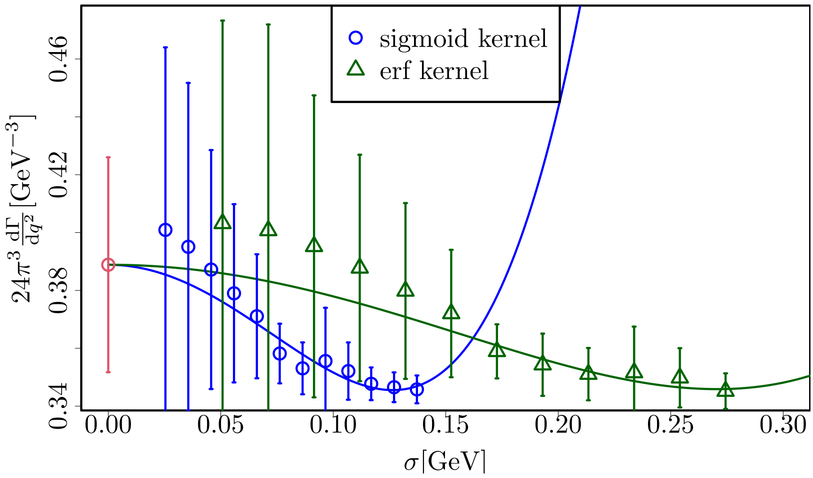
$$\Delta_{\text{sys}}(\sigma, q^2) = \left| x(L) - x\left(\frac{3L}{2}\right)\right| \cdot \text{erf}\left(\frac{\left|P_L(\sigma, q^2)\right|}{\sqrt{2}}\right)$$

Decay rate: Continuum limit



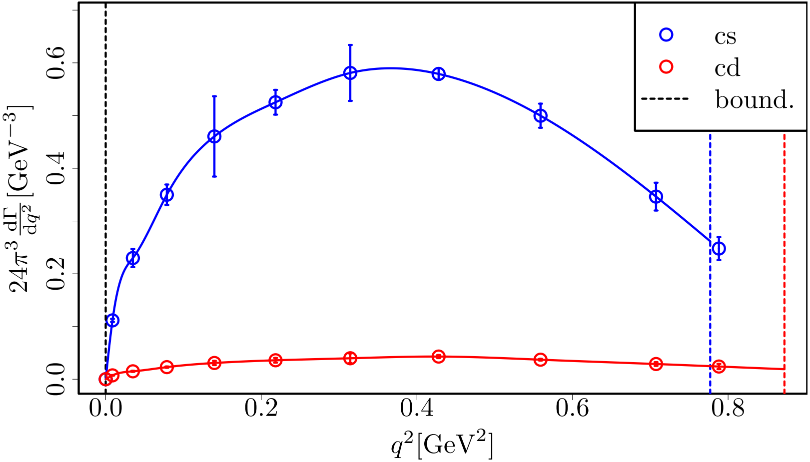
- weight: $\exp\left(-\frac{1}{2}\left(\chi^2 + n_{\text{par}}\right)\right)$
- flat limit
- Z_0
- $q^2 = 0.31\,\text{GeV}^2$
- $\sigma = 0.1\,\text{GeV}$
- $fg = cs$
- sigmoid integration kernel

Decay rate: Smearing limit



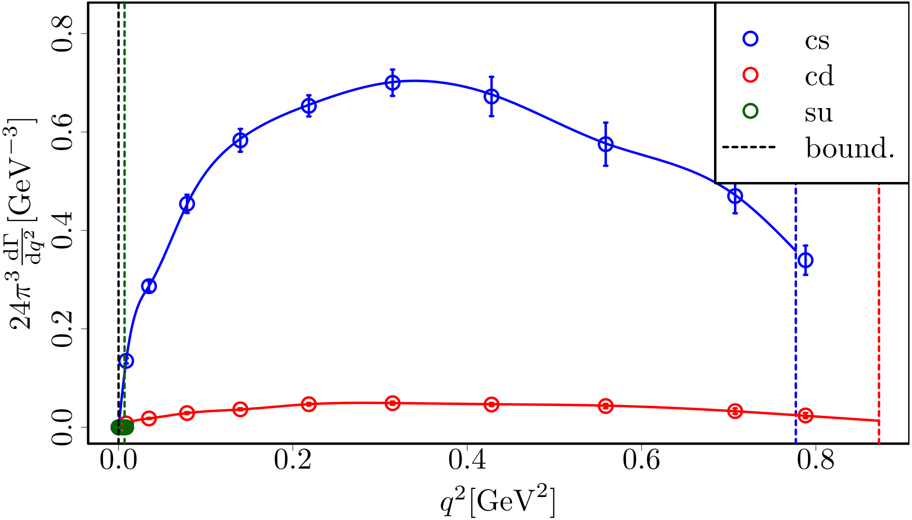
- Z_0
- $q^2 = 0.31\,\mathrm{GeV}^2$
- $fg = cs$
- smooth extrapolations for all contributions
- even powers of σ
- combined fit

Calculation total decay rates



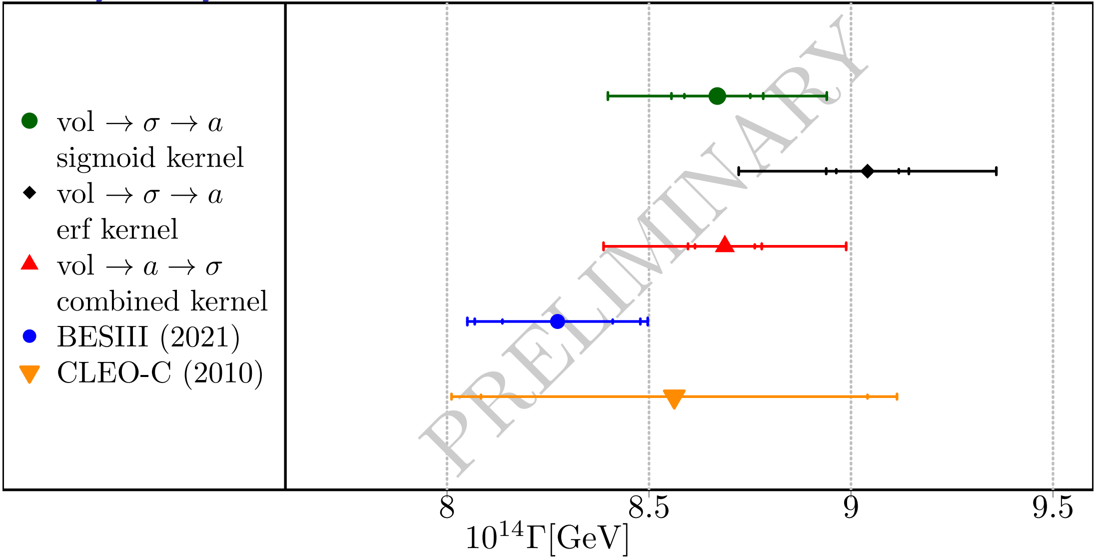
- after all limits
- stat., sys., vol. error
- $Z_0 + Z_1 + Z_2$
- vol $\to a \to \sigma$
- sigmoid kernel
- interpolation with cubic splines
- piecewise integration
- different momenta regions for different decay channels

Contribution $fg = su$



B64, statistical error, $Z_0 + Z_1 + Z_2$ total contribution $su < 10^{-5}$%

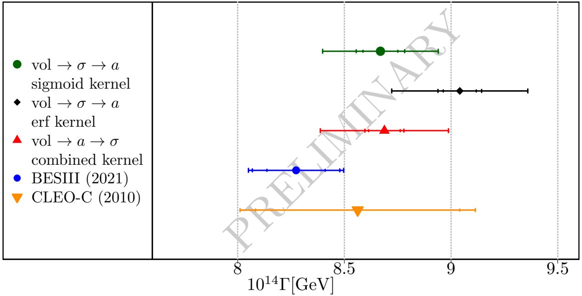
Summary Decay Rate



Summary

Summary

- HLT method well suited
- systematics under control
- good agreement with experimental results
- decay rate and lepton energy moment



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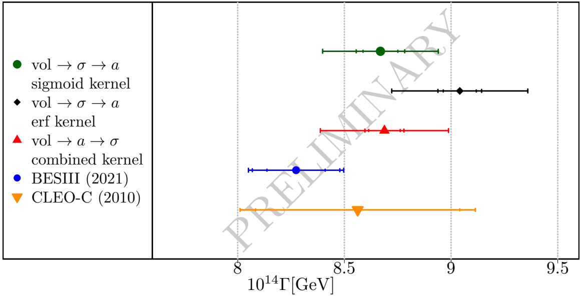
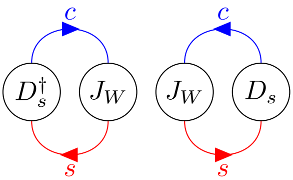


Outlook

- ✓ Quark Mass Dependence
- ✓ Disconnected Diagrams
- ✓ second lepton energy moment
- ✓ Exclusive Contributions
- ! next step: B-decay



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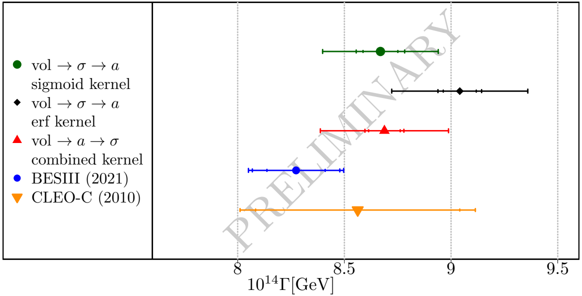
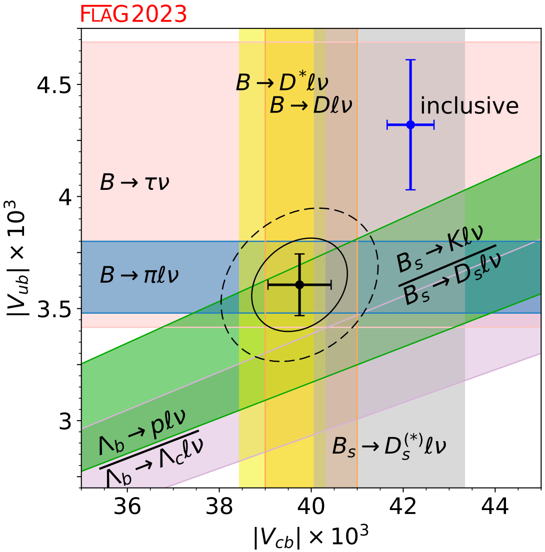
Thank you for your attention!



Relevance of B-decay



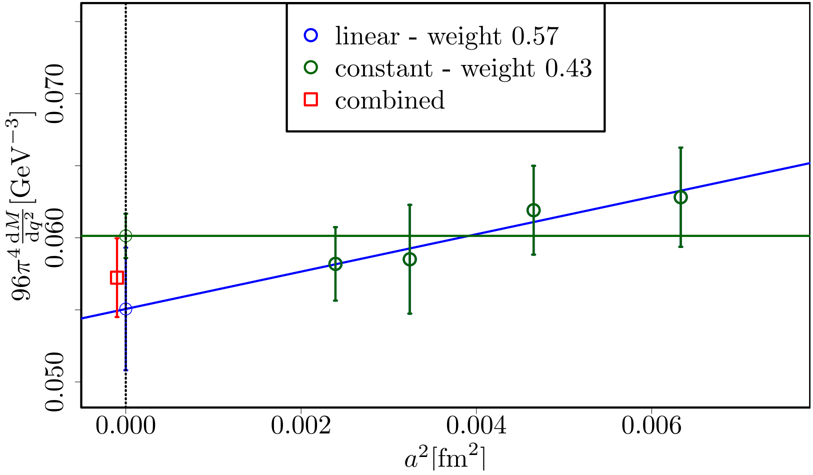
Aoki et al. 2022 and see references there

Lepton energy moment

$$M^{(n)} = \int \mathrm{d}e_l \mathrm{d}q_0 \mathrm{d}\boldsymbol{q}^2 e_l^n \frac{\mathrm{d}\Gamma}{\mathrm{d}e_l \mathrm{d}q_0 \mathrm{d}\boldsymbol{q}^2}$$

Experimental results in Gambino and Kamenik 2010, private communication with P. Gambino
Almost no additional computation time required
Additional moments would be possible

Lepton energy moment: Continuum limit



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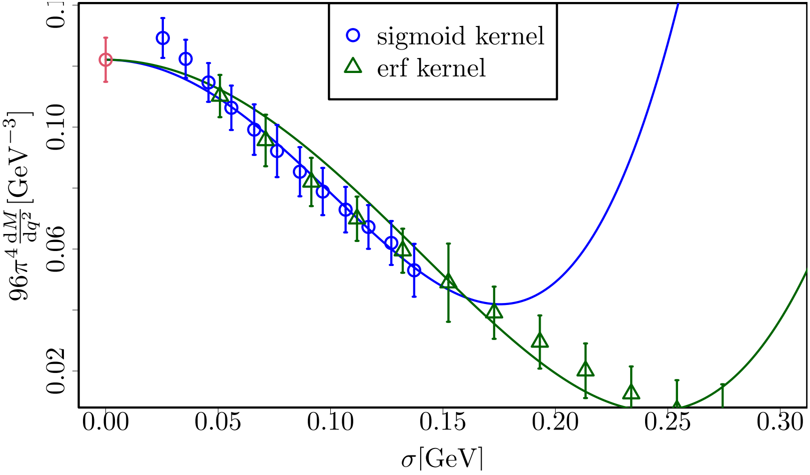
Lepton energy moment: Smearing limit



- Z_1
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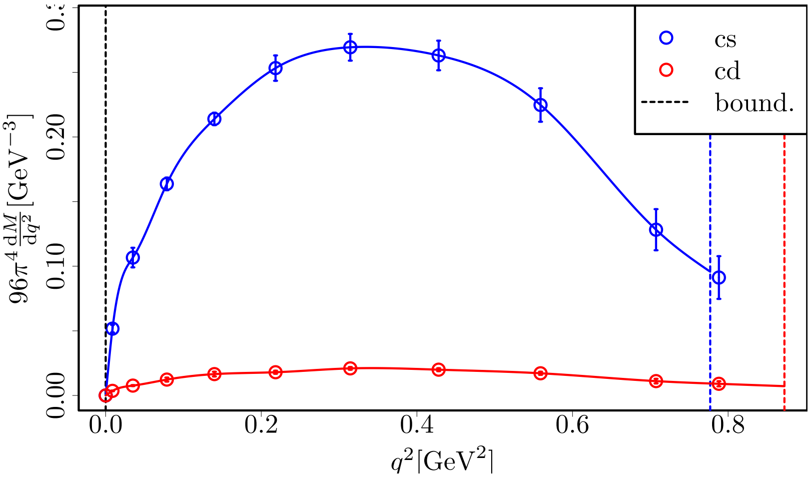
Calculation total lepton energy moment



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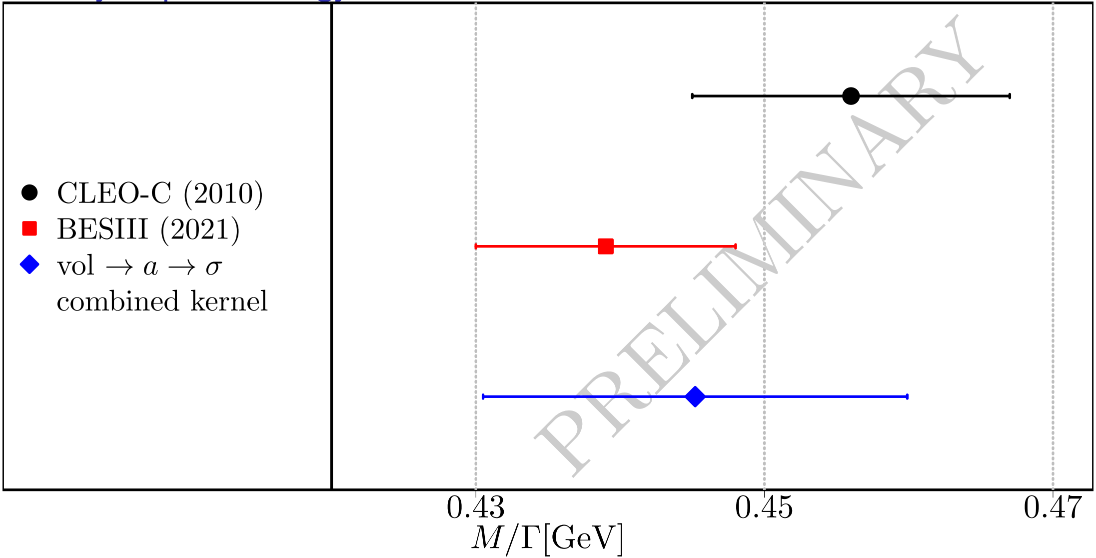


integration boundaries

$$w = \frac{q}{m_D s}$$

$$e_\ell \in \left[\frac{1 - w_0 - |\vec{w}|}{2}, \frac{1 - w_0 + |\vec{w}|}{2}\right]$$

$$w_0 \in \left[\sqrt{r_{gf}^2 + \vec{w}^2}, 1 - \sqrt{\vec{w}^2}\right] \; , \qquad r_{gf} = \frac{m_{gf}}{m_{D_s}}$$

$$\vec{w}^2 \in \left[0, \frac{\left(1 - r_{gf}^2\right)^2}{4}\right]$$

lightest particles:

- $c \to s : \quad \eta_s$
- $c \to d : \quad K$
- $s \to u : \quad D$
- disconnected: π

Definition of Z_n

$$Z_0 \equiv Y_2 + Y_3 - 2Y_4 \qquad Z_1 \equiv 2(Y_3 - 2Y_1 - Y_4) \qquad Z_2 \equiv Y_3 - 2Y_1$$

Form factors decomposition of the hadronic tensor

$$m_{D_s}^3 H^{\mu\nu}(p, p_x) = g^{\mu\nu} m_{D_s}^2 h_1 + p^\mu p^\nu h_2 + (p - p_X)^\mu (p - p_X)^\nu h_3$$
$$+[p^\mu (p - p_X)^\nu + (p - p_X)^\mu p^\nu] h_4 - i\varepsilon^{\mu\nu\alpha\beta} p_\alpha (p - p_X)_\beta h_5$$

$$Y_1 = -m_{D_s} \sum_{ij} \hat{n}^i \hat{n}^j H^{ij} = h_1$$

$$Y_2 = m_{D_s} H^{00} = h_1 + h_2 + \left(1 - \frac{q_0}{m_{D_s}}\right)^2 h_3 + 2\left(1 - \frac{q_0}{m_{D_s}}\right) h_4$$

$$Y_3 = m_{D_s} \sum_{ij} \hat{q}^i \hat{q}^j H^{ij} = -h_1 m_{D_s}^2 + |\boldsymbol{q}|^2 h_3$$

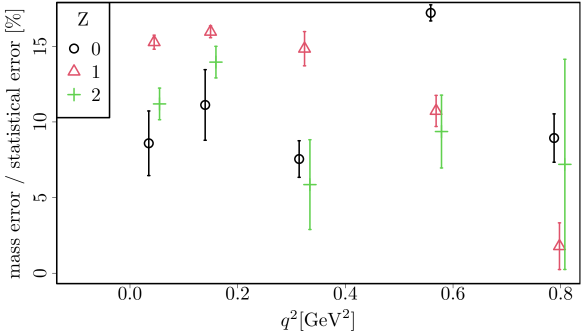
$$Y_4 = -m_{D_s} \sum_i \hat{q}^i H^{0i} = \left(1 - \frac{q_0}{m_{D_s}}\right) |\boldsymbol{q}| h_3 + |\boldsymbol{q}| h_4$$

$$Y_5 = \frac{i m_{D_s}}{2} \sum_{ijk} \varepsilon^{ijk} \hat{q}^k H^{ij} = |\boldsymbol{q}| h_5$$

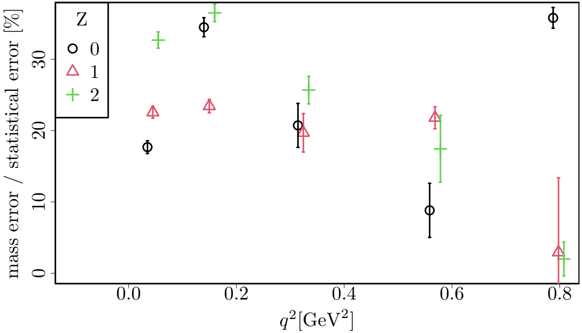
$$\hat{n}^2 = 1$$
$$\hat{n} \cdot \boldsymbol{q} = 0$$
$$\hat{\boldsymbol{q}} = \boldsymbol{q}/|\boldsymbol{q}|$$

Contribution of different strange and charm quark mass

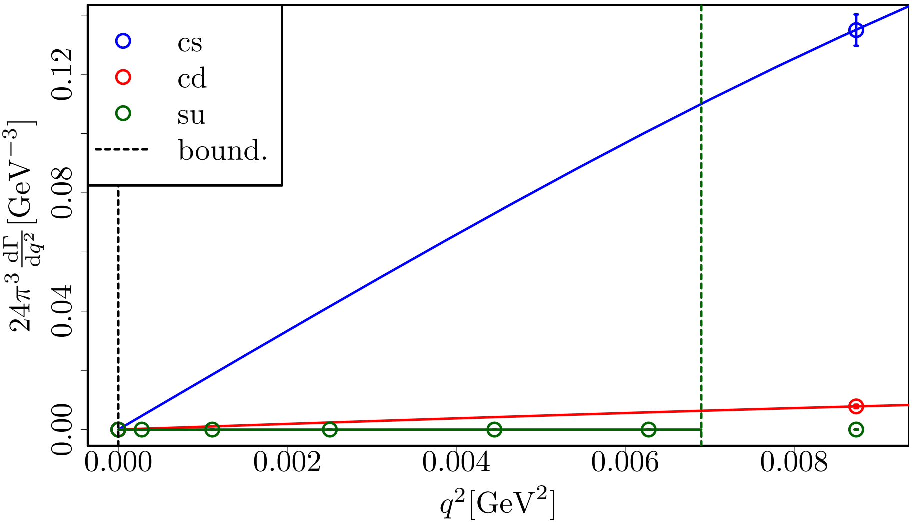
cd channel



cs channel



Contribution $fg = su$



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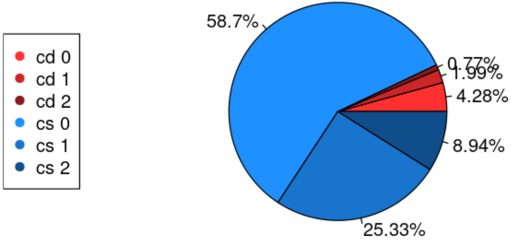
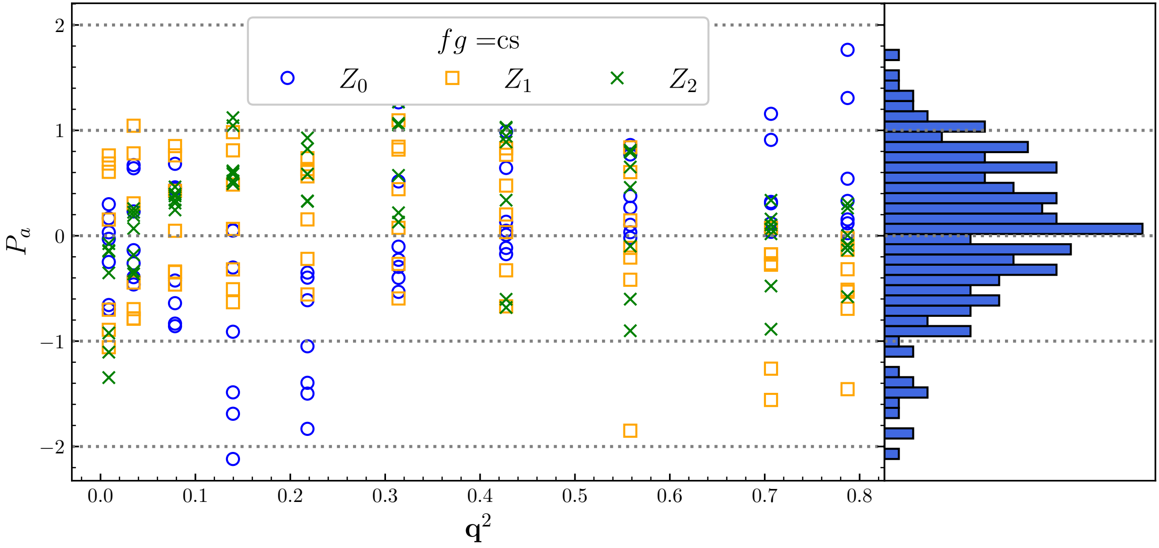


cs	cd	su
93%	7%	$< 10^{-5}\%$

systematics from Continuum Limit



Explanation HLT

Some slides explaining the HLT method by Alessandro De Santis

In general we want to extract $\rho_\sigma = \int \mathrm{d}\omega\, K_\sigma(\omega)\rho(\omega)$ from $C(t) = \int_0^\infty \mathrm{d}\omega\, e^{-\omega t}\rho(\omega)$

- A linear estimator for the solution can be written by **approximating the target smearing (Schwartz) kernel**

$$\rho_\sigma = \sum_{\tau=1}^{T} g_\tau C(a\tau)$$

$$K_{\sigma,T}^{\mathrm{approx}} = \sum_{\tau=1}^{T} g_\tau(T) e^{-a\omega\tau}$$

- The estimator is **model independent and unbiased** in the limits $T \mapsto \infty$ and vanishing statistical errors

$$\lim_{T\mapsto\infty} K_{\sigma,T}^{\mathrm{approx}} = K_\sigma$$

For $T < \infty$ one needs to estimate the residual **systematic** uncertainty due to the kernel approximation in addition to **statistical** error

- The coefficients $\boldsymbol{g}$ are calculated by minimizing

$$W[\lambda, \boldsymbol{g}] = (1-\lambda)\frac{A[\boldsymbol{g}]}{A[0]} + \lambda B[\boldsymbol{g}]$$

- Suppression of the **statistical error**

$$B[\boldsymbol{g}] = \boldsymbol{g}^T \cdot \hat{\mathrm{COV}}\big[C(t)\big] \cdot \boldsymbol{g} \equiv \left(\delta\rho\right)^2$$

- **Accuracy of the approximated** kernel

$$A[\boldsymbol{g}] = \int_{E_0}^{\infty} \mathrm{d}\omega \left\{ \sum_{\tau=1}^{T} g_\tau e^{-a\omega\tau} - K_\sigma^{\mathrm{target}} \right\}^2 \qquad E_0 \sim 0.9 \cdot q_0^{\mathrm{min}}$$

Bibliography I

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