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Inflationary and post-inflationary scalar dark matter production

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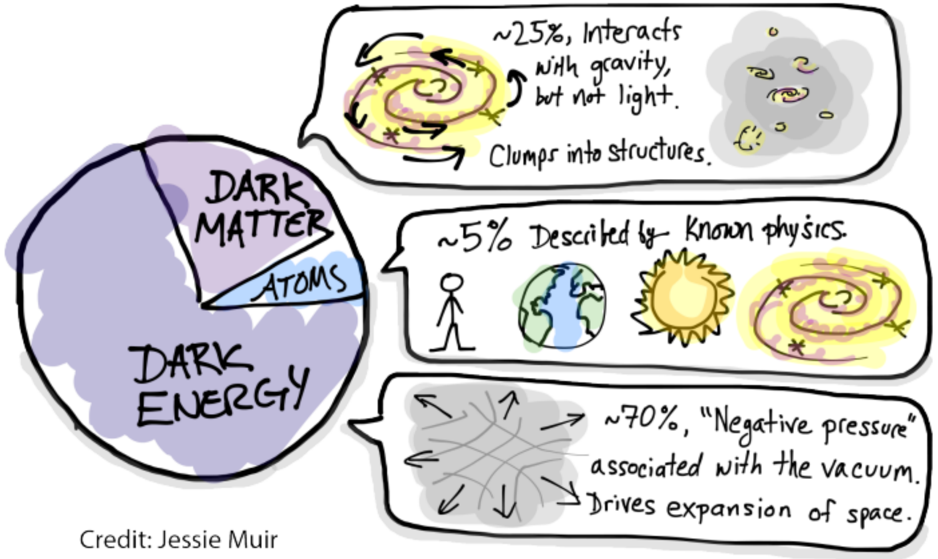
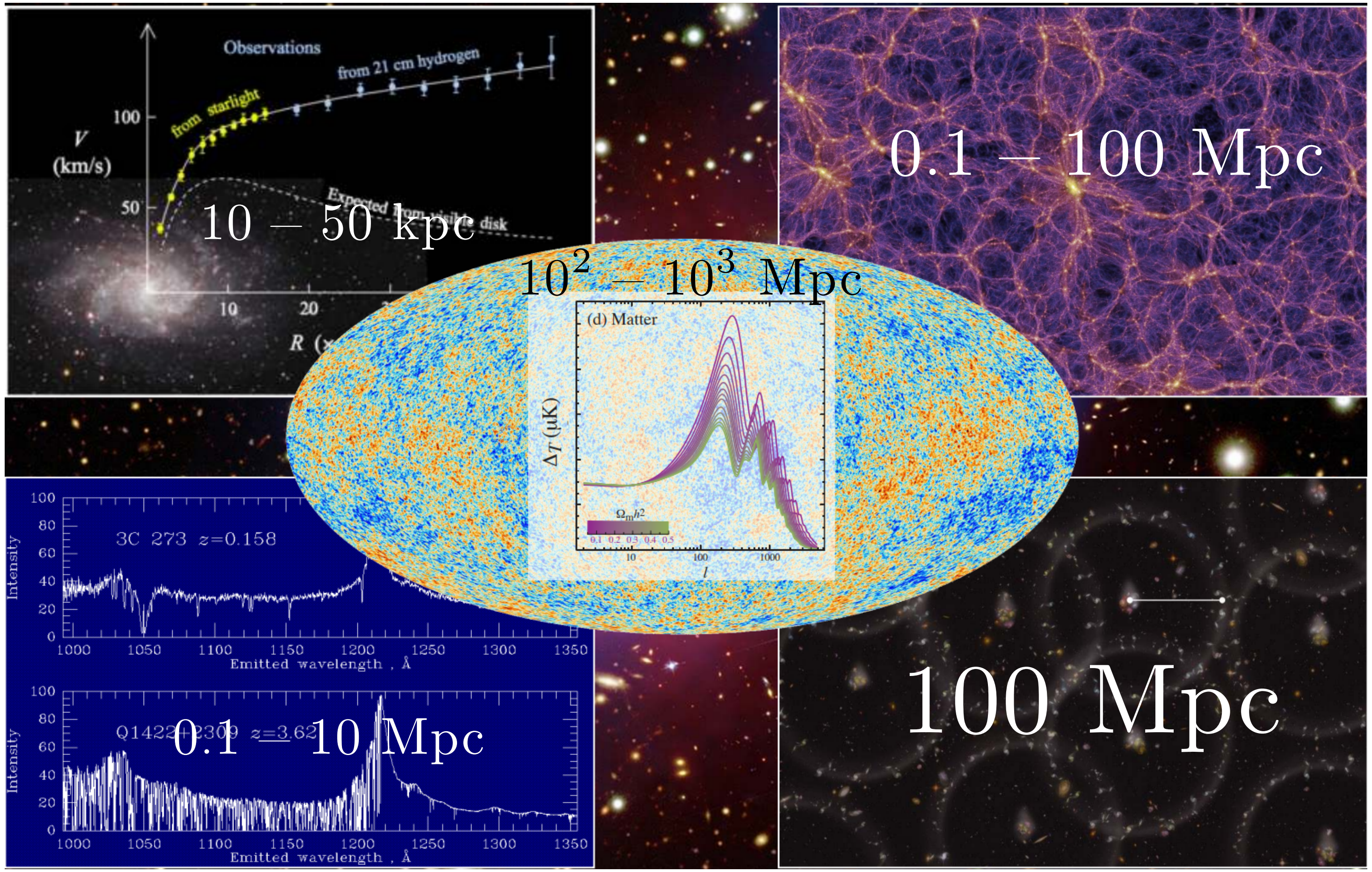
High Energy Theory Seminar - Bonn University

Based (mostly) on
[arXiv: 2206.08940 – 2303.07359 – 2305.14446]
with **M. A. G. Garcia** & **S. Verner**
+ many co-authors

Introduction



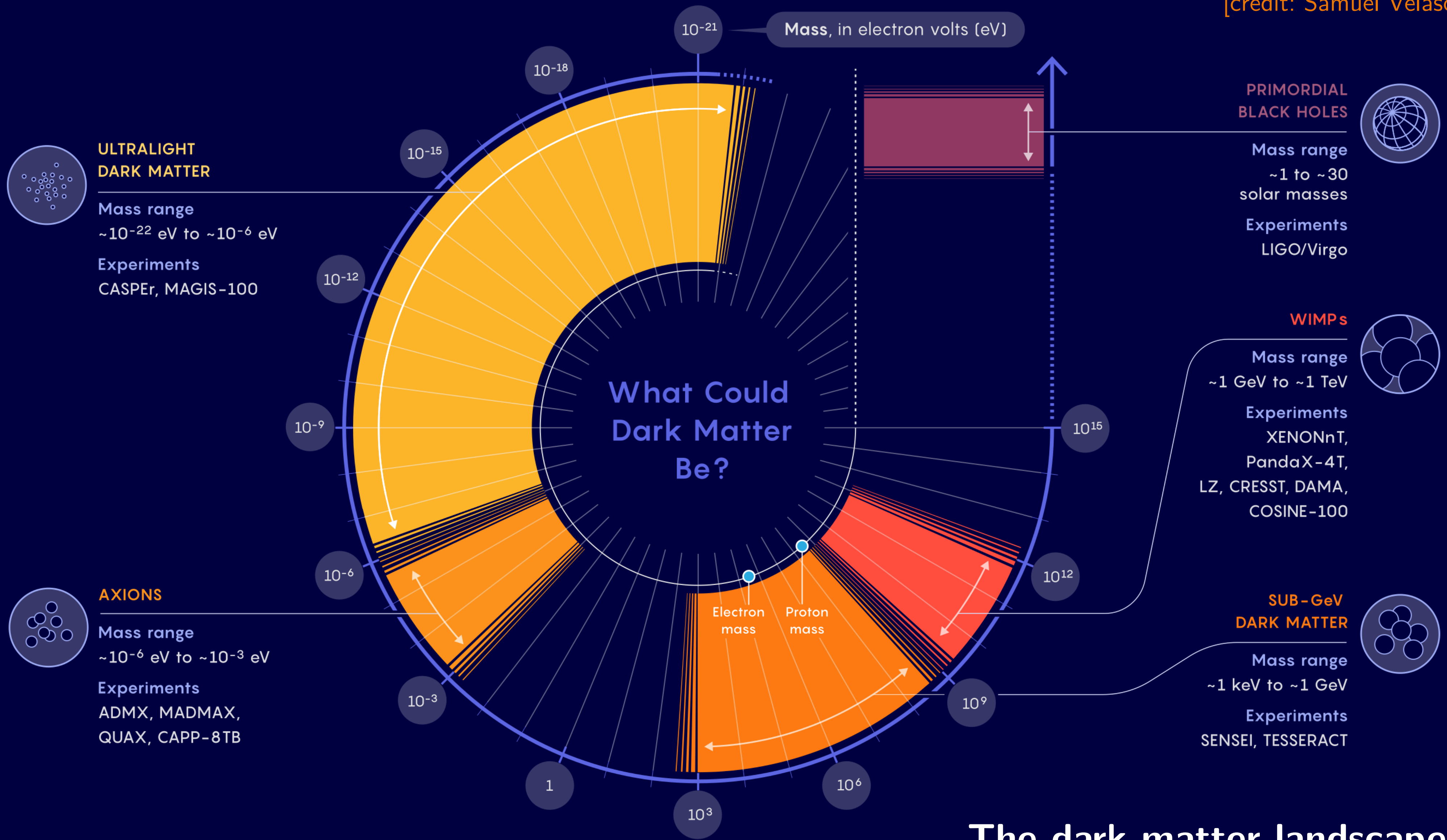
Credit: Jessie Muir



"A 40 σ detection of non-baryonic dark matter" Carlos S. Frenk, *Paris-Saclay astroparticle symposium 2023*

Introduction

[credit: Samuel Velasco/Quanta Magazine]

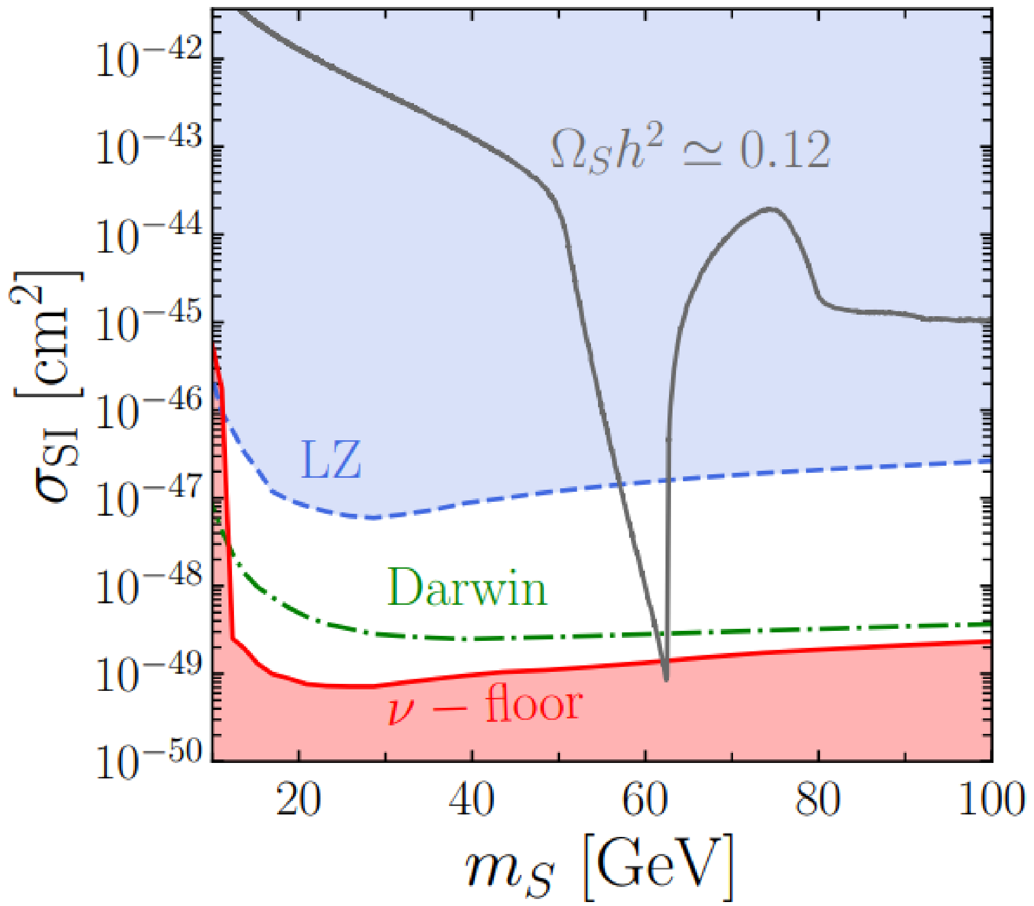


The dark matter landscape, circa 2024

The waning of the WIMP?

- **W**eakly **I**nteracting **M**assive **P**articles (**WIMP**) **most considered** DM candidates
  Simplest example: introduce **scalar dark matter**

$$\mathcal{L} \supset S^2 |H|^2$$



[T. Biekotter & **MP** - EPJC 82 (2022) 11, 1026]

- **Sizable coupling** with SM $\Longrightarrow$ **freeze-out**

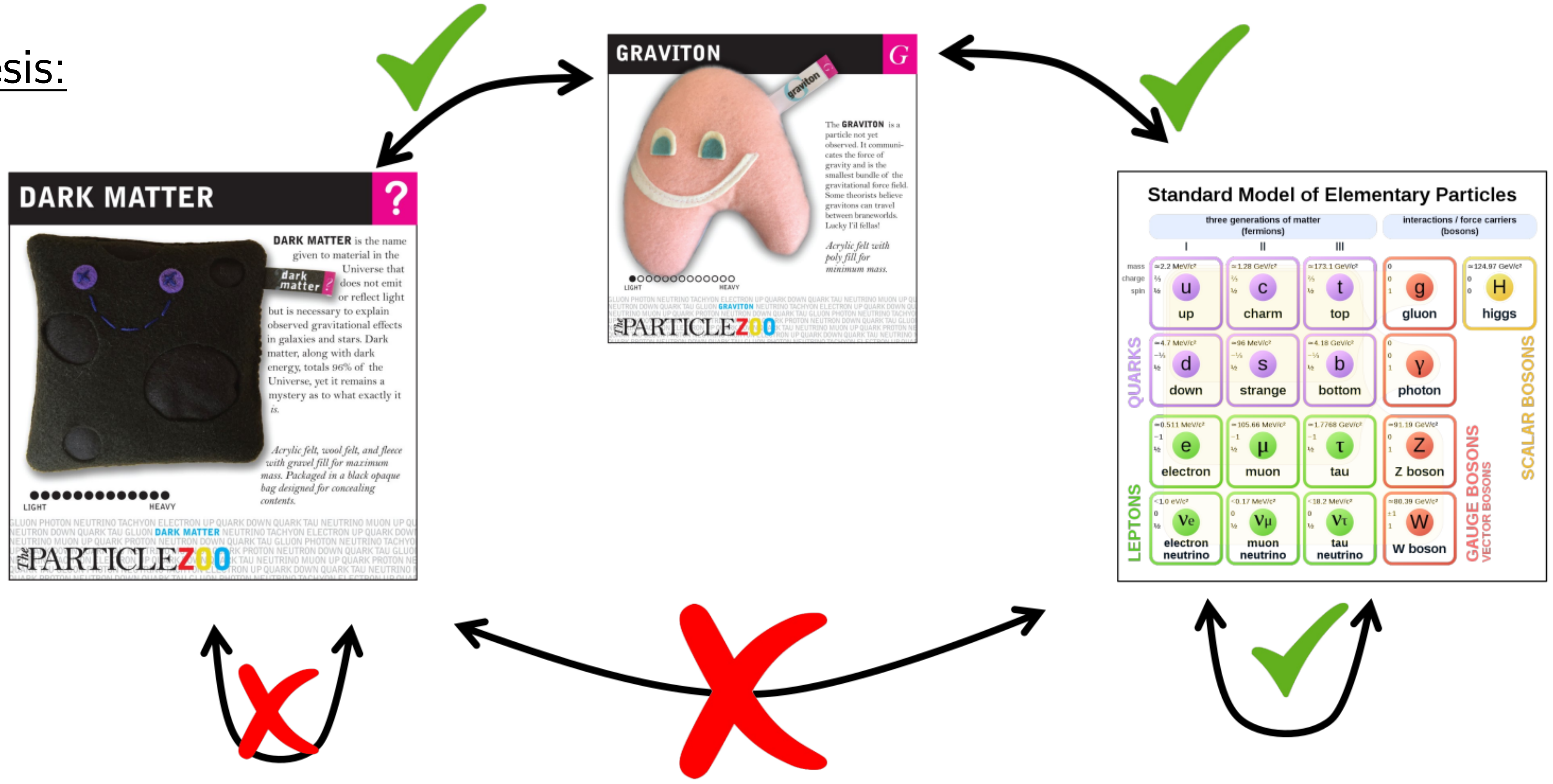
- **Minimalistic** WIMP models **under siege!**

[M. Escudero, A. Berlin, D. Hooper, M.-X. Lin - JCAP 12 (2016) 029]
[G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, **MP**, S. Profumo, F. S. Queiroz EPJC 78 (2018) 203]
[G. Arcadi, A. Djouadi, M. Raidal - Phys.Rept. 842 (2020) 1-180]
[T. Biekotter & **MP** - EPJC 82 (2022) 11, 1026]

- **Interactions** with SM might not be **responsible** for DM **production**

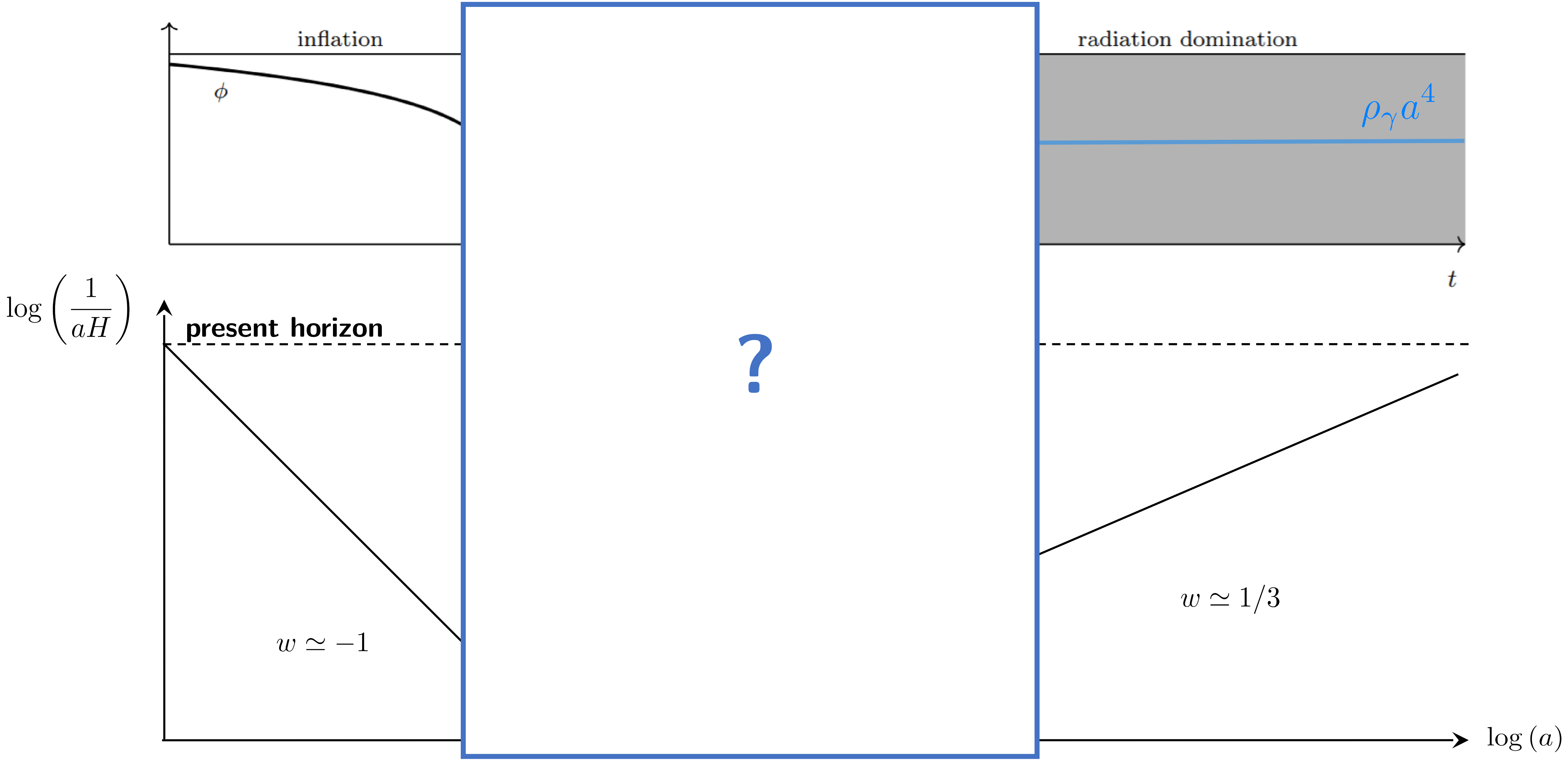
Introduction

Hypothesis:



How can such dark matter candidates be produced?

Introduction



Inflation in a nutshell

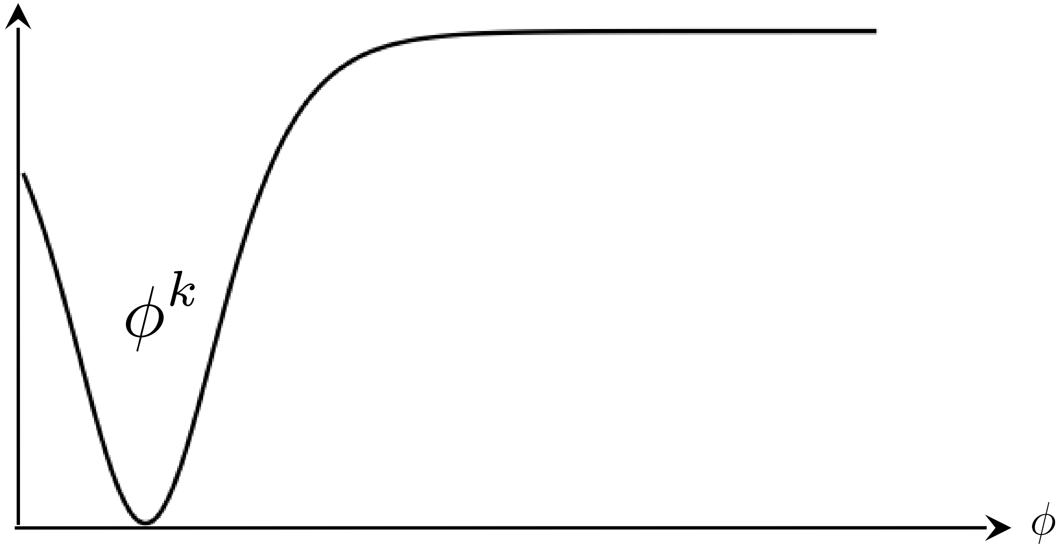
For concreteness, **consider**

$$\mathcal{S} \;=\; \int \mathrm{d}^4x\,\sqrt{-g}\left(\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right)$$

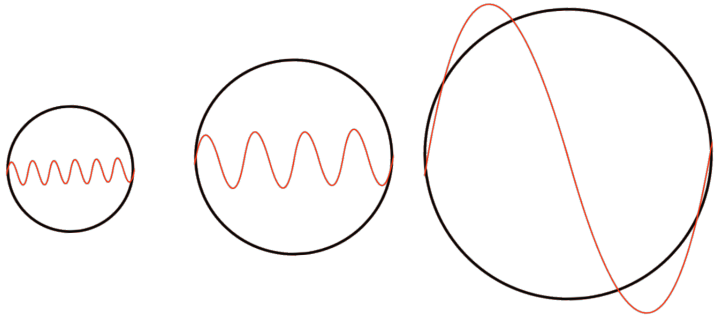
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$

$$V(\phi) \;=\; \lambda M_P^4\left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k$$

[Kallosh & Linde arXiv:1306.5220]



$$\lambda \;\simeq\; \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \;\sim\; 10^{-11}$$



$$\mathcal{P}_\mathcal{R} = \frac{H_*^4}{4\pi^2\dot{\phi}_*^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$$r \equiv \frac{\mathcal{P}_\mathcal{T}(k=k_*)}{\mathcal{P}_\mathcal{R}(k=k_*)}$$

$$\mathcal{P}_\mathcal{T} = \frac{2H_*^2}{\pi^2}\left(\frac{k}{aH}\right)^{n_t}$$

ϕ : inflaton

Inflation in a nutshell

For concreteness, **consider**

[J. Ellis, M. A. G. Garcia, D. V. Nanopoulos, K. A. Olive & S. Verner - arXiv:2112.04466]

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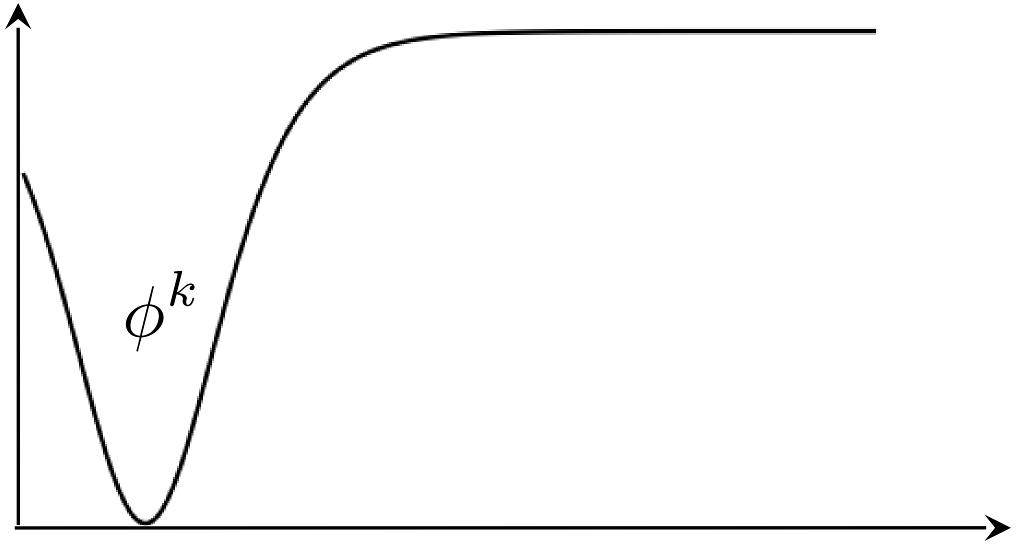
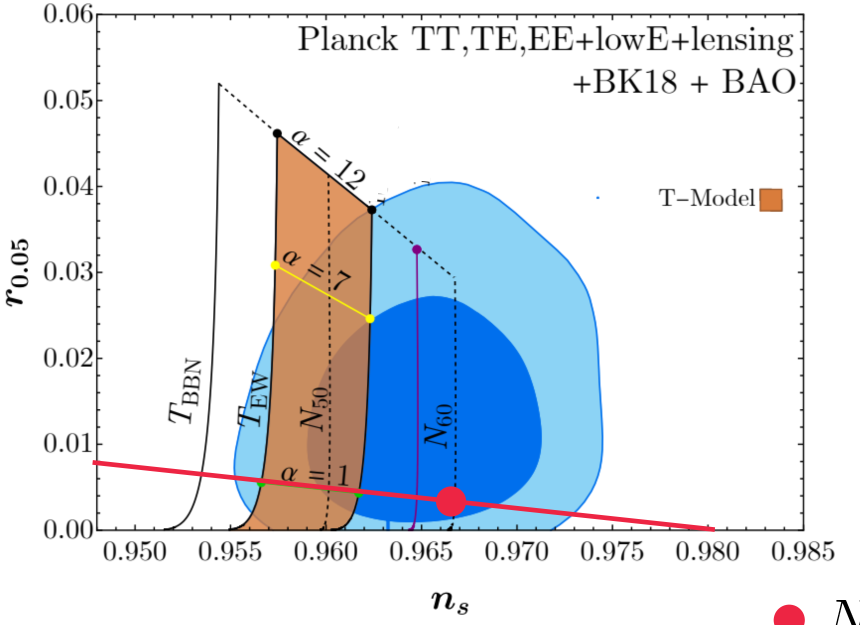


$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$

$k_* = 0.05\,\mathrm{Mpc}^{-1}$



● $N_\star = 60$

$$n_s \simeq 1 - 6\epsilon_\star + 2\eta_\star \simeq 1 - \frac{2}{N_\star}$$

$$r \simeq 16\epsilon_\star \simeq \frac{12}{N_\star^2}$$

$$A_s(k_\star) \simeq 2.1 \times 10^{-9}$$ [Planck 18']

ϕ : inflaton

Inflation in a nutshell

For concreteness, **consider**

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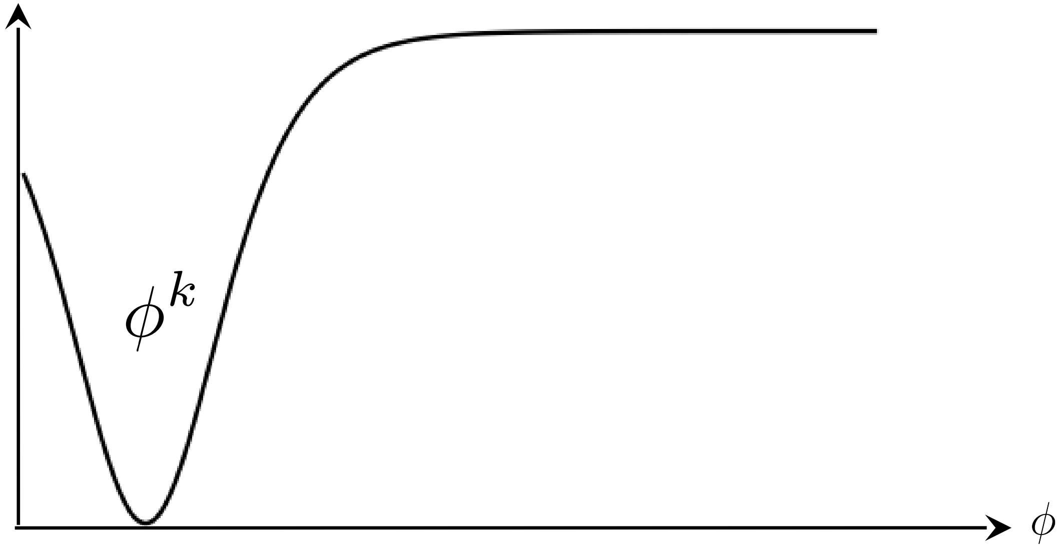
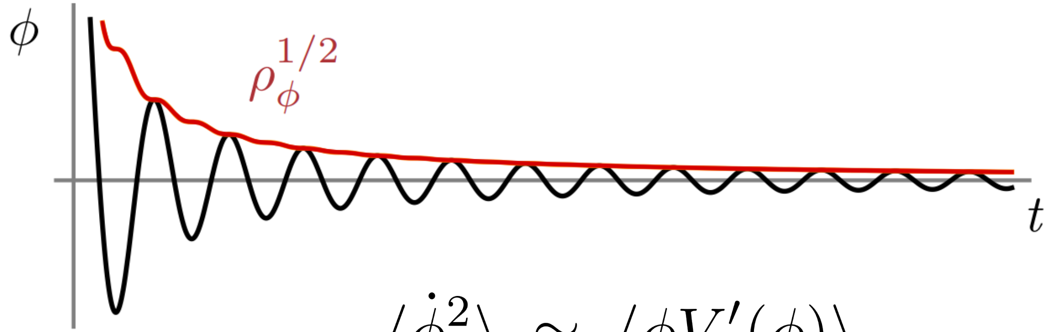


$$\lambda \simeq \frac{18\pi^2 A_s(k_\star)}{6N_*^2} \sim 10^{-11}$$

Close to the minimum $(k=2)$

$$V(\phi) \simeq \frac{1}{2}m_\phi^2\phi^2 = \lambda\phi^2 M_P^2 \quad (\phi \ll M_P)$$



$$\langle\dot{\phi}^2\rangle \simeq \langle\phi V'(\phi)\rangle$$

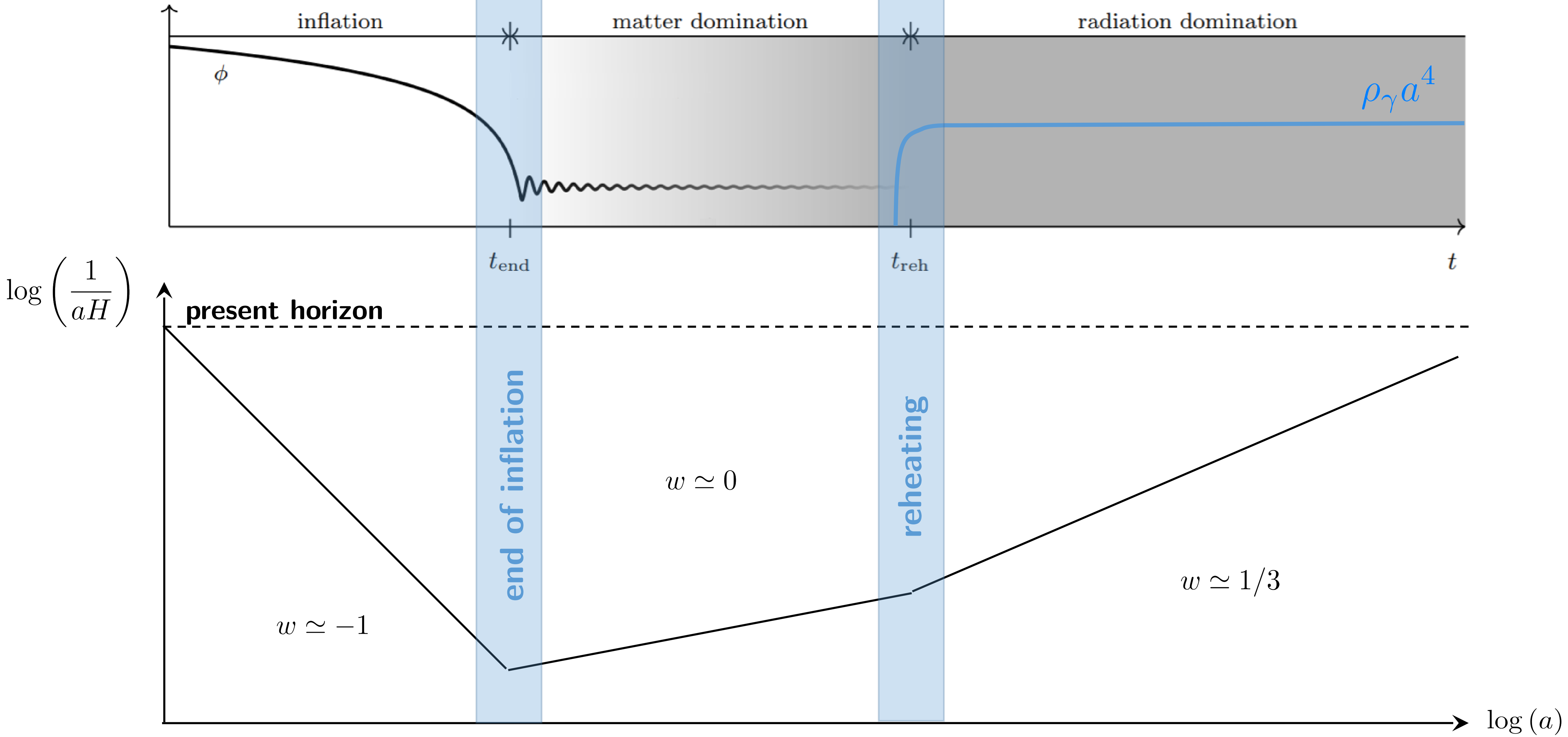
➡ $\phi(t) \simeq \phi_0(t)\cos(m_\phi t) \qquad \phi_0(t) \sim a(t)^{-3/2}$

$$\langle P_\phi\rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle \simeq 0$$

$$\langle\rho_\phi\rangle = \frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle \simeq V(\phi_0)$$

$$\langle w_\phi\rangle \simeq 0$$

Inflation and post-inflationary dynamics



Scalar production during and after inflation

Minimal scalar dark matter

non-minimal coupling — [arXiv:2303.07359] - M. A. G. Garcia, **MP** & S. Verner

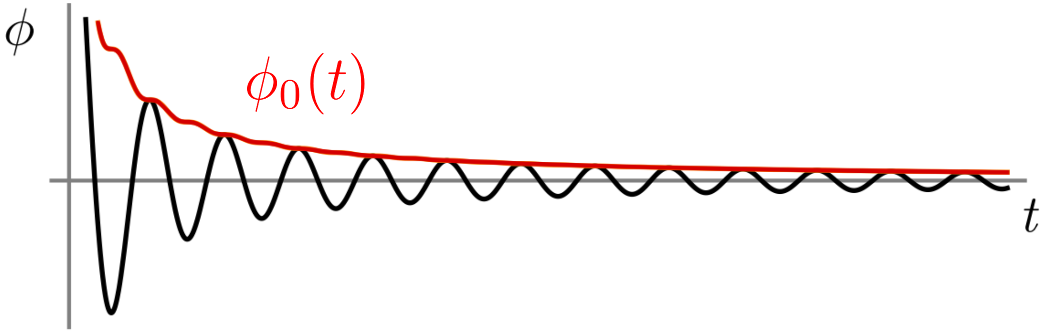
ϕ-coupling — [arXiv:2206.08940] M. A. G. Garcia **MP** & S. Verner

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(M_P^2 - \xi\chi^2)R + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}\sigma\phi^2\chi^2\right.$$

$$\left. + \frac{1}{2}(\partial_\mu\phi)^2 - 6\lambda M_P^4\tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right) - y\phi\bar{\psi}\psi + \mathcal{L}_{\rm SM}\right]$$

T-model inflation

reheating



$$T_{\rm reh} \simeq \left(\frac{9\lambda}{20\pi^4 g_{\rm reh}}\right)^{1/4} y M_P$$

Effective mass $m_{\chi,\rm eff}^2(t) = m_\chi^2 + \sigma\phi^2(t)$

→ **Cosmological gravitational particle production and its implications for cosmological relics**

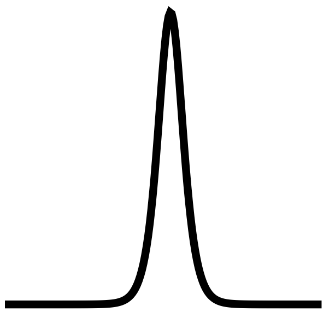
Edward W. Kolb[1,*] and Andrew J. Long[2,†]

[arXiv:2312.09042]

Particle production: perturbative approach

- **Phase space distribution** from $$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial|\boldsymbol{P}|} = \mathcal{C}[f_\chi(|\mathbf{P}|,t)]$$

$$\mathcal{C}[f_\chi(|\mathbf{P}|,t)] = \frac{1}{P^0}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 n_\phi}\frac{\mathrm{d}^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(k-P-P')|\overline{\mathcal{M}}|^2_{\phi\phi\to\chi\chi}$$
$$\times\left[f_\phi(k)(1+f_\chi(P))(1+f_\chi(P')) - f_\chi(P)f_\chi(P')(1+f_\phi(k))\right]$$



$$f_\phi(\boldsymbol{k},t) = (2\pi)^3 n_\phi(t)\delta^{(3)}(\boldsymbol{k})$$

- **Collision term** given by:

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial|\boldsymbol{P}|} = \frac{\pi^2}{\beta^2 m_\phi^3}\rho_\phi\Gamma_{\phi\phi\to\chi\chi}\delta\left(|\boldsymbol{P}| - m_\phi\beta(t)\right)(1+2f_\chi(|\boldsymbol{P}|))$$ Bose enhancement

$$\beta(t) \equiv \sqrt{1-\frac{m^2_{\text{eff}}(t)}{m_\phi^2}}$$: kinematic blocking

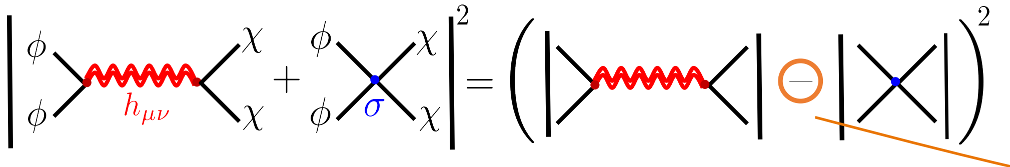
[M. A. G. Garcia, K. Kaneta, K. Olive, Y. Mambrini, S. Verner - arXiv: 2109.13280]

Particle production: perturbative approach

- Treat **inflaton** as **coherently oscillating condensate**

$$\phi(t) \simeq \phi_0(t) \sum_{n=-\infty}^{\infty} \mathcal{P}_n e^{-in\omega_\phi t} \qquad E_n = n\omega_\phi \quad \Rightarrow \quad \Gamma_{\phi\phi\to\chi\chi} = \frac{1}{8\pi(1+w_\phi)\rho_\phi} \sum_{n=1}^{\infty} E_n \beta_n |\mathcal{M}_n|^2$$

$$|\mathcal{M}|^2_{\phi\phi\to\chi\chi} =$$



$$\Rightarrow \quad \Gamma_{\phi\phi\to\chi\chi} = \frac{1}{32\pi}\frac{\rho_\phi^2(t)}{m_\phi^3}\left[\sigma - \lambda\left(1+\frac{m_\chi^2}{2m_\phi^2}\right)\right]^2 \beta_2$$

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2\frac{h_{\mu\nu}}{M_P}$$

Interferences!

- **Approximate solution for** $\beta \simeq 1,\ t_{\rm end} < t < t_{\rm reh}$

[arXiv:2206.08940] M. A. G. Garcia, **MP** & S. Verner

$$f_\chi(q,t) \sim q^{-9/2}\theta(q-1)\theta\left(\frac{a}{a_{\rm end}} - q\right)$$

$$q \equiv \frac{P}{T_\star}\left(\frac{a}{a_0}\right)$$

$$T_\star \equiv m_\phi\left(\frac{a_{\rm end}}{a_0}\right)$$

- Equivalent to treat **inflaton** as **collection of particles (for quadratic potential)**

[arXiv:2206.08940] M. A. G. Garcia, **MP** & S. Verner

- $$n_\chi\left(\frac{a}{a_{\rm end}}\right)^3 = \frac{m_\phi^3}{2\pi^2}\int {\rm d}q\, q^2 f_\chi(q,t)$$ **: time independent** when DM production **stops**

Scalar production: the field picture

- **Treat dark matter as quantum field in curved space-time**

Equation of motion
$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \frac{\nabla^2}{a^2} + 3H\frac{\mathrm{d}}{\mathrm{d}t} + m_\chi^2 + \sigma\phi^2 - \xi R\right)\chi = 0$$

- **Quantize** the (rescaled) field
$$X(\tau, \boldsymbol{x}) \equiv a\chi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^{3/2}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left[X_p(\tau)\hat{a}_{\boldsymbol{p}} + X_p^*(\tau)\hat{a}^\dagger_{-\boldsymbol{p}}\right]$$

- **Harmonic oscillator** with **time-dependent** frequency

$$X_p'' + \omega_p^2 X_p = 0$$

$$\omega_p^2(t) = p^2 + a^2(t)\hat{m}^2_{\text{eff}}(t) \qquad \hat{m}^2_{\text{eff}} = m_\chi^2 + \sigma\phi^2 + \frac{1}{6}(1-6\xi)R$$

Gravity!

$\xi = 1/6$: **Conformal** coupling

$$' \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}$$
$$\mathrm{d}t = a\,\mathrm{d}\tau$$

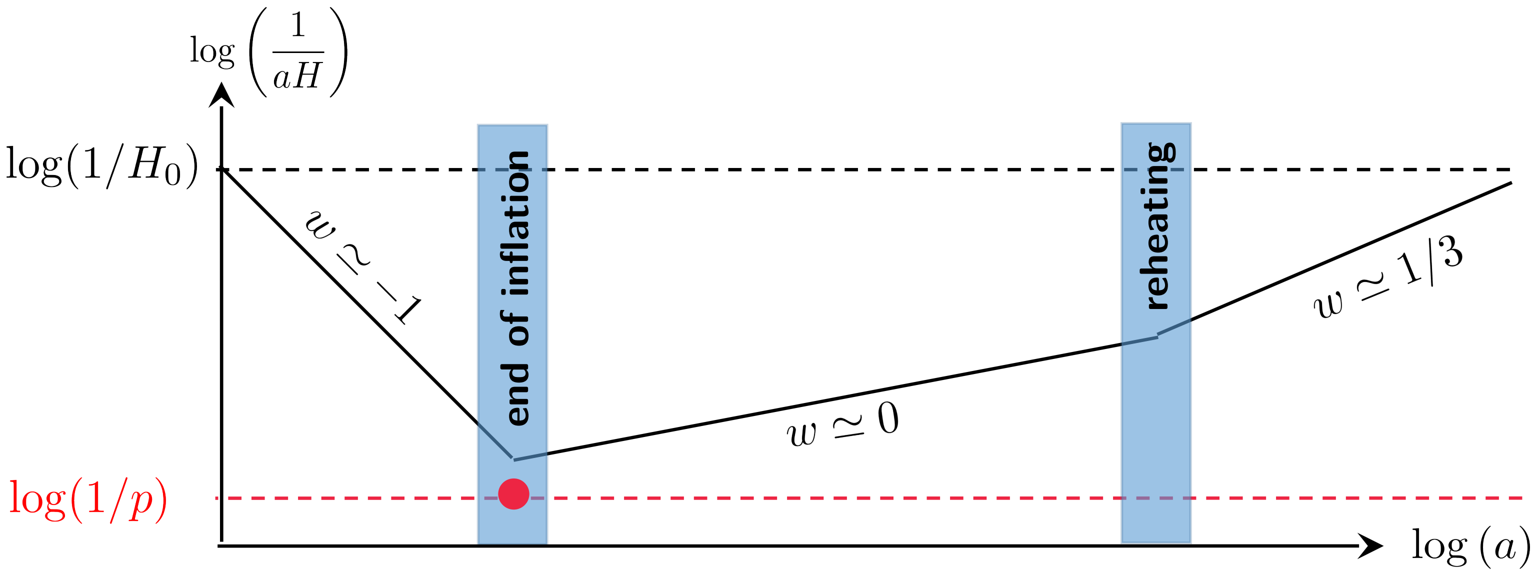
[L. Kofman, A. Linde, A. Starobinsky - arXiv:9704452 – arXiv:9405187]

Scalar production: the field picture

- **Initial conditions: Bunch-Davies** vacuum $$X_p(\tau_0) = \frac{1}{\sqrt{2\omega_p}} \qquad X'_p(\tau_0) = -\frac{i\omega_p}{\sqrt{2\omega_p}}$$

- **Light scalar fields unstable during inflation!** $$\omega_p^2 = p^2 + 2(aH)^2\left[\frac{m_\chi^2}{2H^2} + \frac{\sigma\phi^2}{2H^2} + 6\xi - 1\right]$$

- **For** small physical scales: **modes** always **inside** horizon $p/(aH) \gg 1$ $\quad \omega_p^2 > 0 \quad$ ● $\tau_0 = \tau_{\rm end}$



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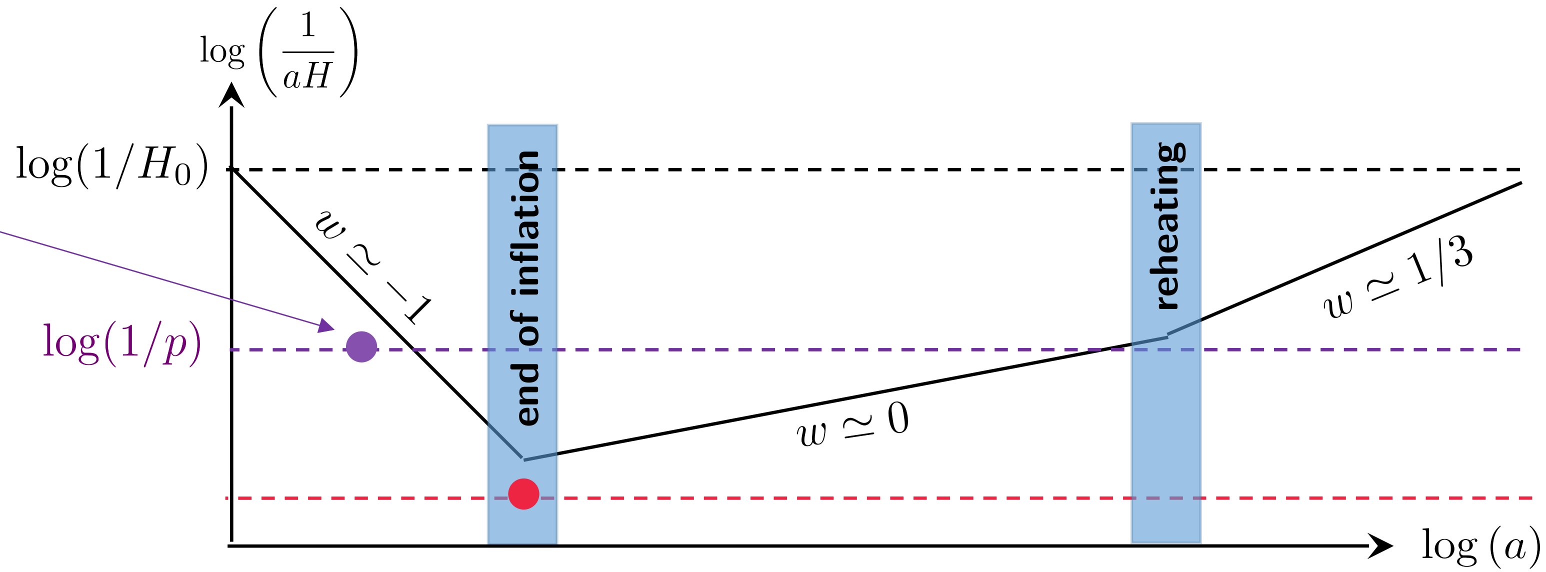
- **For** small physical scales: **modes** always **inside** horizon $p/(aH) \gg 1 \qquad \omega_p^2 > 0 \qquad$ ● $\tau_0 = \tau_{\text{end}}$

- **For** $m_\chi^2 < 2H^2, \sigma/\lambda \ll 1, \xi < 1/6$ $\qquad$ superhorizon **modes** experience $\omega_p^2(t_{\text{end}}) < 0$

● $\tau_0 < \tau_{\text{end}}$

$p^2 \gg a(\tau_0)^2 H(\tau_0)^2$

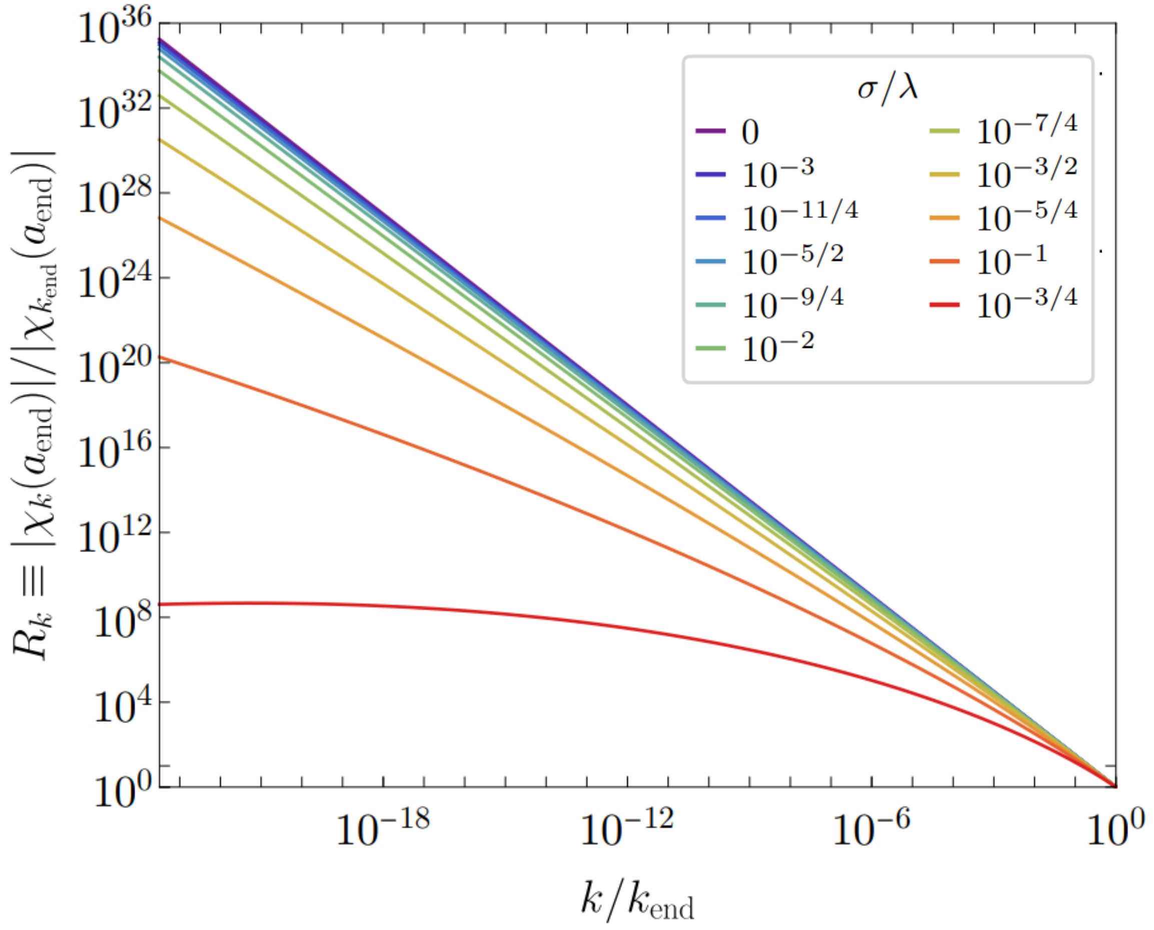
➡ **Particle production!**



Scalar production: the field picture

Solution to EOM during inflation

$$\chi_k(k \ll aH) \simeq \frac{1}{2\sqrt{\pi}a^{3/2}\sqrt{H}} e^{i\frac{\pi}{2}(\nu - \frac{1}{2})} \left(e^{-i\pi\nu}\Gamma(-\nu)\left(\frac{k}{2aH}\right)^{\nu} + \Gamma(\nu)\left(\frac{k}{2aH}\right)^{-\nu} \right)$$



with $\nu \equiv \sqrt{\frac{9}{4} - 12\xi - \frac{m_\chi^2 + \sigma\phi^2}{H^2}}$

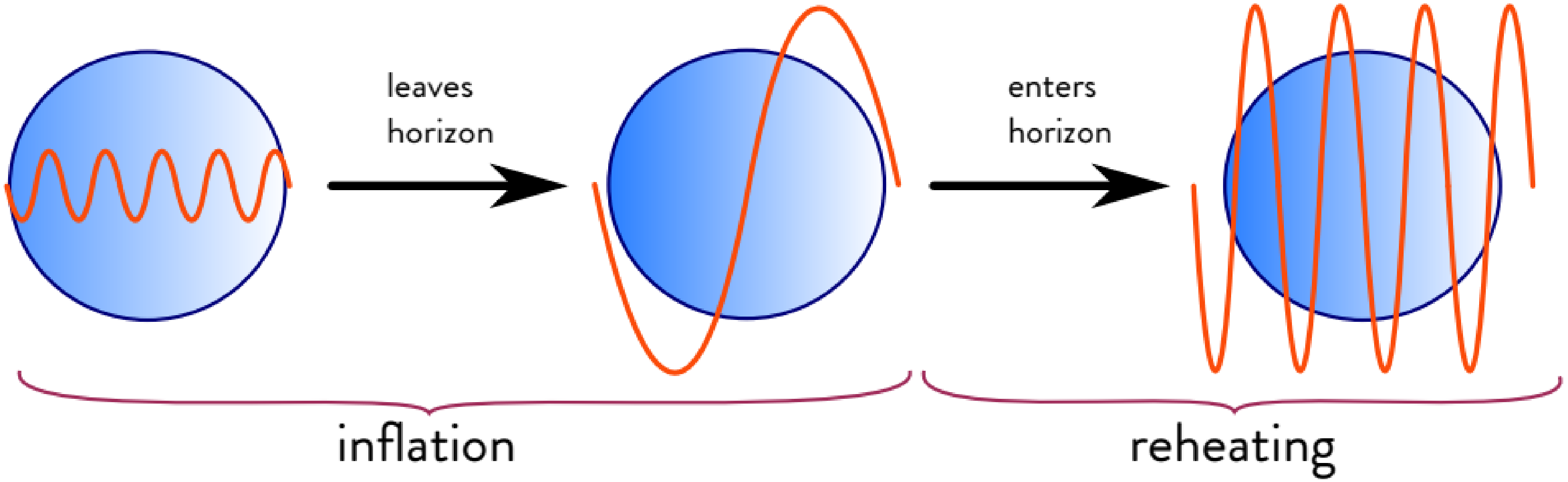
For $\nu \lesssim \frac{3}{2}$ $\quad R_k \equiv \frac{|\chi_k(a_{\rm end})|}{|\chi_{k_{\rm end}}(a_{\rm end})|} \simeq \left(\frac{k}{a_{\rm end}H_{\rm end}}\right)^{-\nu_k}$

$$\nu_k \equiv \nu(k = aH)$$

➡ **Superhorizon enhancement**

(if mass at horizon crossing is smaller than H)

Scalar production: the field picture



$\langle \chi \rangle = 0$ $\xrightarrow[\text{"Random walk"}]{}$ $\langle \chi \rangle = 0$ $\longrightarrow$ $$|\beta_p|^2 = \frac{1}{2\omega_p}|\omega_p X_p - iX_p'|^2$$

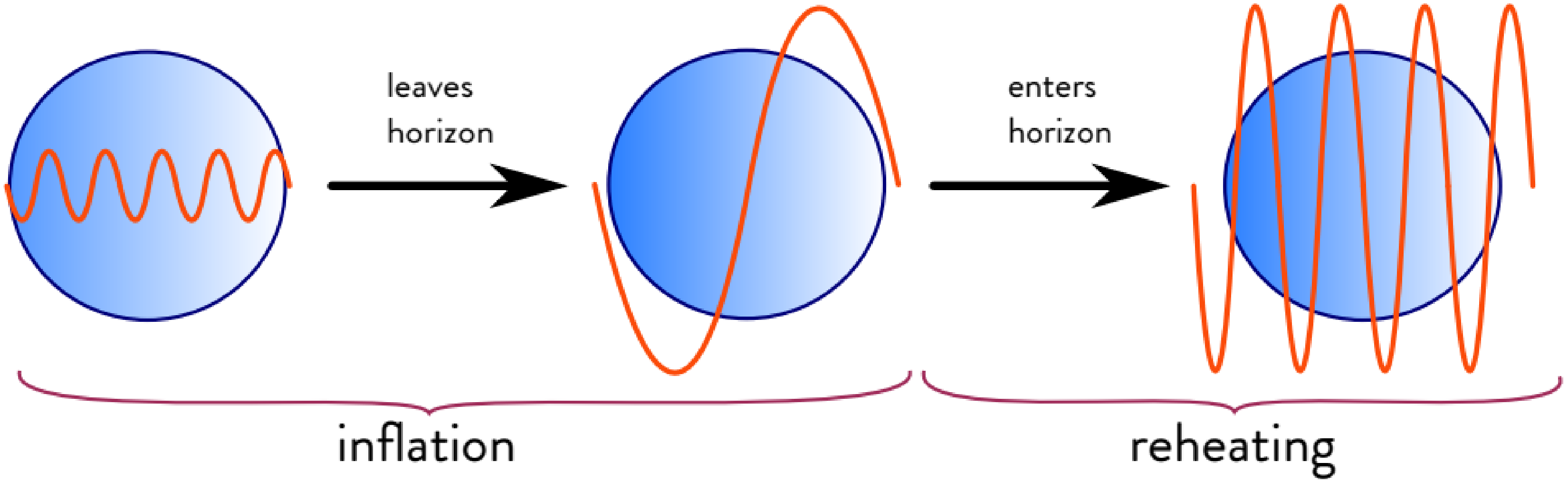
$\langle \chi^2 \rangle = 0$ $\langle \chi^2 \rangle \neq 0$

“Random walk”

Bogolubov coefficient

In curved space one must rely on **correlation functions**!

Scalar production: the field picture



$$\langle \chi \rangle = 0 \qquad \langle \chi^2 \rangle = 0$$

"Random walk" $\longrightarrow$

$$\langle \chi \rangle = 0 \qquad \langle \chi^2 \rangle \neq 0$$

$\longrightarrow$

$$|\beta_p|^2 = \frac{1}{2\omega_p} |\omega_p X_p - i X'_p|^2$$

Bogolubov coefficient

➡ Match **particle interpretation** only at **later times**

Phase space distribution

$$f_\chi(p) = |\beta_p|^2$$

Comoving number density

$$a^3 n_\chi = \int \mathrm{d}\log p \, \frac{p^3}{2\pi^2} |\beta_p|^2$$

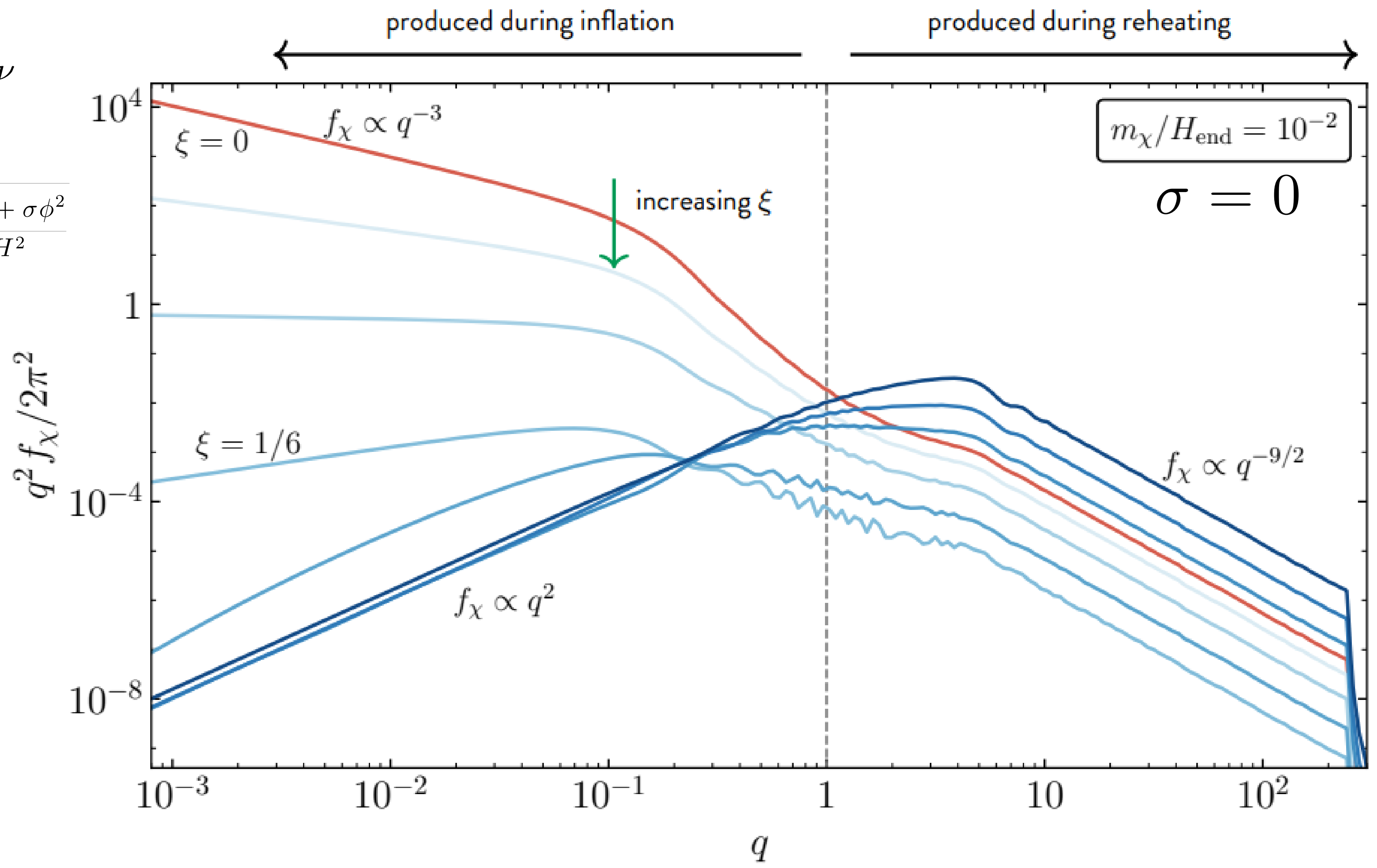
Energy density

$$\rho_\chi \simeq \frac{m_\chi^2}{2} \langle \chi^2 \rangle$$

Inflationary and post-inflationary gravitational production

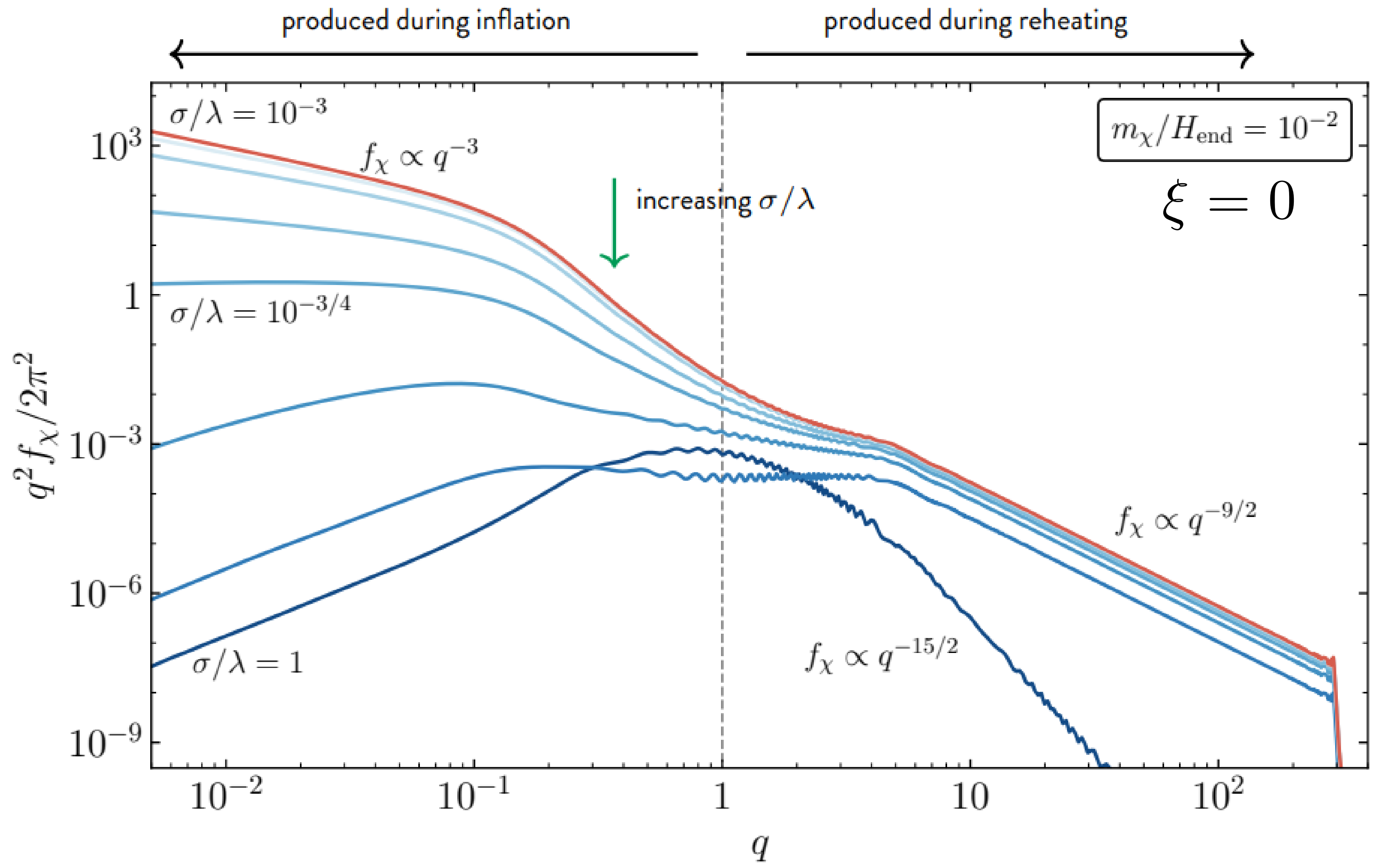
$$f_\chi \propto q^{-2\nu}$$

$$\nu \equiv \sqrt{\frac{9}{4} - 12\xi - \frac{m_\chi^2 + \sigma\phi^2}{H^2}}$$



- Recover **perturbative regime** at **large *q***
- **Conformal** coupling $\xi = 1/6$: same as **fermions**

Inflationary and post-inflationary direct production $0 < \sigma/\lambda < 1$



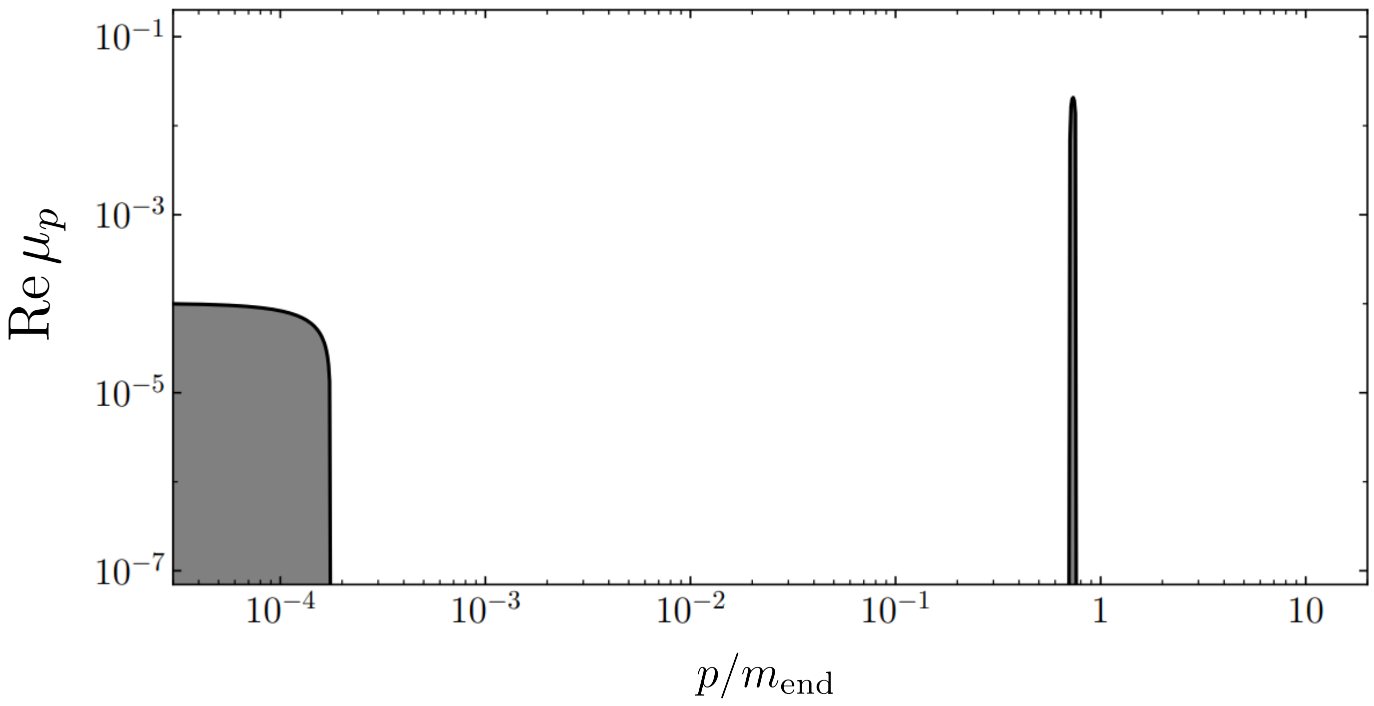
➡ Gravitational/direct interferences: **minimal** amount of dark matter **always produced**

Parametric resonances

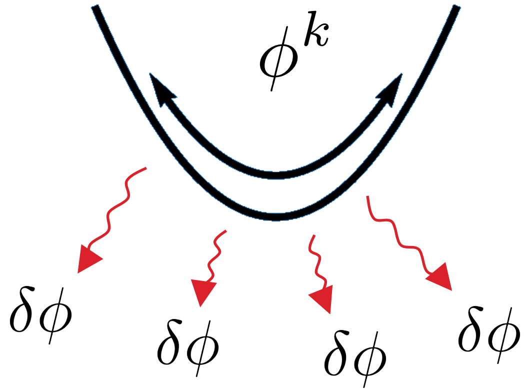
Considering $k = 4$ and dimensionless time $z \equiv m_{\rm end}(\tau - \tau_{\rm end})$ with $m^2_{\rm end} \equiv V_{\phi\phi}(\phi_{\rm end})$ the EOM for **inflaton fluctuations** is

$$\frac{d^2 X_p}{dz^2} + \left[\left(\frac{p}{m_{\rm end}}\right)^2 + {\rm sn}^2\left(\frac{z}{\sqrt{6}}, -1\right)\right] X_k = 0$$

Jacobi elliptic function



$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k$$



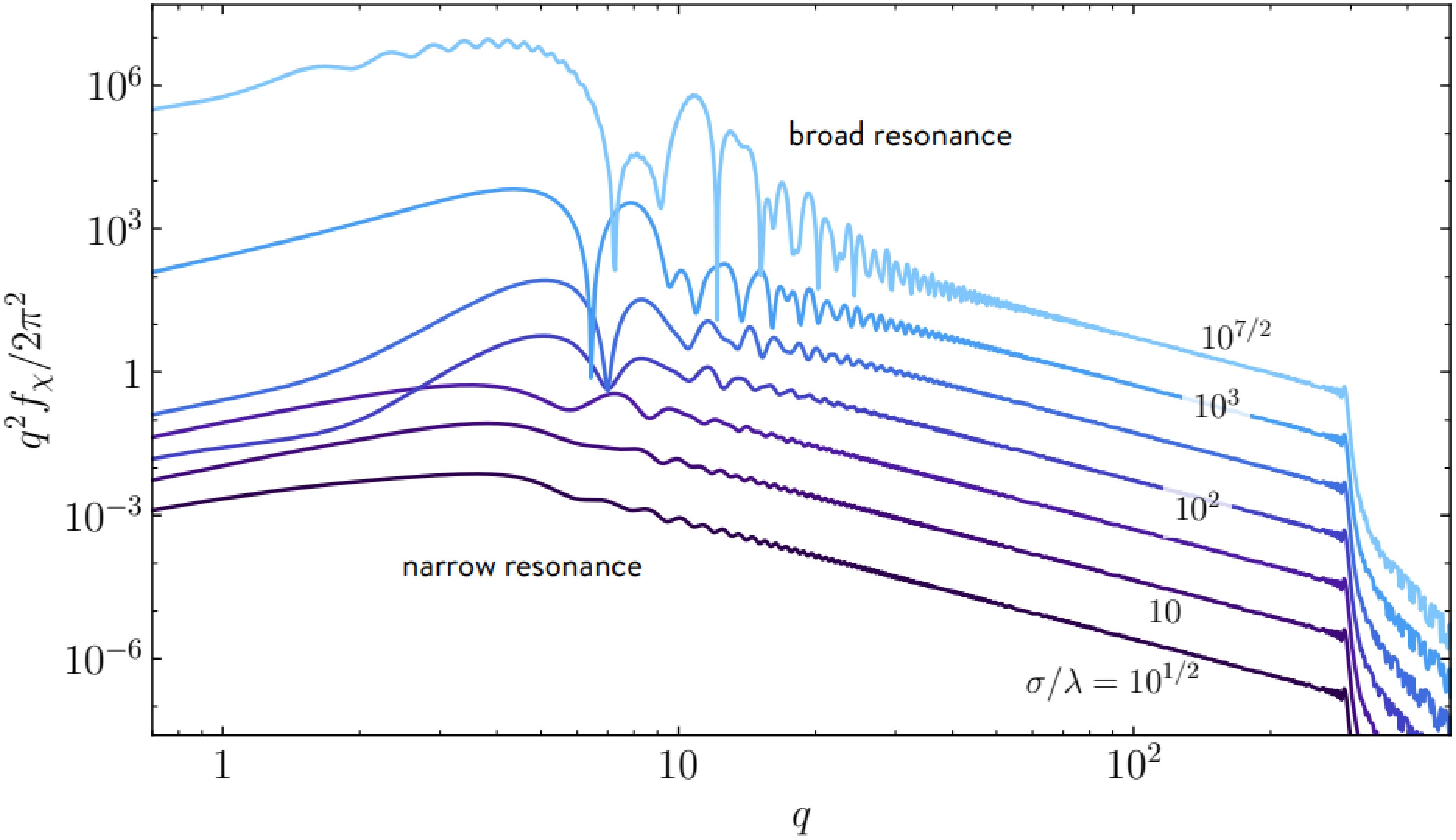
Solutions given in terms of **Floquet index**

$$X_p(\tau) = e^{\mu_p \tau} g_1(\tau) + e^{-\mu_p \tau} g_2(\tau)$$

Floquet chart is **time-dependent** for **non-quartic** potentials $k \neq 4$

Parametric resonances affect all **scalar quantities ("preheating")**

Sizable coupling: preheating $1 < \sigma/\lambda < 10^4$



Parametric resonances at large couplings: $f_\chi(p) \sim e^{2\mu_p m_\phi t}$

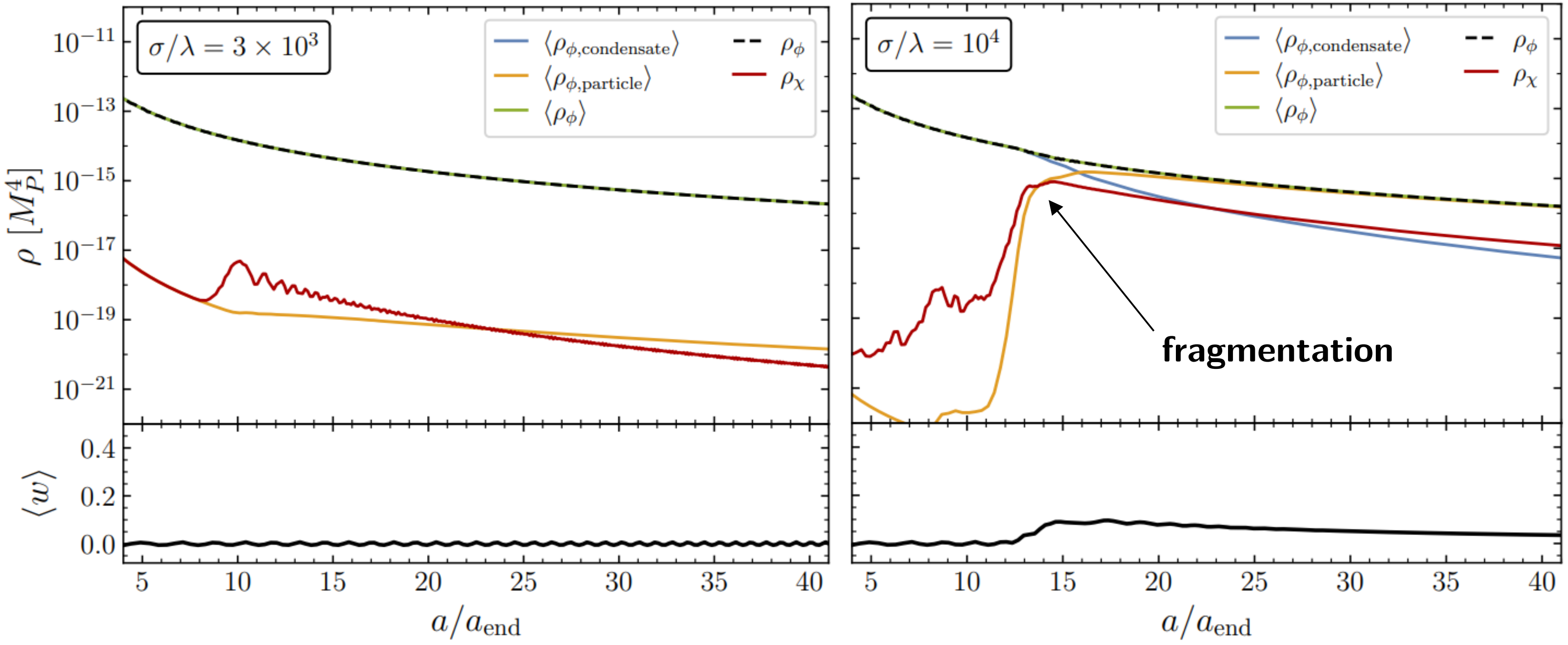
Cannot be accounted for by **perturbative** approach (even with Bose enhancement)

Large couplings $\sigma/\lambda > 10^4$

Copiously produced **dark matter disrupts inflaton condensate**

➡ **Hartree approximation** $$\ddot{\phi} + 3H\dot{\phi} + V_\phi + \sigma\phi^2\langle\chi^2\rangle = 0$$

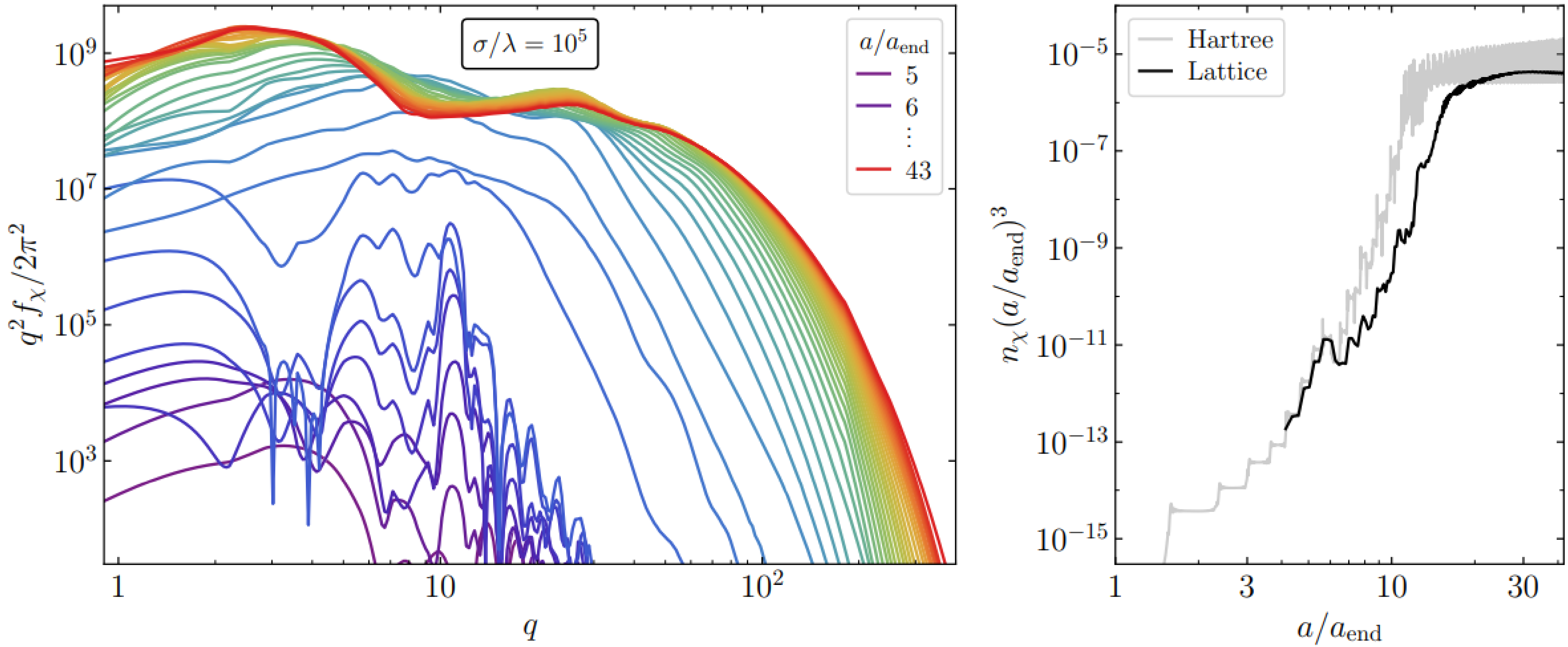
➡ **Real space lattice simulations** *CosmoLattice* — *A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe* [D. G. Figueroa, A. Florio, F. Torrenti, W. Valkenburg, arXiv:2102.01031]



$$\rho_{\phi,\mathrm{condensate}} \equiv \frac{1}{2}\dot{\bar{\phi}}^2 + V(\bar{\phi})$$

$$\rho_{\phi,\mathrm{particle}} \equiv \rho_\phi - \rho_{\phi,\mathrm{condensate}}$$

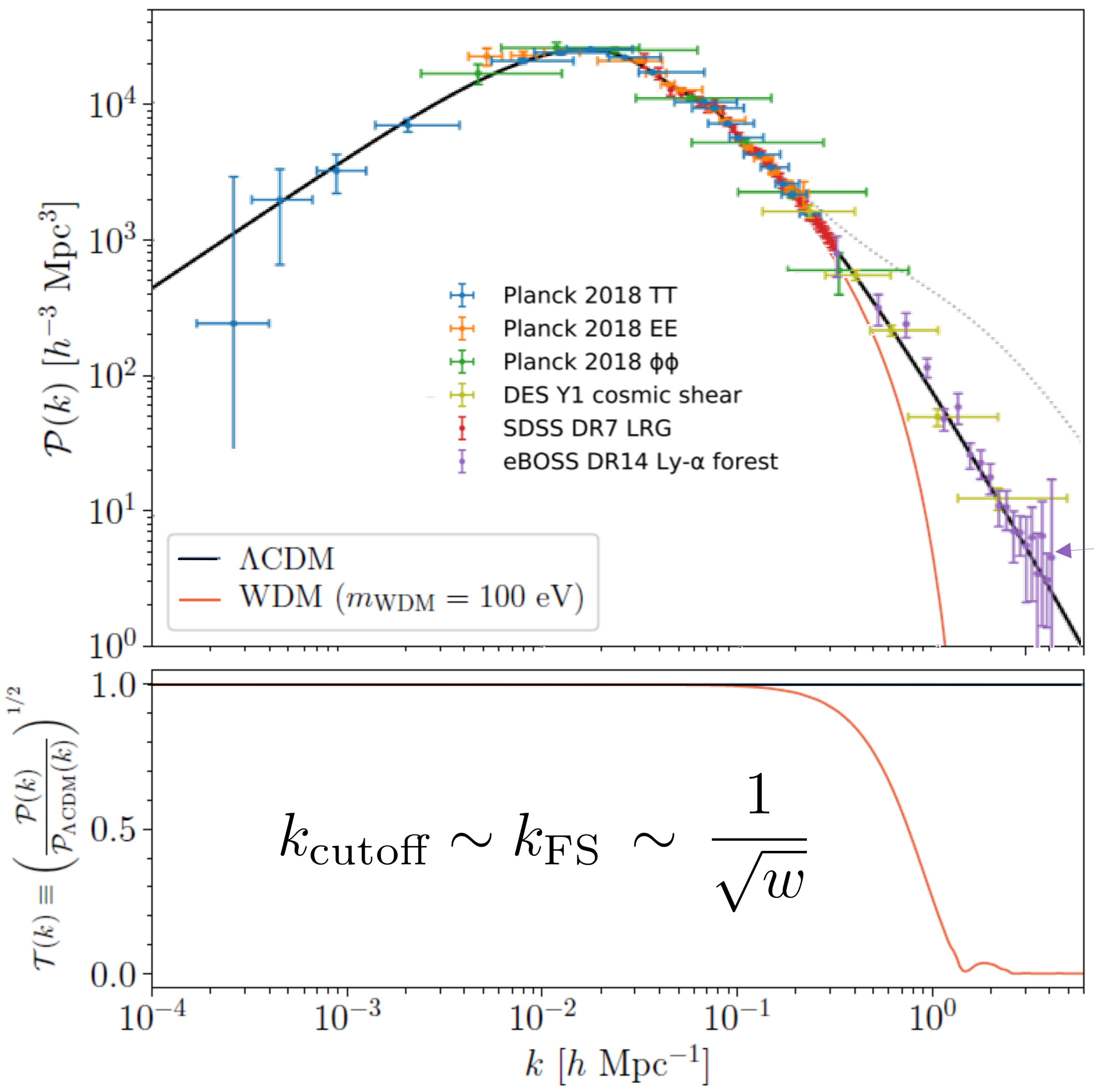
Large couplings $\sigma/\lambda > 10^4$



➡ **Exponential** tails from **quasi-equilibrium** ➡ **Saturation** of number density ➡ Hartree **fails** at **backreaction**

Limits, parameter space & prospects

Constraints on non-cold dark matter



[S. Chabanier, M. Millea, N. Palanque-Delabrouille, MNRAS 489 (2019) 2, 2247-2253]

- **Cutoff** determined by **equation-of-state** parameter

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_\star^2}{3m_\chi^2}\frac{\langle q^2\rangle}{a^2}$$

- **Find mass** that reproduces **cutoff** constrained by **Lyman-α**

[G. Ballesteros, M A. G. Garcia, **MP**, JCAP 03 (2021) 101]

$$m_\chi > 7.5\ \text{keV}\left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\ \text{keV}}\right)^{4/3}\left(\frac{T_\star}{T_\gamma^0}\right)\sqrt{\langle q^2\rangle}$$

$w-$matching

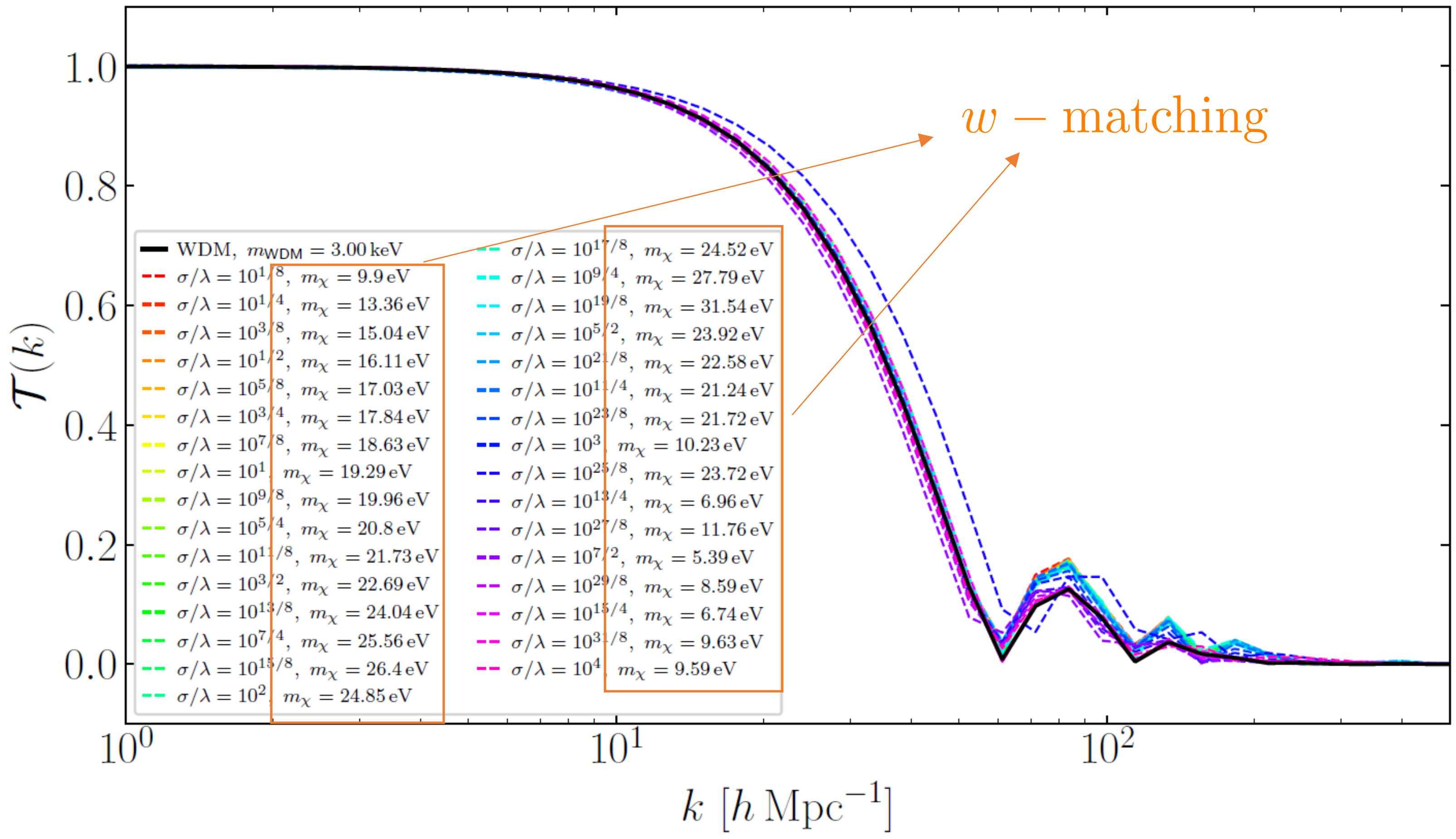
Photon temperature now

$$\langle q^2\rangle \equiv \frac{\int dq q^4 f(q)}{\int dq q^2 f(q)}$$

Constraints on preheating production $1 < \sigma/\lambda < 10^4$

- **Power spectrum** computed **numerically with CLASS**

[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]

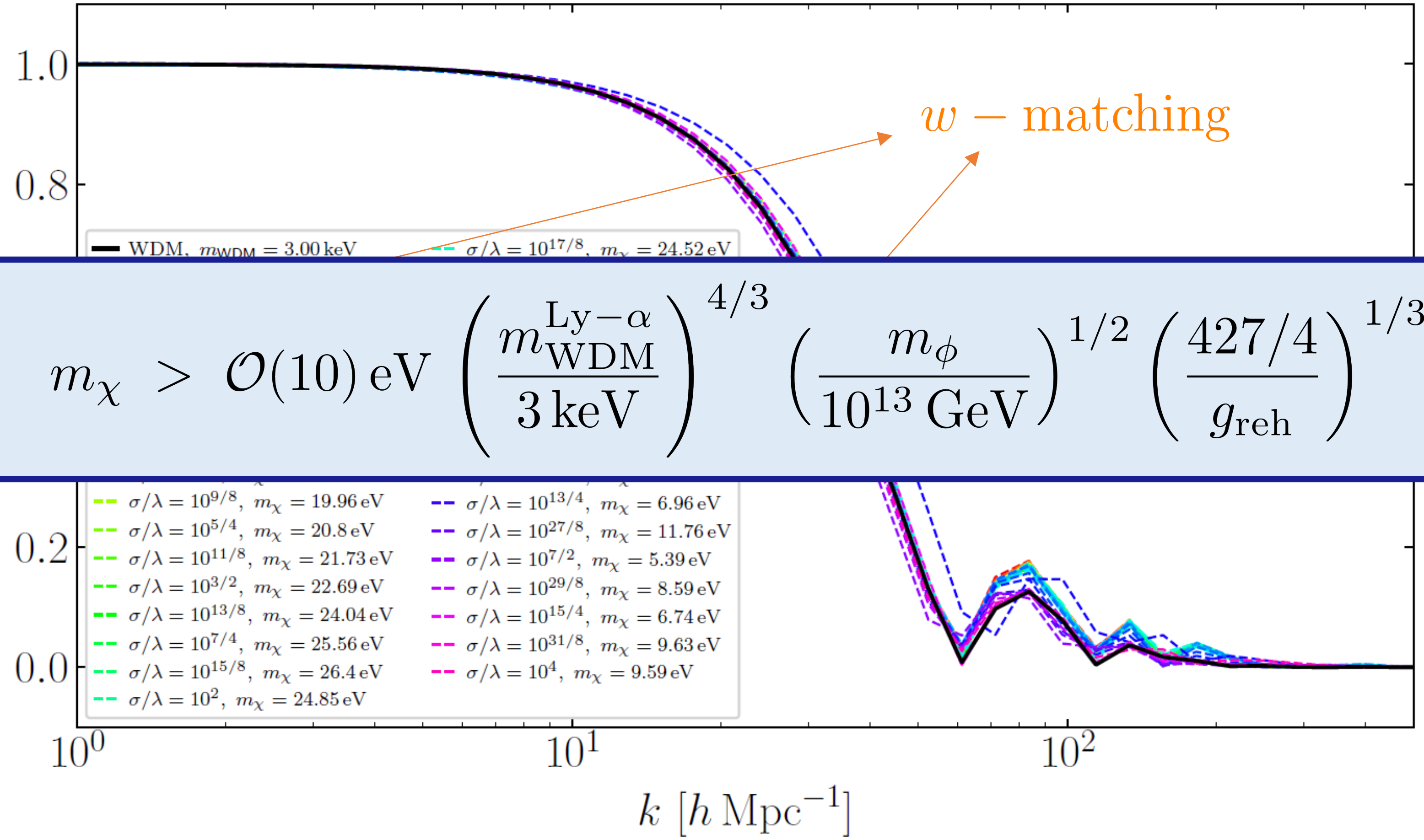


- **Excellent agreement** with $w -$ matching for **all distributions**! Even the nasty ones!

Constraints on preheating production $1 < \sigma/\lambda < 10^4$

- **Power spectrum** computed **numerically with CLASS**

[D. Blas, J. Lesgourgues & T. Tram JCAP 07 (2011) 034 - J. Lesgourgues & T. Tram, JCAP 09 (2011) 032]



$$m_\chi \; > \; \mathcal{O}(10)\,\text{eV} \left(\frac{m_{\rm WDM}^{\rm Ly-\alpha}}{3\,\text{keV}}\right)^{4/3} \left(\frac{m_\phi}{10^{13}\,\text{GeV}}\right)^{1/2} \left(\frac{427/4}{g_{\rm reh}}\right)^{1/3}$$

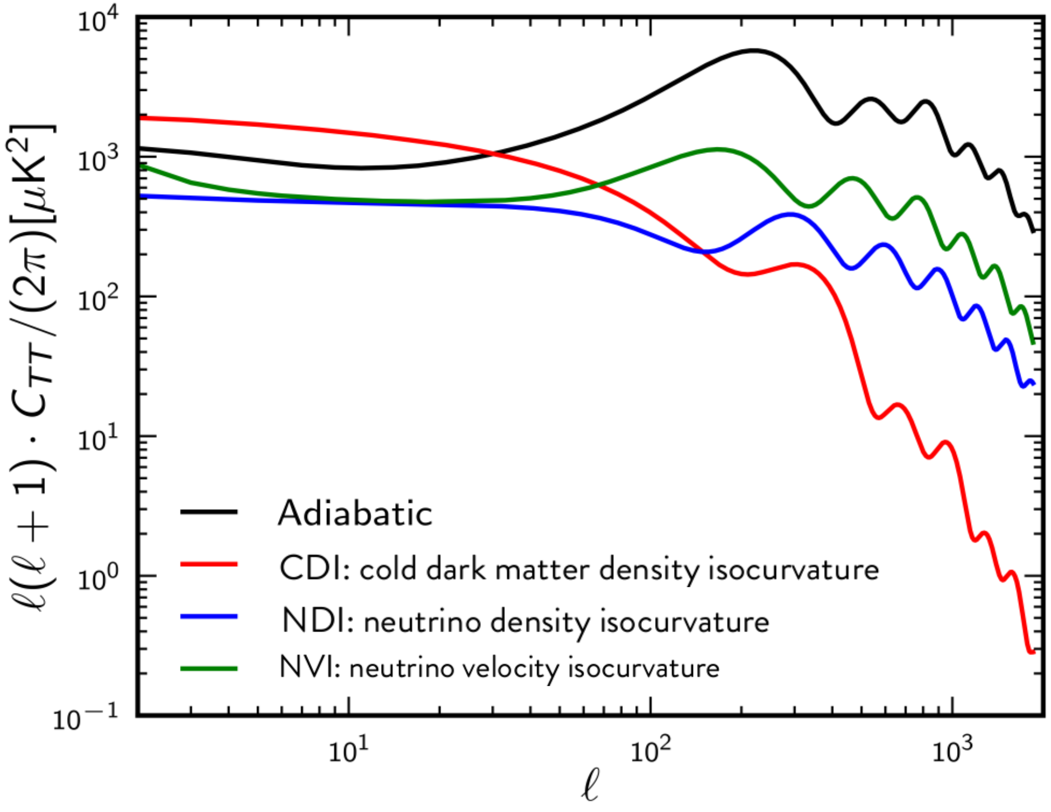
- **Excellent agreement** with $w-\text{matching}$ for **all distributions**! Even the nasty ones!

Isocurvature perturbations

$i, j = \gamma, \nu, \chi, b$

- **Single field** inflation predicts **adiabatic** perturbations $\frac{\delta\rho_i(\boldsymbol{x},t)}{\dot{\bar{\rho}}_i(t)} = \frac{\delta\rho_j(\boldsymbol{x},t)}{\dot{\bar{\rho}}_j(t)} \quad k \ll aH$

- Adiabatic perturbations share "single clock" $\delta\tau(\boldsymbol{x},t)$ **:** $\rho_i(\boldsymbol{x},t) \simeq \bar{\rho}_i(t) + \dot{\bar{\rho}}_i(t)\delta\tau(\boldsymbol{x},t)$



- $\delta\tau(\boldsymbol{x},t) \Leftrightarrow$ **curvature perturbation** $\mathcal{R}(\boldsymbol{x},t)$

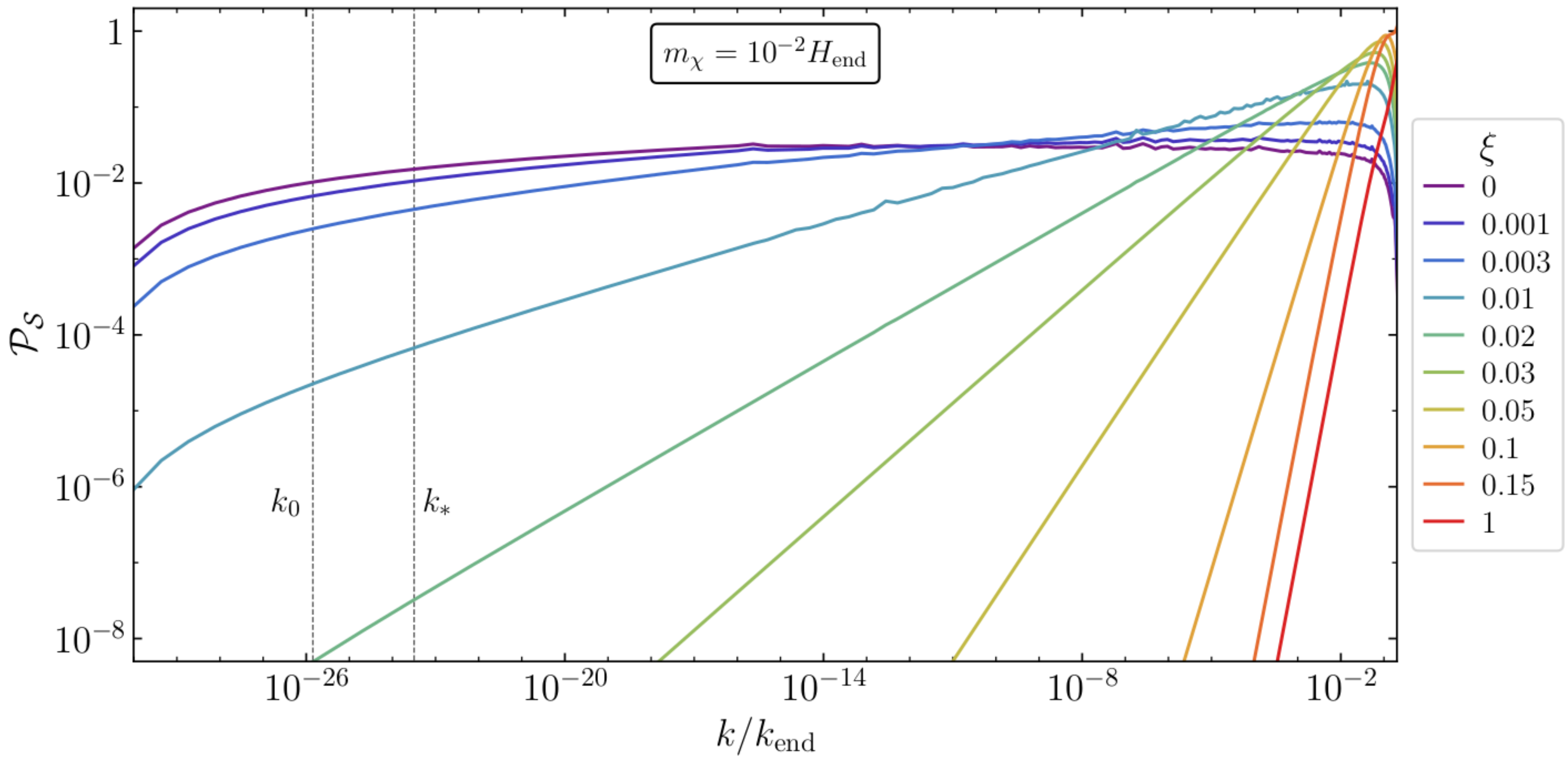
- **Deviations** to adiabatic perturbations are **isocurvature** perturbations

$$\mathcal{S}_{ij} = 3H\left(\frac{\delta\rho_i(\boldsymbol{x},t)}{\dot{\bar{\rho}}_i(t)} - \frac{\delta\rho_j(\boldsymbol{x},t)}{\dot{\bar{\rho}}_j(t)}\right)$$

$$\beta_{\rm iso} = \frac{\mathcal{P}_\mathcal{S}}{\mathcal{P}_\mathcal{R} + \mathcal{P}_\mathcal{S}} < \begin{cases} 2.5\% \text{ (CDI)} \\ 7.4\% \text{ (NDI)} \\ 6.8\% \text{ (NVI)} \end{cases}$$

from Planck at $k_* = 0.002\,\mathrm{Mpc}^{-1}$

Isocurvature: gravitational production



$$m_\chi \gtrsim 0.54\, H_{\rm inf}$$
$$\xi \gtrsim 0.03$$

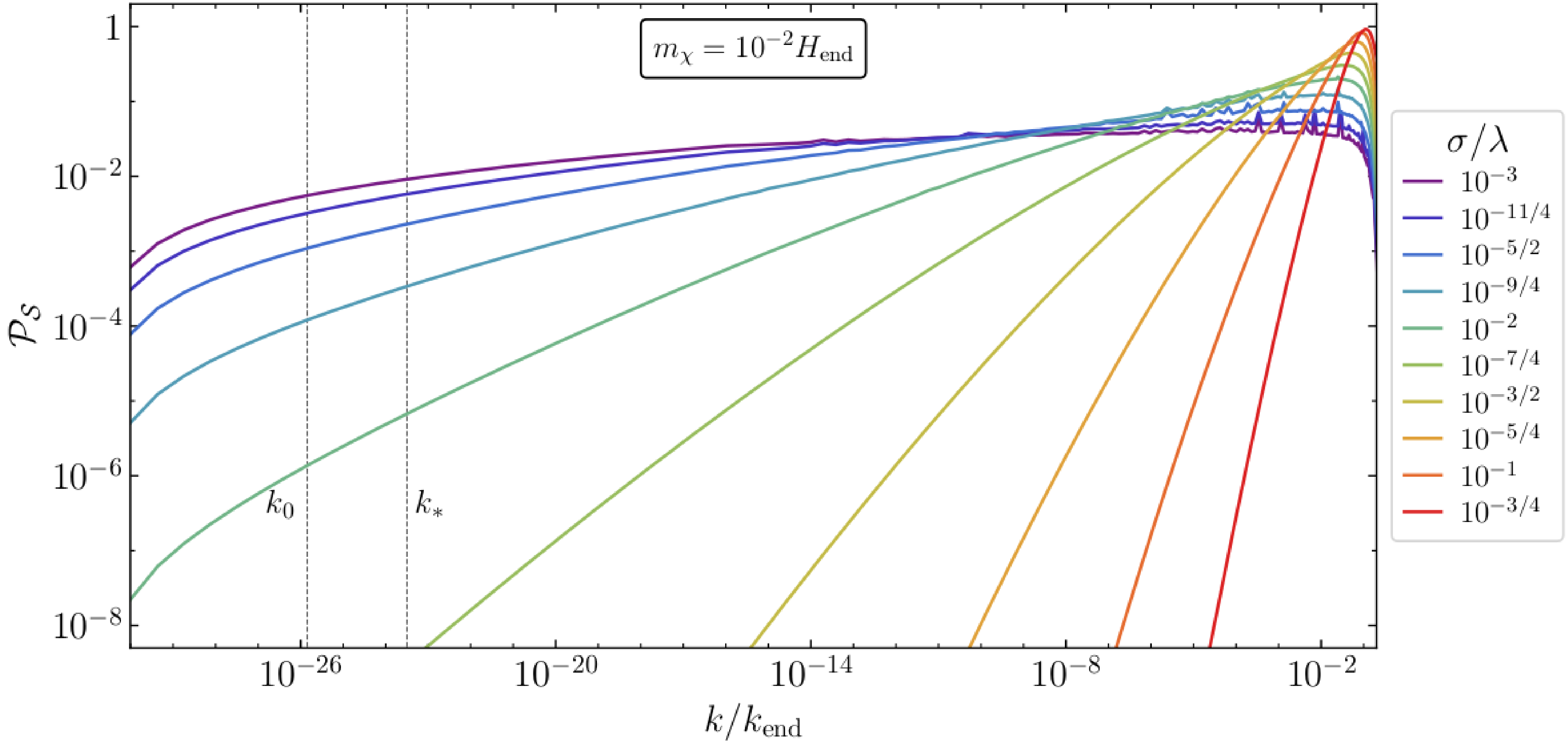
Lower bound!

Planck constraints

$$\mathcal{P}_S(k) = \frac{k^3}{(2\pi)^5 \rho_\chi^2 a^8} \int \mathrm{d}^3\boldsymbol{p}\; P_X(p, |\boldsymbol{p}-\boldsymbol{k}|)$$

$$P_X(p,q) = |X_p'|^2|X_q'|^2 + a^4 m_\chi^4 |X_p|^2|X_q|^2 + a^2 m_\chi^2 \left[(X_p X_p'^*)(X_q X_q'^*) + \text{h.c.}\right]$$

Isocurvature: direct production



$$m_\chi \gtrsim 0.54\, H_{\rm inf}$$

$$\sigma/\lambda \gtrsim 0.02$$

Lower bound!

Planck constraints

$$\mathcal{P}_S(k) = \frac{k^3}{(2\pi)^5 \rho_\chi^2 a^8} \int \mathrm{d}^3\boldsymbol{p}\; P_X(p, |\boldsymbol{p}-\boldsymbol{k}|)$$

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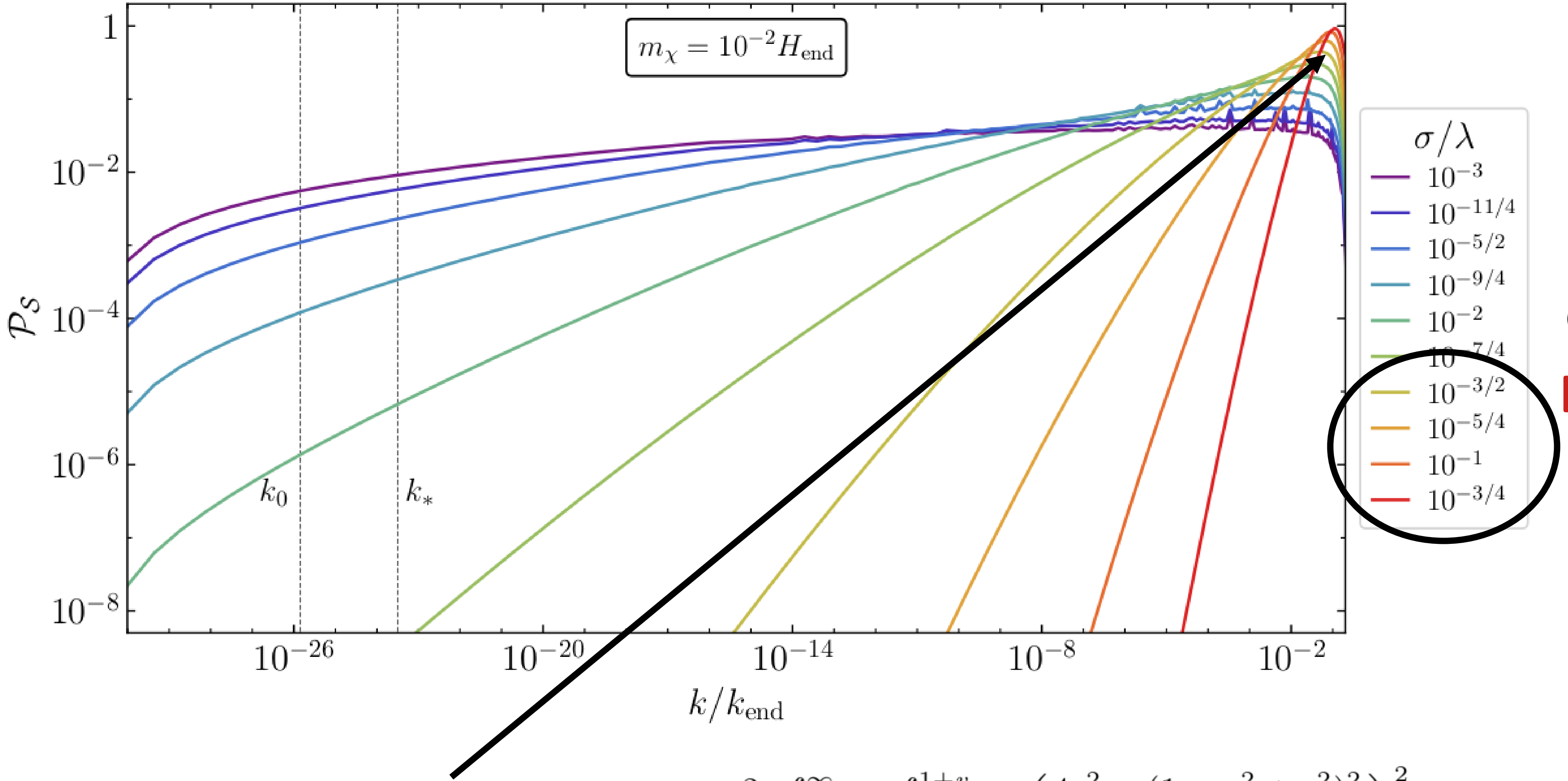
Isocurvature: direct production



$$m_\chi \gtrsim 0.54\, H_{\rm inf}$$

$$\sigma/\lambda \gtrsim 0.02$$

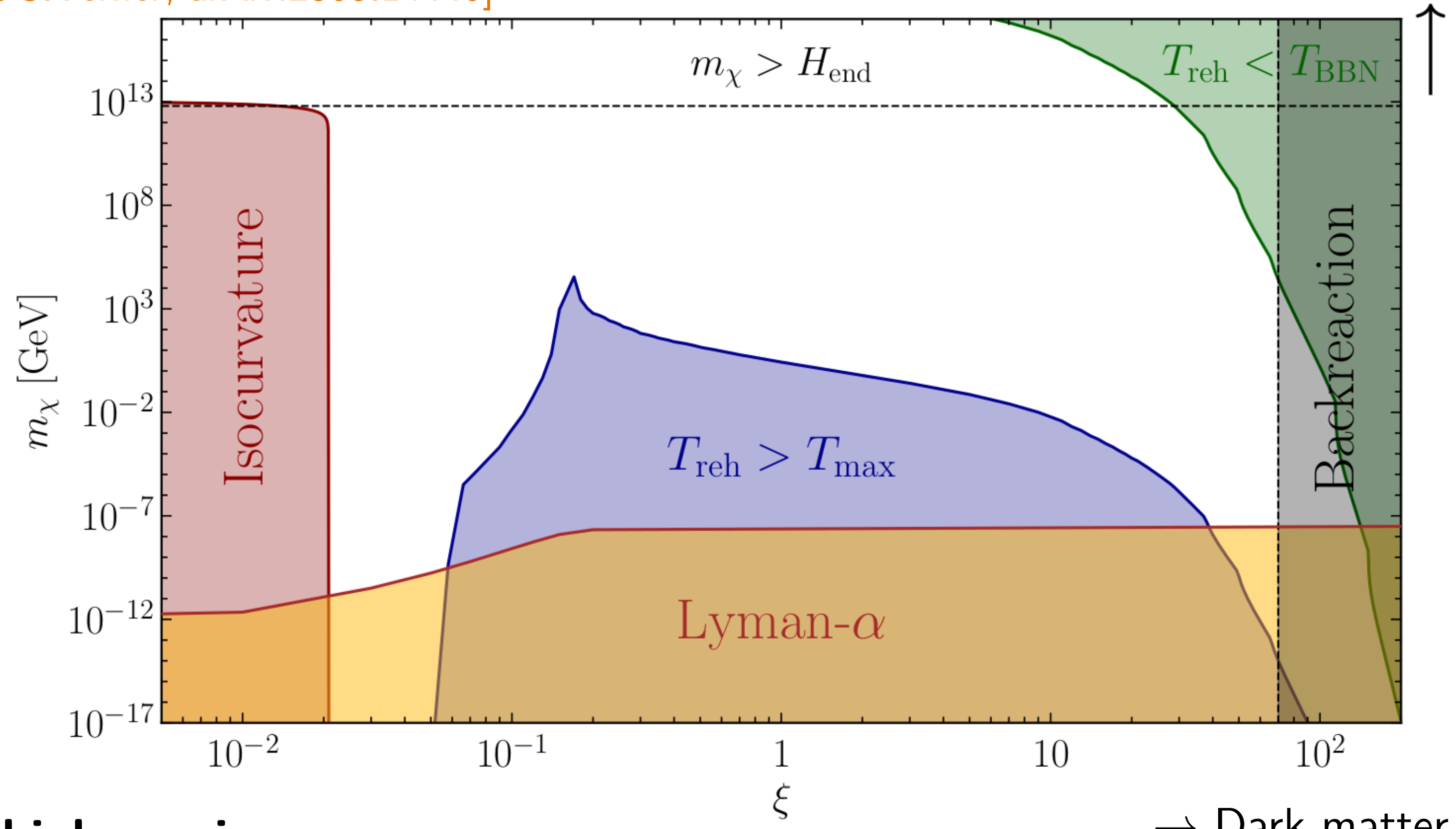
Lower bound!

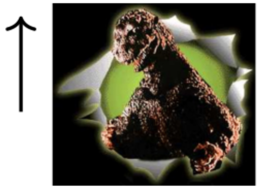
- **Gravitational waves?** $\Omega_{\rm GW,c}(k) = \frac{2}{3}\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1-u^2+v^2)^2}{4uv}\right)^2 \overline{I^2(x_c,k,u,v)}\mathcal{P}_S(ku)\mathcal{P}_S(kv)$
- **Primordial black holes?**

[Work in progress, **M. A. G. Garcia**, **S. Verner & M.P.**]

Gravitational production: summary

[M. A. G. Garcia, **MP** & S.Verner, arXiv:2305.14446]





WIMPZILLAS

[E.W. Kolb, D.J.H. Chung, A. Riotto]

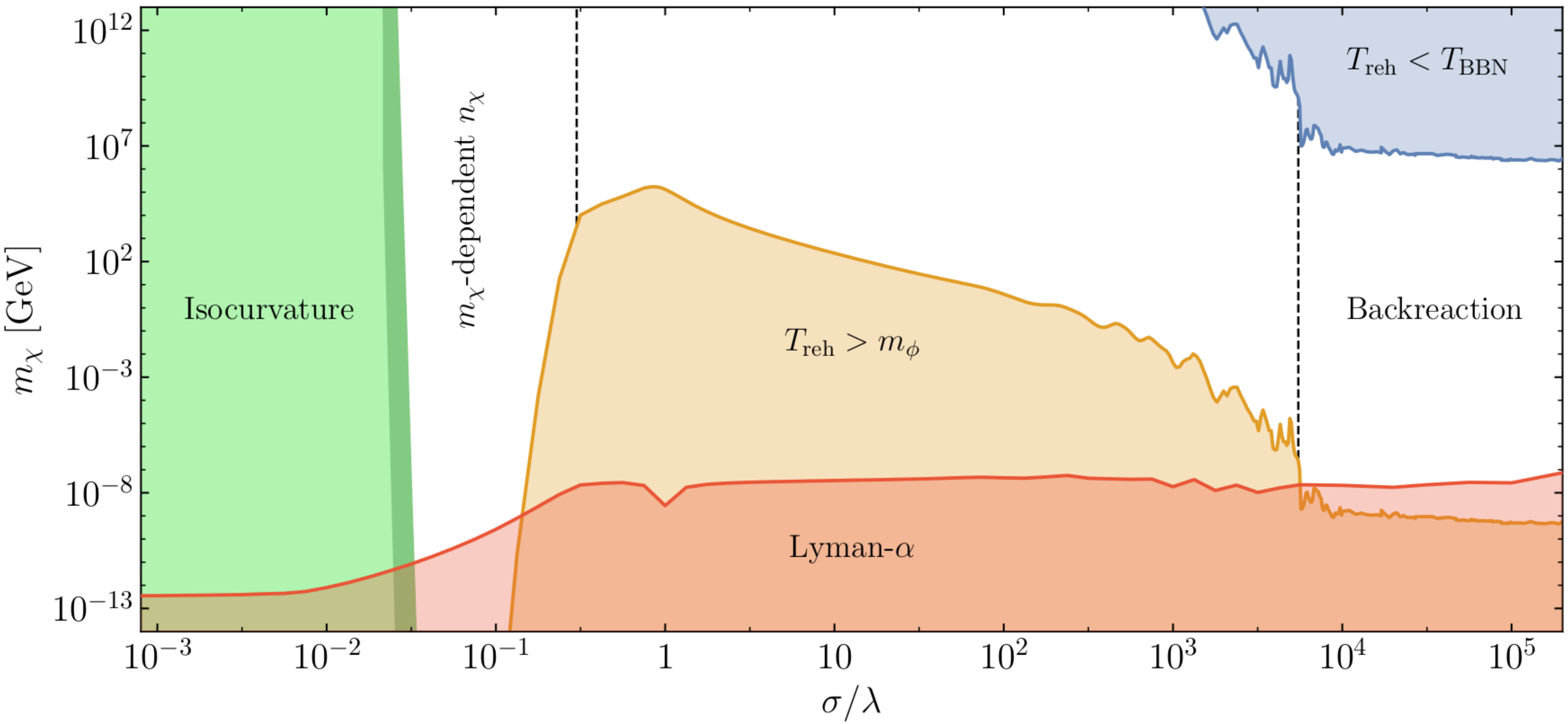
- **Generalization to higher spins**
  - **Spin ½:** if $m_\chi \to 0$: **conformally** coupled to gravity $\leftrightarrow$ scalar $\xi = 1/6$
  - **Spin 1:** Transverse: $\leftrightarrow$ **conformally** coupled scalar $\xi = 1/6$
    Longitudinal: $\leftrightarrow$ **minimally** coupled scalar $\xi = 0$ if $m_\chi \to 0$
  - **Spin 3/2, 2:** [E. W. Kolb & A. Long arXiv:2312.09042]

$\rightarrow$ Dark matter **can** be produced **gravitationally**

$\rightarrow$ Non-minimal coupling to gravity **mimics** direct coupling to the inflation

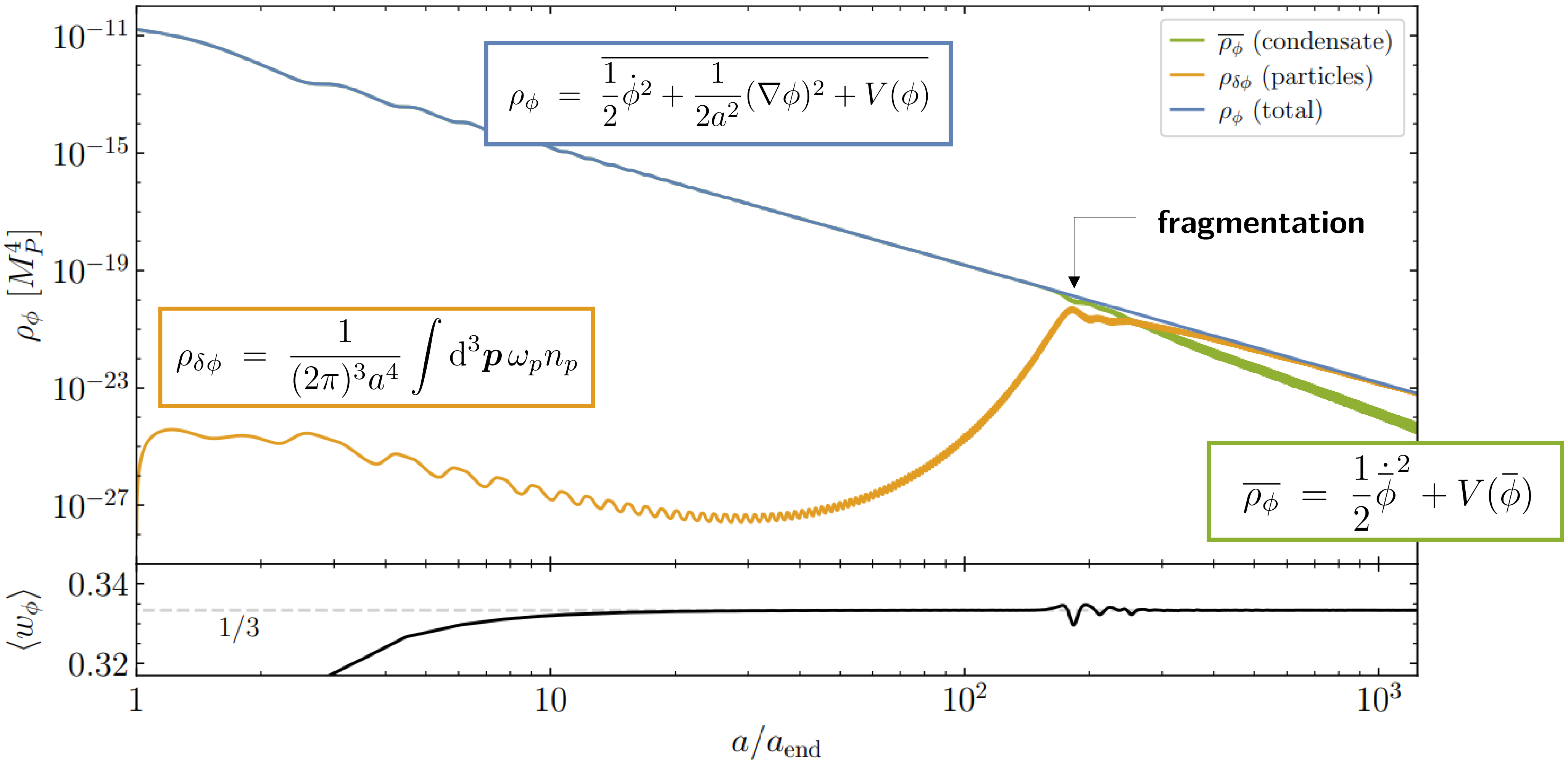
$$\xi \leftrightarrow \sigma/\lambda$$

Direct production: summary



Digestive: reheating after inflaton fragmentation

Self-fragmentation: quartic case $k = 4$



- **The condensate subsists!** ➡ **generic for larger *k***

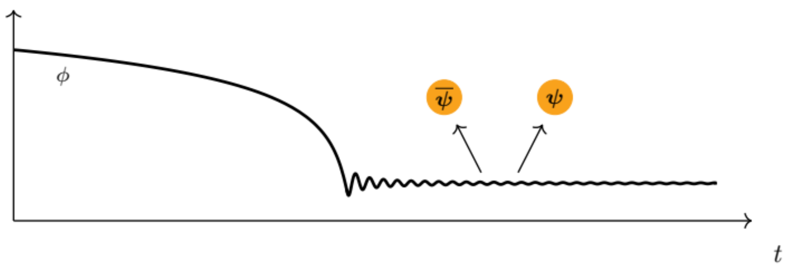
Reheating and inflaton fragmentation

Consider **coupling to fermions** $\mathcal{L} \supset -y\phi\bar{\psi}\psi$

$$\dot{\rho}_\psi + 4H\rho_\psi = \underbrace{R_\phi + R_{\delta\phi}}_{\text{production rate}}$$

$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi = -(R_\phi + R_{\delta\phi})$$

Condensate contribution

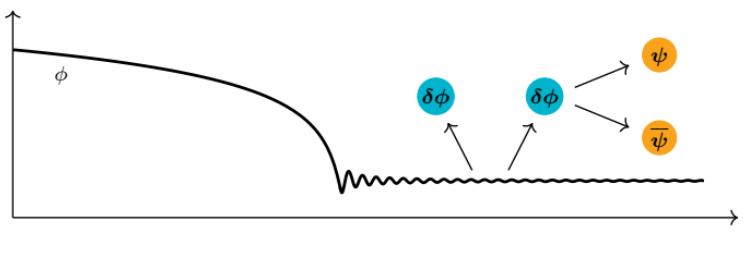


$$\Gamma_\phi = \frac{1}{8\pi(1+w_\phi)\rho_\phi}\sum_{n=1}^{\infty}\langle|\mathcal{M}_n|^2 E_n \beta_n\rangle \simeq \alpha^2 \frac{y^2}{8\pi} m_\phi(t)$$

efficiency

$$R_\phi = \frac{4}{3}\Gamma_\phi \overline{\rho_\phi}$$

Quanta contribution



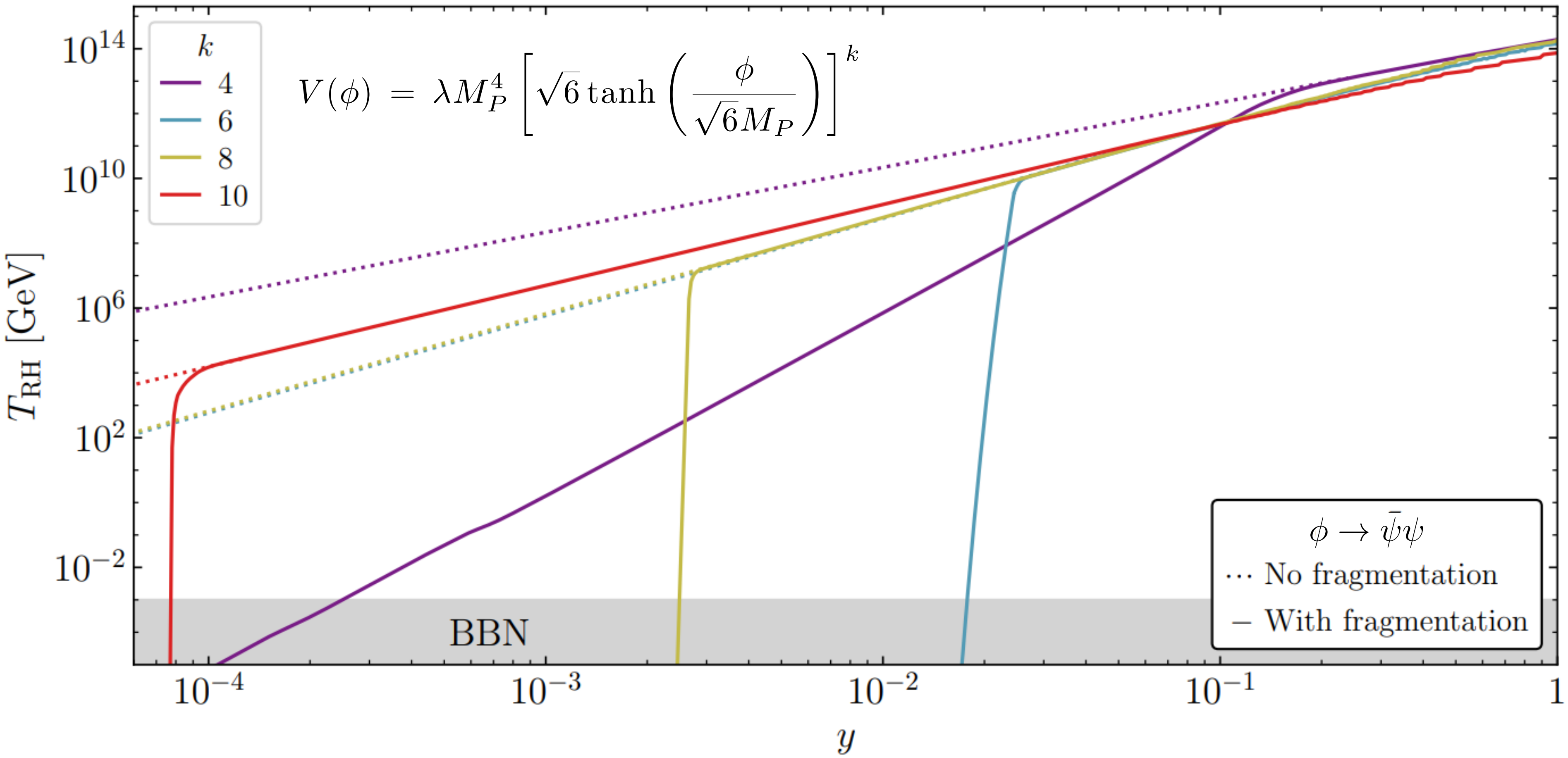
$$R_{\delta\phi}(t) = \Gamma_{\delta\phi} m_\phi n_{\delta\phi}$$

$$\Gamma_{\delta\phi} = \frac{|\mathcal{M}_{\delta\phi\to\bar{\psi}\psi}|^2}{16\pi m_\phi}\sqrt{1 - \frac{4m_\psi^2}{m_\phi^2}} \simeq \frac{y^2}{8\pi} m_\phi(t)$$

- Estimate **number density** from the **lattice**
- **Mass term** induced by **leftover condensate:**
  - ➡ **allow** quanta to decay!

[M. A. G. Garcia & **MP**, JCAP 11 (2023) 004]

Effect on reheating temperature



- **Large** (non-perturbative) **couplings** required
- At large ***k***, **post-fragmentation** decays **extremely suppressed**

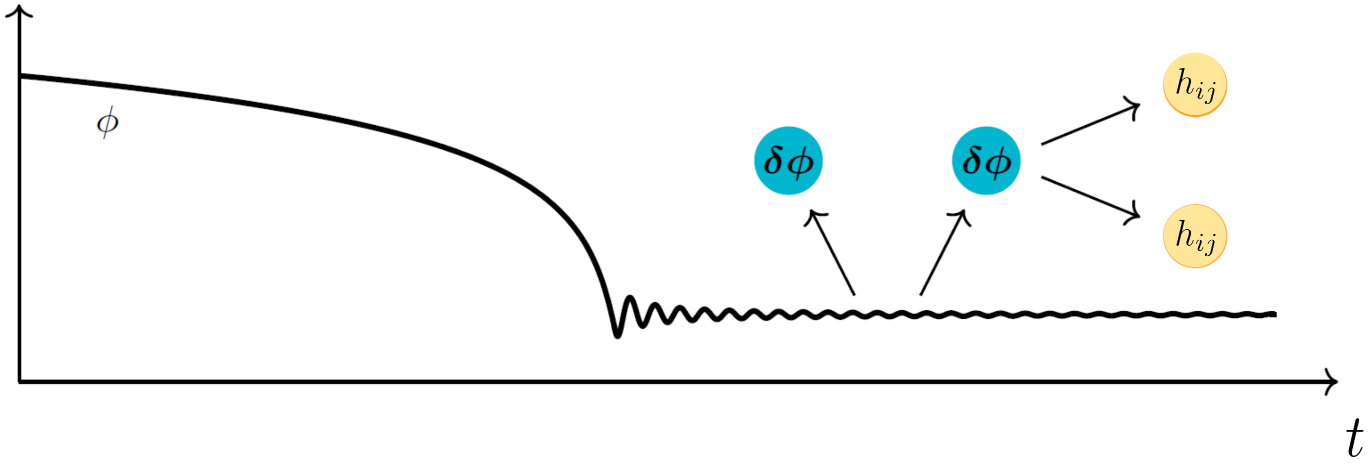
[M. A. G. Garcia, M. Gross, Y. Mambrini, K. Olive, **MP** & J-H Yoon, JCAP 12 (2023) 028]

Gravitational waves: quartic case $k = 4$

- **Tensor** perturbations of the metric $\mathrm{d}s^2 = a(\tau)^2 \left[\mathrm{d}\tau^2 - \left(\delta_{ij} + h_{ij} \right) \mathrm{d}x^i \, \mathrm{d}x^j \right]$

- Sourced by Tranceverse-Traceless (TT) scalar **inhomogeneities**

$$h''_{ij}(\boldsymbol{p},\tau) + 2\mathcal{H}h'_{ij}(\boldsymbol{p},\tau) + k^2 h_{ij}(\boldsymbol{p},\tau) = \frac{2}{M_P^2}\left[\int \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^{3/2}}\, q_i\, q_j\, \phi(\boldsymbol{q},\tau)\, \phi(\boldsymbol{p}-\boldsymbol{q},\tau)\right]^{\mathrm{TT}}$$



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- Use Boltzmann **approach** to predict spectrum of inflaton fluctuations $\phi \to \delta\phi\,\delta\phi$

$$f_{\delta\phi}(|\boldsymbol{p}|,t) \simeq \frac{\pi}{c^2}\left(\frac{m_{\mathrm{end}}}{H_{\mathrm{end}}}\right)\left(\frac{a(t)}{a_{\mathrm{end}}} - 1\right)\sum_{n=1}^{\infty}\frac{|\hat{\mathcal{P}}_n|^2}{n^2\beta_n}\delta\left(\frac{|\boldsymbol{p}|}{m_{\mathrm{end}}} - \frac{1}{2}nc\beta_n\right)$$

series of peaks!

energy levels of inflaton potential

$$\beta_n \equiv \sqrt{1 - \frac{4m_\phi^2}{n^2\omega_\phi^2}} = \sqrt{1 - \left(\frac{2}{nc}\right)^2}$$

$$c \equiv \sqrt{\frac{2\pi}{3}}\frac{\Gamma(3/4)}{\Gamma(1/4)}$$

[M. A. G. Garcia & **MP** JCAP 11 (2023) 004]

Gravitational waves: quartic case $k = 4$

- **Tensor** perturbations of the metric $\mathrm{d}s^2 = a(\tau)^2\left[\mathrm{d}\tau^2 - \left(\delta_{ij} + h_{ij}\right)\mathrm{d}x^i\,\mathrm{d}x^j\right]$
- Sourced by Tranceverse-Traceless (TT) scalar **inhomogeneities**

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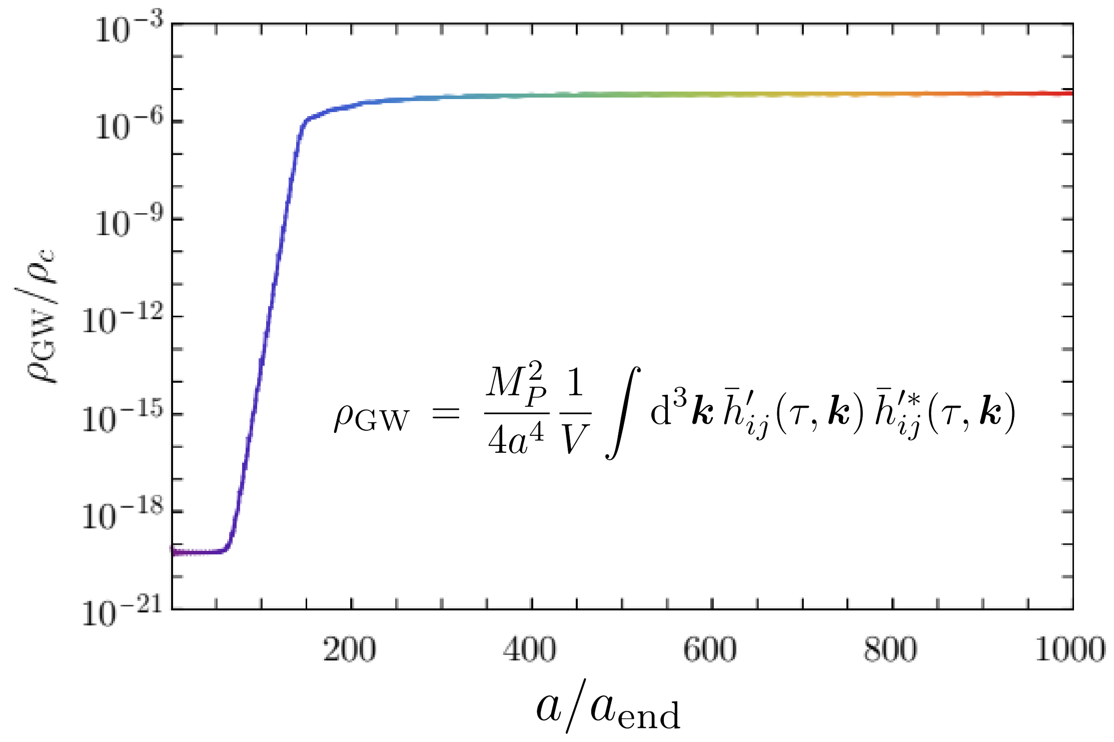
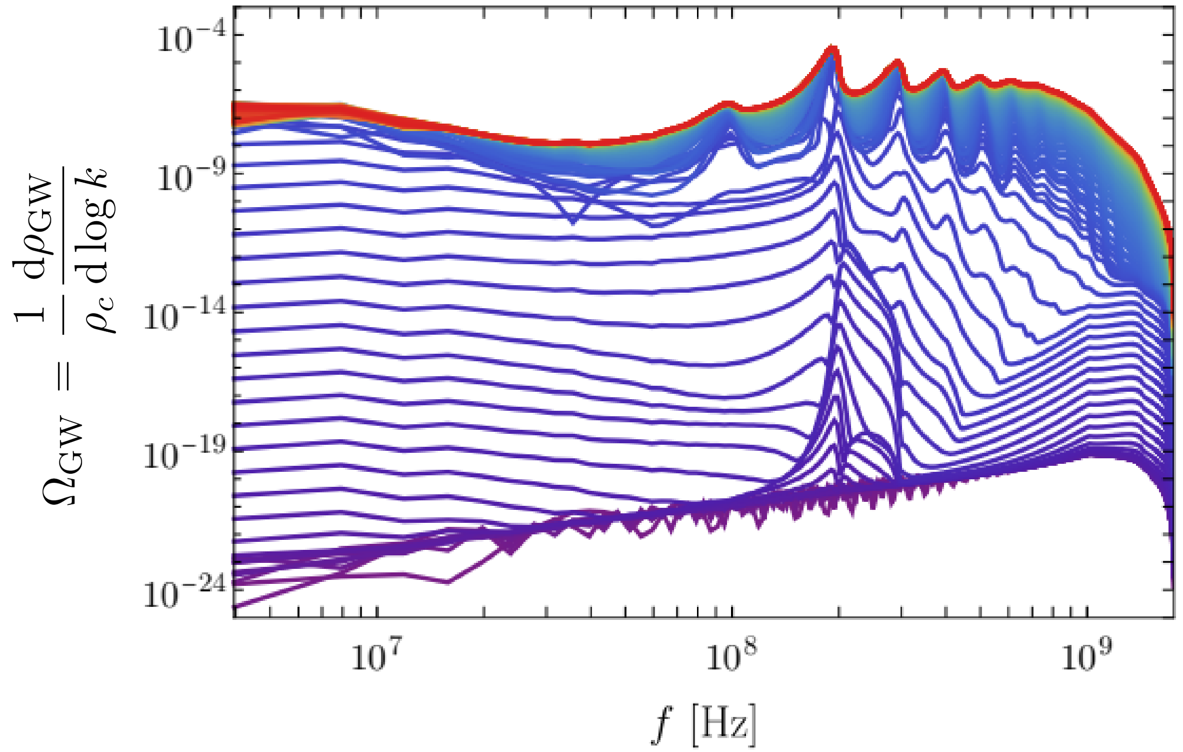
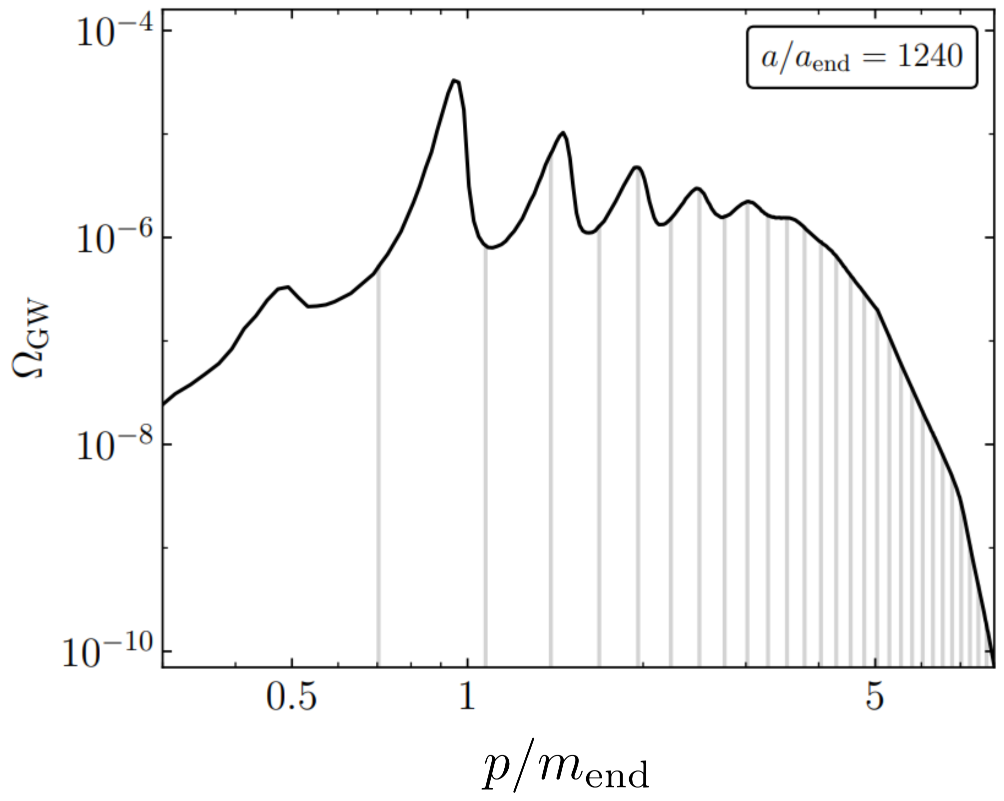
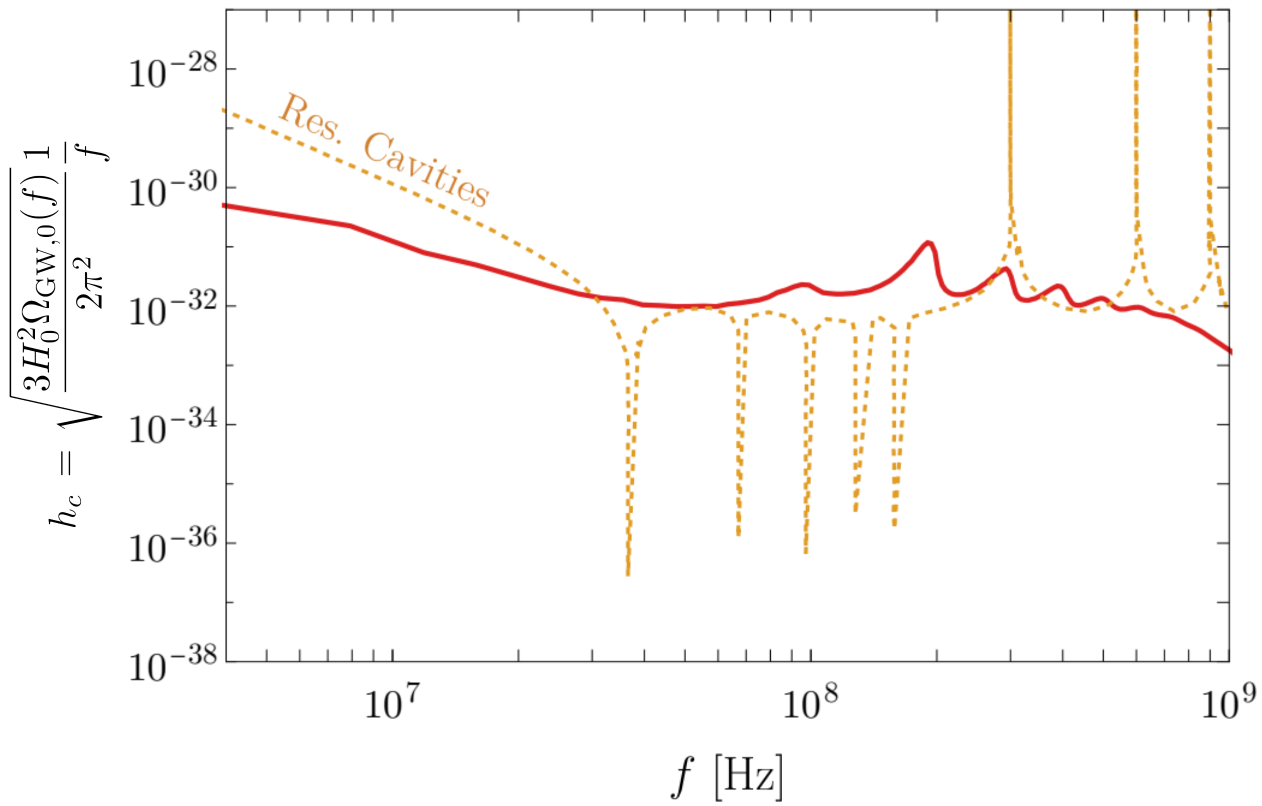




Gravitational waves: quartic case $k = 4$



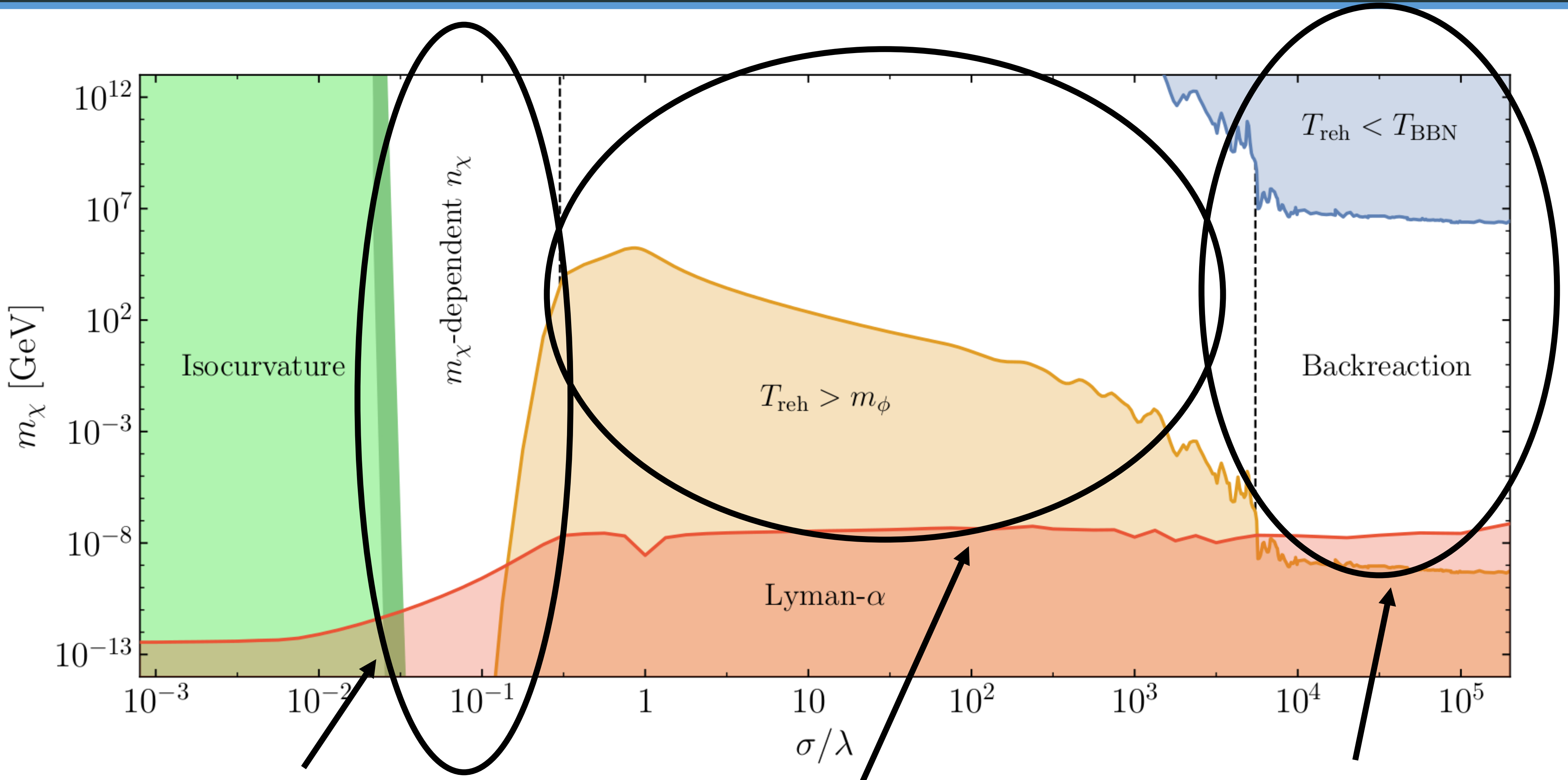


- Peak **interval** from Boltzmann **approach** accurate!
- Detectable via future resonant EM cavities?
  [N. Herman, L. Lehoucq, A. Fűzfa, arXiv:2203.15668]

Inflaton spectroscopy

[M. A. G. Garcia & **MP**, JCAP 11 (2023) 004]

Direct production: summary & prospects

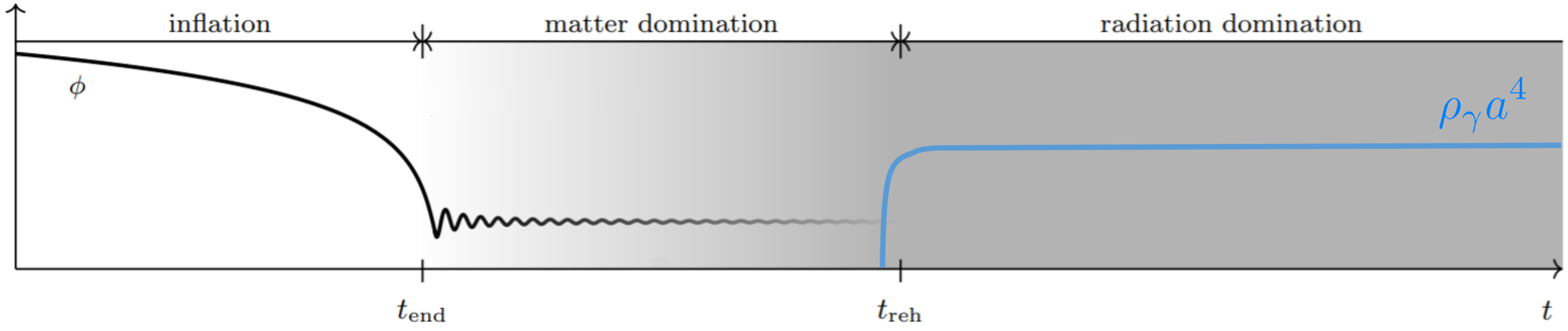


GW and PBH from isocurvature?

Non-perturbative reheating?

GW and PBH from inflaton fragmentation?

Take home message



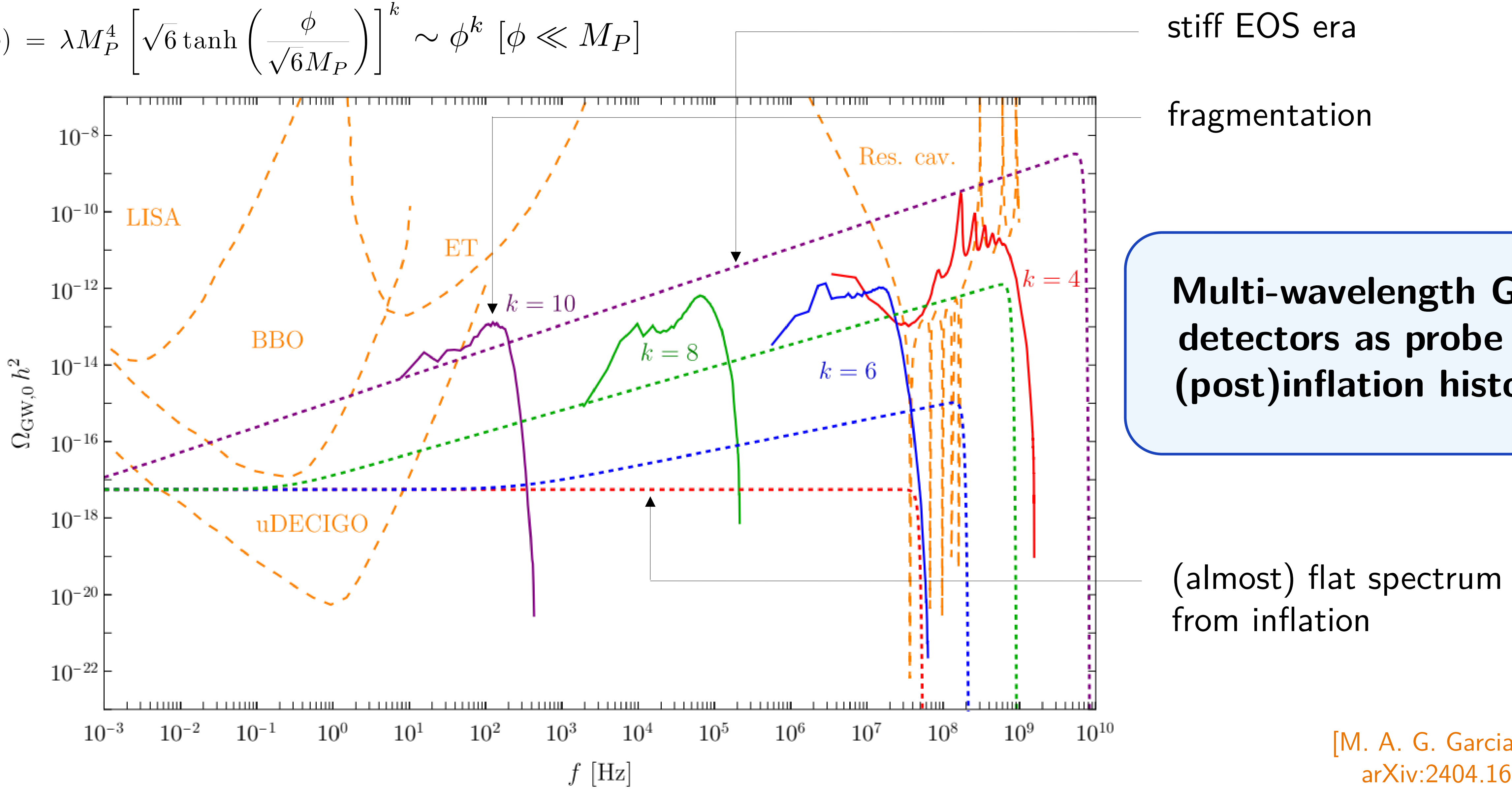
- The **expanding universe** as a **source** for **particle production**
- (Post)inflation **dynamics** offers a **rich spectrum** of phenomenological implications
- **Inhomogeneities** might **reveal** (post-)inflationary dark matter **production**

Thank you for your attention

Back up slides

Gravitational waves from post-fragmentation reheating

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k \sim \phi^k \ [\phi \ll M_P]$$



Multi-wavelength GW detectors as probe of (post)inflation history

[M. A. G. Garcia & **MP**
arXiv:2404.16932]

Perturbative reheating

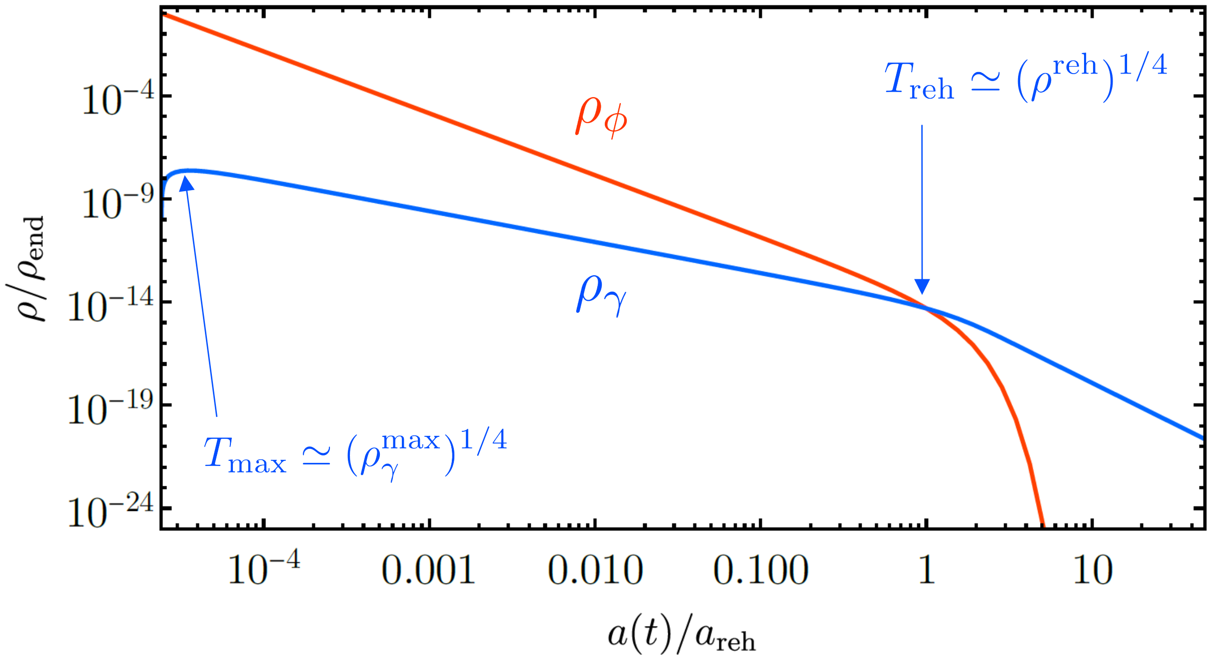
- In **fluid picture**: transition to radiation era via **dissipation** term $\equiv \Gamma_\phi \rho_\phi (1 + w_\phi)$

$$T^{\mu\nu}_{\rm tot} = T^{\mu\nu}_\phi + T^{\mu\nu}_\gamma \qquad \nabla_\mu T^{\mu\nu}_{\rm tot} = 0 \qquad \nabla_\mu T^{\mu\nu}_\phi = -\nabla_\mu T^{\mu\nu}_\gamma$$

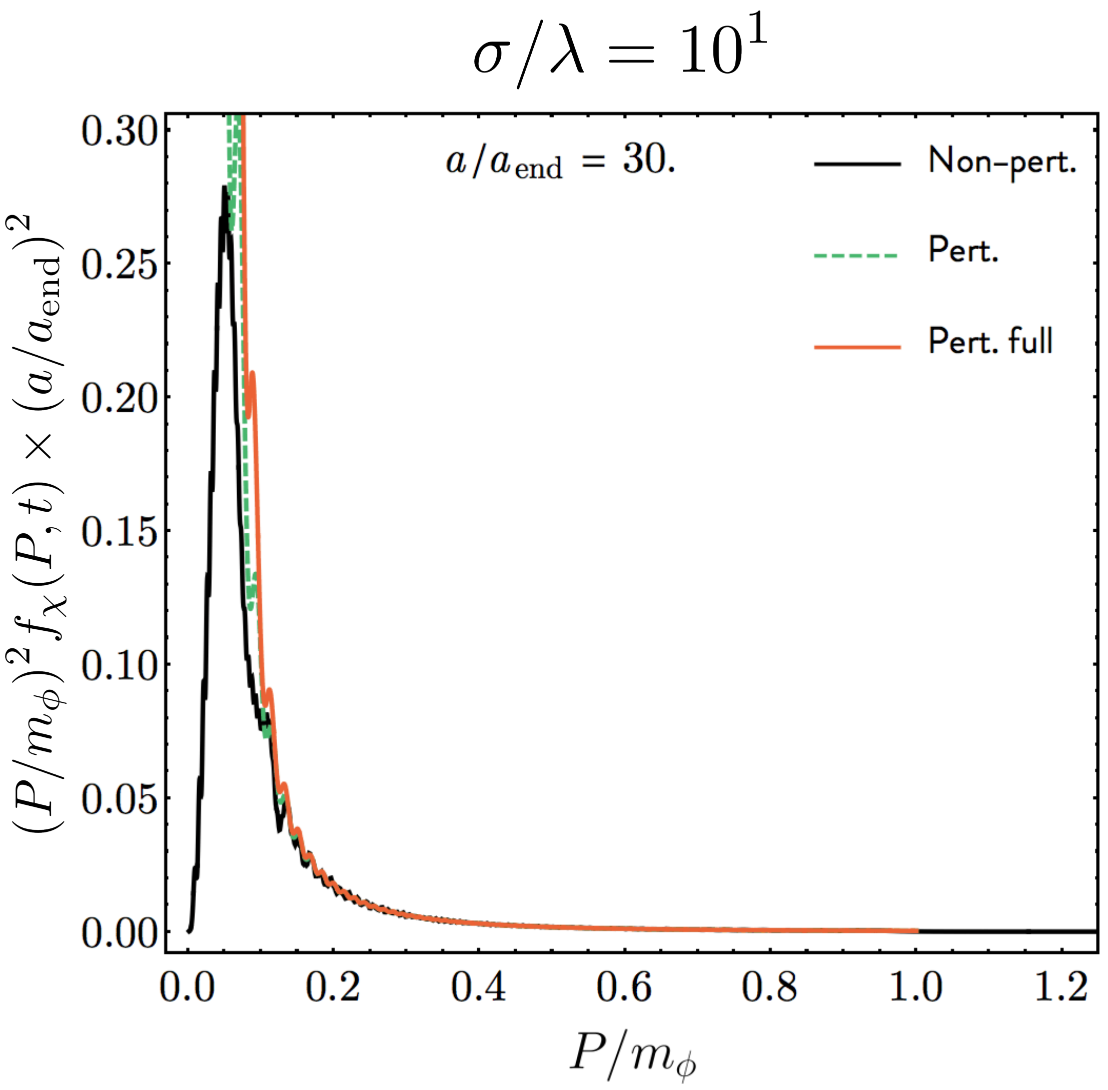
- System of **Friedmann-Boltzmann equations**

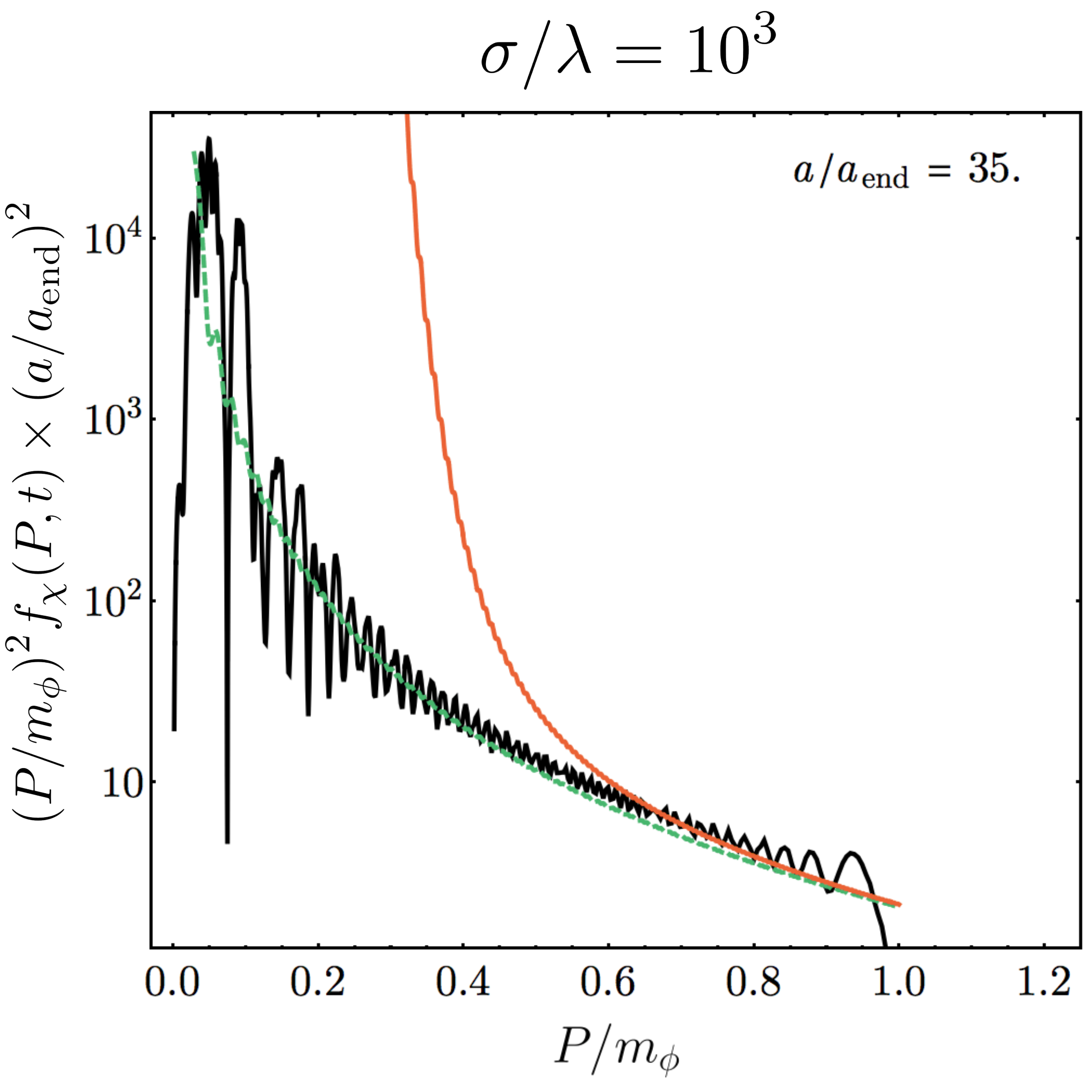
$$\begin{aligned}\dot\rho_\phi + 3H(1+w_\phi)\rho_\phi &= -\Gamma_\phi \rho_\phi (1+w_\phi)\\ \dot\rho_\gamma + 4H\rho_\gamma &= \Gamma_\phi \rho_\phi (1+w_\phi)\\ H^2 &= \frac{1}{3M_{\rm Pl}}(\rho_\phi + \rho_\gamma)\end{aligned}$$

$\Gamma_\phi \ll m_\phi$

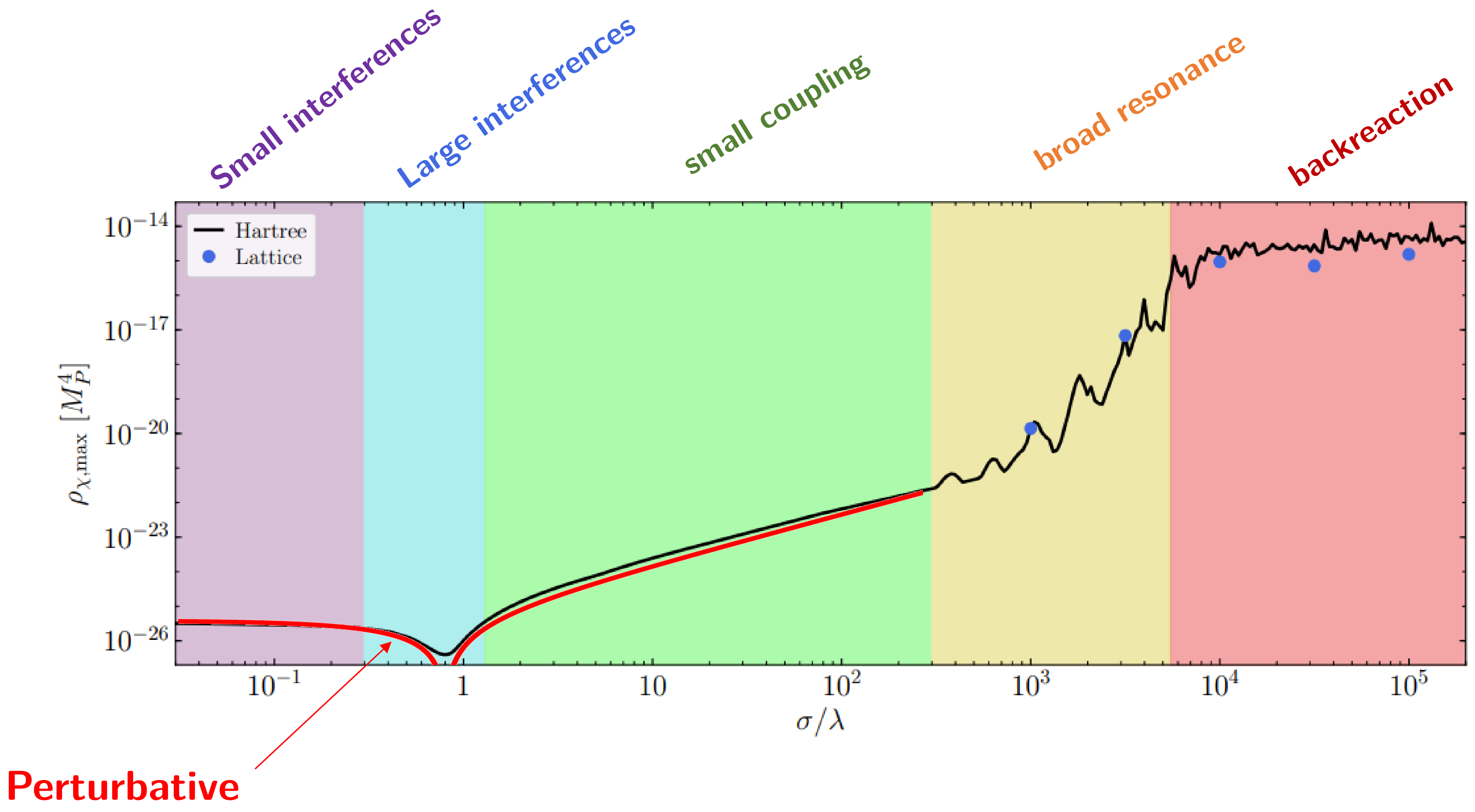


Scalar production: the phase space distribution

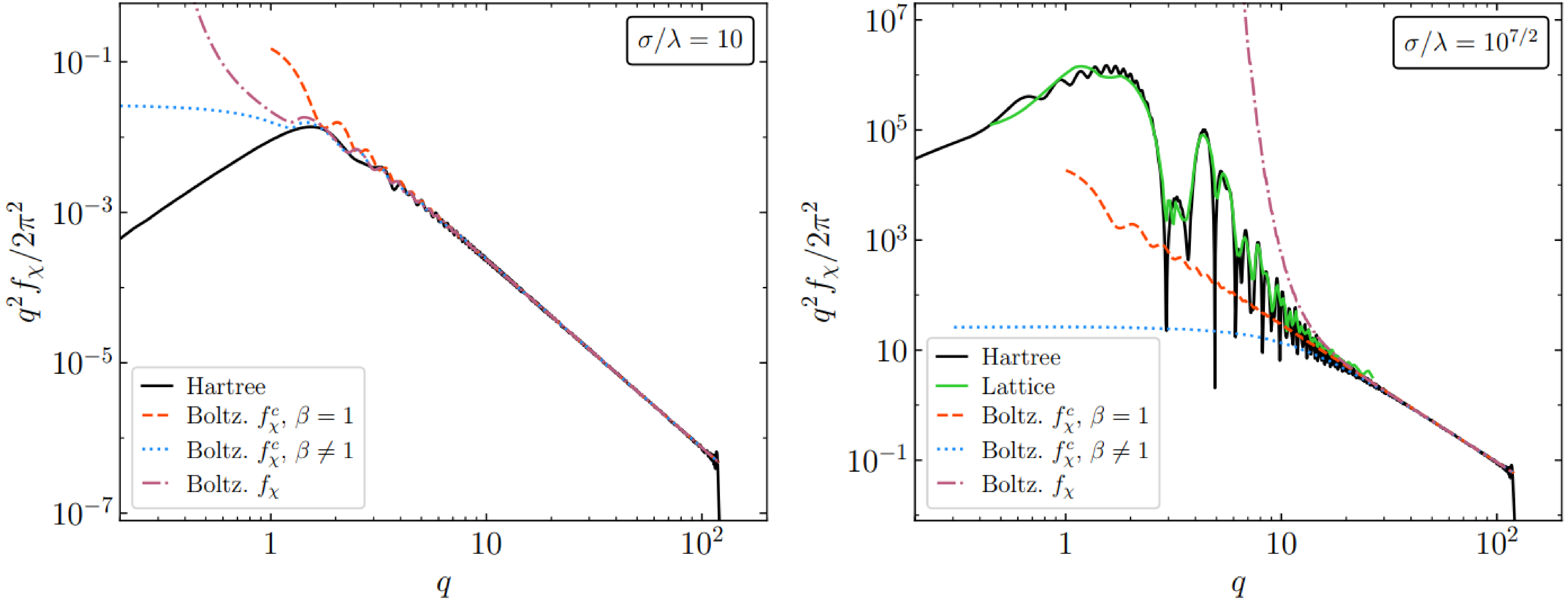




Scalar preheating phases

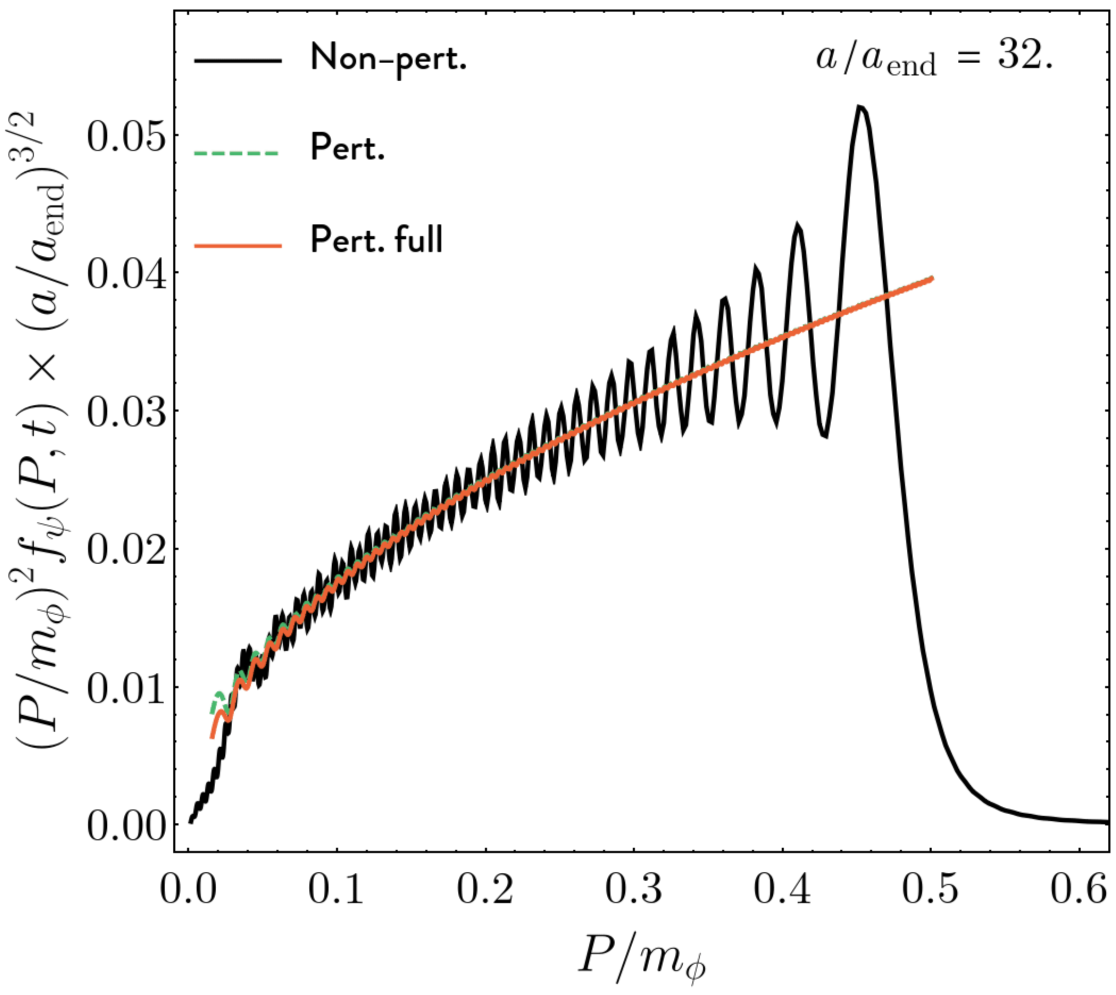


Scalar preheating

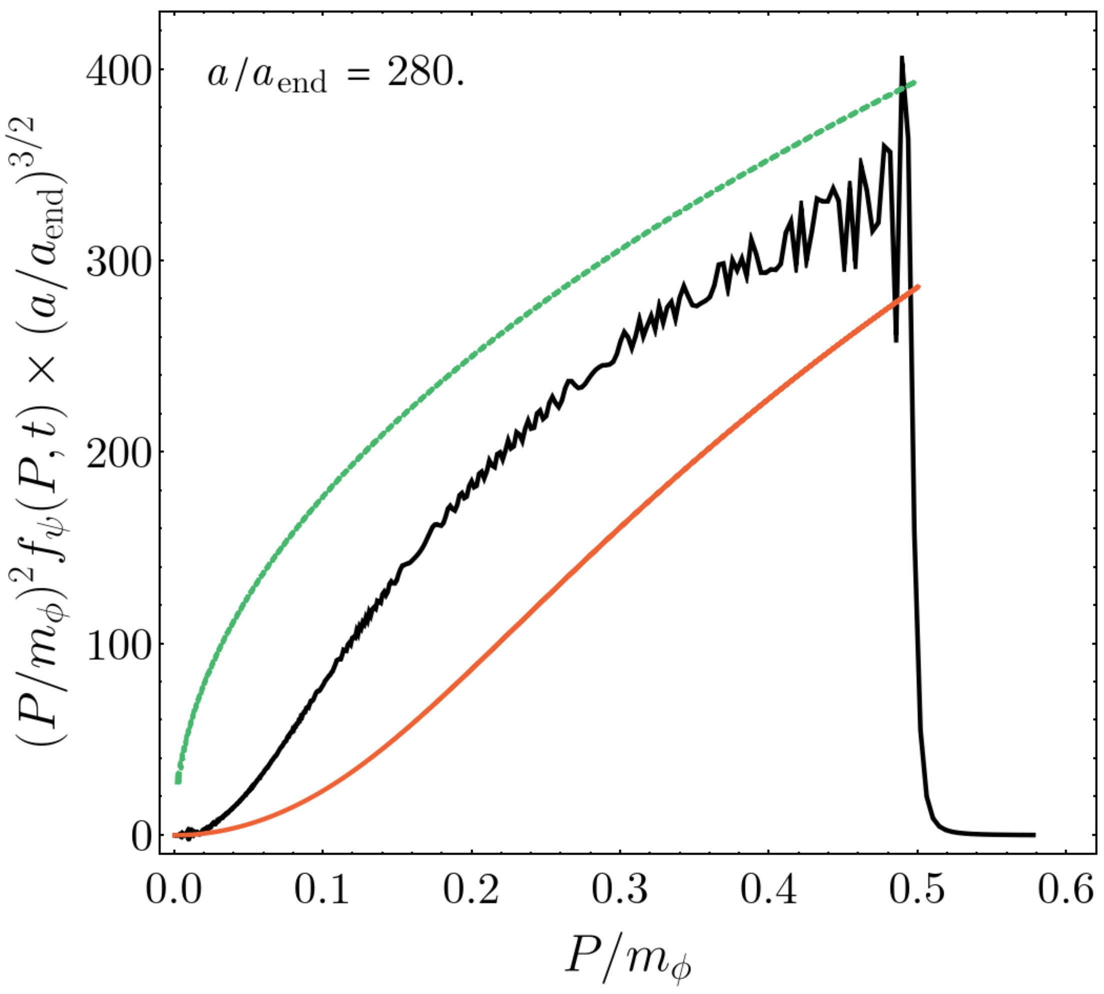


Fermion production: the phase space distribution

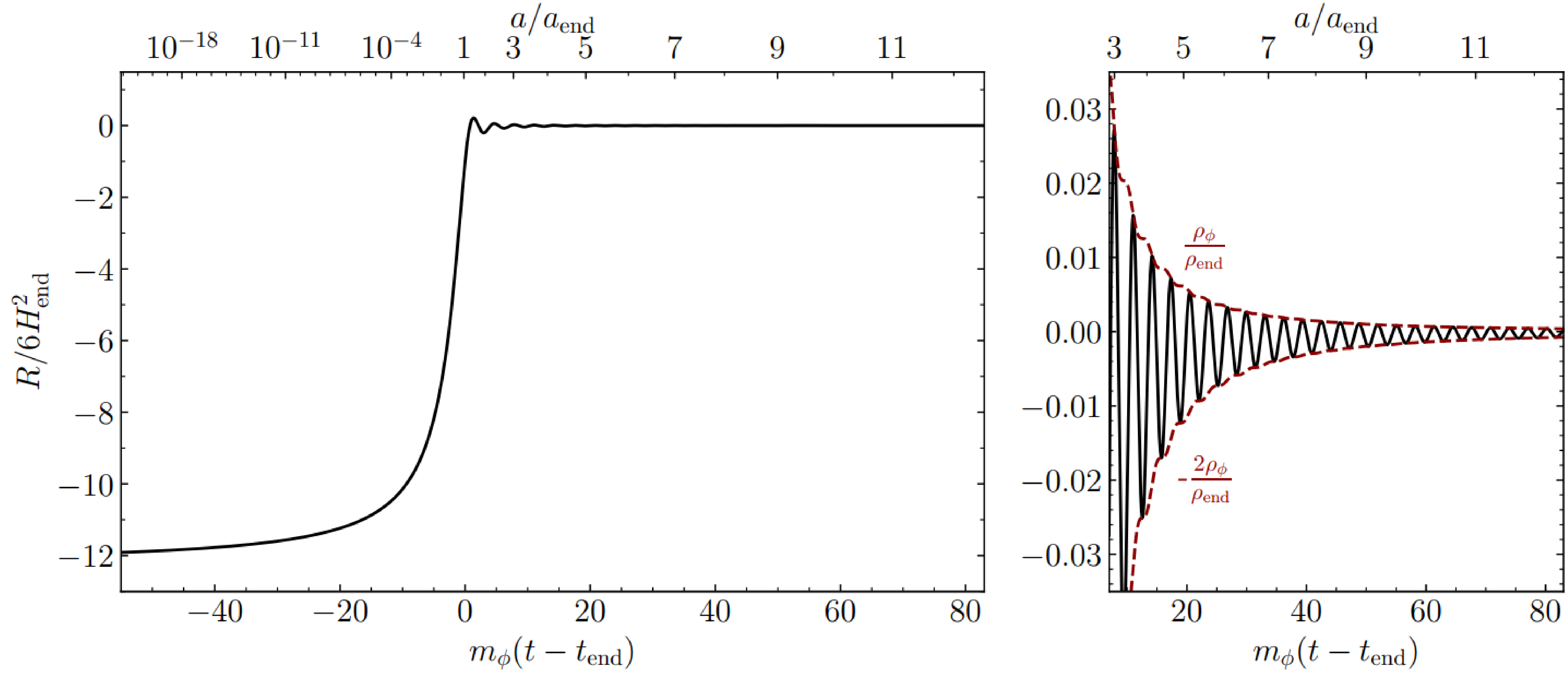
$y = 10^{-6}$



$y = 10^{-4}$



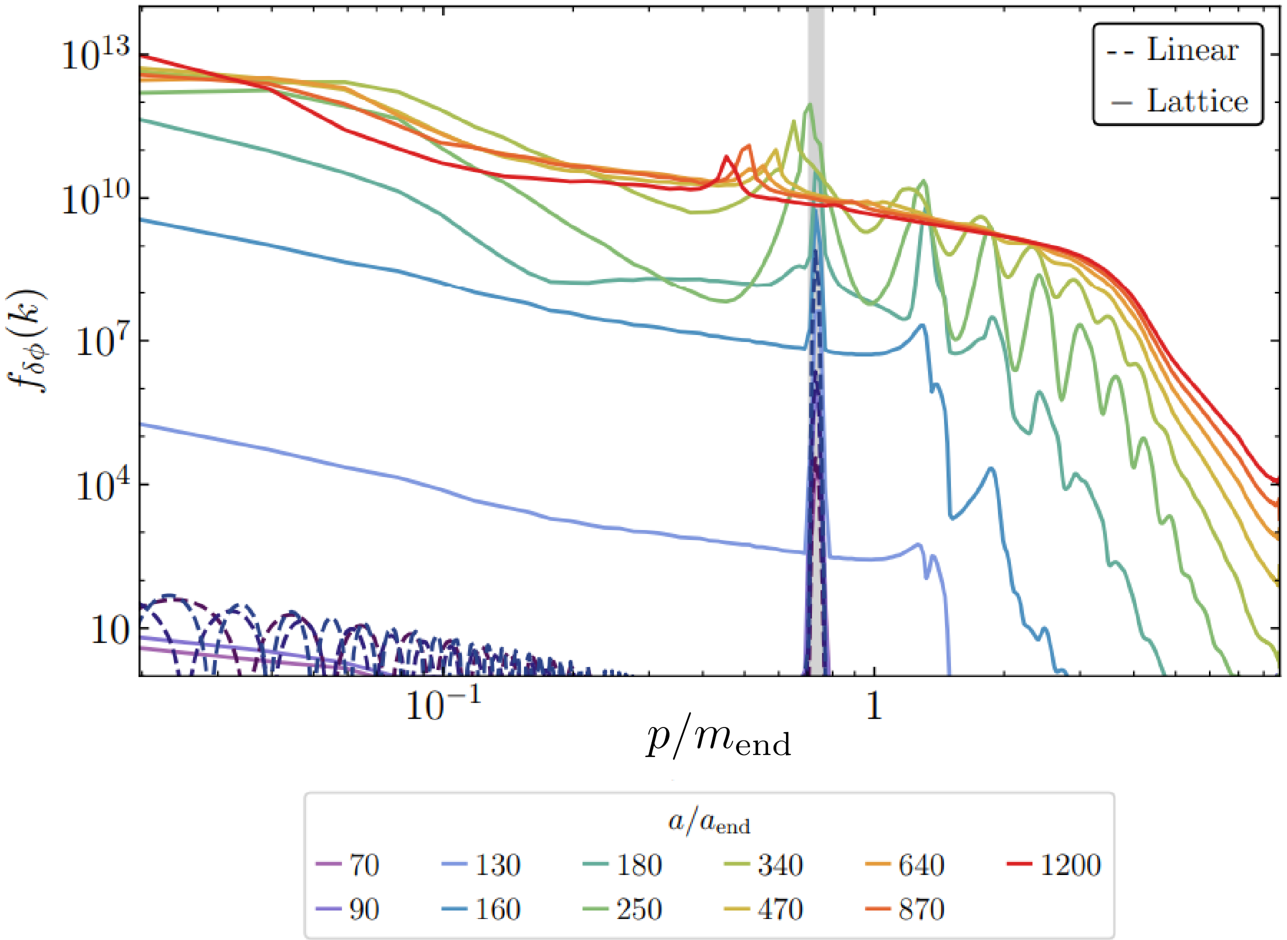
Gravitational contribution to the effective mass

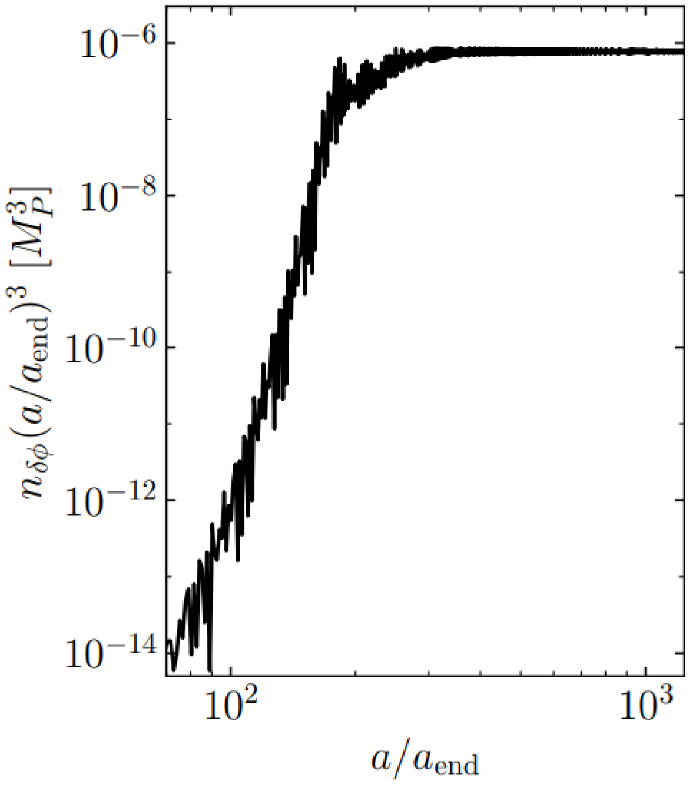


Growth of inhomogeneities: quartic case $k = 4$

- **Simulations** with $\mathcal{C}$osmo$\mathcal{L}$attice [D. G. Figueroa, A. Florio, F. Torrenti, and W. Valkenburg, "*CosmoLattice*" arXiv:2102.01031]

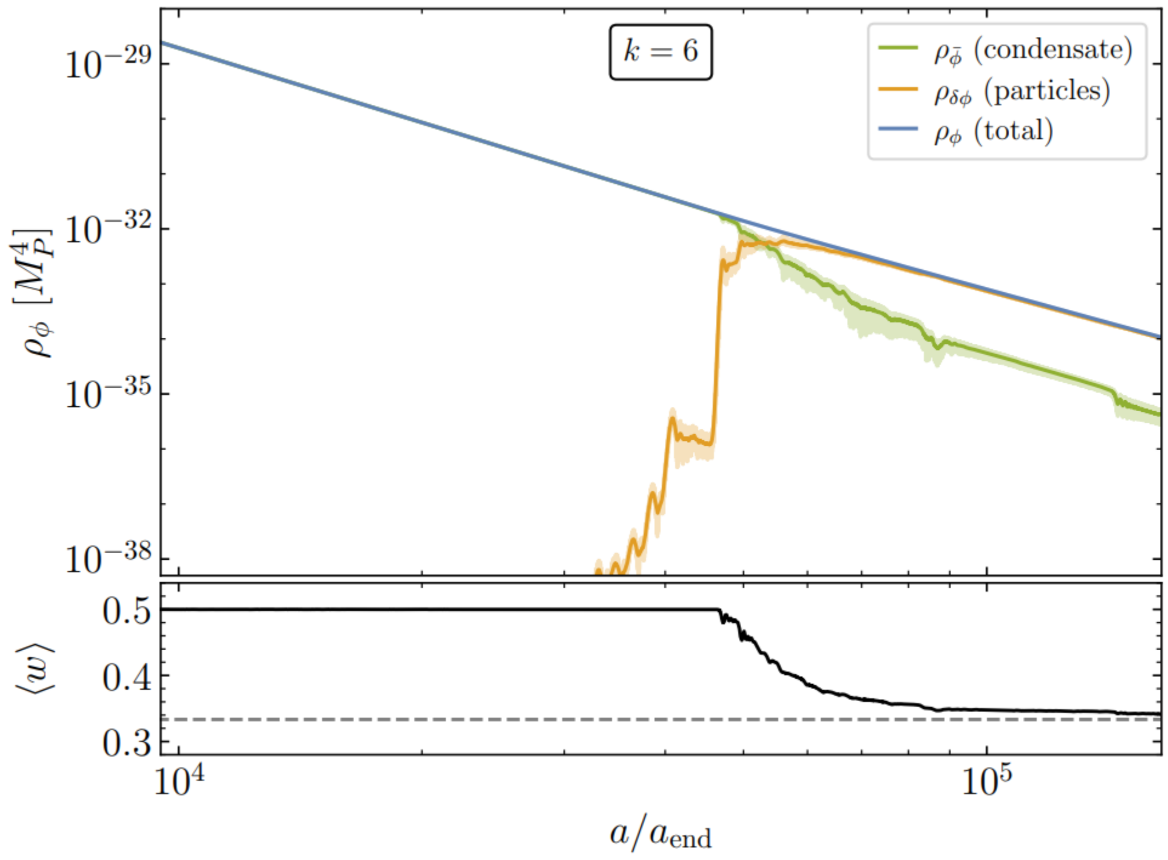
- Estimate the **occupation number** (PSD) $f_{\delta\phi}(p,t) = n_p = \frac{1}{2\omega_p}\left|\omega_p X_p - iX'_p\right|^2$

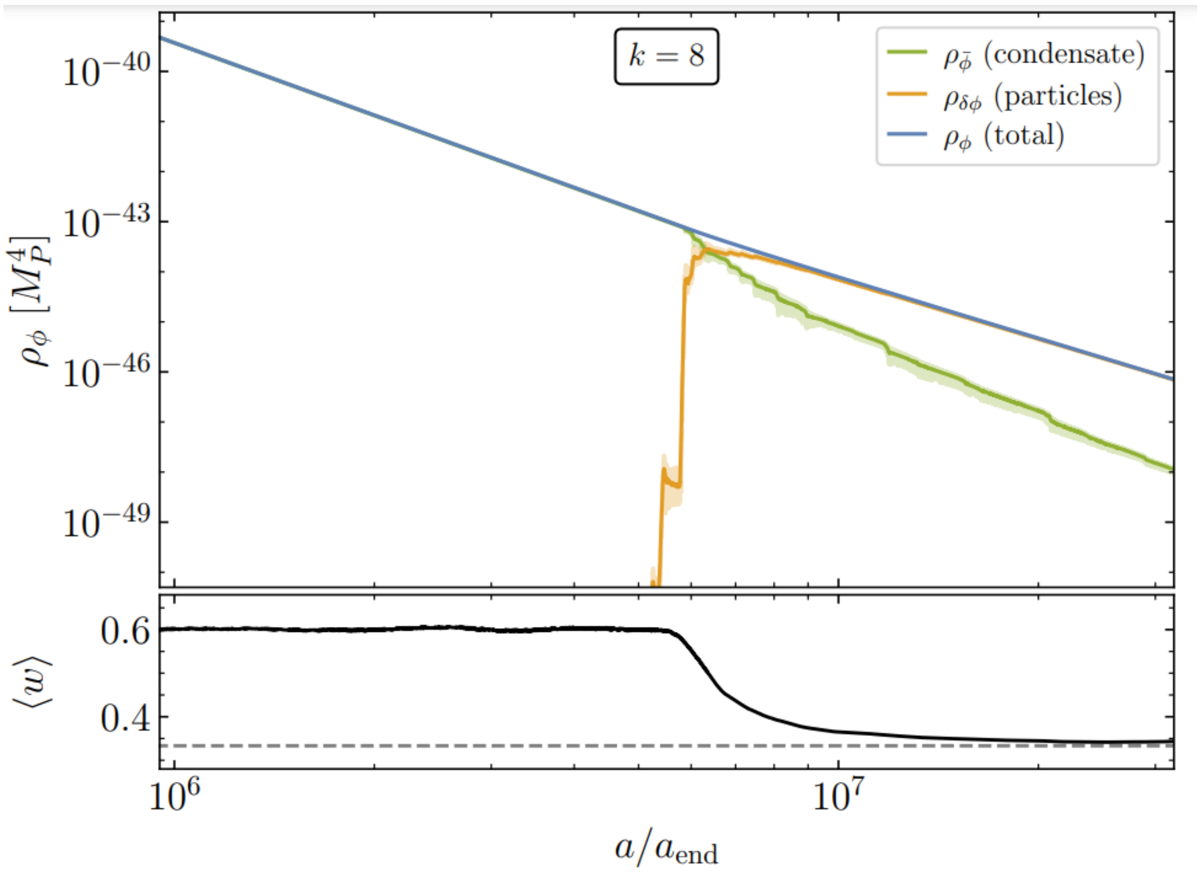




$$n_{\delta\phi} = \frac{1}{(2\pi)^3 a^3}\int d^3\boldsymbol{p}\, n_p$$

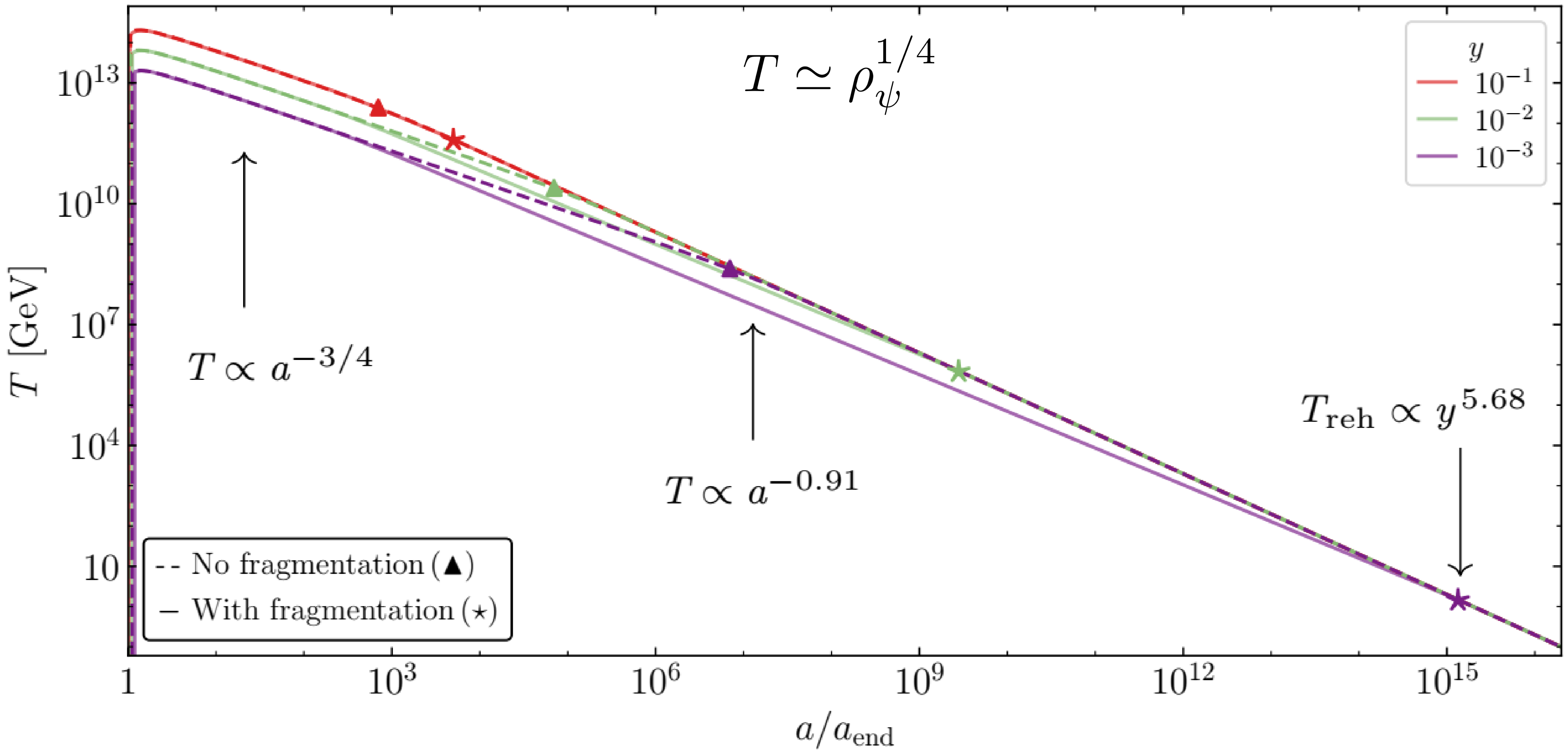
Fragmentation: $k > 4$





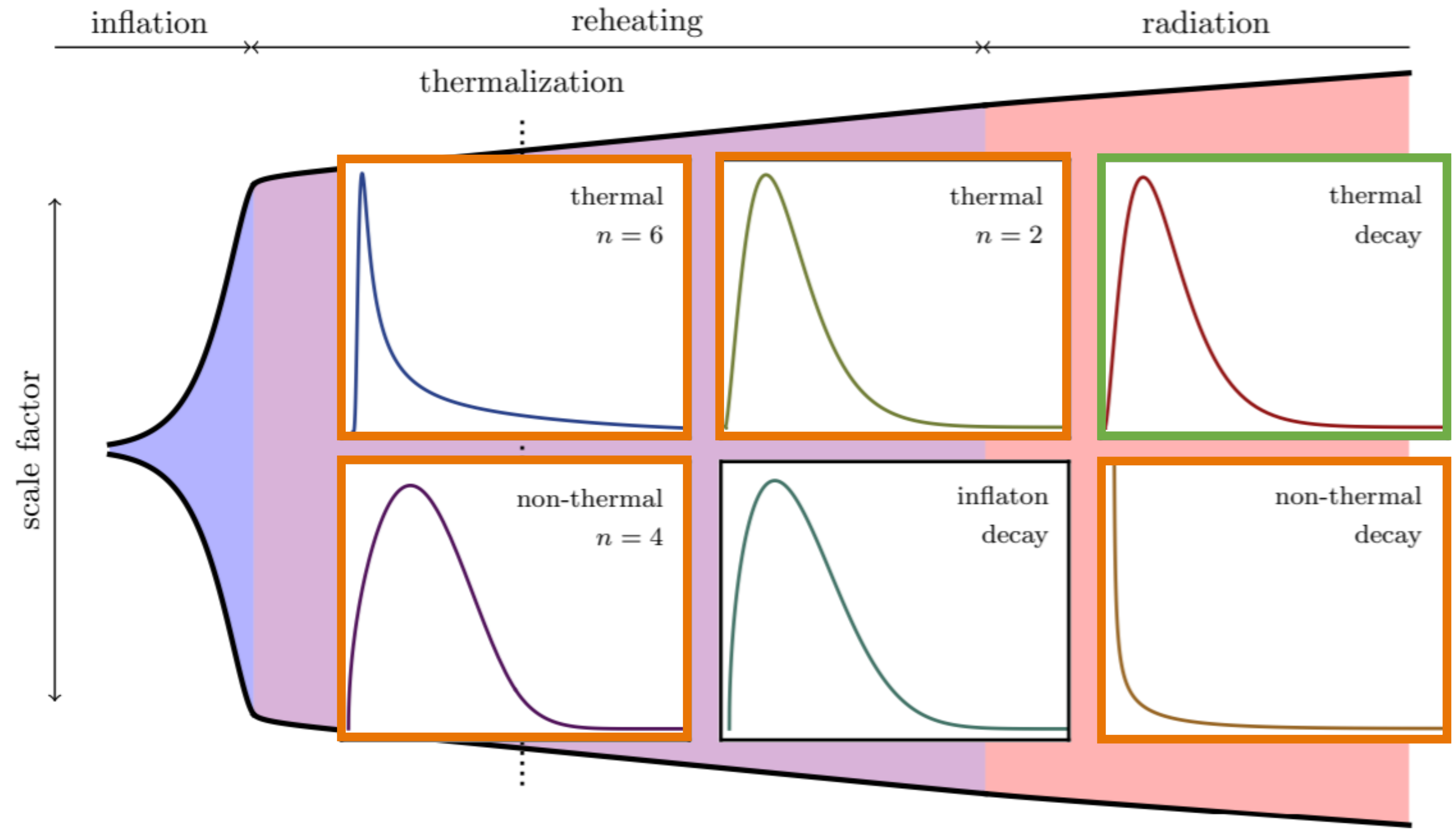
- Transition to **radiation-like** era at the onset of **fragmentation**
- Fragmentation takes **longer** for larger ***k*** but the condensate **subsists**!

Effect on reheating temperature for $k = 4$



- Fragmentation **suppresses efficiency** of **reheating** process

DM phase space distribution from freeze-in scenarios



[G. Ballesteros, M A. G. Garcia, **MP**, JCAP 03 (2021) 101]

Freeze-in via scattering

Freeze-in via decay

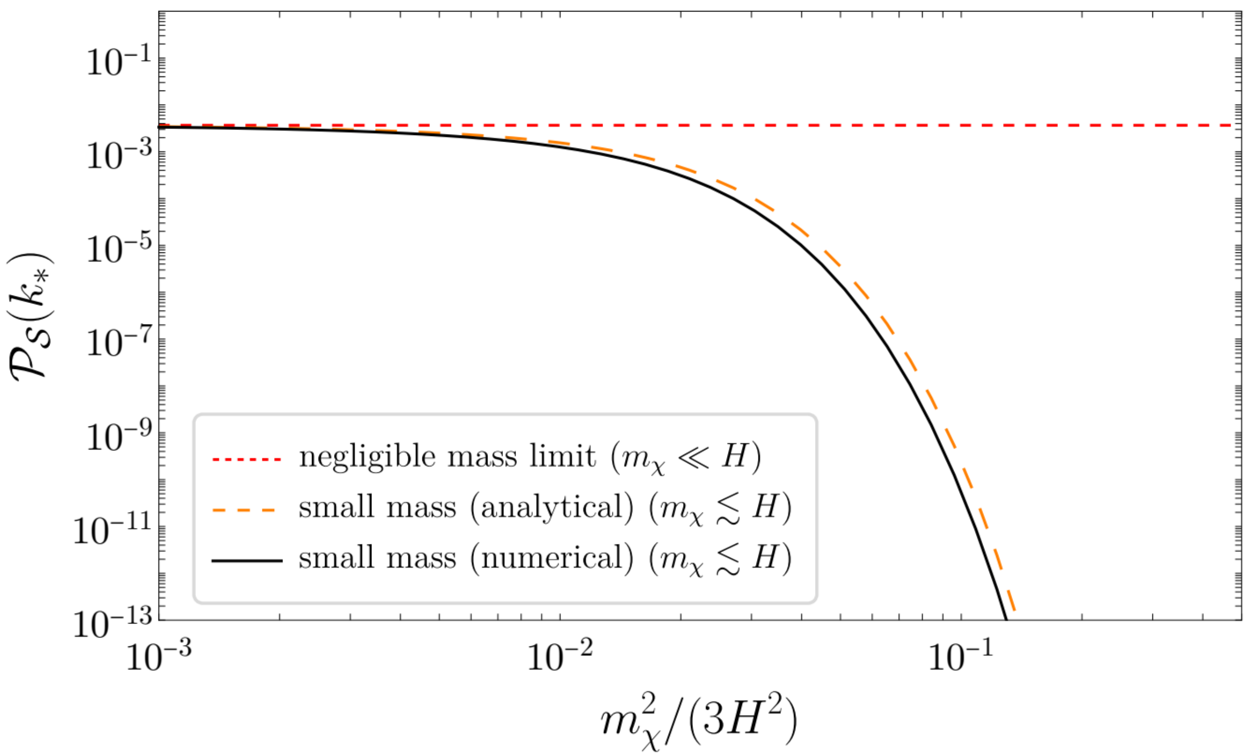
➡ Previous analysis for cases with "**well-behaved**" distributions

$$f(q) \propto q^{\alpha} \exp\left(-\beta\, q^{\gamma}\right)$$

Isocurvature constraints

- **Single field inflation** predicts ***adiabatic*** perturbations $$\frac{\delta\rho_i}{\dot{\rho}_i} = \frac{\delta\rho_j}{\dot{\rho}_j}$$

- Significant **DM production** during inflation departs from "**single clock**" inflation:
  DM ***isocurvature*** perturbations constrained by CMB

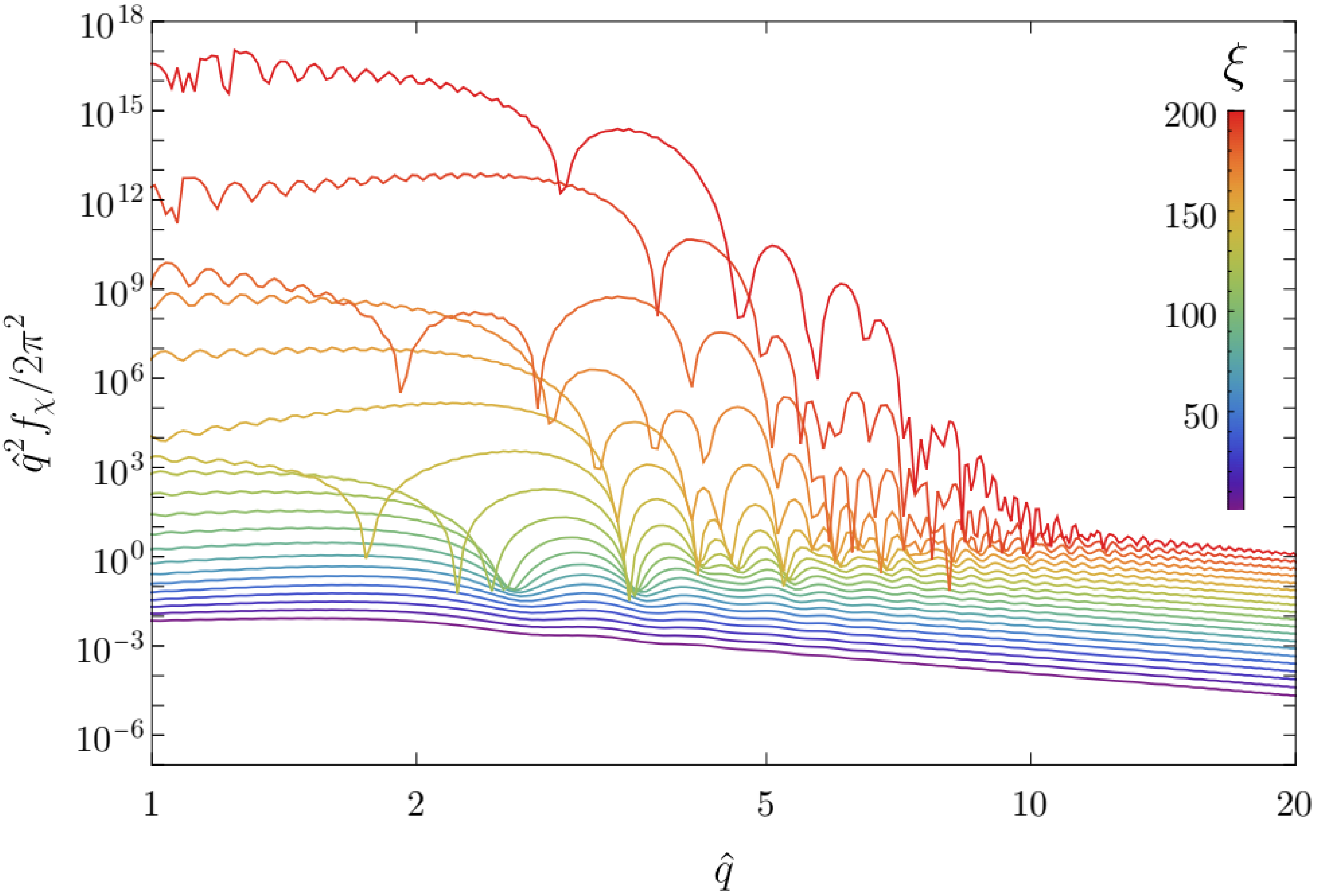


$$\beta_{\rm iso} \simeq \mathcal{P}_\mathcal{S}(k_*)/\mathcal{P}_\mathcal{R}(k_*) < \mathcal{O}(1\%)$$

$$\mathcal{P}_\mathcal{S}(k) = \frac{k^3}{(2\pi)^5 \rho_\chi^2 a^8} \int \mathrm{d}^3\boldsymbol{p}\; P_X(p, |\boldsymbol{p} - \boldsymbol{k}|)$$

$$P_X(p,q) = |X_p'|^2 |X_q'|^2 + a^4 m_\chi^4 |X_p|^2 |X_q|^2 + a^2 m_\chi^2 \left[(X_p X_p'^*)(X_q X_q'^*) + \text{h.c.}\right]$$

Gravitational production



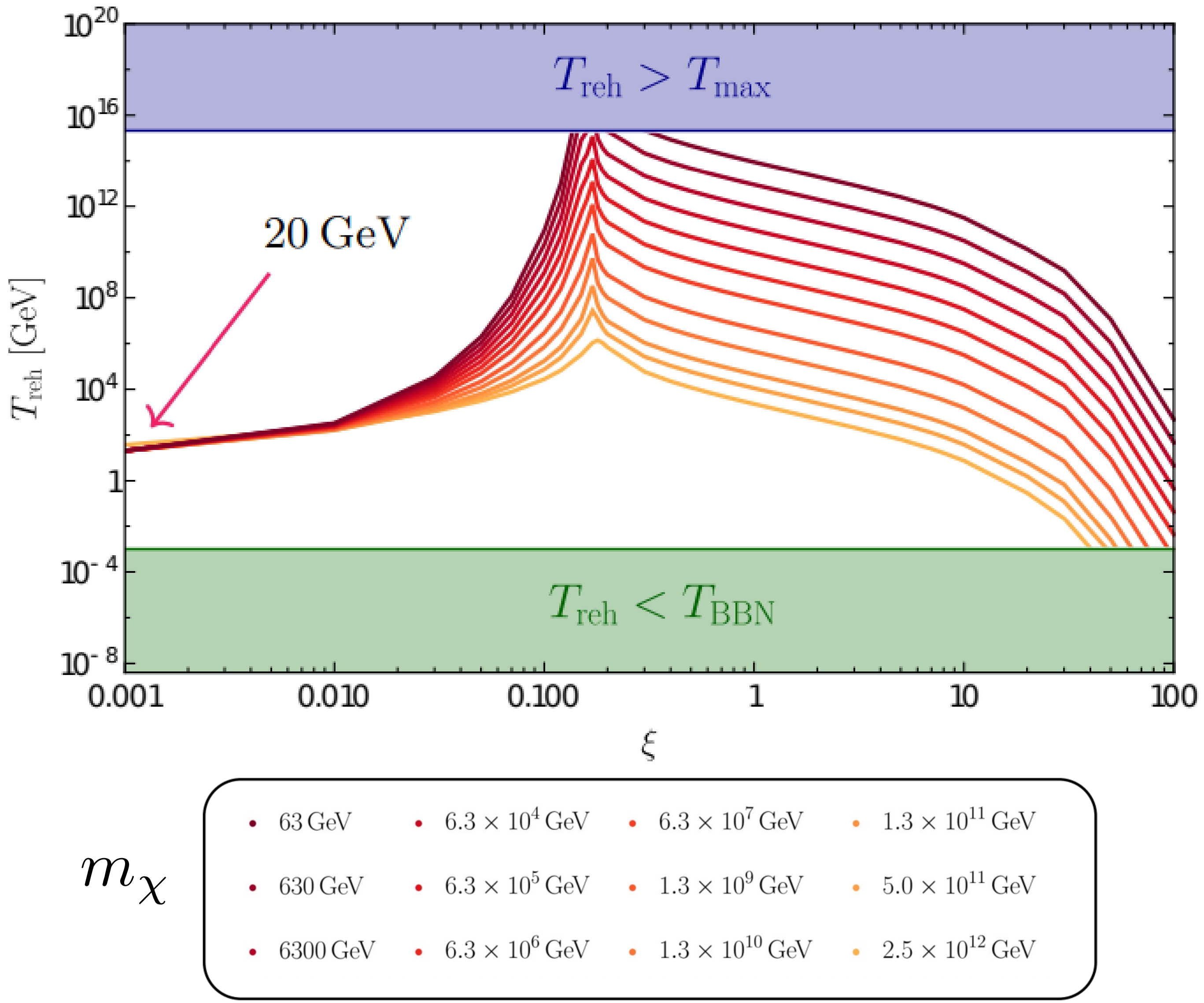
Gravitational production

If $m_\chi \ll H_{\rm end}$ and $\xi \ll 1$,

$$\Omega_{\rm DM} \simeq \frac{\rho_\chi}{\rho_c}$$

$$\propto \frac{m_\chi T_{\rm reh}}{M_P^2} \underbrace{\int dq\, q^2 f_\chi(q)}_{}$$

$$\propto m_\chi^{-1}$$



Gravitational production

If $m_\chi \ll H_{\rm end}$ and $\xi \gtrsim 1/6$,

$$\Omega_{\rm DM} \simeq \frac{\rho_\chi}{\rho_c}$$

$$\propto \frac{m_\chi T_{\rm reh}}{M_P^2} \underbrace{\int dq\, q^2 f_\chi(q)}_{F(\xi)}$$