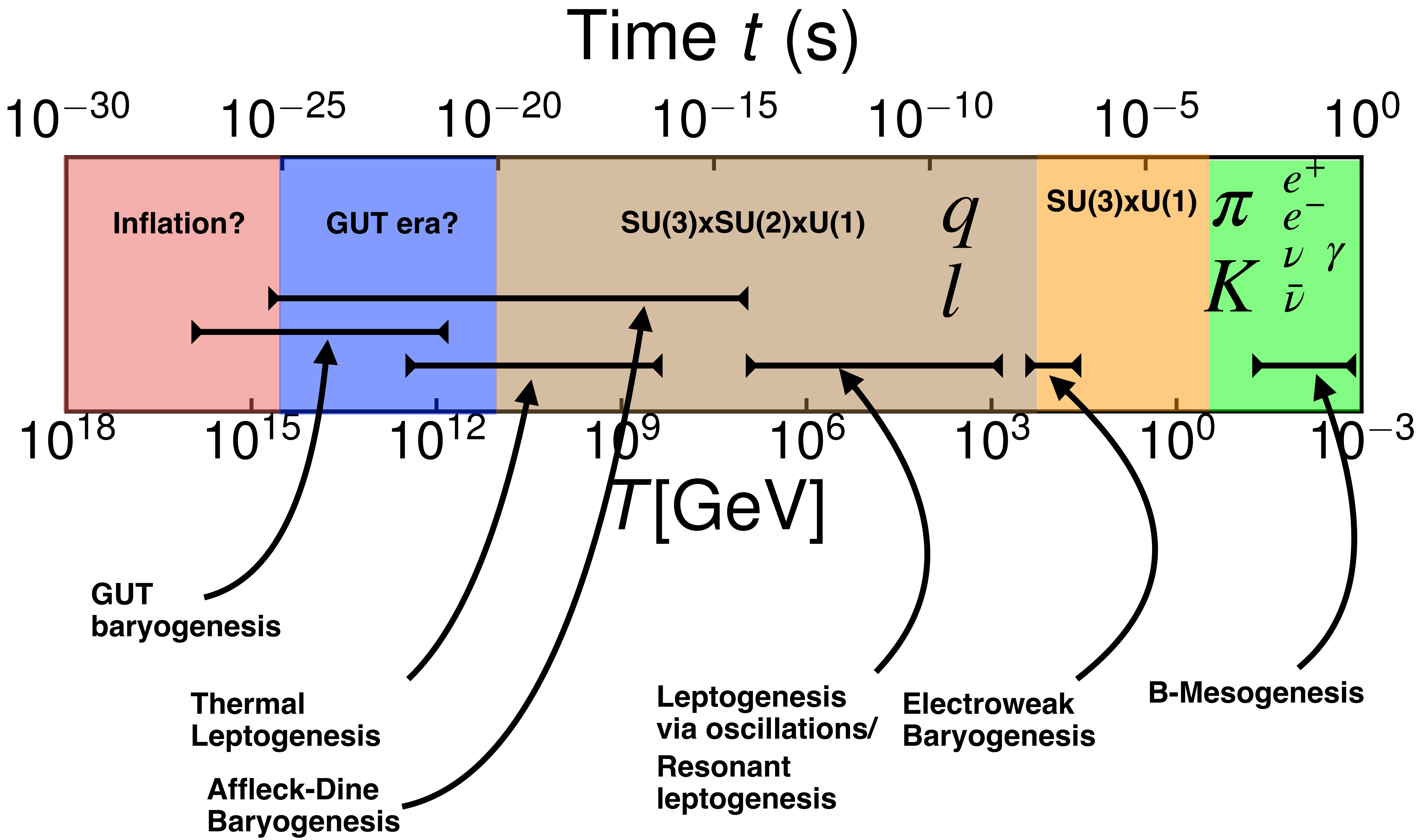
Models for Baryogenesis

Miguel Escudero Abenza
miguel.escudero@cern.ch

Lecture II

Color meets Flavor
Bad Honnef Physics School
22 March 2024

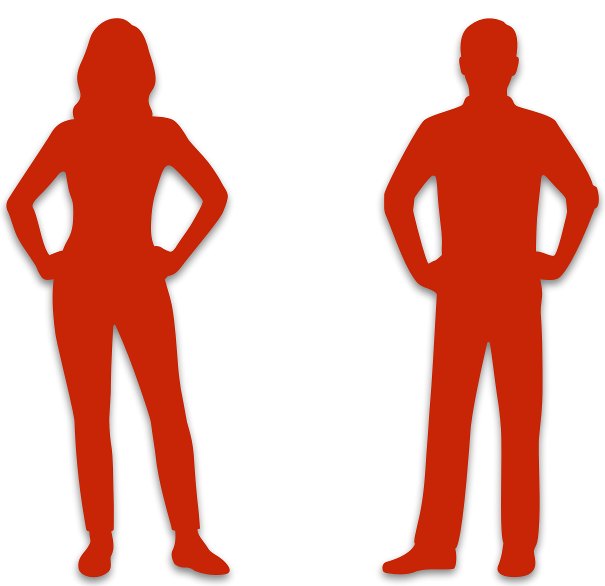
The Baryogenesis Landscape



Set Up

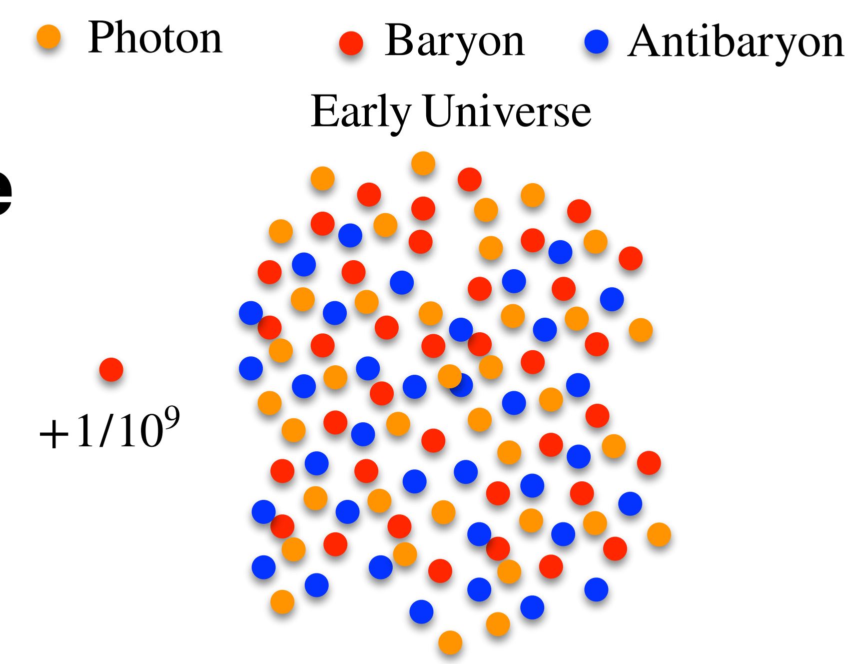
The plan is to discuss and learn as much as possible. Therefore:

Questions and Comments are most welcome, at any time!!!!



Summary & Takeaways of Lecture 1

- **Our observable Universe appears to be made out of only matter**



- **We can explain this if in the early Universe there were $+1$ baryon out of every 10^9 particle antiparticle pairs**

- **Ultra relativistic particles are expected to dominate the energy density of the early Universe**

expansion rate: $H \simeq \sqrt{\rho}/M_{\rm Pl}$

$$\rho \simeq T^4 \quad n \simeq T^3 \quad (T \gg m)$$

- **SM particles were efficiently interacting in the early Universe** $\Gamma > H$
  - 🙁 bad news for baryogenesis because departures from thermal equilibrium will be small
  - 😃 good news for baryogenesis because an array of BSM particles could have been produced!
  - 😃 once baryogenesis happens, B violating processes should not be active!

The Sakharov Conditions

The Three Sakharov Conditions (1967):

1) C and CP violation

SM: Present and potentially seizable

BSM: Generically present

2) Out of equilibrium

SM: Not present

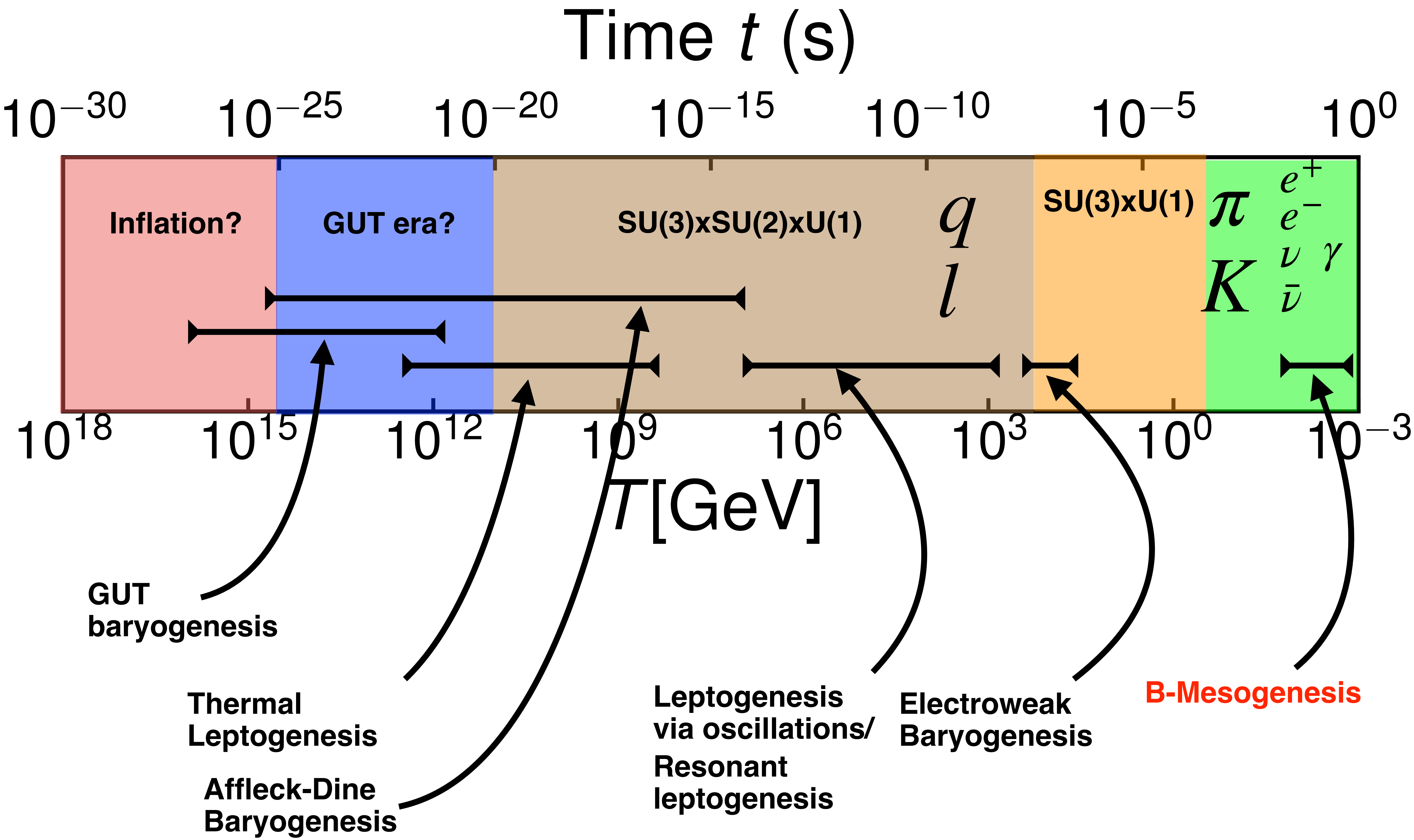
BSM: Need to generate it

3) Baryon number violation

SM: Present and active at $T \gtrsim 130\,\mathrm{GeV}$
(but still need to generate a B-L asymmetry)

BSM: Need to generate it

The Baryogenesis Landscape



Why Baryogenesis with B-Mesons?

1) B-Mesons are heavy

B-Mesons can decay into baryons $m_B > 2m_p$

2) Large CP violation in the neutral B mesons:

Already in the SM, the mixing induced CP asymmetry is:

$$|A_{\rm CP}| \simeq 10^{-5} - 10^{-3}$$

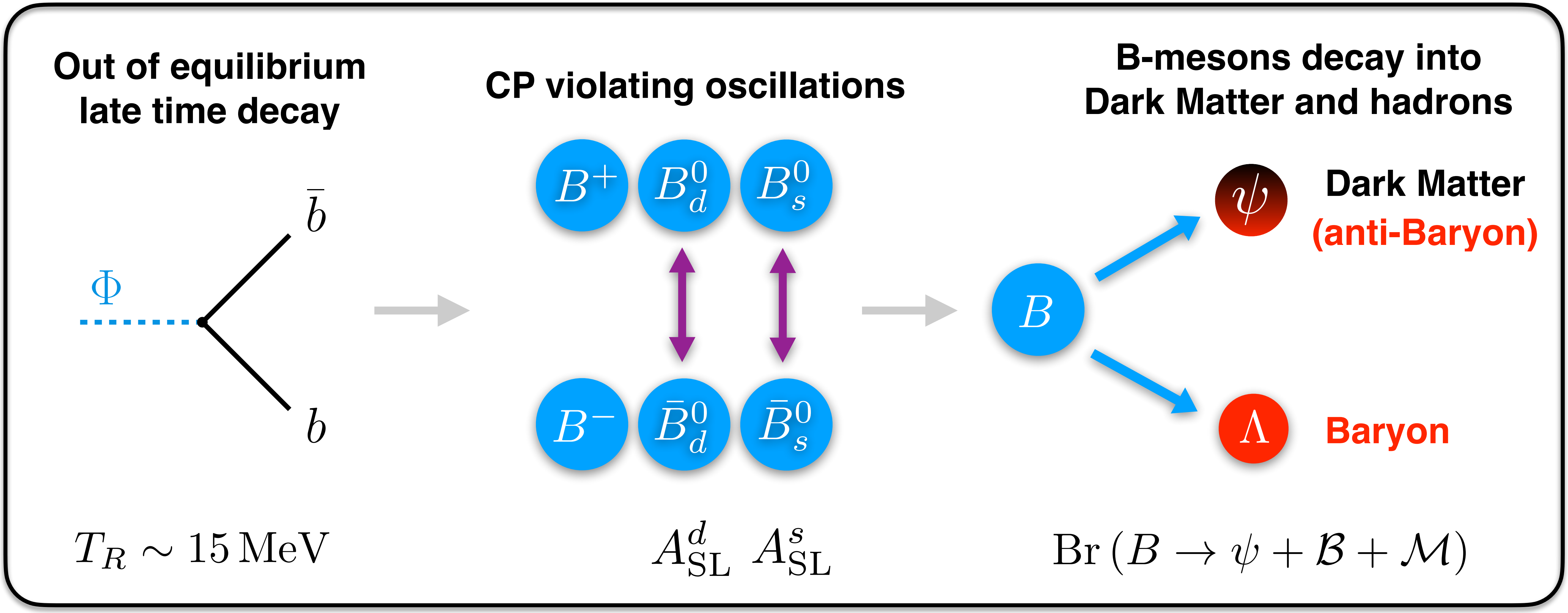
and we want to explain a Universe with $\dfrac{n_b - n_{\bar{b}}}{n_\gamma} \simeq 6 \times 10^{-10}$

3) Some of its decays are fairly unconstrained!

Back in 2018: $\text{BR}(B \to \text{Baryon} + \text{missing energy}) \lesssim 10\,\%$

As of today: $\text{BR}(B \to \text{Baryon} + \text{missing energy}) \lesssim 0.5\,\%$

B-Mesogenesis



based on: arXiv:1810.00880 with: Gilly Elor & Ann Nelson
and on: arXiv:2101.02706 Gonzalo Alonso-Álvarez & Gilly Elor

see also: **Alonso-Álvarez, Elor & Nelson [1907.10612]**
Nelson & Xiao [1901.08141]
Aitken, McKeen, Neder, Nelson [1708.01259]
Ghalsasi, McKeen, Nelson [1508.05392]

Baryogenesis from B Mesons

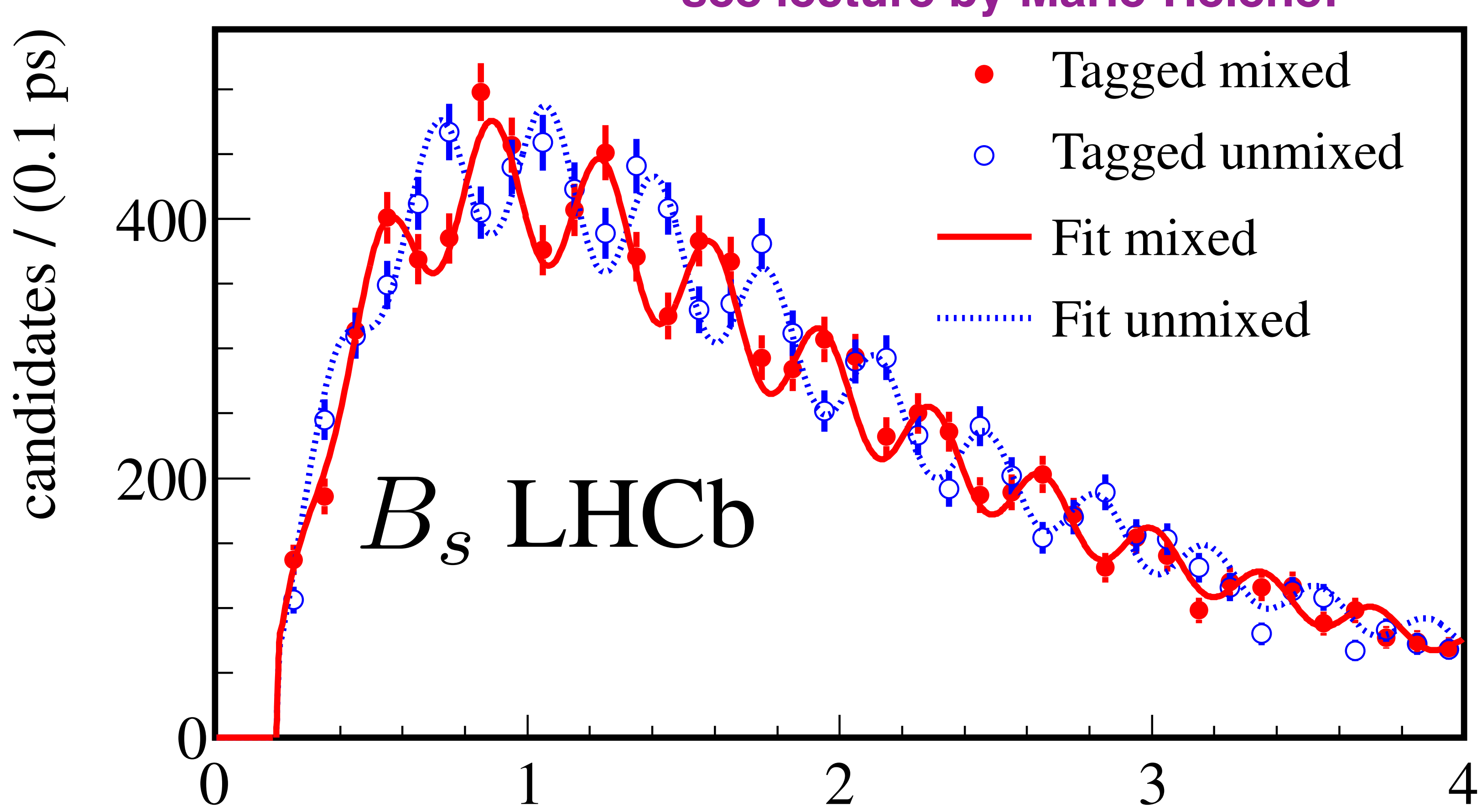
1) C and CP violation

Neutral and CP violating oscillating systems in the SM:

Kaons and D mesons cannot decay into baryons $m_{K^0} < 2m_p$, $m_{D^0} < 2m_p$

Neutral B Mesons are the perfect system: $m_B \simeq 5.3\,\mathrm{GeV}$

see lecture by Marie-Helene!

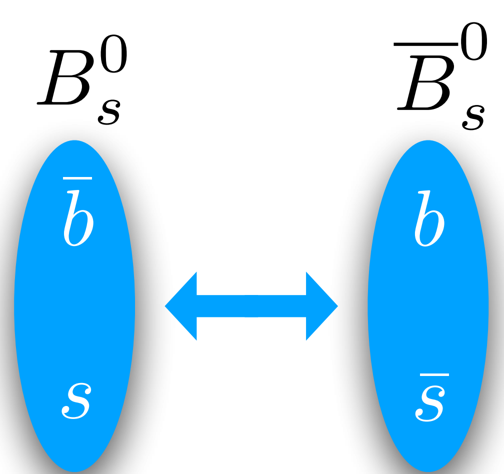


1304.4741

$$\tau_B = 1.52\,\mathrm{ps}$$

$$t_{\rm osc}/\tau_B|_{\rm B_s} = 8.56$$

$$t_{\rm osc}/\tau_B|_{\rm B_d} = 0.24$$



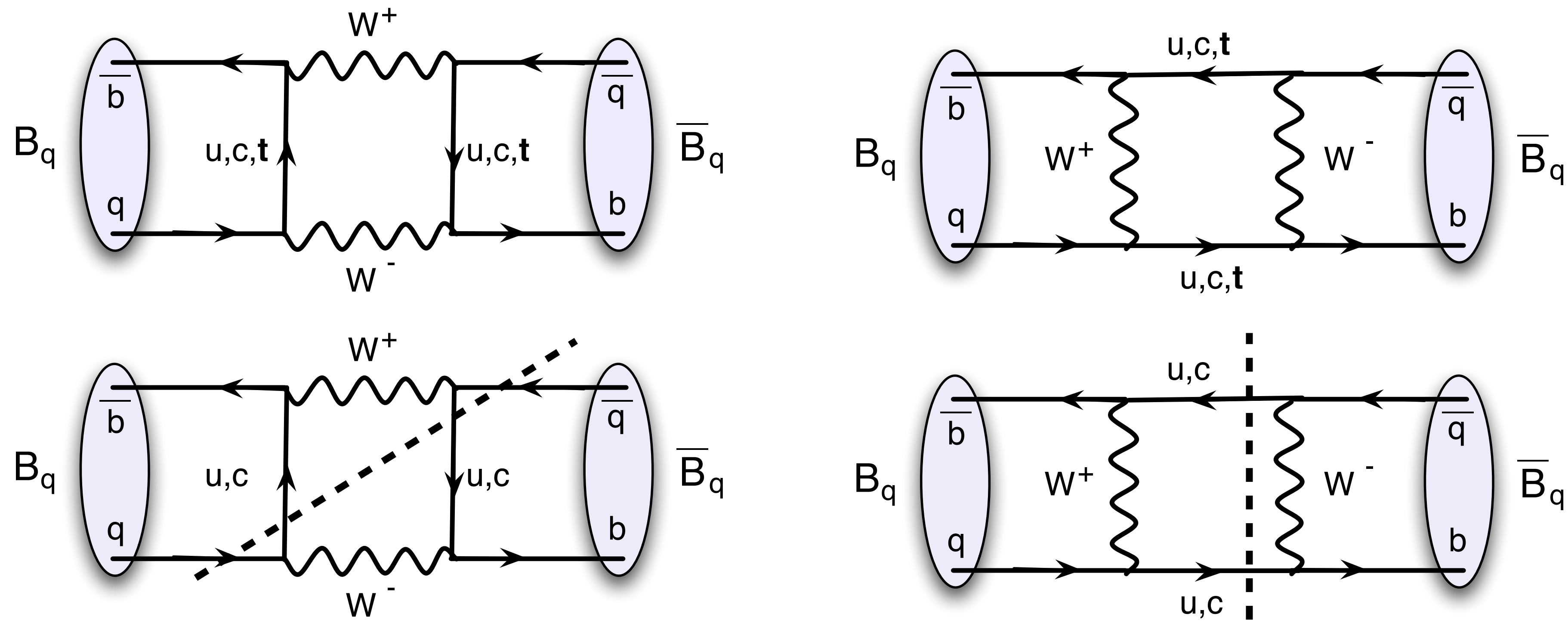
Baryogenesis from B Mesons

1) CP violation in the Meson System

SM: Box Diagrams

$M_{12} \propto$

$\Gamma_{12} \propto$



CP violating mixing requires a relative phase between Γ_{12} and M_{12}

BSM?

***Z'* models (even at tree level), Leptoquarks etc ...**

see *e.g.* Nir 9911321

Baryogenesis from B Mesons

CP violation in the neutral B-meson system

The key quantity: the semileptonic asymmetry,

$$A^q_{\rm SL} = {\rm Im}\left(\frac{\Gamma^q_{12}}{M^q_{12}}\right) = \frac{\Gamma(\overline{B}^0_q \to B^0_q \to f) - \Gamma(B^0_q \to \overline{B}^0_q \to \bar{f})}{\Gamma(\overline{B}^0_q \to B^0_q \to f) + \Gamma(B^0_q \to \overline{B}^0_q \to \bar{f})}$$

Standard Model

Lenz & Tetlalmatzi-Xolocotzi 1912.07621

$$A^d_{\rm SL}|_{\rm SM} = (-4.7 \pm 0.4) \times 10^{-4}$$

$$A^s_{\rm SL}|_{\rm SM} = (2.1 \pm 0.2) \times 10^{-5}$$

small because $(m_c/m_t)^2$ is small

Measurements

$$A^d_{\rm SL} = (-2.1 \pm 1.7) \times 10^{-3}$$

$$A^s_{\rm SL} = (-0.6 \pm 2.8) \times 10^{-3}$$

World averages (HFLAV)

- **Year ago there were plenty of BSM models that can enlarge the asymmetries up to 10^{-4}: SUSY, Extradim, LR, 2HDM, new generations, Leptoquarks, *Z′* models (see e.g. Artuso, Borissov & Lenz 1511.09466, Nebot et al. 1402.1181). Currently under investigation with Carlos Miró & Miguel Nebot**

Baryogenesis from B Mesons

2) Out of equilibrium and production of B Mesons

- **Require the presence of an out of equilibrium particle that dominates the energy density of the Universe and reheats it to a temperature of**

$$T_{RH} = \mathcal{O}(10\,\mathrm{MeV})$$

- **This particle should be very weakly coupled, with lifetimes**

$$\tau_\Phi = \mathcal{O}(10^{-3}\,\mathrm{s})$$

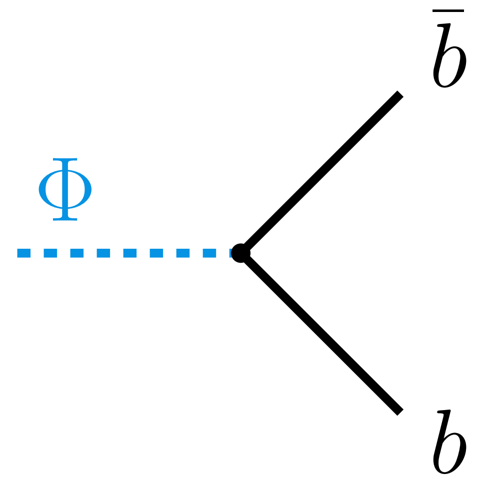
- **The decays don't spoil BBN or the CMB provided** $T_{\rm RH} \gtrsim 4.7\,\mathrm{MeV}$

de Salas *et al*. 1511.00672
Hasegawa *et al*. 1908.10189

Baryogenesis from B Mesons

2) Out of equilibrium and production of B Mesons

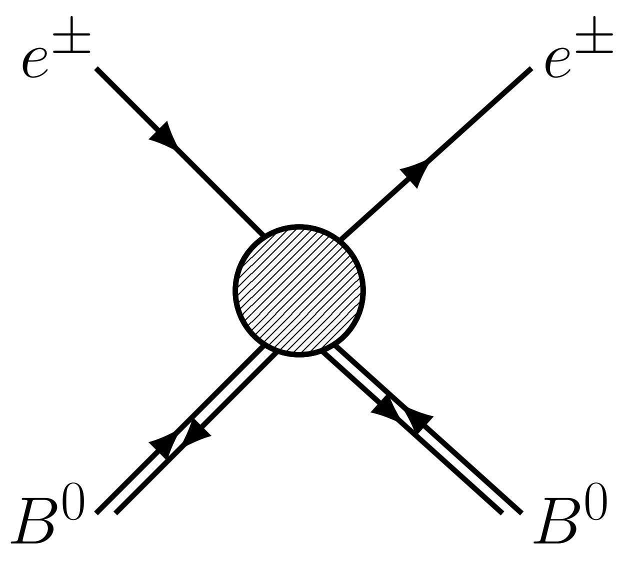
- **Scalar particle with** $M_\Phi \in 11 - 100\,\mathrm{GeV}$ **and** $\tau_\Phi = \mathcal{O}(10^{-3}\,\mathrm{s})$ **generically decays into b-quarks**



- **b-quarks hadronize at** $T < T_{\rm QCD} \sim 200\,\mathrm{MeV}$

- **Coherent oscillations in the B^0 system are maintained in the early Universe for temperatures*:**

$$T \lesssim 20\,\mathrm{MeV}$$



***In preparing these lectures noticed that $\gamma + B_0 \to B_0^\star$ could be a source of decoherence (under investigation)**

Baryogenesis and DM from B Mesons

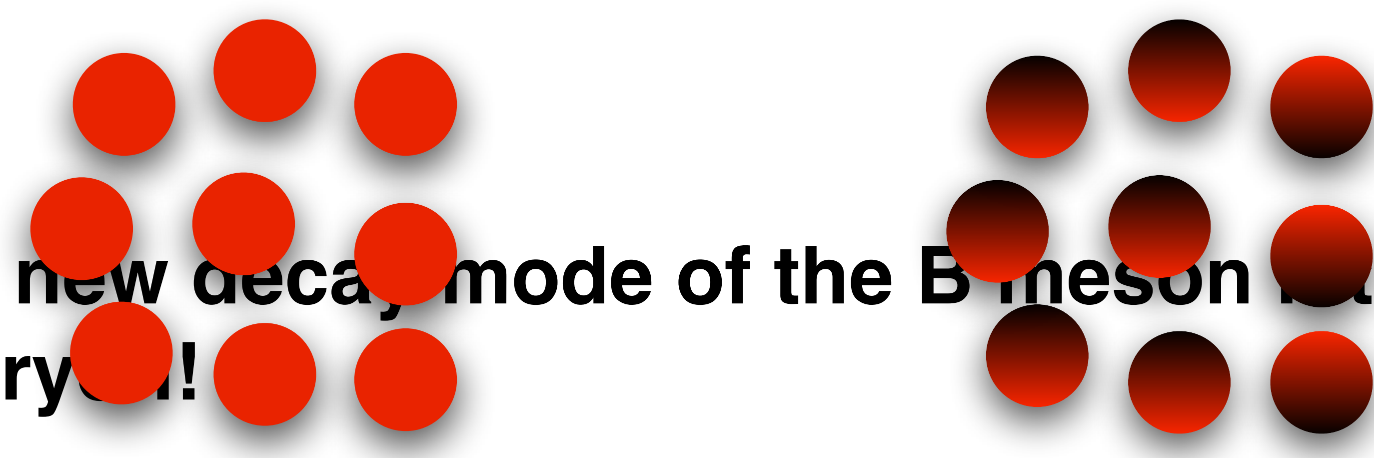
3) Baryon number violation?

- **Baryon number is conserved in our scenario:** $\Delta B = 0$

  In a similar spirit to *Hylogenesis* by Davoudiasl, Morrissey, Sigurdson, Tulin 1008.2399

- **We make Dark Matter an anti-Baryon and generate an asymmetry between the two sectors thanks to the CP violating oscillations and subsequents decays of B-mesons.**

- **Require a new decay mode of the B meson into DM and a visible Baryon!**



Visible Sector (Baryons)

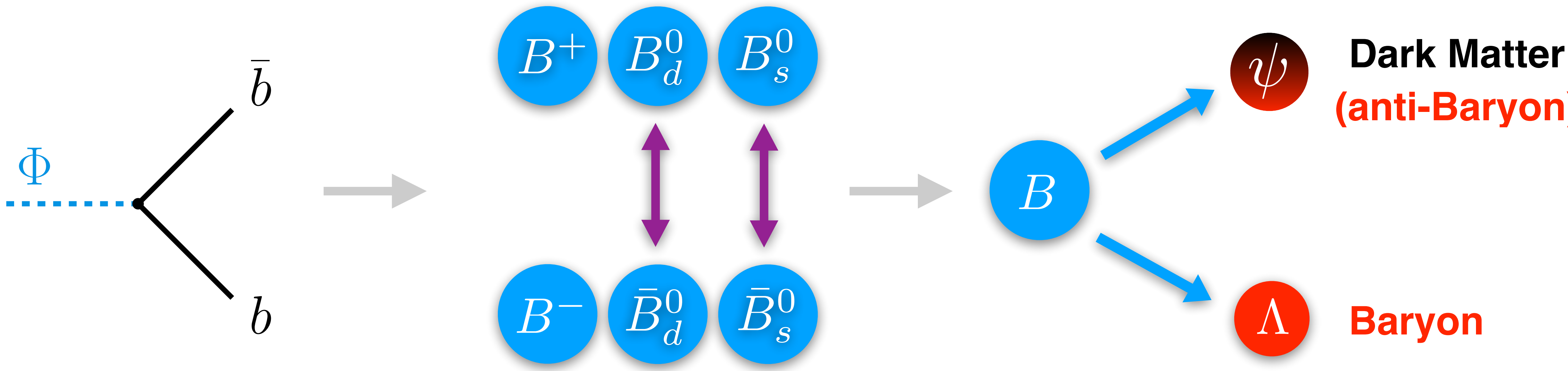
Dark Sector (anti-Baryons)

A Summary of the Mechanism

Out of equilibrium late time decay

CP violating oscillations

B-mesons decay into Dark Matter and hadrons



$T_R \sim 15\,\text{MeV}$

$A_{\text{SL}}^d \;\; A_{\text{SL}}^s$

$\text{Br}\,(B \to \psi + \mathcal{B} + \mathcal{M})$

Baryogenesis and **Dark Matter**

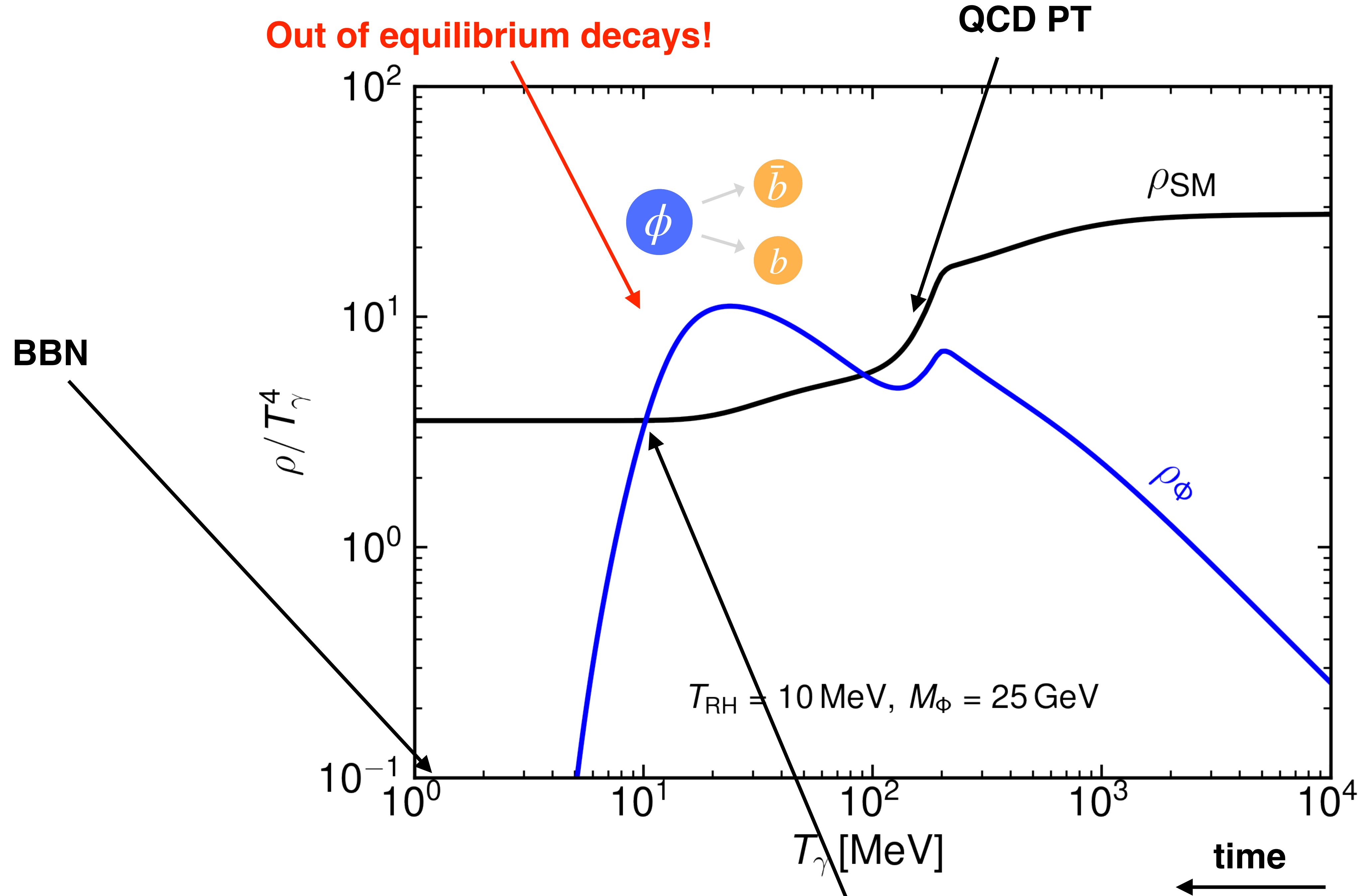
$Y_B = 8.7 \times 10^{-11}$

$\Omega_{\text{DM}} h^2 = 0.12$

With:

$$Y_B \simeq 8.7 \times 10^{-11} \frac{\text{Br}(B \to \psi + \mathcal{B} + \mathcal{M})}{10^{-2}} \sum_q \alpha_q \frac{A_{\text{SL}}^q}{10^{-4}}$$

Early Universe Picture



$$n_\Phi = \rho_\Phi / M_\Phi = g_\star (\pi^2/30) T_{\rm RH}^4 / M_\Phi$$

$$n_\gamma = 2(\pi^2/30) T_{\rm RH}^3$$

$$n_\Phi / n_\gamma = \frac{g_\star}{2} \frac{T_{\rm RH}}{M_\Phi}$$

Parameter Space

$$\eta_b \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_\Phi \times \mathrm{BR} \times \epsilon_{\mathrm{CP}}}{n_\gamma} \simeq \mathrm{BR}(B \to \psi + \mathrm{Baryon} + \mathrm{X}) \times \frac{f_b}{2} \times A_{\mathrm{SL}} \frac{1}{1+(\Gamma/\Delta M)^2} \frac{T_{\mathrm{RH}}}{M_\Phi}$$

Excercise: show that by taking the formulae from Lecture 2 of Alexey

$$g_\pm(t) = \frac{1}{2}\left(\exp\left(-\frac{\Gamma_H t}{2} - i m_H t\right) \pm \exp\left(-\frac{\Gamma_L t}{2} - i m_L t\right)\right)$$

$$\mathrm{TIP}(M^0 \to M^0) = \int_0^\infty |g_+(t)|^2 dt$$

$$\mathrm{TIP}(M^0 \to \bar{M}^0) = \int_0^\infty |g_-(t)|^2 (1 - A_{\mathrm{SL}}) dt$$

$$\mathrm{TIP}(\bar{M}^0 \to \bar{M}^0) = \int_0^\infty |g_+(t)|^2 dt$$

$$\mathrm{TIP}(\bar{M}^0 \to M^0) = \int_0^\infty |g_-(t)|^2 (1 + A_{\mathrm{SL}}) dt$$

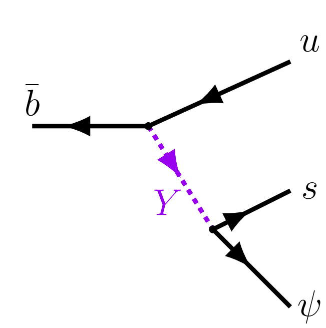
With:

$$Y_B \simeq 8.7 \times 10^{-11} \, \frac{\mathrm{Br}(B \to \psi + \mathcal{B} + \mathcal{M})}{10^{-2}} \sum_q \alpha_q \frac{A^q_{\mathrm{SL}}}{10^{-4}}$$

Constrained directly by B-factories

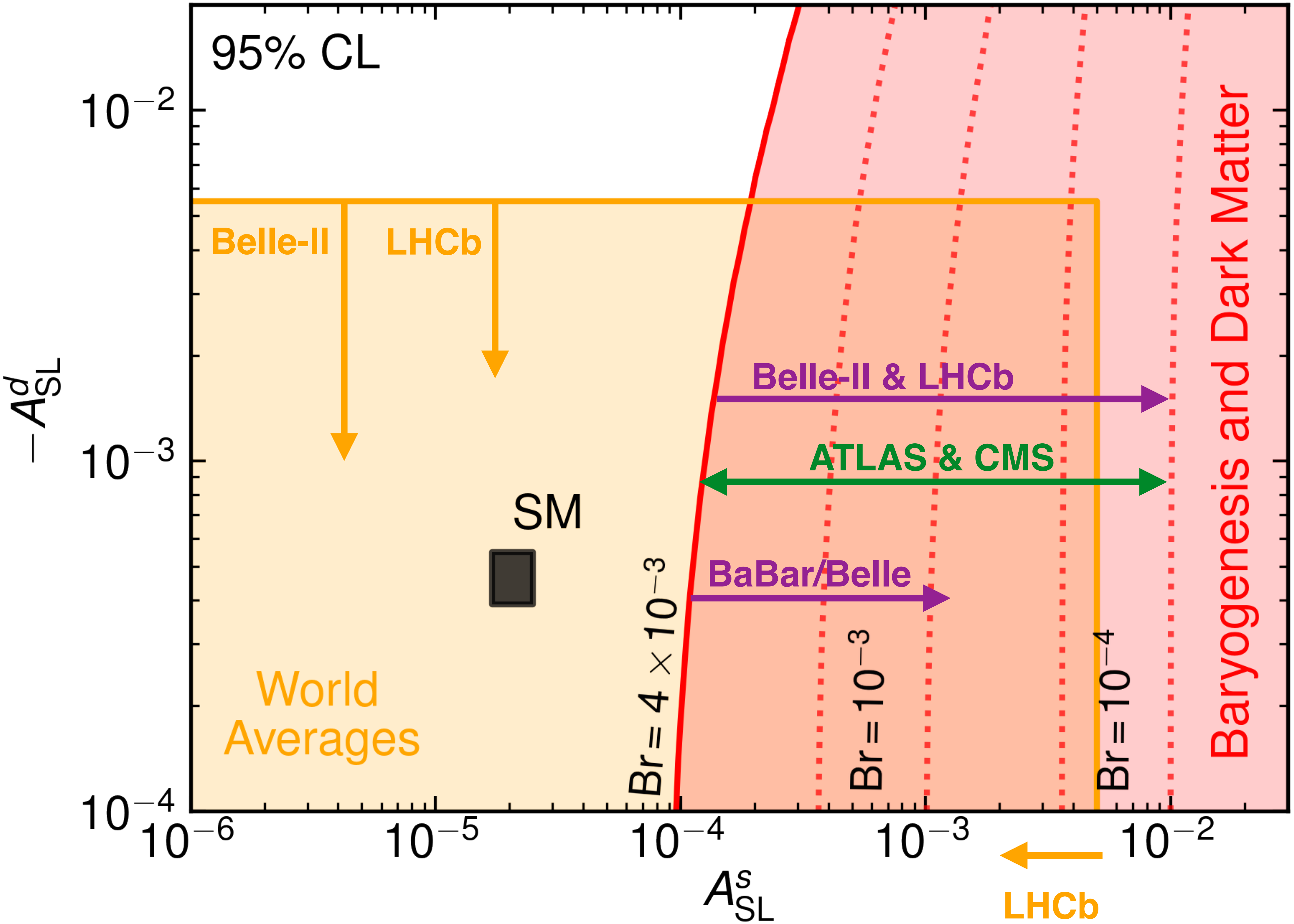
(Analysis of Belle and BaBar already constrain some of the operators)

Constrained indirectly by the LHC



Constrained directly by hadron colliders and B-factories

Parameter Space



Signatures

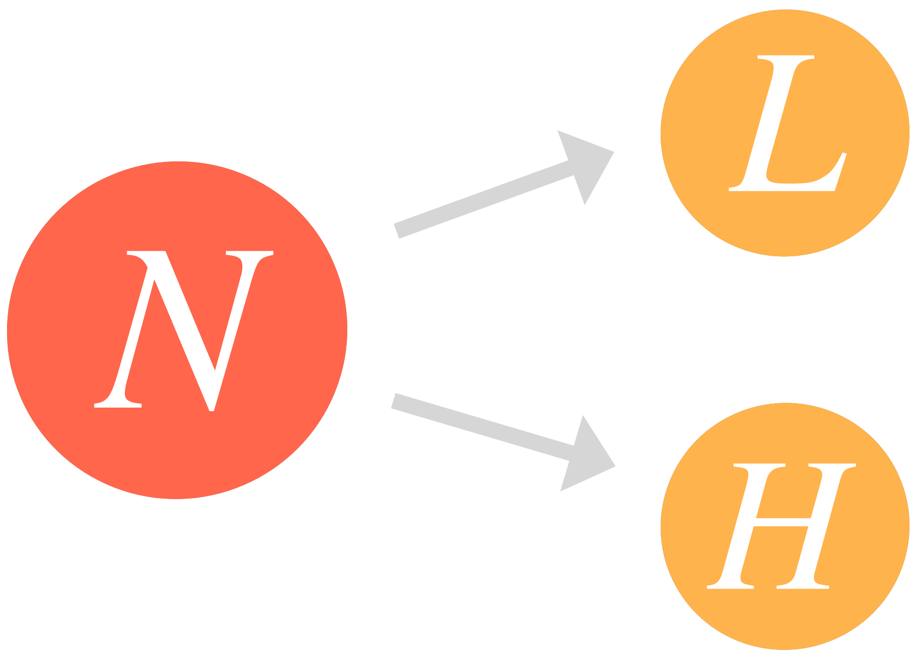
- CP violation
- New b decays
- High pT jets and missing energy

Conclusion: **B-Mesogenesis will be tested in the upcoming years!**

Thermal Leptogenesis

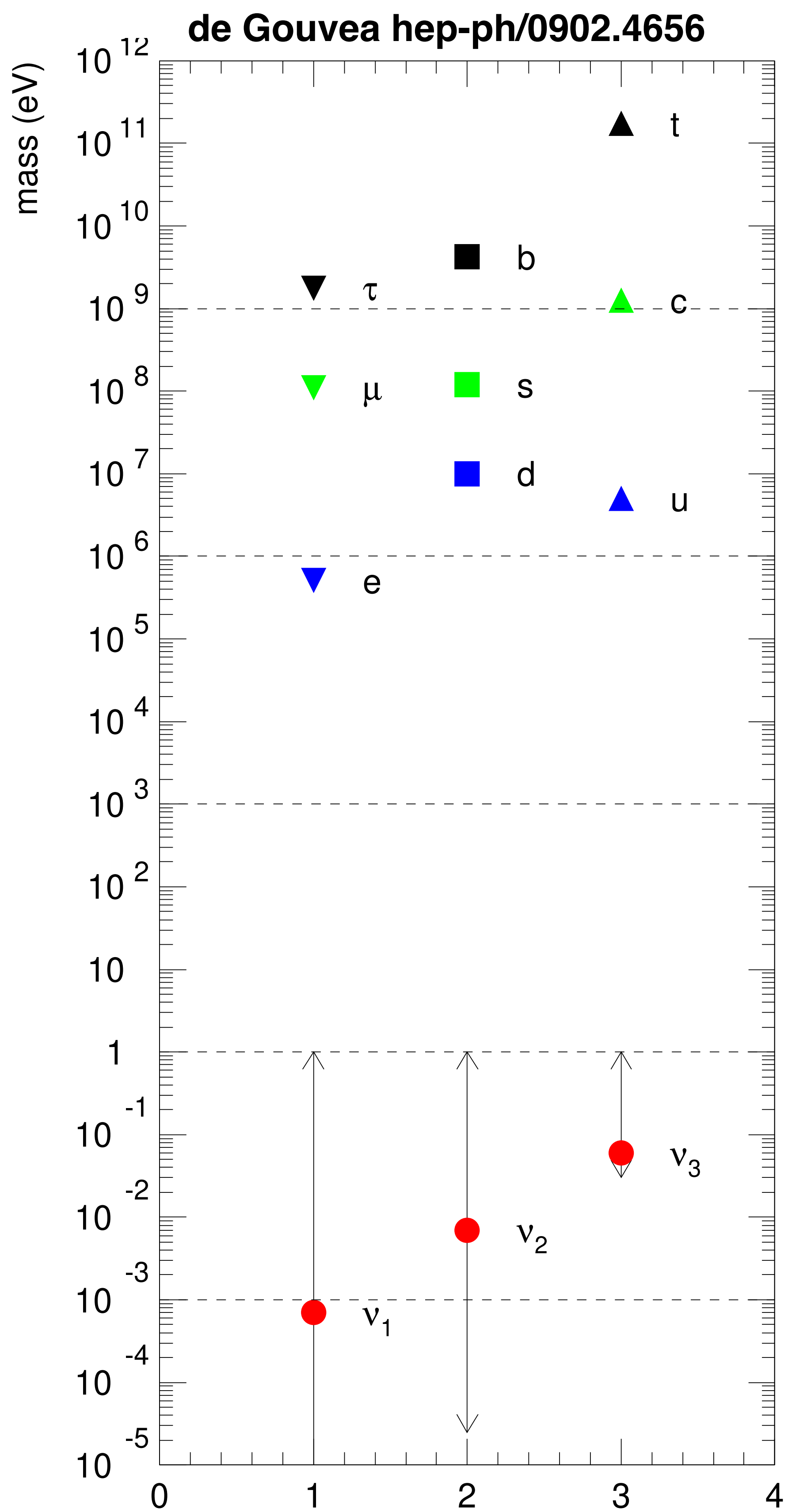
Originally proposed by Fukugita & Yanagida, Phys. Lett. B 174 (1986) 45 but maybe the most thoroughly investigated mechanism in the literature, see reviews of

> ***Leptogenesis for pedestrians,* Buchmüller, Di Bari & Plümacher, hep-ph/0401240**
>
> ***Leptogenesis,* Davidson, Nardi & Nir hep-ph/0802.2962**



$$\Delta L = 1$$

Neutrino Masses



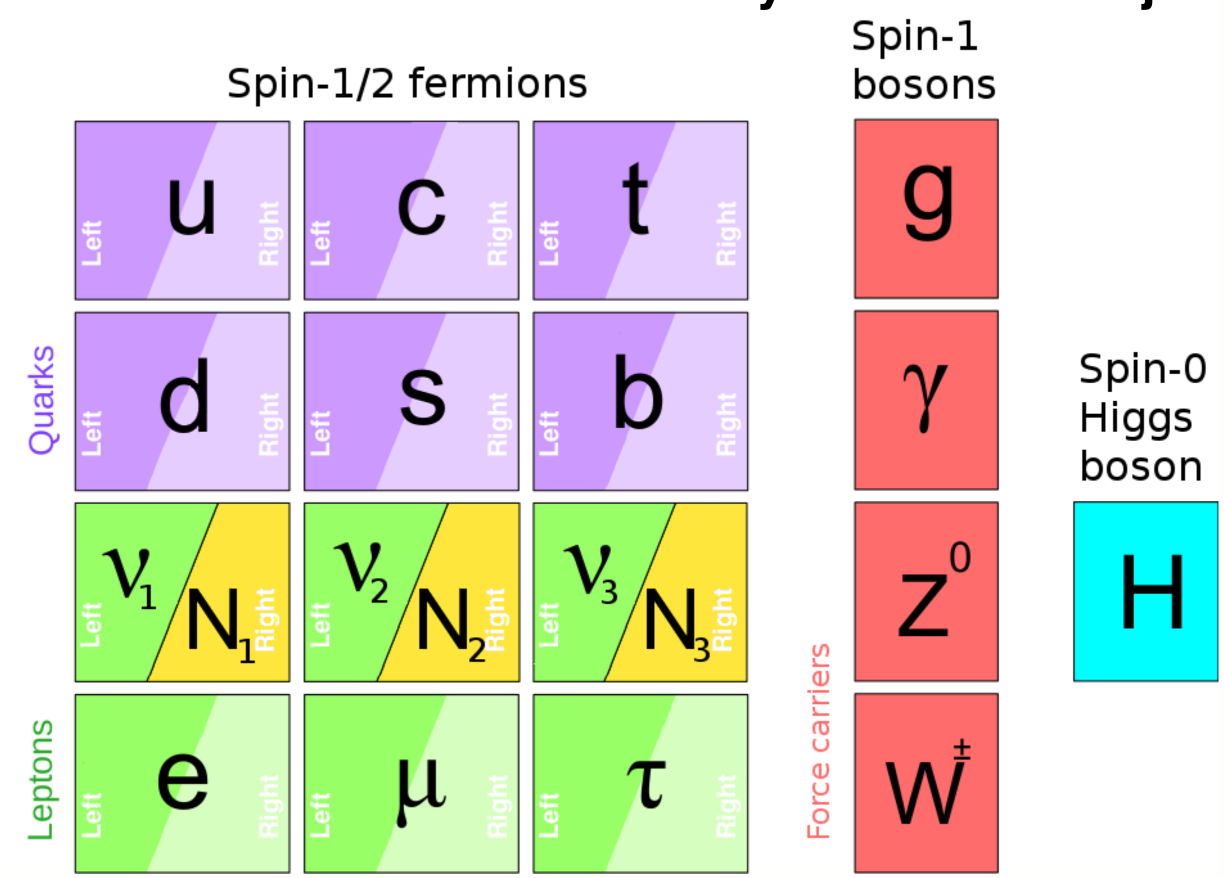
- **We know that neutrinos oscillate and therefore are massive.**
- **Neutrino masses are the only laboratory evidence of physics beyond the Standard Model. There are no right handed neutrinos in the SM!**
- **The smallness of neutrino masses suggest a different mechanism than the Higgs mechanism**

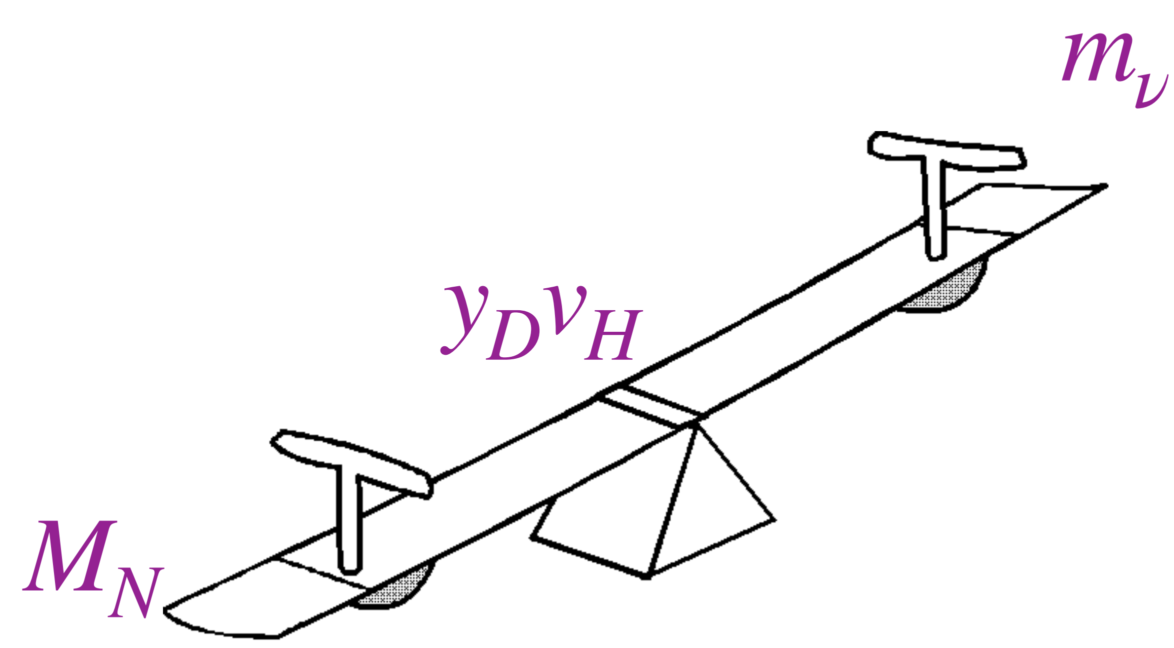
Type-I Seesaw Mechanism

- **A very plausible scenario is the type-I seesaw mechanism**

Minkowski, Ramond, Gell-Mann, Slansky, Yanagida, Mohapatra, Senjanovic ‘79

Complements the SM with right handed Majorana neutrinos and allows to understand the lightness of neutrinos and tell us that they should be Majorana fermions too





$$\mathscr{L} = -\, y_D \bar{L} H^c N_R - M_N \overline{N}_R N_R^c/2 + \mathrm{h.c.}$$

$$M_\nu = \begin{pmatrix} 0 & y_D v_H \\ y_D^t v_H & M_N \end{pmatrix}$$

$$m_\nu \simeq y_D^2 v_H^2 / M_N$$

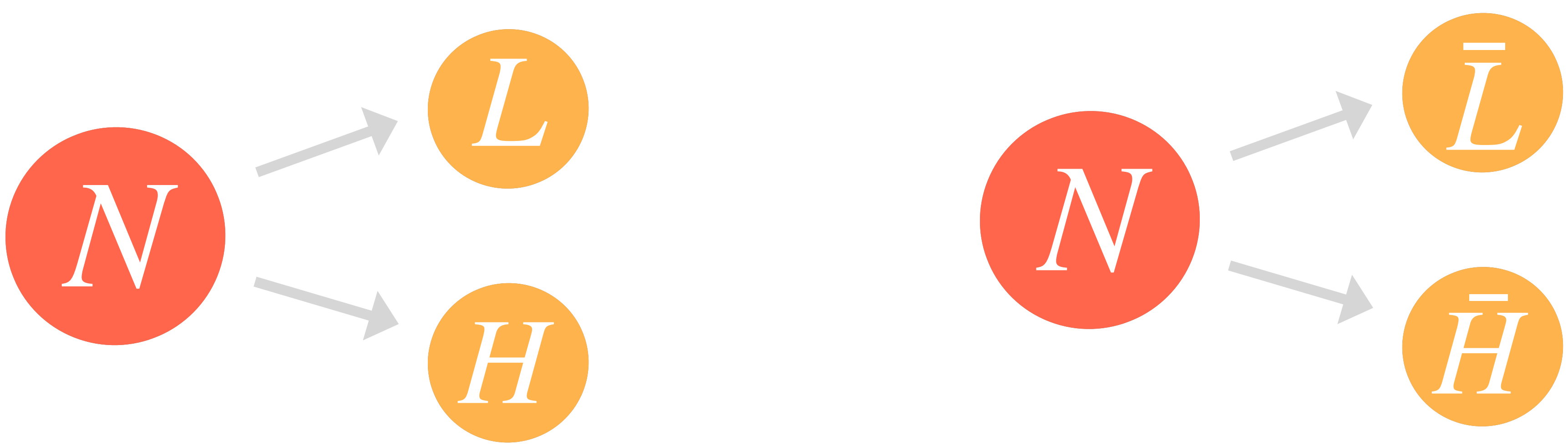
example:

$$m_\nu = 0.1\,\mathrm{eV}$$
$$y_D = 1$$
$$M_N \simeq 10^{13}\,\mathrm{GeV}$$

Type-I Seesaw Mechanism & Leptogenesis

- **Key element that heavy Majorana neutrinos introduce is Lepton number violation! (which thanks to the Sphalerons could lead to a B asymmetry)**

Sakharov #3

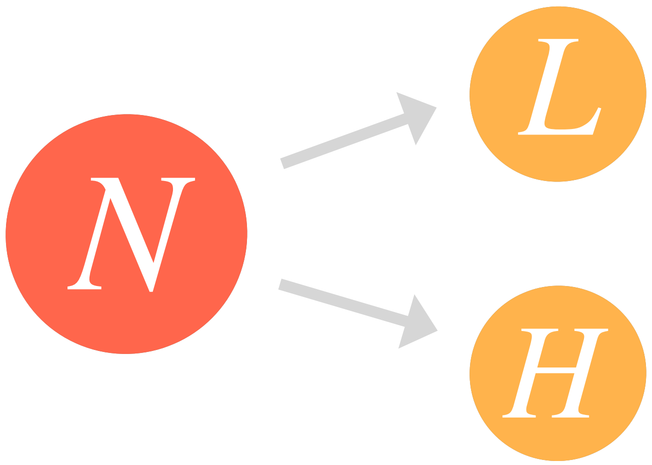


$\Delta L = 1$ $\Delta L = -1$

Type-I Seesaw Mechanism & Leptogenesis

- **In addition, these sterile neutrinos appear to interact just in the right way to be out of equilibrium**

Sakharov #2



$$\Gamma(N \to LH) = \frac{y_D^2}{8\pi} M_N$$

$$m_\nu \simeq y_D^2 v_H^2 / M_N \qquad y_D^2 \simeq m_\nu M_N / v_H^2$$

$$\Gamma(N \to LH) \simeq H(T = M_N)$$

mildly out of equilibrium!

Type-I Seesaw Mechanism & Leptogenesis

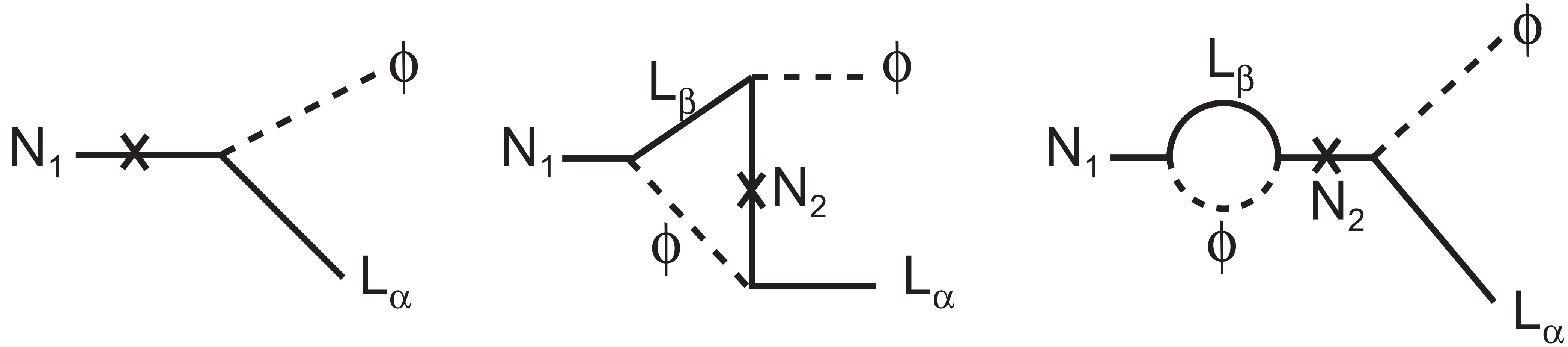
- **In addition, there are new sources of CP violation in their interactions**

Sakharov #1

$$\mathscr{L} = - y_D^{ij} \bar{L}_i H^c N_{Rj} + \mathrm{h.c.}$$

$$\epsilon_{\mathrm{CP}}^{L-\mathrm{break}} = \frac{\Gamma[N \to L\, H] - \Gamma[N \to \bar{L}\, \bar{H}]}{\Gamma[N \to L\, H] + \Gamma[N \to \bar{L}\, \bar{H}]} \neq 0$$

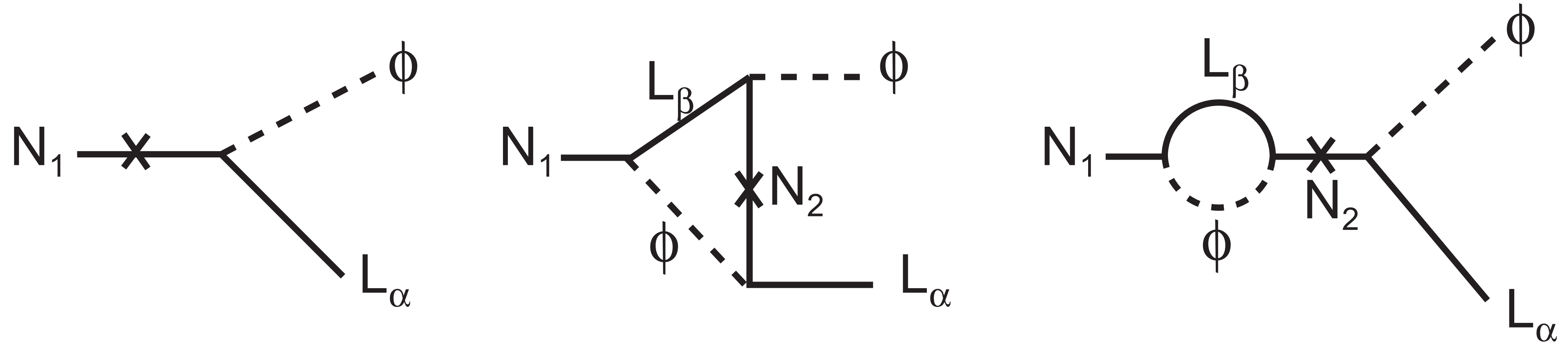
In particular, these diagrams generate the contribution to the CP asymmetry: **Covi, Roulet & Vissani [hep-ph/9605319]**



Nir [hep-ph/0702199]

Type-I Seesaw Mechanism & Leptogenesis

- **Key aspects of CP violation**



1) It appears at the 1-loop level

2) It requires complex coupling constants

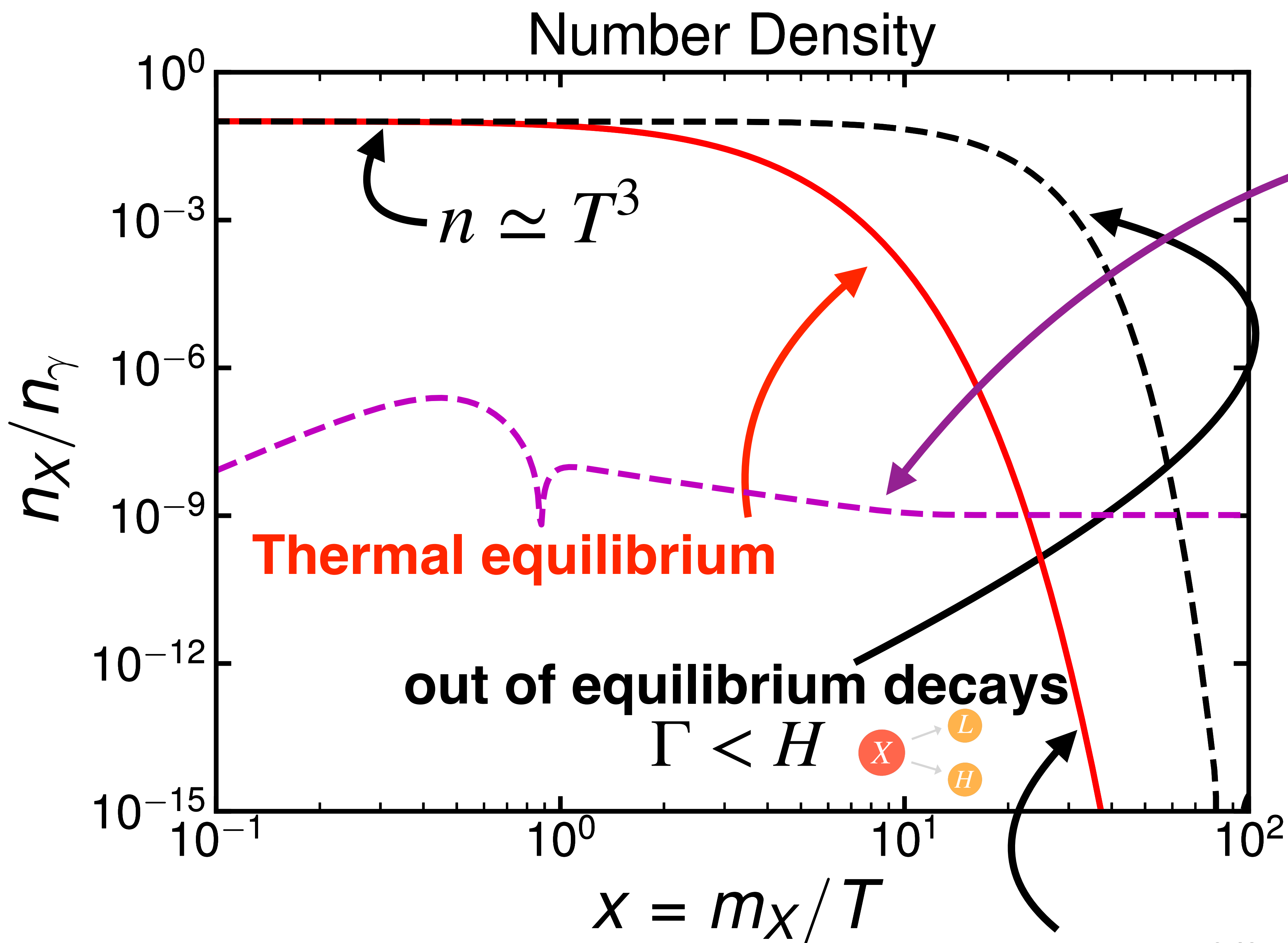
3) It requires on-shell particles mediating in the loops

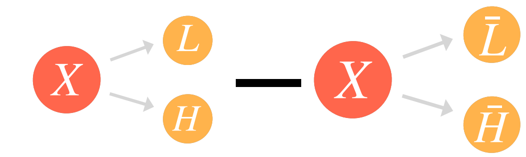
Under the assumption of hierarchical sterile neutrinos the CP asymmetry is bounded!

$$|\epsilon_{CP}^{N_1}| \leq \frac{3}{8\pi}\frac{M_1(m_3 - m_1)}{v_H^2}$$

Davidson & Ibarra [hep-ph/0202239]

Number density of particles





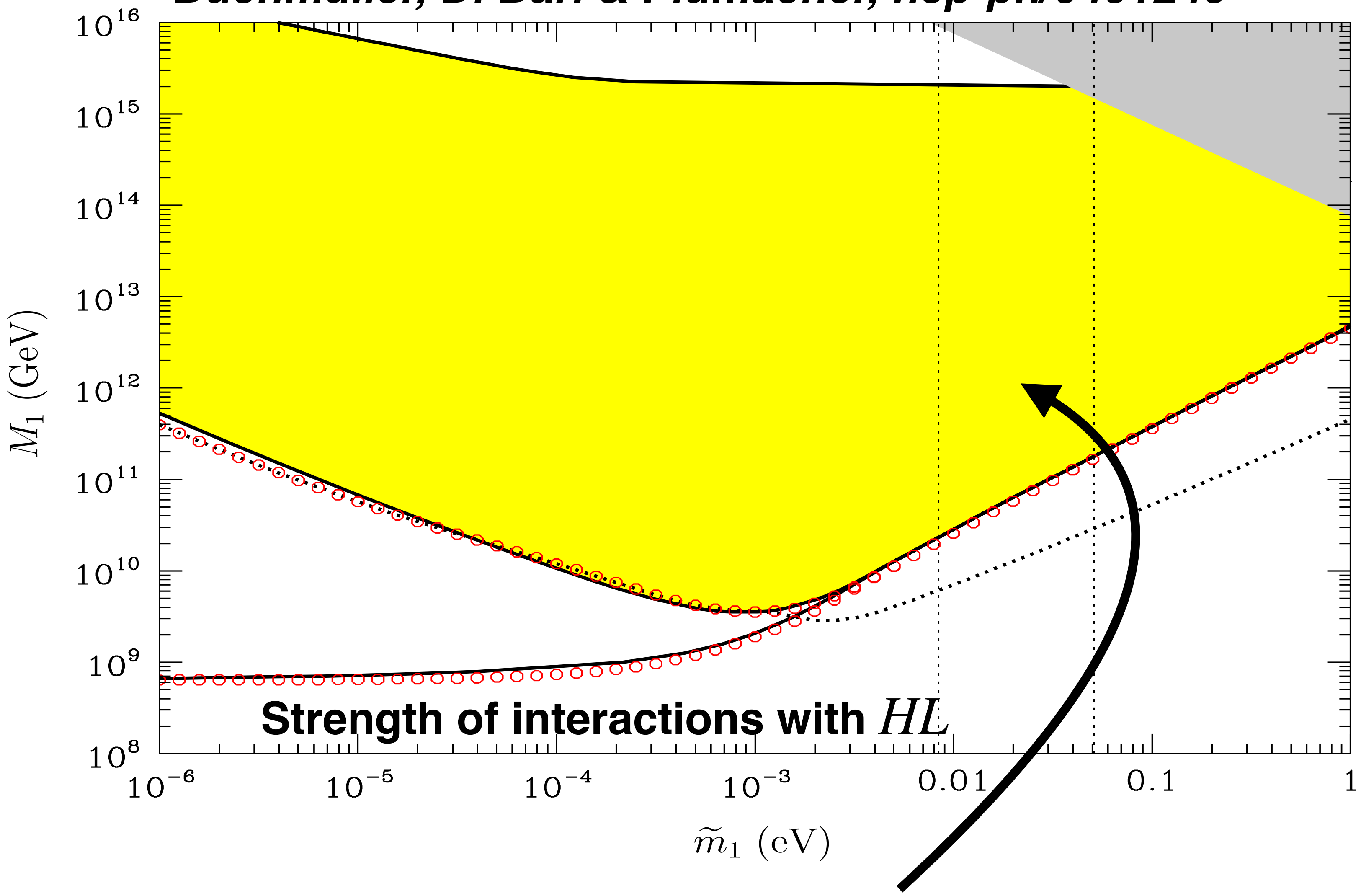
$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

$$\eta_B \simeq 10^{-3}\,\epsilon_{\rm CP}\,{\rm BR}$$

$$n \simeq (mT)^{3/2} e^{-m/T}$$

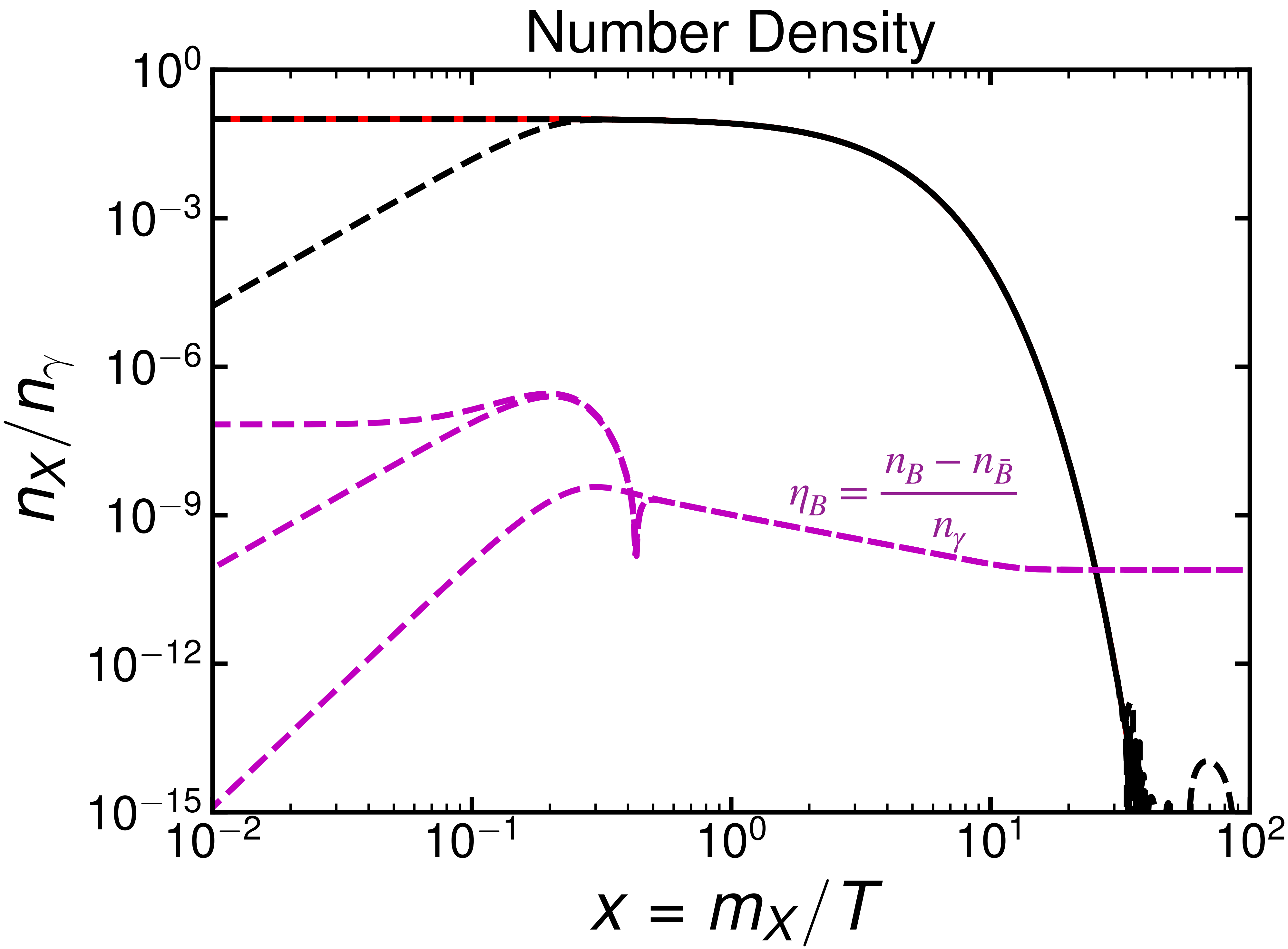
Parameter Space

Buchmüller, Di Bari & Plümacher, hep-ph/0401240

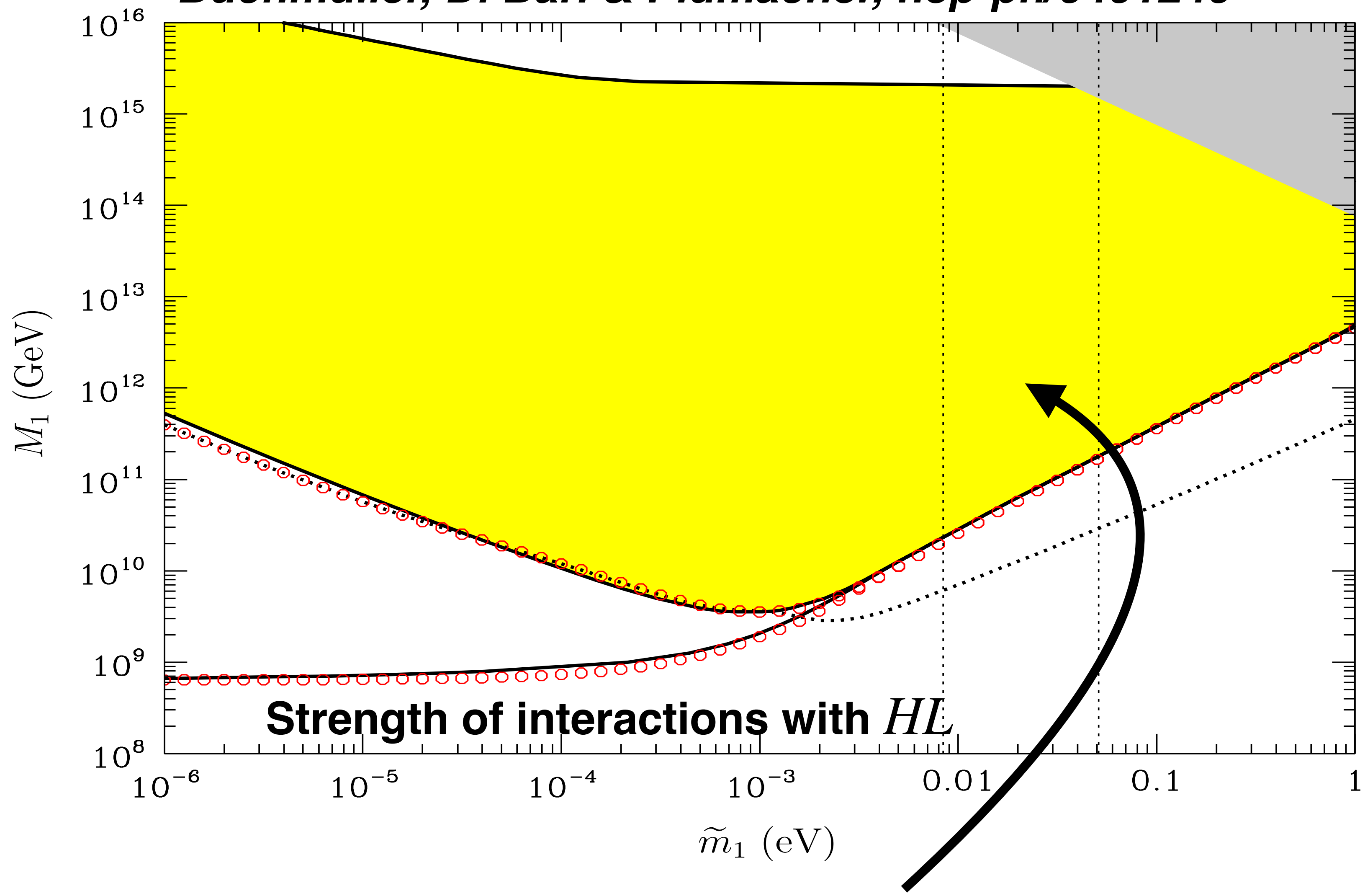


Particularly nice because it works in a regime where previous asymmetries are erased!

Thermal Leptogenesis



Parameter Space



Particularly nice because it works in a regime where previous asymmetries are erased!

Pros: 😃 **Explains neutrino masses and mixings!**

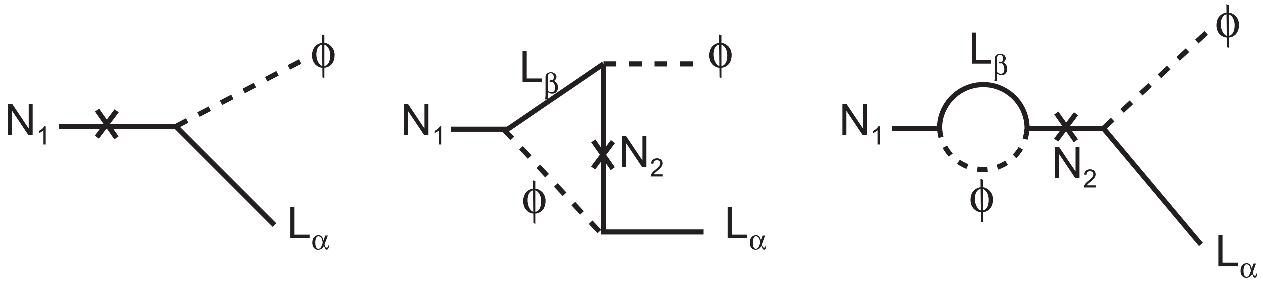
😃 **Works out of the box — These sterile neutrino have just the right properties to do baryogenesis!**

Con?: 😐 **Operates at very high energies and is hard (or impossible) to test**

Other Leptogenesis Mechanisms

Resonant Leptogenesis

Pilaftsis & Underwood [hep-ph/0506107]



Same as thermal leptogenesis but with strong mass degeneracy

Allows to enhance the CP asymmetries by orders of magnitude and works for $M_N > \mathrm{TeV}$

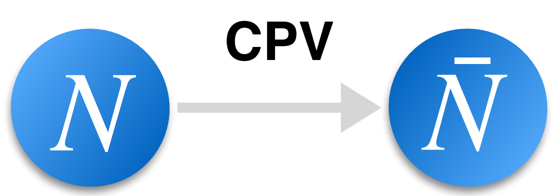
Could be testable at future colliders

Leptogenesis via neutrino oscillations

Akhmedov, Rubakov & Smirnov, hep-ph/9803255
See also Asaka & Shaposhnikov, hep-ph/0505013

$M_N \gtrsim 0.1\,\mathrm{GeV}$ $\quad T_{\mathrm{Lepto}} \sim 10^5\,\mathrm{GeV}$

No N's produced during reheating **+**



+ **Sphalerons** **=** ΔB 😃

sizable regions of parameter space testable at future colliders

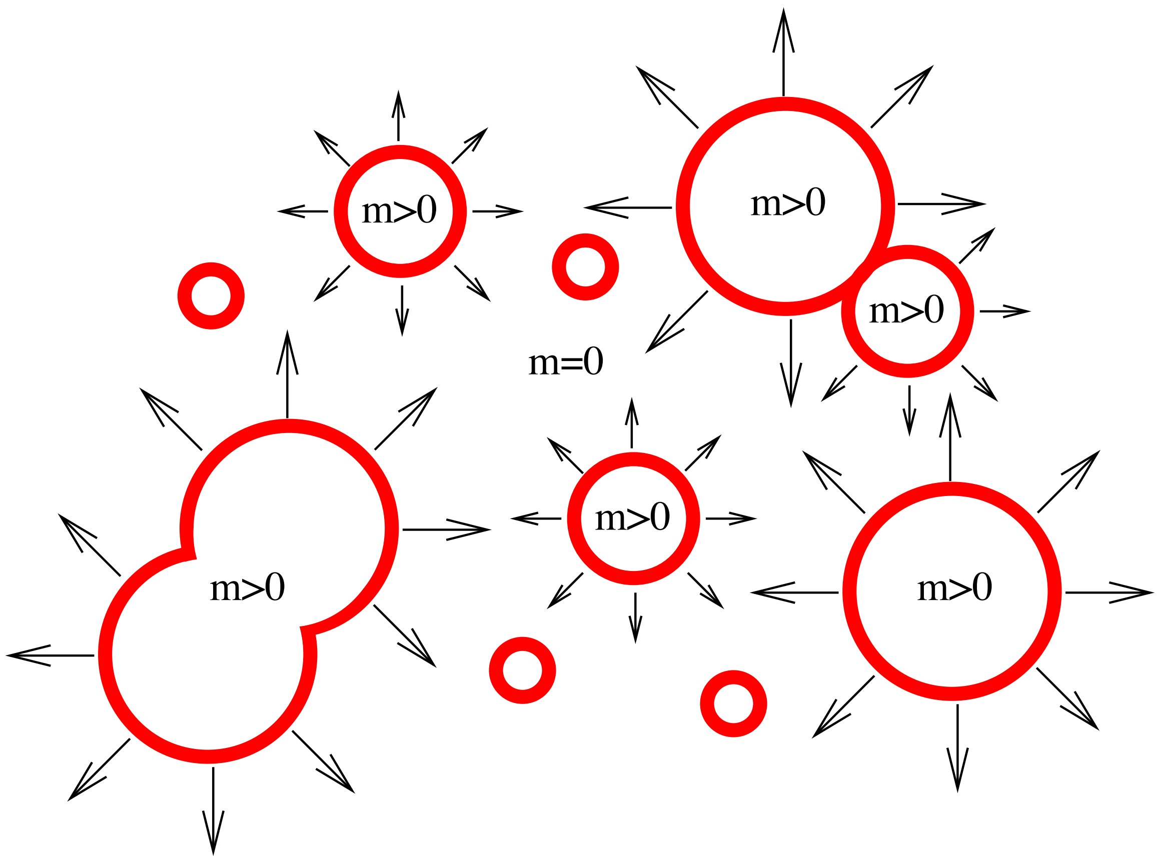
Electroweak Baryogenesis

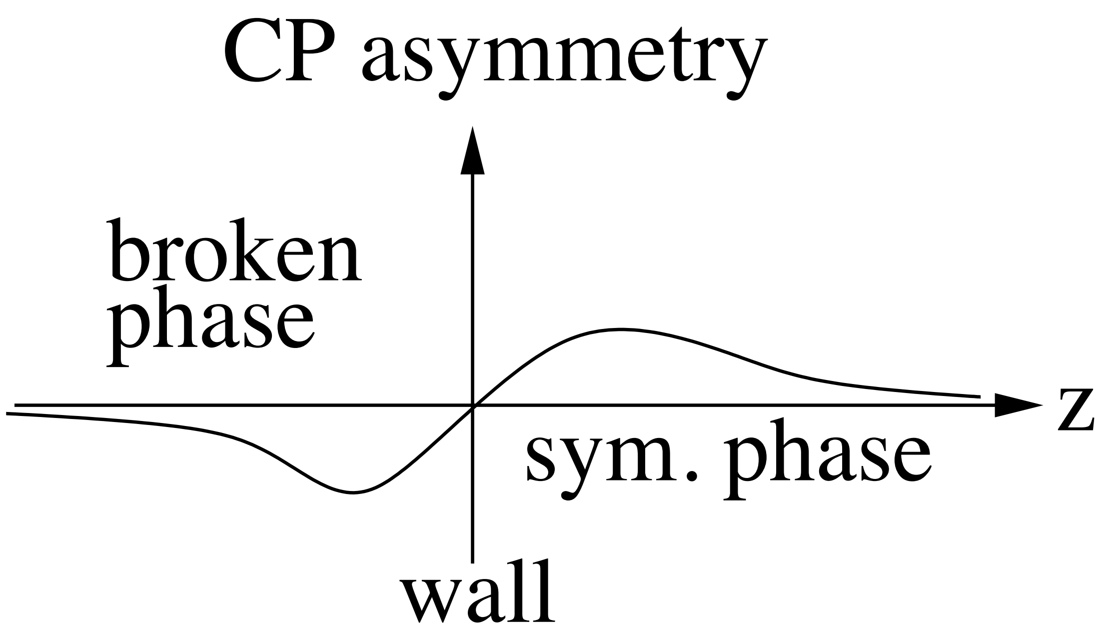
Generate a baryon asymmetry at a first order phase transition

Kuzmin, Rubakov & Shaposhnikov '85
Cohen, Kaplan & Nelson '90

see Morrissey & Ramsey-Musolf [1206.2942] for a review

*Figures taken from Cline hep-ph/0609145





- **Need a source for a first order phase transition BSM, e.g. singlet scalar**
- **Need new sources of CP violation (that can be easily obtained BSM)**
  - 😐 **Typically they lead to contributions to Electric Dipole moments**

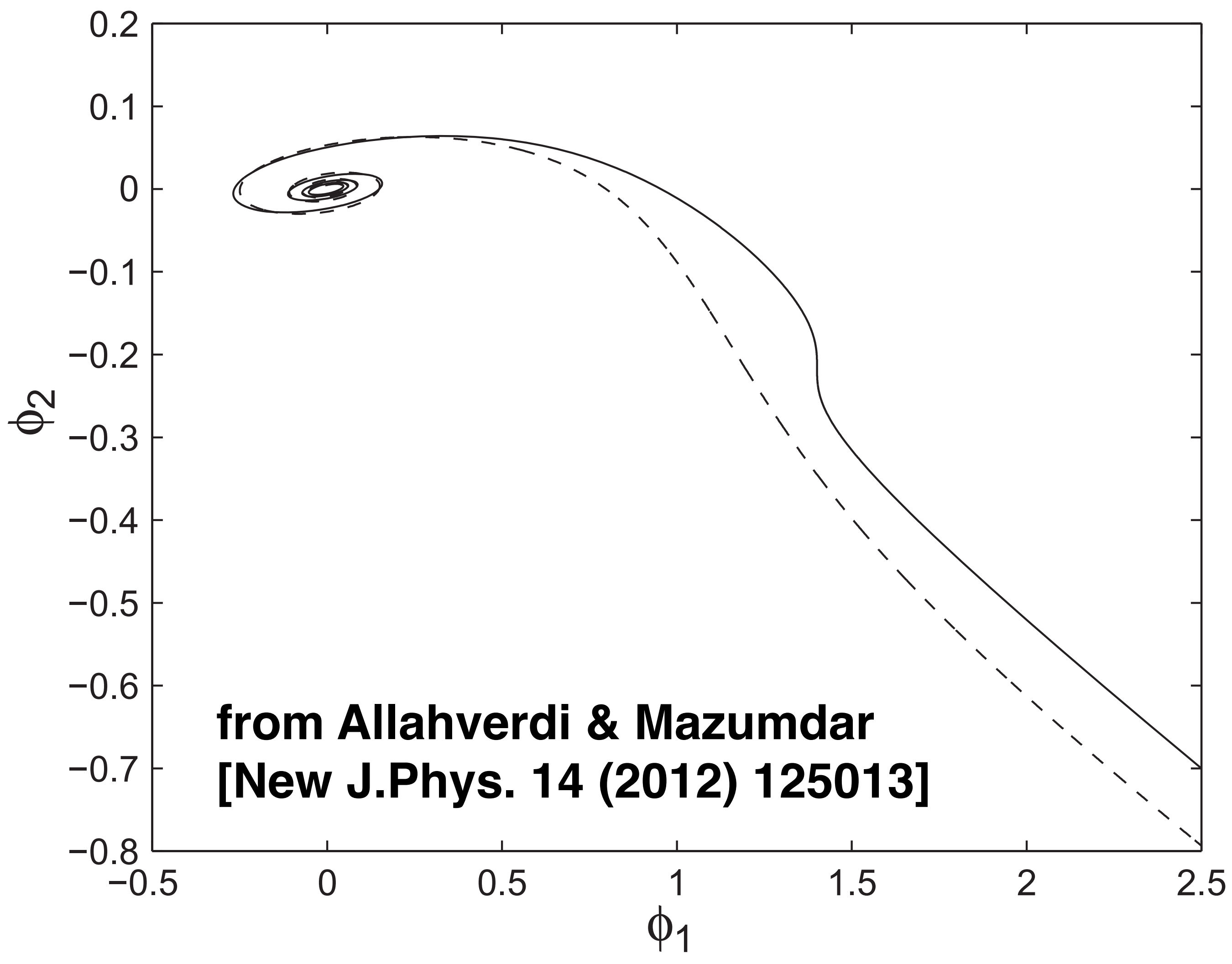
😃 **Testable!**

Affleck-Dine Baryogenesis

Main player: Scalar field carrying baryon number

Affeck-Dine '85, check out review by Dine & Kusenko [hep-ph/0303065]

- **These particles appear naturally in SUSY, e.g. s-quarks**
- **One needs new sources of CP and baryon/lepton number violation (which are not hard to find)**
- **Generation of a Baryon asymmetry arises from the evolution of the scalar condensate in an expanding Universe with very large initial field values as generated by inflation**



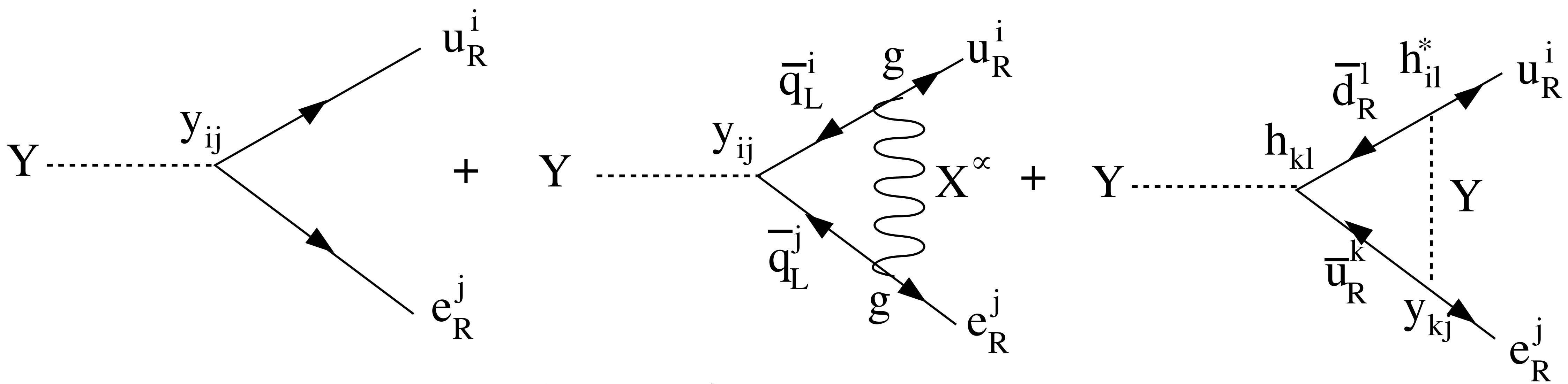
Turns out that the baryon asymmetry of the Universe is tends to be much larger than the observed one

Would require further entropy damping later in the evolution of the Universe

GUT Baryogenesis

Main players: Gauge bosons and scalars of GUT groups

Originally developed in '78-80, Yoshimura, Toussaint et al., Weinberg, Dimopoulos & Susskind, Papastamatiou & Parker, Ignatiev et al., Ellis et al. …



Cline hep-ph/0609145

Fig. 6. Tree plus 1-loop contributions to $Y \to e_R u_R$

- **Main idea: Use out-of equilibrium decays from these bosons. (which naturally decay in a baryon violating way)**
- 😃 **Well motivated particle physics scenario**
- 😐 **Typically in conflict with standard versions of Inflation**
- 😐 **Standard SU(5) does not work because it preserves $B - L$**
- 😐 **Leads to baryon violation, e.g. $p \to \pi^0 e^+$**

Curiosity

BARYON NUMBER GENERATION IN THE EARLY UNIVERSE*

Edward W. KOLB[1]

W.K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA

Stephen WOLFRAM[2]

Theoretical Physics Laboratory, California Institute of Technology, Pasadena, California 91125, USA

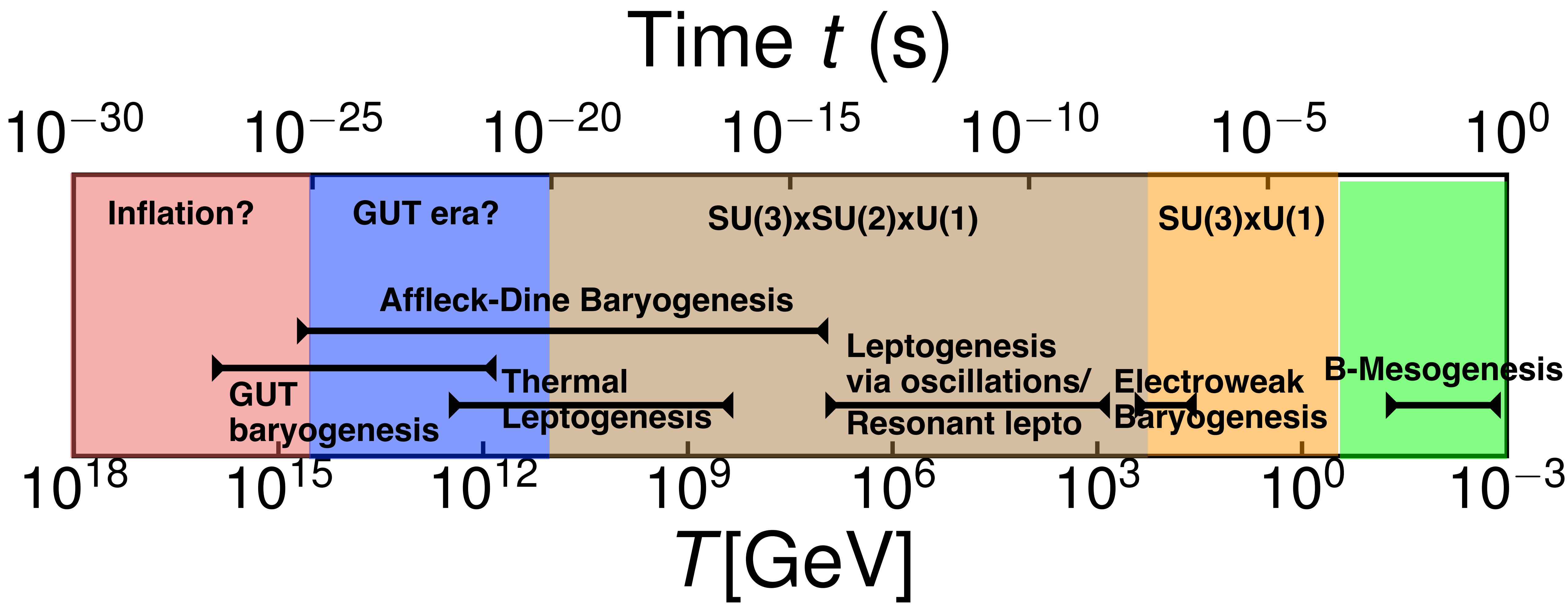
Received 29 November 1979
(Final version received 10 March 1980)

I haven't written any evolution equation, but they are sometimes hard to deal with.

Wolfram (the Mathematica guy) had to deal with these types of equations himself!

Summary & Conclusions

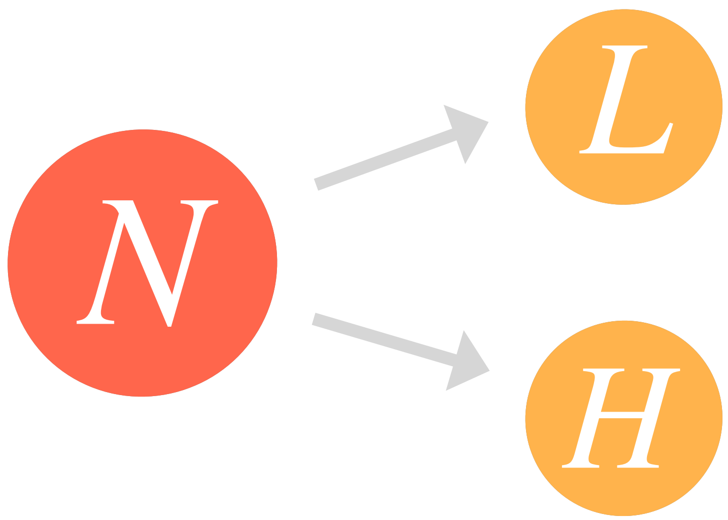
- 60 years have passed since the concept of Baryogenesis was considered seriously, Sakharov 1967.
- The observed baryon asymmetry of the Universe is a clear call of physics beyond the Standard Model
- We have many different mechanisms to explain it:



Summary & Conclusions

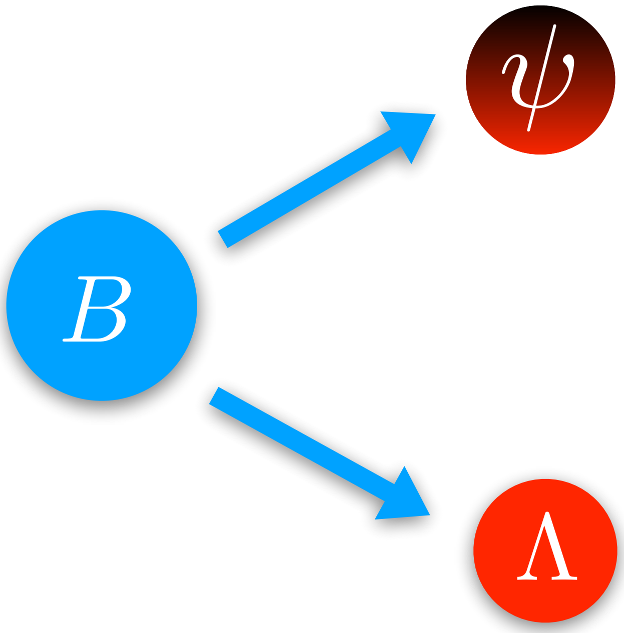
- Some of them are better motivated than others, some can be tested and some others cannot be

Thermal Leptogenesis:



- Neutrino Masses
- Works out of the box
- Hard to test

B-Mesogenesis:

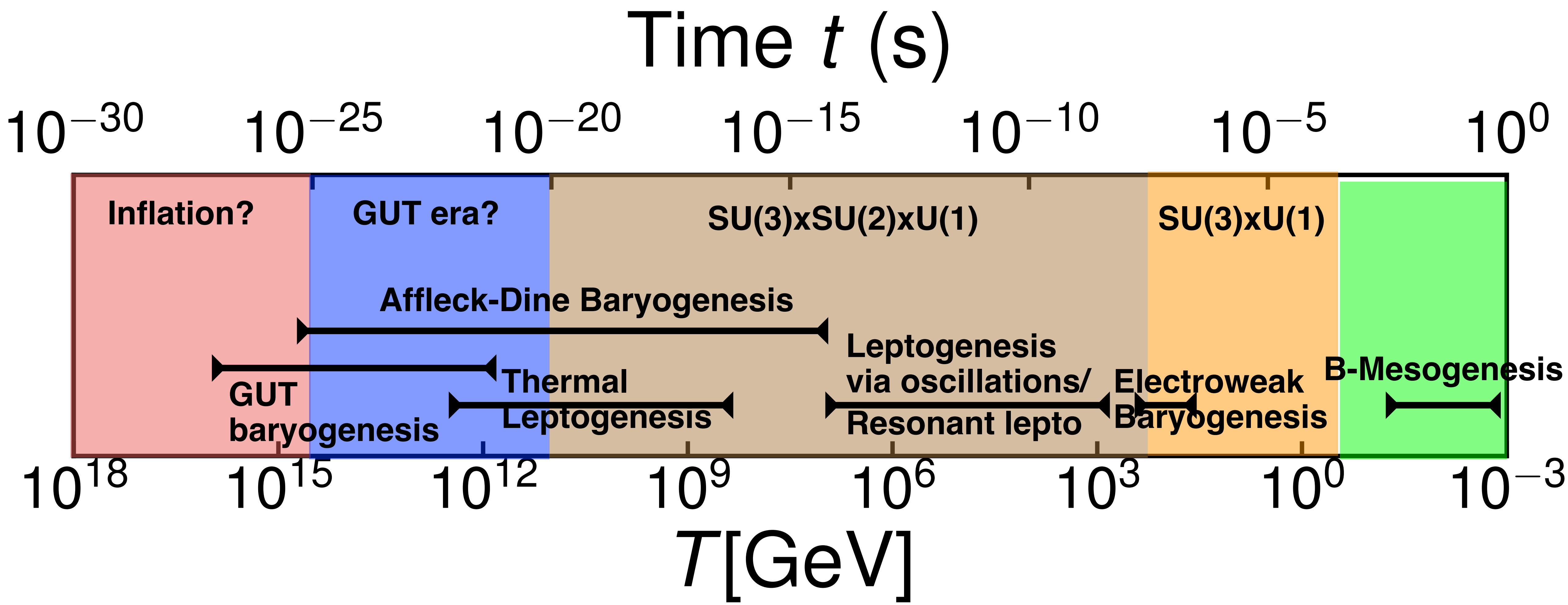


- Testable and provides Dark Matter!
- Uses a naturally occurring CP violating system
- Requires use of non-standard cosmologies

Future:
— More mechanisms to come
— Await for potential laboratory signals of lepton/baryon non-conservation!

The End

End of Lecture II



Thank you for your attention!

miguel.escudero@cern.ch

A Small Tribute to Ann Nelson

Ann Nelson passed away ~5 years ago in a climbing accident

Who was she?

A role model

A leader of the community

An outstanding theoretical physicist

e.g.: Sakurai Prize winner 2018!

For comments from the community see:

https://physicstoday.scitation.org/do/10.1063/pt.6.4.20190808a/full/

including **David B. Kaplan, Howard Georgi, Lisa Randall, Nima Arkani-Hamed, Michael Dine, Kathryn Zurek & Mary K Gaillard**

A Small Tribute to Ann Nelson

Some of Ann's contributions:

Solving the Strong CP problem with spontaneous CP violation	Naturally Weak CP Violation (#10) Ann E. Nelson (Harvard U.) (Dec, 1983) Published in: *Phys.Lett.B* 136 (1984) 387-391
Electroweak Baryogenesis	Progress in electroweak baryogenesis (#10) Andrew G. Cohen (Boston U.), D.B. Kaplan (UC, San Diego), A.E. Nelson (UC, San Diego) (Jan, 1993) Published in: *Ann.Rev.Nucl.Part.Sci.* 43 (1993) 27-70 • e-Print: hep-ph/9302210 [hep-ph]
Little Higgs	The Littlest Higgs (#3) N. Arkani-Hamed (Harvard U., Phys. Dept.), A.G. Cohen (Boston U.), E. Katz (Washington U., Seattle), A.E. Nelson (Washington U., Seattle) (Jun, 2002) Published in: *JHEP* 07 (2002) 034 • e-Print: hep-ph/0206021 [hep-ph]
IR-UV connections in gravity	Effective field theory, black holes, and the cosmological constant (#4) Andrew G. Cohen (Boston U.), David B. Kaplan (Washington U., Seattle), Ann E. Nelson (Washington U., Seattle) (Mar, 1998) Published in: *Phys.Rev.Lett.* 82 (1999) 4971-4974 • e-Print: hep-th/9803132 [hep-th]
Dynamical SUSY breaking	Dynamical supersymmetry breaking at low-energies (#9) Michael Dine (UC, Santa Cruz), Ann E. Nelson (UC, San Diego) (Mar, 1993) Published in: *Phys.Rev.D* 48 (1993) 1277-1287 • e-Print: hep-ph/9303230 [hep-ph]
Dark Energy-Neutrino Connection!	Dark energy from mass varying neutrinos (#1) Rob Fardon (Washington U., Seattle, Astron. Dept.), Ann E. Nelson (Washington U., Seattle, Astron. Dept.), Neal Weiner (Washington U., Seattle, Astron. Dept.) (Sep, 2003) Published in: *JCAP* 10 (2004) 005 • e-Print: astro-ph/0309800 [astro-ph]

A Small Tribute to Ann Nelson

How did I meet Ann?

She gave an amazing seminar at Fermilab about Baryogenesis in 2017. I thought, wow, that's who I would like to be when I am old!

I got funding to visit her and so I did for a month in April 2018

She introduced me to Baryogenesis and we wrote a paper of Baryogenesis and Dark Matter mechanism using a naturally occurring CP violating system in the Standard Model: the neutral B meson system:

Baryogenesis and dark matter from *B* mesons

Gilly Elor,[1,*] Miguel Escudero,[2,3,†] and Ann E. Nelson[1,‡]

[1]*Department of Physics, Box 1560, University of Washington, Seattle, Washington 98195, USA*
[2]*Department of Physics, King's College London, Strand, London WC2R 2LS, United Kingdom*
[3]*Instituto de Física Corpuscular (IFIC), CSIC-Universitat de València, Paterna E-46071, Valencia, Spain*

My experience at the UW with her was incredibly illuminating. She was the most brilliant physicist I have met to date, but also a very generous, inclusive, and friendly person.

We deeply miss her

Motivation, Method and Philosophy

Ann E. Nelson (1958-2019)

A sentence from her "Commentary: Diversity in physics: Are you part of the problem?" in Physics Today that I find very motivating:

I often get asked, "Why are there so few women in physics?" That anyone would ask that question shows how oblivious many people are to the sexism and bias that permeate our society and physics culture. I may not be able to fully answer the question, but I can tell you why there are women like me in physics. Because we love math and nature. Because we like doing computations and figuring things out, step by systematic step. We love the flashes of insight and the excitement of revelations from new data. We revel in breathtaking moments of awe. And we had support, mentors, encouragement, opportunities, and colleagues who gave us a positive view of ourselves as physicists.

Back Up

BACK UP

Asymmetries

In our scenario, this is precisely as a flavor specific decay, in the sense that our decays are:

$$\bar{b} \to u d \psi \to B \to \psi X\,, \tag{12}$$
$$b \to \bar{u}\bar{d}\bar{\psi} \to \bar{B} \to \bar{\psi} X \tag{13}$$

and the CP conjugate cannot happen. In this case, given a number of $\Phi \to b\bar{b}$ decays we have the following number of ψ final states as:

$$\frac{N(\psi)}{N(\Phi \to b\bar{b})} = f_u \mathrm{BR}(B^+ \to \psi X) \tag{14}$$
$$+ f_{\Lambda_b} \mathrm{BR}(\bar{\Lambda}_b{}^0 \to \psi X)$$
$$+ f_d \Gamma(B_d^0 \to \psi X)\left[\mathrm{TIP}(B_d^0(t) \to B_d^0) + \mathrm{TIP}(\bar{B}_d^0(t) \to B_d^0)\right]$$
$$+ f_s \Gamma(B_s^0 \to \psi X)\left[\mathrm{TIP}(B_s^0(t) \to B_s^0) + \mathrm{TIP}(\bar{B}_s^0(t) \to B_s^0)\right]$$

where we have assumed that we have the same probability of hadronizing particle and antiparticle pairs. Similarly, the final state containing $\bar{\psi}$ reads:

$$\frac{N(\bar{\psi})}{N(\Phi \to b\bar{b})} = f_u \mathrm{BR}(B^- \to bar\psi X) \tag{15}$$
$$+ f_{\Lambda_b} \mathrm{BR}(\Lambda_b{}^0 \to \bar{\psi} X)$$
$$+ f_d \Gamma(\bar{B}_d^0 \to \bar{\psi} X)\left[\mathrm{TIP}(\bar{B}_d^0(t) \to \bar{B}_d^0) + \mathrm{TIP}(B_d^0(t) \to \bar{B}_d^0)\right]$$
$$+ f_s \Gamma(\bar{B}_s^0 \to \bar{\psi} X)\left[\mathrm{TIP}(\bar{B}_s^0(t) \to \bar{B}_s^0) + \mathrm{TIP}(B_s^0(t) \to \bar{B}_s^0)\right]$$

Further, by assuming that the lifetimes and the exclusive decay rates of all the b-flavored hadrons are the same, then we can write:

$$\frac{N(\psi) - N(\bar{\psi})}{N(\Phi \to b\bar{b})} = \frac{\mathrm{BR}(\bar{b} \to \psi X)}{2} \tag{16}$$
$$\times \left(f_d \Gamma_b \left[\mathrm{TIP}(B_d^0(t) \to B_d^0) + \mathrm{TIP}(\bar{B}_d^0(t) \to B_d^0)\right.\right. \tag{17}$$
$$\left. - \mathrm{TIP}(\bar{B}_d^0(t) \to \bar{B}_d^0) - \mathrm{TIP}(B_d^0(t) \to \bar{B}_d^0)\right] \tag{18}$$
$$+ f_s \Gamma_b \left[\mathrm{TIP}(B_s^0(t) \to B_s^0) + \mathrm{TIP}(\bar{B}_s^0(t) \to B_s^0)\right. \tag{19}$$
$$\left.\left. - \mathrm{TIP}(\bar{B}_s^0(t) \to \bar{B}_s^0) - \mathrm{TIP}(B_s^0(t) \to \bar{B}_s^0)\right]\right) \tag{20}$$

The time evolution of the B-meson systems is the following:

$$|M_0(t)\rangle = g_+(t)\,|M^0\rangle - \frac{q}{p} g_-(t)\,|\bar{M}^0\rangle \tag{21}$$
$$|\bar{M}_0(t)\rangle = g_+(t)\,|\bar{M}^0\rangle - \frac{p}{q} g_-(t)\,|M^0\rangle \tag{22}$$

where

$$g_\pm(t) = \frac{1}{2}\left(\exp\left(-\frac{\Gamma_H t}{2} - i m_H t\right) \pm \exp\left(-\frac{\Gamma_L t}{2} - i m_L t\right)\right) \tag{23}$$

Since the B-mesons decay fast, we integrate over this evolution to find the time integrated probabilities as:

$$\mathrm{TIP}(M^0 \to M^0) = \int_0^\infty |g_+(t)|^2 dt \tag{24}$$
$$\mathrm{TIP}(M^0 \to \bar{M}^0) = \int_0^\infty |g_-(t)|^2 (1 - A_{\mathrm{SL}}) dt \tag{25}$$
$$\mathrm{TIP}(\bar{M}^0 \to \bar{M}^0) = \int_0^\infty |g_+(t)|^2 dt \tag{26}$$
$$\mathrm{TIP}(\bar{M}^0 \to M^0) = \int_0^\infty |g_-(t)|^2 (1 + A_{\mathrm{SL}}) dt \tag{27}$$

where here we have expanded $|p/q|^2$ as simplified by Eq. (11). We then can calculate the relevant TIP combination to find:

$$\mathrm{TIP}(M^0(t) \to M^0) + \mathrm{TIP}(\bar{M}^0(t) \to M^0) \tag{28}$$
$$- \mathrm{TIP}(\bar{M}^0(t) \to \bar{M}^0) - \mathrm{TIP}(B_d^0(t) \to \bar{M}^0) \tag{29}$$
$$= A_{\mathrm{SL}} \frac{\Gamma\left(\Delta\Gamma^2 + 4\Delta M^2\right)}{(4\Gamma^2 - \Delta\Gamma^2)(\Gamma^2 + \Delta M^2)} \tag{30}$$
$$= \frac{A_{\mathrm{SL}}}{\Gamma} \frac{1}{1 + (\Gamma/\Delta M)^2} \tag{31}$$

where in the last step we have expanded again over small $\Delta\Gamma$, and where here $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$.

Finally, collecting everything, we find

$$\frac{N(\psi) - N(\bar{\psi})}{N(\Phi \to b\bar{b})} = \frac{f_d A_{\mathrm{SL}}^d}{2} \frac{1}{1 + (\Gamma_d/\Delta M_d)^2} + \frac{f_s A_{\mathrm{SL}}^s}{2} \frac{1}{1 + (\Gamma_s/\Delta M_s)^2} \tag{32}$$

Experimentally, we have $\Delta M_s/\Gamma_s = 27$ and $\Delta M_d/\Gamma_d = 0.77 \simeq 1/\sqrt{2}$ and this finally allows us to find:

$$\frac{N(\psi) - N(\bar{\psi})}{N(\Phi \to b\bar{b})} \simeq \frac{f_d A_{\mathrm{SL}}^d}{6} + \frac{f_s A_{\mathrm{SL}}^s}{2} \tag{33}$$

Note that Eq. (32) is precisely what was found for meson oscillations and in perfect agreement with the result of Eq. 18 of [2].