Theoretical Foundations of Flavor Physics II

CP-violation and heavy quarks

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Color meets Flavor, Bad Honnef

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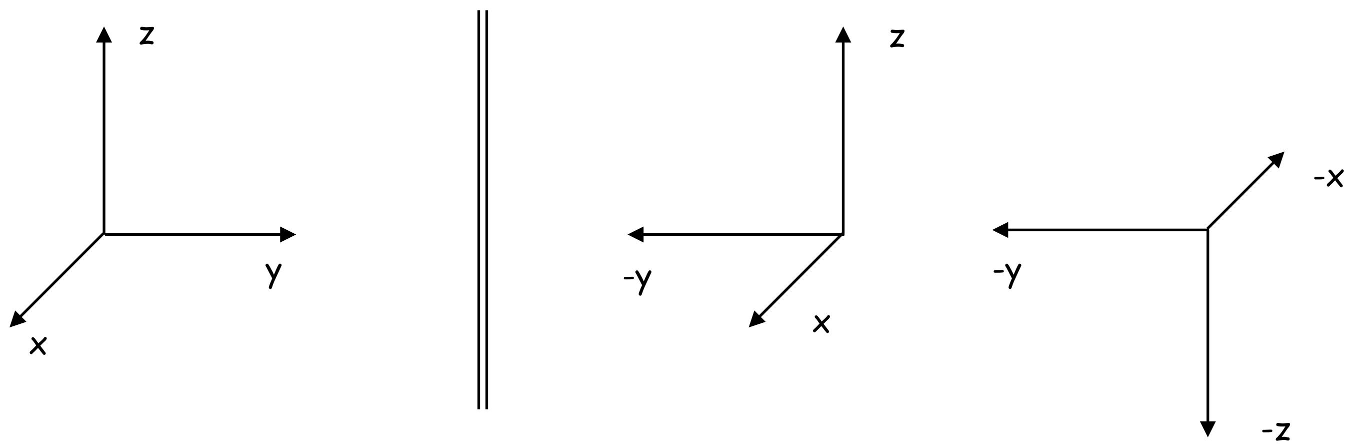
★ The meaning of discrete symmetries in classical mechanics

- Parity [P] transformation: $\vec{r} \rightarrow -\vec{r}$ Refection through a mirror, followed by a rotation of π around an axis defined by the mirror plane.



- Time-reversal [T] transformation: $t \rightarrow -t$ Flips the arrow of time
- Charge-conjugation [C] transformation Changes particles into antiparticles (*)

Introduction: what are C,P, & T classically?

★ The meaning of discrete symmetries in classical mechanics

Parity [P] transformation: $\vec{r} \to -\vec{r}$ || Time-reversal [T] transformation: $t \to -t$

$\vec{v} = \dfrac{d\vec{r}}{dt}$	odd under P	odd under T	
$\vec{p} = m\vec{v}$	odd under P	odd under T	
$\vec{F} = \dfrac{d\vec{p}}{dt}$	odd under P	even under T	
$\vec{L} = \vec{r} \times \vec{p}$	even under P	odd under T	(so is spin)

Q: how is this supposed to work for quantum mechanics with $[r_i, p_k] = i\delta_{ik}$?

- Lorentz force allows us to see how electric and magnetic fields react upon application of P and T

$$\vec{F}_{Lorentz} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$\vec{F}$ and $\vec{v}$ are odd under P:
$\vec{E} \to -\vec{E}$ and $\vec{B} \to \vec{B}$

$\vec{F}$ is even and and $\vec{v}$ is odd under T:
$\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$

Introduction: what are C,P, & T classically?

★ The meaning of discrete symmetries in classical electrodynamics

- We can now see how equations of motion change under P and T

Under P:
$$\vec{E}(\vec{r},t) \to -\vec{E}(-\vec{r},t)$$
$$\vec{B}(\vec{r},t) \to \vec{B}(-\vec{r},t)$$
$$\nabla \to -\nabla$$
$$\vec{j}(\vec{r},t) \to -\vec{j}(-\vec{r},t)$$

Under T:
$$\vec{E}(\vec{r},t) \to \vec{E}(\vec{r},-t)$$
$$\vec{B}(\vec{r},t) \to -\vec{B}(\vec{r},-t)$$
$$\partial/\partial t \to -\partial/\partial t$$
$$\vec{j}(\vec{r},t) \to -\vec{j}(\vec{r},-t)$$

Equation	P	T	C	CPT
$\nabla \cdot \mathbf{E} = 4\pi\rho$	+	+	−	−
$\nabla \cdot \mathbf{B} = 0$	−	−	−	−
$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}$	−	−	−	−
$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$	+	+	−	−

Q: What about $\vec{E} \cdot \vec{B}$?

- Technically, there is no C-parity in classical physics (no antiparticles)…

Under C:
$$\rho(\vec{r},t) \to -\rho(\vec{r},t), \quad \vec{j}(\vec{r},t) \to -\vec{j}(\vec{r},t)$$
$$\vec{E}(\vec{r},t) \to -\vec{E}(\vec{r},t), \quad \vec{B}(\vec{r},t) \to -\vec{B}(\vec{r},t)$$

(fields changed signs since their sources changed signs)

Discrete symmetries are conserved in classical E&M. Need quantum mechanics?

C,P, &T in Quantum Field Theory

★ The meaning of discrete symmetries in Quantum Field Theory

- C and P are unitary operators: $C^\dagger = C^{-1}$ and $P^\dagger = P^{-1}$

  - … and if they are good symmetries, they commute with the Hamiltonian,

$$[C, \mathcal{H}] = 0 \quad \text{and} \quad [P, \mathcal{H}] = 0$$

  - for the scattering matrix $S = 1 + iT$,

$$CSC^\dagger = S \quad \text{and} \quad PSP^\dagger = S$$

  - note, however that weak interactions break both, so $[C, \mathscr{H}_W] \neq 0$, $[P, \mathscr{H}_W] \neq 0$

- … but T is anti-unitary:

$$i\frac{\partial \psi}{\partial t} = -\frac{\vec{\nabla}^2}{2m}\psi$$

T-odd T-even

only possible if T also switched $i \to -i$, and $\psi \to \psi^*$!

  - recall that an anti-unitary operator A=UK, where $U^\dagger = U^{-1}$ and $K[\alpha|\psi_1\rangle + \beta|\psi_2\rangle] = \alpha^*|\psi_1^\dagger\rangle + \beta^*|\psi_2^\dagger\rangle$

  - it interchanges in- and out- states in the S-matrix: $TST^{-1} = S^\dagger$

C,P, &T in Quantum Field Theory

★ The meaning of discrete symmetries in Quantum Field Theory

- Quantum fields in QFT are Hermitian operators

  - written as linear combinations of creation/annihilation operators

$$[CP]\phi(\vec{r},t)[CP]^\dagger = exp(i\alpha)\phi^\dagger(-\vec{r},t)$$

$$[CP]\psi(\vec{r},t)[CP]^\dagger = exp(i\beta)\gamma_0 C A^T \psi^{\dagger T}(-\vec{r},t)$$

$$[CP]\overline{\psi}(\vec{r},t)[CP]^\dagger = -exp(-i\beta)\psi^T(-\vec{r},t)C^{-1}\gamma_0$$

$$A\gamma_\mu = \gamma_\mu^\dagger A$$
$$\gamma_\mu C = -C\gamma_\mu^T$$

- We can summarize actions of discrete symmetries on fermonic currents:

	P	T	C	CP	CPT
$\overline{\psi}\chi$	$\overline{\psi}\chi$	$\overline{\psi}\chi$	$\overline{\chi}\psi$	$\overline{\chi}\psi$	$\overline{\chi}\psi$
$\overline{\psi}\gamma_5\chi$	$-\overline{\psi}\gamma_5\chi$	$\overline{\psi}\gamma_5\chi$	$\overline{\chi}\gamma_5\psi$	$-\overline{\chi}\gamma_5\psi$	$-\overline{\chi}\gamma_5\psi$
$\overline{\psi}\gamma_L\chi$	$\overline{\psi}\gamma_R\chi$	$\overline{\psi}\gamma_L\chi$	$\overline{\chi}\gamma_L\psi$	$\overline{\chi}\gamma_R\psi$	$\overline{\chi}\gamma_R\psi$
$\overline{\psi}\gamma_R\chi$	$\overline{\psi}\gamma_L\chi$	$\overline{\psi}\gamma_R\chi$	$\overline{\chi}\gamma_R\psi$	$\overline{\chi}\gamma_L\psi$	$\overline{\chi}\gamma_L\psi$
$\overline{\psi}\gamma^\mu\chi$	$\overline{\psi}\gamma_\mu\chi$	$\overline{\psi}\gamma_\mu\chi$	$-\overline{\chi}\gamma^\mu\psi$	$-\overline{\chi}\gamma_\mu\psi$	$-\overline{\chi}\gamma^\mu\psi$
$\overline{\psi}\gamma^\mu\gamma_5\chi$	$-\overline{\psi}\gamma_\mu\gamma_5\chi$	$\overline{\psi}\gamma_\mu\gamma_5\chi$	$\overline{\chi}\gamma^\mu\gamma_5\psi$	$-\overline{\chi}\gamma_\mu\gamma_5\psi$	$-\overline{\chi}\gamma^\mu\gamma_5\psi$
$\overline{\psi}\gamma^\mu\gamma_L\chi$	$\overline{\psi}\gamma_\mu\gamma_R\chi$	$\overline{\psi}\gamma_\mu\gamma_L\chi$	$-\overline{\chi}\gamma^\mu\gamma_R\psi$	$-\overline{\chi}\gamma_\mu\gamma_L\psi$	$-\overline{\chi}\gamma^\mu\gamma_L\psi$
$\overline{\psi}\gamma^\mu\gamma_R\chi$	$\overline{\psi}\gamma_\mu\gamma_L\chi$	$\overline{\psi}\gamma_\mu\gamma_R\chi$	$-\overline{\chi}\gamma^\mu\gamma_L\psi$	$-\overline{\chi}\gamma_\mu\gamma_R\psi$	$-\overline{\chi}\gamma^\mu\gamma_R\psi$
$\overline{\psi}\sigma^{\mu\nu}\chi$	$\overline{\psi}\sigma_{\mu\nu}\chi$	$-\overline{\psi}\sigma_{\mu\nu}\chi$	$-\overline{\chi}\sigma^{\mu\nu}\psi$	$-\overline{\chi}\sigma_{\mu\nu}\psi$	$\overline{\chi}\sigma^{\mu\nu}\psi$

Branco, Lavoura, Silva

Example of CP-violating operators

★ In any quantum field theory CP-symmetry can be broken

- recall terms like $\overrightarrow{E} \cdot \overrightarrow{B}$ for E&M; can write a similar one for QCD!

$$\mathcal{L} = \mathcal{L}_{QCD} + \frac{\theta g^2}{32\pi^2} G_a^{\mu\nu} \widetilde{G}_{a\mu\nu}$$

- ... but this is a problem, as a combination

$$\bar{\theta} = \theta + Arg\left[det M\right] \quad \text{with} \quad -\mathcal{L}_M = \overline{q_{Ri}} M_{ik} q_{Lk} + h.c.$$

- ...is observable as an electric dipole moment of a neutron:

$$d_n \simeq e m_q \bar{\theta}/M_n^2 \approx 10^{-16} \bar{\theta}\ ecm$$

★ A variety of proposed solutions exist (axions, anthropic, etc)

How can CP-violation probed with flavor physics?

B physics

- How can CP-violation be observed with b-quarks?
  - can be observed by comparing CP-conjugated decay rates in various ways, both with and w/out time dependence

$$a_{CP}(f) = \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})}$$

  - can manifest itself in charm ΔB=1 transitions (direct CP-violation)

$$\Gamma(B \to f) \neq \Gamma(CP[B] \to CP[f])$$ **dCPV**

  - or in ΔB=2 transitions (indirect CP-violation): mixing $|B_{1,2}\rangle = p|B^0\rangle \pm q|\overline{B^0}\rangle$

$$R_m^2 = |q/p|^2 = \left|\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma}\right|^2 = 1 + A_m \neq 1$$ **CPVmix**

  - or in the interference b/w decays (ΔB=1) and mixing (ΔB=2)

$$\lambda_f = \frac{q}{p}\frac{\overline{A_f}}{A_f} = R_m e^{i(\phi+\delta)}\left|\frac{\overline{A_f}}{A_f}\right|$$ **CPVint**

Recall from this morning: CP-violation in the SM

★ CP-violation in the SM is related to a single phase of the CKM matrix

- there are MULTIPLE ways to parameterize CKM matrix
  - Wolfenstein parameterization (parameters: λ ~ 0.22, A ~ 0.83, ρ ~ 0.15, η ~ 0.35)

$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

  - Buras-Wolfenstein parameterization (with $\bar\rho = \rho(1-\lambda^2/2)$ and $\bar\eta = \eta(1-\lambda^2/2)$)

$$V = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

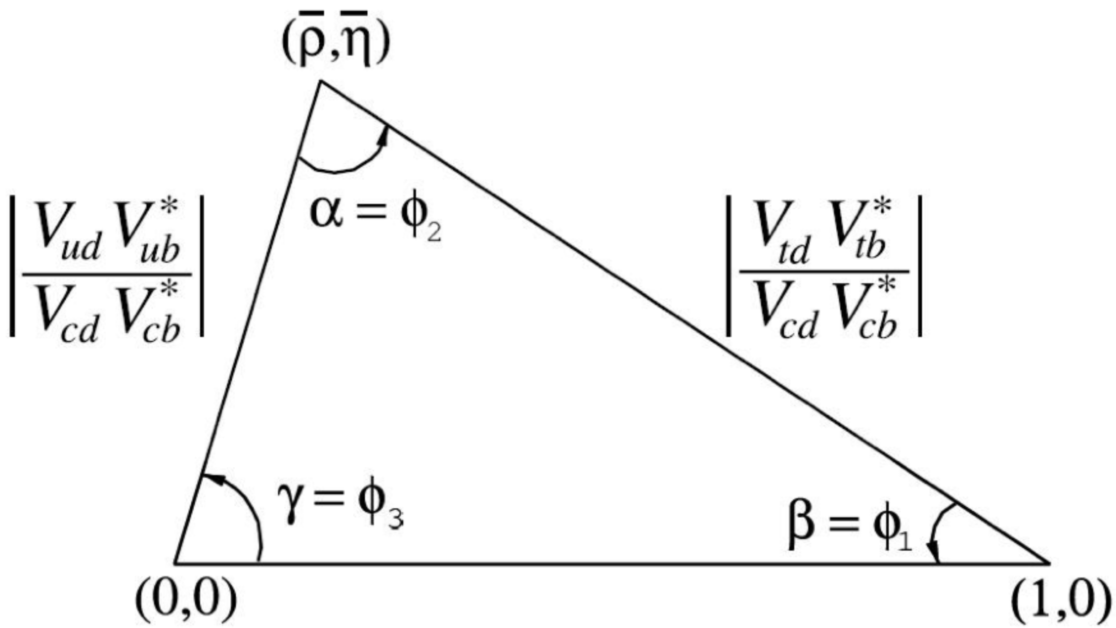
(note $\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$**)**

- off-diagonal terms in relations $VV^+=1$ look like triangles in a complex plane

★ CP-violation in flavor transitions can be learned by studying the CKM matrix

★ There is a single phase of the CKM matrix for 3-generation SM

- off-diagonal terms in unitarity relations VV⁺=1 look like triangles in a complex plane (ρ,η), e.g. $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ Each term is $\mathcal{O}(\lambda^3)$



$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- angles are

$$\phi_1(\beta) = arg\left[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*\right]$$ phase of V_{td} in Wolfenstein param

$$\phi_2(\alpha) = arg\left[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*\right]$$

$$\phi_3(\gamma) = arg\left[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*\right]$$ phase of V_{ub} in Wolfenstein param

2. Time-independent (direct) CP-violation

★ Direct CP-violating asymmetries probe CP-violation in ΔB=1 amplitudes

- CP-asymmetries compare partial rates of CP-conjugated decays

$$a_{CP}(f) = \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})}$$

(both charged and neutral B's)

- a non-vanishing decay asymmetry requires that a decay amplitude
  - contain several components each of which has its own strong and weak phases
  - strong phases: do not change under CP transformation of the decay amplitude
  - weak phases: flip sign under CP transformation of the decay amplitude

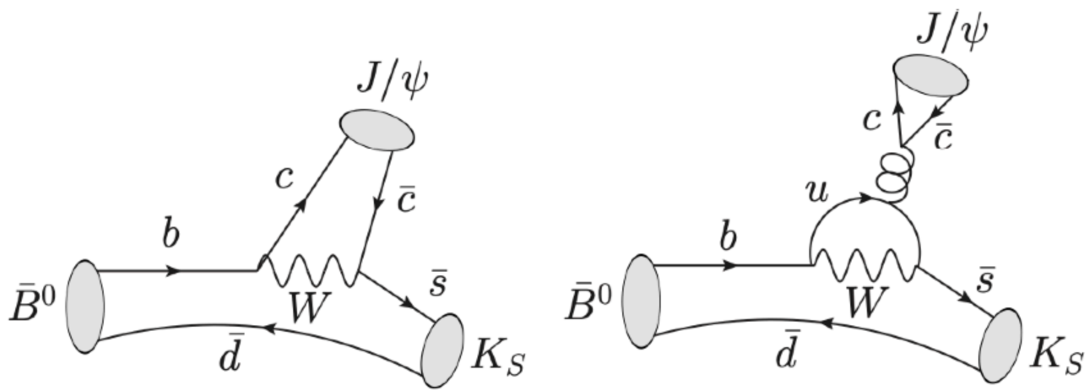
$$A(B \to f) \equiv A_f = |A_{f1}| e^{i\delta_1} e^{i\theta_1} + |A_{f2}| e^{i\delta_2} e^{i\theta_2}$$

- Now we can form the CP-asymmetry

$$a_{CP}(f) = 2 r_f \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2) \quad \text{with} \quad r_f = \left| \frac{A_{f2}}{A_{f1}} \right|$$

weak strong

- How can one compute the amplitudes (especially the strong phase difference)
  - QCD factorization (with Bander-Silverman-Soni mechanism)



  - experimental fits to flavor flow/flavor SU(3) amplitude basis

Loopless studies of CP-violation: CKM angle ϕ_3

★ There are ways to study CP-violation without penguin loops

- cleanest signals involve interference of $b \to c\bar{u}s$ and $b \to u\bar{c}s$

➢ via $B^\pm \to D\left(\left[D^0, \overline{D^0}, D_{CP}\right] \to f\right)K^\pm$ GWS (Gronau, Wyler, London)

➢ via $B^\pm \to D\left(\left[D^0, \overline{D^0}\right] \to K\pi\right)K^\pm$ ADS(Atwood, Dunietz, Soni)

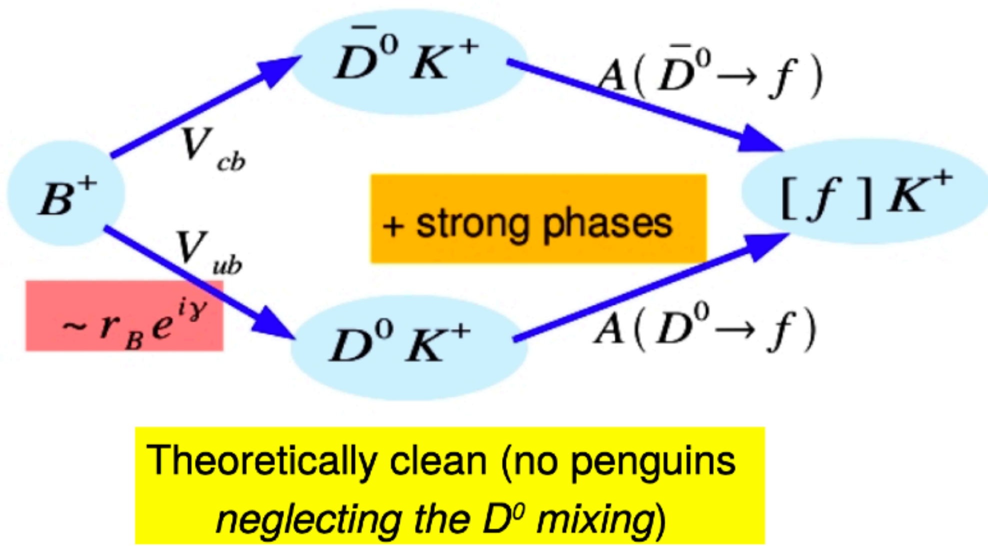
➢ via $B^\pm \to D\left(D \to KK^*\right)K^\pm$ GLS (Grossman, Ligeti, Soffer)

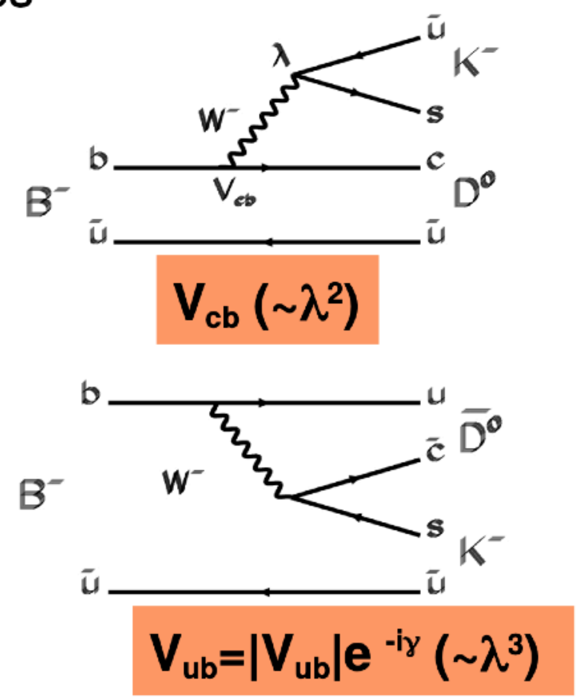
➢ via $B^\pm \to D\left(D \to \text{multibody}\right)K^\pm$ (Giri, Grossman, Soffer, Zupan, Atwood, Soni)

	Process	Observable	Theory	Sys. dom. (Discovery) [ab^{-1}]	vs LHCb	vs Belle	Anomaly	NP
●	GGSZ	ϕ_3	★★★	>50	★★	★★★	★	★★
●	GLW	ϕ_3	★★★	>50	★★	★★★	★	★★
●	ADS	ϕ_3	★★	>50	★★	★★★	★	★★★
●	Time-dependent	$\phi_3 - \phi_2$	★★	-	★★	★★	★	★

CKM angle ϕ_3: final state triangles

- $D^{(*)}K^{(*)}$ decays: from BRs and BR ratios, no time-dependent analysis, just rates
- the phase γ is measured exploiting interferences: two amplitudes leading to the same final states
- some rates can be really small: ~ 10^{-7}





M. Bona

CKM angle ϕ_3: final state triangles

- Let us define the following observables

$$A_{CP} = \frac{\Gamma(B^- \to D^0_{CP} K^-) - \Gamma(B^+ \to D^0_{CP} K^+)}{\Gamma(B^- \to D^0_{CP} K^-) + \Gamma(B^+ \to D^0_{CP} K^+)} = \frac{\pm 2 r_B \sin(\delta_B) \sin(\gamma)}{1 + r_B^2 \pm 2 r_B \cos(\delta_B) \cos(\gamma)}$$

$$R_{CP} = \frac{\Gamma(B^- \to D^0_{CP} K^-) + \Gamma(B^+ \to D^0_{CP} K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to D^0 K^+)} = 1 + r_B^2 \pm 2 r_B \cos(\delta_B) \cos(\gamma)$$

Note: $|D_{CP\pm}\rangle = \left(|D^0\rangle \pm |\overline{D}^0\rangle\right)/\sqrt{2}$

- The state $|D_{CP}\rangle$ is defined by the final state: $\pi^+\pi^-$, K^+K^- (CP=+), $K_s^+\pi^0$ (CP=-) (assuming CP-conservation in D-decay)

CKM angle ϕ_3: final state triangles

GLW(*Gronau, London, Wyler*) method:

more sensitive to r_B

uses the CP eigenstates $D^{(*)0}_{CP}$ with final states:

K^+K^-, $\pi^+\pi^-$ (CP-even), $K_s\pi^0$ (ω,ϕ) (CP-odd)

$$R_{CP\pm} = 1 + r_B^2 \pm 2r_B \cos\gamma \cos\delta_B \qquad A_{CP\pm} = \frac{\pm 2r_B \sin\gamma \sin\delta_B}{1 + r_B^2 \pm 2r_B \cos\gamma \cos\delta_B}$$

ADS(*Atwood, Dunietz, Soni*) method: B^0 and $\overline{B}^0$ in the same final state with $D^0 \to K^+\pi^-$ (suppr.) and $\overline{D}^0 \to K^+\pi^-$ (fav.)

$$R_{ADS} = r_B^2 + r_{DCS}^2 + 2r_B r_{DCS} \cos\gamma \cos(\delta_B + \delta_D)$$

the most sensitive way to γ

D^0 Dalitz plot with the decays $B^- \to D^{(*)0}[K_S\pi^+\pi^-]\,K^-$

three free parameters to extract: γ , r_B and δ_B

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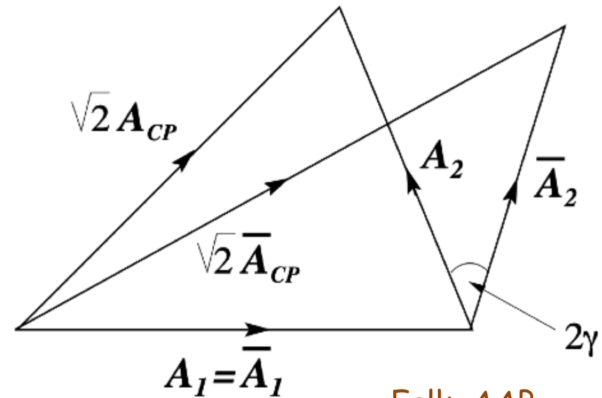
CKM angle ϕ_3: initial state triangles

★ One can also use a fact that initial state at Belle II is quantum coherent

- which means that initial state can be CP-tagged
- can be done for both B_d (at $\Upsilon(4S)$) or B_s (at $\Upsilon(5S)$). For B_s

$$A_{\rm CP} = A(B_s^{\rm CP} \to D_s^- K^+) = (A_1 + A_2)/\sqrt{2}\,,$$
$$\overline{A}_{\rm CP} = A(B_s^{\rm CP} \to D_s^+ K^-) = (\overline{A}_1 + \overline{A}_2)/\sqrt{2}\,.$$



Falk, AAP

- measuring all amplitudes,

$$\alpha = \frac{2|A_{\rm CP}|^2 - |A_1|^2 - |A_2|^2}{2|A_1||A_2|}\,,$$
$$\overline{\alpha} = \frac{2|\overline{A}_{\rm CP}|^2 - |\overline{A}_1|^2 - |\overline{A}_2|^2}{2|\overline{A}_1||\overline{A}_2|}\,,$$

$$\sin 2\gamma = \pm\left(\alpha\sqrt{1-\overline{\alpha}^2} - \overline{\alpha}\sqrt{1-\alpha^2}\right)$$

- analysis is similar for $B_d \to D\pi$ is similar, but coefficients are e time-dependent

3. Time-dependent CP-asymmetries

★ Time-dependent CP-asymmetries probe CP-violation in ΔB=2 amplitudes

- SM: CP-violation in ΔB=2 and ΔB=1 transitions have the same origin, this fact does not have to be true in general NP model

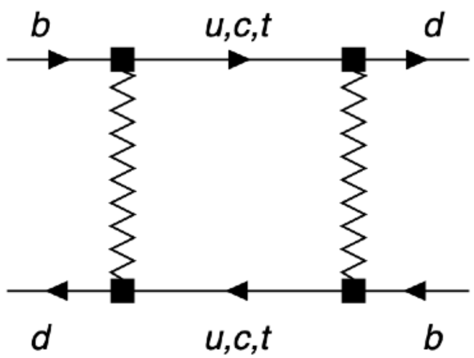
- it most conveniently can be probed in transitions that involve mixing
  - use time-dependent CP asymmetries due to the interference between B-mixing and B decay amplitudes
  - interference between the two neutral B meson evolution eigenstates generates the time-dependent CP asymmetry

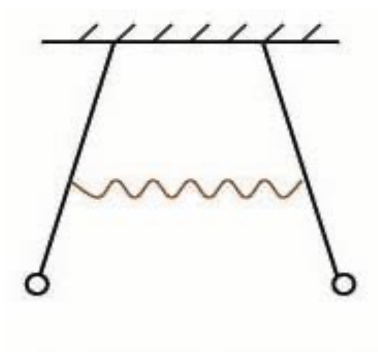
$$a_{CP}(f,t) = \frac{\Gamma(B(t) \to f) - \Gamma(\overline{B}(t) \to \overline{f})}{\Gamma(B(t) \to f) + \Gamma(\overline{B}(t) \to \overline{f})}$$

- Need to develop a formalism for time-dependent decays

Time dependent decay amplitudes

★ In the SM, neutral B-mesons can mix via weak interaction diagrams





Coupled oscillators

- only at one loop in the Standard Model, so can be sensitive to possible quantum effects due to new physics particles
- $\Delta B = 2$ interactions couple dynamics of B^0 and $\bar{B}^0$
- We need to study simultaneous time evolution,

$$|B(t)\rangle = \left[\begin{array}{c} a(t) \\ b(t) \end{array} \right] = a(t)|B^0\rangle + b(t)|\overline{B}^0(t)\rangle$$

- This is very similar to the case of coupled pendula in classical mechanics

Time dependent decay amplitudes

- Time dependence: coupled Schrodinger equations
  - note that CPT-invariance requires that $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

$$i\frac{d}{dt}|B(t)\rangle = \left[M - i\frac{\Gamma}{2}\right]|B(t)\rangle \equiv \left[\begin{array}{cc} A & p^2 \\ q^2 & A \end{array}\right]|B(t)\rangle$$

Q: this Hamiltonian is clearly non-hermitian! What is goin on?

- Non-diagonal Hamiltonian: need to diagonalize the mass matrix

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$
$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$

("switch from flavor to mass eigenstates")

- In the mass basis the mass matrix is diagonal, i.e.

$$Q^{-1}\left[M - i\frac{\Gamma}{2}\right]Q = \left(\begin{array}{cc} M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2 \end{array}\right)$$

- ... with mass and lifetime differences: $\Delta M = M_H - M_L$ & $\Delta\Gamma = \Gamma_L - \Gamma_H$

Note that $m = \frac{M_H + M_L}{2} = M_{11} = M_{22}$ & $\Gamma = \frac{\Gamma_L + \Gamma_H}{2} = \Gamma_{11} = \Gamma_{22}$

Time dependent decay amplitudes

- The transformation matrices that diagonalize the Hamiltonian are

$$Q = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \quad \text{and} \quad Q^{-1} = \frac{1}{2pq}\begin{pmatrix} q & p \\ q & -p \end{pmatrix}$$

- To find the time development of the flavor eigenstates one needs to transform the evolution equation back to the flavor basis

$$\begin{bmatrix} |B^0(t)\rangle \\ |\overline{B}^0(t)\rangle \end{bmatrix} = Q \begin{pmatrix} e^{-iM_L - \Gamma_L/2} & 0 \\ 0 & e^{-iM_H - \Gamma_H/2} \end{pmatrix} Q^{-1} \begin{bmatrix} |B^0\rangle \\ |\overline{B}^0\rangle \end{bmatrix}$$

- … which gives for the time evolution matrix in the flavor basis

$$Q \begin{pmatrix} e^{-iM_L t - \Gamma_L t/2} & 0 \\ 0 & e^{-iM_H t - \Gamma_H t/2} \end{pmatrix} Q^{-1} = \begin{pmatrix} g_+(t) & \frac{q}{p} g_-(t) \\ \frac{p}{q} g_-(t) & g_+(t) \end{pmatrix}$$

Nierste

with

$$g_+(t) = e^{-imt}\, e^{-\Gamma t/2} \left[\cosh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} - i \sinh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right],$$

$$g_-(t) = e^{-imt}\, e^{-\Gamma t/2} \left[-\sinh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} + i \cosh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right].$$

Time dependent decay amplitudes

- This procedure provides a picture of how B-states evolve due to flavor oscillations,

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\overline{B}^0\rangle$$
$$|\overline{B}^0(t)\rangle = \frac{p}{q} g_-(t)|B^0\rangle + g_+(t)|\overline{B}^0\rangle$$

with

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \left[\cosh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} - i \sinh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right],$$
$$g_-(t) = e^{-imt} e^{-\Gamma t/2} \left[-\sinh\frac{\Delta\Gamma\, t}{4} \cos\frac{\Delta M\, t}{2} + i \cosh\frac{\Delta\Gamma\, t}{4} \sin\frac{\Delta M\, t}{2} \right].$$

- The only thing left is to relate q/p, ΔM and $\Delta\Gamma$ to original parameters of H

secular equation: $$(\Delta M + i\frac{\Delta\Gamma}{2})^2 = 4\left(M_{12} - i\frac{\Gamma_{12}}{2}\right)\left(M_{12}^* - i\frac{\Gamma_{12}^*}{2}\right)$$

Re

Im

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4\,|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Delta M\,\Delta\Gamma = -4\,\mathrm{Re}\,(M_{12}\Gamma_{12}^*)$$

- Finally, the ratio $$\frac{q}{p} = -\frac{\Delta M + i\,\Delta\Gamma/2}{2M_{12} - i\,\Gamma_{12}} = -\frac{2M_{12}^* - i\,\Gamma_{12}^*}{\Delta M + i\,\Delta\Gamma/2}$$

Phases and amplitudes

- The B-meson states can have an arbitrary phase, so only relative phase is physical, which implies that there are three quantities that define B-mixing

$$|M_{12}|, \qquad |\Gamma_{12}|, \qquad \text{and} \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

- … which gives for the mixing parameters

$$\Delta M \simeq 2\,|M_{12}| \quad \text{and} \quad \Delta\Gamma \simeq 2\,|\Gamma_{12}|\cos\phi$$

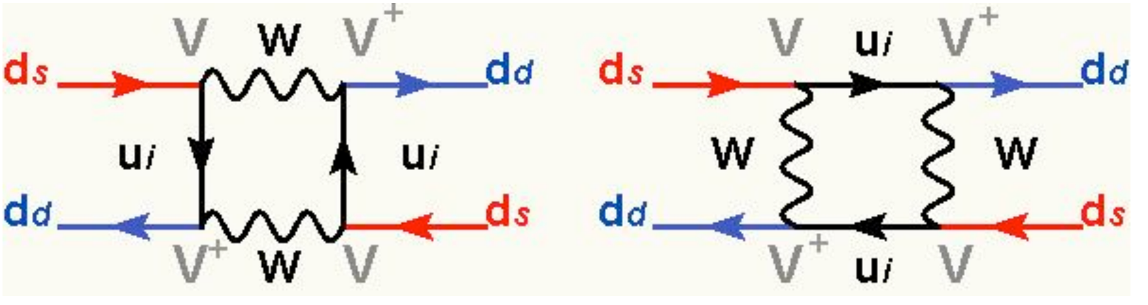
- … and, up to a good approximation, to the phase of the box diagram,

$$\frac{q}{p} = -\frac{M_{12}^*}{M_{12}} = \frac{V_{tb}^* V_{tq}}{V_{tb} V_{tq}^*} \qquad \text{and} \qquad \left|\frac{q}{p}\right|^2 = 1 - a = 1 - Im\frac{\Gamma_{12}}{M_{12}}$$

We can calculate B-mixing parameters in the SM: any sign of New Physics?

Glashow-Iliopoulos-Maiani (GIM) **mechanism**

- There are no ΔQ=2 interactions in the Standard Model…
- … but we can make them via a "two-step process" (loop diagram):

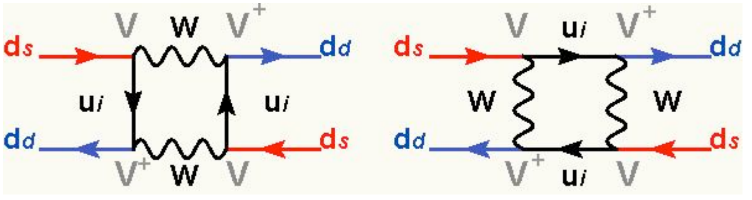


- Let's calculate them! For each internal quark type we get

$$\sim g^4 \left(V_{is} V_{id}^\dagger V_{js} V_{jd}^\dagger\right) \int \frac{d^4k}{(4\pi)^4} \frac{(\text{some gamma matrices})\ (k^2)}{(\not{k} - m_i)(\not{k} - m_j)(k^2 - m_W^2)^2}$$

Divergent: not good…

- However, CKM matrix is unitary:
- contribution of different internal flavors comes with different signs!



top: $\left(V_{tb}V^\dagger_{td}V_{tb}V^\dagger_{td}\right) \sim (1\times A\lambda^3)(1\times A\lambda^3)$

top-charm: $\left(V_{tb}V^\dagger_{td}V_{cb}V^\dagger_{cd}\right) \sim (1\times A\lambda^3)(A\lambda^2\times(-\lambda))$

- Thus, in the limit where k >> m_i, m_j, M_W:

top: $$g^4\left(A\lambda^3\right)^2\int\frac{d^4k}{(4\pi)^4}\frac{(\text{some gamma matrices})(k^2)}{(\not k)(\not k)(k^2)^2}$$

top-charm: $$-g^4\left(A\lambda^3\right)^2\int\frac{d^4k}{(4\pi)^4}\frac{(\text{some gamma matrices})(k^2)}{(\not k)(\not k)(k^2)^2}$$

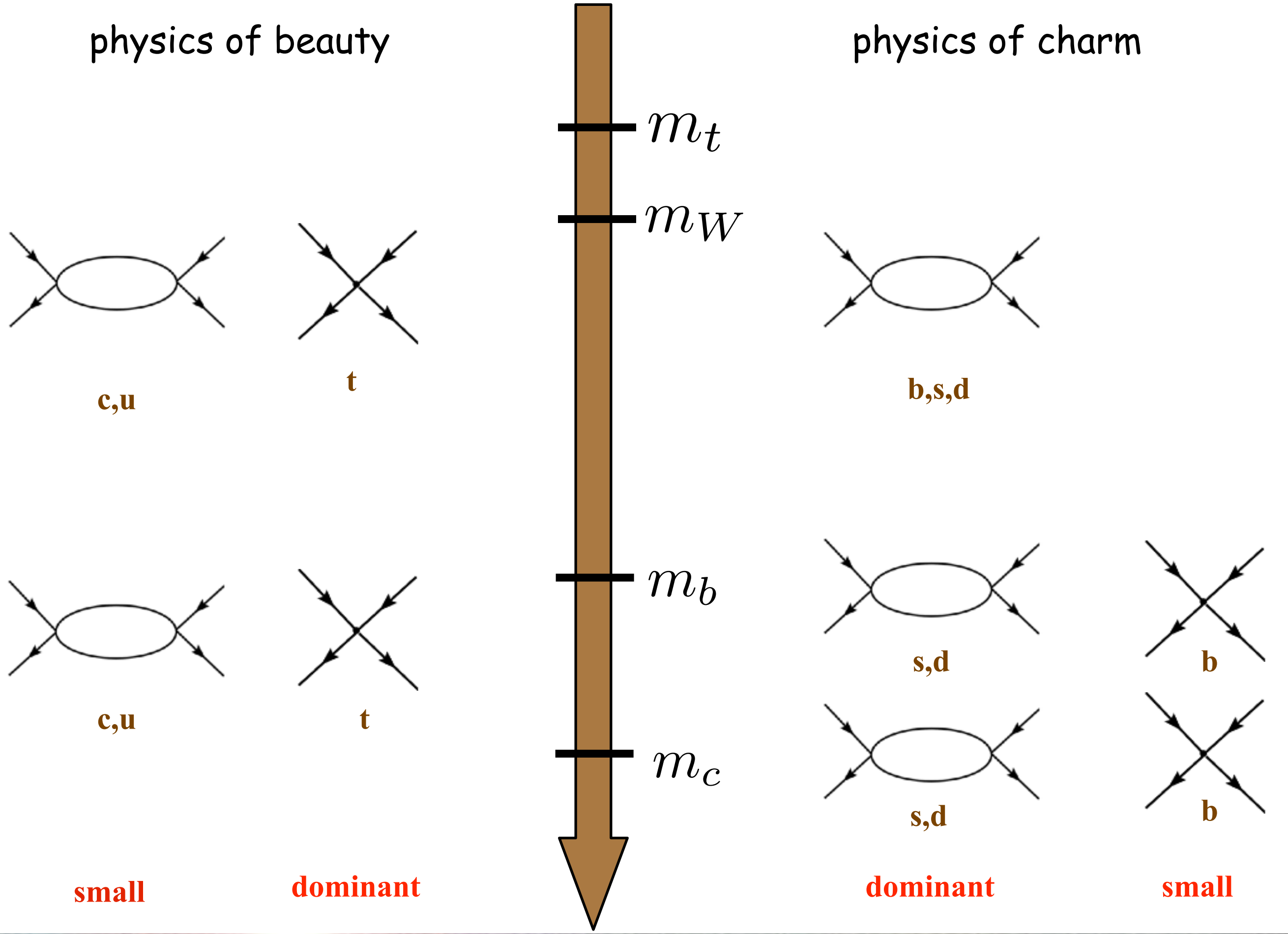
… and similarly for other quarks Cancellation of divergences!

$$A \propto \sum_i m_i^2 (V_{is}V^*_{ib})^2 g_k(m_i^2)$$

Glashow-Iliopulous-Maiani

★ Modern approach to flavor physics calculations: effective field theories

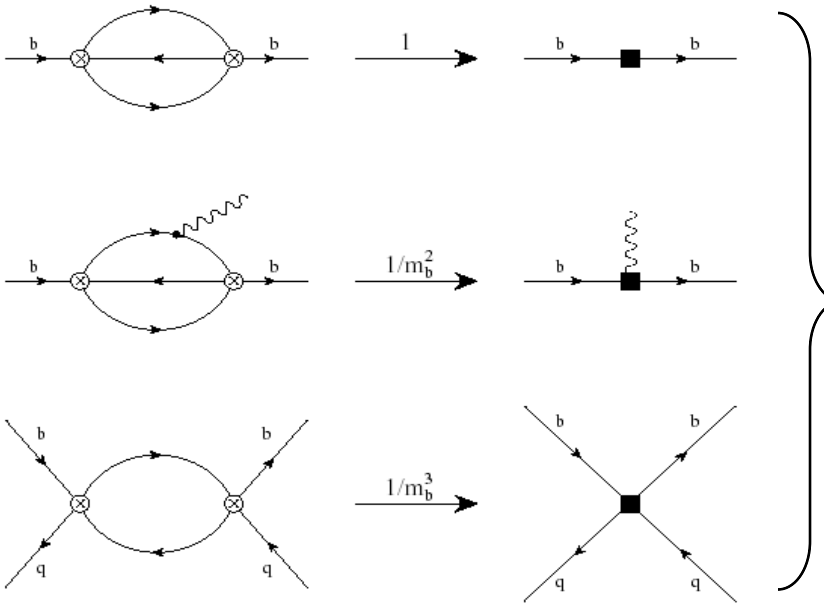
★ It is important to understand relevant energy scales for the problem at hand



- Assume quark-hadron duality: relate width to forward matrix element

$$\Gamma(H_b) = \frac{1}{2M_b}\langle H_b | T | H_b \rangle = \frac{1}{2M_b}\langle H_b | \mathrm{Im}\, i \int d^4x\, T\left\{ H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0) \right\} | H_b \rangle$$

- This correlator can be expanded using OPE



I. Bigi, M. Shifman, A. Vainshtein, M. Voloshin, N. Uraltsev, A. Falk, A. Manohar, M. Wise, M. Neubert, C. Sachrajda, P. Colangelo, F. de Fazio, …

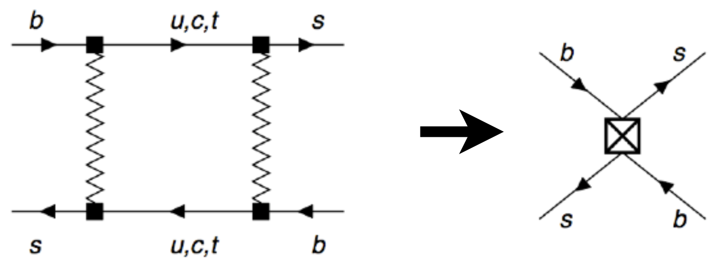
$$\Gamma(H_b) = \sum_k \frac{C_k(\mu)}{m_b^k} \langle H_b | O_k^{\Delta B=0}(\mu) | H_b \rangle$$

What are the results?

Standard Model contributions

Both ΔM_{Bs} and $\Delta\Gamma_{Bs}$ can be computed in the limit $m_b \to \infty$:

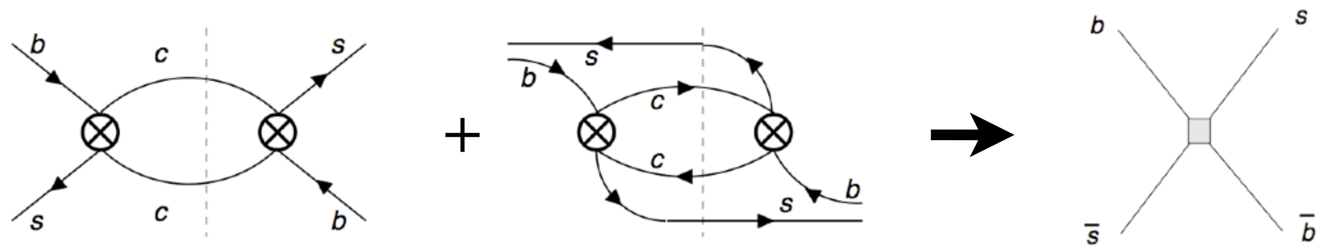
ΔM_{Bs}:



A.Buras, M.Jamin, P.Weisz

$$M_{12}(B_s) = \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 \left(V_{tb}V_{ts}^*\right)^2 \hat{\eta}_B S_0(x_t) f_{B_s}^2 B$$

$\Delta\Gamma_{Bs}$:



A. Lenz, U. Nierste

$$\Gamma_{21}(B_s) = \sum_k \frac{C_k(\mu)}{m_b^k} \langle B_s|\mathcal{O}_k^{\Delta B=2}(\mu)|\overline{B}_s\rangle.$$

$$\frac{\Delta\Gamma_s}{\Gamma_s} \approx 0.137 \pm 0.027$$

$\Delta\Gamma_{Bs}$: a calculation yields:

$$\Gamma_{21}(B_s) = -\frac{G_F^2 m_b^2}{12\pi(2M_{B_s})}(V_{cb}^* V_{cs})^2 \left[[F(z)+P(z)]\langle Q\rangle + [F_S(z)+P_S(z)]\langle Q_S\rangle + \delta_{1/m} + \delta_{1/m^2} \right]$$

★ ... with operators

WC (incl. pQCD corr): Beneke et al, Ciuchini et al

$$Q = (\bar b_i s_i)_{V-A}(\bar b_j s_j)_{V-A}, \quad Q_S = (\bar b_i s_i)_{S-P}(\bar b_j s_j)_{S-P}$$
$$\tilde Q = (\bar b_i s_j)_{V-A}(\bar b_j s_i)_{V-A}, \quad \tilde Q_S = (\bar b_i s_j)_{S-P}(\bar b_j s_i)_{S-P}$$

$$\langle Q\rangle = 2\frac{1+N_c}{N_c} f_{B_s}^2 M_{B_s}^2 B$$
$$\langle Q_S\rangle = \frac{1-2N_c}{N_c}\frac{M_{B_s}^4}{(m_b+m_s)^2} f_{B_s}^2 B_S$$

★ ... so the result (up to $1/m_b^2$) is:

$$\begin{aligned}\Delta\Gamma_{B_s} = & \Big[0.0005B + 0.1732B_s + 0.0024B_1 - 0.0237B_2 - 0.0024B_3 - 0.0436B_4 \\ & + 2\times10^{-5}\alpha_1 + 4\times10^{-5}\alpha_2 + 4\times10^{-5}\alpha_3 + 0.0009\alpha_4 - 0.0007\alpha_5 \\ & + 0.0002\beta_1 - 0.0002\beta_2 + 6\times10^{-5}\beta_3 - 6\times10^{-5}\beta_4 - 1\times10^{-5}\beta_5 \\ & - 1\times10^{-5}\beta_6 + 1\times10^{-5}\beta_7 + 1\times10^{-5}\beta_8\Big] \quad (\mathrm{ps}^{-1}).\end{aligned}$$

A.Badin, F. Gabbiani, A.A.P.
Phys. Lett. B653, 230 (2007)

Time-dependent CP-asymmetries

★ Time-dependent CP-asymmetries probe CP-violation in ΔB=2 amplitudes

- Now we know how to deal with time-dependent rates

$$\Gamma(M(t) \to f) = \mathcal{N}_f \, |\langle f|S|M(t)\rangle|^2$$
$$\Gamma(\overline{M}(t) \to f) = \mathcal{N}_f \, \left|\langle f|S|\overline{M}(t)\rangle\right|^2$$

- … which can be calculated using the developed formalism, $\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f}$

$$\begin{aligned}\Gamma(M(t) \to f) &= \mathcal{N}_f \, |A_f|^2 \, e^{-\Gamma t} \left\{ \frac{1+|\lambda_f|^2}{2} \cosh \frac{\Delta\Gamma \, t}{2} + \frac{1-|\lambda_f|^2}{2} \cos(\Delta M \, t) \right. \\ &\left. -\mathrm{Re}\,\lambda_f \, \sinh \frac{\Delta\Gamma \, t}{2} - \mathrm{Im}\,\lambda_f \, \sin(\Delta M \, t) \right\},\end{aligned}$$

$$\begin{aligned}\Gamma(\overline{M}(t) \to f) &= \mathcal{N}_f \, |A_f|^2 \, \frac{1}{1-a} \, e^{-\Gamma t} \left\{ \frac{1+|\lambda_f|^2}{2} \cosh \frac{\Delta\Gamma \, t}{2} - \frac{1-|\lambda_f|^2}{2} \cos(\Delta M \, t) \right. \\ &\left. -\mathrm{Re}\,\lambda_f \, \sinh \frac{\Delta\Gamma \, t}{2} + \mathrm{Im}\,\lambda_f \, \sin(\Delta M \, t) \right\}.\end{aligned}$$

Time-dependent CP-asymmetries

★ Various time-dependent CP-asymmetries can now be formed

- The flavor-specific CP-asymmetry (aka semileptonic CP asymmetry)

$$a_{\rm fs} \equiv \frac{\Gamma(\overline{M}(t) \to f) - \Gamma(M(t) \to \overline{f})}{\Gamma(\overline{M}(t) \to f) + \Gamma(M(t) \to \overline{f})} = \frac{1-(1-a)^2}{1+(1-a)^2} = a + \mathcal{O}(a^2).$$

- CP-asymmetry for decays to CP-eigenstates (such as $f_{CP} = J/\psi K_S$, etc.)

$$a_{f_{\rm CP}}(t) = \frac{\Gamma(\overline{M}(t) \to f_{\rm CP}) - \Gamma(M(t) \to f_{\rm CP})}{\Gamma(\overline{M}(t) \to f_{\rm CP}) + \Gamma(M(t) \to f_{\rm CP})}$$

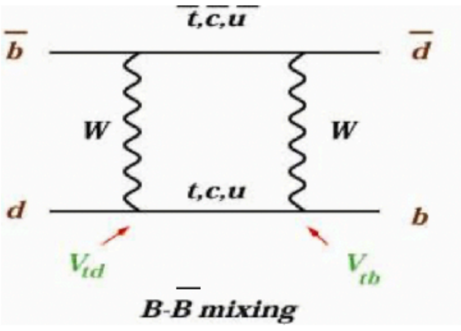
$$= -\frac{A_{CP}^{\rm dir}\cos(\Delta M\, t) + A_{CP}^{\rm mix}\sin(\Delta M\, t)}{\cosh(\Delta\Gamma\, t/2) + A_{\Delta\Gamma}\sinh(\Delta\Gamma\, t/2)} + \mathcal{O}(a)$$

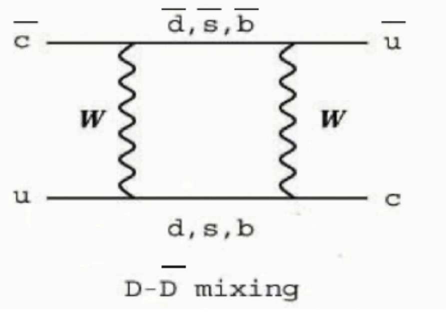
where $A_{CP}^{\rm dir} = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$, $A_{CP}^{\rm mix} = -\frac{2\,{\rm Im}\,\lambda_f}{1+|\lambda_f|^2}$ and $A_{\Delta\Gamma} = -\frac{2\,{\rm Re}\,\lambda_f}{1+|\lambda_f|^2}$

Ex.: Belle II studies for time-dependent CPV

	Process	Observable	Theory	Sys. dom. (Discovery) [ab^{-1}]	vs LHCb	vs Belle	Anomaly	NP
●	$B \to J/\psi K_S^0$	ϕ_1	$\star\star\star$	5-10	$\star\star$	$\star\star$	$\star$	$\star$
●	$B \to \phi K_S^0$	ϕ_1	$\star\star$	>50	$\star\star$	$\star\star\star$	$\star$	$\star\star\star$
●	$B \to \eta' K_S^0$	ϕ_1	$\star\star$	>50	$\star\star$	$\star\star\star$	$\star$	$\star\star\star$
●	$B \to \rho^\pm \rho^0$	ϕ_2	$\star\star\star$	>50	$\star$	$\star\star\star$	$\star$	$\star$
●	$B \to J/\psi \pi^0$	ϕ_1	$\star\star\star$	>50	$\star$	$\star\star\star$	-	-
●	$B \to \pi^0 \pi^0$	ϕ_2	$\star\star$	>50	$\star\star\star$	$\star\star\star$	$\star\star$	$\star\star$
●	$B \to \pi^0 K_S^0$	$S_{\rm CP}$	$\star\star$	>50	$\star\star\star$	$\star\star\star$	$\star\star$	$\star\star$

- We discuss how CP-violation can be studied with B-mesons
  - would D-mixing be different?
  - would CP-violation studies in charm be different?





$\overline{D^0} - D^0$ mixing	$\overline{B^0} - B^0$ mixing
• intermediate down-type quarks • SM: b-quark contribution is negligible due to $V_{cd}V_{ub}^*$ • $rate \propto f(m_s) - f(m_d)$ (zero in the SU(3) limit) Falk, Grossman, Ligeti, and A.A.P. Phys.Rev. D65, 054034, 2002 2^{nd} order effect!!!	• intermediate up-type quarks • SM: t-quark contribution is dominant • $rate \propto m_t^2$ (expected to be large)
1. Sensitive to long distance QCD 2. Small in the SM: New Physics! (must know SM x and y)	1. Computable in QCD (*) 2. Large in the SM: CKM!

(*) up to matrix elements of 4-quark operators

Charm physics

- How can CP-violation be observed in charm system?
  - can be observed by comparing CP-conjugated decay rates in various ways, both with and w/out time dependence

$$a_{\rm CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})}$$

  - can manifest itself in charm ΔC=1 transitions (direct CP-violation)

$$\Gamma(D \to f) \neq \Gamma(CP[D] \to CP[f])$$ **dCPV**

  - or in ΔC=2 transitions (indirect CP-violation): mixing $|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D^0}\rangle$

$$R_m^2 = |q/p|^2 = \left| \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma} \right|^2 = 1 + A_m \neq 1$$ **CPVmix**

  - or in the interference b/w decays (ΔC=1) and mixing (ΔC=2)

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} = R_m e^{i(\phi+\delta)} \left| \frac{\overline{A}_f}{A_f} \right|$$ **CPVint**

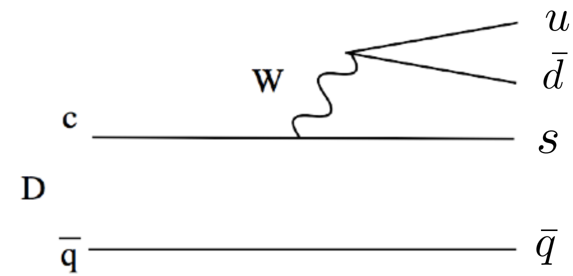
Introduction: charm-specific lingo

★ Can be classified by SM CKM suppression of tree amplitude ($V_{us} \sim \lambda$)

★ Cabibbo-favored (CF: λ^0) decay

- originates from $c \to s u\bar{d}$
- examples: $D^0 \to K^- \pi^+$

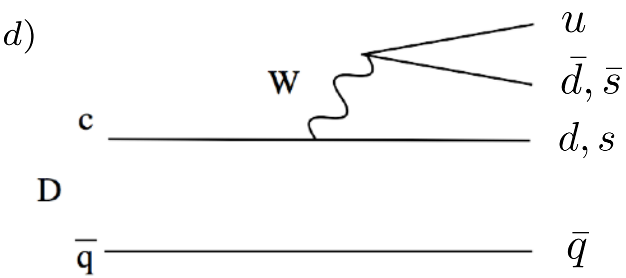
$V_{cs} V_{ud}^*$



★ Singly Cabibbo-suppressed (SCS: λ^1) decay

- originates from $c \to q u\bar{q}$
- examples: $D^0 \to \pi\pi$ and $D^0 \to KK$

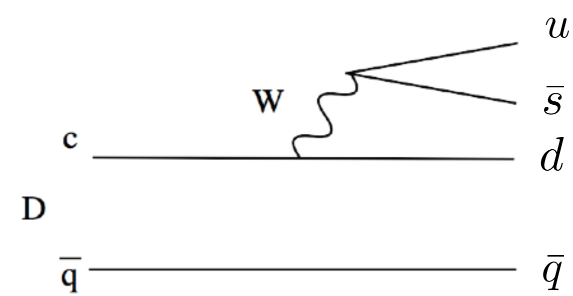
$V_{cs(d)} V_{us(d)}^*$



★ Doubly Cabibbo-suppressed (DCS: λ^2) decay

- originates from $c \to d u\bar{s}$
- examples: $D^0 \to K^+ \pi^-$

$V_{cd} V_{us}^*$

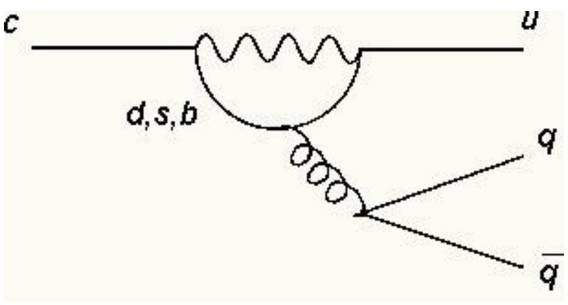


★ We shall concentrate on SCS decays. Why is that?

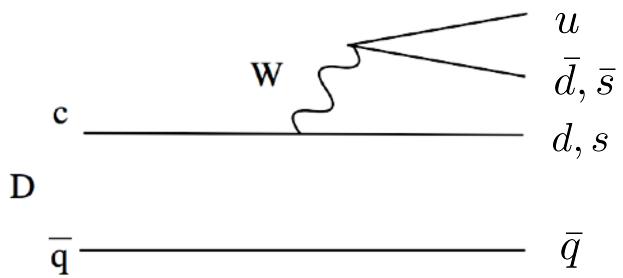
Generic expectations for sizes of CPV effects

★ Generic expectation is that CP-violating observables in the SM are small

$\Delta c = 1$ amplitudes allow to reach third -generation quarks!



"Penguin" amplitude/contraction



"Tree" amplitude

★ The Unitarity Triangle relation for charm:

$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$

$$\sim\lambda \qquad \sim\lambda \qquad \sim\lambda^5$$

With b-quark contribution neglected: only **2** generations contribute $\Rightarrow$ **real 2x2 Cabibbo matrix**

Any CP-violating signal in the SM will be small, at most $O(V_{ub}V_{cb}^*/V_{us}V_{cs}^*) \sim 10^{-3}$

Thus, O(1%) CP-violating signal can provide a "smoking gun" signature of New Physics

2. Time-independent (direct) CP-violation

★ Direct CP-violating asymmetries probe CP-violation in ΔC=1 amplitudes

- CP-asymmetries compare partial rates of CP-conjugated decays

$$a_{CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})}$$ (both charged and neutral D's)

- a non-vanishing decay asymmetry requires that a decay amplitude
  - contain several components each of which has its own strong and weak phases
  - strong phases: do not change under CP transformation of the decay amplitude
  - weak phases: flip sign under CP transformation of the decay amplitude

$$A(D \to f) \equiv A_f = |A_{f1}| e^{i\delta_1} e^{i\theta_1} + |A_{f2}| e^{i\delta_2} e^{i\theta_2}$$

- Now we can form the CP-asymmetry

$$a_{CP}(f) = 2 r_f \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$ with $$r_f = \left| \frac{A_{f2}}{A_{f1}} \right|$$

weak strong

Direct CP-violation in charm: realities of life

★ IDEA: consider the DIFFERENCE of decay rate asymmetries: D →ππ vs D → KK! For each final state the asymmetry

D^0: no neutrals in the final state!

$$a_f = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})} \Rightarrow a_f = a_f^d + a_f^m + a_f^i$$

direct mixing interference

★ A reason: $a^m_{KK} = a^m_{\pi\pi}$ and $a^i_{KK} = a^i_{\pi\pi}$ (for CP-eigenstate final states), so, ideally, mixing asymmetries cancel ($r_f = P_f / A_f$)!

$$a_f^d = 2 r_f \sin\phi_f \sin\delta_f$$

★ ... and the resulting DCPV asymmetry is $\Delta a_{CP} = a^d_{KK} - a^d_{\pi\pi} \approx 2 a^d_{KK}$ (double!)

$$A_{KK} = \frac{G_F}{\sqrt{2}} \lambda \left[(T + E + P_{sd}) + a \lambda^4 e^{-i\gamma} P_{bd} \right]$$

$$A_{\pi\pi} = \frac{G_F}{\sqrt{2}} \lambda \left[(-(T + E) + P_{sd}) + a \lambda^4 e^{-i\gamma} P_{bd} \right]$$

★ ... so it is doubled in the limit of $SU(3)_F$ symmetry

SU(3) is badly broken in D-decays

Experimental analysis from LHCb

★ Since we are comparing rates for D0 and anti-D0: need to tag the flavor at production

$$D^{*+} \to D^0 \pi_s^+$$

"D*-trick" -- tag the charge of the slow pion (or muon for D's produced in B-decays)

★ The difference Δa_{CP} is also preferable experimentally, as

$$a_f^{\text{raw}} = a_f^{CP} + a_f^{\text{detect, D}} + a_D^{\text{detect, }\pi_s} + a_{D*}^{\text{prod}}$$

- a_f^{CP}: physics
- $a_f^{\text{detect, D}}$: detection asymmetry of D^0
- $a_D^{\text{detect, }\pi_s}$: detection asymmetry of soft pion
- a_{D*}^{prod}: production asymmetry of D^{*+}

★ D* production asymmetry and soft pion asymmetries are the same for KK and ππ final states-- they cancel in Δa_{CP}!

★ Integrate over time,

$$a_{CP,\ f} = \int_0^\infty a_{CP}(f;t) D(t) dt = a_f^d + \frac{\langle t \rangle}{\tau} a_f^{ind}$$

$D(t)$: distribution of proper decay time

★ Viola! Report observation!

- Experimental results
  - note that while the new result does constitute an observation of CP-violation in the difference…

$$\Delta a_{CP}^{dir} = a_{CP}(K^-K^+) - a_{CP}(\pi^-\pi^+) = (-0.156 \pm 0.029)\%$$

LHCb 2019

  - … it is not yet so for the individual decay asymmetries

$$\begin{aligned} a_{CP}(K^-K^+) &= (0.04 \pm 0.12\ (\text{stat}) \pm 0.10\ (\text{syst}))\%\,, \\ a_{CP}(\pi^-\pi^+) &= (0.07 \pm 0.14\ (\text{stat}) \pm 0.11\ (\text{syst}))\%\,. \end{aligned}$$

LHCb 2017

- Need confirmation from other experiments (Belle II)
- What does this result mean? New Physics? Standard Model?

Theoretical troubles

ΔA_{CP} within the Standard Model and beyond

Mikael Chala, Alexander Lenz, Aleksey V. Rusov and Jakub Scholtz

*Institute for Particle Physics Phenomenology, Durham University,
DH1 3LE Durham, United Kingdom*

Implications on the first observation of charm CPV at LHCb

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[1]*Institute of Physics, Academia Sinica,*

Taipei, Taiwan 11529, Republic of China

The Emergence of the $\Delta U = 0$ Rule in Charm Physics

Yuval Grossman[*] and Stefan Schacht[†]

Department of Physics, LEPP, Cornell University, Ithaca, NY 14853, USA

Revisiting CP violation in $D \to PP$ and VP decays

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Cheng-Wei Chiang

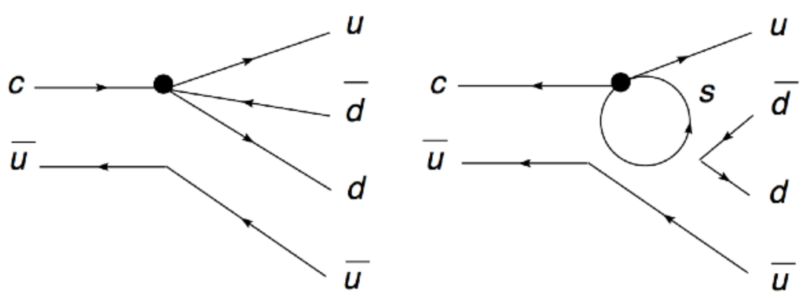
Department of Physics, National Taiwan University, Taipei, Taiwan 10617, ROC

Theoretical troubles

★ These asymmetries are notoriously difficult to compute

★ In the Standard Model

- need to estimate size of penguin/penguin contractions vs. tree



- unknown penguin contributions
  - SU(3) analysis: some ME are enhanced?
    Golden & Grinstein PLB 222 (1989) 501; Pirtshalava & Uttayarat 1112.5451
  - could expect large $1/m_c$ corrections (E/PE/PA/…)
    Isidori et al PLB 711 (2012) 46; Brod et al 1111.5000
  - flavor-flow diagrams
    Broad et al 1203.6659; Bhattacharya et al PRD 85 (2012) 054014; Cheng & Chiang 1205.0580; 1909.03063; Gronau, Rosner

★ General comments on SU(3)/flavor flow — type analyses

- fit both SM and (possible) NP parts of the amplitudes: can one claim SM-only?
- many parameters: can one claim $O(10^{-4})$ precision if rates are known to $O(10^{-2})$?

★ Need direct calculations of amplitudes/CPV-asymmetries

- QCD sum rule calculations of Δa_{CP} — Khodjamirian, AAP; Lenz, Piscopo, Rush
- SU(3) breaking analyses of $D \to PV, VV$
- constant (but slow) lattice QCD progress in $D \to \pi\pi, \pi\pi\pi$ — Hansen, Sharpe

- Effective Hamiltonian for singly Cabibbo-suppressed (SCS) decays
  - drop all "penguin" operators (Q_i for i ≥ 3) as C_i are small, $\lambda_q = V_{uq} V^*_{cq}$,

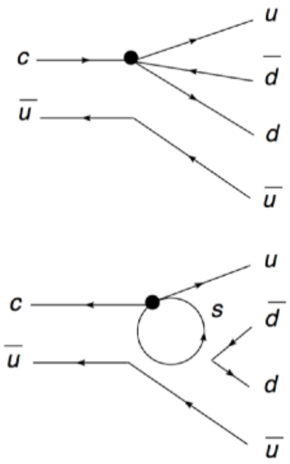
$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left(C_1 Q_1^q + C_2 Q_2^q \right) - \lambda_b \sum_{i=3,\dots,6,8g} C_i Q_i \right]$$

$$Q_1^q = (\bar u \Gamma_\mu q)\,(\bar q \Gamma^\mu c)\,, \qquad Q_2^q = (\bar q \Gamma_\mu q)\,(\bar u \Gamma^\mu c)$$

  - recall that $\sum_{q=d,s,b} \lambda_q = 0$ or $\lambda_d = -(\lambda_s + \lambda_b)$ and $\mathcal{O}^q \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i Q_i^q\,, \quad$ with $q = d, s.$

without QCD

with QCD



dCPV predictions

- As a result... $\langle \pi^+\pi^-|\widetilde{\mathcal{Q}}_2^s|D^0\rangle = (9.50 \pm 1.13) \times 10^{-3} \exp[i(-97.5^o \pm 11.6)]\,\mathrm{GeV}^3$

$$\langle K^+K^-|\widetilde{\mathcal{Q}}_2^d|D^0\rangle = (13.9 \pm 2.70) \times 10^{-3} \exp[i(-71.6^o \pm 29.5)]\,\mathrm{GeV}^3$$

- Thus, $r_\pi = \frac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \frac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015$

and with $\Delta a_{CP}^{dir} = -2 r_b \sin\gamma (r_K \sin\delta_K + r_\pi \sin\delta_\pi)$

- Phases of $r_{\pi\pi(KK)}$ are given by the phases of $\mathcal{P}^{s(d)}_{\pi\pi(KK)}$?

No:

$$\left|a_{CP}^{dir}(\pi^-\pi^+)\right| < 0.012 \pm 0.001\%,$$
$$\left|a_{CP}^{dir}(K^-K^+)\right| < 0.009 \pm 0.002\%,$$
$$\left|\Delta a_{CP}^{dir}\right| < 0.020 \pm 0.003\%\,.$$

Yes:

$$a_{CP}^{dir}(\pi^-\pi^+) = -0.011 \pm 0.001\%,$$
$$a_{CP}^{dir}(K^-K^+) = 0.009 \pm 0.002\%.$$
$$\Delta a_{CP}^{dir} = 0.020 \pm 0.003\%\,.$$

Khodjamirian, AAP; Lenz, Piscopo, Rush

- Again, experiment: $\Delta a_{CP}^{dir} = (-0.156 \pm 0.029)\%$

4. CP-violation in charmed baryons

- Other observables can be constructed for baryons, e.g.

$$A(\Lambda_c \to N\pi) = \bar{u}_N(p,s)\left[A_S + A_P\gamma_5\right]u_{\Lambda_c}(p_\Lambda, s_\Lambda)$$

These amplitudes can be related to "asymmetry parameter" $\alpha_{\Lambda_c} = \dfrac{2\,\mathrm{Re}\left(A_S^* A_P\right)}{\left|A_S\right|^2 + \left|A_P\right|^2}$

… which can be extracted from $\dfrac{dW}{d\cos\vartheta} = \dfrac{1}{2}\left(1 + P\alpha_{\Lambda_c}\cos\vartheta\right)$

Same is true for $\overline{\Lambda}_c$-decay

If CP is conserved $\alpha_{\Lambda_c} \overset{CP}{\Rightarrow} -\bar{\alpha}_{\Lambda_c}$, thus CP-violating observable is

$$A_f = \frac{\alpha_{\Lambda_c} + \overline{\alpha}_{\Lambda_c}}{\alpha_{\Lambda_c} - \overline{\alpha}_{\Lambda_c}}$$

FOCUS[2006]: $A_{\Lambda\pi}$=-0.07±0.19±0.24

Things to take home: charm

- Computation of charm amplitudes is a difficult task
  - no dominant heavy dof, as in beauty decays
  - light dofs give no contribution in the flavor SU(3) limit
  - D-mixing is a second order effect in SU(3) breaking ($x,y \sim 1\%$ in the SM)
- For indirect CP-violation studies
  - constraints on Wilson coefficients of generic operators are possible, point to the scales much higher than those directly probed by LHC
  - consider new parameterizations that go beyond the "superweak" limit
- For direct CP-violation studies
  - unfortunately, large DCPV signal is no more; need more results in individual channels, especially including baryons
  - hit the "brown muck": future observation of DCPV does not give easy interpretation in terms of fundamental parameters
  - need better calculations: lattice?
- Lattice calculations can, in the future, provide a result for a_{CP}!
- Need to give more thought on how large SM CPV can be…


EXTRA!

How to observe CP-violation: easy

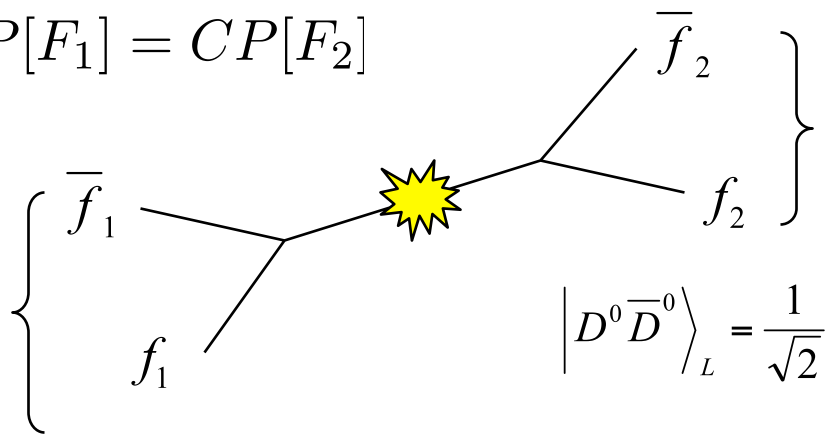
τ-charm factory

★ Recall that CP of the states in $D^0\overline{D^0} \to (F_1)(F_2)$ are anti-correlated at ψ(3770):

★ a simple signal of CP violation: $\psi(3770) \to D^0\overline{D^0} \to (CP_\pm)(CP_\pm)$

I. Bigi, A. Sanda; H. Yamamoto; Z.Z. Xing; D. Atwood, AAP

$$CP[F_1] = CP[F_2]$$



CP eigenstate F_2

CP eigenstate F_1

$$\left|D^0\overline{D}^0\right\rangle_L = \frac{1}{\sqrt{2}}\left[\left|D^0(k_1)\overline{D}^0(k_2)\right\rangle + (-1)^L\left|D^0(k_2)\overline{D}^0(k_1)\right\rangle\right]$$

$$\Gamma_{F_1F_2} = \frac{\Gamma_{F_1}\Gamma_{F_2}}{R_m^2}\left[\left(2+x^2+y^2\right)\left|\lambda_{F_1}-\lambda_{F_2}\right|^2 + \left(x^2+y^2\right)\left|1-\lambda_{F_1}\lambda_{F_2}\right|^2\right]$$

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f}$$

★ CP-violation in the **rate** → of the second order in CP-violating parameters.

★ Cleanest measurement of CP-violation!

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