# Challenges in Lattice Field Theory

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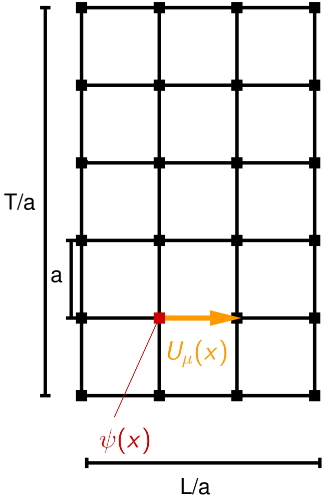
# Lattice Field Theory

- Numerical simulation of gauge (field) theories with fermions using Feynman's path integral
- Defined in terms of a Lagrangian (gauge action, fermion action)
- Simulations possible at weak coupling (perturbative) and strong coupling (nonperturbative)
- Path integral requires a probability interpretation
  → Wick-rotate to Euclidean time $t \to i\tau$

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

- Discretize space-time and set up a hypercube of finite extent $(L/a)^3 \times T/a$ and spacing $a$
- Path integral is now a huge but finite dimensional integral
- Stochastic procedure requiring statistical data analysis



# Steps of Lattice Simulations

1. Generate ensembles of gauge field configurations
   - Sea-sector or QCD vacuum (gluons, fermion bubbles)
   - Hybrid Monte Carlo updating MD + accept/reject)

2. Valence-sector "measurements"
   - Read-in gauge field to calculate matrix elements of operators describing the process of interest

3. Data processing, statistical data analysis
   → Jackknife, Bootstrap
   → $\chi^2$ fits, model-averaging
   → Machine-learning

- Costs of the simulation dominated by inverting Dirac operator
  → Dirac operator: diagonally dominant huge sparse matrix
  → Implementation depends on chosen discretization
  → Size proportional to $L^3 \times T \times (4\ \textit{dimensions}) \times (4\ \textit{spinor}) \times (3\ \textit{color})$

# Challenges:

- GPU machines significantly boost computational power, while network performance is stagnating
  - ⇒ Communication avoiding algorithms
    - → Domain-decomposition (DD-HMC) [Lüscher CPC 165 (2005) 199] [Del Debbio et al. JHEP 02 (2007) 082], ...
    - → Compress information to be communicated (lower precision, low-/high-modes, reconstruction)

- Simulations on larger and finer lattices
  - ⇒ Accelerating algorithms (Deflation, multi-grid, ...)
    - → Calculate low-modes, eigenvectors of the Dirac operator and re-use
  - ⇒ Critical slowing down, freezing of topological charge (open BC, RM-HMC,...)

- Simulations at very strong coupling (BSM models or huge boxes, many particle simulations)
  - ⇒ Bulk phase transitions (lattice artifacts)
  - → Modify action e.g. add bosonic PV fields (integrated out in the continuum)