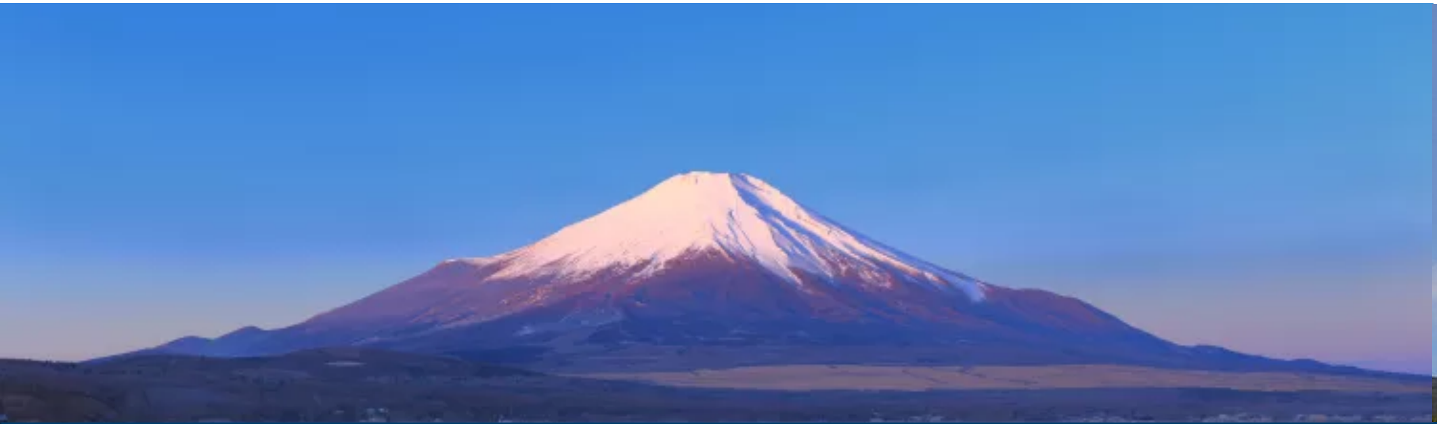


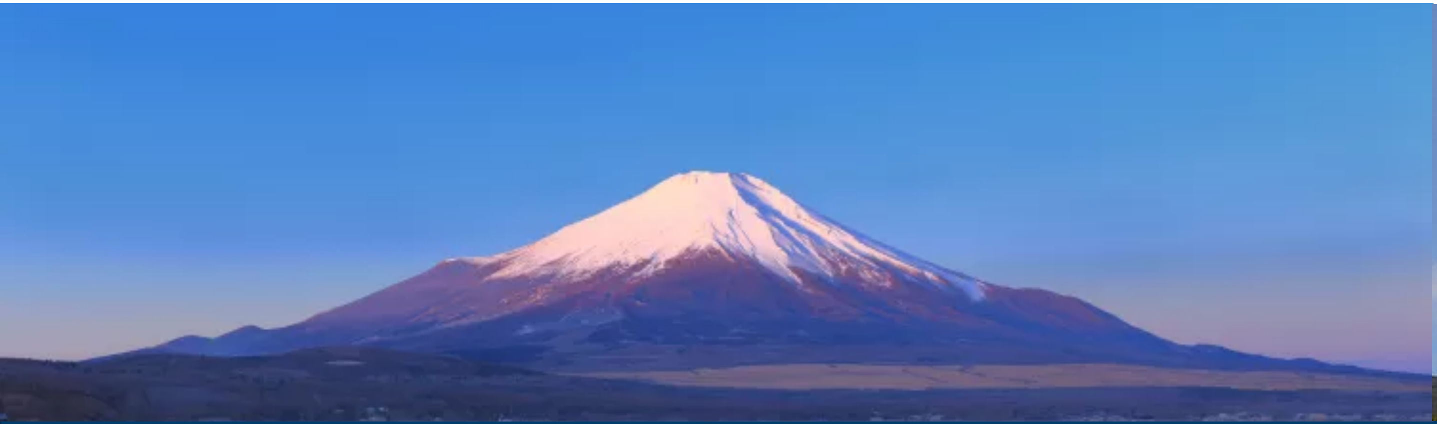
EFFECTIVE (FIELD) THEORIES FOR LOW-DIMENSIONAL SYSTEMS

JUNE 5, 2024, CLOSING MEETING

CRC110 “SYMMETRIES AND THE EMERGENCE OF STRUCTURE IN QCD”

THOMAS LUU, IAS-4/FZJ

JÜLICH Forschungszentrum



EFFECTIVE (FIELD) THEORIES FOR LOW-DIMENSIONAL SYSTEMS

JUNE 5, 2024, CLOSING MEETING

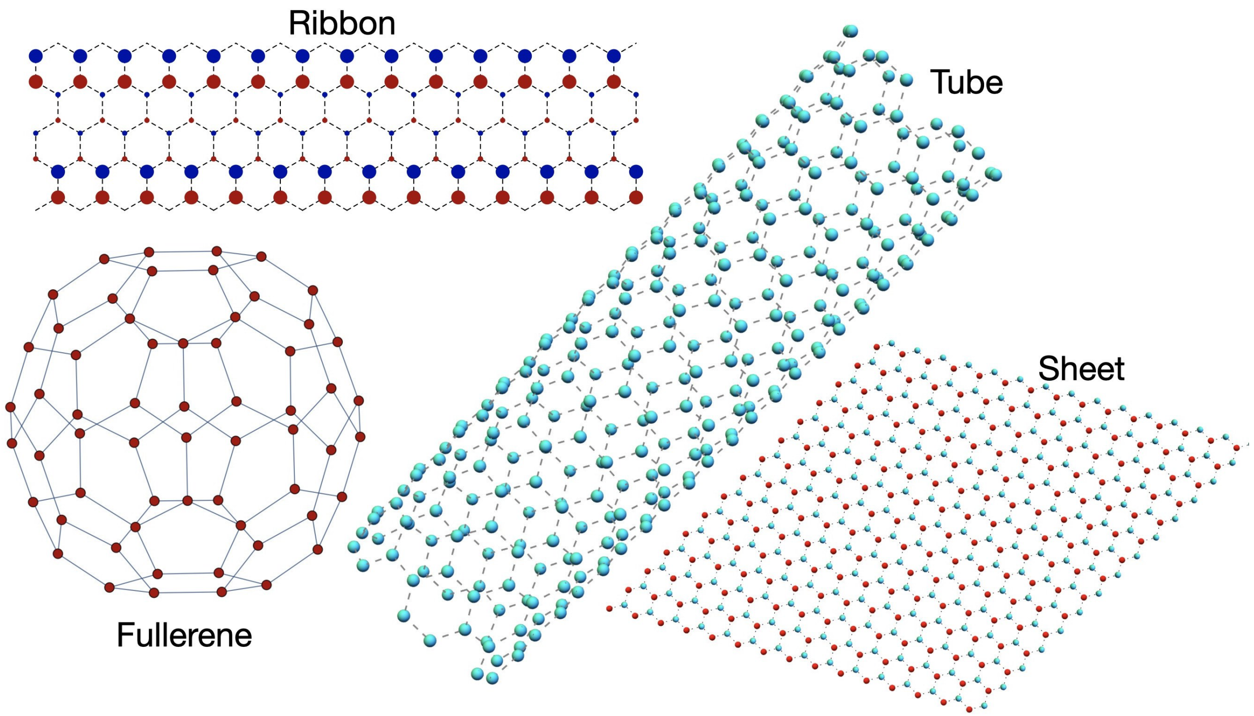
CRC110 “SYMMETRIES AND THE EMERGENCE OF STRUCTURE IN ~~QCD~~”

THOMAS LUU, IAS-4/FZJ

JÜLICH
Forschungszentrum

WHY LOW-D MATERIALS?

- At least one of the dimensions of the material is small (~ nanoscale)
- Quantum effects and strong correlations induce novel phenomena
- Novel quantum electronics
- Fault tolerant quantum computing



WHY DO WE EXPECT STRONG CORRELATIONS IN LOW -D?

Isn't QED perturbative?

Let's first assume a quadratic dispersion relation:

Average kinetic energy of electron

$$E_k \approx n_d^{2/d}(2m^*) \quad : \quad n_d = 1/l^d$$

Density of electrons (n_d); Effective mass (m^*); Mean distance between electrons (l); Dimension of system (d)

Dielectric constant (depends on medium) (ϵ_0)

Average Coulomb energy of electron

$$E_C \approx e^2 n_d^{1/d}/\epsilon_0 \quad : \quad V(r) = \frac{e^2}{\epsilon_0 r}$$

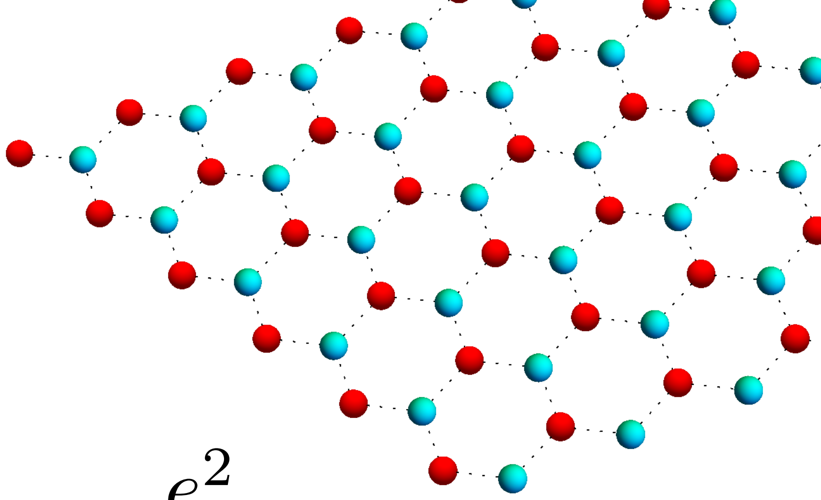
$$\Gamma = \frac{E_C}{E_K} \propto (n_0/n_d)^{1/d} \quad : \quad n_0 = \left(m^* e^2/\epsilon_0\right)^d$$

Fiducial density

Strength of electron correlations depends on density of electrons and dimensionality of system

2D EXAMPLE: GRAPHENE

Honeycomb lattice



Use linear dispersion and set d = 2:

$$E_K \approx v_F n^{1/2} \quad \longrightarrow \quad \Gamma = \frac{e^2}{\epsilon_0 v_F}$$

Independent of electron density!

Let's plug in some numbers:

$$v_F \approx \frac{c}{300} \quad \longrightarrow \quad \Gamma \approx 2 - 3$$

The electrons in graphene are strongly interacting!

In general, lower dimensions enhance correlations.

SYMMETRIES RELEVANT FOR LOW-D MATERIALS

- Time-reversal symmetry $T: \quad T^2 = \pm 1$

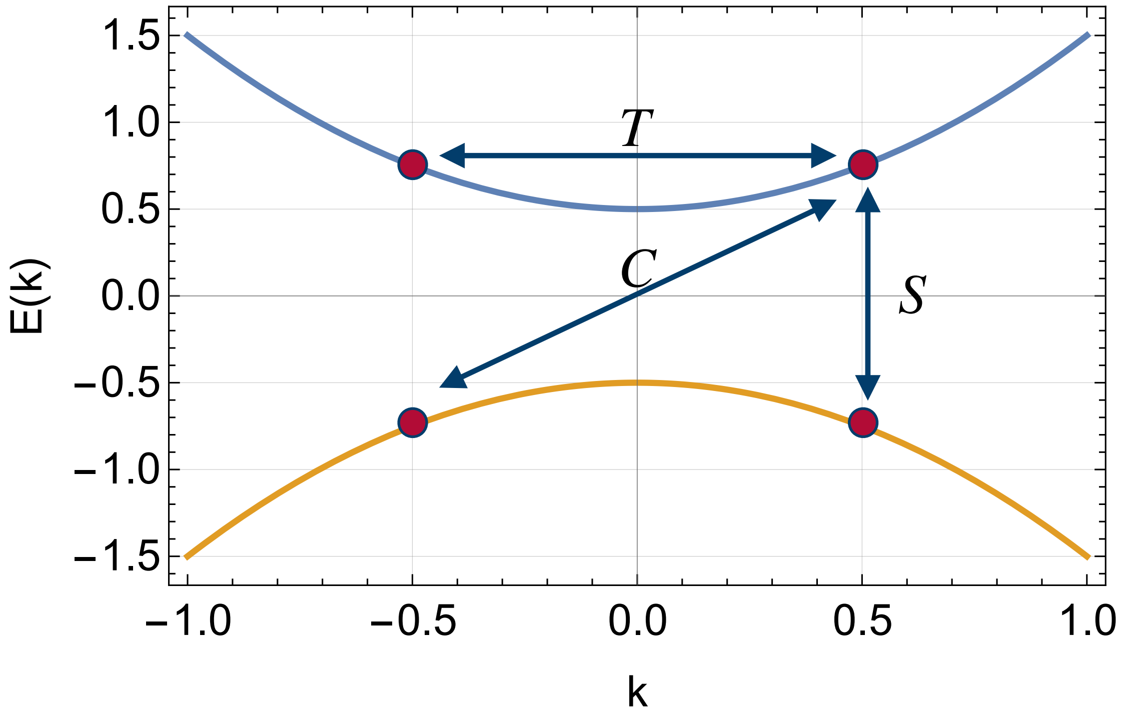
$$t \rightarrow -t \implies E(k) = E(-k)$$

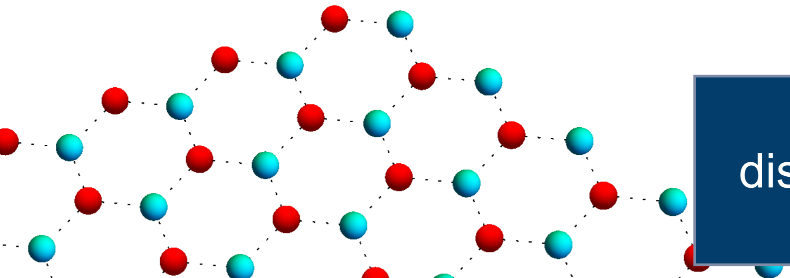
- Charge conjugation symmetry (or *particle-hole* symmetry) $C: \quad C^2 = \pm 1$

Spectrum symmetric about zero: $E_+(k) = -E_-(-k)$

- Chiral symmetry (or *sublattice* symmetry)

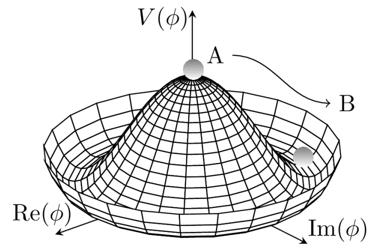
$S: \quad S^2 = S \qquad E_+(k) = -E_-(k)$





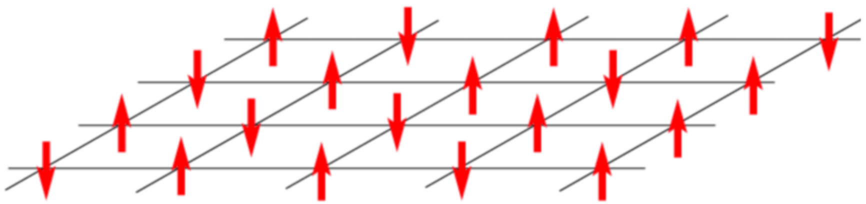
Normally, different phases of matter are distinguished by their ground-state symmetries (and lack thereof)

SYMMETRY BREAKING AND PHASES OF MATTER

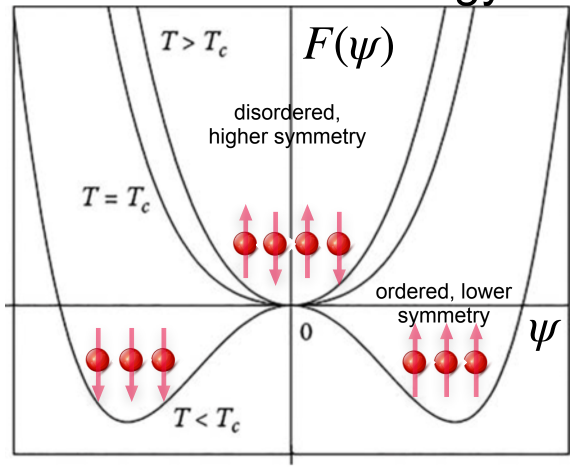


. . . and the transitions between them . . .

Ising Model



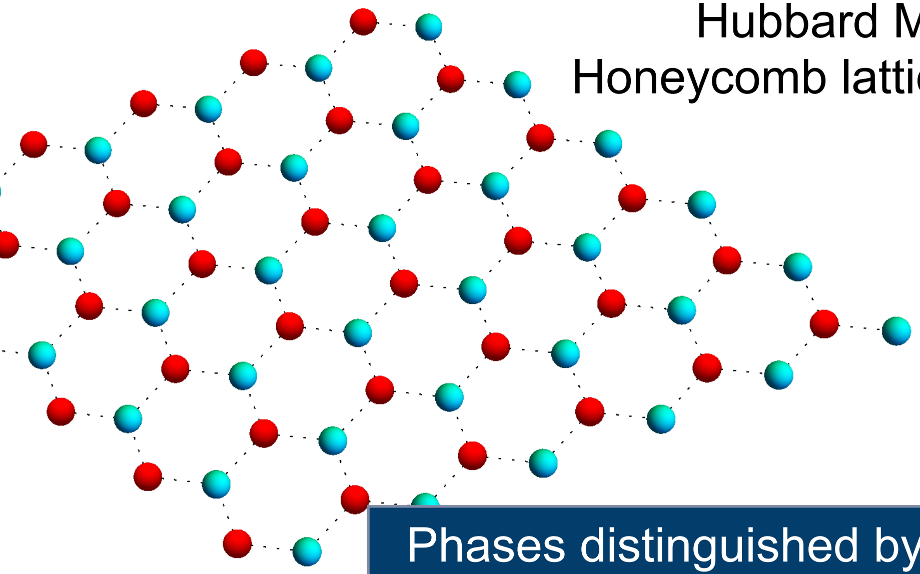
Landau Free Energy

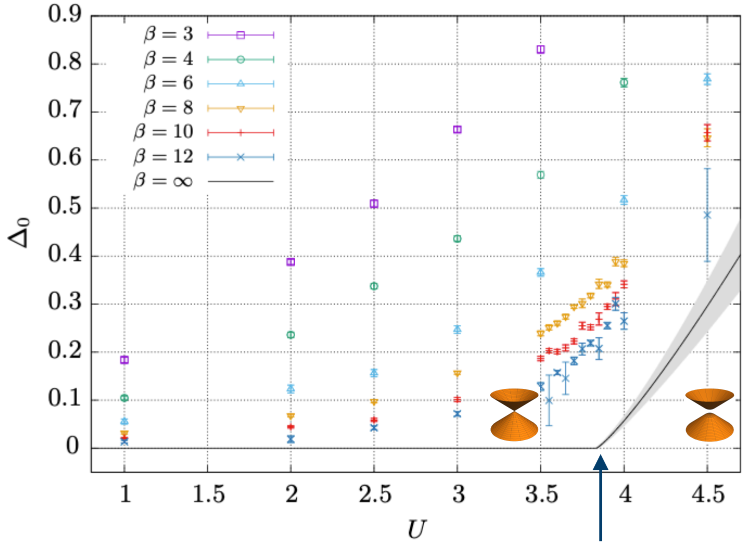


Continuous order parameter in a discrete system?

J. Ostmeyer, **TL**, et al. [arXiv:1912.03278] Comput.Phys.Commun. **265** (2021) 107978

Hubbard Model on Honeycomb lattice (graphene)





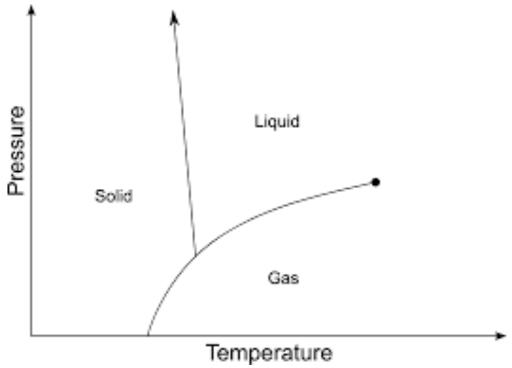
Most accurate prediction of critical coupling: $U_c = 3.834(14)$

Phases distinguished by different symmetries

J. Ostmeyer, **T.L.**, C. Urbach et al. [arXiv:2005.11112] Phys.Rev.B **102** (2020) 245105
J. Ostmeyer, **T.L.**, C. Urbach et al. [arXiv:2105.06936] Phys.Rev.B **104** (2021) 155142

PHASES OF MATTER THAT SHARE THE SAME SYMMETRIES

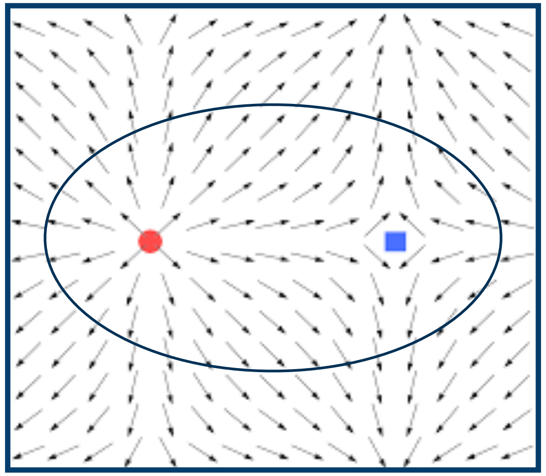
Classic example: liquid/gas transition



Another example: BKT transition (XY-Model)

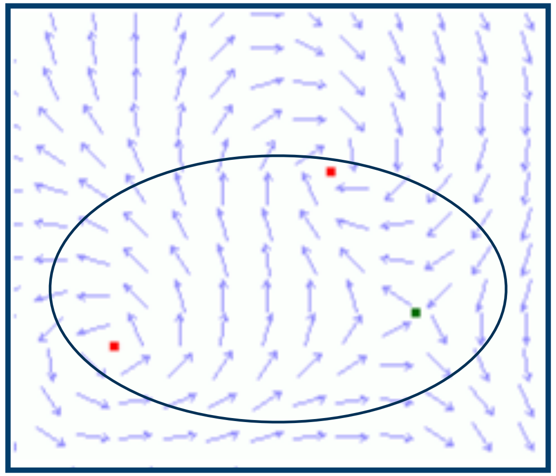
Phases classified by local topological invariant

$\pi_1(S^1) = \mathbb{Z}$ (ie winding number)



$J < J_c$

$|\nu| = 0 \in \mathbb{Z}$



$J > J_c$

$|\nu| = 1 \in \mathbb{Z}$

Phases are distinct, but the ground states do not break the symmetry of the system

MERMIN-WAGNER THEOREM

continuous symmetries cannot be spontaneously broken at *finite temperature* in systems with sufficiently short-range interactions in dimensions $d \leq 2$

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↓

no goldstone modes!

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no goldstone modes!

Phases of matter classified topologically

THUS TOPOLOGY CAN ALSO DISTINGUISH PHASES OF MATTER

But what does “topology” mean in this case?

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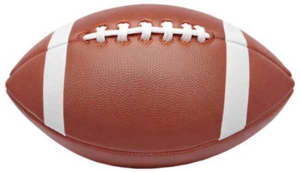
Topological Geometry

$\sim$

$\nsim$

$\sim$



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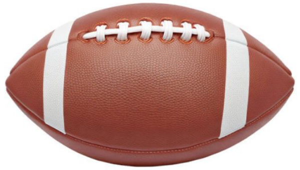
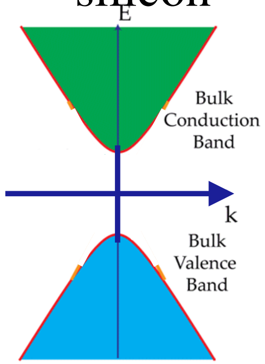
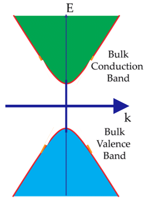


Topological Matter

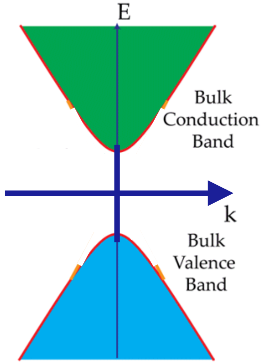
$H^{\text{solid}}_{\text{silicon}}$



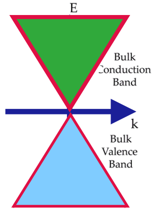
$\sim$



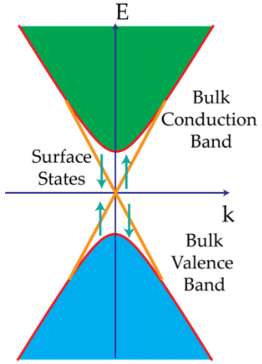
$H^{\text{solid}}_{\text{argon}}$



$\nsim$

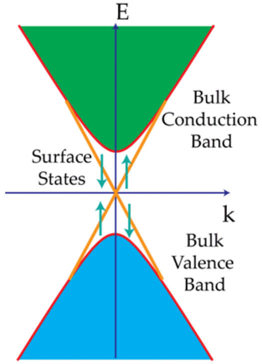


$H_{\text{Be}_2\text{Te}_3}$



$\sim$

$H_{\text{Sb}_2\text{Te}_3}$



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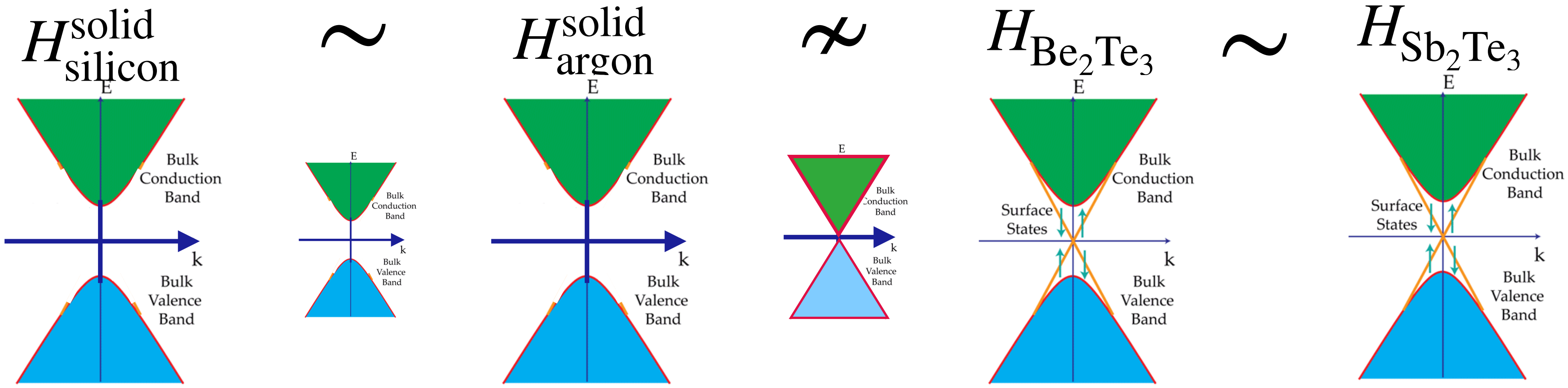
Topological Geometry

Topological Matter



 Topological invariants: $\mathbb{Z}$, $\mathbb{Z}_2$

CLASSIFICATION OF MATTER: *THE TEN-FOLD WAY*

. . . aka having ‘particle-physics envy’ . . .

	Symmetry			Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

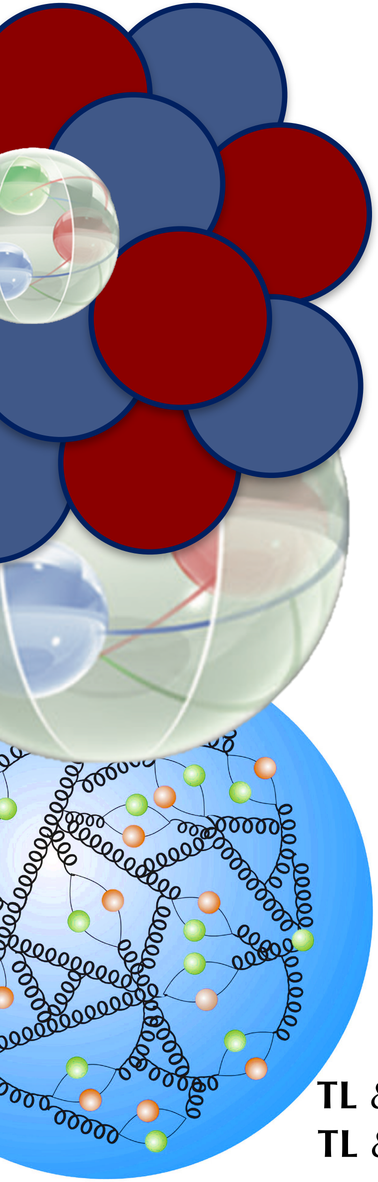
Bott periodicity

Atland & Zirnbauer, arXiv:cond-mat/9602137, https://doi.org/10.48550/arXiv.cond-mat/9602137
Dyson, J.Math.Phys. **3** (1962) 1199

NOVEL FORMS OF EMERGENT PHENOMENA

CASCADE

“Strong Emergentism” vs “Weak Emergentism”



$\mathcal{L}_{NN\ldots N}$

~ 1-8 x 10^{-15} m

$\mathcal{L}_N$

~ 10^{-15} m

$\mathcal{L}_{\text{QCD}}$

< 10^{-15} m

TL & U. Meiβner, [arXiv:2007.10062] Found.Phys. **50** (2020) 1140

TL & U. Meiβner, [arXiv:1910.13770] **Top-Down Causation & Emergence**, Springer Verlag (2021), pgs.101-114

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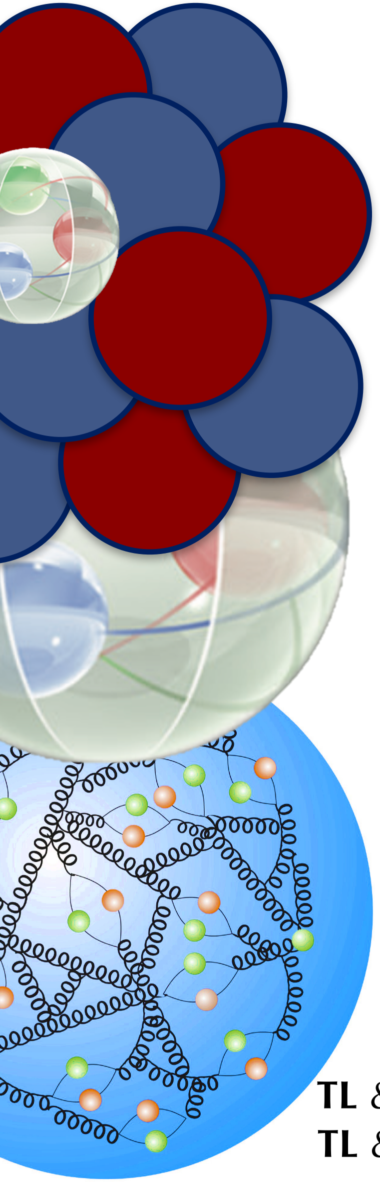


$\mathcal{L}_{NN...N}$

~ 1-8 x 10^{-15} m

$\mathcal{L}_{N}$

~ 10^{-15} m

$\mathcal{L}_{\text{QCD}}$

< 10^{-15} m

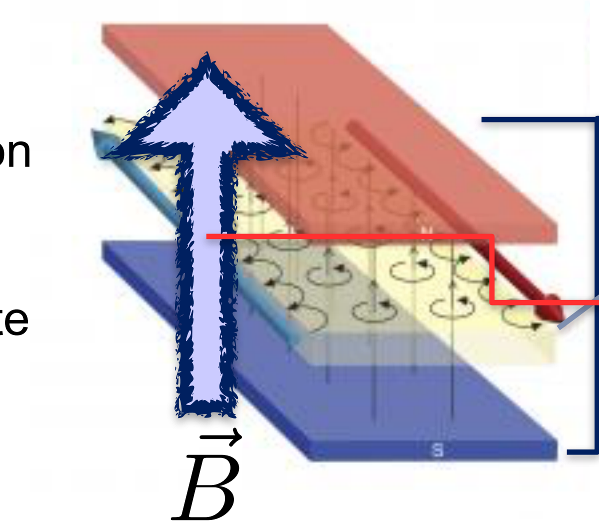
The (Fractional) Quantum Hall effect

Representation 1

QHE: Quantized cyclotron orbits

FQHE: Anyons (composite electrons)

Representation 2



Macro-state boundaries

Electrons in 2-dimensions

Macro-state magnetic field

All systems described by micro-level states, i.e. QED

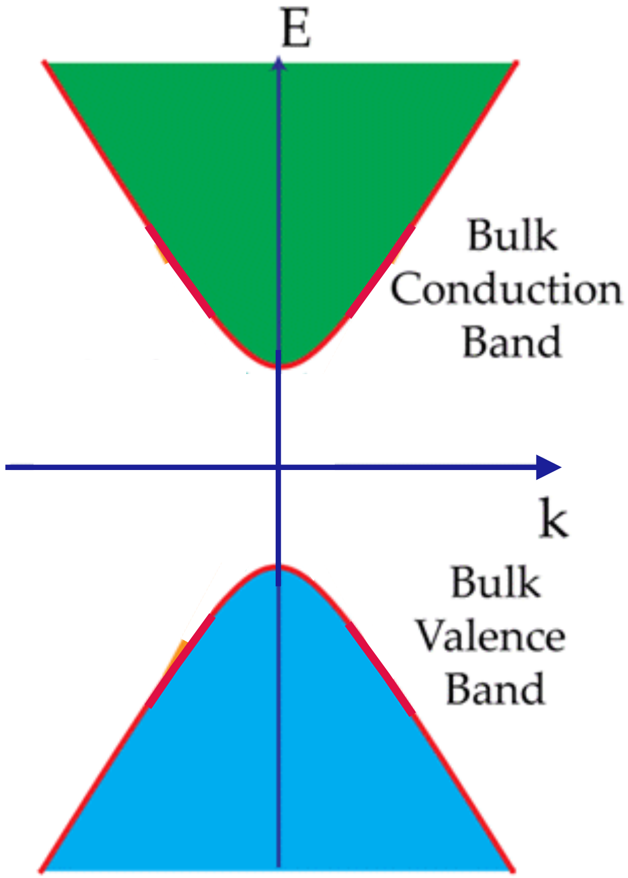
Weak emergence is “. . . reducible in principle, but also *in principle irreducible in practice*” (M. Bedau, 2010)

TL & U. Meiβner, [arXiv:2007.10062] Found.Phys. **50** (2020) 1140

TL & U. Meiβner, [arXiv:1910.13770] **Top-Down Causation & Emergence**, Springer Verlag (2021), pgs.101-114

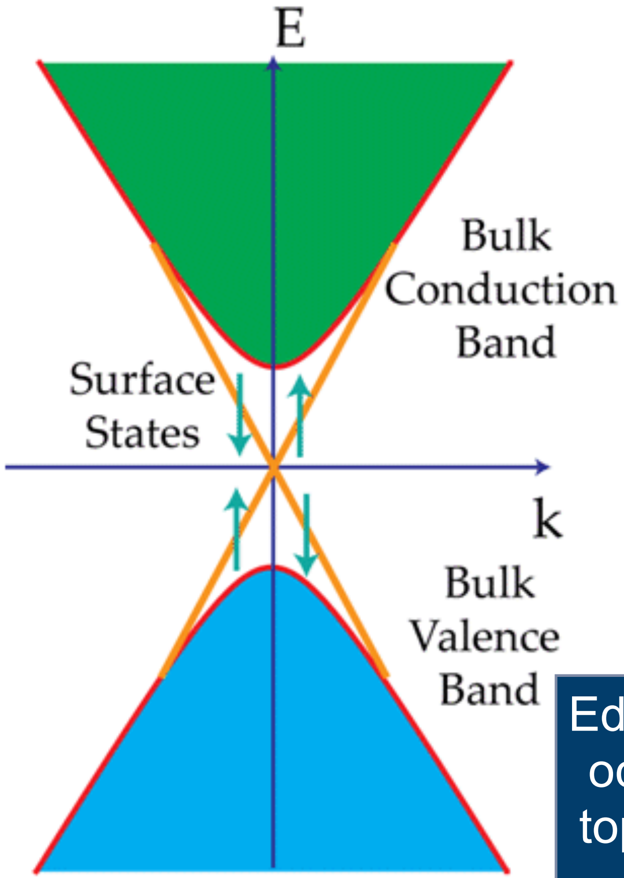
TOPOLOGICAL INSULATORS . . .

. . . and the bulk-boundary correspondence

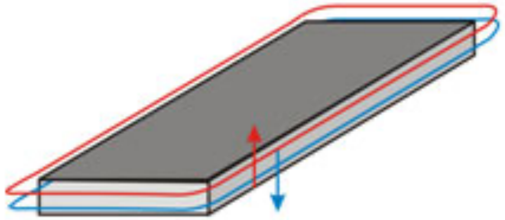


trivial insulator

≁



topological insulator



2D topological insulator

Edge/surface (localized) states occur at boundaries between topologically distinct materials

Localized states are "symmetry-protected"

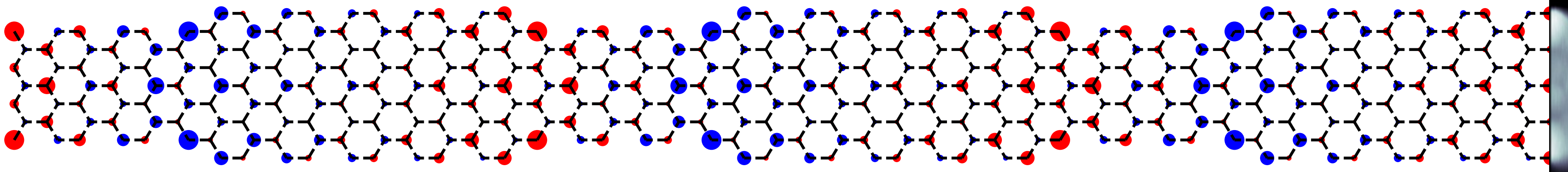
Mong & Shivamoggi, https://doi.org/10.48550/arXiv.1010.2778

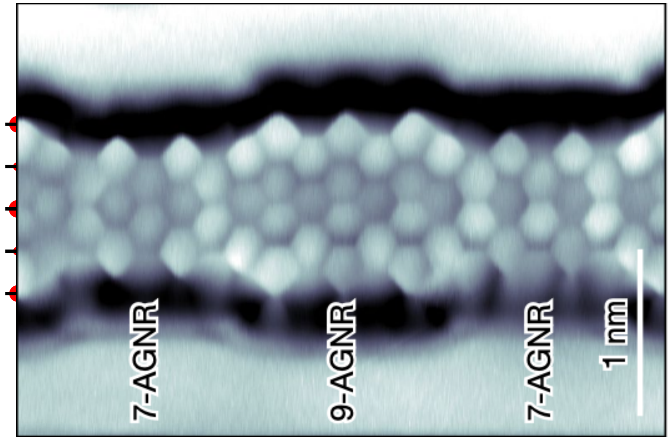
ANOTHER EXAMPLE OF LOCALIZATION

Hybrid nanoribbons

What is an hybrid nanoribbon?

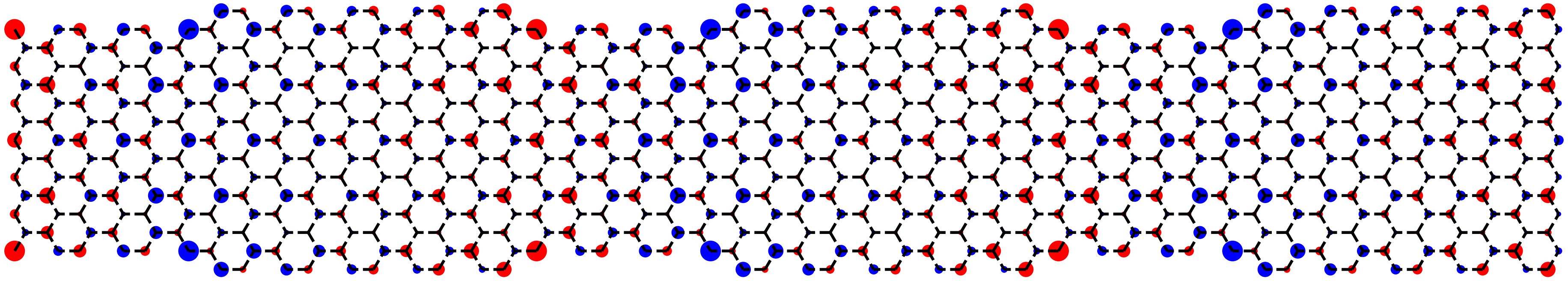
7/9 ribbon



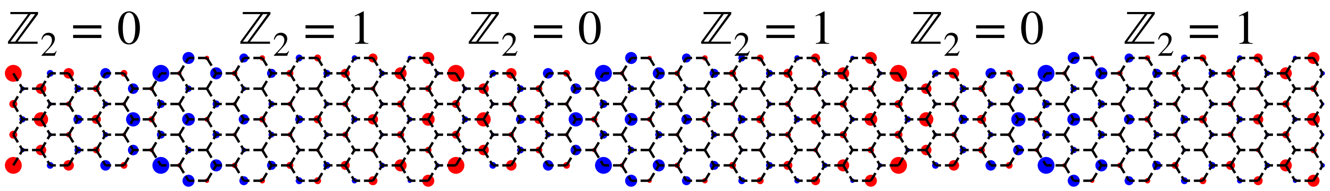


Rizzo, D.J., Veber, G., Cao, T. *et al.* Topological band engineering of graphene nanoribbons. *Nature* **560**, 204–208 (2018)

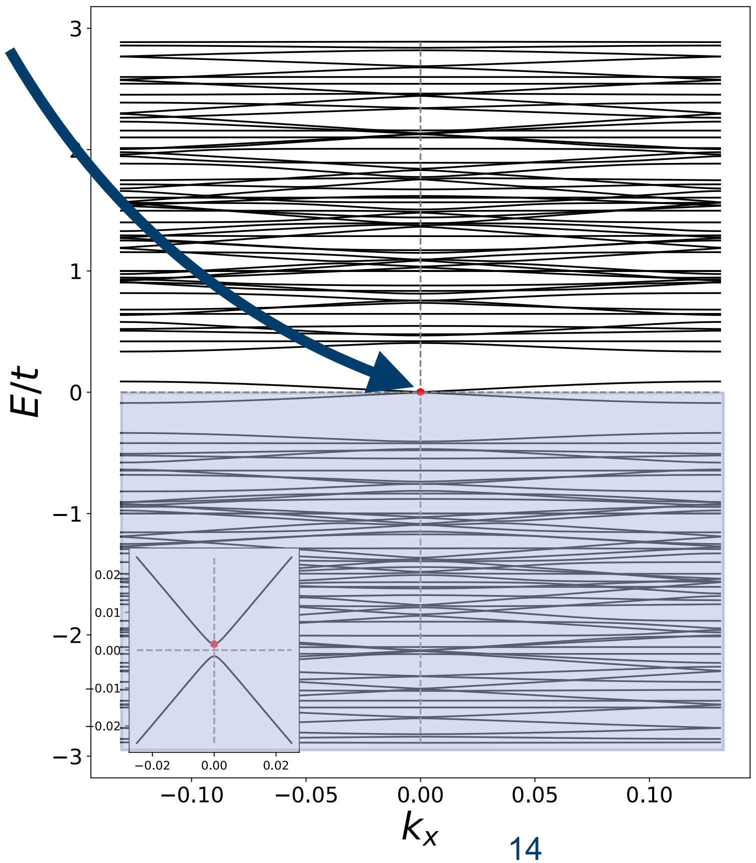
13/15 ribbon



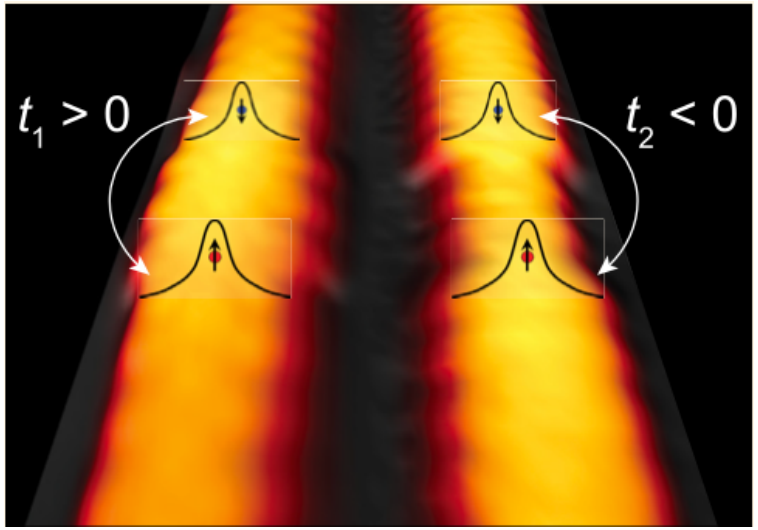
LOWEST ENERGY STATE EXHIBITS "LOCALIZATION"



Cao *et al*., Phys. Rev. Lett. **119**, 076401 (2017)



Experimental evidence

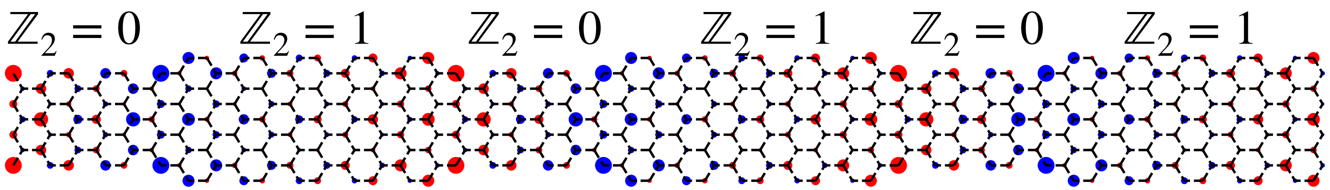


Rizzo *et al.,* ACS Nano 2021, 15, 12, 20633–20642

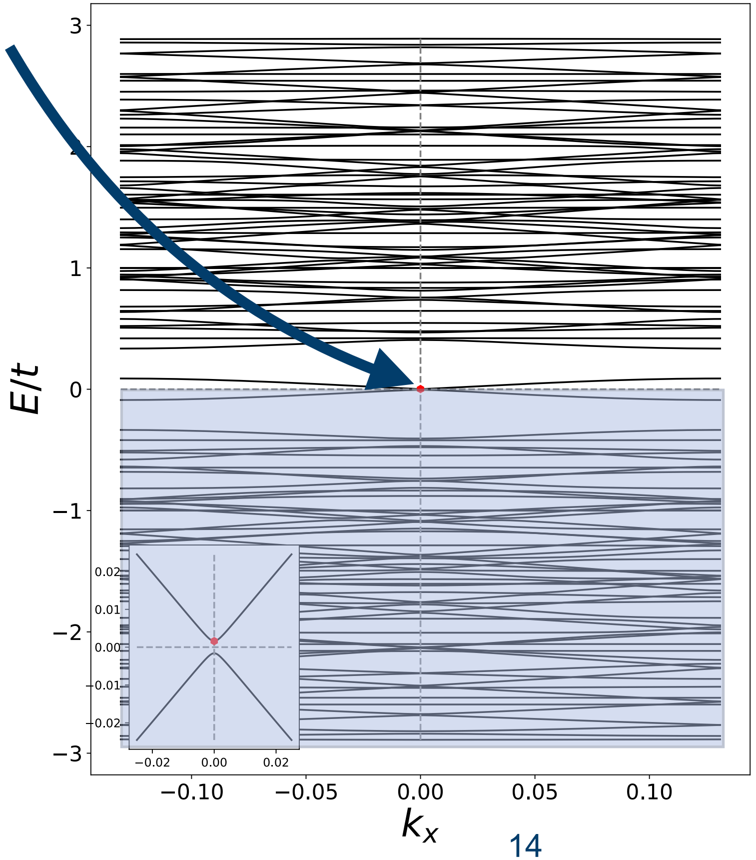
Potential application: Topological Quantum Dots

. . . and fault-tolerant quantum computing (one day)

LOWEST ENERGY STATE EXHIBITS "LOCALIZATION"

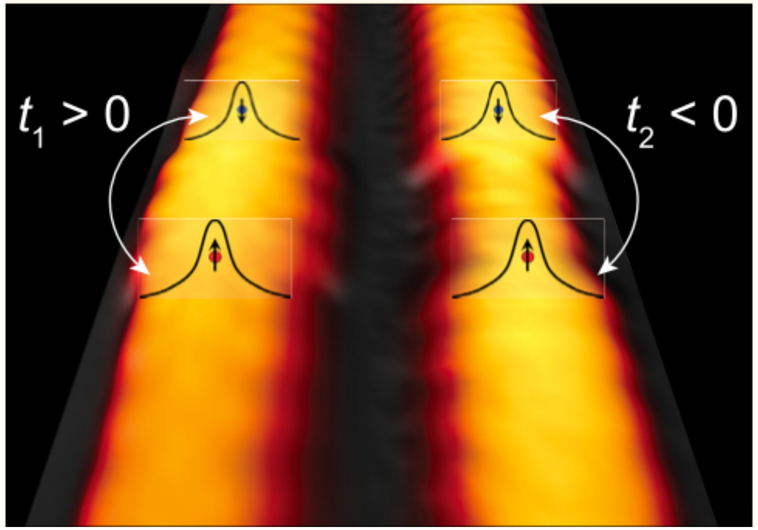


Cao *et al*., Phys. Rev. Lett. **119**, 076401 (2017)



All theoretical analysis is based off *non-interacting* dynamics!

Experimental evidence



Rizzo *et al.,* ACS Nano 2021, 15, 12, 20633–20642

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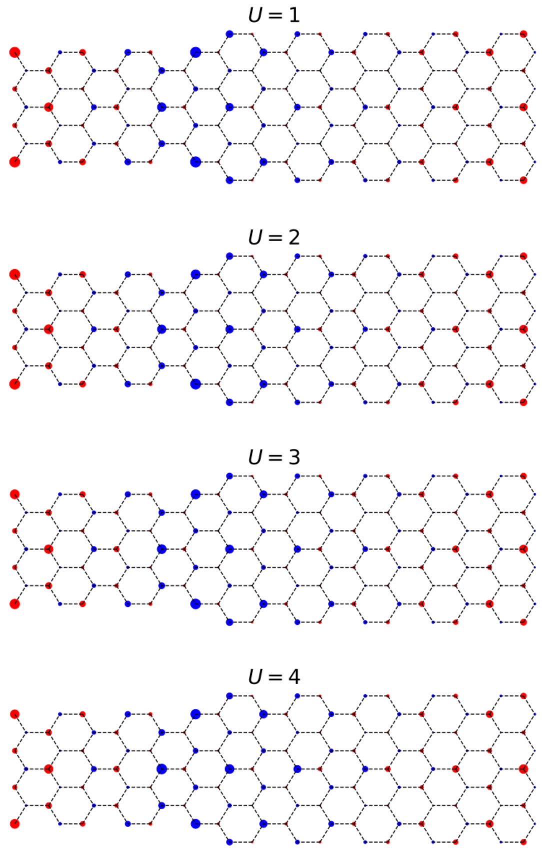
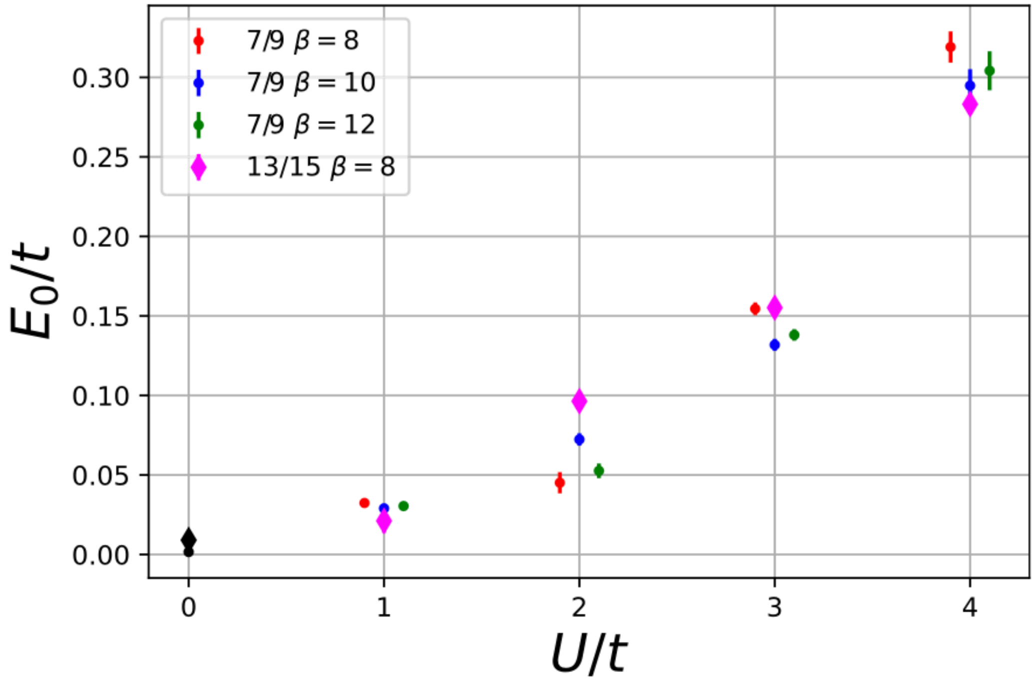
HOW DOES INTERACTION CHANGE THINGS?

Simulations with Quantum Monte Carlo

$$H = -t \sum_{\langle i,j\rangle,\sigma=\uparrow,\downarrow} (a^\dagger_{i,\sigma} a_{j\sigma} + \mathrm{H.c}) + U \sum_i \left(n_{i,\uparrow} - \frac{1}{2}\right)\left(n_{i,\downarrow} - \frac{1}{2}\right)$$



Localizations persist with strong interactions

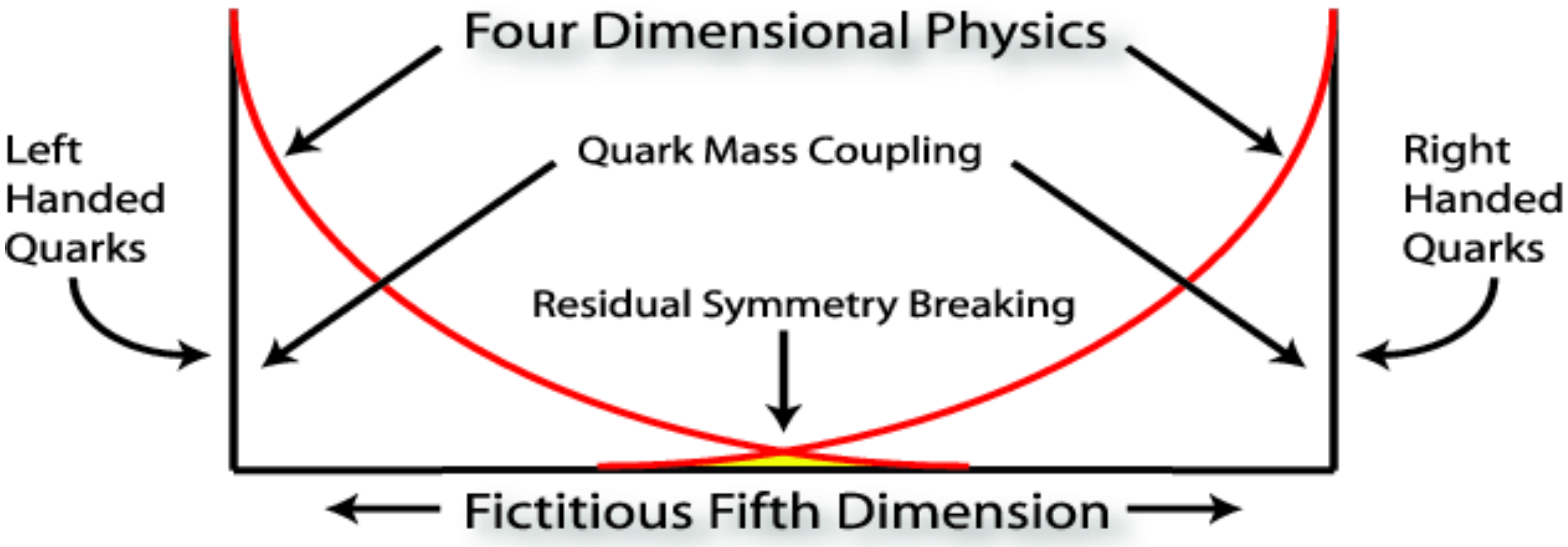


But energy depends on U!

TL, U.-G. Meißner & L. Razmadze, [arXiv:2204.02742] Phys.Rev.B **106** (2022) 195422

DO SUCH LOCALISATIONS LOOK FAMILIAR?

Domain-Wall fermions in LQCD



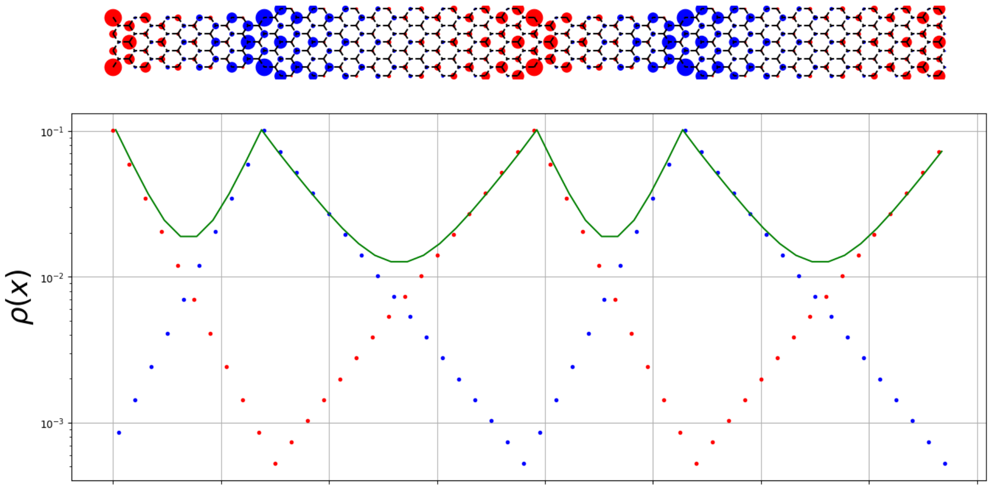
Hybrid ribbons provide physical manifestation of domain wall fermions

D.B.Kaplan, A Method for simulating chiral fermions on the lattice, Phys. Lett. B 288, 342 (1992)
D.B.Kaplan, Chiral gauge theory at the boundary between topological phases, [arXiv:2312.01494] Phys.Rev.Lett. **132** (2024) 141603

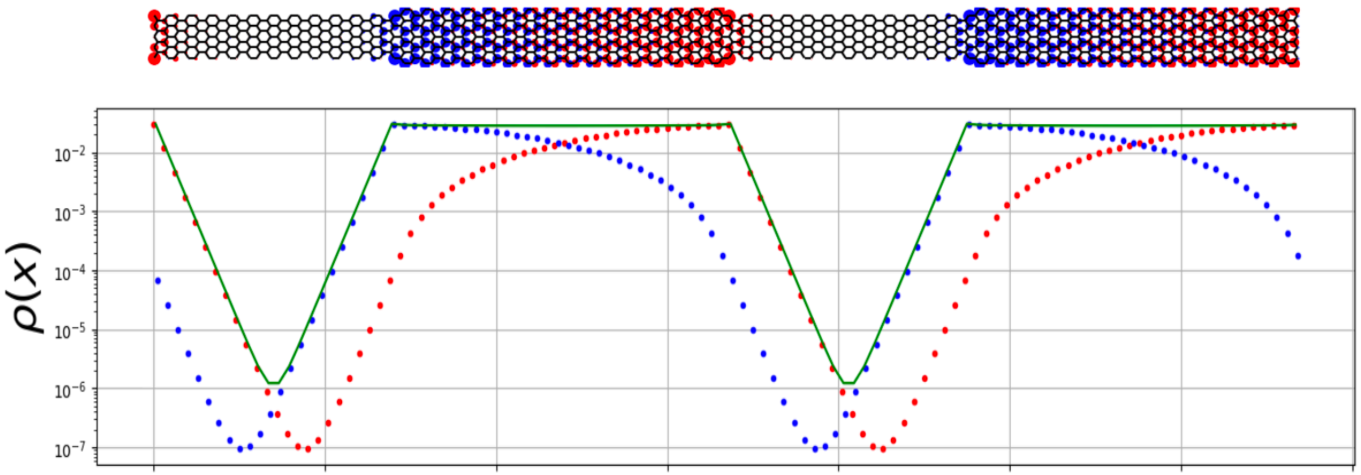
OUR INVESTIGATIONS LEAD US TO A NEW TYPE OF LOCALIZATION

Fuji vs Kilimanjaro J. Ostmeyer, L. Razmadze, E. Berkowitz, **TL** & U.-G. Meißner, [arXiv:2401.04715] Phys.Rev.B 109 (2024) 195135

7/9 hybrid



9/11 hybrid

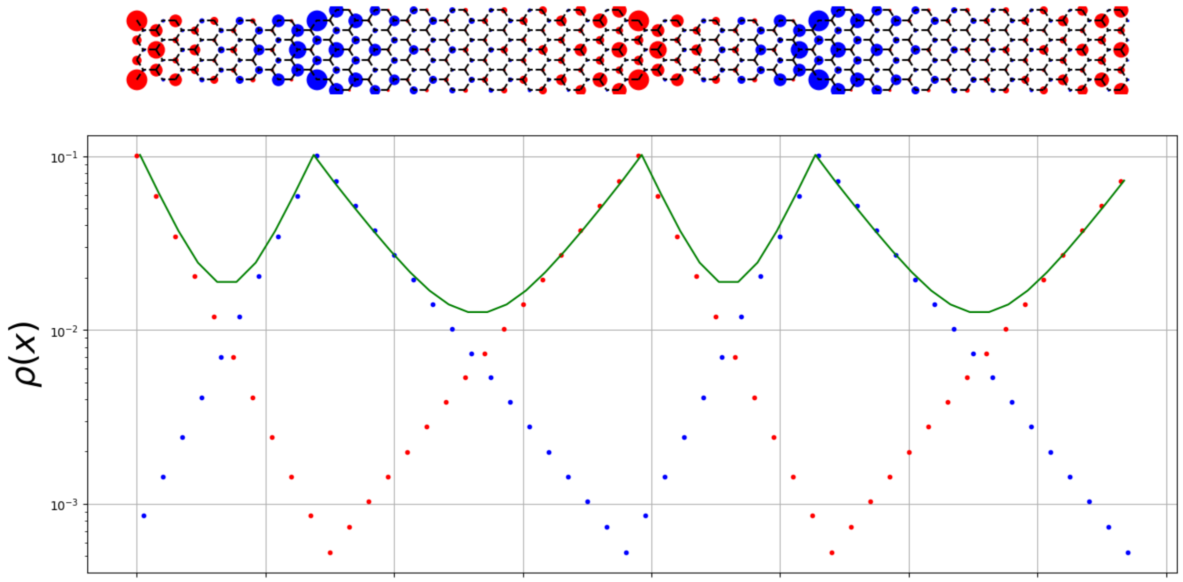


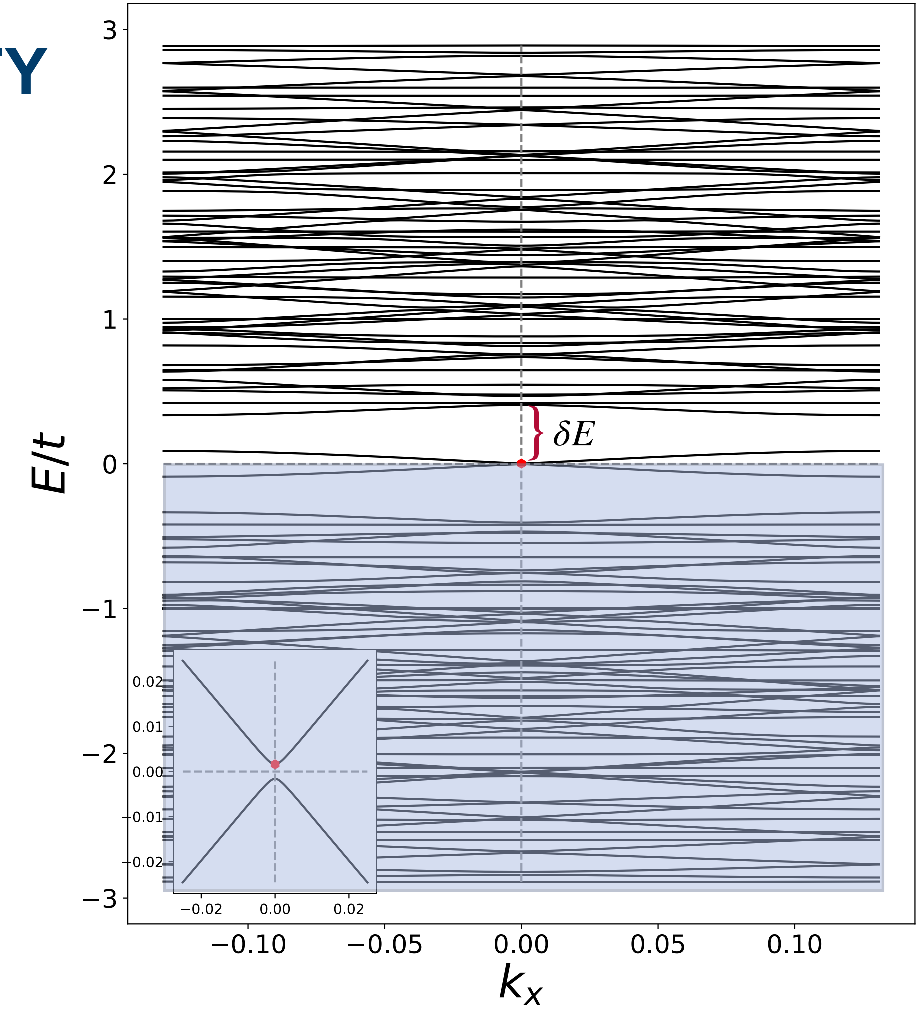
Predicted from Cao *et al.*, Phys. Rev. Lett. **119**, 076401 (2017)

Our addition to Cao *et al.*, Phys. Rev. Lett. **119**, 076401 (2017)

SUCH LOCALIZATIONS ALLOW US TO SIMPLIFY OUR THEORY

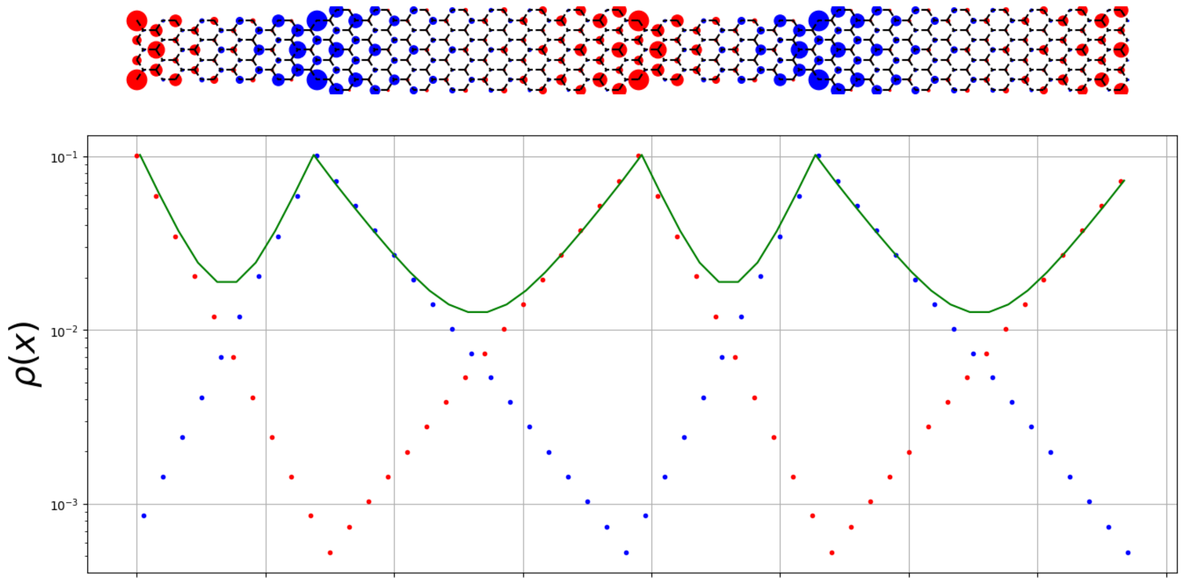
1-D effective theory

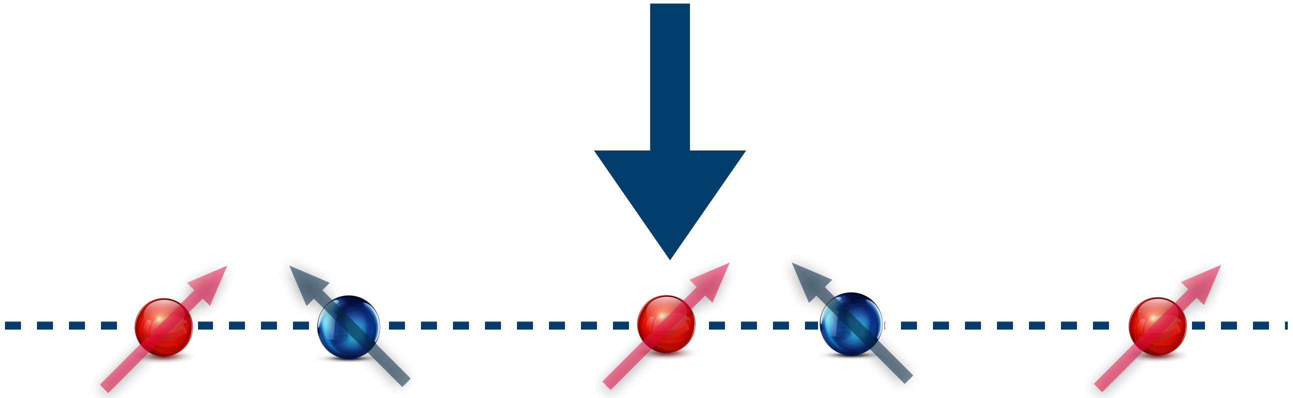


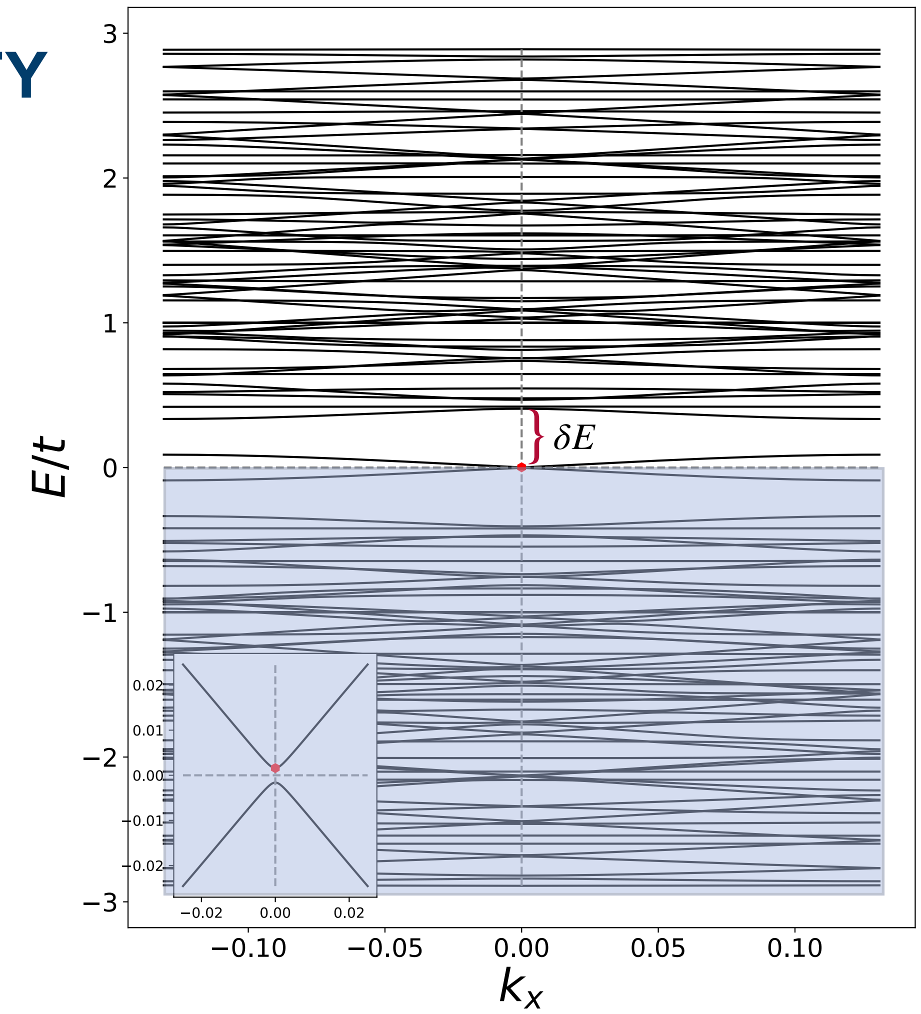


SUCH LOCALIZATIONS ALLOW US TO SIMPLIFY OUR THEORY

1-D effective theory







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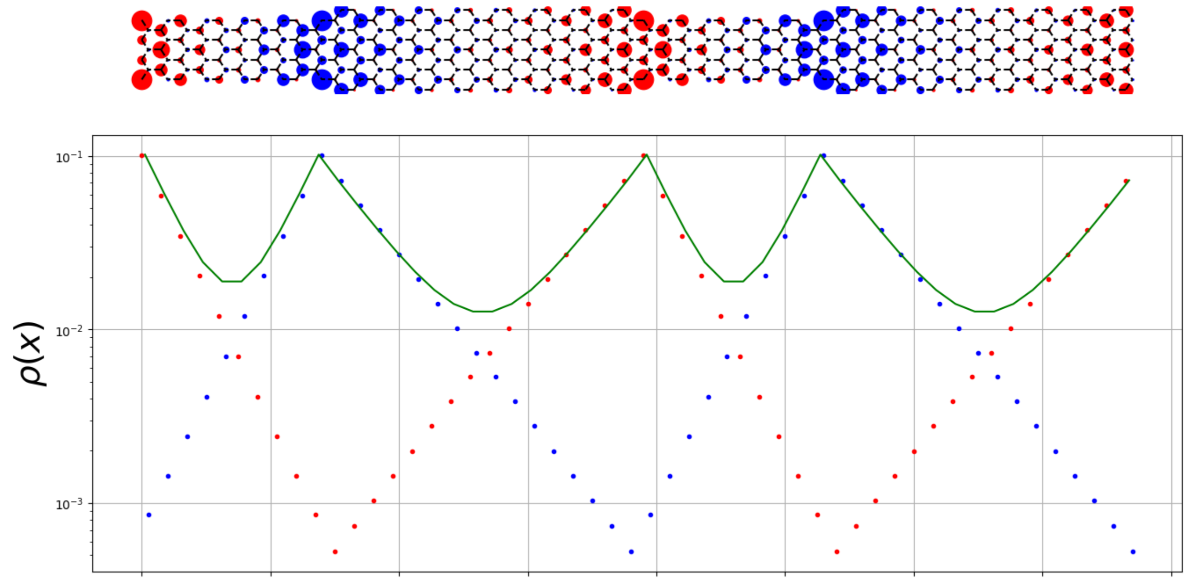
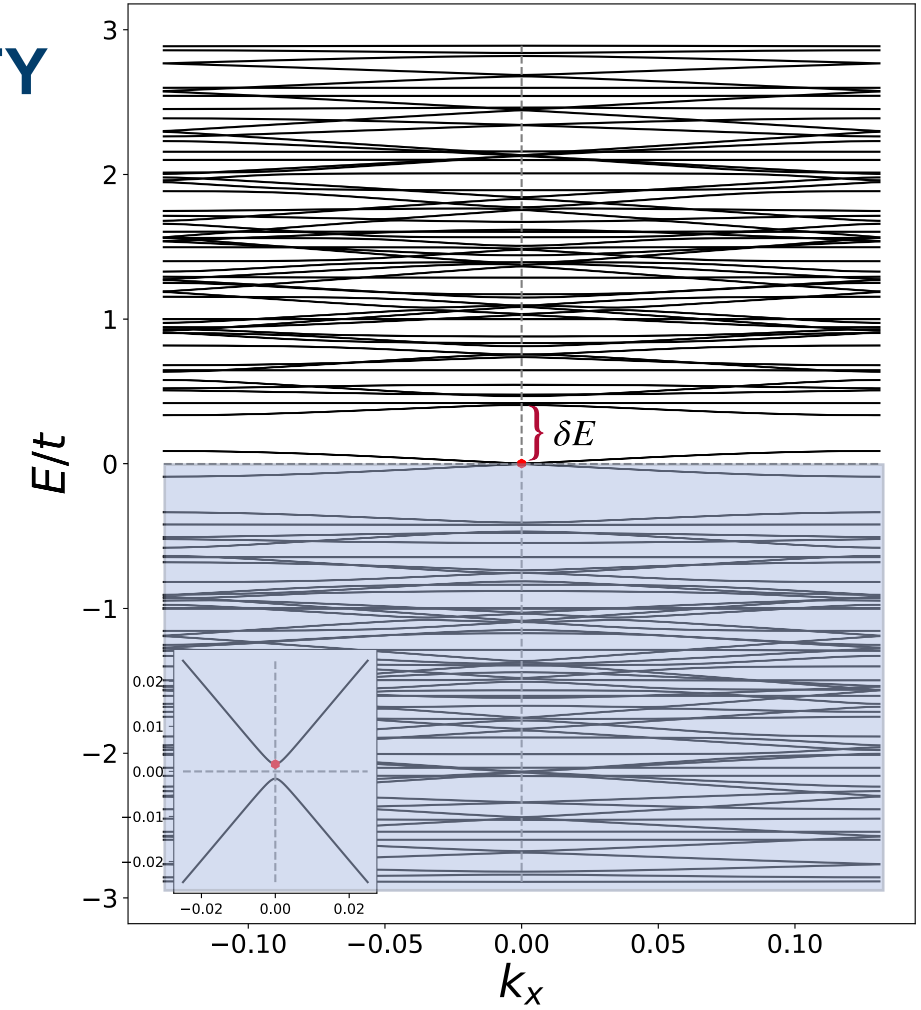
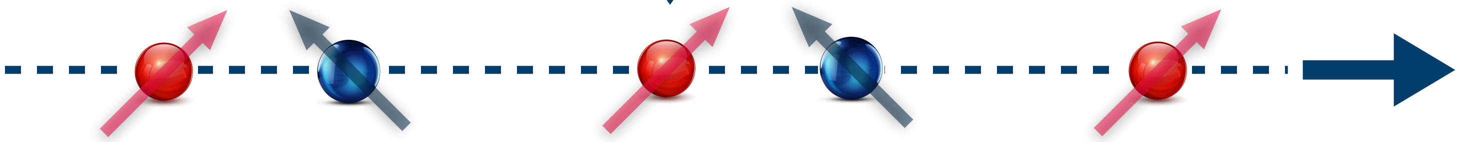






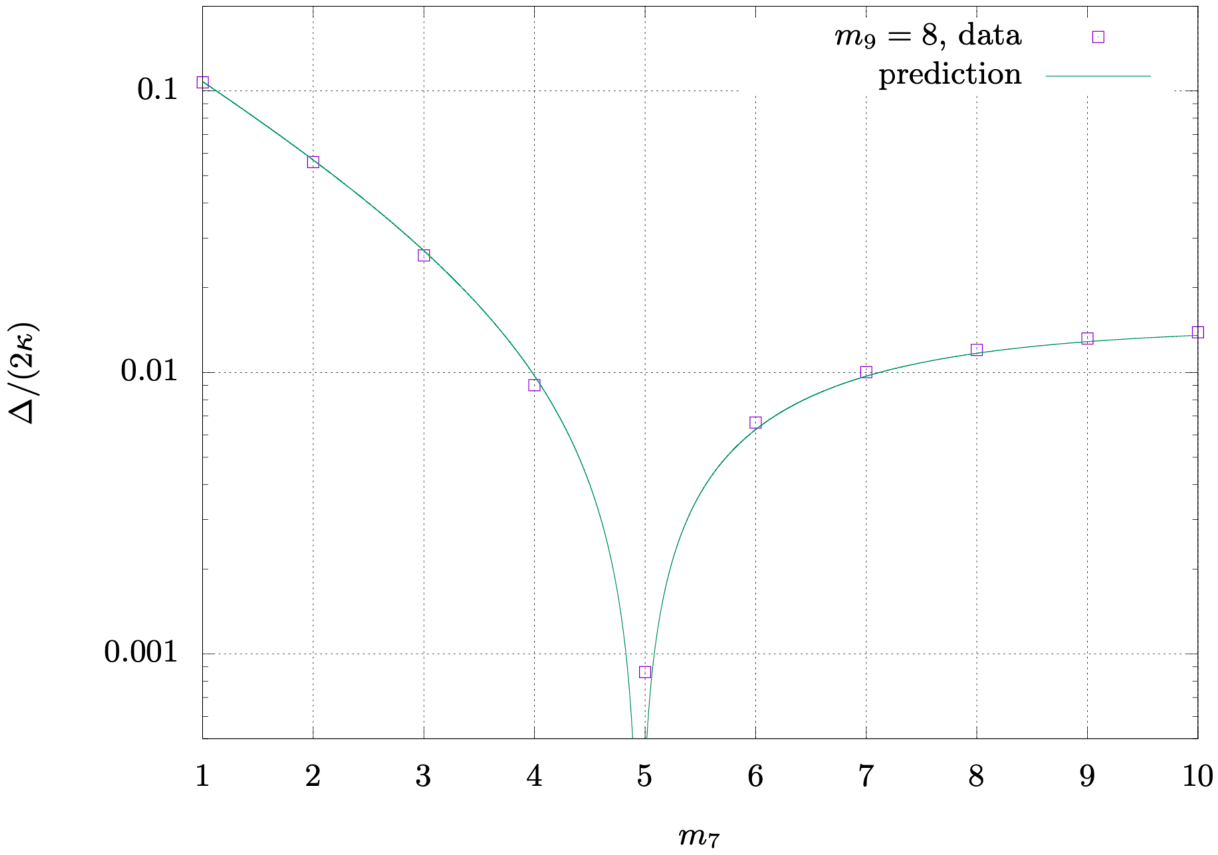
Low-energy ET

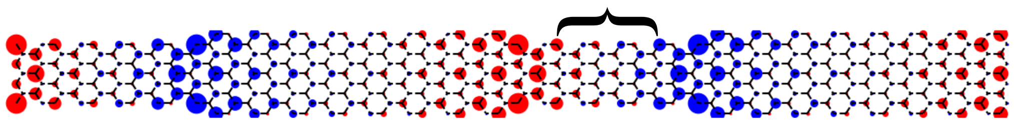
$$H_{1D} = -\sum_i \left(t_A a^\dagger_{2i} a_{2i-1} + t_B a^\dagger_{2i+1} a_{2i+2} + \mathrm{H.c.} \right)$$

TUNING LOW-ENERGY CONSTANTS (LECS)

$$H_{1D} = -\sum_i \left(t_A a^\dagger_{2i} a_{2i-1} + t_B a^\dagger_{2i+1} a_{2i+2} + \mathrm{H.c.} \right) = -\sum_k a^\dagger_k \begin{pmatrix} 0 & t_A e^{ik} + t_B e^{-ik} \\ t_A e^{-ik} + t_B e^{ik} & 0 \end{pmatrix} a_k$$

- Match t_A and t_B to underlying theory with a particular geometry
- Predict low-energy spectrum of different geometries





⟨NUMERIQS⟩

INCLUDING INTERACTIONS WITHIN OUR ET

- Localization persists in the presence of interactions
- Energy gap is symmetric about Fermi energy
  - particle/hole & chiral symmetries
  - $\Longrightarrow$Inclusion of staggered mass $m_s\sigma_3$ (LEC) into ET

$$H_{1D} = -\sum_k a_k^\dagger \begin{pmatrix} m_s & t_A e^{ik} + t_B e^{-ik} \\ t_A e^{-ik} + t_B e^{ik} & -m_s \end{pmatrix} a_k$$

J. Ostmeyer, **TL,** U.-G. Meißner, *et al.*, [arXiv:2401.04715] Phys.Rev.B 109 (2024) 195135

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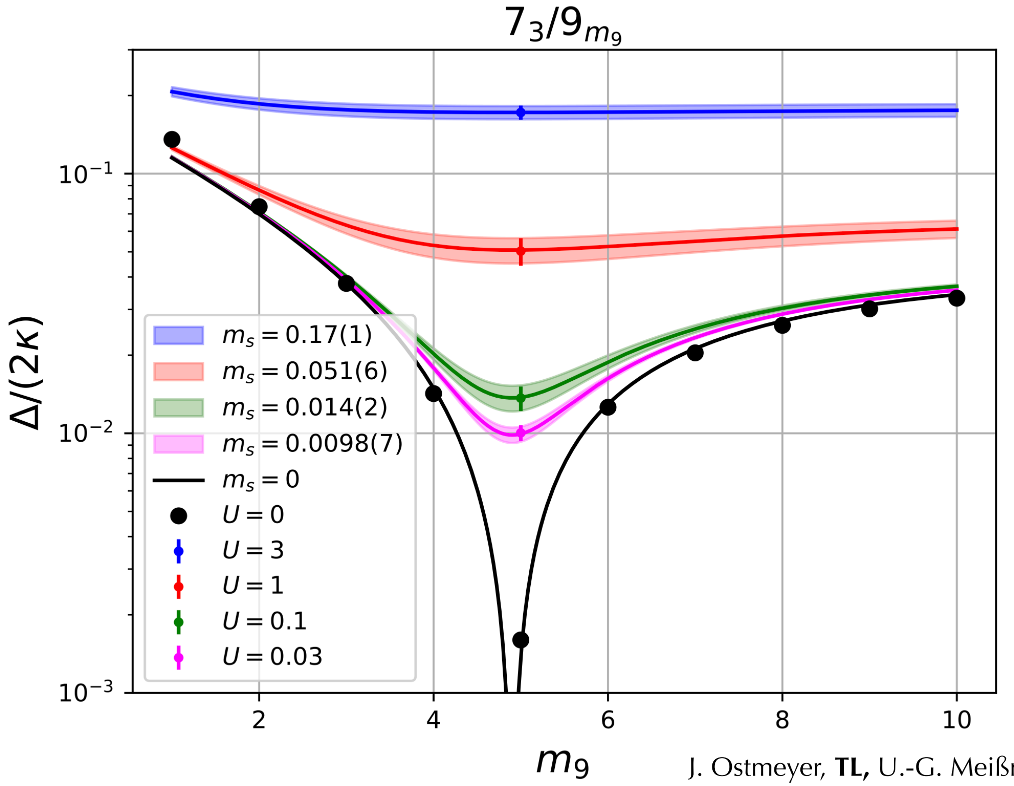
Tune m_s to underlying theory



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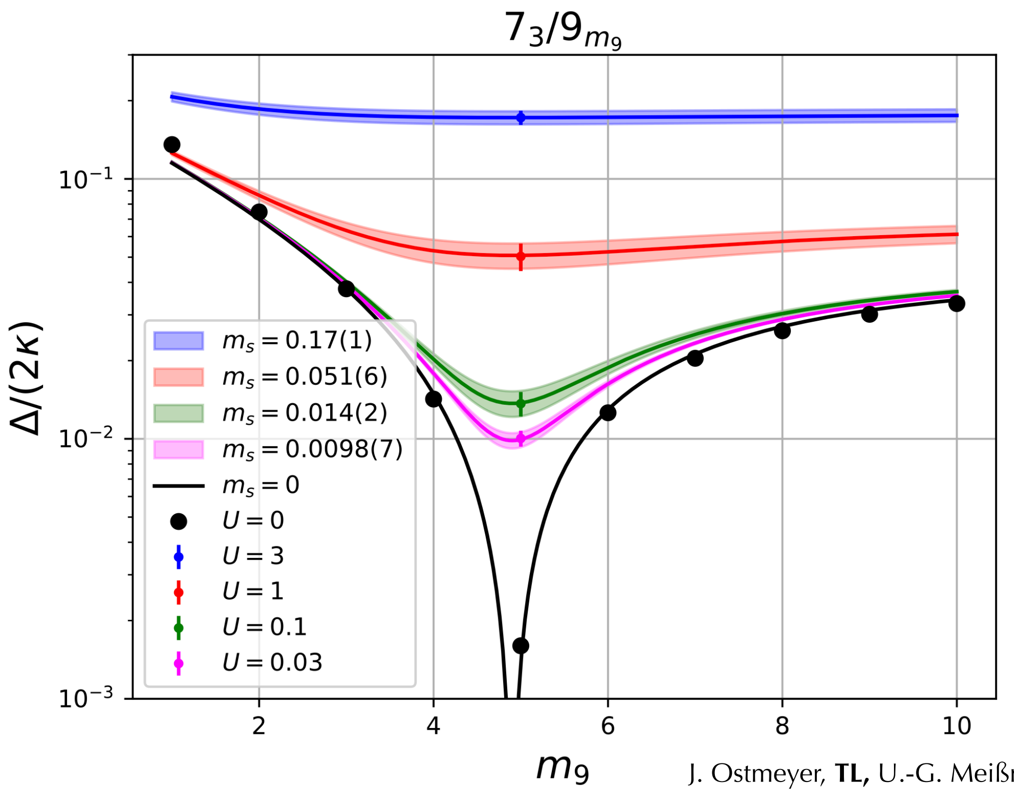
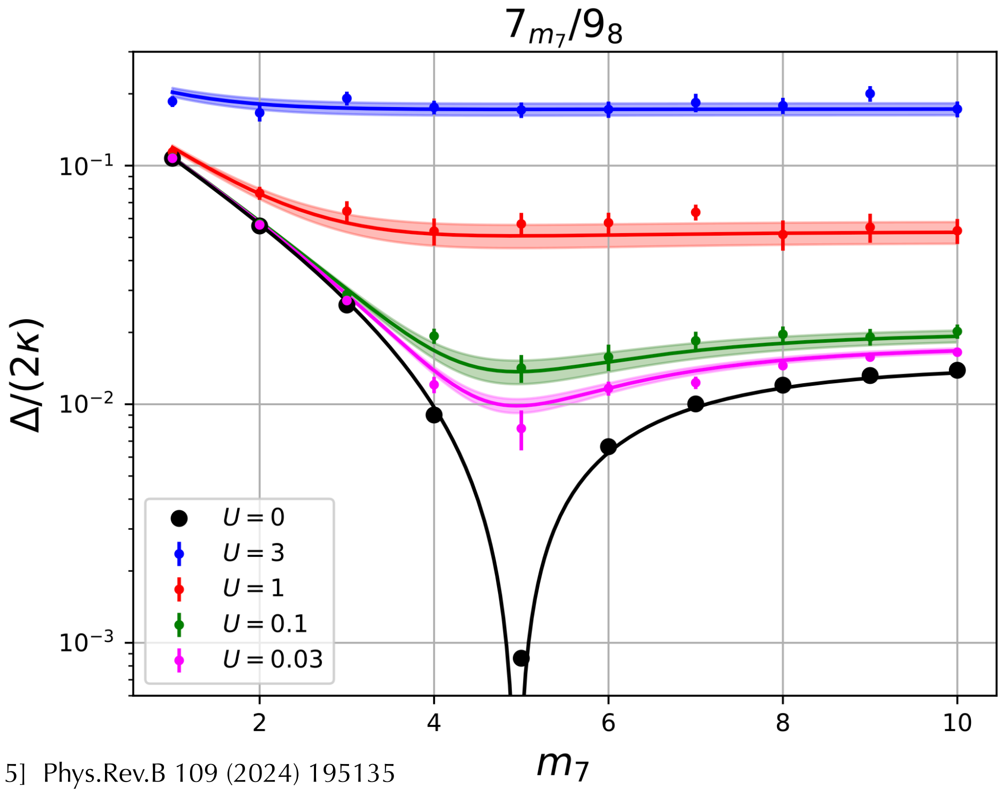


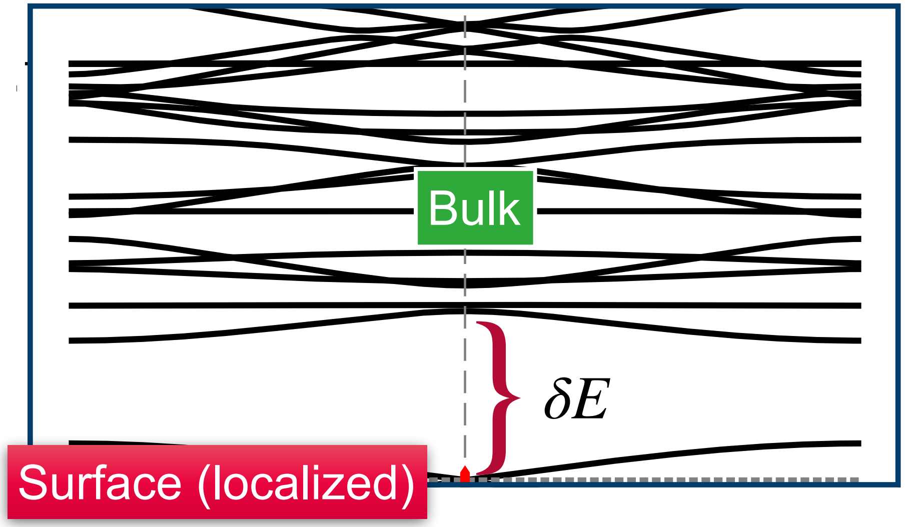
Predict spectrum of new geometries



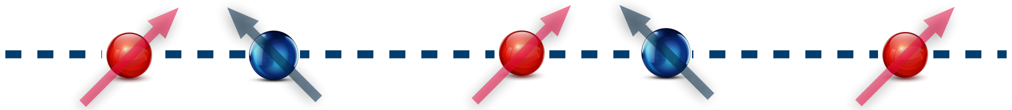
J. Ostmeyer, **TL,** U.-G. Meißner, *et al.*, [arXiv:2401.04715] Phys.Rev.B 109 (2024) 195135

INGREDIENTS FOR AN EFT

- Separation of scales (ie energy gap to bulk states)



- Identification of relevant low-energy degrees of freedom

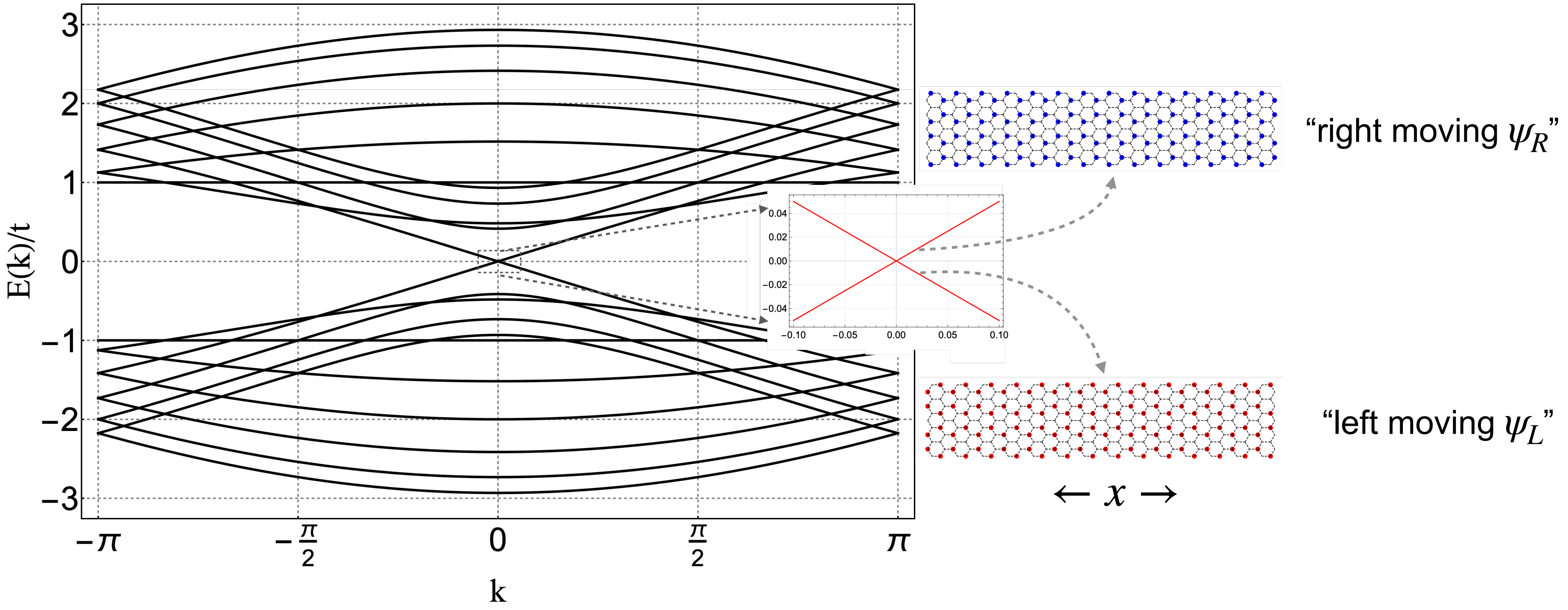


- Interactions terms constrained by symmetries

$$\delta H^{i}_{T,C,S} + \mathcal{O}\left(\left(\frac{q}{\delta E}\right)^{i+1}\right)$$

ANOTHER EXAMPLE

Pure armchair nano ribbon (w/ width = 11)



Low-energy (non-interacting) dispersion $E(k) = \pm v_f\ k$

LOW-ENERGY EFT

. . . of a quantum wire . . .

- Low-energy degrees of freedom in two-component form

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

- Lagrangian that captures correct low-energy dispersion

$$\mathscr{L}_{\text{EFT}} = \bar{\psi}\left(i\gamma_0\partial_t + iv_f\gamma_1\partial_x\right)\psi$$

$$\gamma_0 = \sigma_2 \quad \gamma_1 = i\sigma_1 \quad \bar{\psi} = \psi^\dagger\gamma_0$$

- States are electrically charged! Can include U(1) vector fields A_μ to describe interactions

$$\mathscr{L}_{\text{EFT}} + \mathscr{L}_{\text{QED}} = \bar{\psi}\left(i\gamma_0(\partial_t - ieA_0(x)) + iv_f\gamma_1(\partial_x - ieA_1(x))\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Bazzanella, Faccioli,Lipparini, https://arxiv.org/pdf/1007.1316

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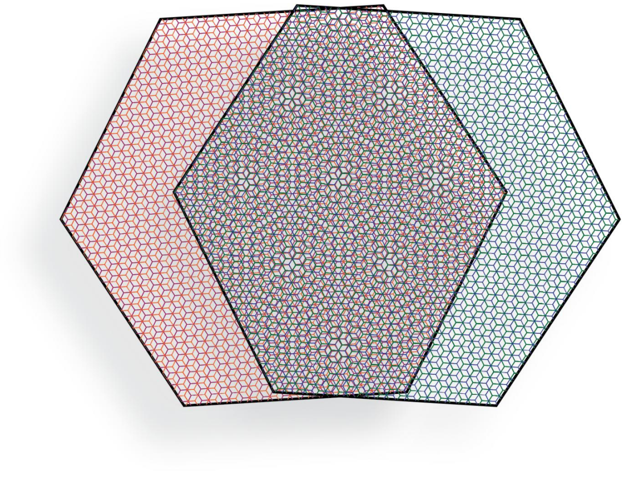
QED in 1+1 dimensions: massless Schwinger model with fermi velocity v_f

$$\Longrightarrow \quad m_s = \frac{ev_f}{\sqrt{\pi}}$$

Bazzanella, Faccioli,Lipparini, https://arxiv.org/pdf/1007.1316

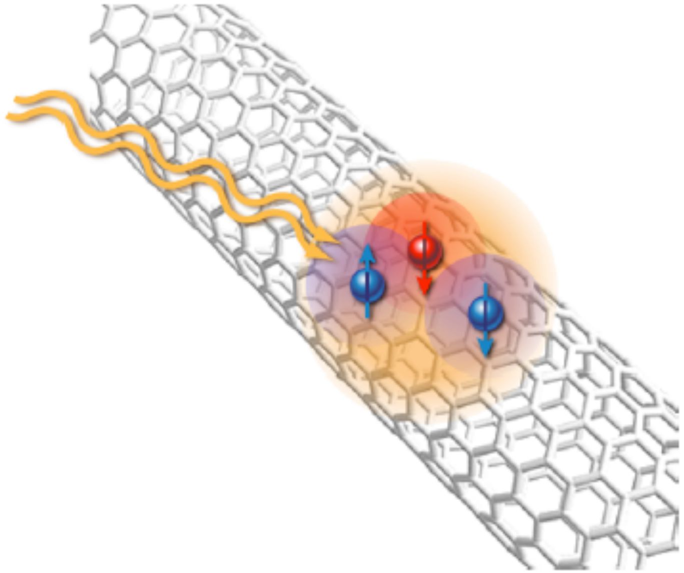
OTHER EXAMPLES OF EMERGENT PHENOMENA

Superconductivity in bilayers with a twist



Andrei, E.Y., MacDonald, A.H. Graphene bilayers with a twist. *Nat. Mater.* **19**, 1265–1275 (2020). https://doi.org/10.1038/s41563-020-00840-0

Bound three-body (trion) state in doped systems

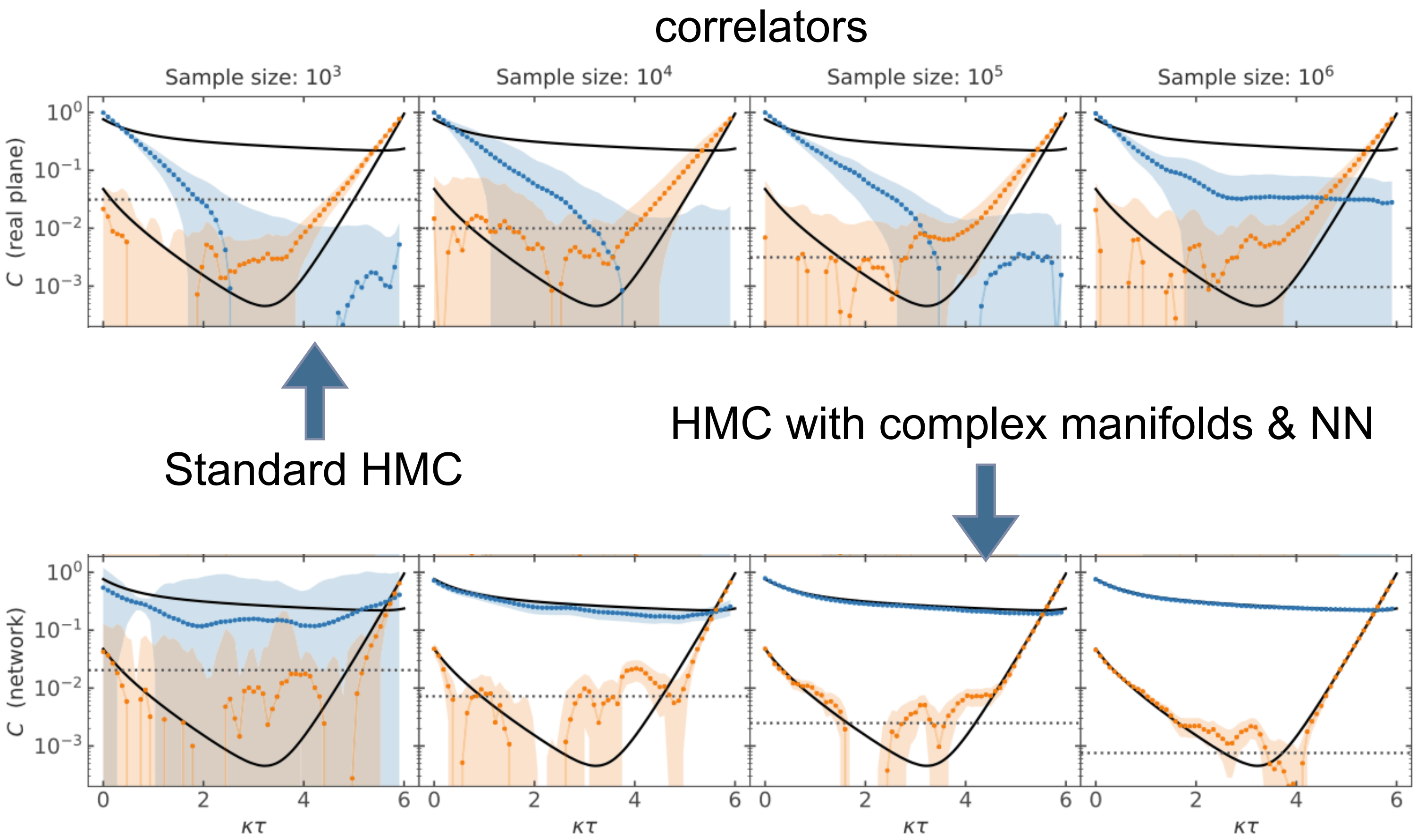


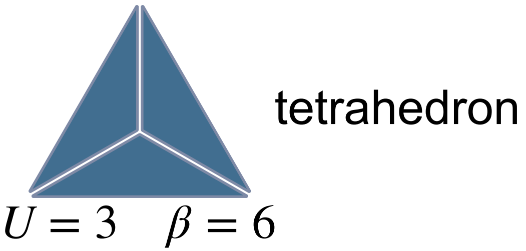
Matsunaga et al., PRL 106, 037404 (2011)

LOW-D SYSTEMS ARE PERFECT TESTBEDS FOR NOVEL ALGORITHMS

Tackling the sign problem

- Stochastic simulations at finite chemical potential
  - Suffer from numerical sign problem
  - Similar situation to LQCD
- Deform path integral contour integral into the complex plane
  - Manifolds comprising Lefschetz thimbles have significantly reduced sign problem
- Test Machine Learning (ML) algorithms to learn these manifolds and alleviate sign problem



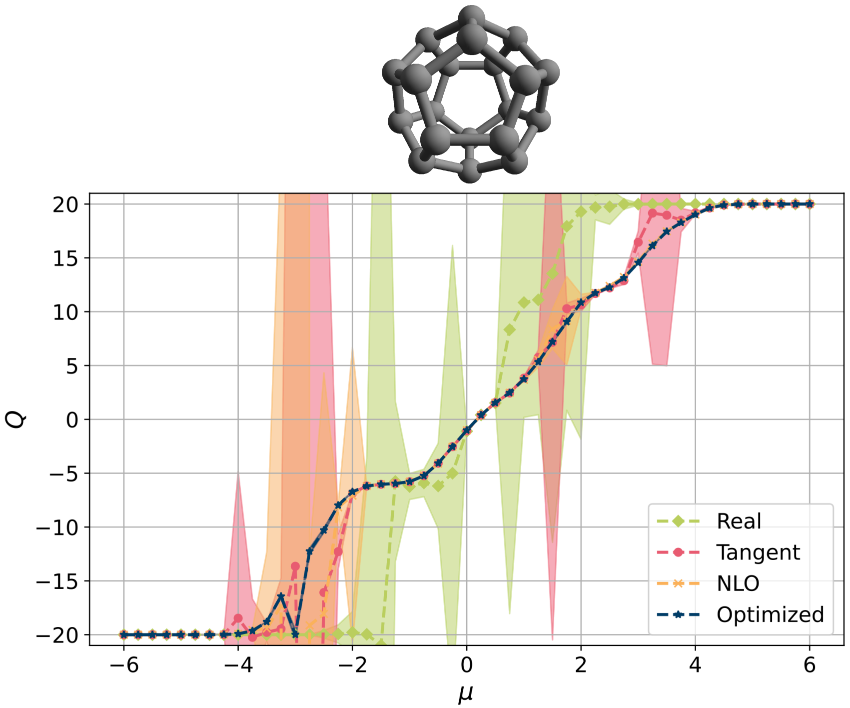


J.-L. Wynen, **TL**, et al.,[arXiv:2006.11221] Phys.Rev.B **103** (2021) 125153

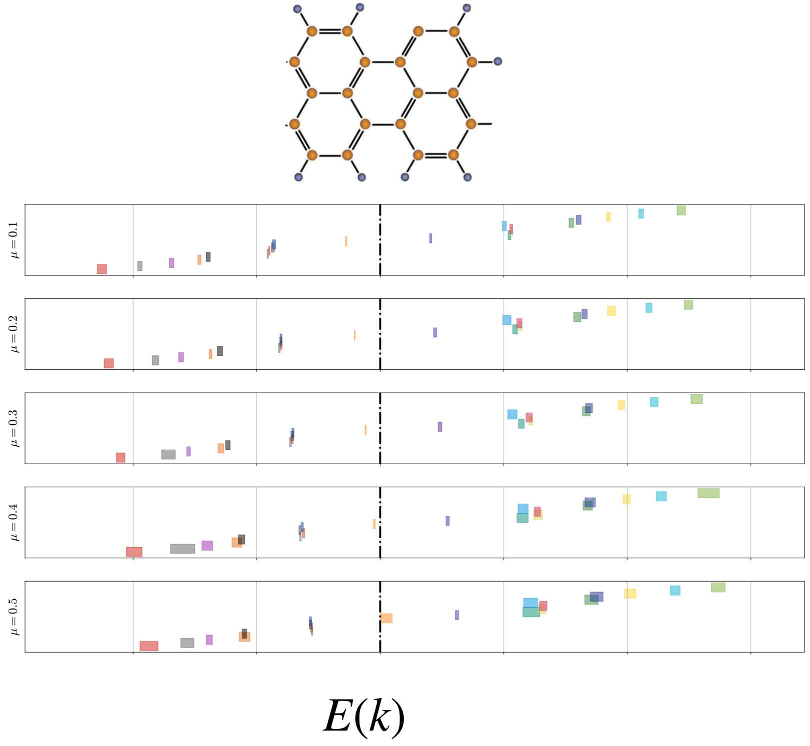
M. Rodekamp, **TL**, et al.[arXiv:2203.00390] Phys.Rev.B **106** (2022) 125139

WE CAN NOW PROBE SYSTEMS NOT AVAILABLE TO US BEFORE

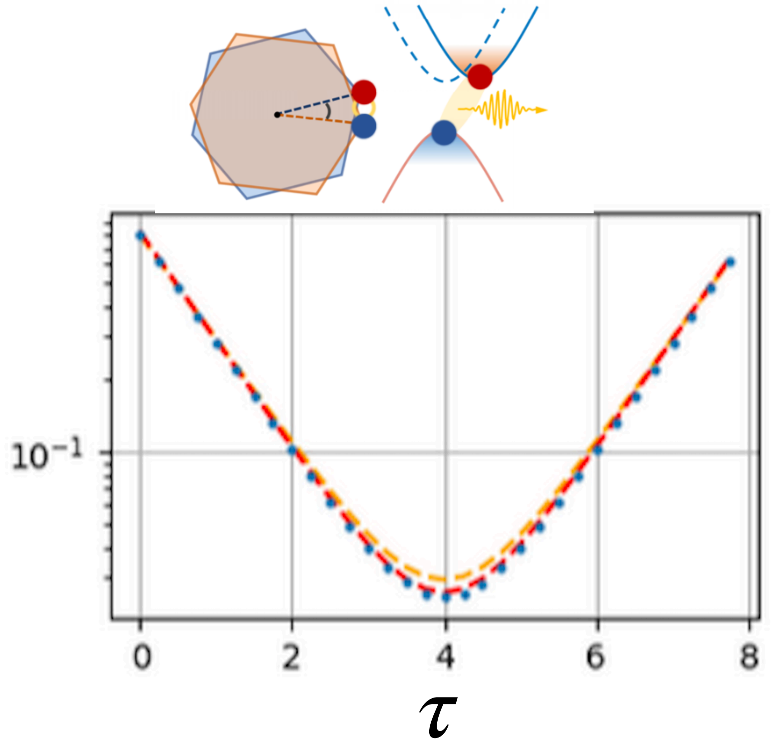
Making predictions. . .



Global charge Q of C_{20} fullerene as a function of doping μ



Spectrum of perylene as a function of doping μ



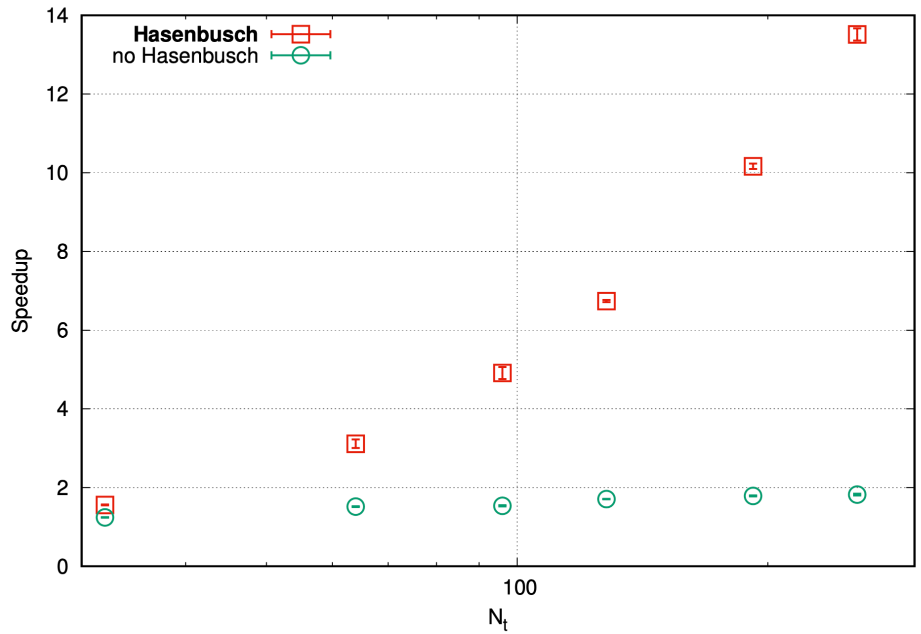
Correlators corresponding to *exciton* (particle/hole) excitations

C. Gäntgen, **TL**, *et al.*, [arXiv:2307.06785] Phys.Rev.B **109** (2024) 195158

M. Rodekamp, **TL**, *et al.*, in preparation

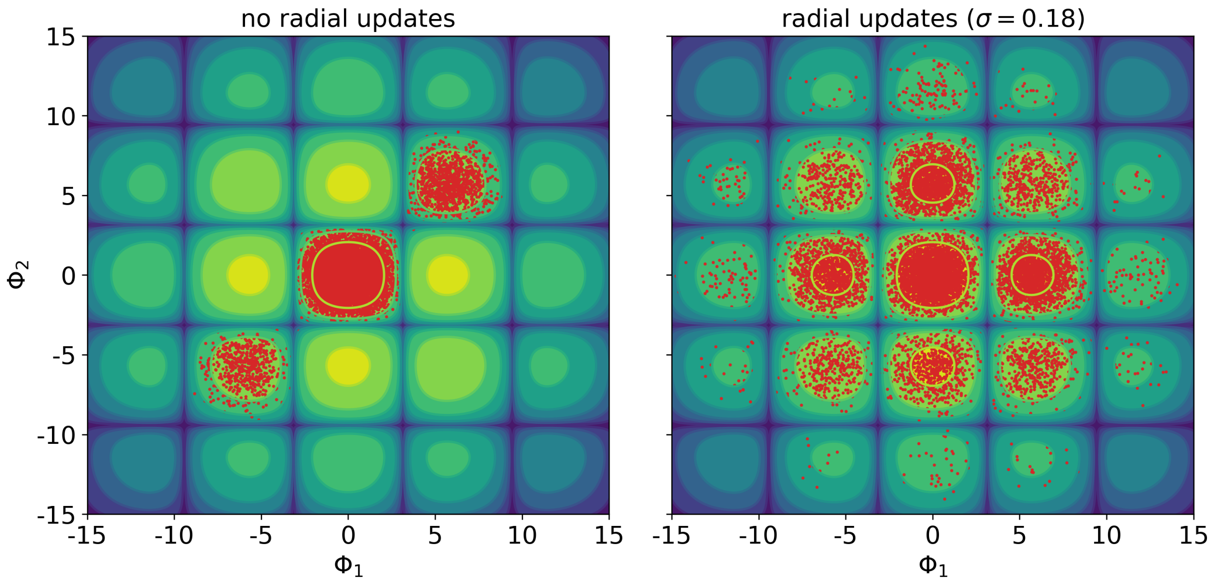
P. Sinilkov., preliminary

OTHER EXAMPLES OF ALGORITHMIC ADVANCEMENTS



10x speedup using Hasenbusch preconditioning (from LQCD)

J. Ostmeyer, **TL**, C. Urbach, *et al.*, [arXiv:1804.07195] Comput.Phys.Commun. **236** (2019) 15-25



Circumventing ergodicity problems with, e.g. radial updates

F. Temmen, preliminary

J.-L. Wynen, **TL**, *et al.*, [arXiv:1812.09268] Phys.Rev. **B100** (2019) 075141

FAZIT

- Low-D materials offer fascinating novel phenomena, but require non-perturbative techniques due to strong correlation effects
- EFT methods applicable
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- Also great testbed for algorithmic testing and development, which already is leading to calculations in novel phase spaces

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My perspectives on “Life after the CRC 110”

CASCADE

NuMeriQS

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Life does indeed go on!