Evgeny Epelbaum, Ruhr University Bochum

Final meeting CRC110, Universitätsclub Bonn, 3-5 June 2024

Nuclear forces from chiral EFT

Based on:

PRL 124 (2020) 082501, PRL 126 (2021) 092501, PRC 98 (2018) 014003, PRC 103 (2021) 054001, PRC 106 (2022) 064002, PRD 109 (2024) L071506, JHEP 10 (2021) 051, EPJA 54 (2018) 86, EPJA 54 (2018) 186, EPJA 55 (2019) 56, EPJA 56 (2020) 152, NPA 1002 (2020) 121980, FBS 63 (2022) 67, e-Print: 2311.10893, e-Print: 2312.13932

+ review articles: Front in Phys. 8 (2020) 98, e-Print: 2405.09807

+ PhD theses: Susanne Strohmeier (TUM, 2020), Patrick Reinert (RUB, 2022), Daniel Möller (RUB, 2024)

+ work in progress

Deutsche Forschungsgemeinschaft DFG

国家自然科学基金委员会
National Natural Science Foundation of China

RUB has joined
the CRC 110 in 2016
(2. funding period)

CRC110 Workshop on

Nuclear Dynamics and Threshold Phenomena

Ruhr-Universität Bochum, April 5-7, 2017

Projects involving Bochum theory groups:
A9, B1, **B7** and B9

PLs: EE (B1, B7, B9), Norbert Kaiser (A9, B7), Hermann Krebs (A9), Jie Meng (B7), Ulf Meißner (B9)

Funded (3. FP): Dr. Arseniy Filin (A9), Daniel Möller (A9),
Herzallah Alharazin (B1), Dr. Jambul Gegelia (B1), Julia Panteleeva (B1),
Dr. Vadim Baru (B7), Patrick Reinert (B7), Victor Springer (B7),
Lukas Bovermann (B9) + Dr. Lu Meng (RUB-fellow)

Outline of the talk

- Introduction
- Chiral EFT for nuclear forces
  - The NN force
  - Three-pion exchange
  - Precision studies in the 2N sector
  - Beyond the 2N system
- Matching nuclear chiral EFT to lattice QCD
- Chiral EFT and the $\Delta(1232)$ isobar
  - The coupled-channel approach
  - Nuclear forces in the small-scale expansion
  - Parity- and time-reversal-violating nuclear interactions
- Chiral EFT using gradient flow
- Summary

Chiral effective field theory

Chiral perturbation theory Weinberg '79; Gasser, Leutwyler '84,'85

QCD in the presence of external sources: $\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \bar{q}(\gamma^\mu \nu_\mu + \gamma_5 \gamma^\mu a_\mu - s - ip)q$

$$\langle 0,\, \text{out} | 0,\, \text{in} \rangle_{\rm v,a,s,p} = Z[v,a,s,p] = \int [DG_\mu][Dq][D\bar{q}]\, e^{i \int d^4x \mathcal{L}(q,\bar{q},G_{\mu\nu};v,a,s,p)} \Big|_{\text{low energy}}$$

$$= \int \underbrace{[DU]}_{\text{pion fields}}\, e^{i \int d^4x \mathcal{L}_{\rm eff}(U;v,a,s,p)} \Big|_{\text{low energy}} \xrightarrow{\text{loop expansion}} \text{S-matrix}$$

(chiral perturbation theory)

Generalization to the single-nucleon sector is straightforward Bernard, Kaiser, Meißner, ...

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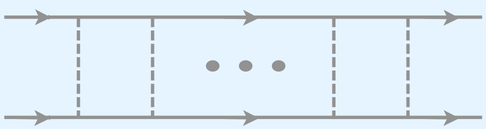
Chiral EFT for nuclear systems Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Machleidt, …

— non-perturbative re-summation of ladder diagrams



$$\left[\left(\sum_{i=1}^{A} \frac{-\vec\nabla_i^2}{2 m_N} + \mathcal{O}(m_N^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}} \right] |\Psi\rangle = E |\Psi\rangle$$

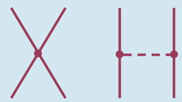
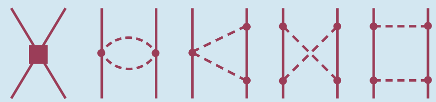
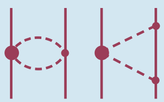
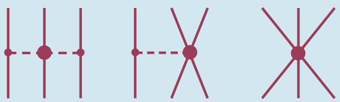
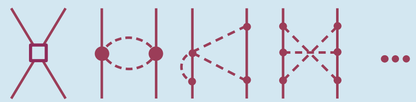
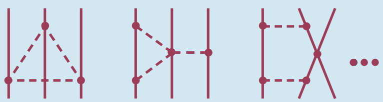
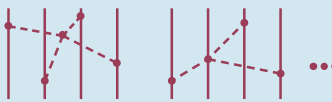
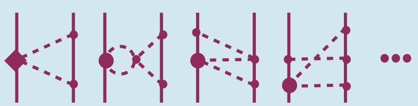
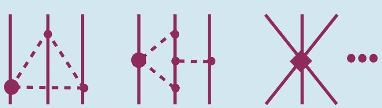
— analytic results for (scheme-dependent!) nuclear forces & currents derived from $\mathscr{L}_{\rm eff}$

— πN LECs from matching to Roy-Steiner eq. Hoferichter et al.'15 $\Rightarrow$ predict large-r interactions

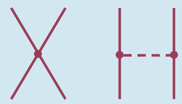
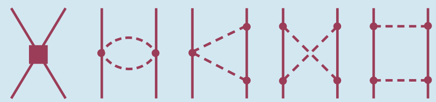
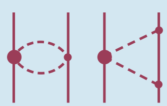
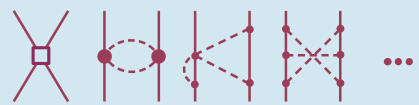
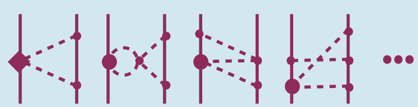
— finite cutoff needed to regularize the Schrödinger equation Lepage, EE, Meißner, Gasparyan, Gegelia

(renormalizability rigorously proven to NLO Ashot Gasparyan, EE, PRC 105 (2022); PRC 107 (2023))

Chiral expansion of the nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N²LO (Q^3)			—
N³LO (Q^4)			
N⁴LO (Q^5)			—

Chiral expansion of the nuclear forces

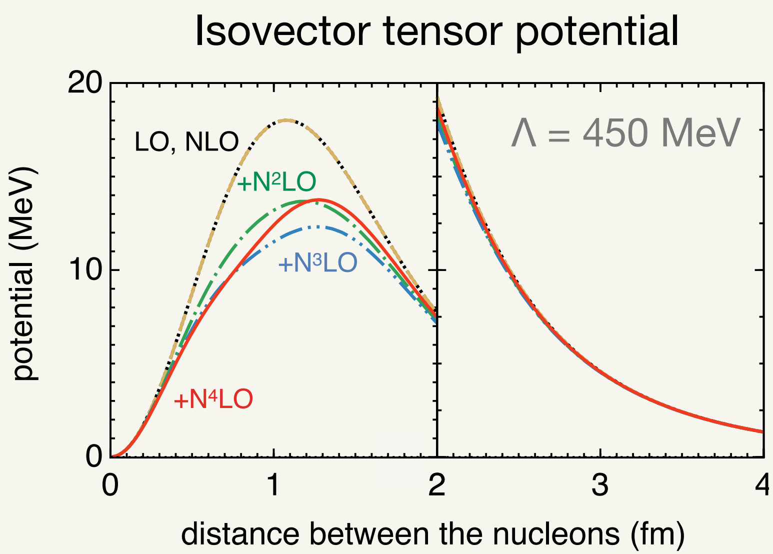
	Two-nucleon force
LO (Q^0)	
NLO (Q^2)	
N^2LO (Q^3)	
N^3LO (Q^4)	
N^4LO (Q^5)	

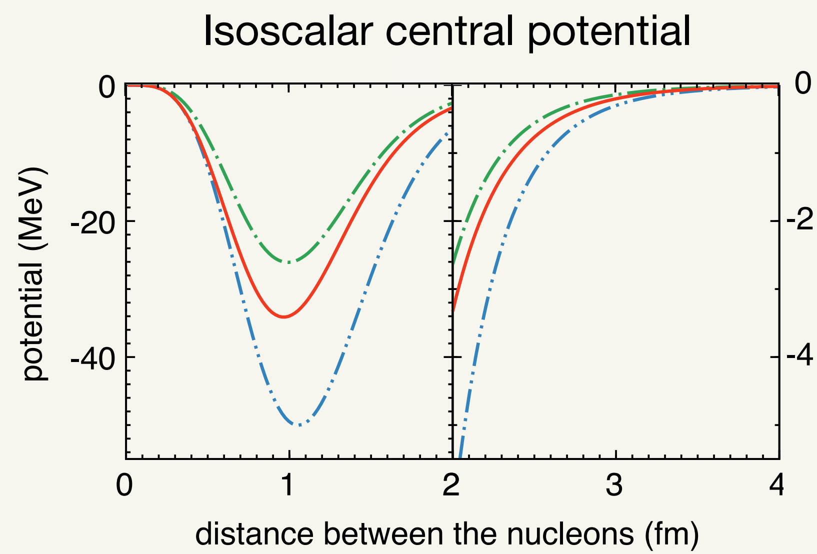
The newest Bochum NN interactions Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \, \mu \, \frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2} \; e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

Chiral expansion of the long-range NN force





— Long-distance behavior of the NN force is **a parameter-free prediction of chiral EFT**

— Agrees with phenomenology (strong intermediate-range attraction from 2π-exchange)

— Reasonable convergence of the chiral expansion (at large r)

— Short-range interactions parametrized by contacts

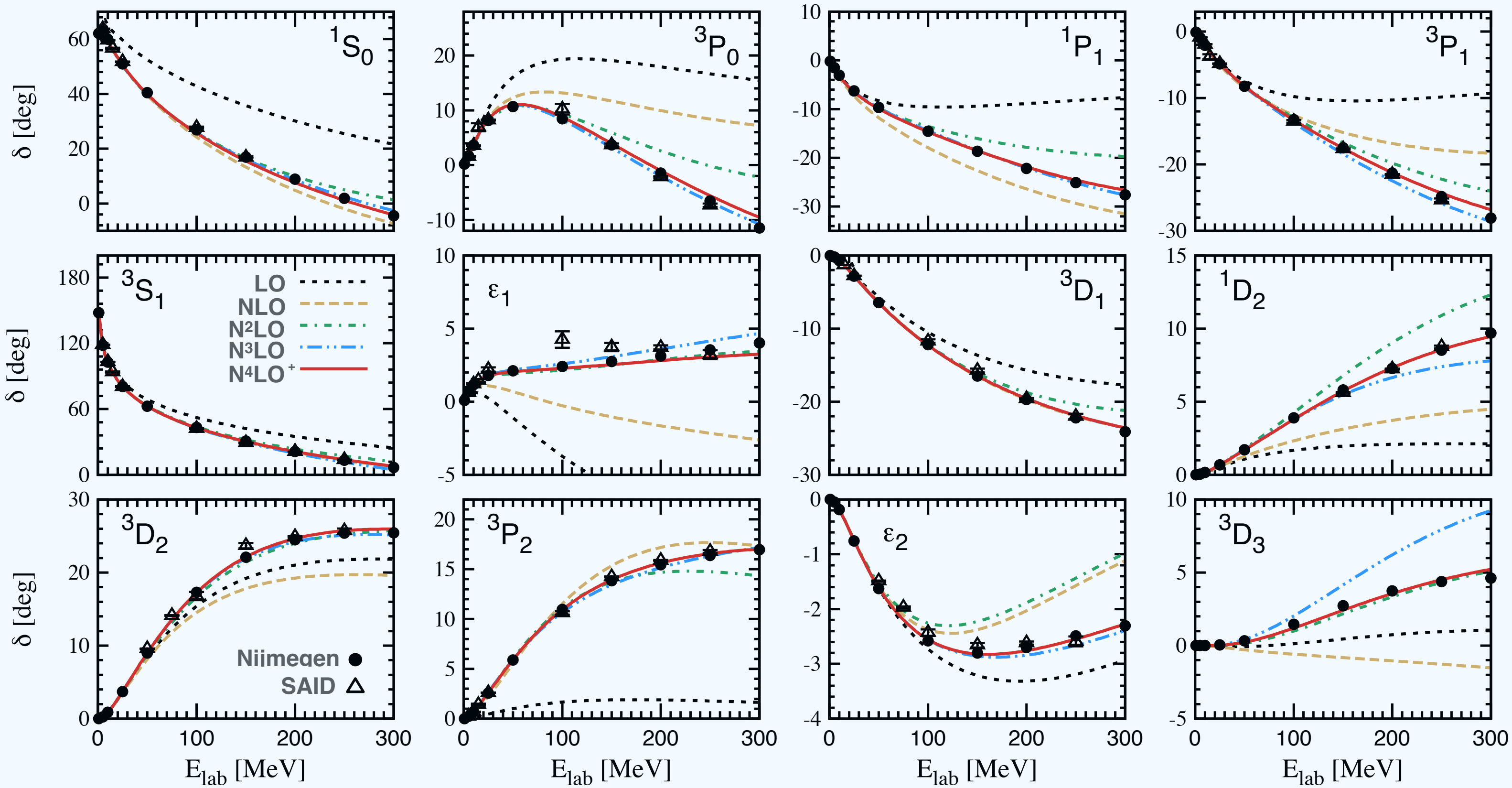
The two-nucleon system

Results for $\Lambda = 450$ MeV

from: P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

	LO (Q^0)	NLO (Q^2)	N^{2}LO (Q^3)	N^{3}LO (Q^4)	N^{4}LO (Q^5)	N^{4}LO$^+$
χ^2/datum (np, 0 – 300 MeV)	75	14	4.1	2.01	1.16	1.06
χ^2/datum (pp, 0 – 300 MeV)	1380	91	41	3.43	1.67	1.00
	2 LECs	**+ 7 + 1 IB LECs**		**+ 12 LECs**	**+ 1 LEC (np)**	**+ 4 LECs**

Chiral expansion of the neutron-proton phase shifts [Λ = 450 MeV]



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Clear evidence of the 2π-exchange (chiral symmetry!)

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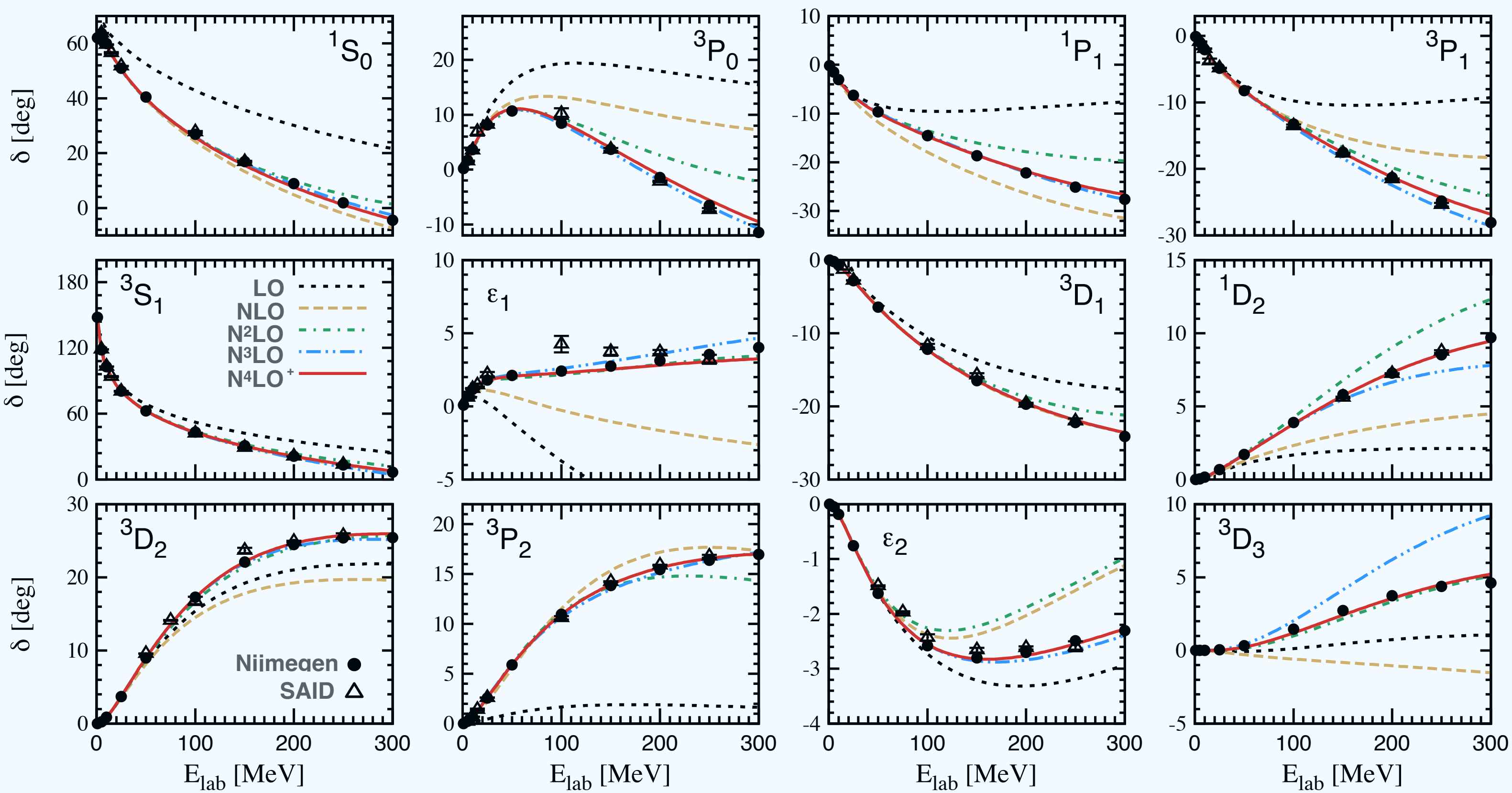


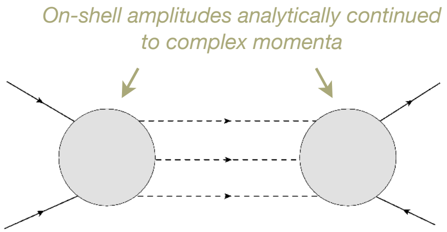
Three-pion exchange

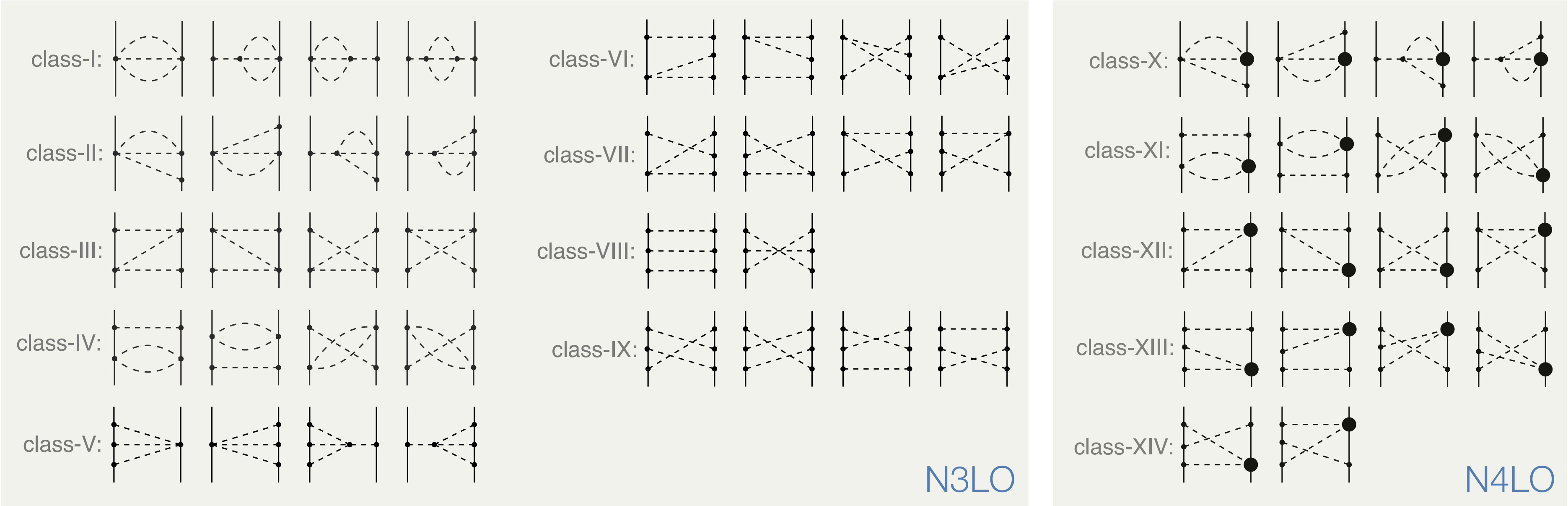
What about the 3π-exchange?

Three-pion exchange

What about the 3π-exchange? Tour-de-force calculation by N. Kaiser using the Cutkosky cutting rules N. Kaiser, PRC61 (2000), PRC62 (2000), PRC63 (2001)

$$\mathrm{Im}\Big[V(q_\mu q^\mu = \mu^2 > 9M_\pi^2)\Big] = \int d\Gamma_3 \, \mathrm{Ampl}_1 \times \mathrm{Ampl}_2$$





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- As one may expect, 3π-exchange is well representable by contacts EE, Krebs, Meißner, PRL115 (2015)

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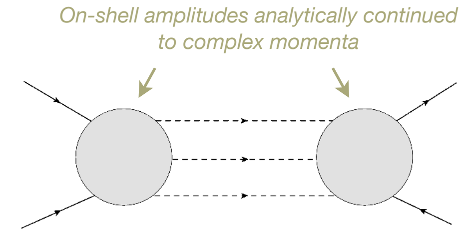
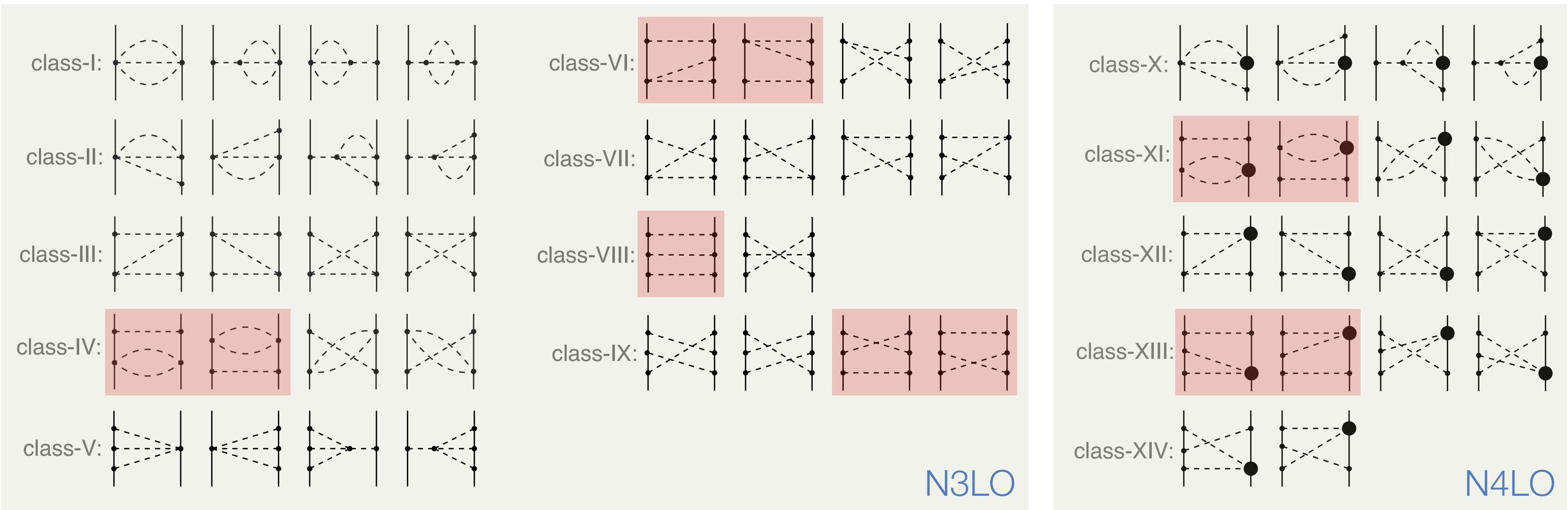




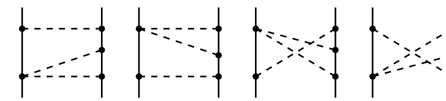
- A bit sparse on detail: *„After a somewhat lengthy calculation we find, from class II,…"*
- As one may expect, 3π-exchange is well representable by contacts EE, Krebs, Meißner, PRL115 (2015)
- Main concern: Potentials from reducible-like diagrams are scheme-dependent. Are the results of Norbert consistent with our potentials obtained using the Method of Unitary Transformation?

3π-exchange using the Method of UT

Victor Springer, Hermann Krebs, EE, in preparation

Re-derived the 3π-exchange using the Method of UT [PhD thesis of Victor Springer]

Find same result as Norbert at N⁴LO, but different expressions at N³LO. For example, for the class-VI:



— Norbert finds the only non-vanishing contributions:

$$\mathrm{Im}\, W_S(i\mu) = \frac{2g_A^4}{(8\pi F_\pi^2)^3} \iint_{z^2 \le 1} d\omega_1 d\omega_2 \left\{ -k_1^2 - \frac{5}{3}\mu\omega_1 + (\mu\omega_1 - M_\pi^2)\left(z + \frac{k_2}{k_1}\right) \frac{\arccos(-z)}{\sqrt{1-z^2}} \right\}, \qquad \mathrm{Im}\, W_T(i\mu) = \ldots$$

where $k_{1,2} = \sqrt{\omega_{1,2}^2 - M_\pi^2}, \qquad z k_1 k_2 = \omega_1\omega_2 - \mu(\omega_1 + \omega_2) + \frac{1}{2}(\mu^2 + M_\pi^2)$

— Method of UT:

$$\delta W_S(r) = \frac{2}{3} V_S(r) = -\frac{g_A^4}{3\,(8\pi F^2)^3} \frac{e^{-3M_\pi r}}{r^5} M_\pi^2 \,(1 + M_\pi r)^2$$

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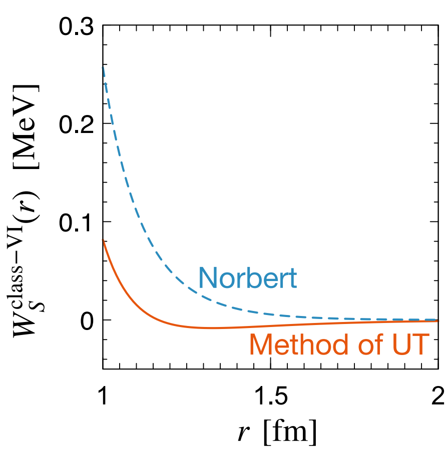
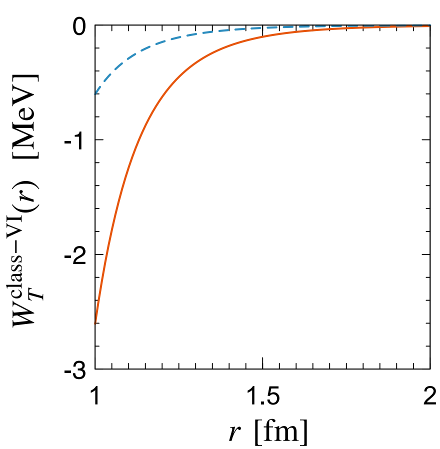


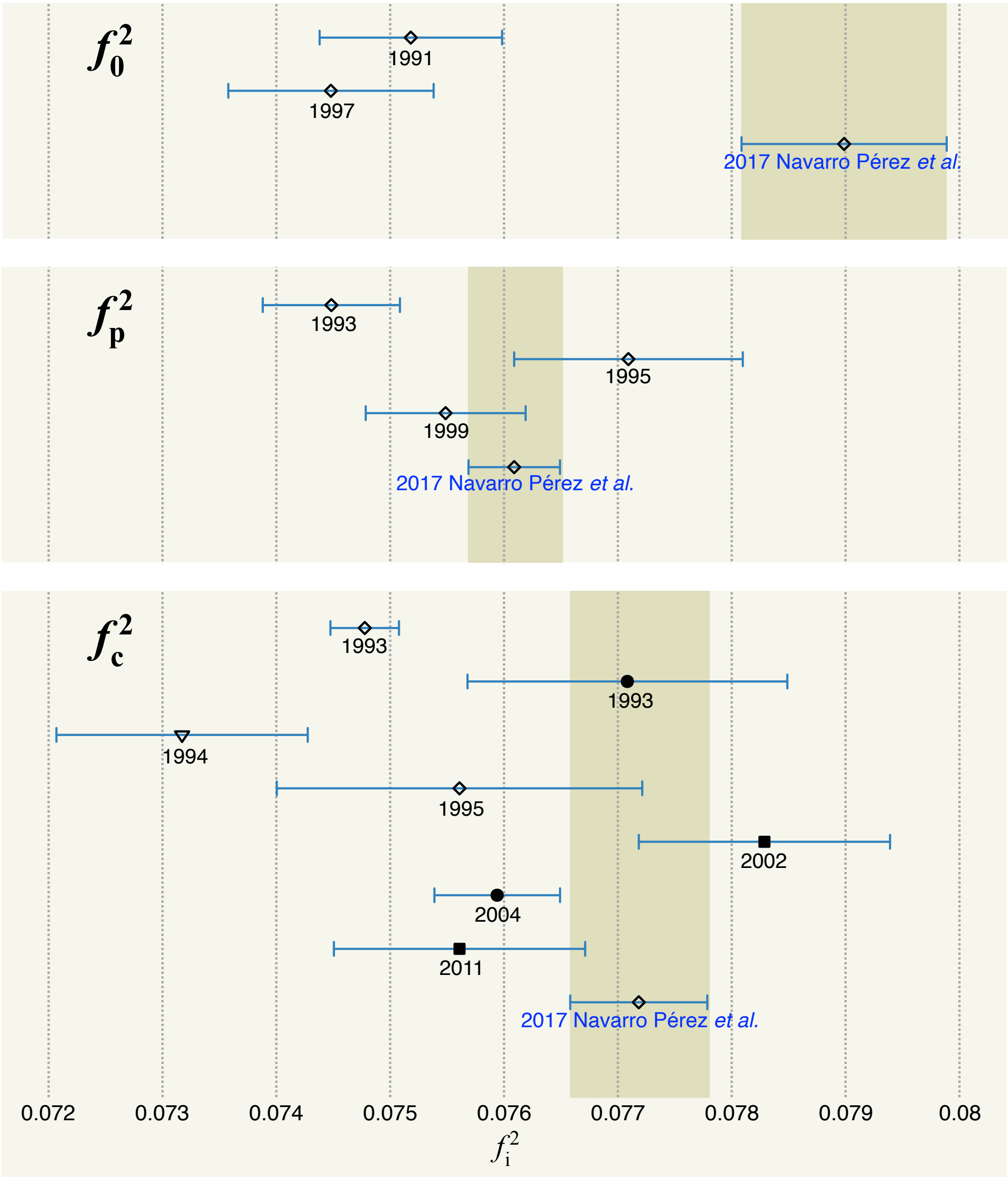


Phenomenological importance of the 3π is still to be explored

Victor Springer, Hermann Krebs, EE, in progress

Precision physics in the 2N sector I: πN couplings

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501



Standard notation ($f_{\pi NN} = \frac{M_{\pi^\pm}}{2\sqrt{4\pi} m_N} g_{\pi NN}$):

$$f_0^2 = -f_{\pi^0 nn}\, f_{\pi^0 pp}$$
$$f_p^2 = f_{\pi^0 pp}\, f_{\pi^0 pp}$$
$$2 f_c^2 = f_{\pi^\pm pn}\, f_{\pi^\pm pn}$$

2017 Granada PWA: claimed to find significant charge dependence of the coupling constants:

$$f_0^2 - f_p^2 = 0.0029(10)$$

Navarro Perez et al., PRC 95 (2017) 6, 064001

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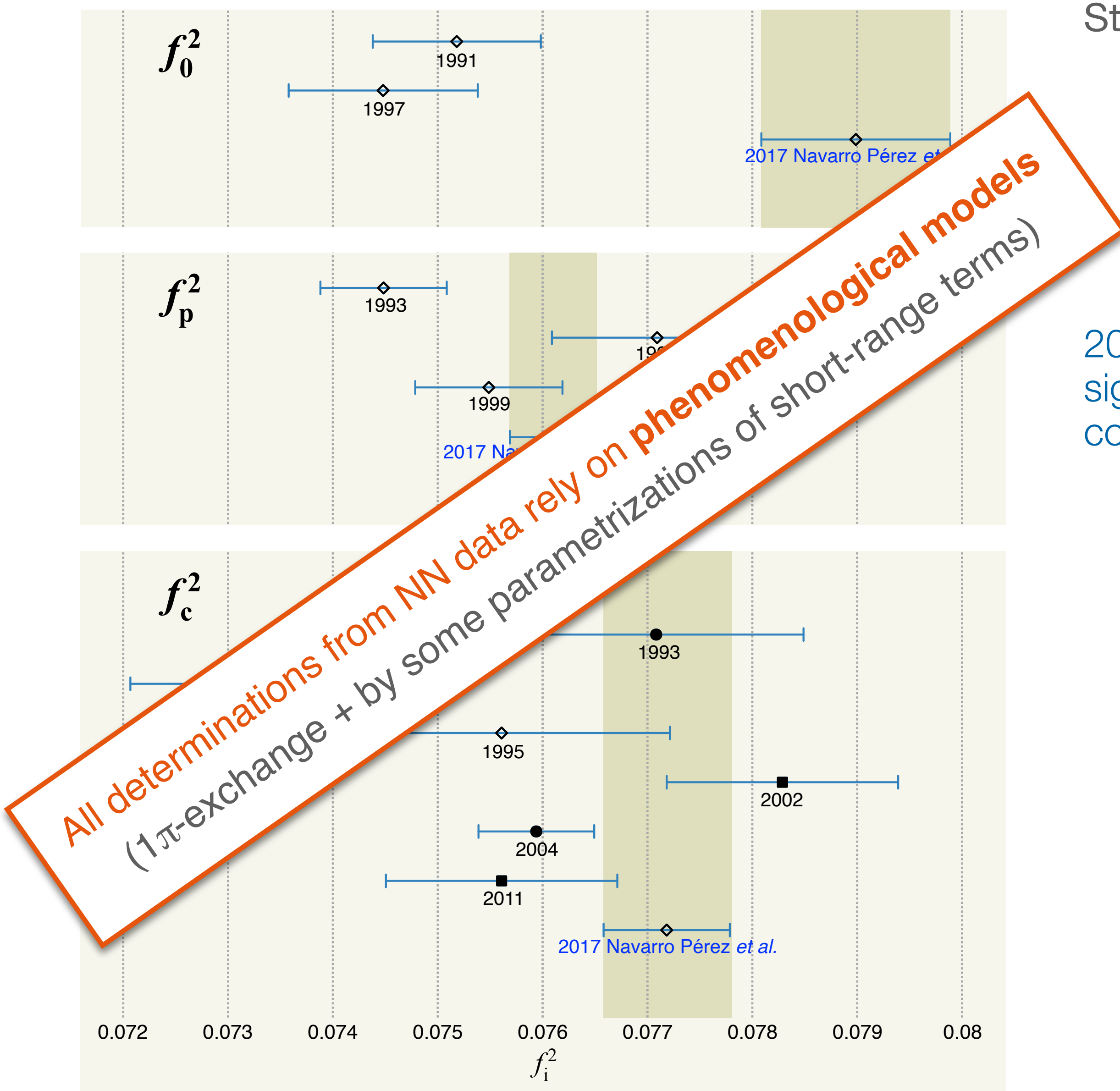

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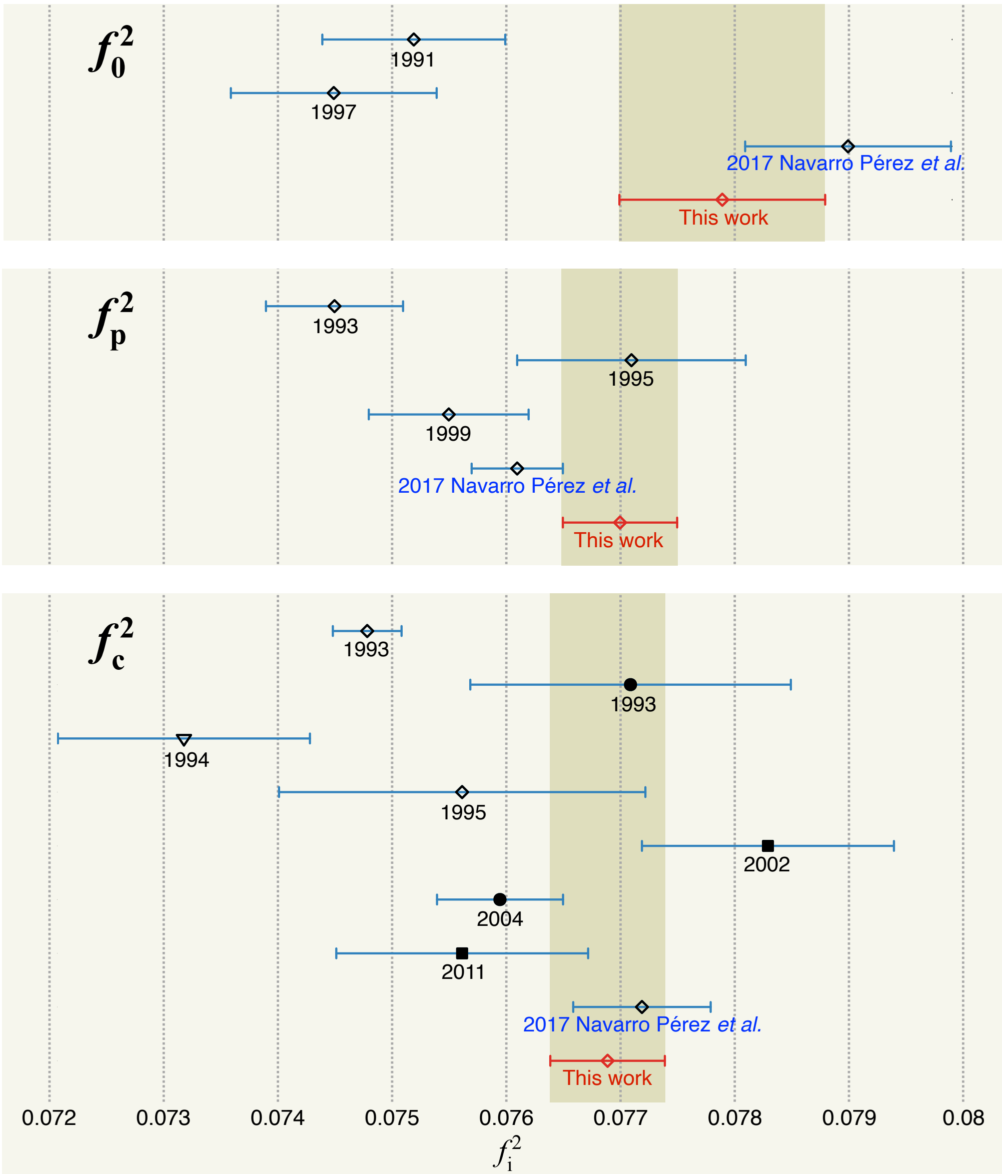

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Our result (χEFT at N^{4}LO):

Bayesian determination; statistical **and systematic** uncertainties.

No evidence for charge dependence of the πN coupling constants

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 092501

Precision physics in the 2N sector I: πN couplings

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

Our $g_{\pi NN}$ value corresponding to f_c^2 reads:

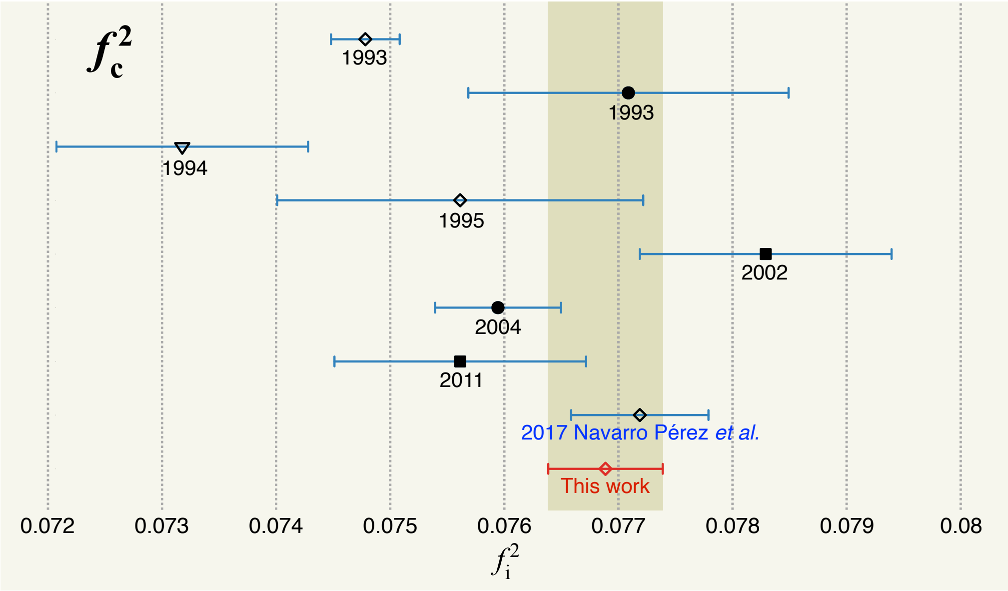
$$g_{\pi NN} = 13.23 \pm 0.04$$

Pionic hydrogen exp. at PSI (GMO sum rule)

[Hirtl et al., Eur. Phys. J. A57 (2021) 2, 70]

$$\epsilon_{1s}^{\pi H} + \epsilon_{1s}^{\pi D}: \quad g_{\pi NN} = 13.10 \pm 0.10$$

$$\Gamma_{1s}^{\pi H}: \quad g_{\pi NN} = 13.24 \pm 0.10$$



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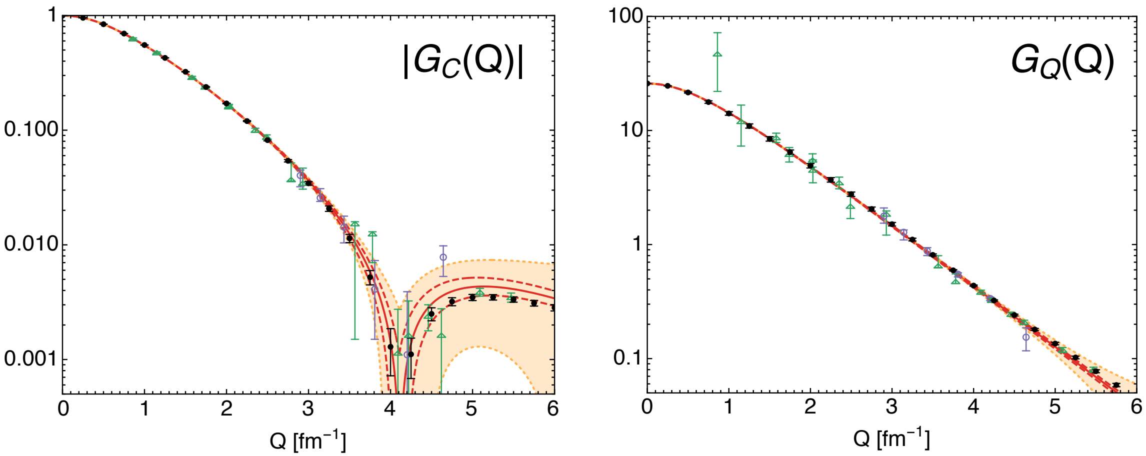
Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 092501

Precision physics in the 2N sector II: Deuteron FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Charge and quadrupole form factors of the deuteron at N⁴LO



Extracted quadrupole moment:

$$Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}\ \text{fm}^2$$

EFT truncation, choice of fitting range, NN, πN and γNN LECs

to be compared with experiment

$$Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)\ \text{fm}^2$$

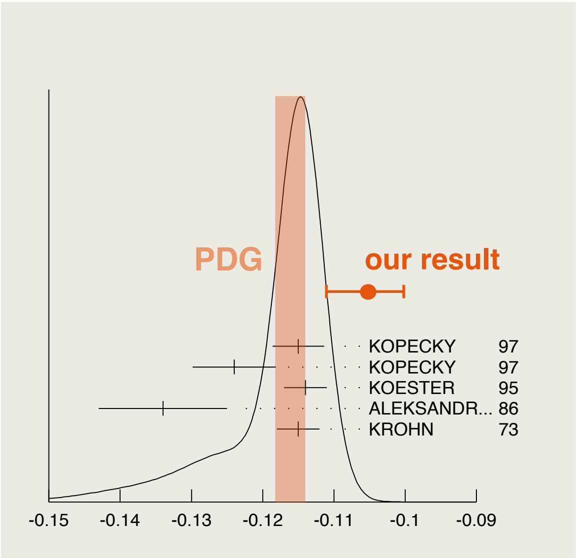
Puchalski et al., PRL 125 (2020)

The charge and structure radius:

$$r_d^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\bigg|_{Q^2=0} = r_{str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

Combining our result $r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}$ fm with very precise isotope-shift spectroscopy data for $r_d^2 - r_p^2$, we determine the neutron m.s. charge radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ \text{fm}^2$$

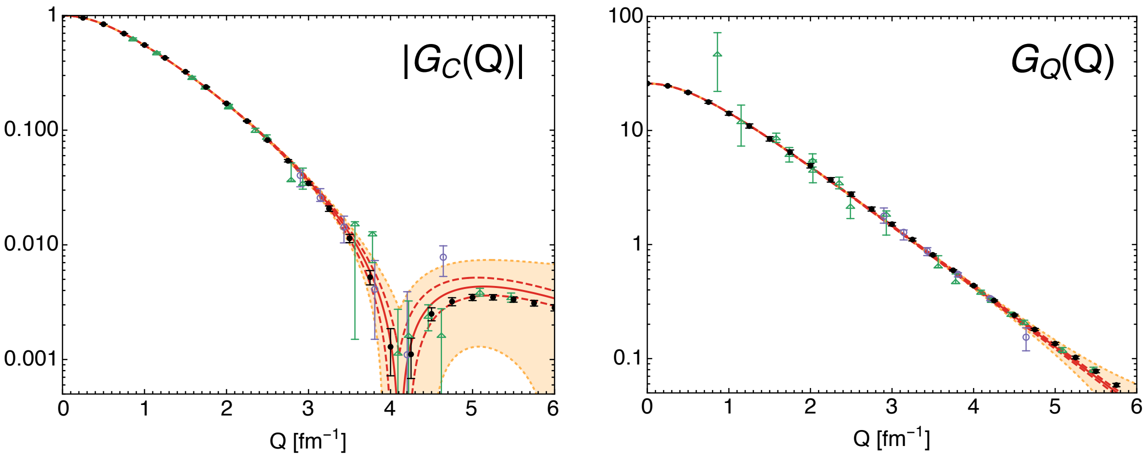


In progress: magnetic (Daniel Möller, PhD thesis) and gravitational (Julia Panteleeva, PhD thesis) FFs of ²H

Precision physics in the 2N sector II: Deuteron FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Charge and quadrupole form factors of the deuteron at N⁴LO



Extracted quadrupole moment:

$$Q_d = 0.2854^{+0.0038}_{-0.0017}\ \text{fm}^2$$

EFT truncation, choice of fitting range, NN, πN and γNN LECs

to be compared with experiment

$$Q_d^{\text{exp}} = 0.285\,699(15)(18)\ \text{fm}^2$$

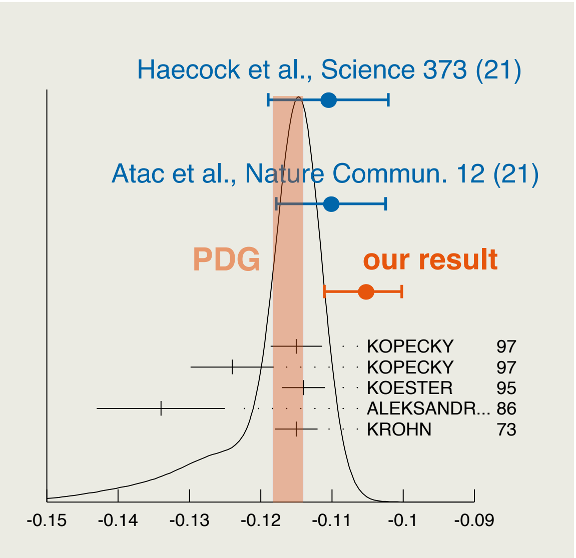
Puchalski et al., PRL 125 (2020)

The charge and structure radius:

$$r_d^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\bigg|_{Q^2=0} = r_{str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

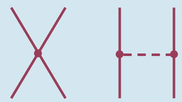
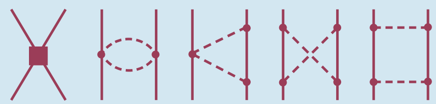
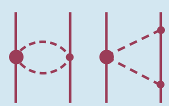
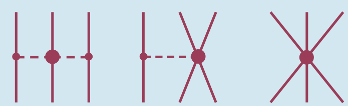
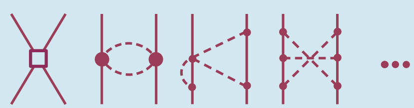
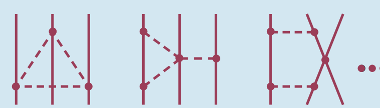
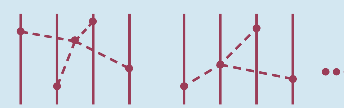
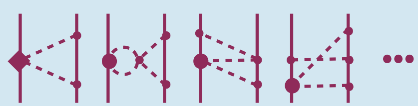
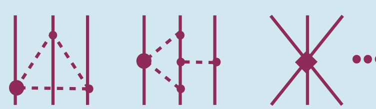
Combining our result $r_{\text{str}} = 1.9729^{+0.0015}_{-0.0012}$ fm with very precise isotope-shift spectroscopy data for $r_d^2 - r_p^2$, we determine the neutron m.s. charge radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ \text{fm}^2$$



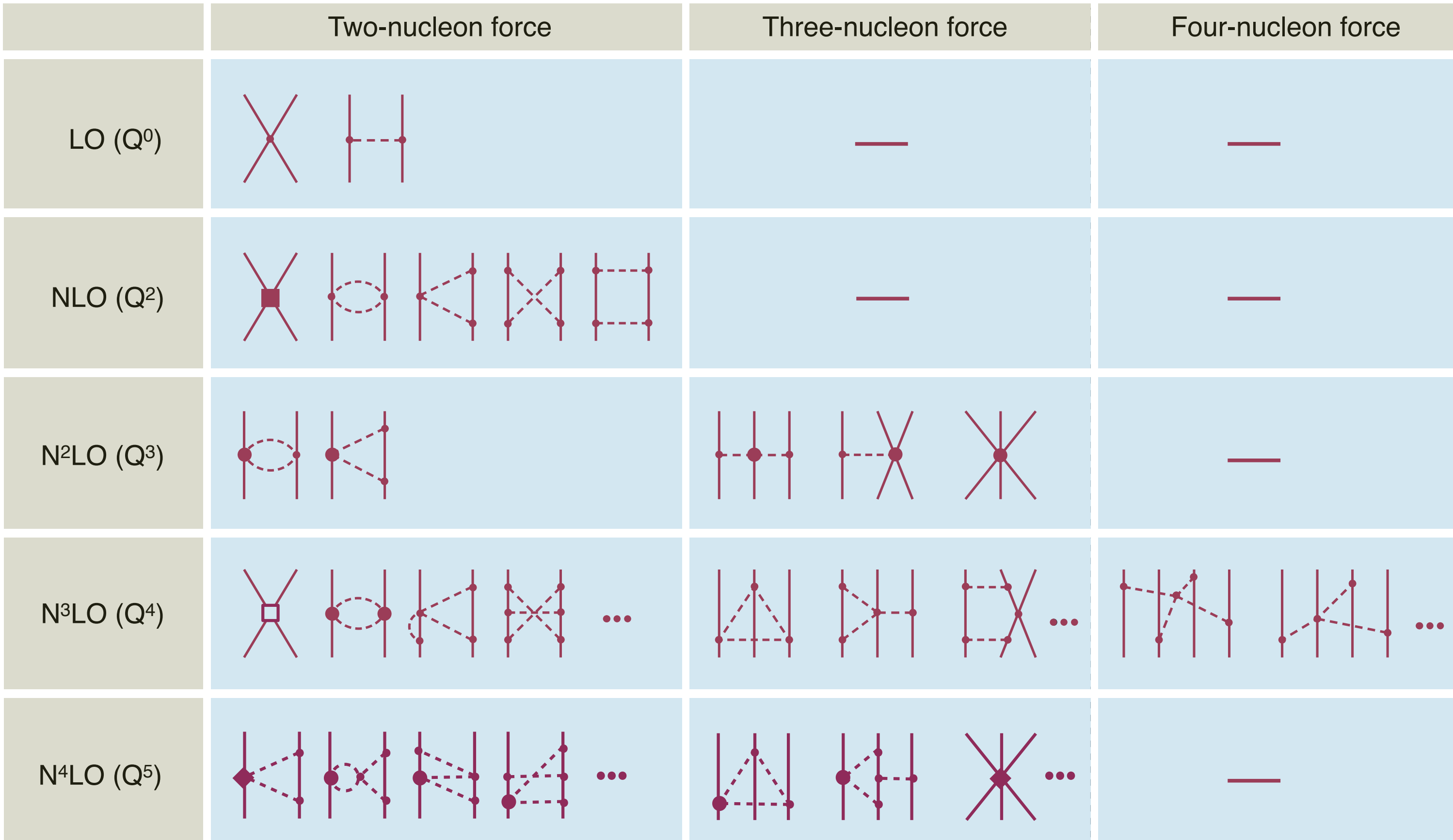
In progress: magnetic (Daniel Möller, PhD thesis) and gravitational (Julia Panteleeva, PhD thesis) FFs of ²H

Beyond the 2N system

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^2LO (Q^3)			—
N^3LO (Q^4)			
N^4LO (Q^5)			—

have been worked out using dimensional regularization

Beyond the 2N system

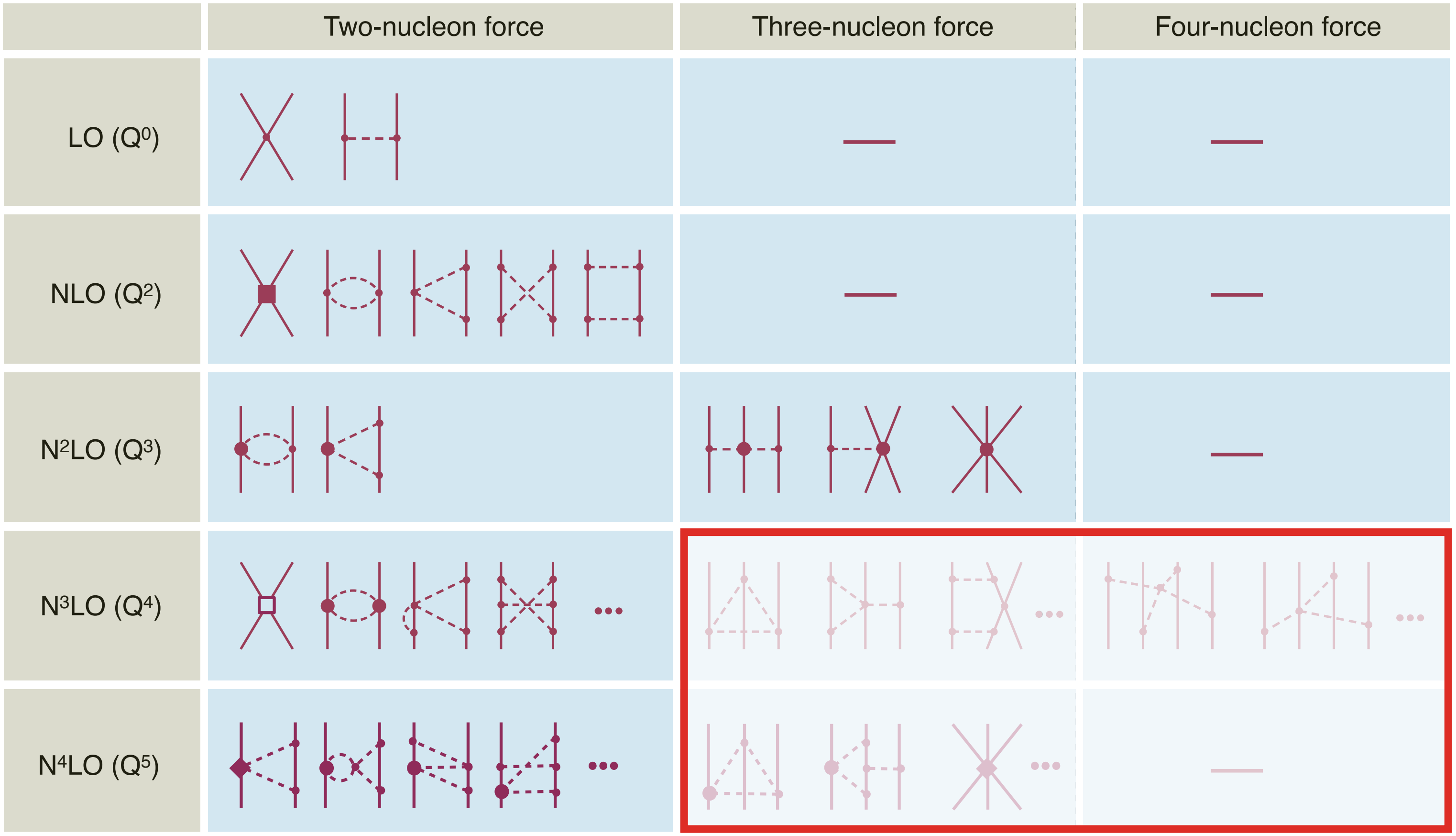


have been worked out using dimensional regularization

Regularization: $V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}}$ + subtraction, $V_{2\pi}(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu \, \mu \frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2} \; e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}}$ + subtractions

+ nonlocal (Gaussian) cutoff for contacts

Beyond the 2N system



have been worked out using dimensional regularization

mixing DimReg with Cutoff violates χ-symmetry (also for current operators)

$\Rightarrow$ need to be re-derived using invariant cutoff regulator

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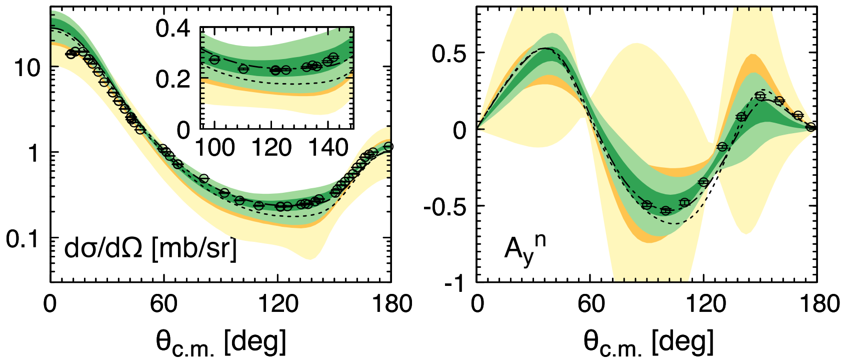
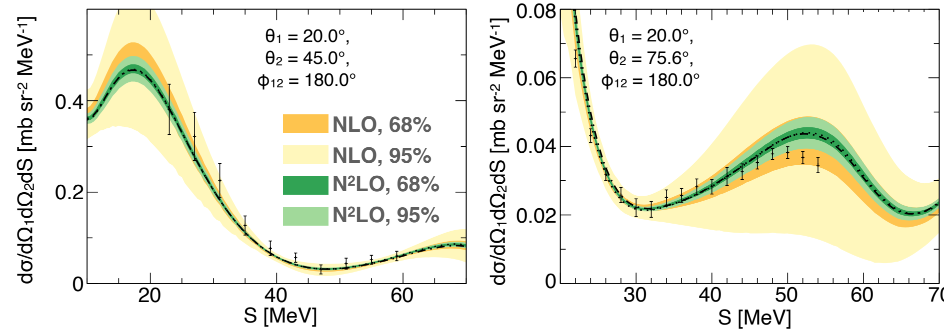
Beyond the 2N system

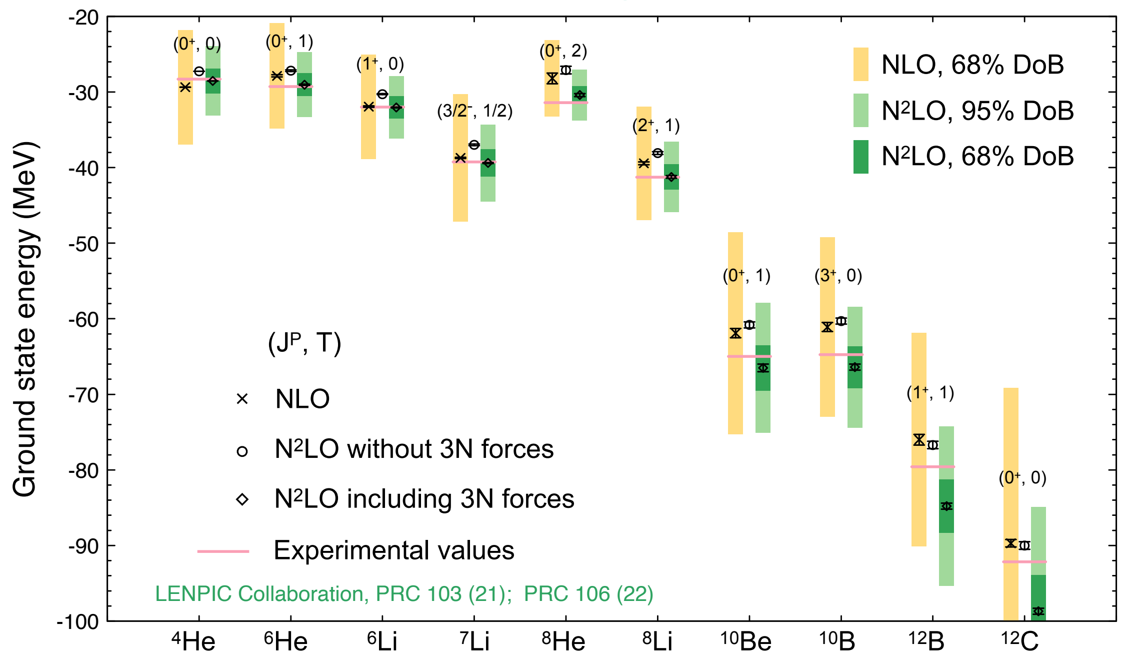
Elastic Nd scattering at 135 MeV



Selected breakup observables at 65 MeV



Predictions for light p-shell nuclei



LENPIC

Matching nuclear χEFT to lattice QCD

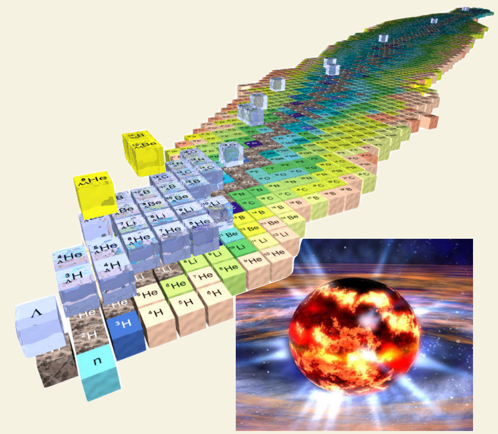
Experiment

Data for 2B & 3B observables at physical m_q

Chiral EFT

(Hyper-) nuclear interactions & current operators derived from the effective chiral Lagrangian

(Hyper-) nuclear physics, neutron stars, …



Matching nuclear χEFT to lattice QCD

Lattice-QCD

input for few-B systems
(crucial for $S \neq 0$, BSM MEs and variable m_q)

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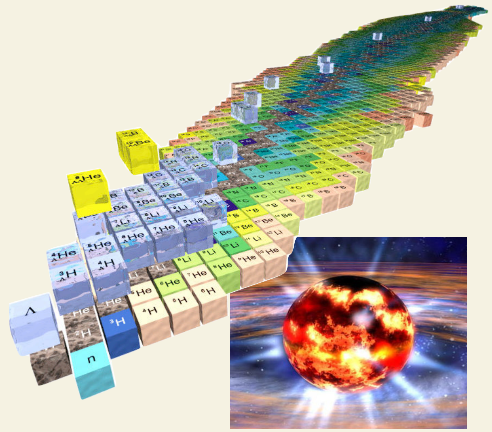

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FAIR Jefferson Lab Exploring the Nature of Matter J-PARC

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FAIR Jefferson Lab J-PARC

(Hyper-) nuclear physics, neutron stars, …

Finite volume energy spectra as an efficient interface between lattice-QCD and chiral EFT

Lu Meng, EE, JHEP 10 (21); Lu Meng, Baru, EE, Filin, Gasparyan, PRD 109 (24)

— infinite-V extrapolations without Lüscher

— solves the t-channel cut problem

— partial wave mixing included

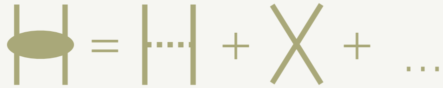
known function of FV energies

$$\det\left[M^{(\Gamma,\boldsymbol{P})}_{ln,l'n'} - \delta_{ll'}\delta_{nn'}\cot\delta_l\right] = 0$$

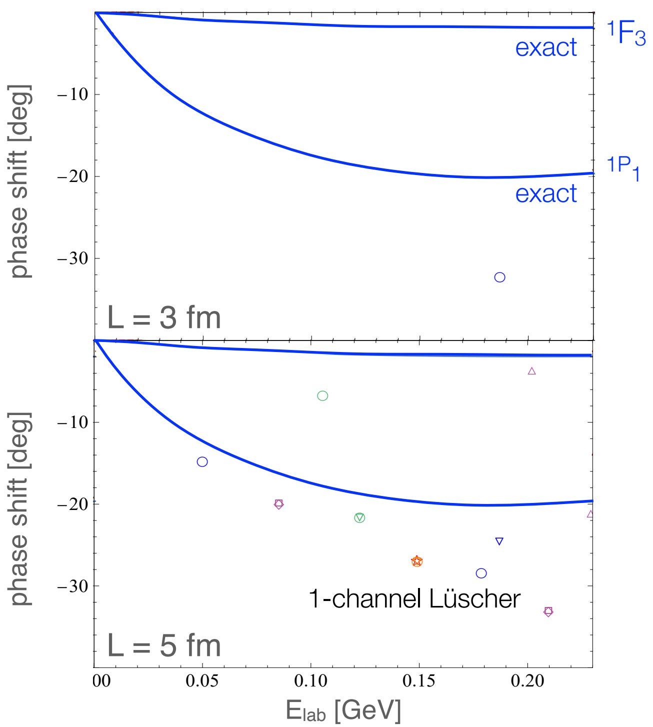
Lüscher's quantization condition is not valid below the left-hand cut

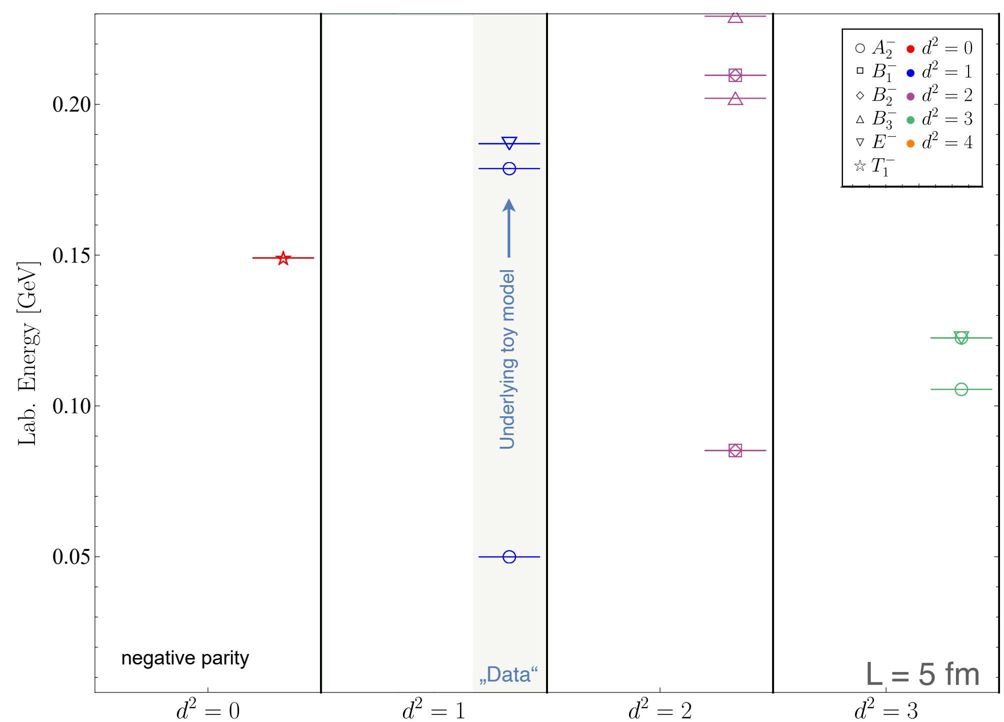
Two nucleons in a finite box (spin-0 channels)

Lu Meng, EE, JHEP 10 (2021) 051

- EFT-based Hamiltonian: 
- Solve the theory in a box using PW basis and fix the LECs from FV energies
- Extract real-world observables by solving the theory in the infinite volume

$$V_{\rm toy} = -\underbrace{\left(\frac{g_A}{2F_\pi}\right)^2 \frac{M_\pi^2}{\boldsymbol{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}_{\textit{long-range}} + \underbrace{(c_{h1}+c_{h2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{1}{\boldsymbol{q^2}+m_h^2}}_{\textit{short-range}} \quad\Rightarrow\quad V_{\rm EFT} = -\left(\frac{g_A}{2F_\pi}\right)^2 \frac{M_\pi^2}{\boldsymbol{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} + V_{\rm cont}^{(4)} + \ldots$$





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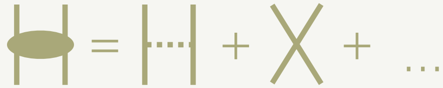
- EFT-based Hamiltonian: 
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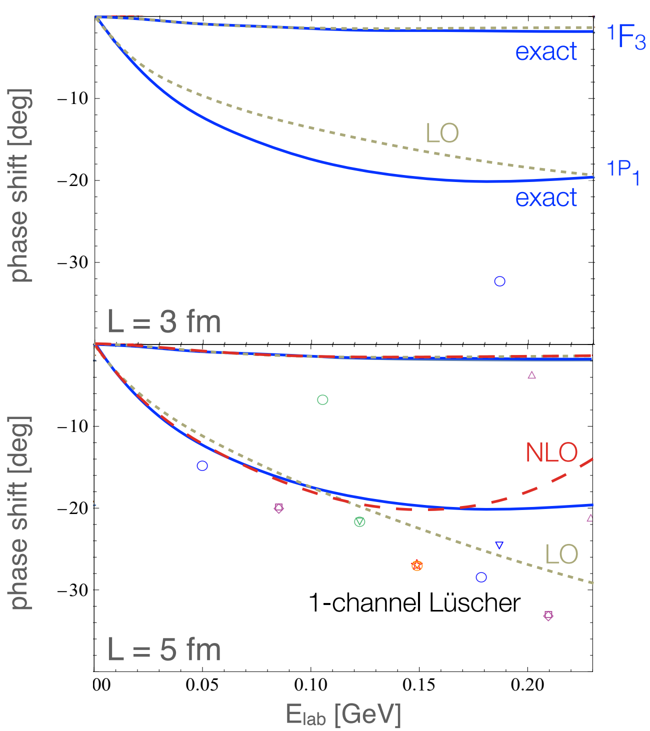
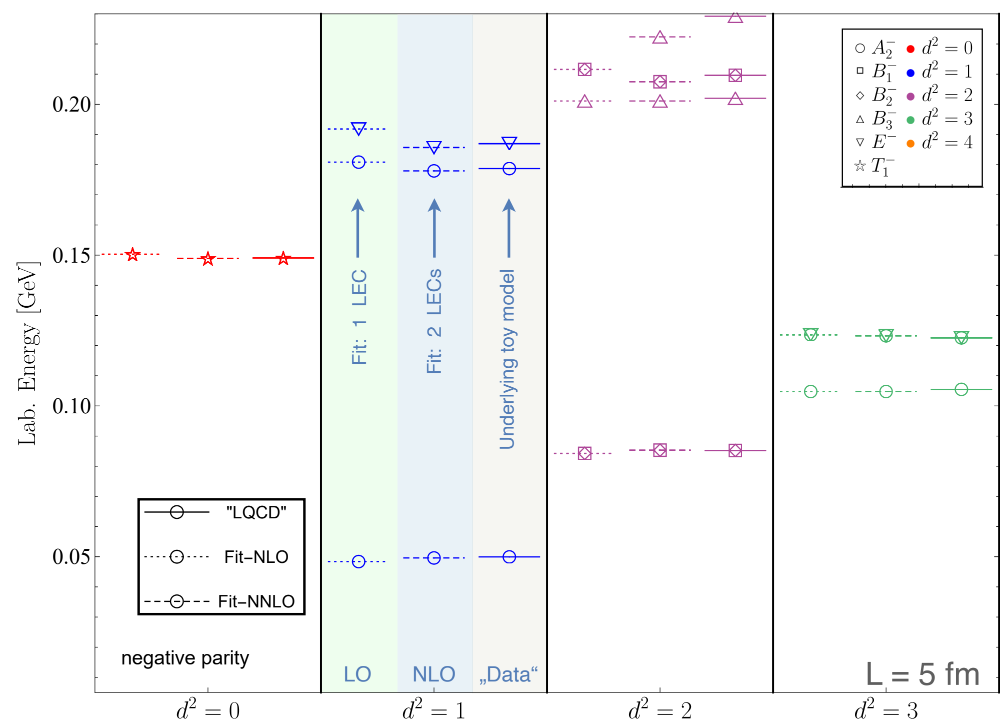
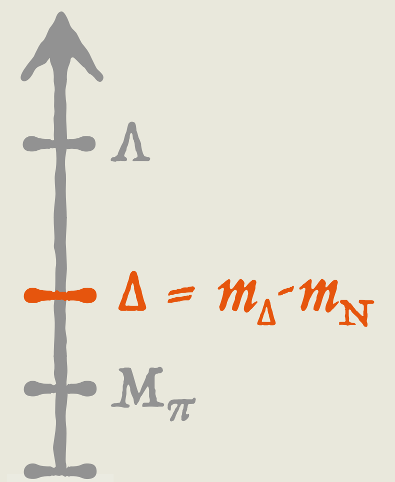
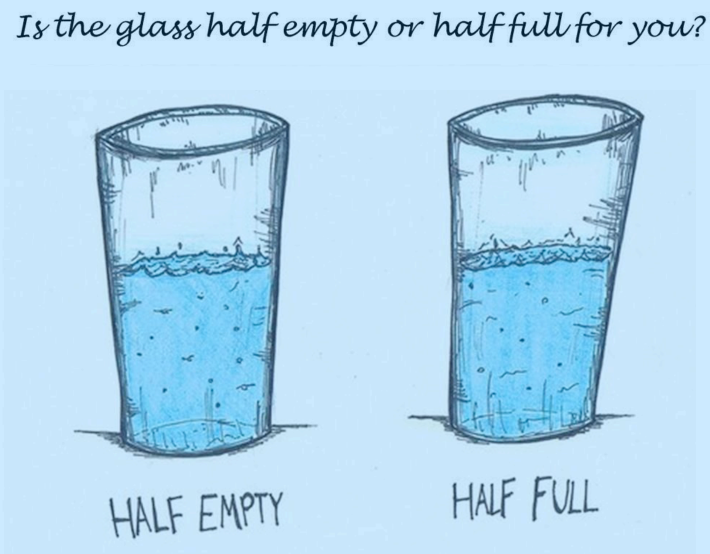




Chiral EFT and the Δ(1232) isobar





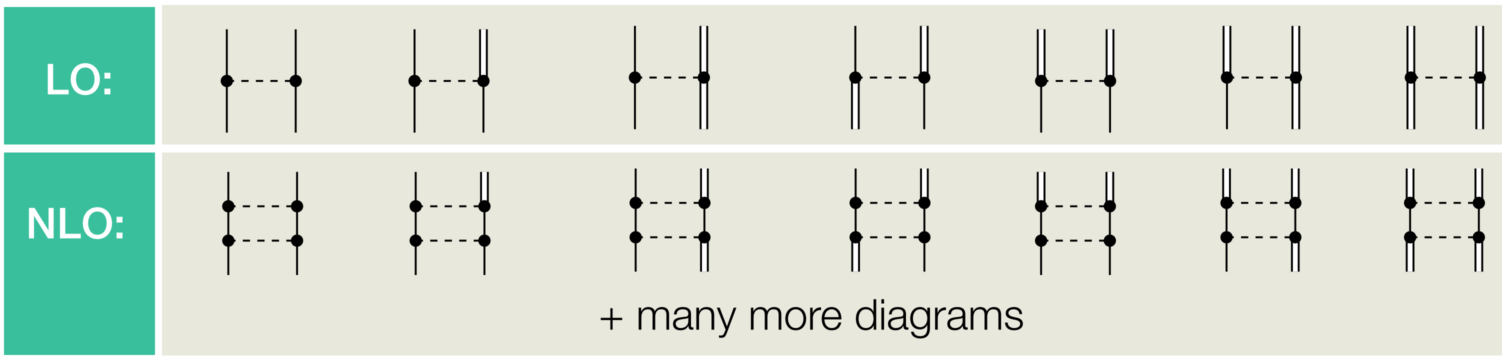
$\Delta \sim M_\pi$

$\Delta \sim \Lambda$

The NN, NΔ and ΔΔ coupled channel dynamics

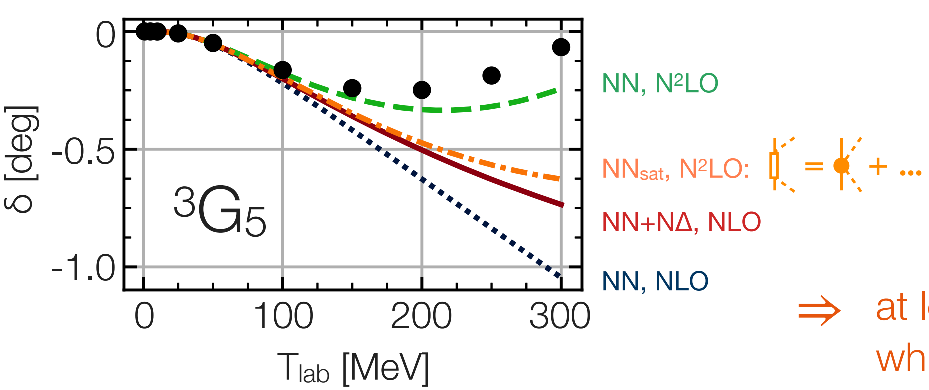
Susanne Strohmeier, Norbert Kaiser, Nucl. Phys. A1002 (2020) 121980

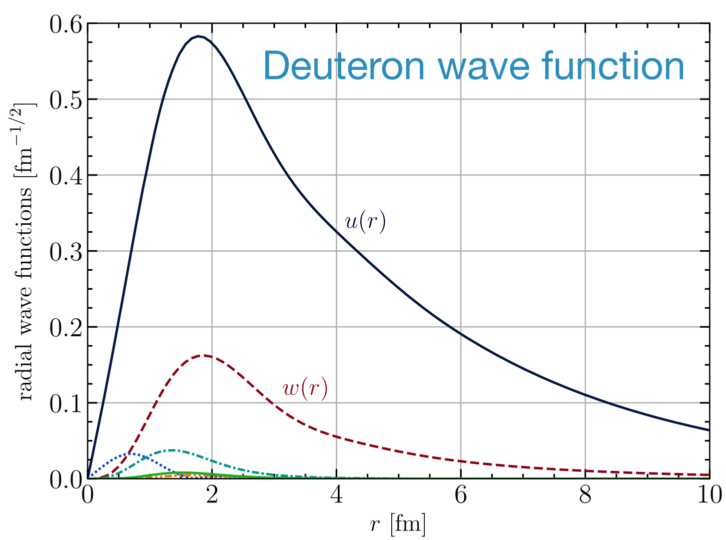
- Worked out the LO and NLO coupled-channel potentials:



- Tuned NN LECs to data (NΔ and ΔΔ contact set to 0) by solving the coupled channel Kadyshevsky equation

- In many cases, the coupled-channel NLO results lie between the purely NN NLO and N²LO ones:





⇒ at low energy, main effects are coming from $p \sim \Delta$, while contributions from momenta $p \sim \sqrt{m_N \Delta}$ (coupled-channel dynamics) are small

The small-scale (ε) expansion [Th. Hemmert et al. '98]

Alternative strategy: Re-sum $1/\Delta^n$-contributions from $p \sim \Delta$ by including Δ(1232) in $\mathscr{L}_{\rm eff}$ and counting $\Delta \sim M_\pi = \mathcal{O}(\epsilon)$ while $\sqrt{m_N \Delta} = \mathcal{O}(1)$ (no coupled channels)

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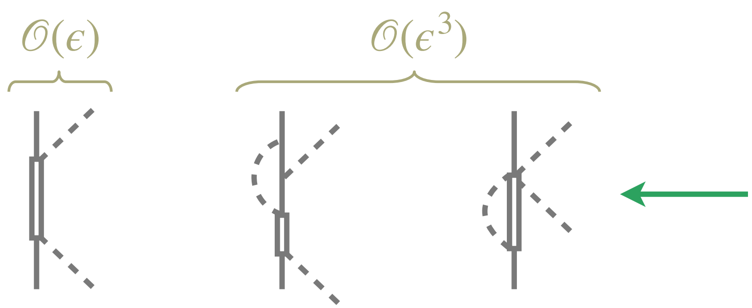
Potential concern: slow(er) convergence of χEFT due to Δ/Λ_b being twice as large as M_π/Λ_b?

The Appelquist-Carrazone decoupling Theorem: Effects of heavy particles go into local terms in an EFT, either in renormalizable or in non-renormalizable suppressed by powers of the heavy mass

Small-scale expansion:



enforce decoupling through finite subtractions

The small-scale (ε) expansion [Th. Hemmert et al. '98]

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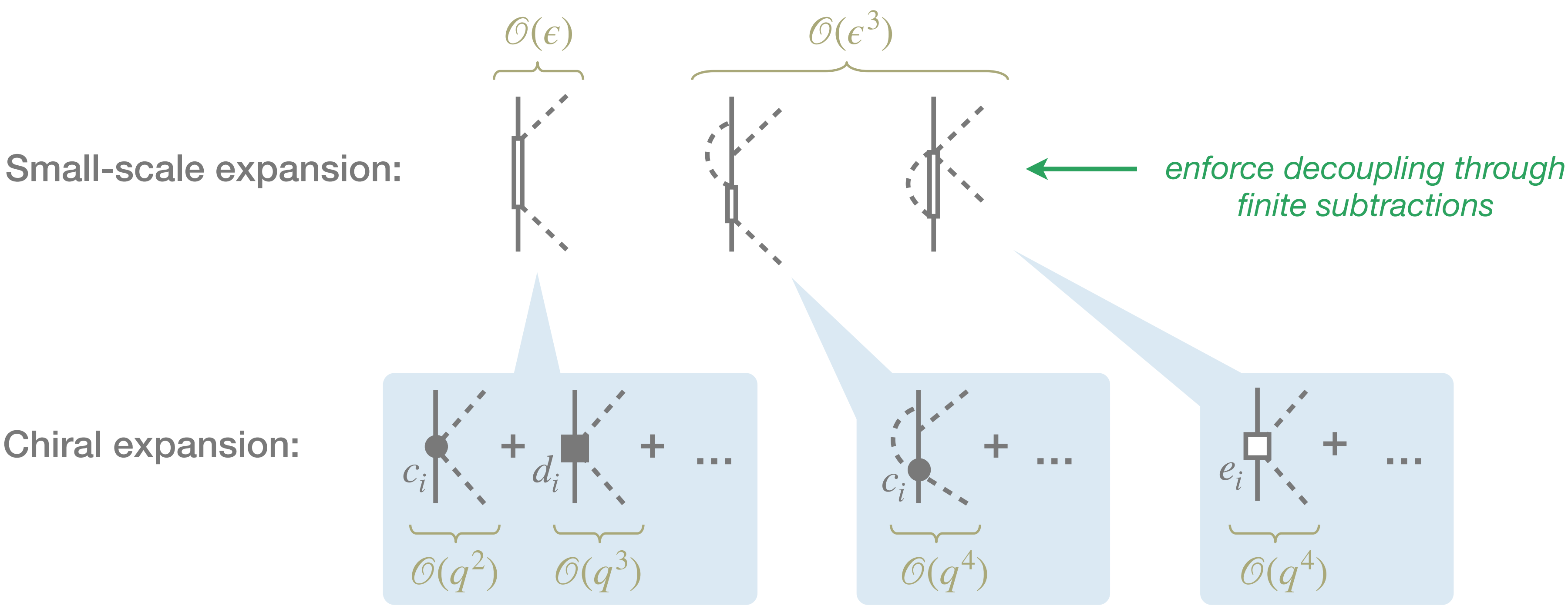
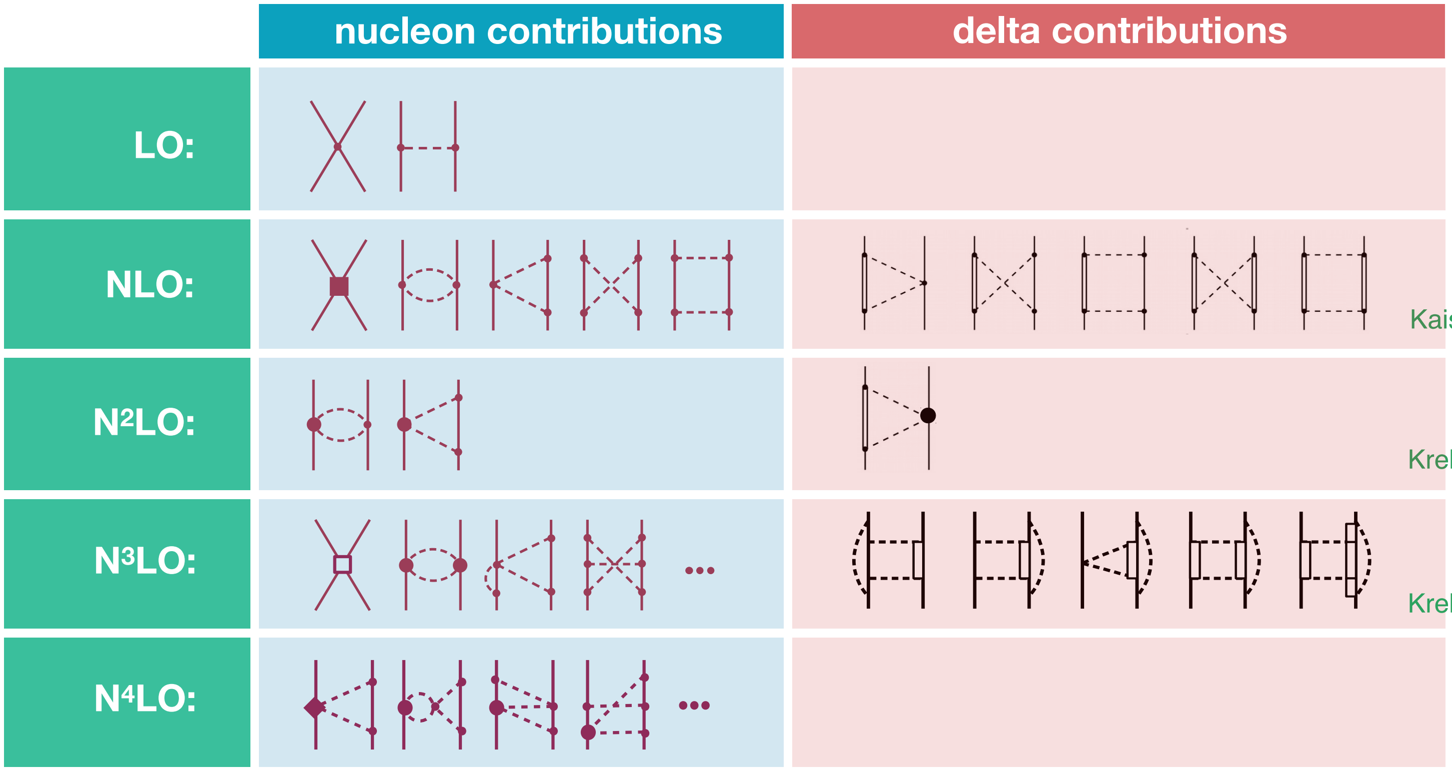


NN force in the small-scale expansion

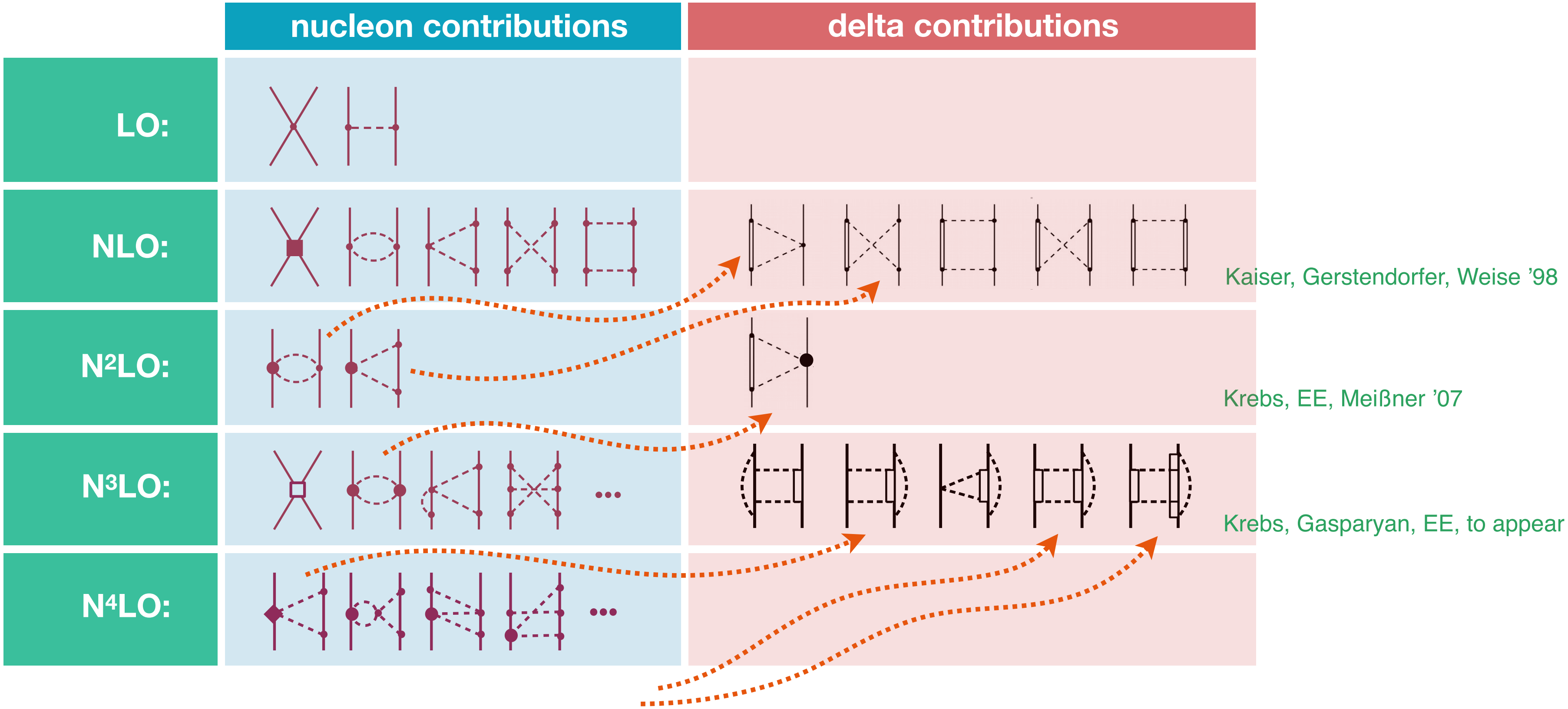


Kaiser, Gerstendorfer, Weise ’98

Krebs, EE, Meißner ’07

Krebs, Gasparyan, EE, to appear

NN force in the small-scale expansion



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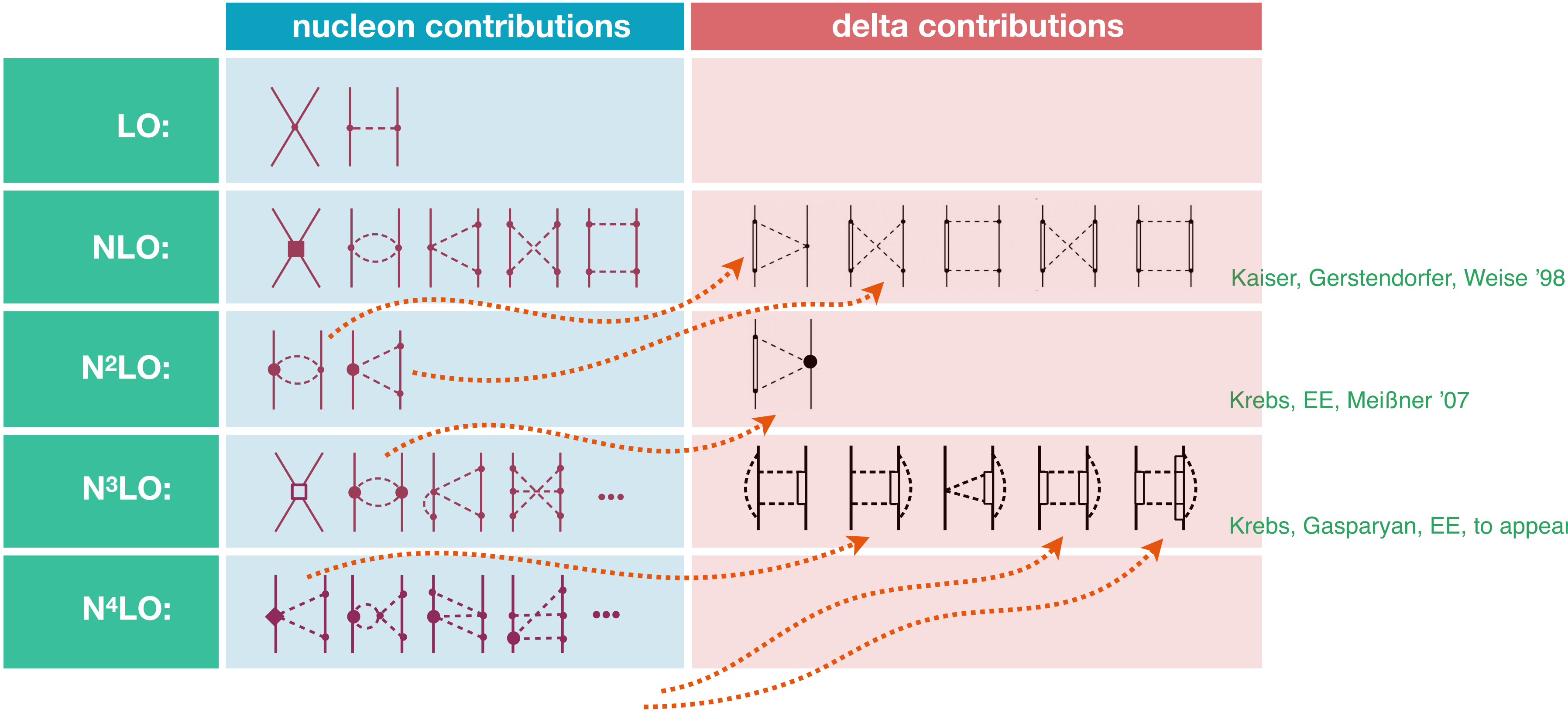
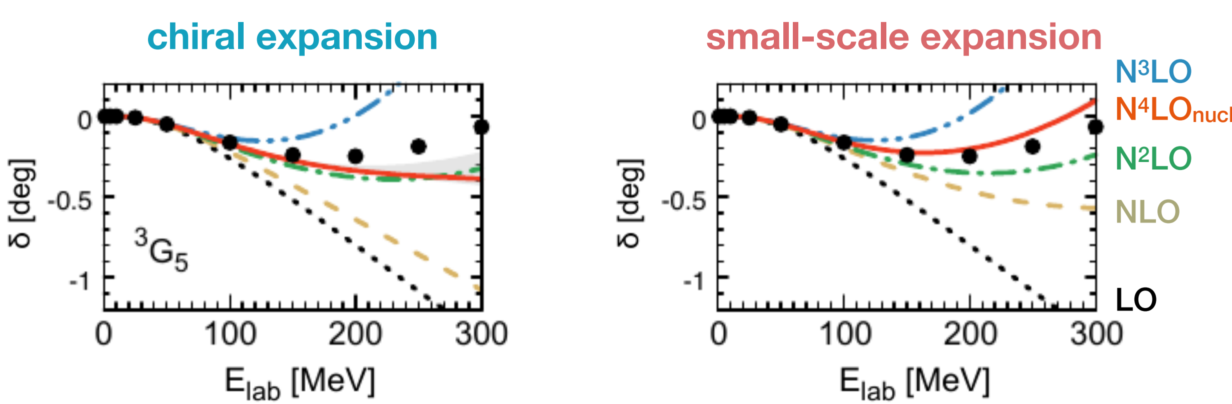
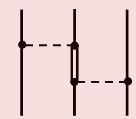
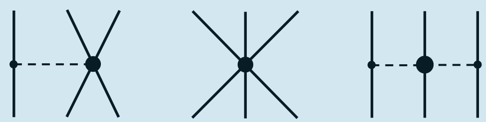
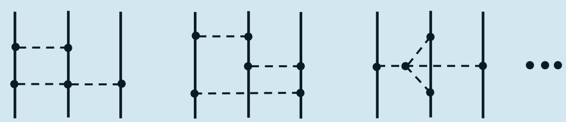
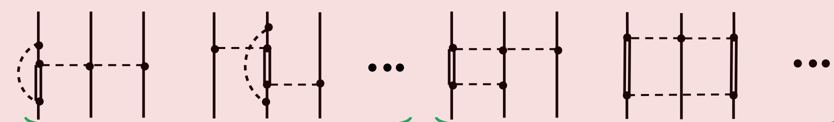
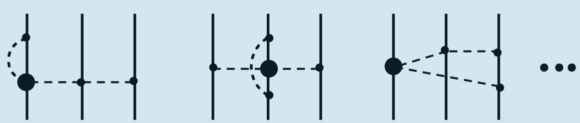
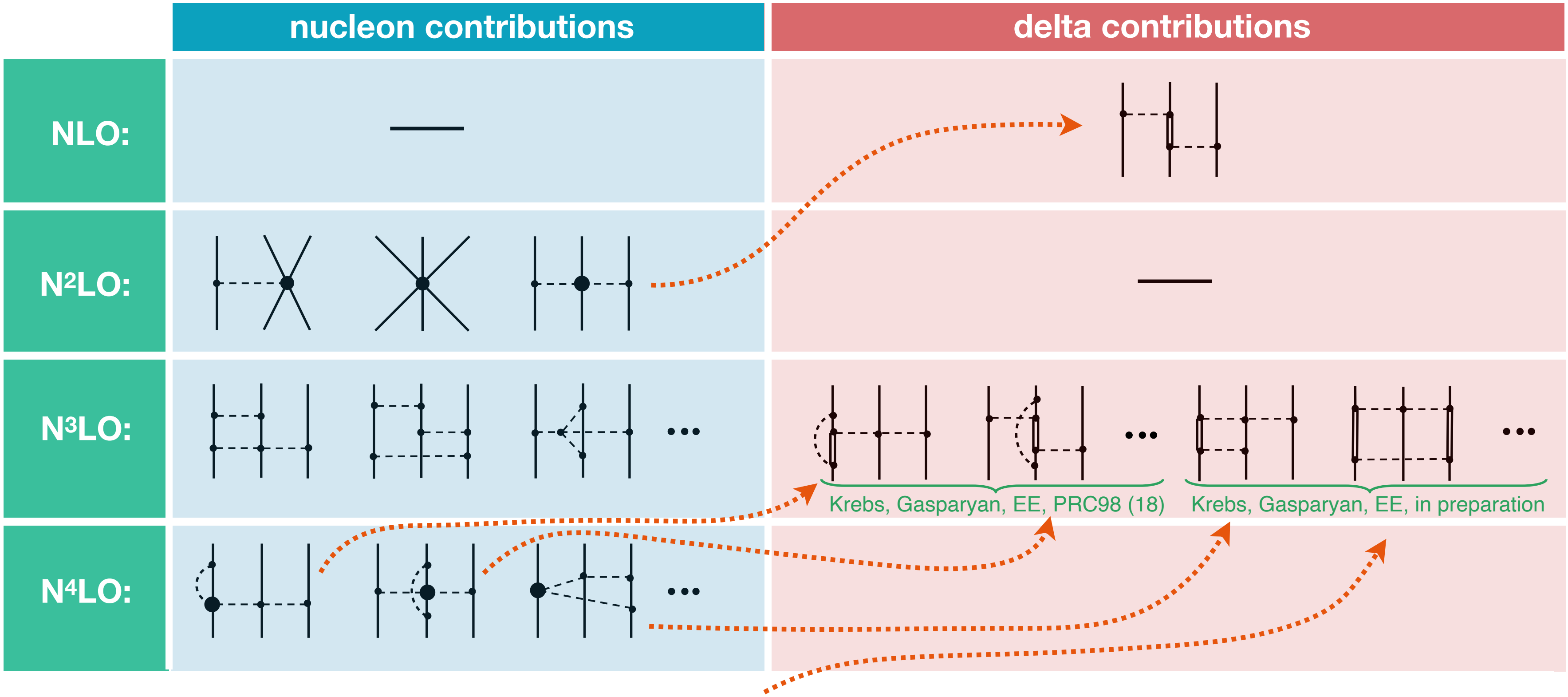




3N force in the small-scale expansion

	nucleon contributions	delta contributions
NLO:	—	
N^{2}LO:		—
N^{3}LO:		 Krebs, Gasparyan, EE, PRC98 (18) · Krebs, Gasparyan, EE, in preparation
N^{4}LO:		

3N force in the small-scale expansion



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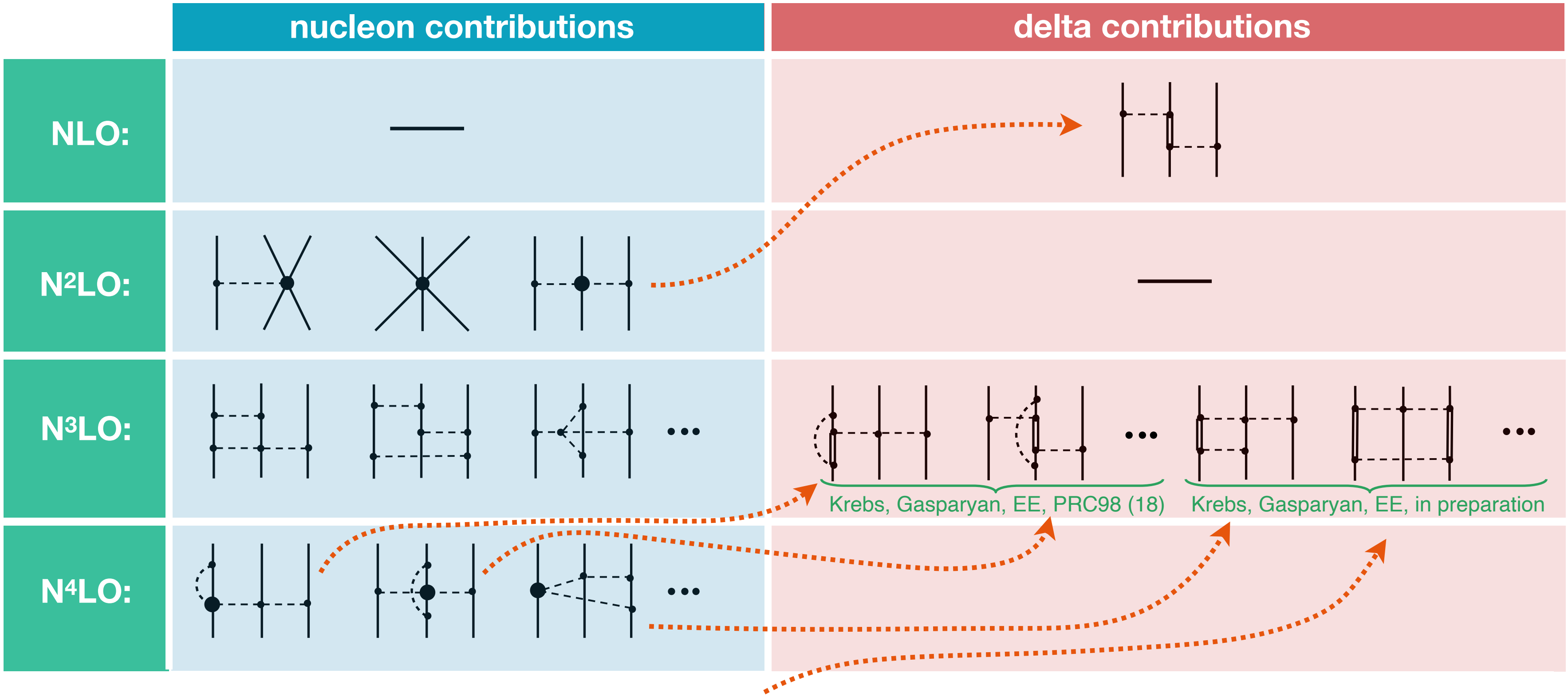
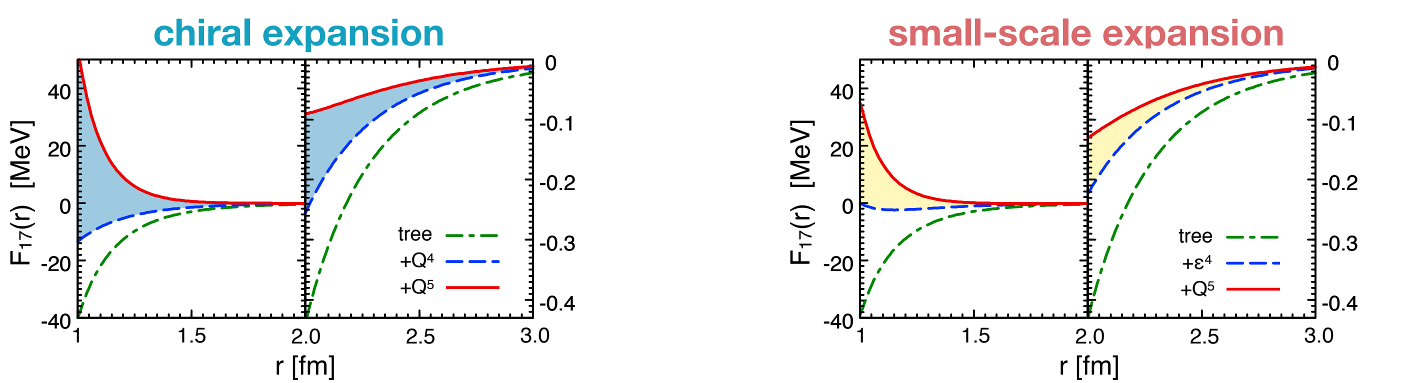




$$V_{3N} \;=\; \ldots \;+\; F_{17}(r_{12}, r_{23}, r_{31})\, \vec{\tau}_1 \cdot \vec{\tau}_3 \; \hat{r}_{23} \cdot \vec{\sigma}_1 \; \hat{r}_{12} \cdot \vec{\sigma}_3 \;+\; \ldots$$

Δ(1232)-contributions to CP-violating nuclear forces

Lukas Gandor, Hermann Krebs and EE, to appear

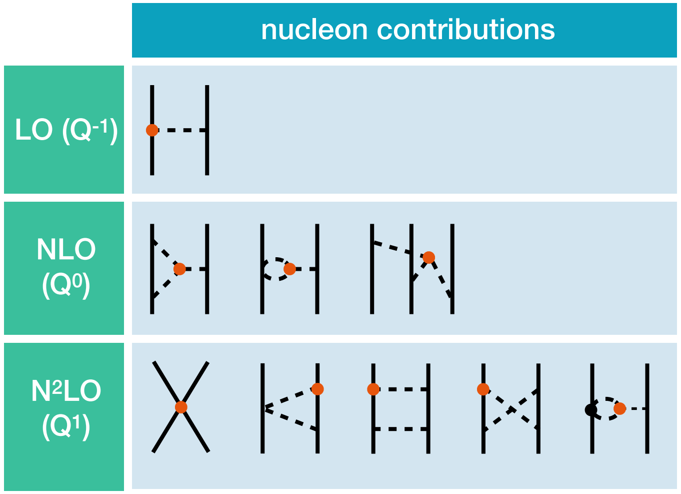
Searches for CP-violation in/beyond the SM with light nuclei (e.g., EDMs)

Bsaisou, Meißner, Nogga, Wirzba, de Vries, Gnech, Viviani, …

LO time-reversal violating vertices:

$$\mathcal{L}_{\rm TRV}^{3\pi\,(0)} = M \Delta_3 \pi_3 \pi^2$$

$$\mathcal{L}_{\rm TRV}^{\pi N\,(0)} = g_0 \overline{\psi} \vec{\pi} \cdot \vec{\tau} \psi + g_1 \overline{\psi} \pi_3 \psi + g_2 \overline{\psi} \pi_3 \tau_3 \psi$$
$$+ \; 5 \; NN \; contact \; terms$$



no sense to go beyond N²LO (more LECs…)

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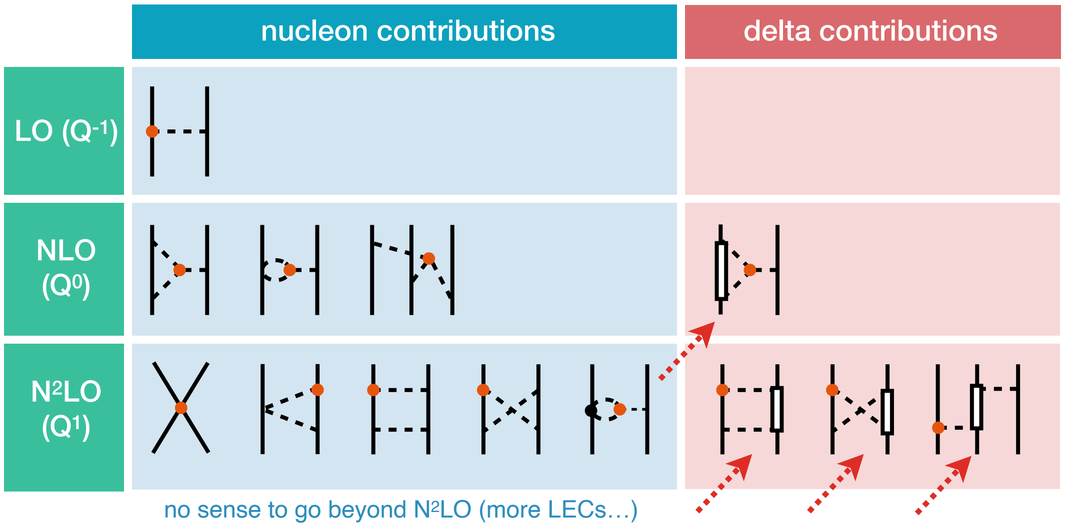


no sense to go beyond N²LO (more LECs…)

No LO CP-violating πNΔ-couplings

⇒ re-sum the 1/Δn-contributions to the PVTV nuclear forces without introducing additional parameters

Δ(1232)-contributions to CP-violating nuclear forces

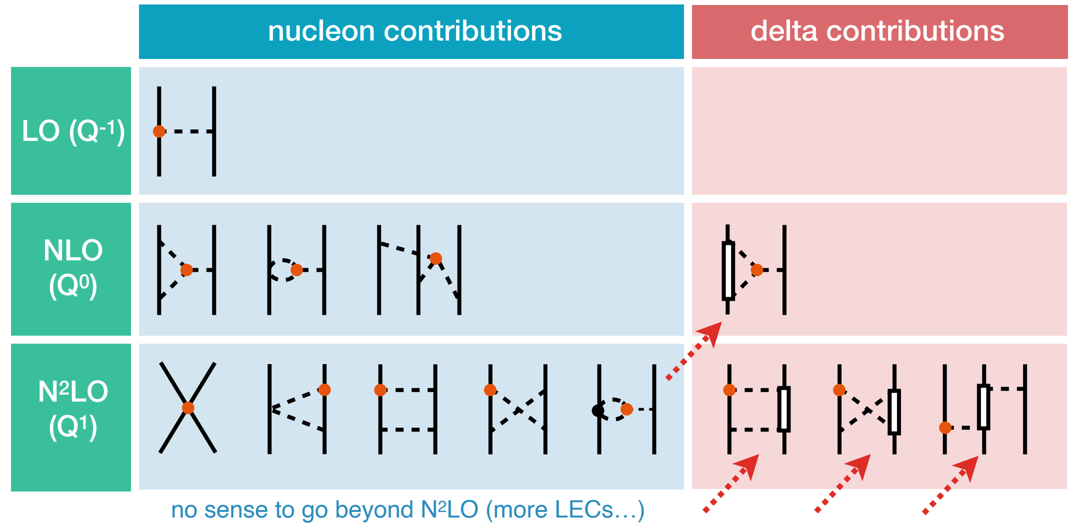
Lukas Gandor, Hermann Krebs and EE, to appear

Searches for CP-violation in/beyond the SM with light nuclei (e.g., EDMs)

Bsaisou, Meißner, Nogga, Wirzba, de Vries, Gnech, Viviani, …

LO time-reversal violating vertices:

$$\mathcal{L}^{3\pi\,(0)}_{\rm TRV} = M\Delta_3\pi_3\pi^2$$
$$\mathcal{L}^{\pi N\,(0)}_{\rm TRV} = g_0\bar{\psi}\vec{\pi}\cdot\vec{\tau}\psi + g_1\bar{\psi}\pi_3\psi + g_2\bar{\psi}\pi_3\tau_3\psi$$
$$+\ 5\ NN\ contact\ terms$$



No LO CP-violating πNΔ-couplings

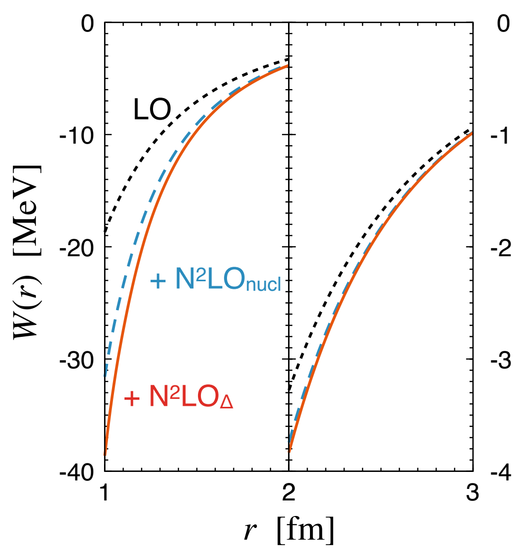
⇒ re-sum the 1/Δn-contributions to the PVTV nuclear forces without introducing additional parameters

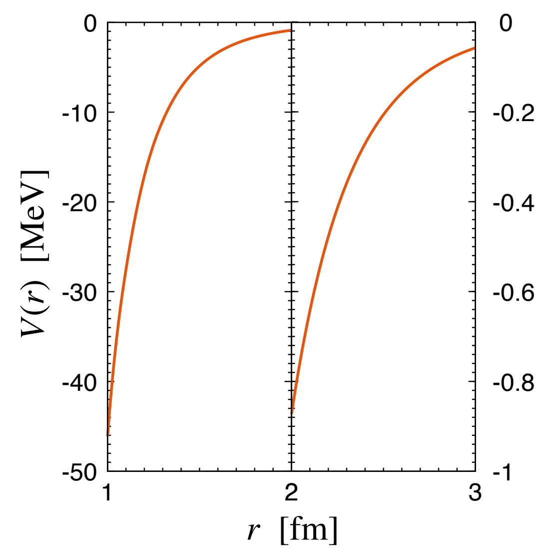
E.g., suppose the main source of CP violation is the QCD θ-term

$$\Rightarrow\quad \Delta_3, g_1, g_2 \ll g_0$$

and the long-range potential involves just two structures:

$$V(r)\ \hat{r}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)$$
$$+\ W(r)\ \vec{\tau}_1\cdot\vec{\tau}_2\ \hat{r}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)$$





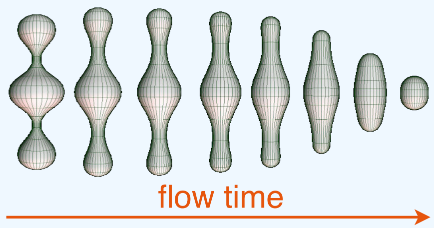
Gradient flow for chiral EFT

Hermann Krebs, EE, e-Print: 2311.10893, 2312.13932

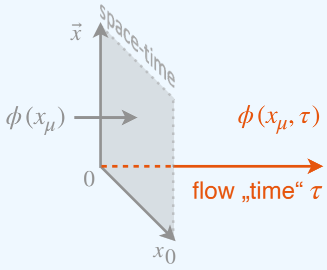
A rigorous approach to regularize nuclear interactions and currents
in harmony with the chiral and gauge symmetries

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $$\frac{\partial}{\partial \tau}\phi(x,\tau) = -\left.\frac{\delta S[\phi]}{\delta \phi(x)}\right|_{\phi(x)\to\phi(x,\tau)}$$

subject to the boundary condition $\phi(x,0) = \phi(x)$

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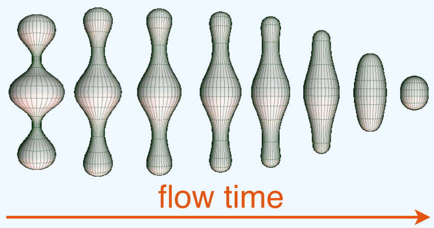

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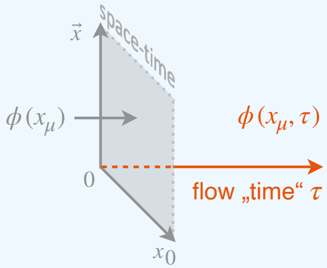

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Free scalar field:

$$G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}$$

$$\left[\partial_\tau - (\partial^x_\mu\partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \underbrace{\int d^4y\, \overbrace{G(x-y,\tau)}\phi(y)}_{\text{heat kernel}} \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$

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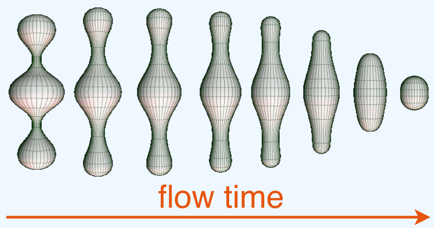

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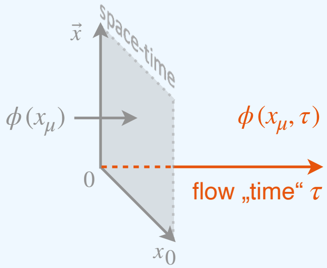

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YM gradient flow Narayanan, Neuberger ’06, Lüscher, Weisz ’11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ ← extensively used in LQCD

Gradient flow

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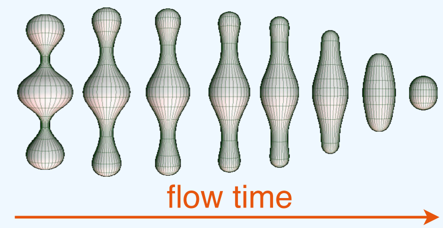

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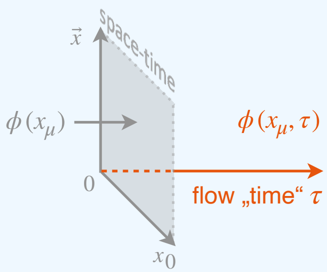

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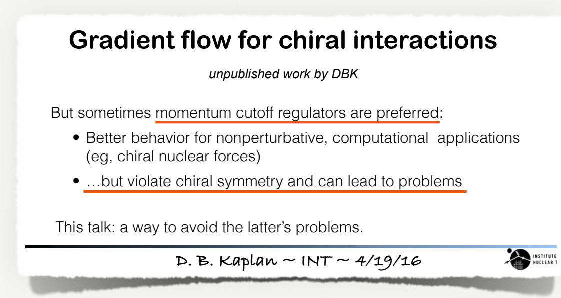
Chiral gradient flow Krebs, EE, 2312.13932

Start with $U(\boldsymbol{\pi}(x)) \in \mathrm{SU}(2) \ \to \ RU(x)L^\dagger$

Generalize $U(x)$ to $W(x,\tau)$: $\partial_\tau W = -i\underbrace{w}_{\sqrt{W}}\overbrace{\mathrm{EOM}(\tau)}^{[D_\mu, w_\mu] + \frac{i}{2}\chi_-(\tau) - \frac{i}{4}\mathrm{Tr}\,\chi_-(\tau)}\, w, \quad W(x,0) = U(x)$

We have proven $\forall\tau$: $W(x,\tau) \in \mathrm{SU}(2), \ \ W(x,\tau) \ \to \ RW(x,\tau)L^\dagger$



Gradient flow for chiral interactions

unpublished work by DBK

But sometimes momentum cutoff regulators are preferred:

- Better behavior for nonperturbative, computational applications (eg, chiral nuclear forces)
- ...but violate chiral symmetry and can lead to problems

This talk: a way to avoid the latter’s problems.

D. B. Kaplan ~ INT ~ 4/19/16

Gradient flow regularization

Solving the chiral gradient flow equation $\partial_\tau W = -iw\,\mathrm{EOM}(\tau)\,w$

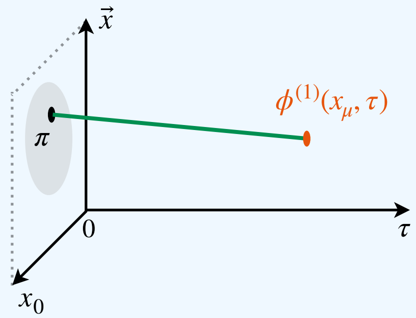
— most general parametrization of $\boldsymbol{U}$: $U = 1 + \frac{i}{F}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\left(1 - \alpha\frac{\boldsymbol{\pi}^2}{F^2}\right) + \mathcal{O}(\boldsymbol{\pi}^4)$

— similarly, write $W = 1 + i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\left(1 - \alpha\,\boldsymbol{\phi}^2\right) - \mathcal{O}(\boldsymbol{\phi}^4)$ and make an ansatz $\boldsymbol{\phi} = \sum_{n=0}^{\infty}\frac{\boldsymbol{\phi}^{(n)}}{F^n}$

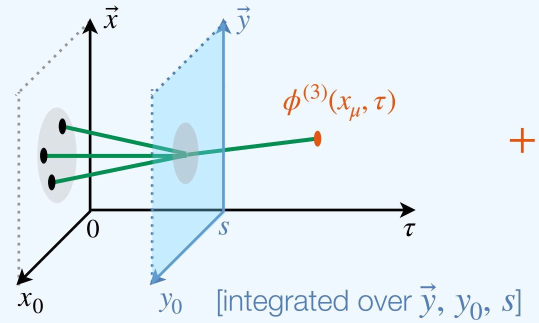
$\Rightarrow$ recursive (perturbative) solution of the GF equation in $1/F$

Gradient flow regularization

$\phi(x_\mu, \tau) \quad =$

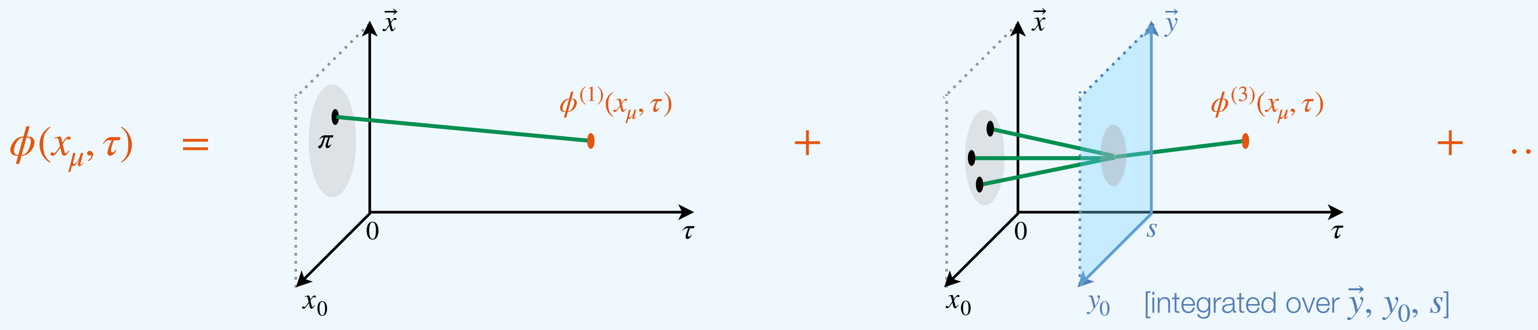


$+$

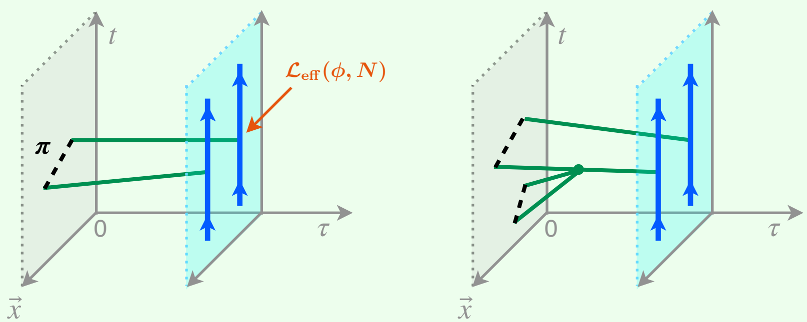


$+ \quad \cdots$

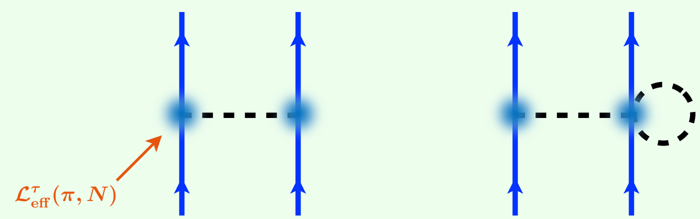
Gradient flow regularization



Local field theory in 5d

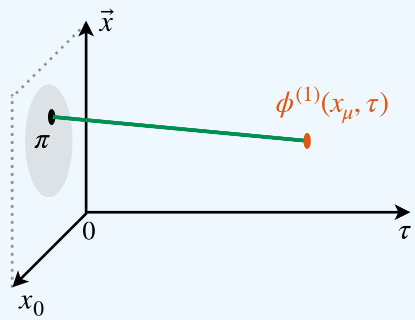


Smeared (non-local) theory in 4d



Gradient flow regularization

$\phi(x_\mu, \tau) \;=$



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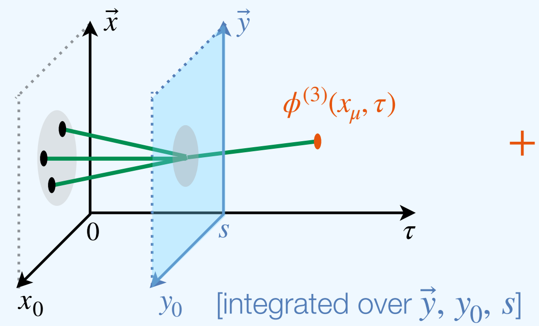
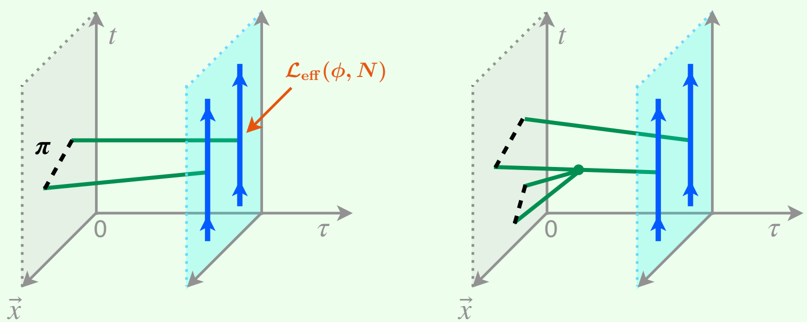


$+ \; \ldots$

Local field theory in 5d



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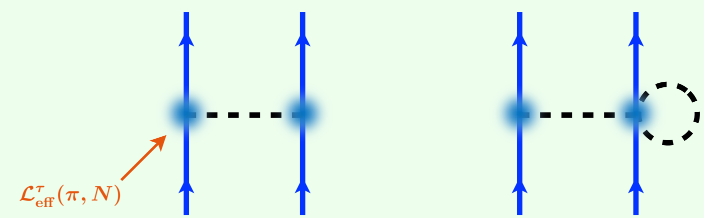


We now have the regularized Lagrangian, but cannot use the canonical-quantization-based UT method to derive nuclear forces ($\partial_0^n \pi$ with arbitrary n…). Path Integral approach [Krebs, EE, e-Print: 2311.10893]:

$$Z[\eta^\dagger, \eta] \;=\; A \int \mathscr{D}N^\dagger \, \mathscr{D}N \, \mathscr{D}\pi \, \exp\left(i S^\Lambda_{\rm eff} + i \int d^4x [\eta^\dagger N + N^\dagger \eta] \right)$$

nonlocal redefinitions of $N, N^\dagger$ → (instantaneous)

$$A \int \mathscr{D}\tilde{N}^\dagger \, \mathscr{D}\tilde{N} \exp\left(i S^\Lambda_{\rm eff,\,N} + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta] \right)$$

The CRC 110 has significantly contributed towards developing nuclear chiral EFT into a precision tool

The horizon is always three miles away

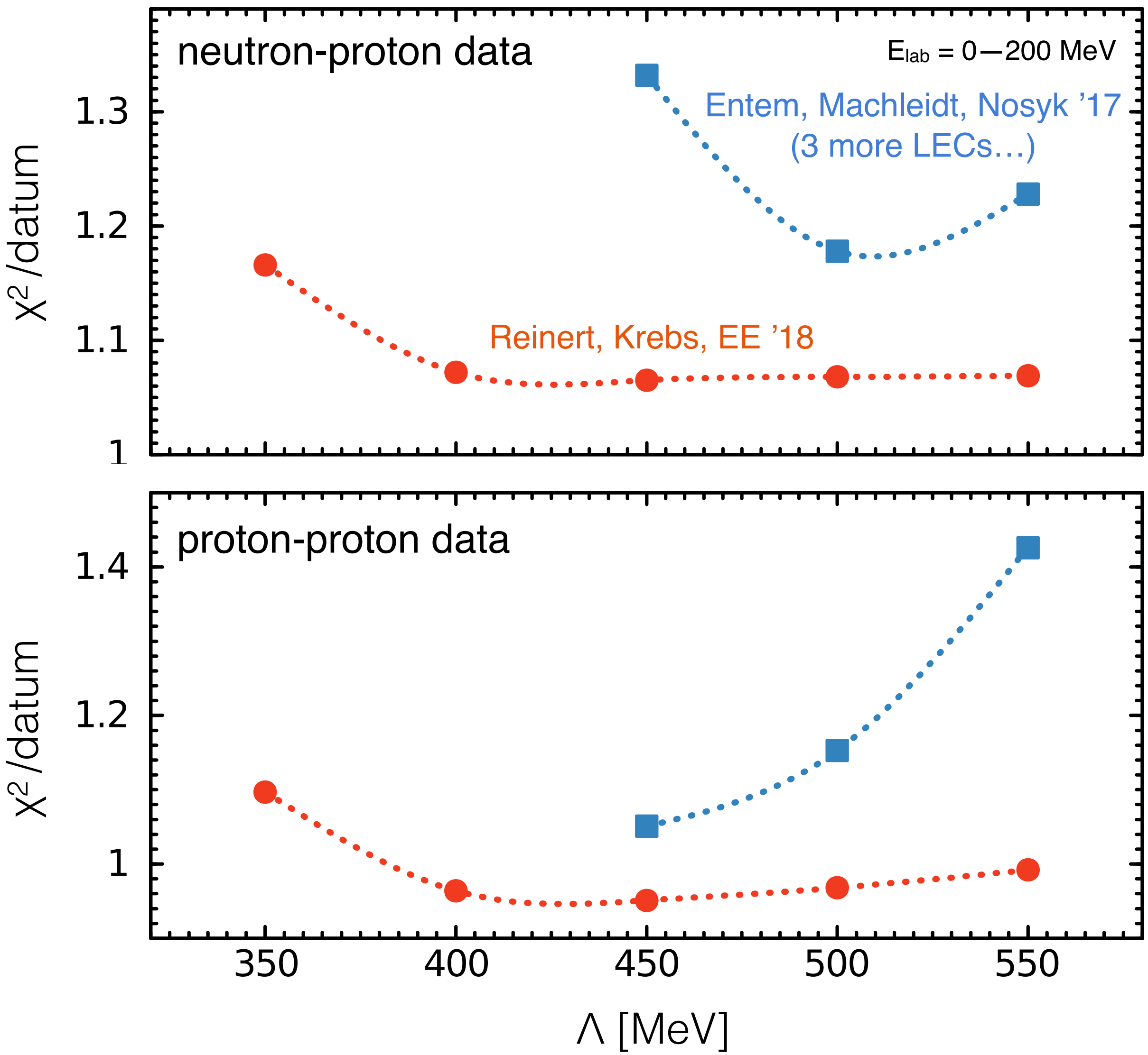
…but is becoming increasingly brighter

Thank you for your attention

Spares

Regulator (in)dependence

χ^2/datum for the description of the NN data in the range of 0 – 200 MeV at N^4LO^+



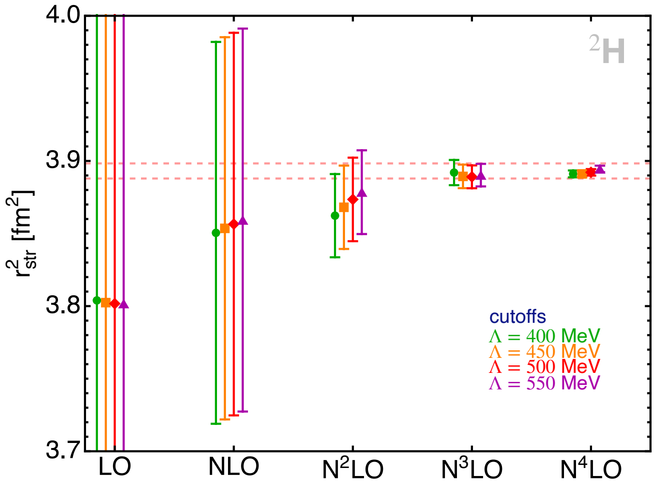
Charge radius and quadrupole moment

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Charge radius and quadrupole moment

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Consistency check
(residual cutoff dependence):



Charge radius and quadrupole moment

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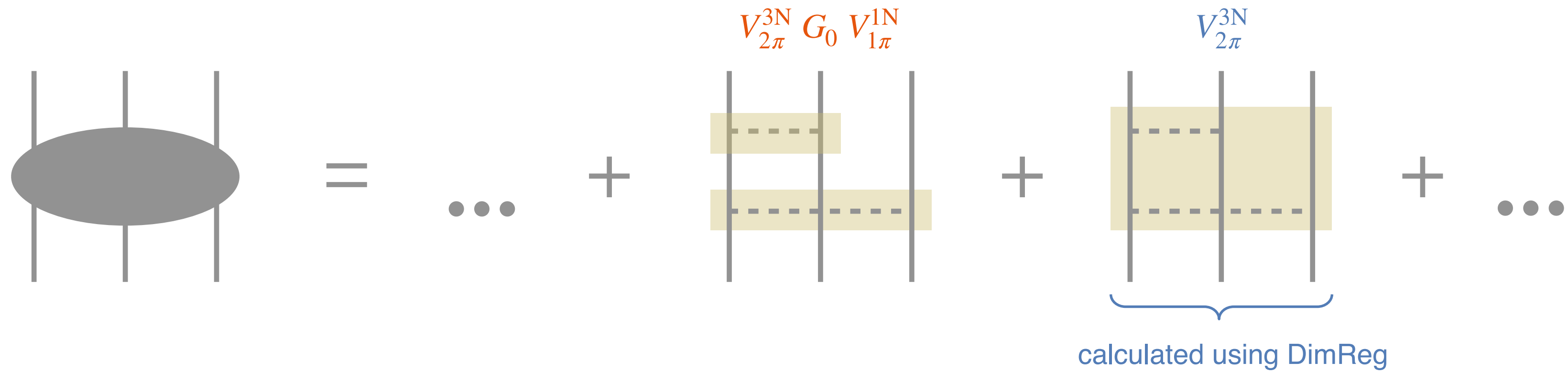
Regularization and symmetry

Nuclear potentials are derived using dim. reg. and supplied with an additional cutoff prior to solving the Schrödinger equation. Consistent?

Faddeev equation for 3N scattering:



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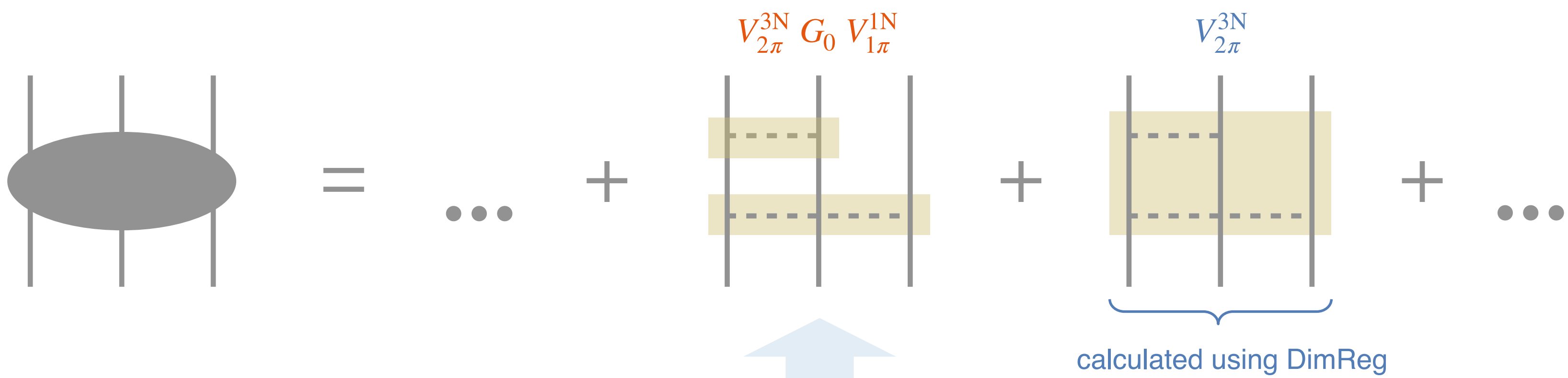

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\Big[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\Big]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ldots$$

absorbable into c_D: violates chiral symmetry

Regularization and symmetry

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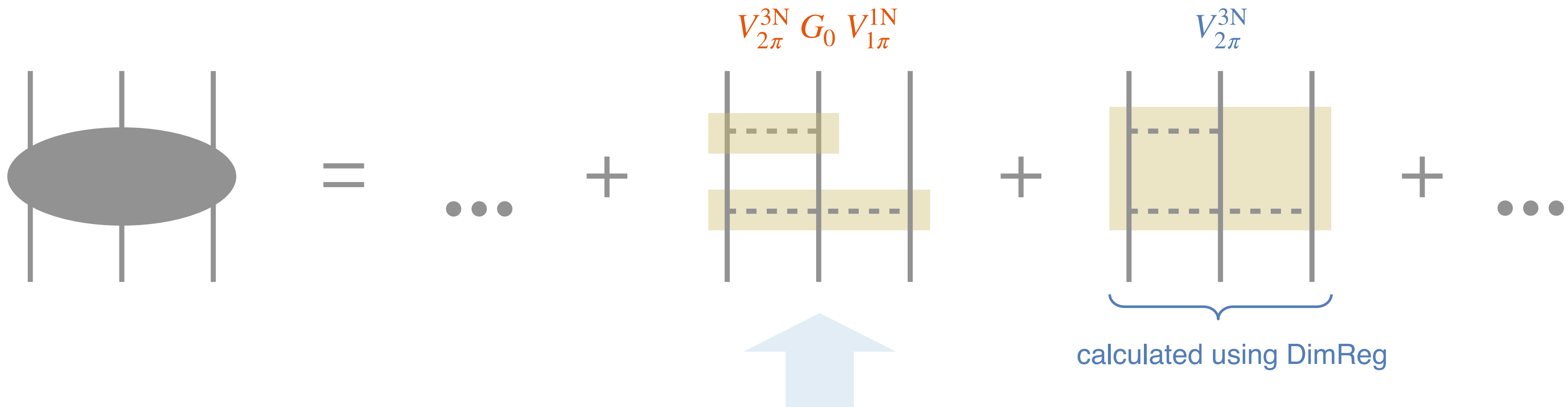

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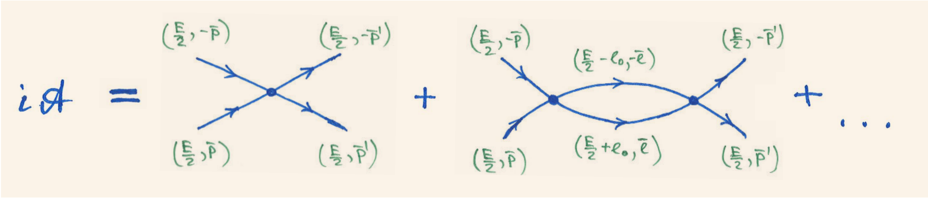
If $V_{2\pi}^{3N}$ were calculated with a cutoff, the problematic divergence would cancel exactly. This issue affects all loop contributions beyond N²LO to 3NF and exchange currents. In contrast, NN forces are not affected (at a fixed M_π).

Warm-up exercise

Pion-less EFT:

$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2}(N^\dagger N)^2 \; + \ldots$$

$$\Rightarrow \quad \mathcal{A}_{\rm tree} = \left[C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots \right]$$



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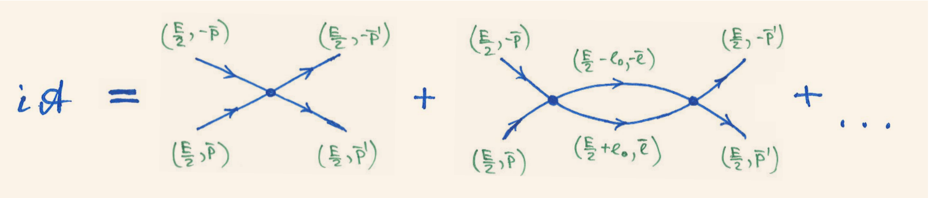


$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2}(N^\dagger N)^2 + \ldots$$

$$\Rightarrow \quad \mathcal{A}_{\text{tree}} = \left[C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots \right]$$

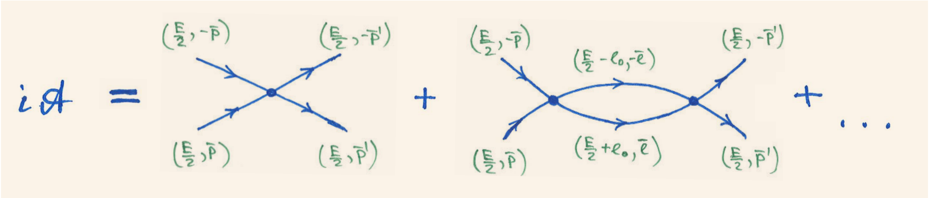
Scattering amplitude to 1 loop:

$$\begin{aligned}
-i\mathcal{A}_{1-\text{loop}} &= \int \frac{d^4 l}{(2\pi)^4} \left[C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots \right] \frac{1}{\left(\frac{E}{2} + l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)\left(\frac{E}{2} - l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)} \left[C_0 + \ldots \right] \\
&= -i \int \frac{d^3 l}{(2\pi)^3} \left[C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots \right] \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} \left[C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots \right]
\end{aligned}$$

All l_0-integrals factorize $\Rightarrow$ Lippmann-Schwinger eq. $\mathcal{A} = \mathcal{V} + \mathcal{V}\, G_0\, \mathcal{A}$ with $\mathcal{V} = -\mathcal{L}_{\text{int}}$

Warm-up exercise

Pion-less EFT:



$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2} (N^\dagger N)^2 + \ldots$$

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$$= -i \int \frac{d^3 l}{(2\pi)^3} [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} [C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots]$$

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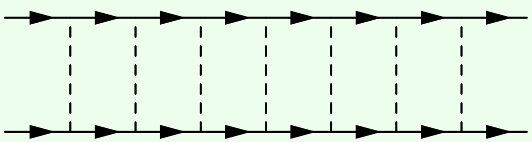
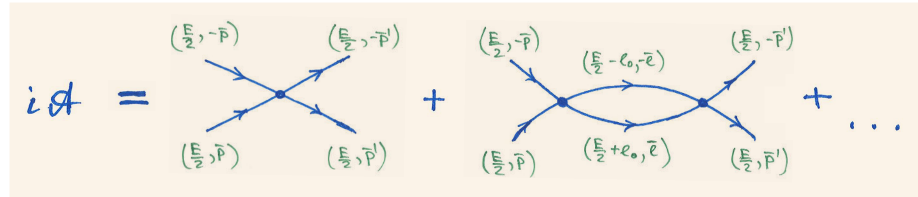


But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

Warm-up exercise

Pion-less EFT:



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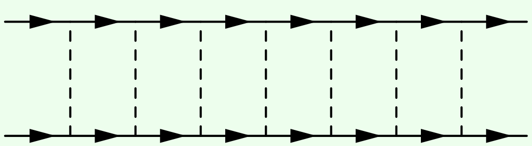

But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

Idea: $Z[\eta^\dagger, \eta] = A \int \mathcal{D}N^\dagger\, \mathcal{D}N\, \mathcal{D}\pi \exp\left(iS_{\text{eff}}^{\Lambda} + i \int d^4x [\eta^\dagger N + N^\dagger \eta] \right)$

nonlocal redefinitions of $N, N^\dagger$ $\longrightarrow$ $A \int \mathcal{D}\tilde{N}^\dagger\, \mathcal{D}\tilde{N} \exp\left(iS_{\text{eff, N}}^{\Lambda} + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta] \right)$

instantaneous ($\to S_{\text{eff, N}}^{\Lambda}$)