

Collaborative Research Center CRC 110
“Symmetries and the emergence of structure in QCD”



Two CRC periods of project B.11 (FP 2 and 3)
“Coupled-channel dynamics”

CRC 110 Final Meeting, Bonn, June 2024

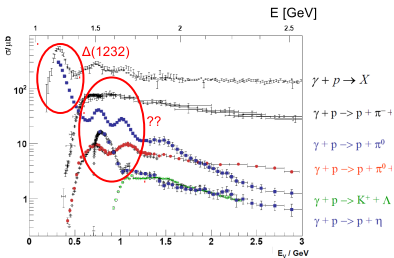
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In collaboration with: M. Döring, J. Hergenrather, M. Mai, T. Mart, Ulf-G. Meißner, S. Rawat, C. Schneider, C.-W. Shen, Y.-F. Wang, Z.-L. Wang, R. Workman |

The excited baryon spectrum:

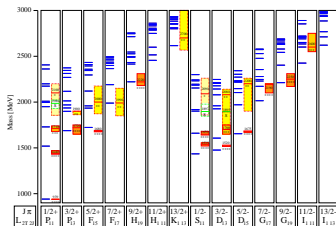
Connection between experiment and QCD in the non-perturbative regime

Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

Theoretical predictions of excited hadrons
e.g. from relativistic quark models:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Goals:

- Extraction of N^* , Δ^* and Y^* resonances in pion-, kaon-, photon- and electron induced reactions using a dynamical coupled-channel (DCC) approach
- Analysis of states in the hidden-charm (-beauty) sector, P_c states

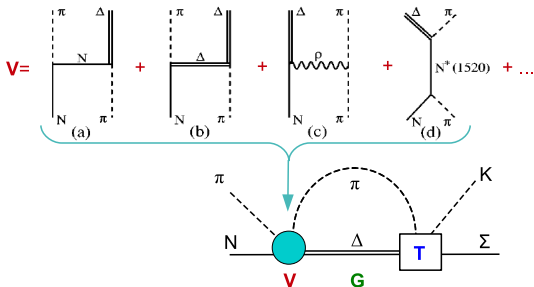
Methods:

- Jülich-Bonn dynamical coupled-channels model ("JüBo DCC")

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p' | T_{\mu\nu}^{IJ} | LS p \rangle = \langle L'S'p' | V_{\mu\nu}^{IJ} | LS p \rangle + \sum_{\gamma, L''S''} \int_0^\infty dq \, q^2 \langle L'S'p' | V_{\mu\gamma}^{IJ} | L''S'' q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S'' q | T_{\gamma\nu}^{IJ} | LS p \rangle$$



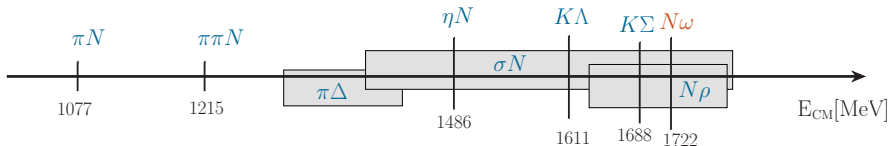
- theoretical constraints on the S matrix: **unitarity** and **analyticity**
- resonances = poles on the 2nd Riemann sheet of T
- hadronic reactions: potentials V constructed from effective \mathcal{L}
- s -channel diagrams: T^P genuine resonance states
- t - and u -channel: T^{NP} (dynamical generation of poles)

Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p' | T_{\mu\nu}^{IJ} | LSp \rangle = \langle L'S'p' | V_{\mu\nu}^{IJ} | LSp \rangle + \sum_{\gamma, L''S''} \int_0^\infty dq \, q^2 \langle L'S'p' | V_{\mu\gamma}^{IJ} | L''S''q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q | T_{\gamma\nu}^{IJ} | LSp \rangle$$

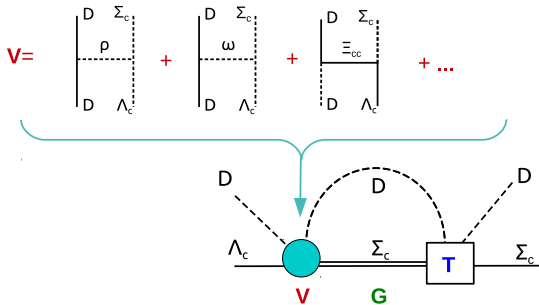
■ hadronic channels ν, μ, γ :



Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p' | T_{\mu\nu}^{IJ} | LSp \rangle = \langle L'S'p' | V_{\mu\nu}^{IJ} | LSp \rangle + \sum_{\gamma, L''S''} \int_0^\infty dq q^2 \langle L'S'p' | V_{\mu\gamma}^{IJ} | L''S''q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q | T_{\gamma\nu}^{IJ} | LSp \rangle$$

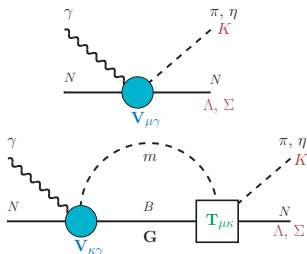


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Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p' | T_{\mu\nu}^I | LSp \rangle = \langle L'S'p' | V_{\mu\nu}^I | LSp \rangle + \sum_{\gamma, L''S''} \int_0^\infty dq \, q^2 \langle L'S'p' | V_{\mu\gamma}^I | L''S''q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q | T_{\gamma\nu}^I | LSp \rangle$$



- theoretical constraints on the S matrix: **unitarity** and **analyticity**
- resonances = poles on the 2nd Riemann sheet of T
- photon/electron reactions: potentials V energy-dependent polynomials
- $T_{\mu\kappa}$: full hadronic T -matrix
→ simultaneous fit of pion- and photon (electron) induced reactions

Progress during the CRC 110 FPs 2 and 3

- Inclusion of the $\pi N \rightarrow \omega N$ channel (Yu-Fei Wang *et al.* PRD 106 (2022))
(ωN channel not yet included in photoproduction fits)

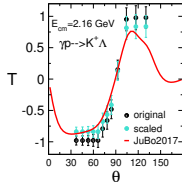
Extension of the JüBo model to $K\Lambda$ photoproduction on the proton (EPJA 54 (2018) 110):

- Simultaneous analysis of $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ and
 $\gamma p \rightarrow \pi N, \eta N, K\Lambda$
- determination of the N^* and Δ^* states, e.g. confirmation of the $N(1900)3/2^+$

Λ decay parameter α_- :

- BESIII measurement ($e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$):
 $\alpha_- = 0.750 \pm 0.009 \pm 0.004$ (Ablikim, Nature (2019))
- independent estimation from $\gamma p \rightarrow K^+\Lambda$ CLAS data using Fierz identities
 $\Rightarrow \alpha_- = 0.721 \pm 0.006 \pm 0.005$ (Ireland *et al.* PRL 123 (2019)) (PDG average until 2019: $\alpha_- = 0.642 \pm 0.013$)

- Analysis of new CLAS eta photoproduction data, P. Collins *et al.* PLB 771, 213 (2017)



data: Paterson (CLAS) PRC 93, 065201
(2016)

Extension to $K\Sigma$ photoproduction on the proton

JüBo2022 Eur.Phys.J.A 58 (2022) 229

Simultaneous analysis of $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ and $\gamma p \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$

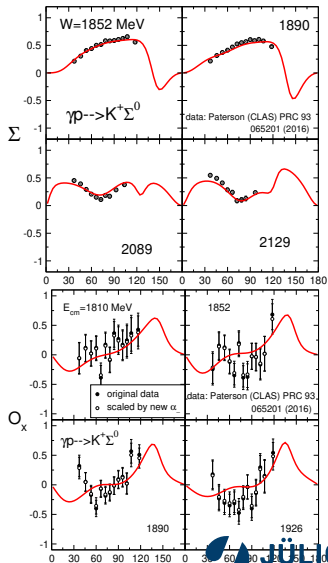
- almost 72,000 data points in total, $W_{\max} = 2.4$ GeV
 - $\gamma p \rightarrow K^+\Sigma^0$: $d\sigma/d\Omega, P, \Sigma, T, C_{x',z'}, O_{x,z} = 5,652$
 - $\gamma p \rightarrow K^0\Sigma^+$: $d\sigma/d\Omega, P = 448$
- polarizations scaled by new Λ decay constant α_- (Ireland PRL 123 (2019), 182301), if applicable
- χ^2 minimization with MINUIT on JURECA [Jülich Supercomputing Centre, JURECA: JLSRF 2, A62 (2016)]

Resonance analysis:

- all PDG 4-star N and Δ states up to $J = 9/2$ are seen (exception: $N(1895)1/2^-$) + some states rated less than 4 stars
- no additional s -channel diagram, but indications for new dyn. gen. poles

New data on $\gamma p \rightarrow K^0\Sigma^+$ by CLAS L. Clark et al. 2404.19404 more (double) polarization observables \rightarrow JüBo2023-1 fit

Selected fit results



Elementary or composite state?

Weinberg's criterion (deuteron) for **elementariness** Z

- from scattering length a and effective range r :

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L) \quad r = -\frac{Z}{1-Z}R + \mathcal{O}(L) ;$$

$R = (2\mu B)^{-1/2} \simeq 4.3$ fm deuteron radius, $L = 1/M_\pi \simeq 1.4$ fm interaction range,

- experiment: $a = -5.41$ fm , $r = 1.75$ fm \rightarrow small $Z \rightarrow$ deuteron composed of two nucleons
- conditions: S -wave, near-threshold, stable

Modern generalizations:

- Higher partial waves, broad resonances, not close to thresholds
- among other approaches: spectral density functions [Baru et al., PLB 586, 53 (2004)]
- apply to comprehensive data driven models \rightarrow meaningful results

Coupled-channel formalism

- coupled channels: elementariness Z disperses into a finite probability distribution $w(z) \sim$ "spectral density function"
- mathematically: $w(z)$ projection of physical scattering state (with energy z) on bare elementary state

$$w_i(z) = -\frac{1}{\pi} \text{Im} D_{ii}(z) \quad D_{ii} \text{ dressed propagator of bare } s\text{-channel state } i, \text{ includes full coupled-channel } T \text{ matrix}$$

$$\mathcal{Z}_i \simeq \frac{\int_{M_R - \Gamma_R}^{M_R + \Gamma_R} w_i(z) dz}{\int_{M_R - \Gamma_R}^{M_R + \Gamma_R} BW(z) dz}, \quad BW(z) \equiv \frac{1}{\pi} \frac{\Gamma_R/2}{(z - E_R)^2 + (\Gamma_R/2)^2}$$

- more than one bare state in a PW: Total elementariness:

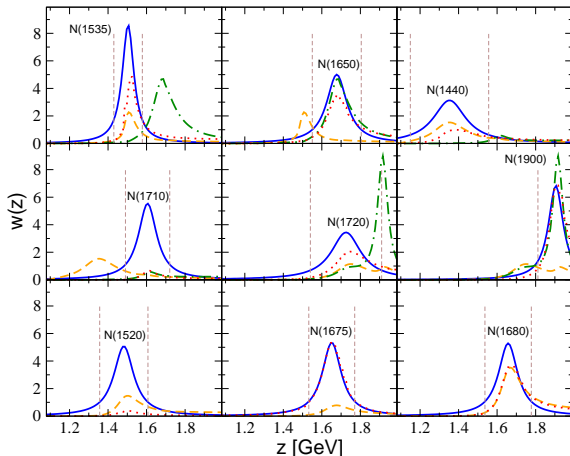
$$\mathcal{Z} = 1 - \prod_i (1 - \mathcal{Z}_i)$$

On the nature of N^* and Δ states

Y.-F. Wang et al. PRC 109 (2024)

Application to the JüBo2022 resonances

Selected results for N^* states: spectral density functions



Blue solid line: BW denominator

Orange dashed (green dash-dotted): 1st
(2nd) spectral density function (model)

Red dotted: locally constructed function

Vertical lines: the integral region

On the nature of N^* and Δ states

Y.-F. Wang et al. PRC 109 (2024)

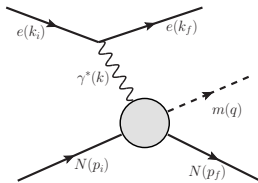
Application to the JüBo2022 resonances

Selected results for N^* states: elementariness

State	Pole position (MeV)	\mathcal{Z}_1	\mathcal{Z}_2	\mathcal{Z}_{tot}	\mathcal{Z}^{lc}
$N(1535) \frac{1}{2}^-$	$1504 - 37i$	24.8%	5.6%	29.0%	50.8%
$N(1650) \frac{1}{2}^-$	$1678 - 64i$	13.4%	91.7%	92.8%	70.5%
$N(1440) \frac{1}{2}^+$	$1353 - 102i$	48.7%	1.7%	49.5%	31.5%
$N(1710) \frac{1}{2}^+$	$1605 - 58i$	11.5%	10.3%	20.6%	10.2%
$N(1720) \frac{3}{2}^+$	$1726 - 93i$	34.1%	68.5%	79.3%	62.5%
$N(1900) \frac{3}{2}^+$	$1905 - 47i$	19.9%	100%	100%	99.9%
$N(1520) \frac{3}{2}^-$	$1482 - 63i$	29.4%	...	29.4%	7.2%
$N(1680) \frac{5}{2}^+$	$1657 - 60i$	67.9%	...	67.9%	69.9%

Green: high compositeness, blue: high elementariness

Electroproduction



Experimental studies of electroproduction:

major progress in recent years, e.g., from JLab, MAMI, ...

- 10^5 data points for πN , ηN , KY , $\pi\pi N$ electroproduction
- access the Q^2 dependence of the amplitude
 - expected to provide a link between perturbative QCD and the region where quark confinement sets in
- so far, no new N^* or Δ^* established from electroproduction: data not yet analyzed on the same level as photoproduction
Reviews: Prog.Part.Nucl.Phys. 67 (2012); Few. Body Syst. 63 (2022) 3, 59

Single-channels analyses, e.g.:

- **MAID:** π , η , kaon electroproduction (EPJA 34, 69 (2007), NPA 700, 429 (2002),)
- **JLab:** π electroproduction covering the resonance region (PRC 80 (2009) 055203)

Coupled-channels analyses:

- **ANL-Osaka:** extension of DCC analysis of pion electroproduction (PRC 80, 025207 (2009)) in progress (Few Body Syst. 59 (2018) 3, 24)
- **Jülich-Bonn-Washington approach** M. Mai *et al.* PRC 103 (2021): $\gamma^* p \rightarrow \pi^0 p, \pi^+ n, \eta p, K \Lambda$

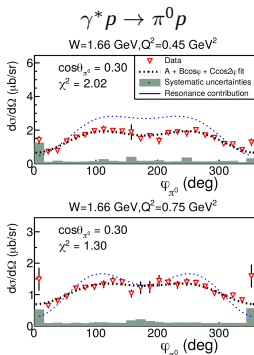


Figure and data from Markov *et al.* (CLAS) PRC 101 (2020),
resonance contribution: JLab/YerPhI

Jülich-Bonn-Washington (JBW) parametrization

M. Mai et al. PRC 103 (2021), PRC 106 (2022), EPJ A 59 (2023)

$$\mathcal{M}_{\mu\gamma^*}(k, W, Q^2) = R_{\ell'}(\lambda, q/q_\gamma) \left(V_{\mu\gamma^*}(k, W, Q^2) + \sum_{\kappa} \int_0^{\infty} dp p^2 T_{\mu\kappa}(k, p, W) G_{\kappa}(p, W) V_{\kappa\gamma^*}(p, W, Q^2) \right)$$

(Pseudo)-threshold behavior with meson/photon momenta

$$\begin{aligned} \lim_{k \rightarrow 0} E_{\ell^+} &= k^{\ell} \\ \lim_{q \rightarrow 0} L_{\ell^+} &= q^{\ell} \\ &\dots \end{aligned}$$

For $Q^2=0$ (real photons) identical to Jülich-Bonn photoproduction amplitude

$$\begin{aligned} V_{\mu\gamma^*}(k, W, Q^2) &= V_{\mu\gamma}^{\text{JUBO}}(k, W) \cdot F_D(Q^2) \cdot \\ &e^{-\beta_\mu^0 Q^2/m_p^2} \left(1 + Q^2/m_p^2 \beta_\mu^1 + (Q^2/m_p^2)^2 \beta_\mu^2 \right) \end{aligned}$$

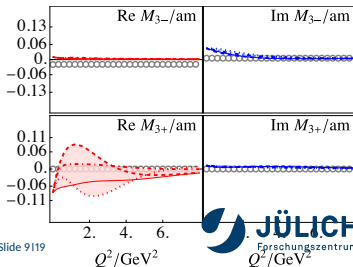
Siegert's theorem [Siegert\(1973\)](#)
[Amaaldi et al.\(1979\)](#)
[Tiator\(2016\)](#)

$$V^{L_{\ell^\pm}} = (\text{const.}) \cdot V^{E_{\ell^\pm}}$$

...at pseudo-threshold

- simultaneous fit to πN , ηN , $K \Lambda$ electroproduction off proton ($W < 1.8$ GeV, $Q^2 < 8$ GeV²)
- 533 fit parameters, 110.281 data points
- Input from JüBo: $V_{\mu\gamma}(k, W, Q^2 = 0)$, $T_{\mu\kappa}(k, p, W)$, $G_{\kappa}(p, W)$
→ universal pole positions and residues (fixed in this study)
- long-term goal: fit pion-, photo- and electron-induced reactions simultaneously

$\gamma^* p \rightarrow K \Lambda$ at $W = 1.7$ GeV



based on most recent JBW, JüBo2022

Q^2 dependence of transition form factors (TFFs) allows conclusions on the nature of resonances.

Here:

- for the first time determined from a **coupled-channel** study of πN , ηN , and $K \Lambda$ electroproduction (+ constraints from photon and pion-induced reactions)
- first estimation of TFFs for **higher excited states**
- from poles, not Breit-Wigner states

TFFs defined independently of the hadronic final state as Workman et al. PRC 87 (2013) :

$$H_h^{l\pm, l}(Q^2) = C_l \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \tilde{R}^{l\pm, l}}} \tilde{\mathcal{H}}_h^{l\pm, l}(Q^2),$$

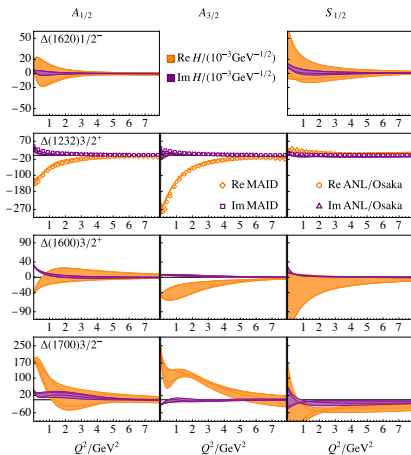
$h = 1/2, 3/2$ helicity, \mathcal{H} ($=\mathcal{A}$ or \mathcal{S}) helicity amplitudes, $\tilde{\mathcal{H}}$, \tilde{R} residues, z_p pole position

Baryon Transition Form Factors

Y.-F. Wang et al. 2404.17444 [nucl-th]

based on most recent JBW, JüBo2022

Δ states:



[ANL/OSAKA: Kamano Few Body Syst. 59, 24 (2018), MAID: Tiator et al. PRC94 (2016)]

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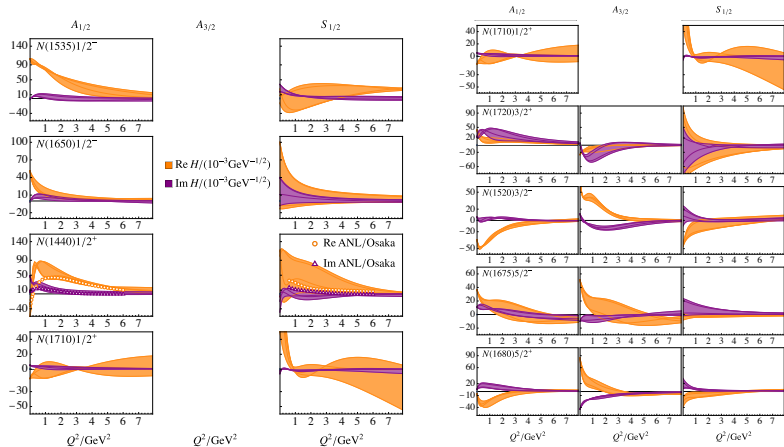
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Baryon Transition Form Factors

Y.-F. Wang et al. 2404.17444 [nucl-th]

based on most recent JBW, JüBo2022

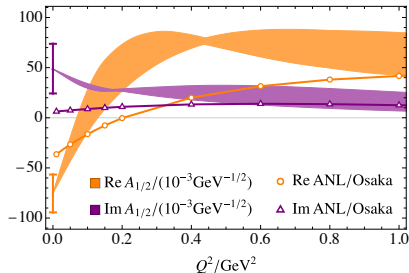
N^* states:



[ANL/OSAKA: Kamano Few Body Syst. 59, 24 (2018), MAID: Tiator et al. PRC94 (2016)]

based on most recent JBW, JüBo2022

The Roper resonance $N(1440)1/2^+$:

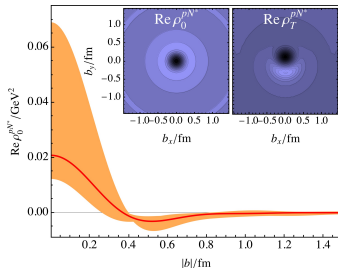


- Zero crossing in $\text{Re}A_{1/2}$ at smaller Q^2 than in Breit-Wigner determinations or in ANL/OSAKA [Kamano, Few Body Syst. 59, 24 (2018)]
- important for quark models, DSE: meson cloud contributions or radial excitation of the nucleon?

Transverse charge density ρ of $p \rightarrow N(1440)$ transition:

following Tiator et al. Chin. Phys. C 33 (2009)

- study flavor decomposition, u and d quark distribution



- Orange band: JBW, red line: MAID 2007.
 Insets: light/dark shades represent negative/positive values
 b : transverse position in xy -plane

Hidden charm reactions

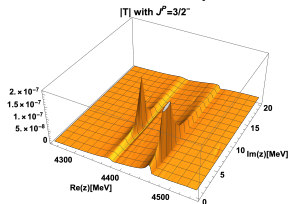
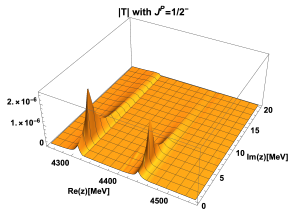
Hidden charm reactions in JüBo Wang, Shen, Rönchen, Meißner, Zou

$\bar{D}^{(*)}\Lambda_c - \bar{D}^{(*)}\Sigma_c^{(*)}$ interactions

EJP C 82, 497 (2022)

- 2017: exploratory study of only two channels, $\bar{D}\Lambda_c$ and $\bar{D}\Sigma_c$ [Shen et al. CPC 42, 023106 \(2018\)](#)
- 2022: extension to a more complete coupled-channel calculation: $\bar{D}\Lambda_c$, $\bar{D}\Sigma_c$, $\bar{D}^*\Lambda_c$, $\bar{D}^*\Sigma_c$, $\bar{D}\Sigma_c^*$
 - more comprehensive picture of the hidden-charm resonance spectrum
 - postdict LHCb pentaquarks by tuning free parameters, predict additional dyn. gen. states

Selected results:



$$J^P = 1/2^-, I = 1/2:$$

- $z_0 = 4312.4 - i2.9$ MeV ($\bar{D}\Sigma_c$ bound state) → LHCb $P_c(4312)$ [Aaij et al. PRL 122, 222001 \(2019\)](#)
- $z_0 = 4439.4 - i2.8$ MeV ($\bar{D}^*\Sigma_c$ bound state) → LHCb $P_c(4440)$ [Aaij et al. PRL 122, 222001 \(2019\)](#)

$$J^P = 3/2^-, I = 1/2:$$

- $z_0 = 4375.9 - i7.6$ MeV ($\bar{D}\Sigma_c^*$ bound state)
- $z_0 = 4460.0 - i3.9$ MeV ($\bar{D}^*\Sigma_c$ bound state) → LHCb $P_c(4457)$ [Aaij et al. PRL 122, 222001 \(2019\)](#)

$$J^P = 1/2^+, I = 1/2:$$

- $z_0 = 4339.3 - i106.3$ MeV → LHCb $P_c(4337)$? [Aaij et al. PRL 128, 062001 \(2022\)](#)

+ several resonances in higher PWS

P_c states from fit to LHCb data

C.-W Shen et al. 2405.02626 [hep-ph]

$J/\psi p$ invariant mass distributions in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay

→ extend framework to $J/\psi N$ channel

- $J/\psi p$ invariant mass distribution:

$$\frac{d\Gamma}{dm_{J/\psi p}} = \frac{\tilde{q}_{J/\psi} q_K}{4(2\pi)^3 m_{\Lambda_b}} \left(\sum_{J=1/2}^{5/2} |T^J|^2 + |T_{bg}|^2 \right)$$

with

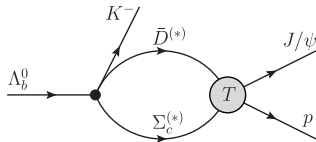
$$T^{1/2} = \sum_{\kappa} g_{\kappa}^{1/2} G_{\kappa} T_{\kappa J/\psi p}^{1/2}$$

$$T^{3/2} = \sum_{\kappa} g_{\kappa}^{3/2} G_{\kappa} T_{\kappa J/\psi p}^{3/2} q_K$$

$$T^{5/2} = \sum_{\kappa} g_{\kappa}^{5/2} G_{\kappa} T_{\kappa J/\psi p}^{5/2} q_K^2$$

g_{κ}^J : free fit parameters

$T_{\kappa J/\psi p}^J, G_{\kappa}$: coupled-channel T -matrix and propagator



Polynomial background term:

$$T_{bg} = a + bs + cs^2$$

s : Mandelstam variable; a, b, c : free fit parameters

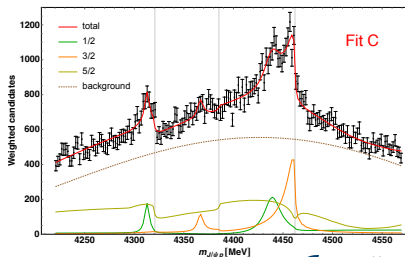
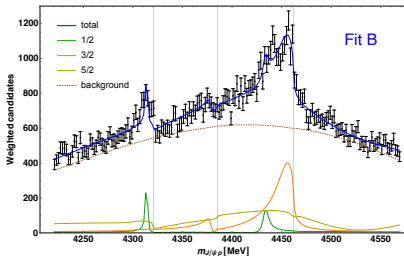
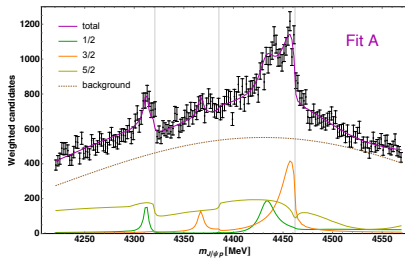
P_C states from fit to LHCb data

C.-W. Shen et al. 2405.02626 [hep-ph]

Three fit results with different starting values

Fit parameters:

- g_κ^J in T^J
 - cut-offs in form factors of coupled-channel T -matrix
 - a, b, c : free fit parameters in T_{bg}
- 33 fit parameters, 175 data points



Data: R. Aaij et al. [LHCb], Phys. Rev. Lett. 122, 222001 (2019).

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Slide 15119

Resonance analysis: 4 prominent narrow states

Fit	z_R, J^P
A	$4312.3 - i2.4, \frac{1}{2}^-$
B	$4314.0 - i0.74, \frac{1}{2}^-$
C	$4312.9 - i2.6, \frac{1}{2}^-$
A	$4366.5 - i4.7, \frac{3}{2}^-$
B	$4378.7 - i4.1, \frac{3}{2}^-$
C	$4365.9 - i4.7, \frac{3}{2}^-$
A	$4433.0 - i9.3, \frac{1}{2}^-$
B	$4433.4 - i3.2, \frac{1}{2}^-$
C	$4438.6 - i10, \frac{1}{2}^-$
A	$4461.4 - i9.7, \frac{3}{2}^-$
B	$4461.5 - i14, \frac{3}{2}^-$
C	$4464.9 - i6.8, \frac{3}{2}^-$

- just below $\bar{D}\Sigma_c$, strong coupling to $\bar{D}\Sigma_c$
→ $\bar{D}\Sigma_c$ boundstate
- $P_c(4312)$

- $\bar{D}\Sigma_c^*$ bound state

- strong coupling to $\bar{D}^*\Sigma_c$, below $\bar{D}^*\Sigma_c$ threshold
→ likely a $\bar{D}^*\Sigma_c$ bound state
- $P_c(4440)$

- strong coupling to $\bar{D}^*\Sigma_c$
- below $\bar{D}^*\Sigma_c$ threshold in Fit A, B, above in Fit C
- $P_c(4457)$

P_c states from fit to LHCb data

C.-W Shen et al. 2405.02626 [hep-ph]

Resonance analysis: poles in higher partial waves

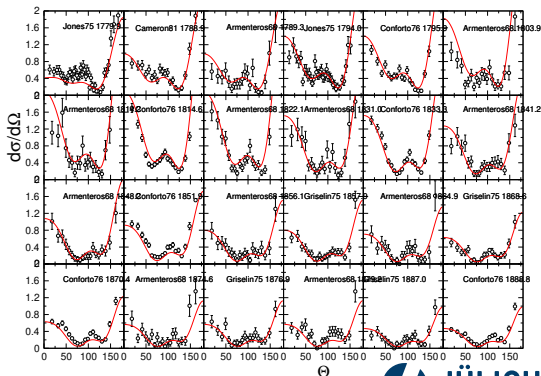
J^P	z_R [MeV]			
	Fit A	Fit B	Fit C	
$\frac{1}{2}^-$	4408.1 - i 181.5	4413.8 - i 182.4	4407.1 - i 178.1	
$\frac{1}{2}^-$	4475.7 - i 143.6	4502.5 - i 140.7	4476.4 - i 143.9	
$\frac{1}{2}^+$	4337.2 - i 76.3	4325.4 - i 85.4	4335.1 - i 75.3	$P_c(4337) ?$
$\frac{3}{2}^+$	4413.0 - i 197.0	4410.9 - i 193.0	4413.3 - i 196.0	
$\frac{3}{2}^+$	4387.9 - i 156.3	-	4387.4 - i 156.1	$P_c(4380) ?$
$\frac{3}{2}^-$	4408.0 - i 168.9	4408.3 - i 175.9	4407.0 - i 167.6	
$\frac{3}{2}^-$	4506.6 - i 136.2	4501.5 - i 132.3	4506.5 - i 133.9	
$\frac{5}{2}^-$	4411.6 - i 180.1	-	4411.6 - i 179.9	
$\frac{5}{2}^+$	4400.4 - i 100.6	4393.3 - i 101.6	4400.9 - i 100.5	
$\frac{5}{2}^+$	4467.1 - i 100.3	4457.8 - i 140.4	4466.7 - i 103.2	
$\frac{5}{2}^+$	4439.1 - i 122.6	-	4437.9 - i 121.9	

Extension of JüBo to $\bar{K}N$ scattering: in progress

PhD topic of S. Rawat (FZJ)

- use $SU(3)$ to adapt $\pi N \rightarrow X$ model to $\bar{K}N \rightarrow X$
- goal: determine hyperon spectrum, i.e. Λ^* and Σ^* states

Selected preliminary fit results $K^- p \rightarrow K^0 n$



- Almost finished:
coupled-channel fit to
 $\bar{K}N \rightarrow \bar{K}N, \pi\Lambda, \pi\Sigma$
- next step: resonance analysis
(pole search)

Summary project B.11 "Coupled-channel dynamics"

Extension of JüBo to

- $K\Lambda$, $K\Sigma$ photoproduction (EPJA 54 (2018) 110, EPJ A 58 (2022) 229, 2404.19404)
- $\pi N \rightarrow \omega N$ (Yu-Fei Wang *et al.* PRD 106 (2022))
- hidden-charm channels (Shen CPC 42 (2018), Wang EPJ C 82 (2022), Shen 2405.02626)
→ P_c states
- Electroproduction of pions, etas, $K\Lambda$: Jülich-Bonn-Washington collaboration (Mai PRC 103 (2021) 065204, PRC 106 (2022), EPJ A 59 (2023))

Further activities:

- Compositeness or elementariness of baryon resonances (Wang PRC 109 (2024))
- Baryon transition form factors beyond Roper and $\Delta(1232)$, many for the first time (Wang 2404.17444)
- JüBo model for $\bar{K}N$ scattering (hyperon resonance spectrum): PhD project S. Rawat, almost finished
- Coupled channel study of a_0 resonances (Z.-L. Wang, B.-S. Zou EJP C 82 (2022) 509)

Thank you for you attention!

Appendix

■ Selection of the resonances

- JüBo2022 solution ($K\Sigma$ photoproduction, no ωN) [Rönchen et. al., EPJA 58, 229 (2022)]
- For $N^* J \leq 5/2$, for $\Delta J \leq 3/2$
- Width $\Gamma_R < 300$ MeV

■ Uncertainties \rightarrow comparison of three results

- spectral density functions directly from the model
- locally constructed spectral density functions only by the pole position and residues (L_κ : loop functions)

$$w^{lc}(z) = -\frac{1}{\pi} \text{Im} \left[z - M_0 - \sum_{\kappa} g_{\kappa}^2 L_{\kappa}(z) \right]^{-1}$$

- complex compositeness of the Gamow state, with naive measure [Sekihara, PRC 104, 035202 (2021)]

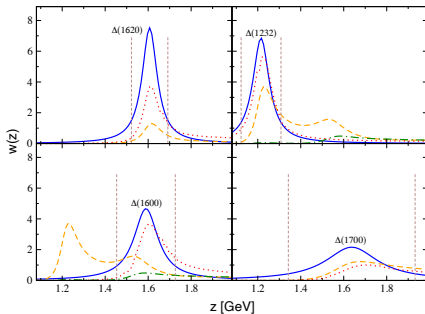
$$\tilde{X}_{\kappa} \equiv \frac{|X_{\kappa}|}{\sum_{\alpha} |X_{\alpha}| + |Z|}, \tilde{Z} \equiv \frac{|Z|}{\sum_{\alpha} |X_{\alpha}| + |Z|}$$

Numerical results

Spectral density functions – Δ states

Blue solid line: the Breit-Wigner denominator. Orange dashed (green dash-dotted) line: the 1st (2nd) spectral density function (model).

Red dotted line: the locally constructed function. Vertical lines: the integral region.



Local construction of the spectral density functions

Slide by Yu-Fei Wang

- Local simulation of the amplitude

$$T_{\alpha\beta}^{\text{lc}}(z) = \frac{c g_{\alpha} g_{\beta} f_{\alpha}^a(q_{\alpha z}) f_{\beta}^c(q_{\beta z})}{z - M_0 - \sum_{\kappa} g_{\kappa}^2 L_{\kappa}(z)} + \dots$$

- Loop functions (f : vertex function in this model)

$$L_{\kappa}(z) \equiv \int_0^{\infty} p^2 dp G_{\kappa}(p, z) f_{\alpha}^a(q_{\kappa z}) f_{\alpha}^c(q_{\kappa z})$$

- Parameters

$$h_{\kappa} \equiv \frac{g_{\kappa}^2}{g_1^2} = \left| \frac{r_{\kappa} f_1^a}{r_1 f_{\kappa}^a} \right|^2,$$
$$g_1^2 = -\frac{\Gamma_R}{2 \sum_{\kappa} h_{\kappa} \text{Im}(L_{\kappa}^{\parallel})},$$
$$M_0 = M_R - g_1^2 \sum_{\kappa} h_{\kappa} \text{Re}(L_{\kappa}^{\parallel}),$$
$$c = \frac{r_1^2}{g_1^2 f_1^a f_1^c} \left(1 - g_1^2 \sum_{\kappa} h_{\kappa} \frac{d}{dz} L_{\kappa}^{\parallel} \Big|_{z=M_R - i\Gamma_R/2} \right).$$

- $g^2 < 0 \rightarrow$ construction fails. Estimation:

$$g_{\kappa}^2 \rightarrow \left| \sqrt{\frac{2\pi\rho_{\kappa}}{\Gamma_R}} r_{\kappa} \right|^2, M_0 \rightarrow M_0 - i\frac{\Gamma_0}{2}$$

Details hidden charm

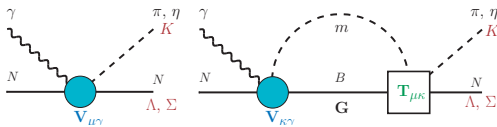
TABLE I: The new isospin factors used in this work.

Process	Exchanged Particle	IF($\frac{1}{2}$)	IF($\frac{3}{2}$)
$\bar{D}^{(*)}\Lambda_c \rightarrow J/\psi N$	D/D^*	1	0
	Λ_c	1	0
$\bar{D}^{(*)}\Sigma_c^{(*)} \rightarrow J/\psi N$	D/D^*	$-\sqrt{3}$	0
	Σ_c	$-\sqrt{3}$	0

Multipole amplitude

$$M_{\mu\gamma}^{IJ} = V_{\mu\gamma}^{IJ} + \sum_{\kappa} T_{\mu\kappa}^{IJ} G_{\kappa} V_{\kappa\gamma}^{IJ}$$

(partial wave basis)



$$m = \pi, \eta, K, B = N, \Delta, \Lambda$$

$T_{\mu\kappa}$: full hadronic T -matrix as in pion-induced reactions

Photoproduction potential: approximated by energy-dependent polynomials (field-theoretical description numerically too expensive)

$$\begin{aligned}
 \mathbf{V}_{\mu\gamma}(E, q) = & \text{Diagram 1} + \text{Diagram 2} \\
 = & \frac{\tilde{\gamma}_{\mu}^a(q)}{m_N} P_{\mu}^{NP}(E) + \sum_i \frac{\gamma_{\mu;i}^a(q) P_i^P(E)}{E - m_i^b}
 \end{aligned}$$

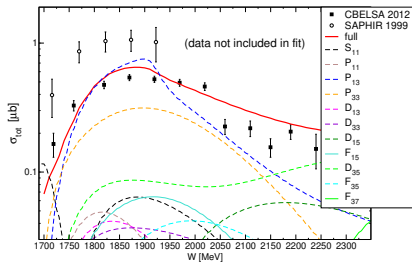
Combined analysis of pion- and photon-induced reactions

Data base

Reaction	Observables (# data points)	p./channel
$\pi N \rightarrow \pi N$	PWA GW-SAID W108 [?] (ED solution)	3,760
$\pi^- p \rightarrow \eta n$	$d\sigma/d\Omega$ (676), P (79)	755
$\pi^- p \rightarrow K^0 \Lambda$	$d\sigma/d\Omega$ (814), P (472), β (72)	1,358
$\pi^- p \rightarrow K^0 \Sigma^0$	$d\sigma/d\Omega$ (470), P (120)	590
$\pi^- p \rightarrow K^+ \Sigma^-$	$d\sigma/d\Omega$ (150)	150
$\pi^+ p \rightarrow K^+ \Sigma^+$	$d\sigma/d\Omega$ (1124), P (551), β (7)	1,682
$\gamma p \rightarrow \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (3287), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (1227), F (397), $C_{x'}$ (74), $C_{z'}$ (26)	26,662
$\gamma p \rightarrow \pi^+ n$	$d\sigma/d\Omega$ (5670), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,457
$\gamma p \rightarrow \eta p$	$d\sigma/d\Omega$ (9112), Σ (535), P (63), T (291), F (144), E (306), G (47), H (56)	10,554
$\gamma p \rightarrow K^+ \Lambda$	$d\sigma/d\Omega$ (2563), P (1663), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	6,072
$\gamma p \rightarrow K^+ \Sigma^0$	$d\sigma/d\Omega$ (4381), P (422), Σ (280), T (127), $C_{x'}$ (94), $C_{z'}$ (94), O_x (127), O_z (127)	5,652
$\gamma p \rightarrow K^0 \Sigma^+$	$d\sigma/d\Omega$ (281), P (167)	448
	in total	67,008

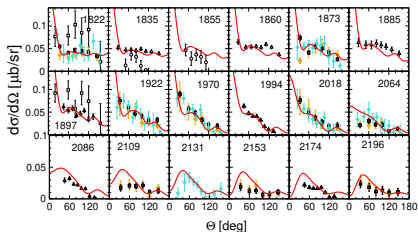
Selected results $\gamma p \rightarrow K^0 \Sigma^+$

JüBo2022 Eur.Phys.J.A 58 (2022) 229

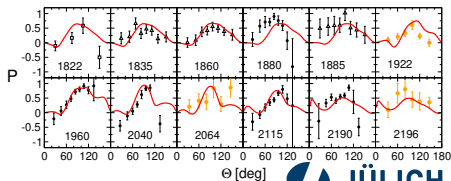


- much less data than for $K^+ \Sigma^0$ (448 vs 5,652 data points)
- in parts inconsistent data very few polarization, no double polarization data
- difficult to achieve a good fit result ($\chi^2 = 3.16$ ($K^0 \Sigma^+$) vs 1.66 ($K^+ \Sigma^0$))
- cusp in σ_{tot} at ~ 2 GeV not reproduced (data not included in fit)

Selected fit results:



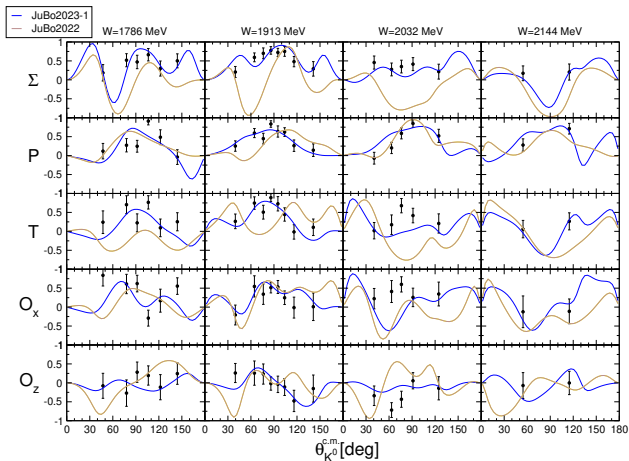
Data: open squares: SAPHIR 1999, cyan: SAPHIR 2005, orange: CBELSA/TAPS 2007, black squares: CBELSA/TAPS 2011, open circles: A2 2018, open triangles: A2 2013, black triangles: Hall B 2003, black circles: CLAS 2013



New data for $\gamma p \rightarrow K^0 \Sigma^+$

Fit JüBo2023-1, together with CLAS Collaboration

L. Clark et al. 2404.19404 [nucl-ex]



- 105 new data points, first ever for T , O_x , O_z
- simultaneous fit to full JüBo data base including new data
- $\chi^2 = 2.01$ ($K^0 \Sigma^+$)

New data for $\gamma p \rightarrow K^0 \Sigma^+$

Fit JüBo2023-1, together with CLAS Collaboration

L. Clark et al. 2404.19404 [nucl-ex]

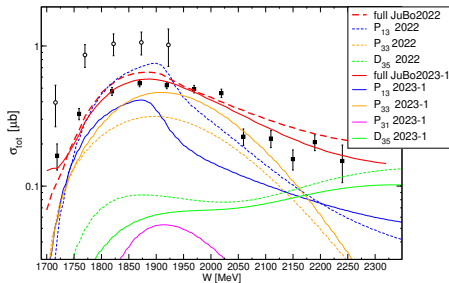
Resonance analysis:

$\Delta(1910) 1/2^+$	Re E_0	$-2\text{Im } E_0$
2032-1	1745	433
2022	1802 (11)	550 (22)
$N(2190) 7/2^-$		
2032-1	1946	162
2022	1965 (12)	288 (66)
$N(1900) 3/2^+$		
2032-1	1893	105
2022	1905 (3)	93 (4)

(all numbers in [MeV])

Partial wave content:

- JüBo2022: dominant PW in $K^0 \Sigma^+$: P_{13} , followed by P_{33}
- JüBo2023-1: order reversed



⇒ (Double) polarization data very important to determine the resonance spectrum!

New data for $\gamma p \rightarrow K^0 \Sigma^+$

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L. Clark et al. 2404.19404 [nucl-ex]

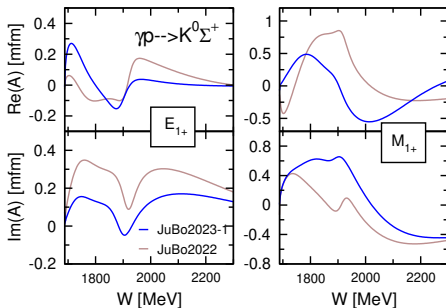
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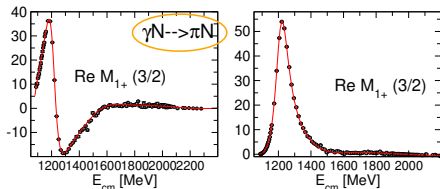
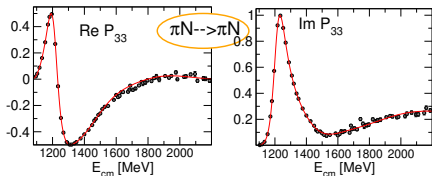
s -, t - and u -channel exchanges

- 21 s -channel states (**resonances**) coupling to πN , ηN , $K\Lambda$, $K\Sigma$, $\pi\Delta$, ρN .
- t - and u -channel **exchanges** ("background", coupling constants fixed from SU(3)):

	πN	ρN	ηN	$\pi\Delta$	σN	$K\Lambda$	$K\Sigma$
πN	$N, \Delta, (\pi\pi)_\sigma, (\pi\pi)_\rho$	$N, \Delta, \text{Ct.}, \pi, \omega, a_1$	N, a_0	N, Δ, ρ	N, π	Σ, Σ^*, K^*	$\Lambda, \Sigma, \Sigma^*, K^*$
ρN		$N, \Delta, \text{Ct.}, \rho$	-	N, π	-	-	-
ηN			N, f_0	-	-	K^*, Λ	Σ, Σ^*, K^*
$\pi\Delta$				N, Δ, ρ	π	-	-
σN					N, σ	-	-
$K\Lambda$						$\Xi, \Xi^*, f_0, \omega, \phi$	Ξ, Ξ^*, ρ
$K\Sigma$							$\Xi, \Xi^*, f_0, \omega, \phi, \rho$

Excited states / Resonances

$$J^P = 3/2^+, I = 3/2$$



Points: SAID 2006 and CM12

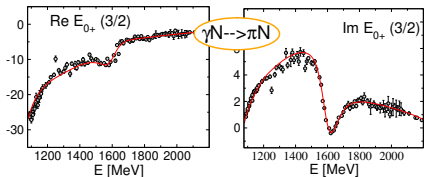
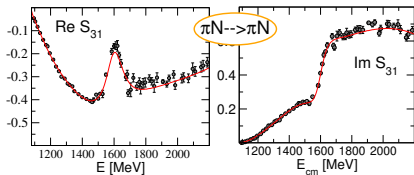
Breit-Wigner parameterization:

$$\mathcal{M}_{ba}^{\text{Res}} = -\frac{g_b g_a}{E^2 - M_{BW}^2 + iE\Gamma_{BW}}$$

- M_{BW} , Γ_{BW} channel dependent
- background? overlapping resonances? thresholds?

Excited states / Resonances

$$J^P = 1/2^-, I = 3/2$$



Points: SAID 2006 and CM12

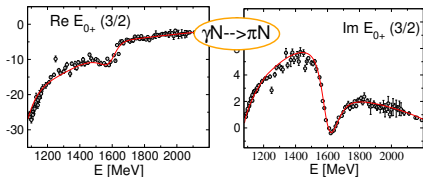
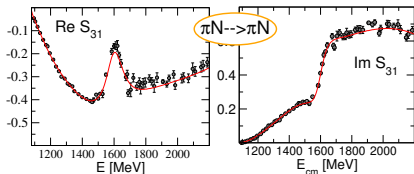
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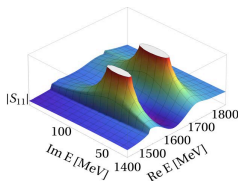
Breit-Wigner parameterization:

$$\mathcal{M}_{ba}^{Res} = -\frac{g_b g_a}{E^2 - M_{BW}^2 + iE\Gamma_{BW}}$$

- M_{BW} , Γ_{BW} channel dependent
- background? overlapping resonances? thresholds?

Resonances: poles in the T -matrix on the 2nd Riemann sheet

- Pole position E_0 is the same in all channels
- thresholds: branch points



$\text{Re}(E_0)$ = "mass"
 $-2\text{Im}(E_0)$ = "width"
 residues \rightarrow branching ratios

Details of the formalism

Polynomials:

$$P_i^P(E) = \sum_{j=1}^n g_{i,j}^P \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{i,n+1}^P (E - E_0)}$$

$$P_\mu^{\text{NP}}(E) = \sum_{j=0}^n g_{\mu,j}^{\text{NP}} \left(\frac{E - E_0}{m_N} \right)^j e^{-g_{\mu,n+1}^{\text{NP}} (E - E_0)}$$

- $E_0 = 1077$ MeV

- $g_{i,j}^P, g_{\mu,j}^{\text{NP}}$: fit parameter

- $e^{-g(E-E_0)}$: appropriate energy behavior

- $n = 3$

high

◀ back

The scattering potential: s -channel resonances

$$V^P = \sum_{i=0}^n \frac{\gamma_{\mu;i}^a \gamma_{\nu;i}^c}{z - m_i^b}$$

- i : resonance number per PW
- $\gamma_{\nu;i}^c$ ($\gamma_{\mu;i}^a$): creation (annihilation) vertex function
with **bare coupling f** (**free parameter**)
- z : center-of-mass energy
- m_i^b : **bare mass** (**free parameter**)

■ $J \leq 3/2$:

$\gamma_{\nu;i}^c$ ($\gamma_{\mu;i}^a$) from effective \mathcal{L}

Vertex	\mathcal{L}_{int}
$N^*(S_{11})N\pi$	$\frac{f}{m_\pi} \bar{\Psi}_{N^*} \gamma^\mu \vec{\tau} \partial_\mu \vec{\pi} \Psi + \text{h.c.}$
$N^*(S_{11})N\eta$	$\frac{f}{m_\pi} \bar{\Psi}_{N^*} \gamma^\mu \partial_\mu \eta \Psi + \text{h.c.}$
$N^*(S_{11})N\rho$	$f \bar{\Psi}_{N^*} \gamma^5 \gamma^\mu \vec{\tau} \vec{\rho}_\mu \Psi + \text{h.c.}$
$N^*(S_{11})\Delta\pi$	$\frac{f}{m_\pi} \bar{\Psi}_{N^*} \gamma^5 \vec{S} \partial_\mu \vec{\pi} \Delta^\mu + \text{h.c.}$

■ $5/2 \leq J \leq 9/2$:

correct dependence on L (centrifugal barrier)

$$\begin{aligned}
 (\gamma^{a,c})_{\frac{5}{2}-} &= \frac{k}{M} (\gamma^{a,c})_{\frac{3}{2}+} & (\gamma^{a,c})_{\frac{5}{2}+} &= \frac{k}{M} (\gamma^{a,c})_{\frac{3}{2}-} \\
 (\gamma^{a,c})_{\frac{7}{2}-} &= \frac{k^2}{M^2} (\gamma^{a,c})_{\frac{3}{2}-} & (\gamma^{a,c})_{\frac{7}{2}+} &= \frac{k^2}{M^2} (\gamma^{a,c})_{\frac{3}{2}+} \\
 (\gamma^{a,c})_{\frac{9}{2}-} &= \frac{k^3}{M^3} (\gamma^{a,c})_{\frac{3}{2}+} & (\gamma^{a,c})_{\frac{9}{2}+} &= \frac{k^3}{M^3} (\gamma^{a,c})_{\frac{3}{2}-}
 \end{aligned}$$

Interaction potential from effective Lagrangian

J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); U.-G. Meißner, Phys. Rept. **161**, 213 (1988); B. Borasoy and U.-G. Meißner, Int. J. Mod. Phys. A **11**, 5183 (1996).

- consistent with the approximate (broken) chiral $SU(2) \times SU(2)$ symmetry of QCD

Vertex	\mathcal{L}_{int}	Vertex	\mathcal{L}_{int}
$NN\pi$	$-\frac{g_{NN\pi}}{m_\pi} \bar{\Psi} \gamma^5 \gamma^\mu \vec{\tau} \cdot \partial_\mu \vec{\pi} \Psi$	$NN\omega$	$-g_{NN\omega} \bar{\Psi} [\gamma^\mu - \frac{\kappa_\omega}{2m_N} \sigma^{\mu\nu} \partial_\nu] \omega_\mu \Psi$
$N\Delta\pi$	$\frac{g_{N\Delta\pi}}{m_\pi} \bar{\Delta}^\mu \vec{S}^\dagger \cdot \partial_\mu \vec{\pi} \Psi + \text{h.c.}$	$\omega\pi\rho$	$\frac{g_{\omega\pi\rho}}{m_\omega} \epsilon_{\alpha\beta\mu\nu} \partial^\alpha \vec{\rho}^\beta \cdot \partial^\mu \vec{\pi} \omega^\nu$
$\rho\pi\pi$	$-g_{\rho\pi\pi} (\vec{\pi} \times \partial_\mu \vec{\pi}) \cdot \vec{\rho}^\mu$	$N\Delta\rho$	$-i \frac{g_{N\Delta\rho}}{m_\rho} \bar{\Delta}^\mu \gamma^5 \gamma^\mu \vec{S}^\dagger \cdot \vec{\rho}_{\mu\nu} \Psi + \text{h.c.}$
$NN\rho$	$-g_{NN\rho} \bar{\Psi} [\gamma^\mu - \frac{\kappa_\rho}{2m_N} \sigma^{\mu\nu} \partial_\nu] \vec{\tau} \cdot \vec{\rho}_\mu \Psi$	$\rho\rho\rho$	$g_{NN\rho} (\vec{\rho}_\mu \times \vec{\rho}_\nu) \cdot \vec{\rho}^{\mu\nu}$
$NN\sigma$	$-g_{NN\sigma} \bar{\Psi} \Psi \sigma$	$NN\rho\rho$	$\frac{\kappa_\rho g_{NN\rho}^2}{2m_N} \bar{\Psi} \sigma^{\mu\nu} \vec{\tau} \Psi (\vec{\rho}_\mu \times \vec{\rho}_\nu)$
$\sigma\pi\pi$	$\frac{g_{\sigma\pi\pi}}{2m_\pi} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} \sigma$	$\Delta\Delta\pi$	$\frac{g_{\Delta\Delta\pi}}{m_\pi} \bar{\Delta}_\mu \gamma^5 \gamma^\nu \vec{T} \Delta^\mu \partial_\nu \vec{\pi}$
$\sigma\sigma\sigma$	$-g_{\sigma\sigma\sigma} m_\sigma \sigma\sigma\sigma$	$\Delta\Delta\rho$	$-g_{\Delta\Delta\rho} \bar{\Delta}_\tau (\gamma^\mu - i \frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta} \sigma^{\mu\nu} \partial_\nu) \cdot \vec{\rho}_\mu \cdot \vec{T} \Delta^\tau$
$NN\rho\pi$	$\frac{g_{NN\pi}}{m_\pi} 2g_{NN\rho} \bar{\Psi} \gamma^5 \gamma^\mu \vec{\tau} \Psi (\vec{\rho}_\mu \times \vec{\pi})$	$NN\eta$	$-\frac{g_{NN\eta}}{m_\pi} \bar{\Psi} \gamma^5 \gamma^\mu \partial_\mu \eta \Psi$
NNa_1	$-\frac{g_{NN\pi}}{m_\pi} m_{a_1} \bar{\Psi} \gamma^5 \gamma^\mu \vec{\tau} \Psi \vec{a}_\mu$	NNa_0	$g_{NNa_0} m_\pi \bar{\Psi} \vec{\tau} \Psi \vec{a}_0$
$a_1\pi\rho$	$-\frac{2g_{\pi a_1 \rho}}{m_{a_1}} [\partial_\mu \vec{\pi} \times \vec{a}_\nu - \partial_\nu \vec{\pi} \times \vec{a}_\mu] \cdot [\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu]$ $+\frac{2g_{\pi a_1 \rho}}{2m_{a_1}} [\vec{\pi} \times (\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu)] \cdot [\partial^\mu \vec{a}^\nu - \partial^\nu \vec{a}^\mu]$	$\pi\eta a_0$	$g_{\pi\eta a_0} m_\pi \eta \vec{\pi} \cdot \vec{a}_0$

Generalization to SU(3)

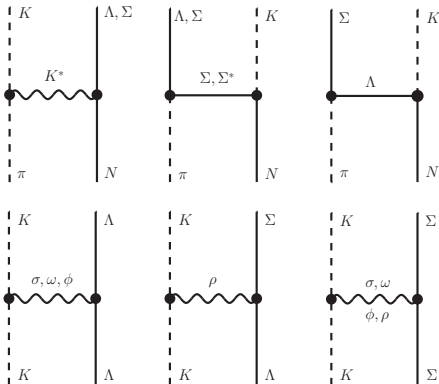
to include KY final states

- t- and u-channel exchange: T^{NP}

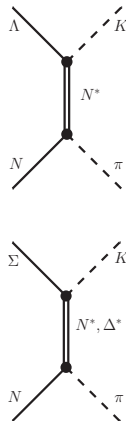
coupling constants fixed from SU(3) symmetry

e.g. $g_{\Lambda NK} = -\frac{\sqrt{3}}{3} g_{NN\pi} (1 + 2\alpha_{BBP})$

J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963) [Erratum-ibid. 37, 326 (1965)].



- s-channel: resonances

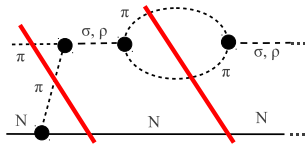


New free parameters:
bare couplings g_{N^*KY} and

Theoretical constraints of the S -matrix

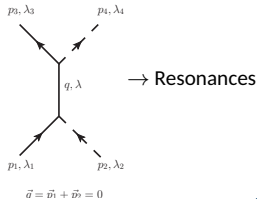
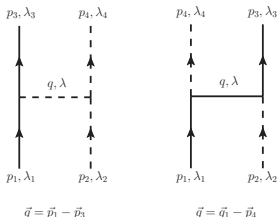
Unitarity: probability conservation

- 2-body unitarity
- 3-body unitarity:
 - discontinuities from t -channel exchanges
 - Meson exchange from requirements of the S -matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]



Analyticity: from unitarity and causality

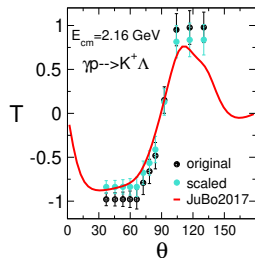
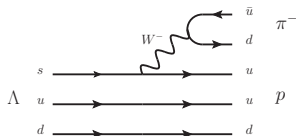
- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut → Baryon u -channel exchange



Λ decay parameter α_-

Advantage in KY photoproduction: self-analysing decay of the hyperons
 \rightarrow measurement of recoil polarization easier

- Λ decays weakly to $\pi^- p$ with decay parameter α_- (PDG average: $\alpha_- = 0.642 \pm 0.013$)
- recent BESIII measurement ($e^+ e^- \rightarrow J/\psi \rightarrow \Lambda \bar{\Lambda}$):
 $\alpha_- = 0.750 \pm 0.009 \pm 0.004$ (Ablikim, Nature (2019))
 \rightarrow polarizations affected by α_- are $\sim 17\%$ too large!
- independent estimation of α_- from $\gamma p \rightarrow K^+ \Lambda$ CLAS data using Fierz identities
 $\Rightarrow \alpha_- = 0.721 \pm 0.006 \pm 0.005$ (Ireland et al. PRL 123 (2019))



data: Paterson (CLAS) PRC 93, 065201 (2016)

Has impact on

- observables P , T , C_x , C_z , O_x , O_z
- reactions $\gamma p \rightarrow K^+ \Lambda$, $K^+ \Sigma^0$ ($\rightarrow K^+ \gamma \Lambda$), $\pi^- p \rightarrow K^0 \Lambda$, $K^0 \Sigma^0$
- resonance spectrum? \Rightarrow **JuBo re-fit to data scaled by new α_-**