Final-state interactions

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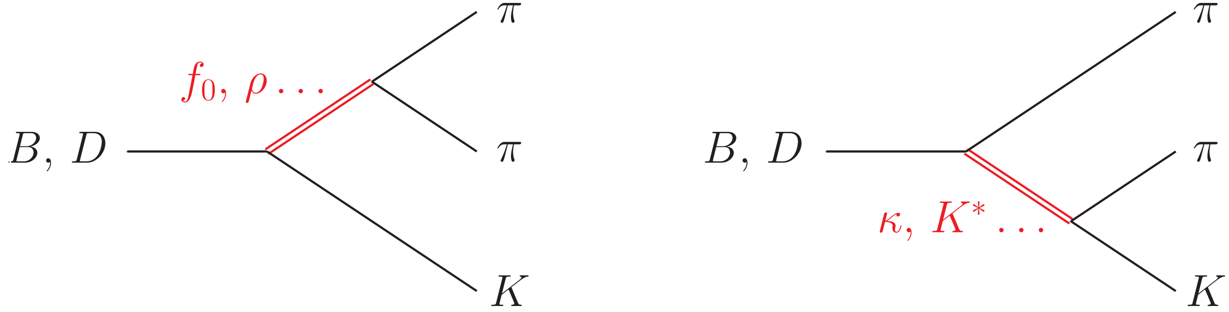
Final-state interactions

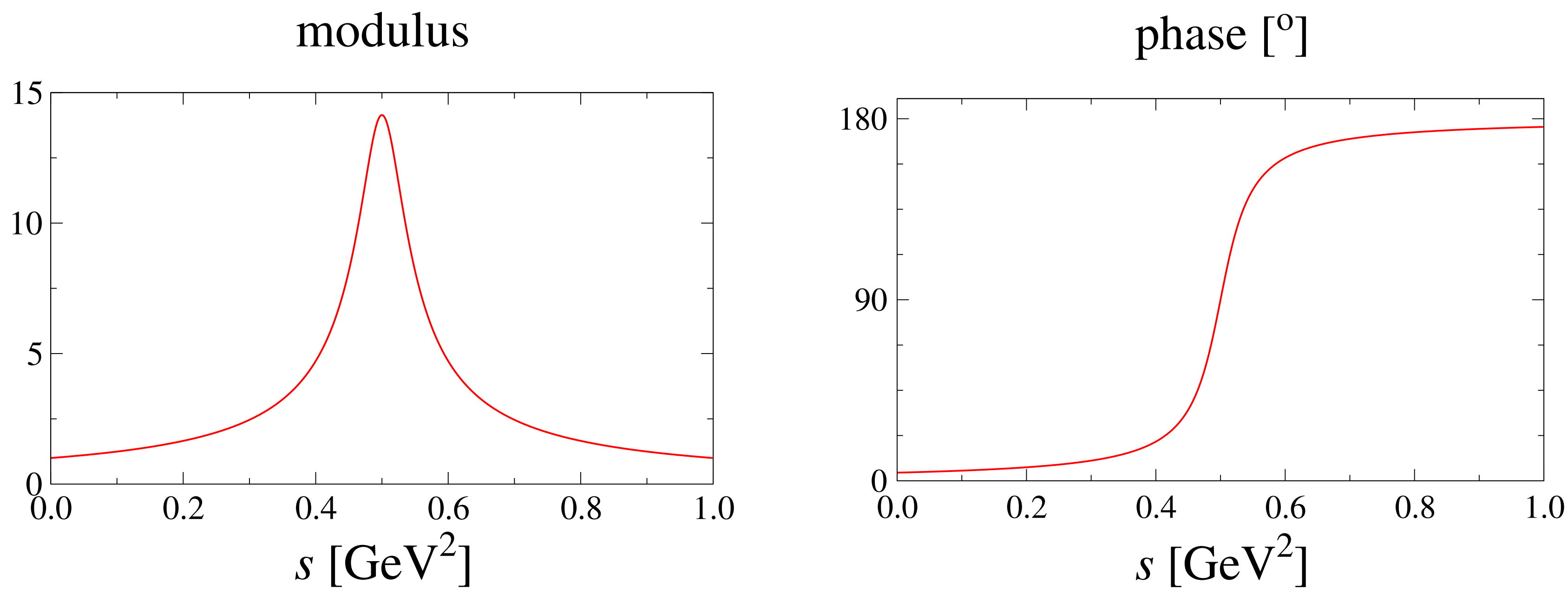
- **Meson–meson final-state interactions**
  - ▷ form factors and Omnès problem: $\bar{B}^0 \to J/\psi\pi\pi$
  - ▷ extension to higher energies: two-potential formalism
  - ▷ pion–kaon: τ decays, Primakoff production
  - ▷ left-hand cuts: Z_b states and $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$

- **Three-meson systems**
  - ▷ diffractive 3π production and Khuri–Treiman: can rescattering be observed?
  - ▷ modified lineshapes beyond 3π: $B \to D\pi\pi$
  - ▷ rescattering and BSM physics: CPV in $\eta \to 3\pi$

Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances





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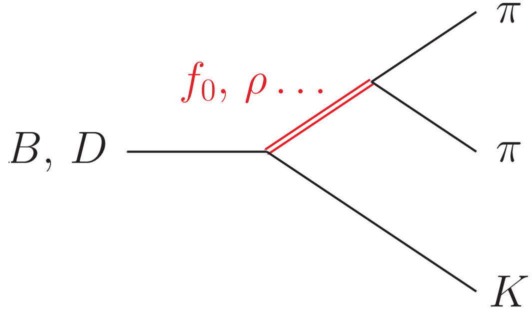
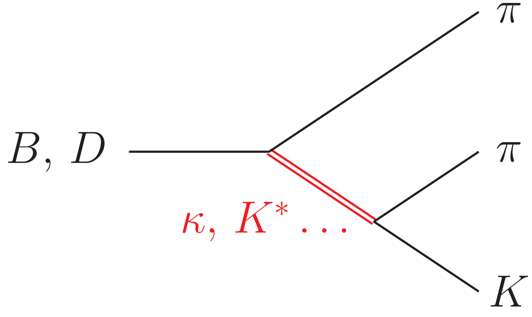
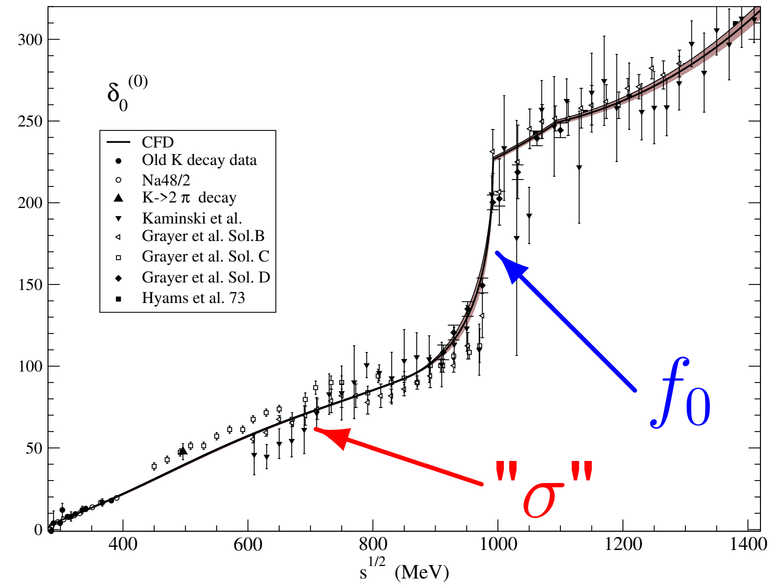




. . . so what's not to like?

- some resonances don't look like Breit–Wigners at all!

  $\longrightarrow$ use exact scattering phase shifts instead



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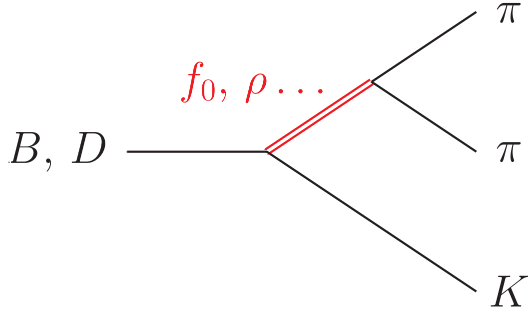
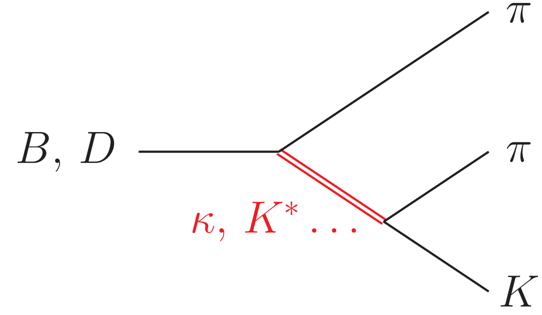


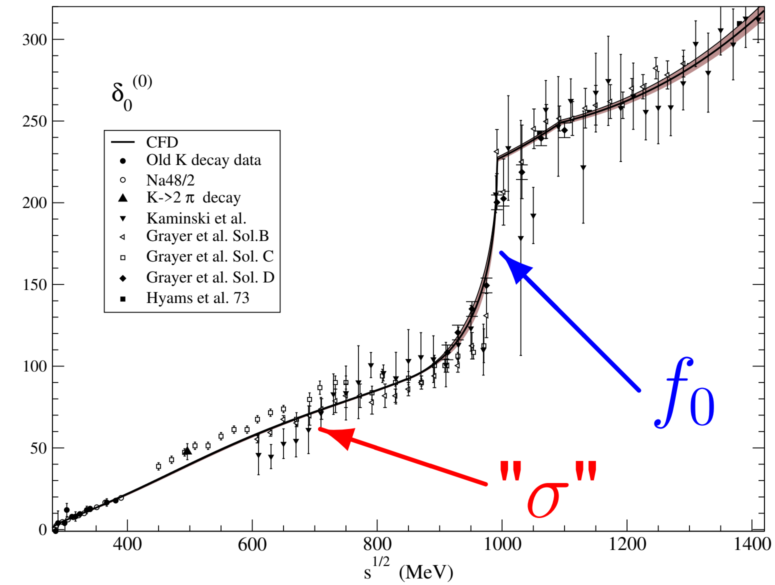


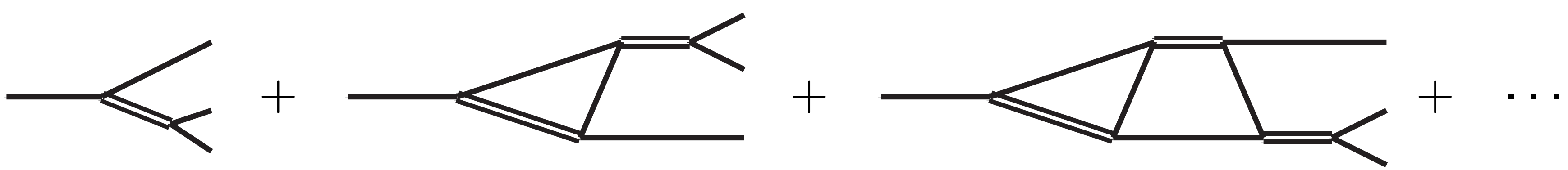
. . . so what's not to like?

- some resonances don't look like Breit–Wigners at all!

  $\longrightarrow$ use exact scattering phase shifts instead

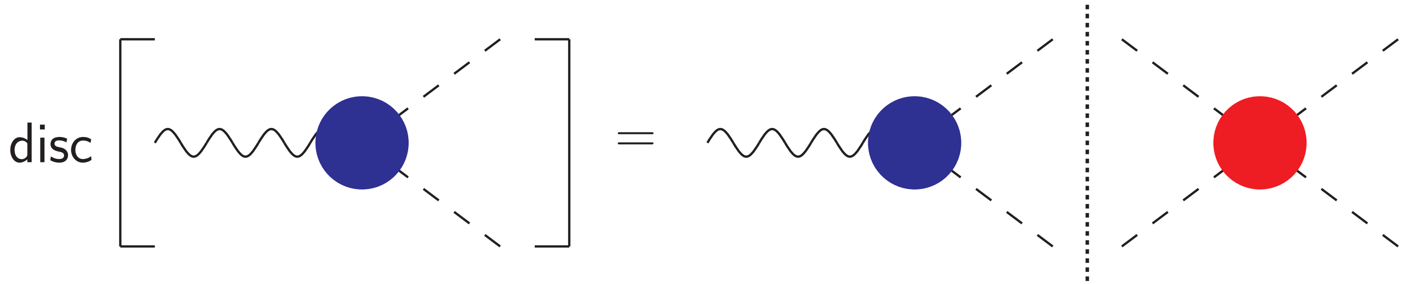
- 3-particle rescattering





Final-state interactions: Omnès formalism

- two-particles FSI: form factor; from unitarity:



$$\frac{1}{2i}\,\text{disc}\,F_I(s) \;=\; \text{Im}\,F_I(s) \;=\; F_I(s)\times\theta(s-4\,M_\pi^2)\times\sin\delta_I(s)\,e^{-i\delta_I(s)}$$

$\longrightarrow$ final-state theorem: phase of $F_I(s)$ is just $\delta_I(s)$ Watson 1954

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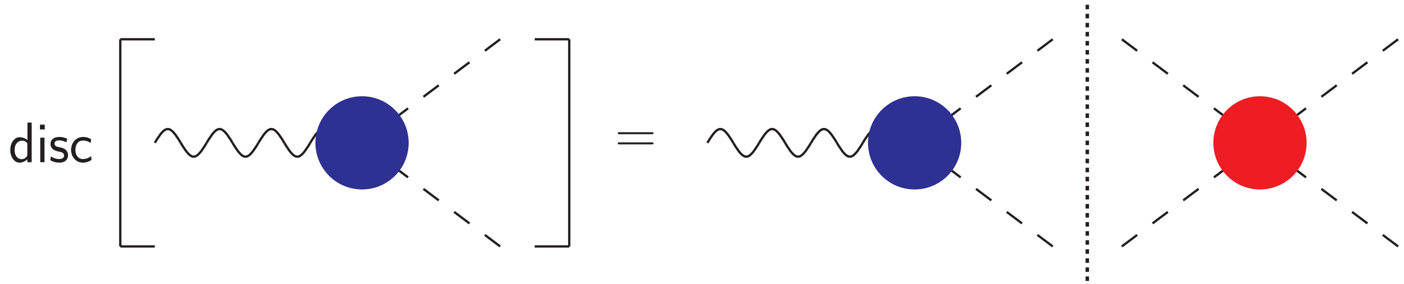

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$\longrightarrow$ final-state theorem: phase of $F_I(s)$ is just $\delta_I(s)$ — Watson 1954

- solution to this homogeneous integral equation known:

$$F_I(s) = P_I(s)\Omega_I(s)\,, \quad \Omega_I(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\delta_I(s')}{s'(s'-s)}\right\}$$

$P_I(s)$ polynomial, $\Omega_I(s)$ Omnès function — Omnès 1958

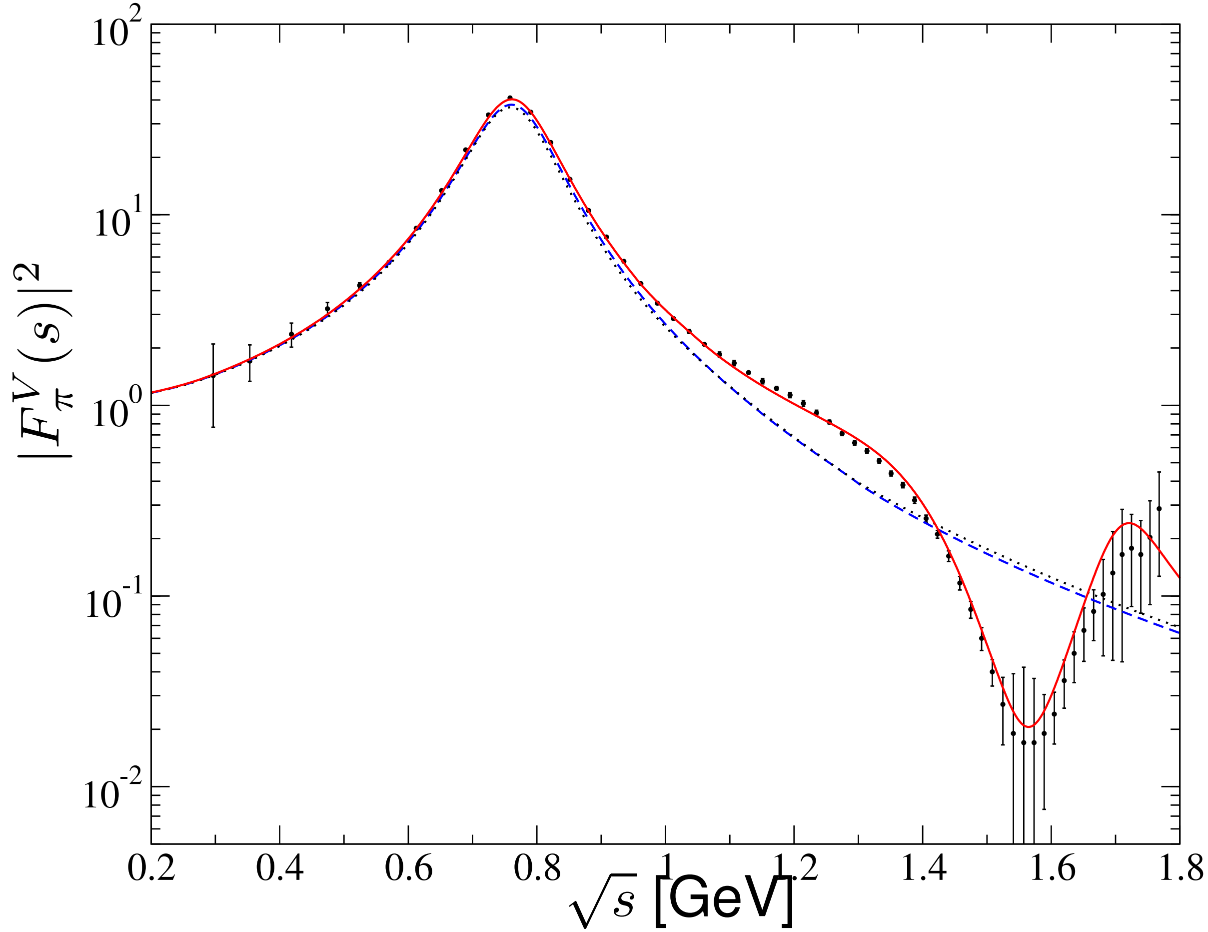
$P_I(s)$ non-universal, needs to be fixed by symmetries, data, ...

- today: high-accuracy $\pi\pi$ phase shifts available

Ananthanarayan et al. 2001, García-Martín et al. 2011, Caprini et al. 2012

Pion vector form factor vs. Omnès representation

Data on pion form factor in $\tau^- \to \pi^- \pi^0 \nu_\tau$ Belle 2008



$\pi\pi$ P-wave phase shift / effective form factor phase incl. ρ', ρ''

Schneider et al. 2012

Scalar form factors: coupled channels

- two scalar isoscalar pion form factors:

$$\langle \pi^+\pi^- | \tfrac{1}{2}(\bar{u}u + \bar{d}d) | 0 \rangle = \mathcal{B}^n \Gamma_\pi^n(s) \qquad \langle \pi^+\pi^- | \bar{s}s | 0 \rangle = \mathcal{B}^s \Gamma_\pi^s(s)$$

- strong inelastic coupling to $\bar{K}K$ near $f_0(980)$
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- three input functions:
  - ▷ $\pi\pi$ S-wave phase shift $\delta(s)$ Caprini, Colangelo, Leutwyler 2012
  - ▷ modulus $|g(s)|$ and phase $\psi(s)$ of $\pi\pi \to \bar{K}K$ amplitude
    Büttiker et al. 2004; Cohen et al. 1980, Etkin et al. 1982
- solution in terms of Omnès matrix

$$\begin{pmatrix} \Gamma_\pi(s) \\ \frac{2}{\sqrt{3}}\Gamma_K(s) \end{pmatrix} = \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \begin{pmatrix} \Gamma_\pi(0) \\ \frac{2}{\sqrt{3}}\Gamma_K(0) \end{pmatrix}$$

- also: new solution strategy to unitarise ChPT
  Shi, Seng, Guo, BK, Meißner, Wang 2021

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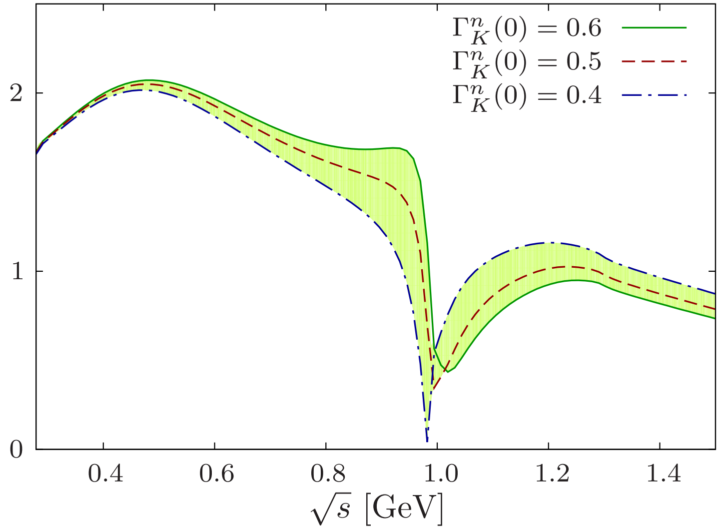
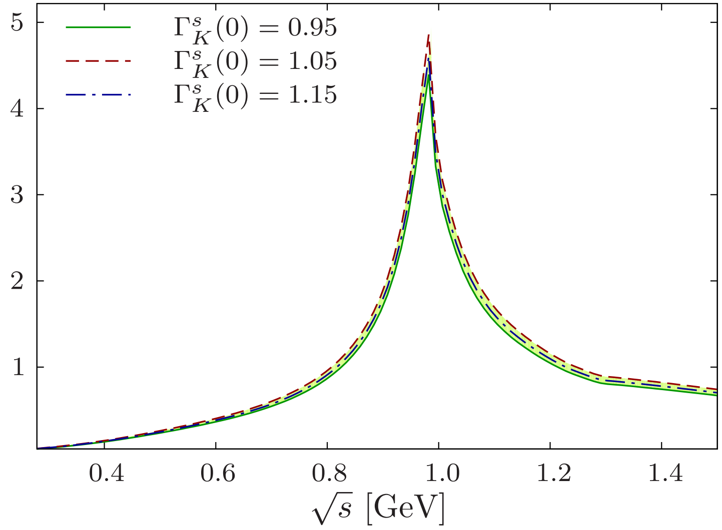
- normalisation fixed by Feynman–Hellmann theorem and ChPT:

$$\Gamma_\pi^n(0) = 0.98,\ \Gamma_K^n(0) = \{0.4...0.6\},\ \Gamma_\pi^s(0) = 0,\ \Gamma_K^s(0) = \{0.95...1.15\}$$





▷ broad bump: $f_0(500)$ / "σ"

▷ dip near $f_0(980)$ pole

▷ prominent peak of $f_0(980)$

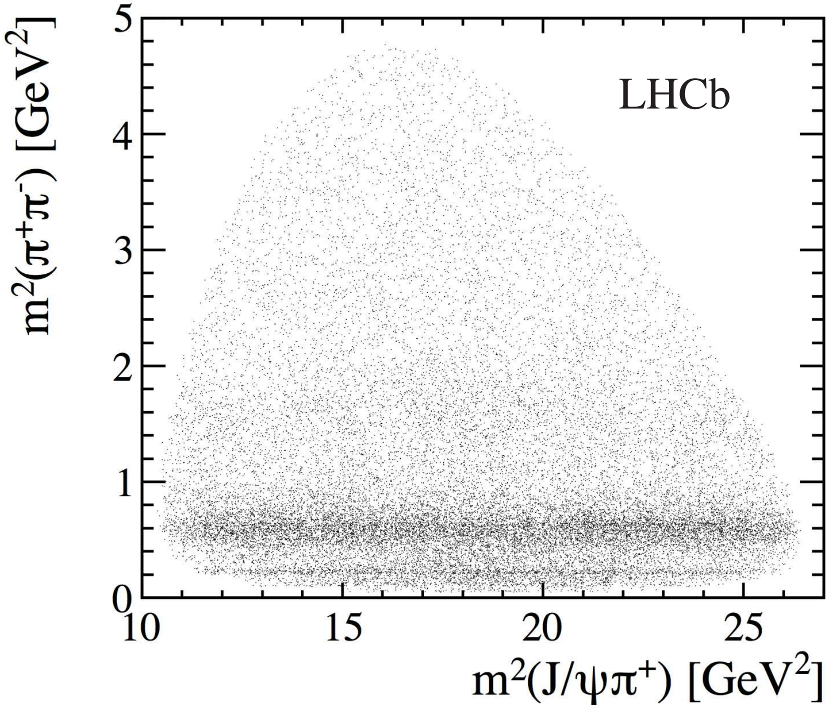
Daub, Hanhart, BK 2015

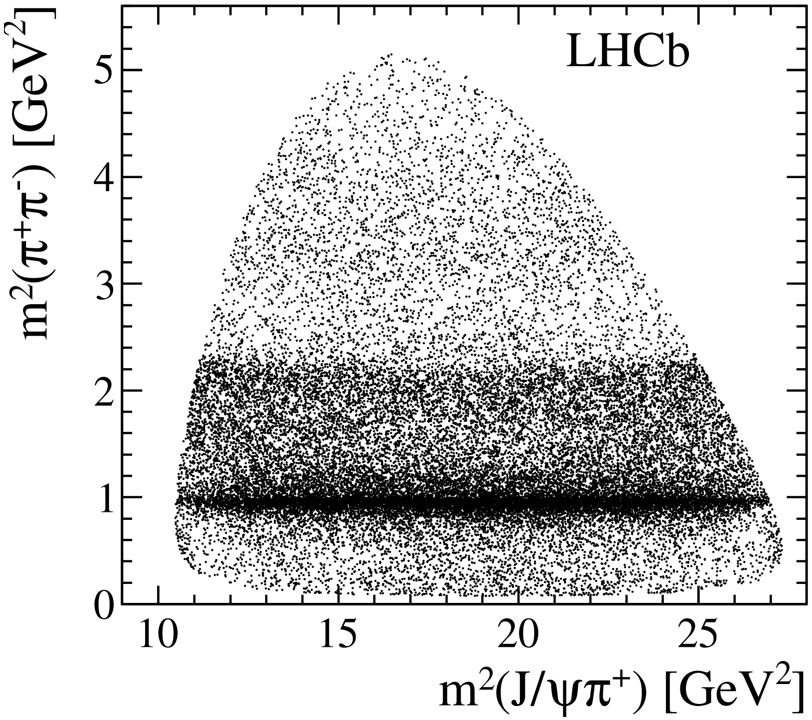
cf. also Daub, Dreiner, Hanhart, BK, Meißner 2013

Scalar form factors from decays: $\bar{B}^0_{d/s} \to J/\psi\pi\pi$

- no scalar source in SM $\longrightarrow$ test scalar form factors in decays
- experimental evidence: only $\pi\pi$ dynamics important

$\bar{B}^0_d \to J/\psi\pi^+\pi^-$ $\bar{B}^0_s \to J/\psi\pi^+\pi^-$

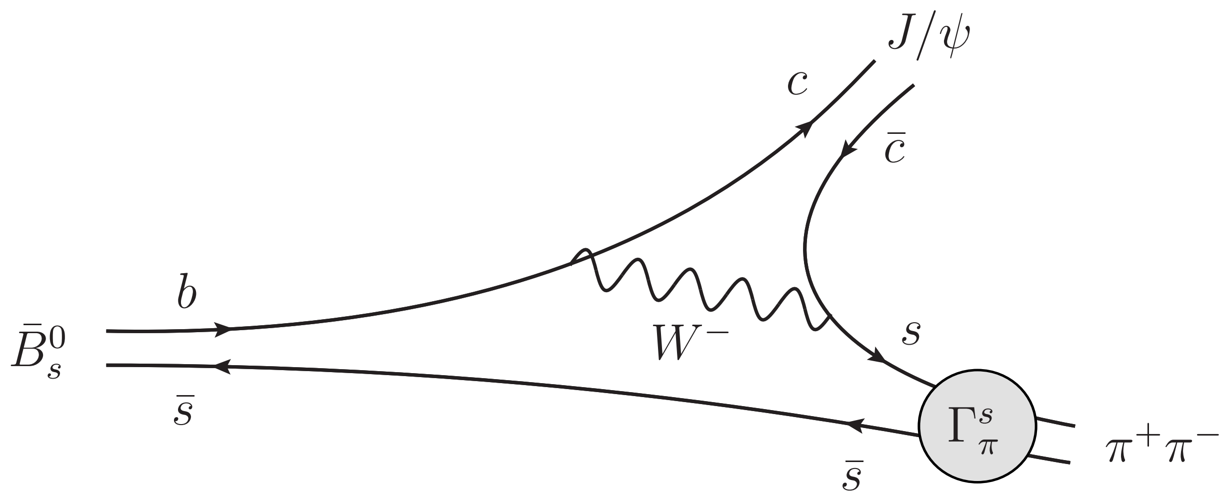




LHCb 2014

Scalar form factors from decays: $\bar{B}^0_{d/s} \to J/\psi\pi\pi$

- no scalar source in SM $\longrightarrow$ test scalar form factors in decays
- experimental evidence: only $\pi\pi$ dynamics important
- $\bar{B}^0_s \to J/\psi\pi^+\pi^-$: clean $\bar{s}s$ source $\longrightarrow$ S-wave dominated



- $\bar{B}^0_d \to J/\psi\pi^+\pi^-$: analogous $\bar{d}d$ source
  $\longrightarrow$ both isoscalar S-wave and isovector P-wave contribute

$\bar{B}_s^0 \to J/\psi\pi^+\pi^-$: extension to higher energies

- $\pi\pi$ and $K\bar{K}$ coupled channels work up to $1.1\,\mathrm{GeV}$
- beyond: strong coupling to $4\pi \longrightarrow$ phase/inelasticity description??
- resonances, e.g. $\mathcal{B}(f_0(1500) \to 4\pi) = (49.5 \pm 3.3)\%$ PDG 2018

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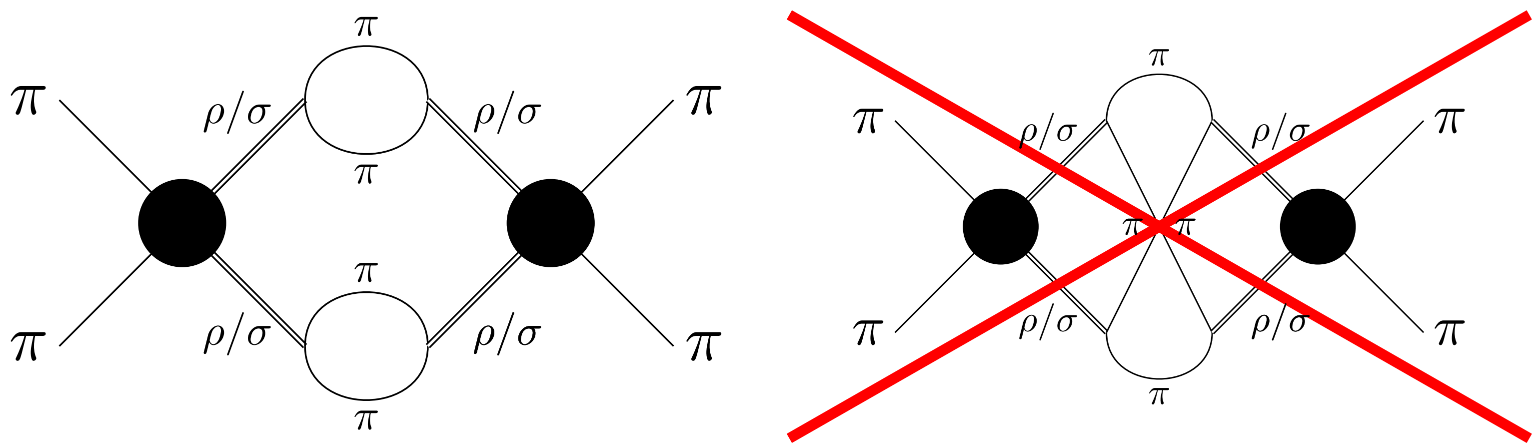
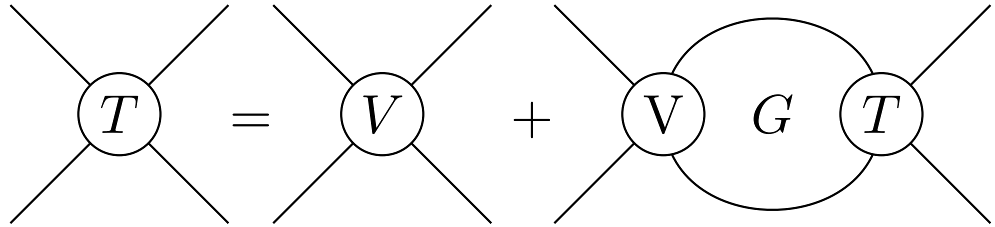
- resonances, e.g. $\mathcal{B}(f_0(1500) \to 4\pi) = (49.5 \pm 3.3)\%$ PDG 2018
- idea: couple to 4π *via* resonances, preserve unitarity Hanhart 2012
  $\longrightarrow$ Omnès at low energies, unitary isobar model above
- 4π in general very complicated; approximation: isobars $\rho\rho$ or $\sigma\sigma$
  Ropertz, Hanhart, BK 2018
- neglect crossed-channel effects:

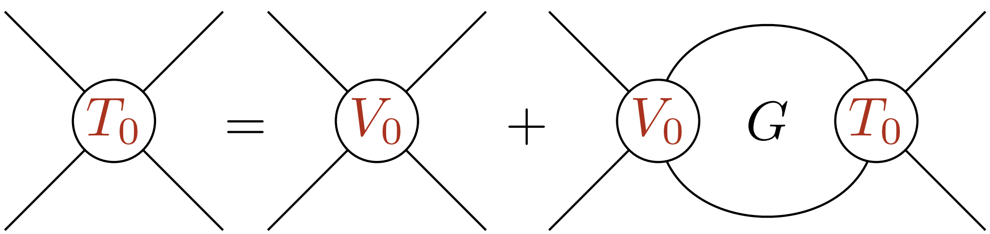


2-potential formalism: partial-wave amplitude

- Bethe–Salpeter equation for partial-wave amplitude T:

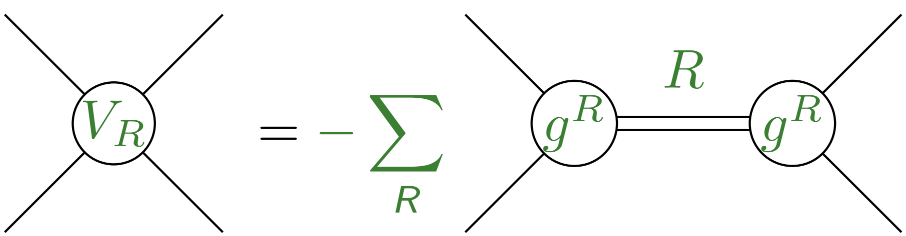


- split scattering kernel $V = V_0 + V_R \longrightarrow T = T_0 + T_R$
- unitary scattering amplitude T_0 (given by known phases and inelasticities)



$$= \begin{pmatrix} \frac{\eta e^{2i\delta}-1}{2i\sigma_\pi} & g e^{i\psi} & 0 \\ g e^{i\psi} & \frac{\eta e^{2i(\psi-\delta)}}{2i\sigma_K} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- resonance-exchange potential V_R



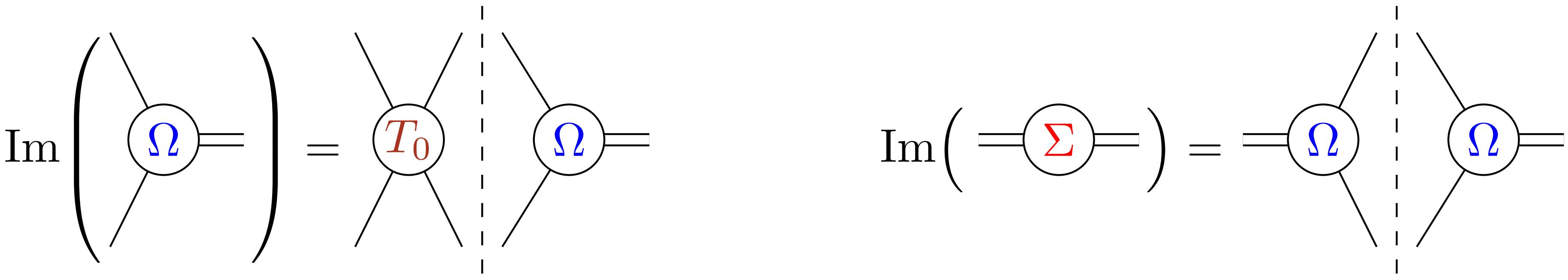
$$= -\sum_R g_i^R \frac{s}{m_R^2\,(s-m_R^2)}\, g_j^R$$

2-potential formalism: partial-wave amplitude

- full parametrisation for scattering matrix T:

$$T = T_0 + \Omega\,[\mathbb{1} - V_R\,\Sigma]^{-1}\;V_R\,\Omega^t$$

vertex factor $\Omega(s)$ self energy $\Sigma(s)$



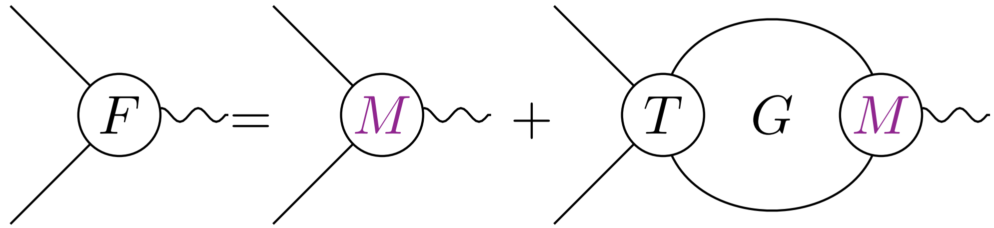
$$\Omega_{ij}(s) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} {\rm d}z\,\frac{(T_0)^*_{ik}(z)\sigma_k(z)\Omega_{kj}(z)}{z-s-i\epsilon} \qquad \Sigma_{ij}(s) = \frac{s}{\pi}\int_{s_{\rm th}}^{\infty}\frac{{\rm d}z}{z}\,\frac{\Omega^*_{ki}(z)\sigma_k(z)\Omega_{kj}(z)}{z-s-i\epsilon}$$

- additional channels: $(T_0)_{ij} = 0 \longrightarrow$

$$\Omega_{ij}(s) = \delta_{ij} \quad \text{and} \quad \Sigma_{ij}(s) = \delta_{ij}\,\frac{s}{\pi}\int_{s_{\rm th}}^{\infty}\frac{{\rm d}z}{z}\,\frac{\sigma_i(z)}{z-s-i\epsilon}$$

2-potential formalism: form factors

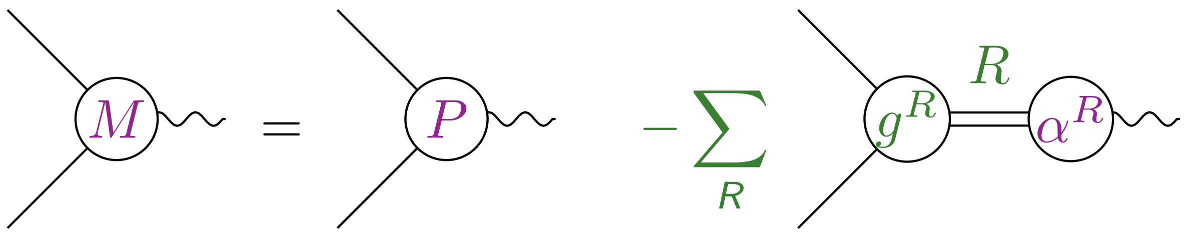
- coupling to a source/current:



- full parametrisation for form factor F:

$$\boxed{F = \Omega\,[\mathbb{1} - V_R\,\Sigma]^{-1}\,M}$$

- source term $M(s)$:



$$M_i(s) = a_i + b_i\,s + \ldots - \sum_R g_i^R\,\frac{s}{s - m_R^2}\,\alpha^R$$

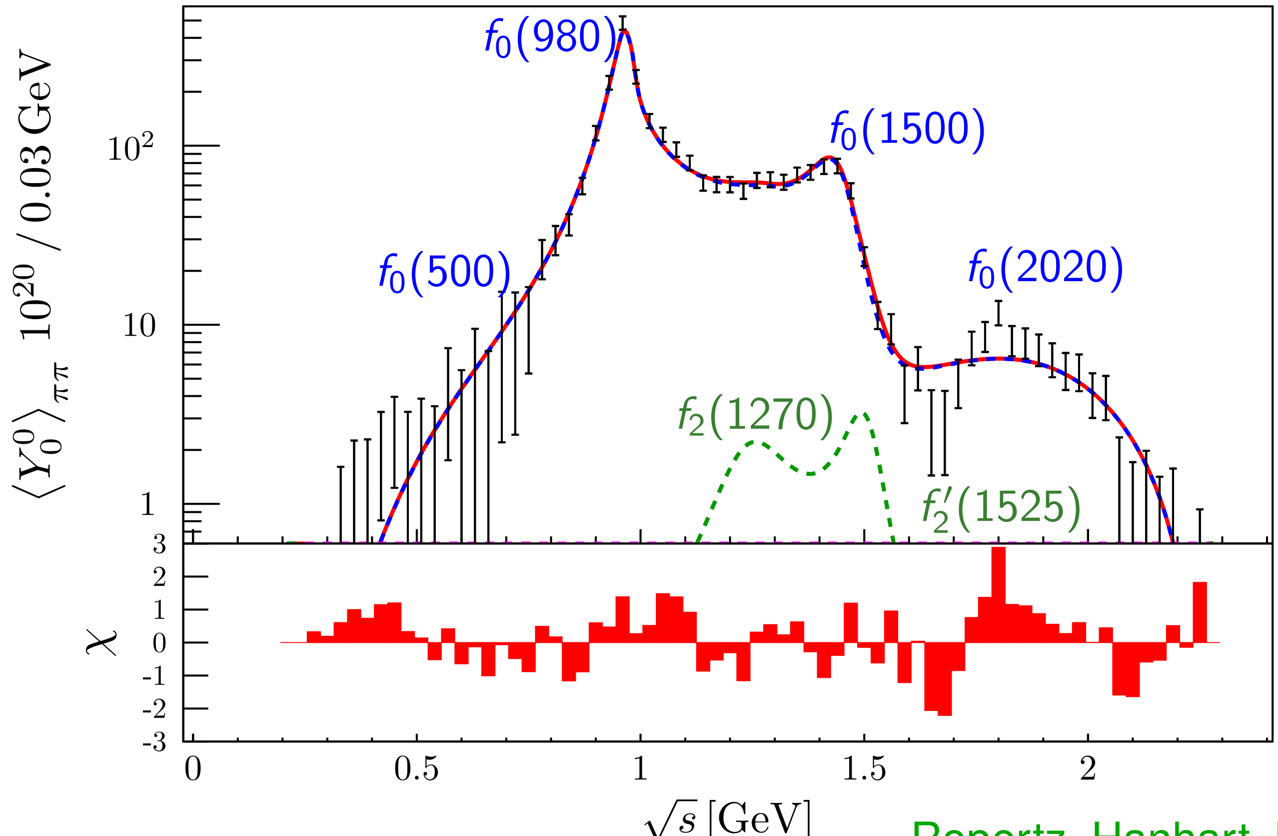
$\longrightarrow$ new parameters: resonance masses (m_R),
resonance–source (α^R) and resonance–channel (g_i^R) couplings

Application to $\bar{B}_s^0 \to J/\psi\pi^+\pi^-$

- fit to angular moments in full energy range — LHCb 2014

$$\langle Y_l^0\rangle(s) = \int_{-1}^{1} \frac{d^2\Gamma}{d\sqrt{s}\,d\cos\theta_\pi} Y_l^0(\cos\theta_\pi)\,d\cos\theta_\pi$$

- higher partial waves modelled by Breit–Wigner functions



Ropertz, Hanhart, BK 2018

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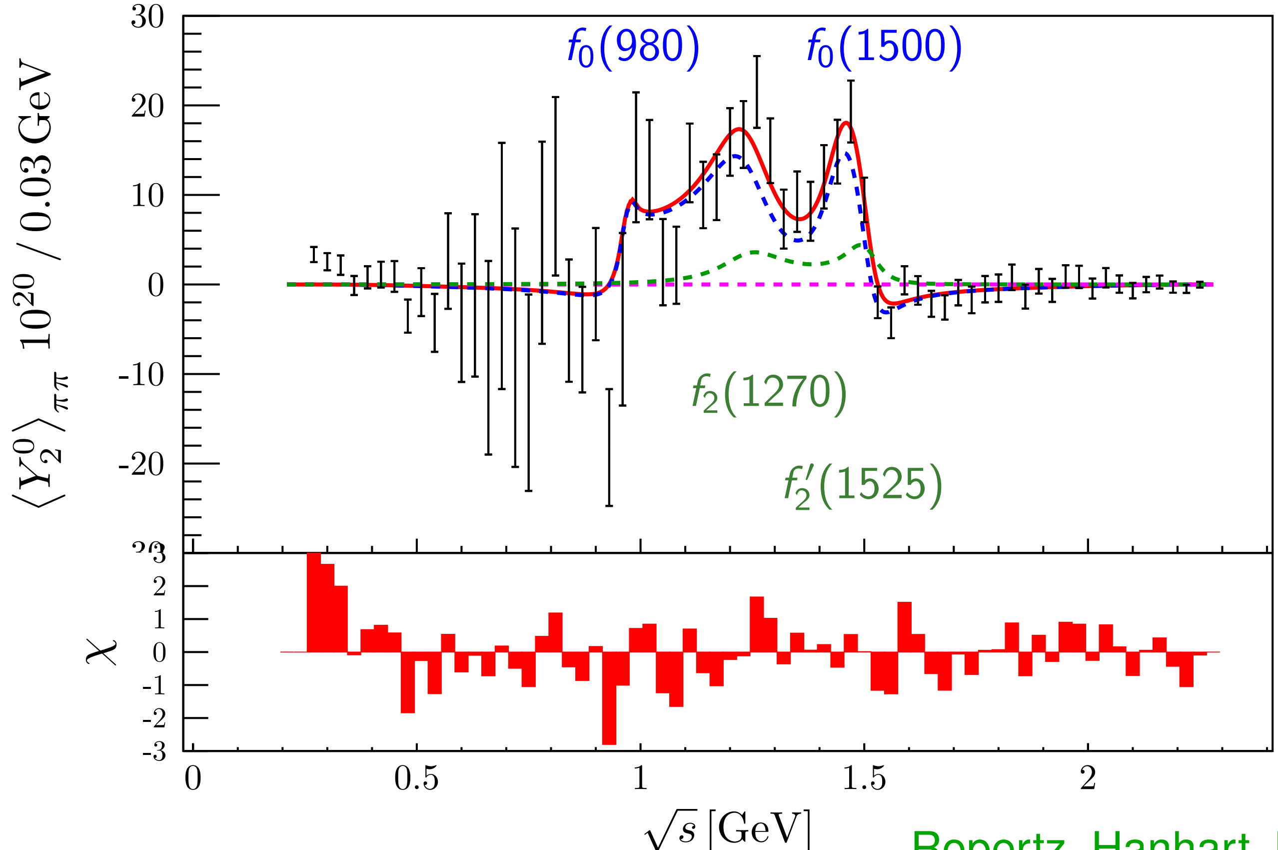
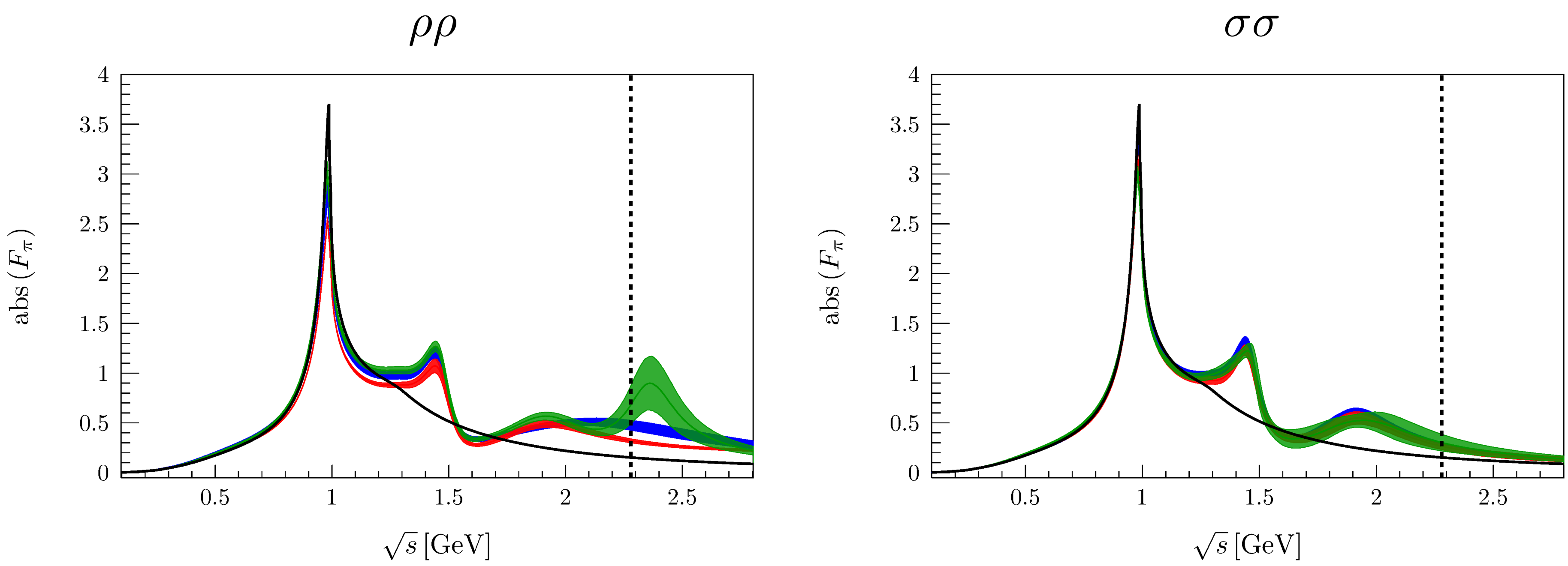


Ropertz, Hanhart, BK 2018

Results: strange scalar pion form factor

- comparison of different fit variants: Ropertz, Hanhart, BK 2018



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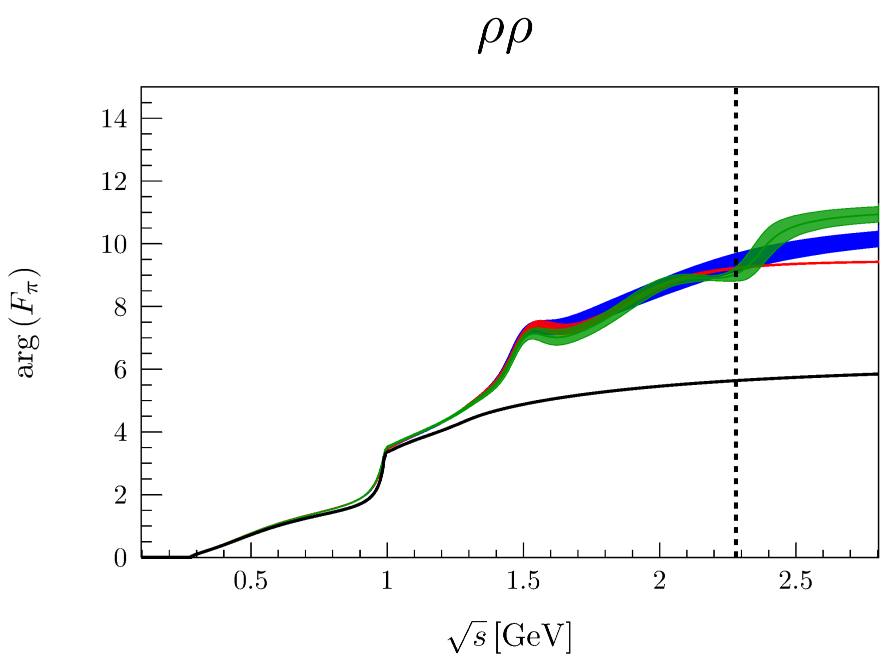

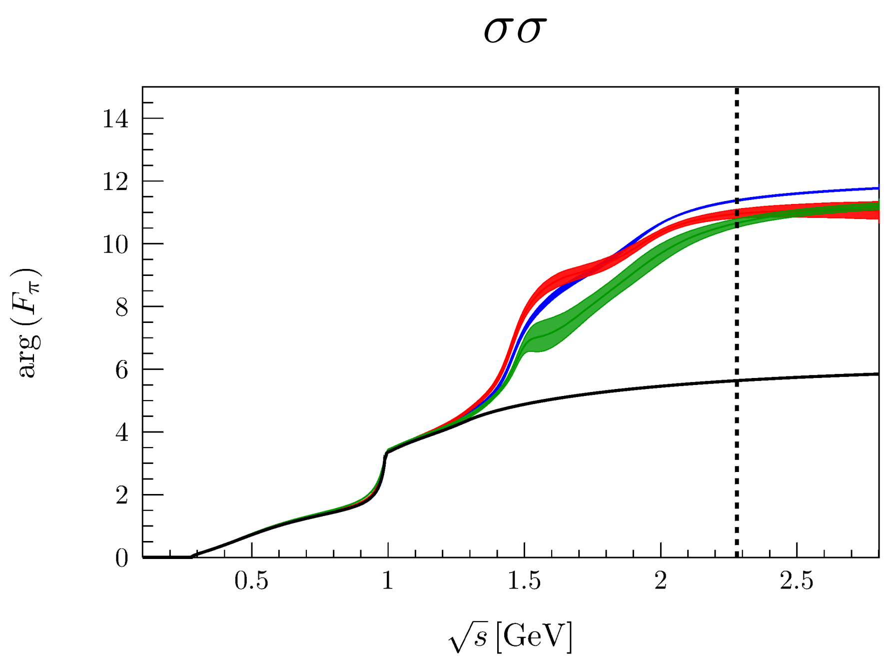

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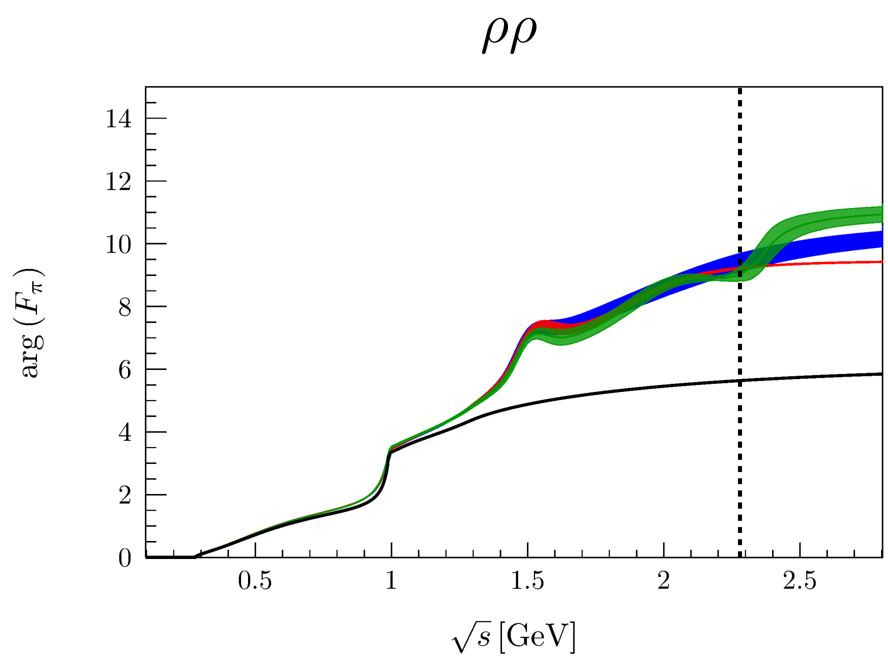

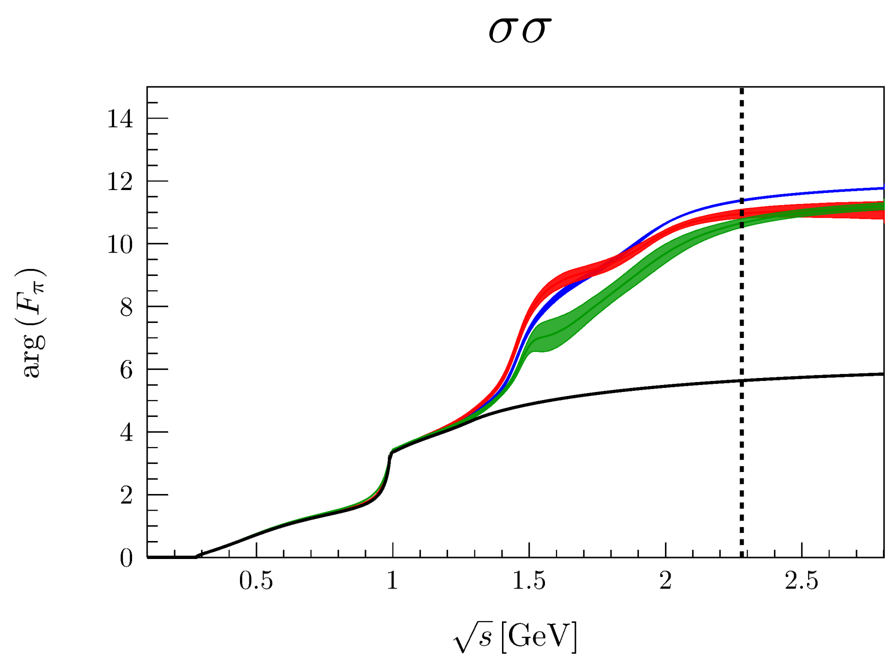

- also: Padé extraction of pole parameters for

$$f_0(500),\ f_0(980),\ f_0(1500),\ f_0(2020)$$

- new application of 2-potential formalism:

pole parameters $\leftrightarrow$ lineshapes

Heuser, Chanturia, Guo, Hanhart, Hoferichter, BK 2024

πK scalar form factor extended to higher energies

- extension from pion–pion to pion–kaon S-wave
- πK: phase shifts from Roy–Steiner equations Peláez, Rodas 2020
  + $\eta' K$ coupled via $K_0^*(1430)$, $K_0^*(1950)$

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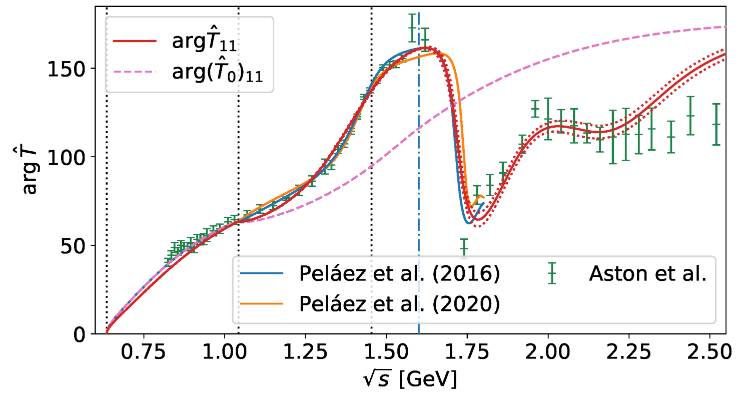
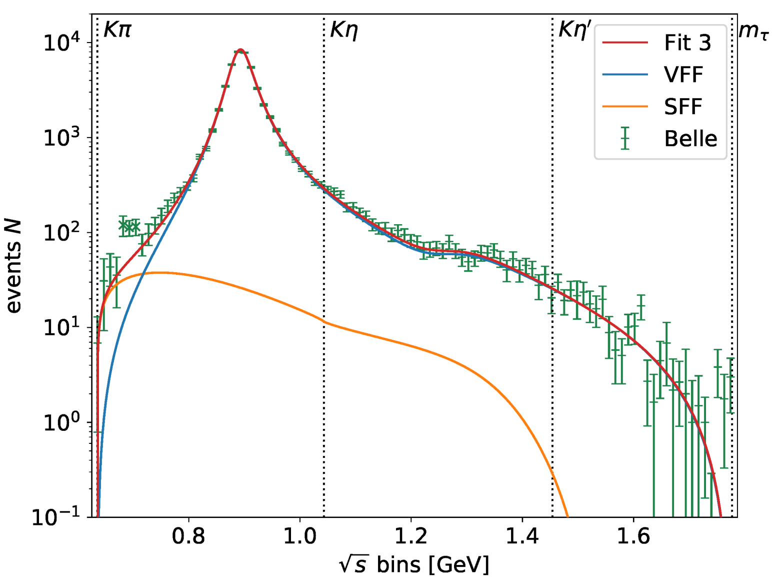
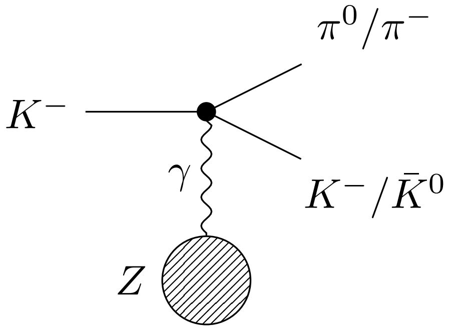


- phase shifts well reproduced
- application to $\tau \to K_s \pi^0 \nu_\tau$:
  scalar ff. beneath dominant vector;
  better separation of $K_0^*(1430)$
  and $K^*(1410)$
- improved estimate of CP-odd
  asymmetry induced by tensor op.
- extraction of K_0^* pole positions
  + residues von Detten et al. 2021



Primakoff reaction $\gamma K \to \pi K$

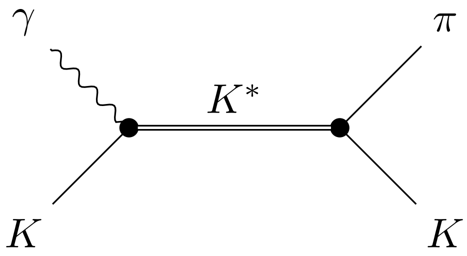
- AMBER: upgrade pion to kaon beam
- pion production in the Coulomb field of a heavy nucleus



- combine knowledge about
  - ▷ chiral anomaly at $s = t = u = 0$

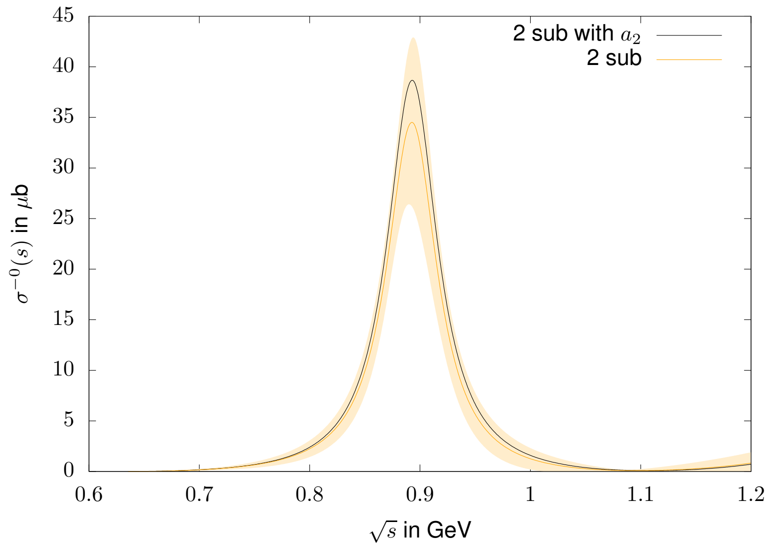
$$F_{KK\pi\gamma} = \frac{e}{4\pi^2 F_\pi^3}$$

  - ▷ radiative $K^*(892)$ couplings



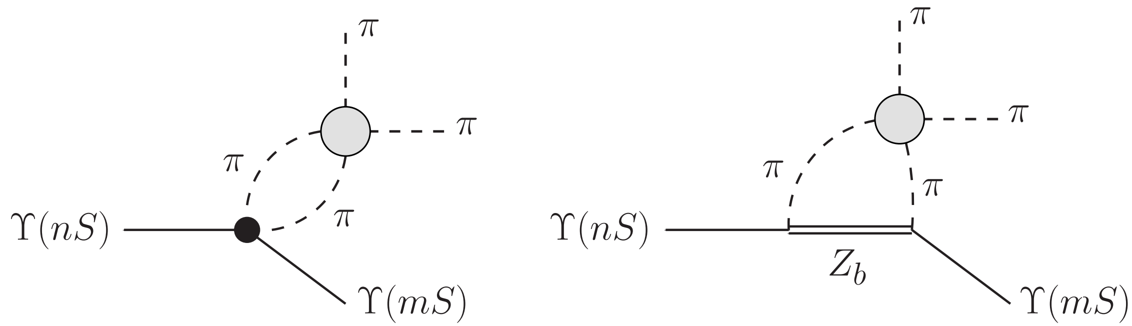
- amplitude representation based on
  πK phase shifts — Peláez, Rodas 2020
  + phenomenological left-hand cuts — Dax, Stamen, BK 2021

$\longrightarrow$ link anomaly to cross sections as for $\gamma\pi \to \pi\pi$ — Hoferichter et al. 2012, 2017



Left-hand cuts: $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$

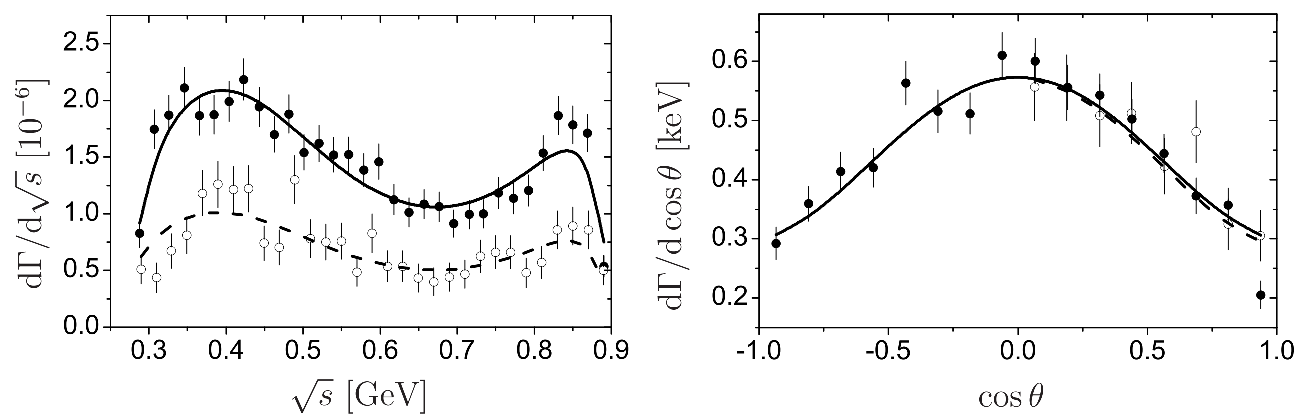
- inclusion of Z_b exchanges as left-hand cuts:



- formally: $\hat{M}(s)$ partial-wave projection of Z_b exchanges

$$M(s)+\hat{M}(s), \qquad M(s) = \Omega(s)\left\{P^{n-1}(s)+\frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx}{x^2}\frac{\hat{M}(x)\sin\delta(x)}{|\Omega(x)|(x-s)}\right\}$$

$\longrightarrow$ e.g. two peaks in $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ reproduced:



Chen, Daub, Guo, BK, Meißner, Zou 2015

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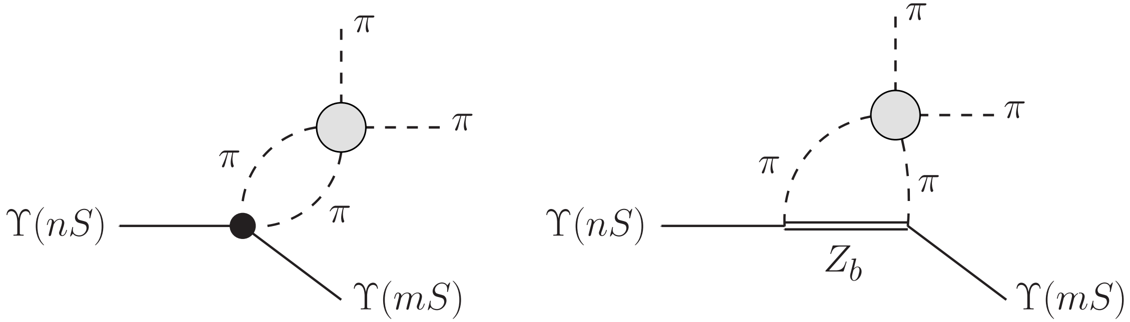

- formally: $\hat{M}(s)$ partial-wave projection of Z_b exchanges

$$M(s) + \hat{M}(s), \qquad M(s) = \Omega(s)\left\{P^{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}x}{x^2}\frac{\hat{M}(x)\sin\delta(x)}{|\Omega(x)|(x-s)}\right\}$$

- generalisation for $\Upsilon(4S)$, $\Upsilon(10860)$ decays:
  - ▷ add open-flavour $(B^{(*)}\bar{B}^{(*)})$ loops to left-hand cuts
  - ▷ generalise to $\pi\pi/K\bar{K}$ coupled channels for *S*-waves

Chen, Cleven, Daub, Guo, Hanhart, BK, Meißner, Zou 2016

Baru, Epelbaum, Filin, Hanhart, Mizuk, Nefediev, Ropertz 2021

Diffractive 3π production at COMPASS

- large data set on
  diffractive $\pi^-\pi^-\pi^+$ production
- $m_{3\pi}$ mass range from 0.5 to $2.5\,\mathrm{GeV}$
  set of 88 partial waves
- question for COMPASS PWA framework:
  to what extent and how are the
  $\pi^+\pi^-$ partial waves modified
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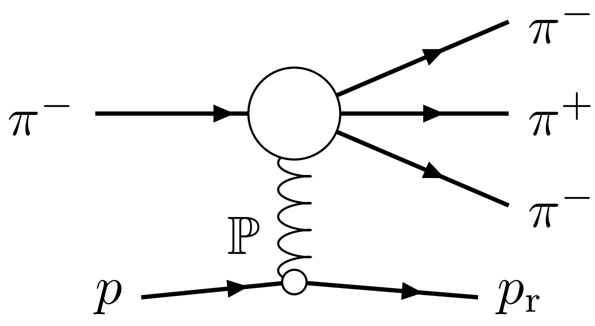

COMPASS 2015

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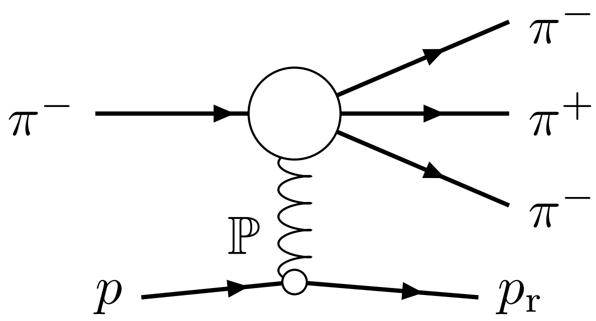
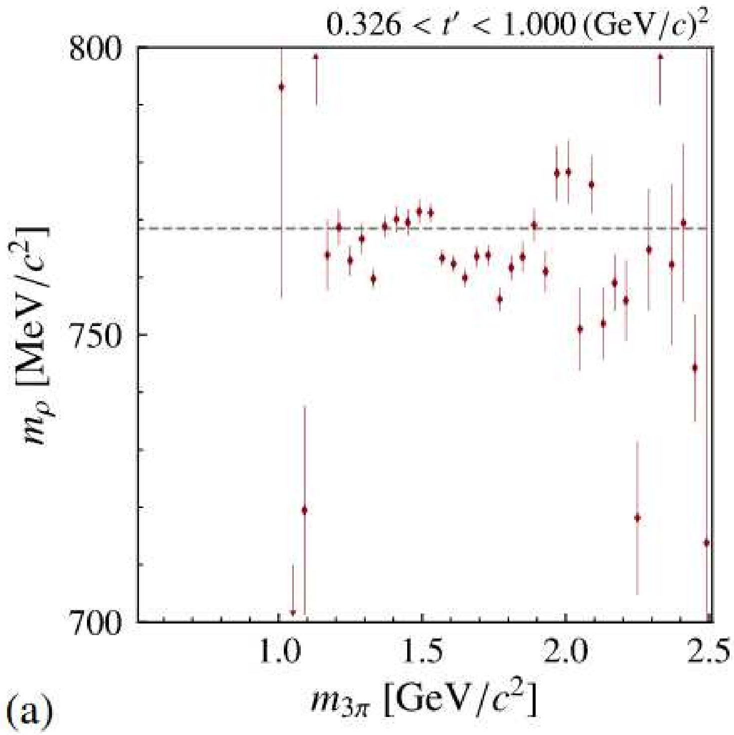
- ρ-mass dependence on $m_{3\pi}$?

COMPASS 2022



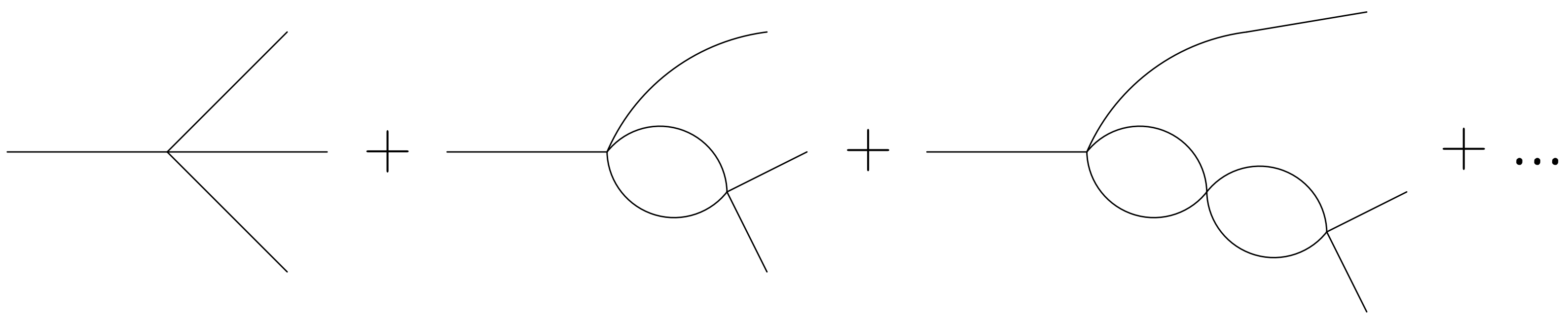
COMPASS 2015



(a)

Rescattering beyond the isobar model

- different decays into $\pi\rho$ final states
- effects of the ρ described by $\pi\pi$ P-wave phase shift



Rescattering beyond the isobar model

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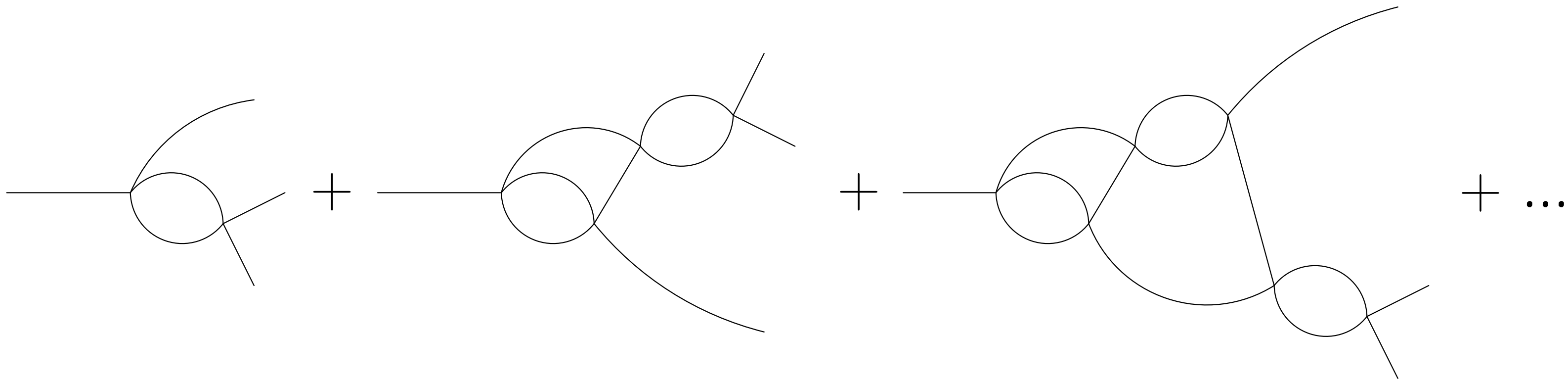
- full rescattering effects via Khuri–Treiman equations

Khuri, Treiman 1960



$\longrightarrow$ what statistics necessary to reject naïve isobar model (≙ undistorted ρ lineshape) at 5σ significance?

Stamen, Isken, BK, Mikhasenko, Niehus 2023

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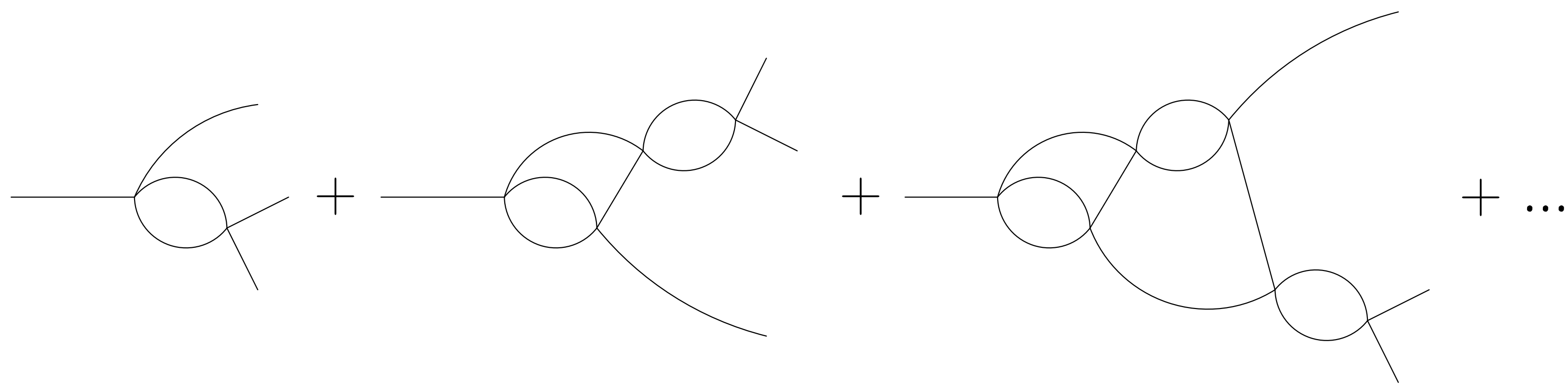

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Stamen, Isken, BK, Mikhasenko, Niehus 2023

- $\pi\rho$ decays of

$$J^{PC} = \quad 0^{--} \text{ [exotic]} \quad 1^{--} \, [\omega, \phi] \quad 1^{-+} \, [\pi_1] \quad 2^{++} \, [a_2]$$

$\longrightarrow 1^{-+}$ and 2^{++} relevant for COMPASS

Khuri–Treiman equations, example: $\omega/\phi \to 3\pi$

- decay amplitude $\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$
- unitarity relation for $\mathcal{F}(s)$:

$$\text{disc}\,\mathcal{F}(s) = 2i\{\underbrace{\mathcal{F}(s)}_{\text{right-hand cut}} + \underbrace{\hat{\mathcal{F}}(s)}_{\text{left-hand cut}}\} \times \theta(s - 4\,M_\pi^2) \times \sin\,\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

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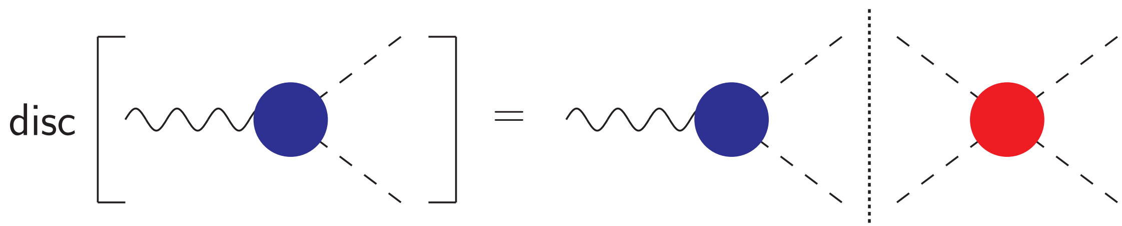
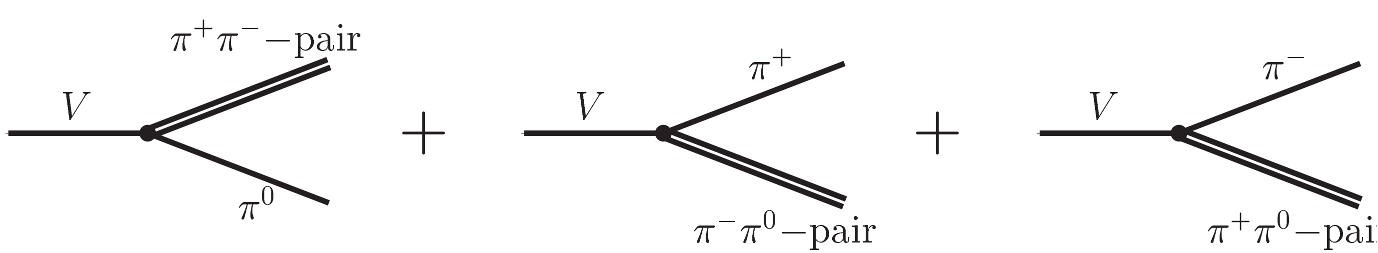


- right-hand cut only $\longrightarrow$ Omnès problem

$$\mathcal{F}(s) = a\,\Omega(s)\;, \qquad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_1^1(s')}{s'-s}\right\}$$



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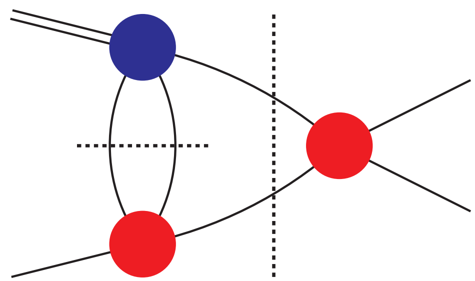
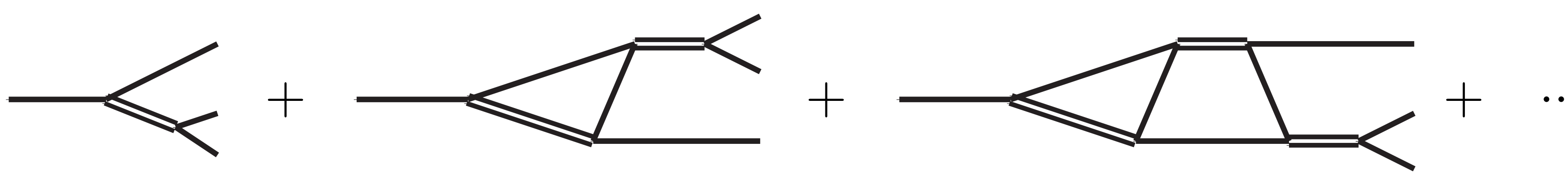
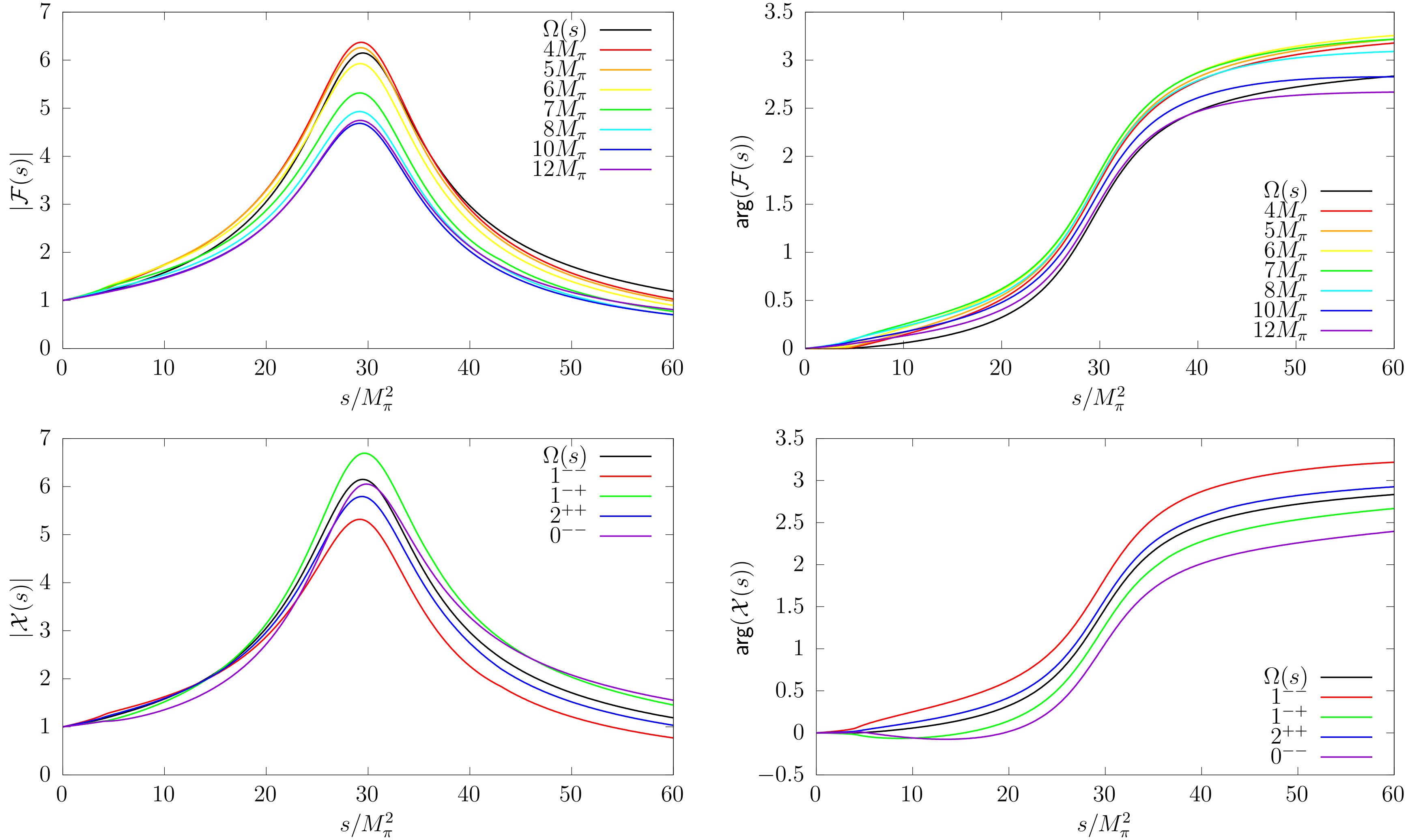


- inhomogeneities $\hat{\mathcal{F}}(s)$: partial-wave projections of $\mathcal{F}(t)$, $\mathcal{F}(u)$

$$\mathcal{F}(s) = a\,\Omega(s)\left\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

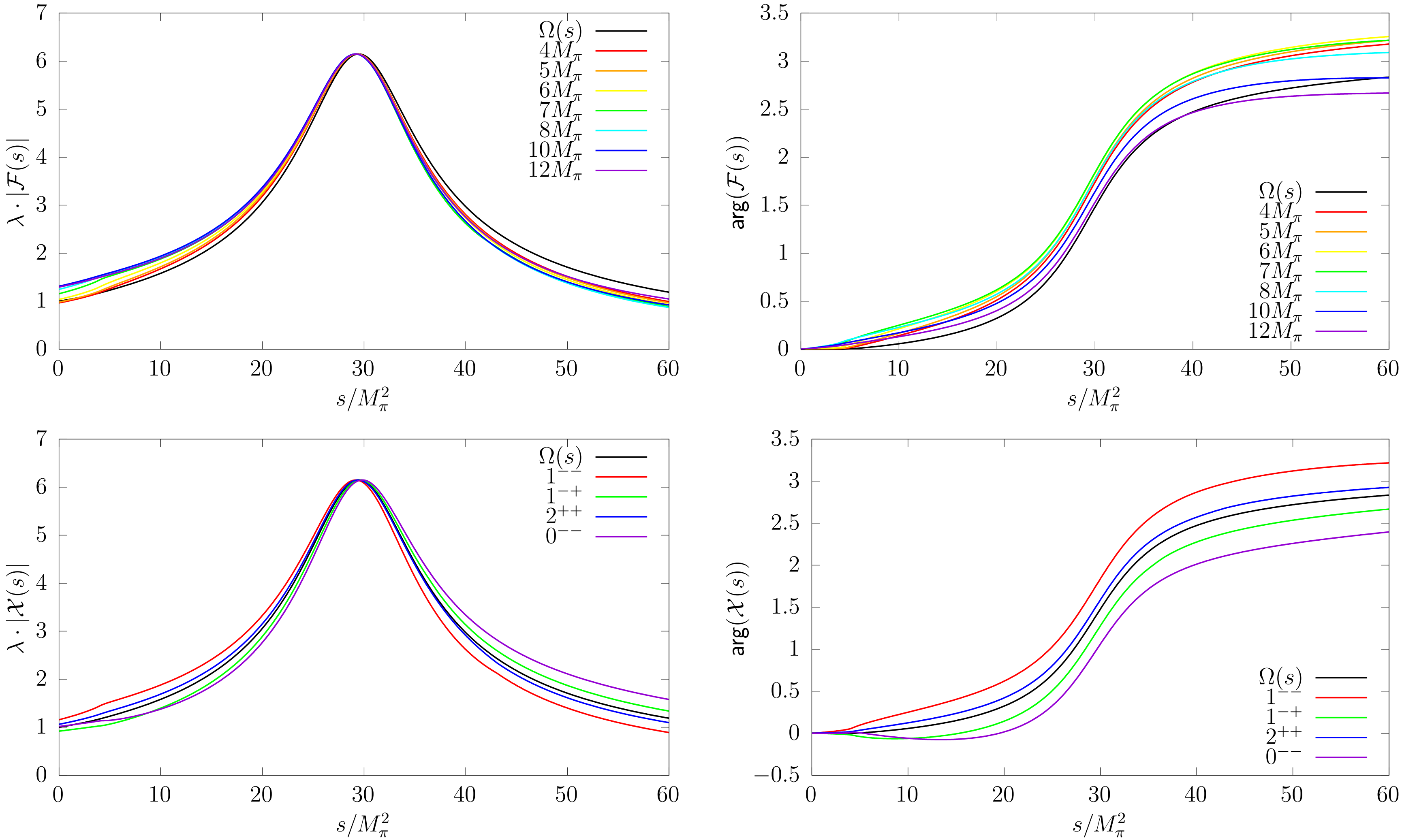


Basis functions (normalised $s = 0$)



Stamen, Isken, BK, Mikhasenko, Niehus 2023

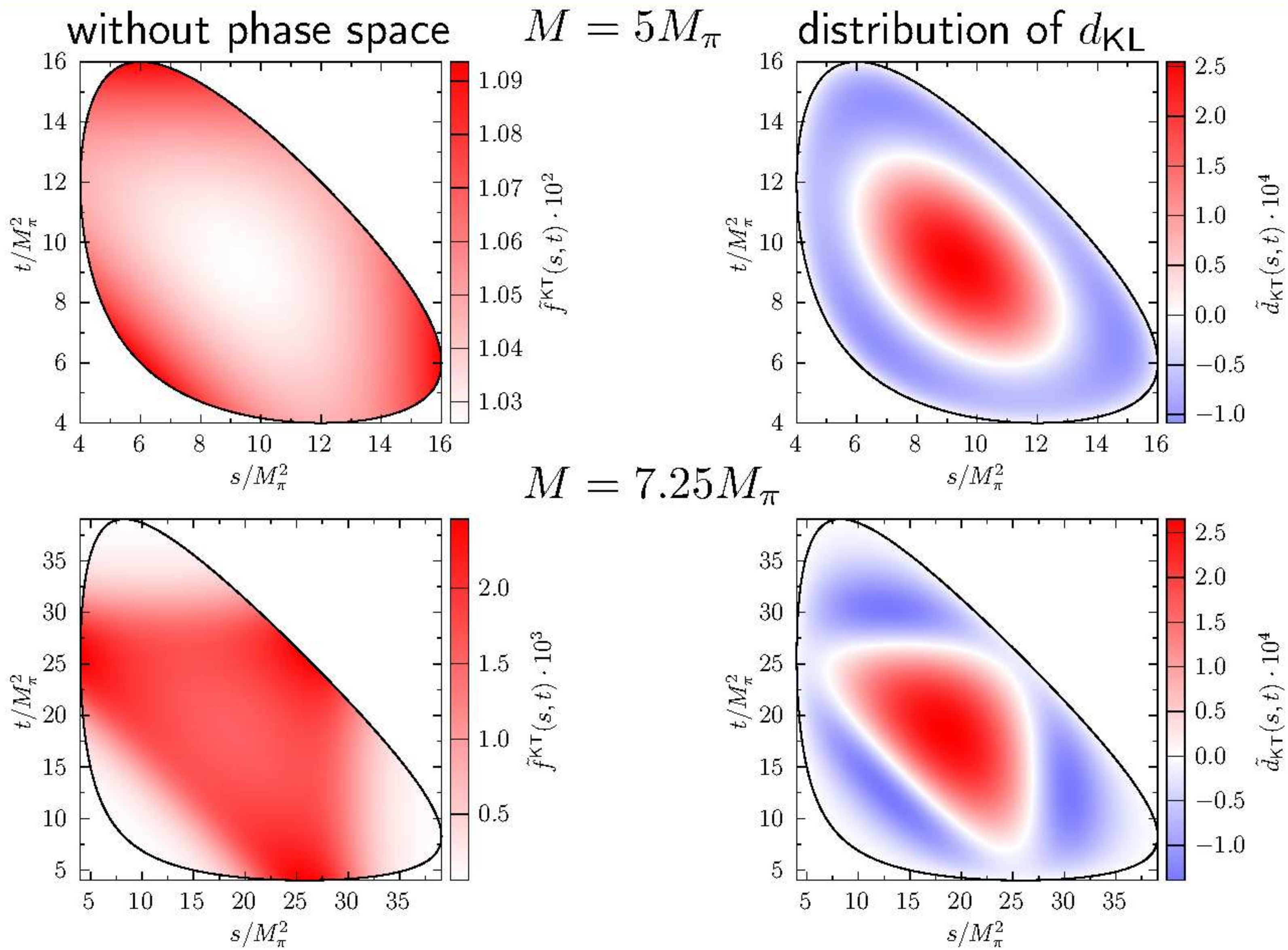
Basis functions (normalised $s = M_\rho^2$)



Stamen, Isken, BK, Mikhasenko, Niehus 2023

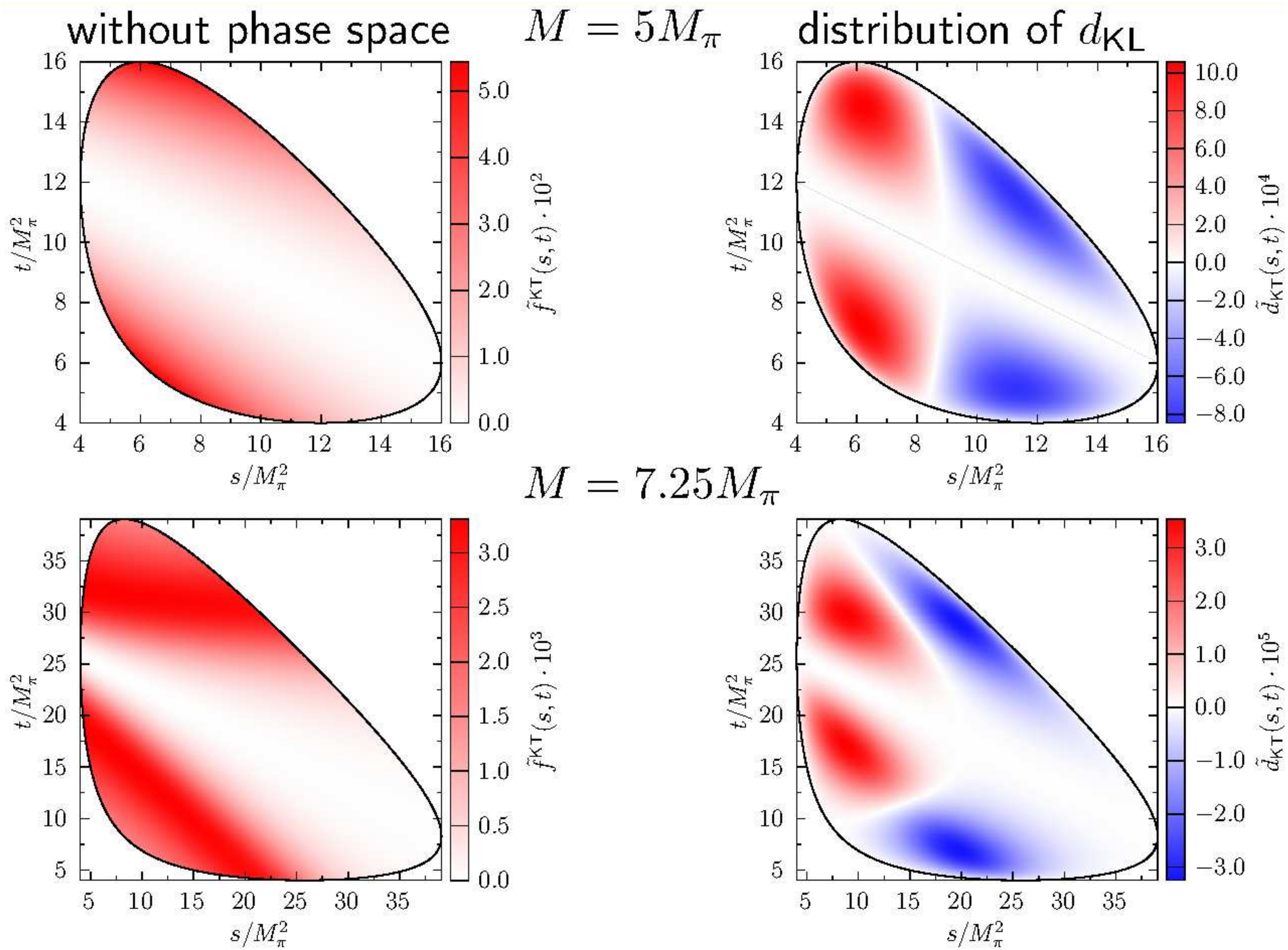
- KT = truth — can Omnès functions reproduce these?

Dalitz plots for 1^{--}: ω, ϕ



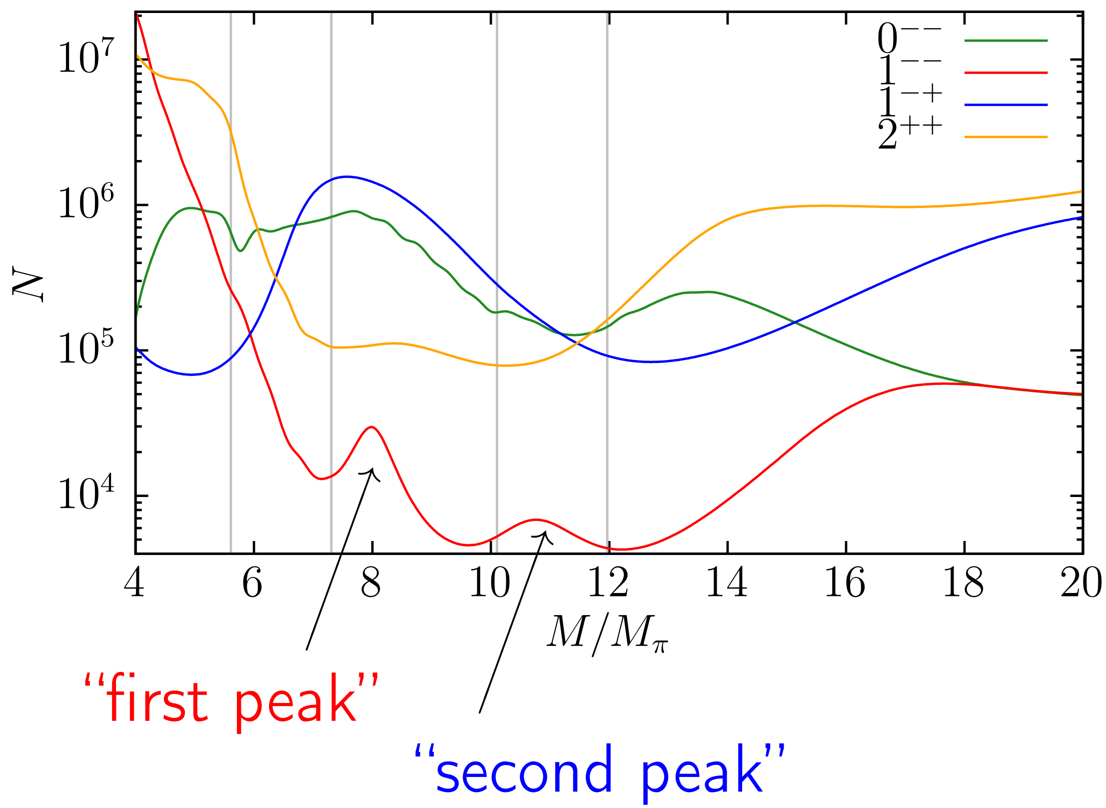
Stamen, Isken, BK, Mikhasenko, Niehus 2023

Dalitz plots for 1^{-+}: π_1



Stamen, Isken, BK, Mikhasenko, Niehus 2023

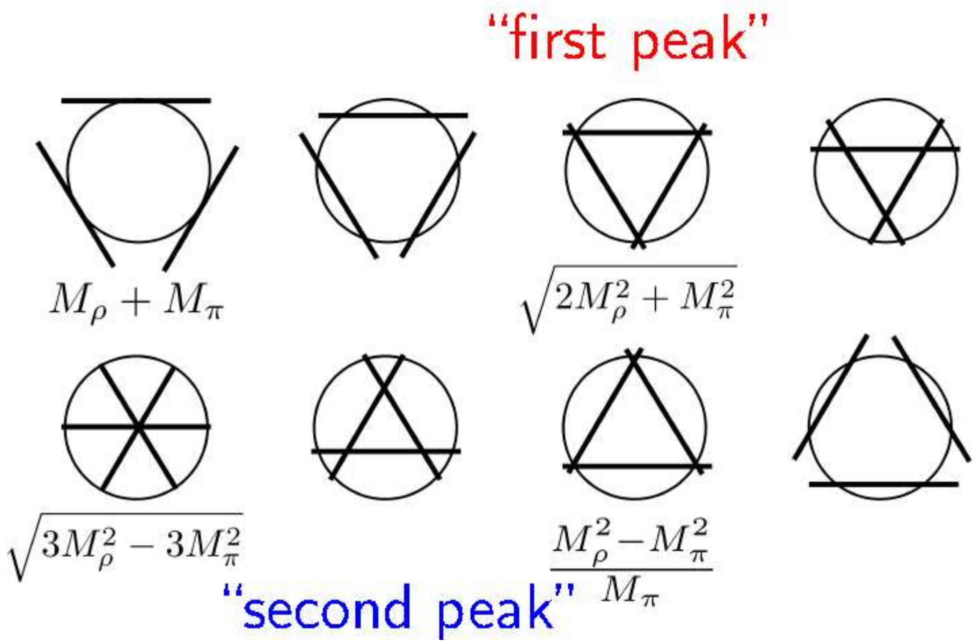
Results for 5σ significance



- dominated by J^{PC}; rise for $M \to \infty$ (but inelastic effects)
- $\mathcal{O}(10^5) - \mathcal{O}(10^6)$ events for all decay masses in 0^{--}, 1^{-+}, 2^{++}
- strong decay-mass dependence for 1^{--}:
  $\omega \to 3\pi$ small, $\phi \to 3\pi$ favourable BESIII 2018; KLOE 2003

Schematic Dalitz plots

- schematic Dalitz plots for different decay masses

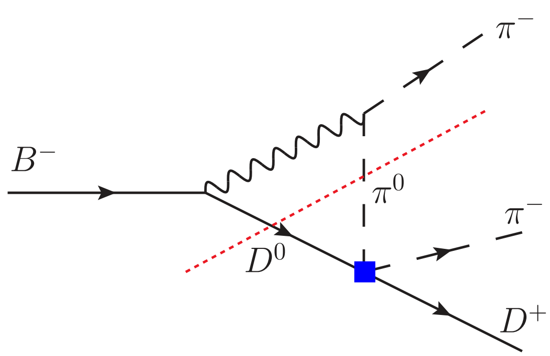


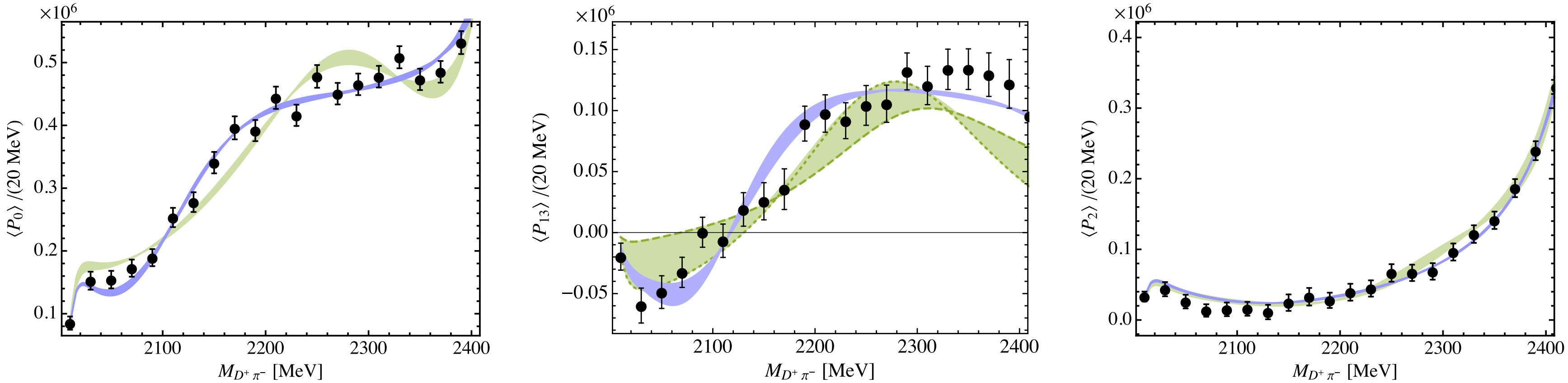
- difference decreases when ρ bands inside the Dalitz plot
- "first peak" at $\approx \sqrt{2M_\rho^2 + M_\pi^2} \approx 1100\,\text{MeV}$
- "second peak" also largely kinematic effect

Stamen, Isken, BK, Mikhasenko, Niehus 2023

Modified lineshapes beyond 3π

- $D\pi$ distribution in $B^- \to D^+\pi^-\pi^-$: Du, Guo, Hanhart, BK, Meißner 2021
  - ▷ strong charge-exchange reaction
    $\mathcal{B}(B^- \to D^0\rho^-) \gg \mathcal{B}(B^- \to D^+\pi^-\pi^-)$
  - ▷ $D\pi$ *S*-wave scattering phase from
    UChPT ($\chi^2/\text{dof} = 1.2$) Du et al. 2018
    vs. BW ($\chi^2/\text{dof} = 2.0$)





- confirms lightest D_0^* near 2.1 GeV
- cf. modified $K\pi$ *S*-wave in $D^+ \to K^-\pi^+\pi^+$ Niecknig, BK 2018

BSM: Dalitz plot asymmetries in $\eta \to \pi^+\pi^-\pi^0$

- $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$ breaks *G*-parity:
  - ▷ SM: *C* conserved, isospin broken (& el.magn. suppressed)
    $\longrightarrow$ ideal process to extract $m_u - m_d$

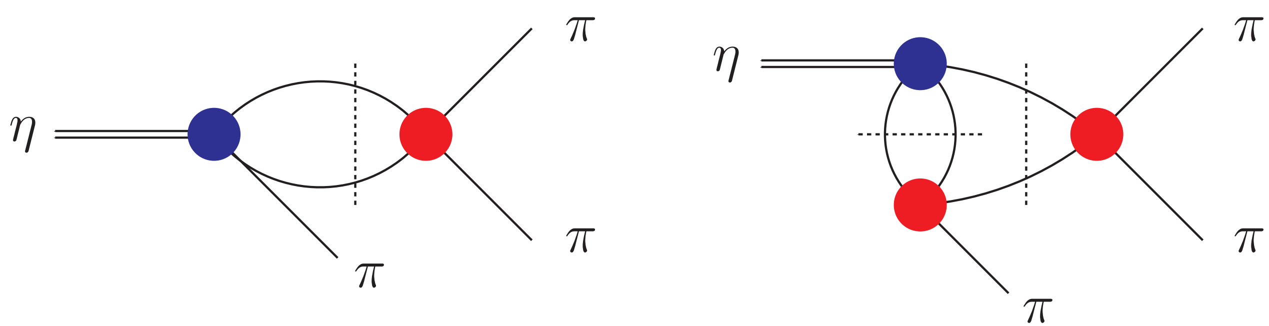
    see e.g. Bijnens, Ghorbani 2007; Colangelo et al. 2018 ...
  - ▷ BSM: *C* broken, isospin either conserved or broken

$$\mathcal{M}(s,t,u) = \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not C}(s,t,u) + \mathcal{M}_2^{\not C}(s,t,u)$$

- interference: $\pi^+ \leftrightarrow \pi^-$ asymmetries linear in BSM couplings

  Gardner, Shi 2019
- follow SM strategy for hadronic amplitudes: Akdag, Isken, BK 2021

  analyse $\mathcal{M}_{0,2}^{\not C}(s,t,u)$ using dispersive Khuri–Treiman framework



$\eta \to \pi^+\pi^-\pi^0$: amplitude decomposition

- Bose symm.: even (odd) $\pi\pi$ isospin $\leftrightarrow$ even (odd) partial waves
- "reconstruction theorem": symmetrised partial-wave expansion

$$\mathcal{M}_1^C(s,t,u) = \mathcal{F}_0(s) + (s-u)\,\mathcal{F}_1(t) + (s-t)\,\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not C}(s,t,u) = (t-u)\,\mathcal{G}_1(s) + (u-s)\,\mathcal{G}_1(t) + (s-t)\,\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not C}(s,t,u) = 2(u-t)\,\mathcal{H}_1(s) + (u-s)\,\mathcal{H}_1(t) + (s-t)\,\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

$\longrightarrow$ rescattering for *S*- and *P*-waves

Gardner, Shi 2019

cf. also Bernard et al. 2024 for $K \to 3\pi$

- note: *C*-even/odd $\leftrightarrow$ even/odd under $t \leftrightarrow u$
- Omnès solutions ($\mathcal{A}_I = \mathcal{F}_I, \mathcal{G}_I, \mathcal{H}_I$):

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx}{x^n}\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(x)}{|\Omega_I(x)|\,(x-s)}\right)$$

▷ $P_{n-1}(s)$: subtraction polynomial, free parameters

$\eta \to \pi^+\pi^-\pi^0$: parameters, data

SM amplitude $\mathcal{M}_1^C$

- minimal subtraction scheme: 3 (real) constants
- "data" fit to
  - ▷ KLOE Dalitz plot $\eta \to \pi^+\pi^-\pi^0$ KLOE 2016
  - ▷ A2 Dalitz plot $\eta \to 3\pi^0$ A2 2018
  - ▷ chiral constraints [at $\mathcal{O}(p^4)$] Colangelo et al. 2018

$\longrightarrow$ χ^2/dof ≈ 1.054, works very well!

BSM amplitude $\mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$

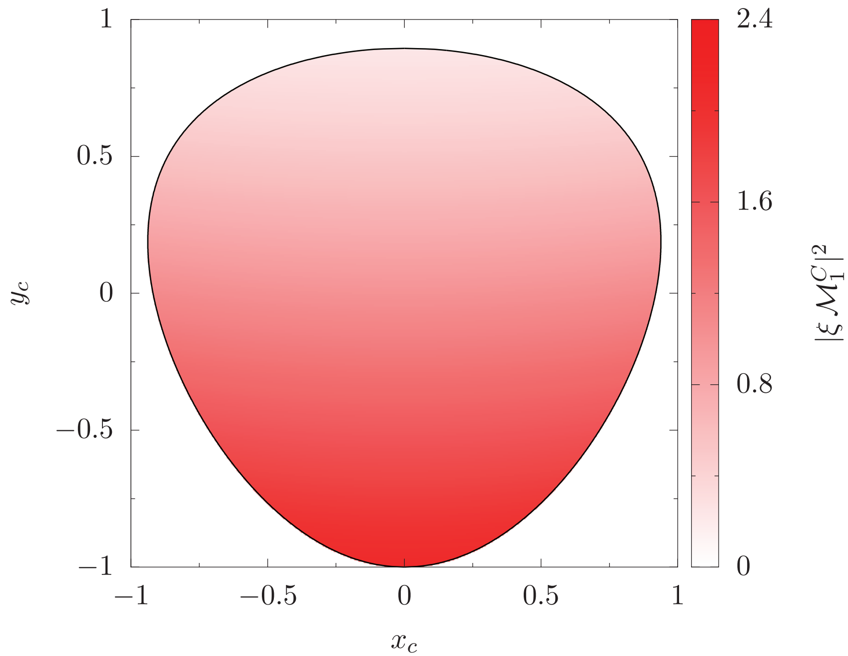
- by same assumptions: 1 imaginary subtraction each for $\mathcal{M}_{0,2}^{\not C}$

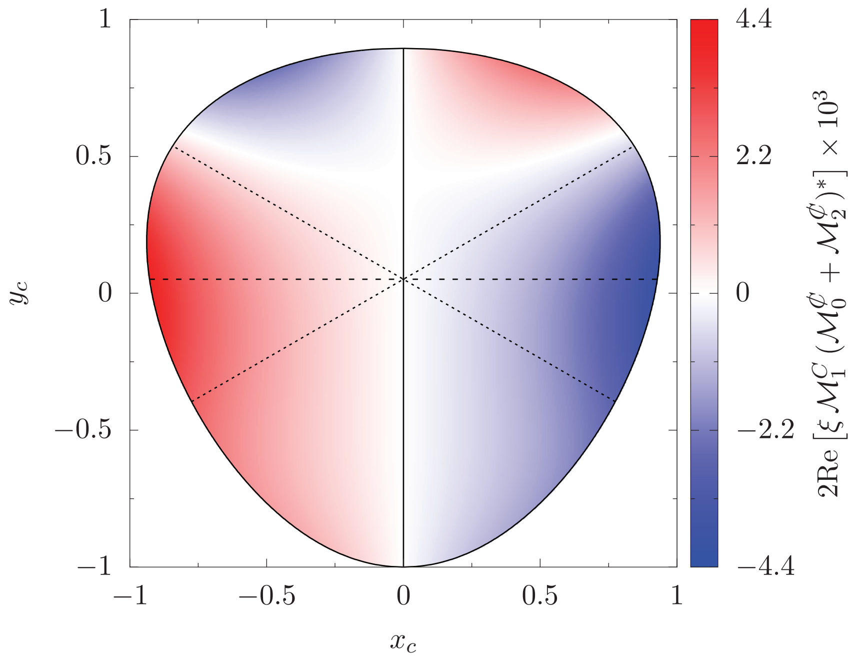
  act as overall normalisation constants $\longrightarrow$ χ^2/dof ≈ 1.048
- all *C*-/*CP*-violating signals vanish within $(1-2)\sigma$

$\eta \to \pi^+\pi^-\pi^0$: Dalitz plot asymmetries

- Dalitz plot decomposition (central fit result)

$$\left|\mathcal{M}_c\right|^2 \approx \left|\mathcal{M}_1^C\right|^2 + 2\mathrm{Re}\left[\mathcal{M}_1^C \left(\mathcal{M}_0^{\not C}\right)^*\right] + 2\mathrm{Re}\left[\mathcal{M}_1^C \left(\mathcal{M}_2^{\not C}\right)^*\right]$$





- asymmetries constrained to the permille level
- nonvanishing interference due to FSI phases!

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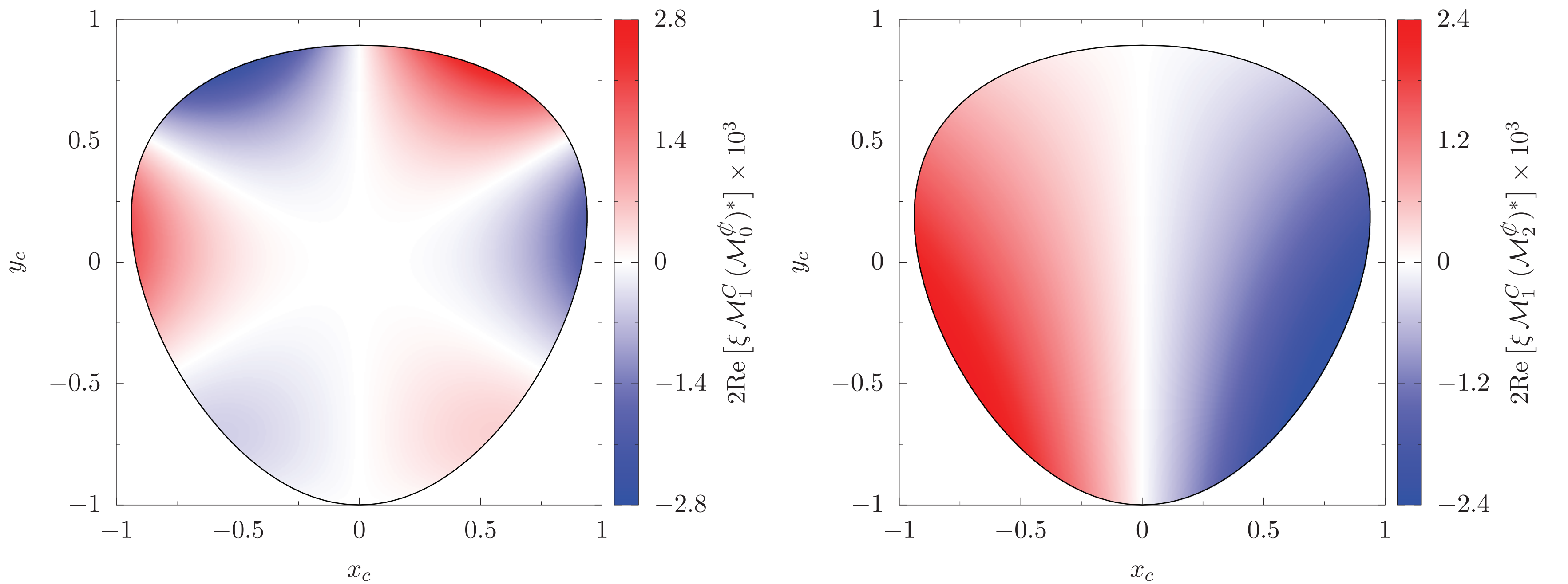


- asymmetries constrained to the permille level
- nonvanishing interference due to FSI phases!
- $\mathcal{M}_0^{\not{C}}$ and $\mathcal{M}_2^{\not{C}}$ lead to different interference patterns

Effective BSM couplings

Akdag, Isken, BK 2021

- polynomial ambiguities $\longrightarrow$ subtractions no good observables
- define unambiguous Taylor invariants & match to these:

$$\mathcal{M}_0^{\not C}(s,t,u) = i\,g_0\,(s-t)(u-s)(t-u) + \mathcal{O}(p^8)$$
$$\mathcal{M}_2^{\not C}(s,t,u) = i\,g_2\,(t-u) + \mathcal{O}(p^4)$$

- fit corresponds to

$$g_0 = -2.8(4.5)\,\text{GeV}^{-6}\,, \quad g_2 = -9.3(4.6)\times 10^{-3}\,\text{GeV}^{-2}$$

$\longrightarrow$ sensitivity $|g_0/g_2| \sim 10^3\,\text{GeV}^{-4} = \mathcal{O}(M_\pi^{-4})$

$\longrightarrow$ theoretical/chiral expectation: $|g_0/g_2| \sim \text{GeV}^{-4}$

- small phase space ($M_\eta - 3M_\pi \sim M_\pi$) reduces sensitivity to $\mathcal{M}_0^{\not C}$
- match couplings via ChPT to LEFT/SMEFT Akdag, BK, Wirzba 2023

Summary

Meson–meson final-state interactions

- great progress over the last 12 years!
- many applications jointly studied—Sino–German collaborations ongoing!
- cross-relations to
  - ▷ (exotic) spectroscopy: crossed channels, diagnostics. . .
  - ▷ triangle singularities
- future activities: application in CP-violating *B*-decays

Three-meson systems

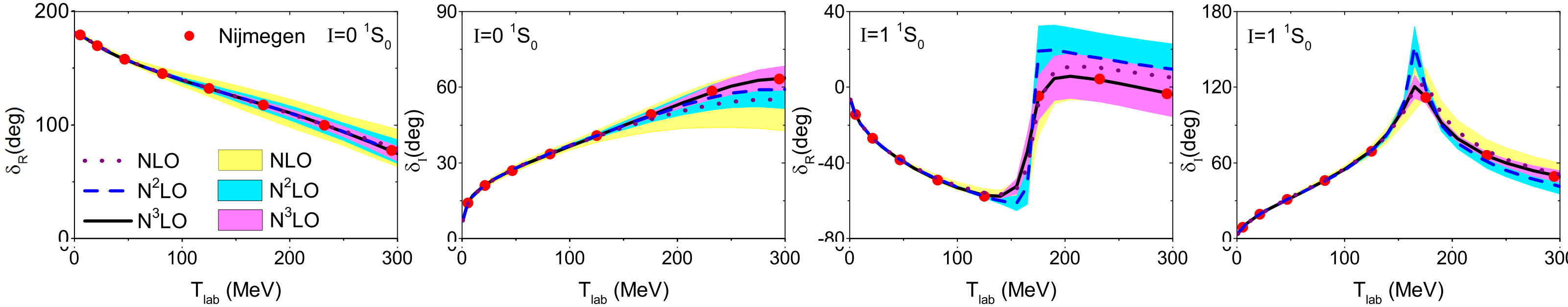
- systematic studies only begun; lots to be understood!
  - ▷ more partial waves, higher subtractions. . .
  - ▷ low-energy Dalitz plots (incl. BSM) well studied

Crimes and omissions

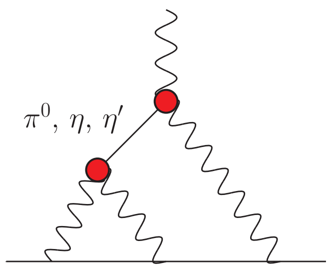
Baryon–antibaryon interactions

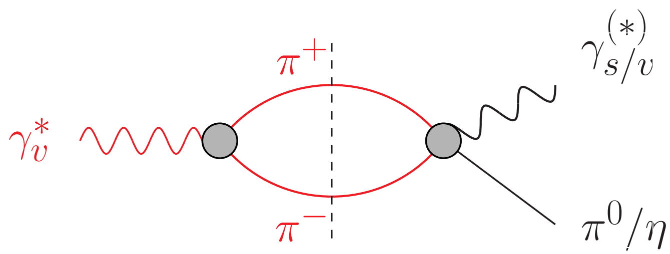
Dai, Haidenbauer, Meißner 2017

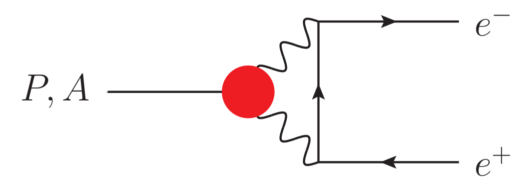
- $J/\psi \to V^0 \bar{p}p$, $e^+e^- \to \bar{p}p$, extension to hyperons. . .



Light transition form factors and $(g-2)_\mu$







Hoid, Holz, Schäfer, Zanke. . .

Heavy-hadron transition form factors

- analyticity & unitarity constraints on B transition form factors

Kürten, van Dyk. . .