Quarkonium production in pNRQCD

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Bibliography

(1) N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Inclusive production of J/ψ, $\psi(2S)$ and Υ states in pNRQCD
JHEP 03 (2023) 242 arXiv:2210.17345

(2) N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Production and polarization of S-wave quarkonia in potential Nonrelativistic QCD
Phys. Rev. D 105 (2022) 11, L111503 arXiv:2203.07778

(3) N. Brambilla, H. S. Chung and A. Vairo
Inclusive production of heavy quarkonia in pNRQCD
JHEP 09 (2021) 032 arXiv:2106.09417

(4) N. Brambilla, H. S. Chung and A. Vairo
Inclusive hadroproduction of P-wave heavy quarkonia in potential Nonrelativistic QCD
Phys. Rev. Lett. 126 (2021) 8 arXiv:2007.07613

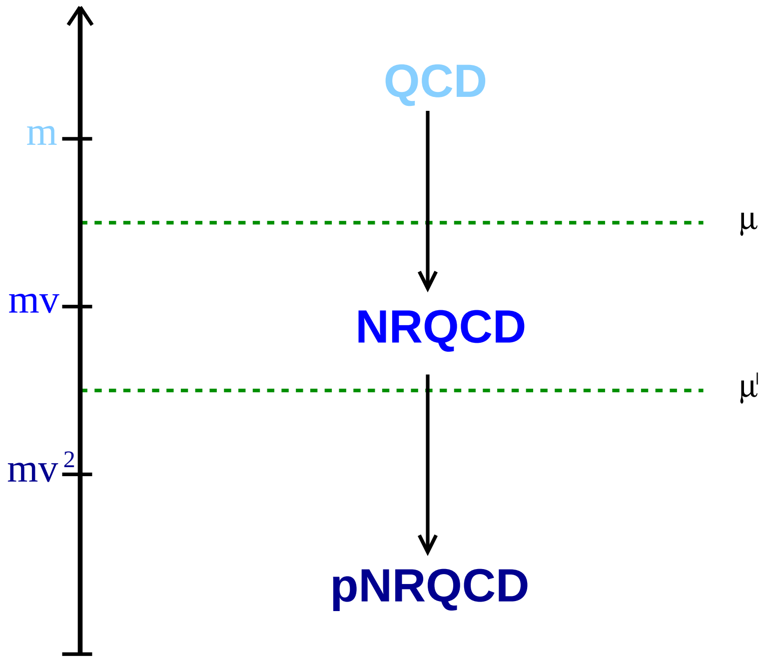
(5) N. Brambilla, H. S. Chung, D. Müller and A. Vairo
Decay and electromagnetic production of strongly coupled quarkonia in pNRQCD
JHEP 04 (2020) 095 arXiv:2002.07462

Inclusive quarkonium production in EFTs

Scales and non relativistic EFTs

Quarkonium physics may be described through non relativistic effective field theories, owing to the hierarchy of scales typical of any nonrelativistic bound state:

$$m \gg mv \gg mv^2$$



○ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

NRQCD

In NRQCD, the production cross sections for a quarkonium $\mathcal{Q}$ factorize

- in short distance coefficients, $\sigma_{Q\bar{Q}(N)}$, encoding contributions from energy scales of order m or larger,
- and in long distance matrix elements (LDMEs), $\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle$, encoding contributions of order mv, mv^2 and $\Lambda_{\rm QCD}$,

so that we can write:

$$\sigma_{\mathcal{Q}+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle\Omega|\mathcal{O}^{\mathcal{Q}}(N)|\Omega\rangle.$$

Note that the NRQCD factorization for hadroproduction has not been proved in general.

○ Bodwin Braaten Lepage PRD 51 (1995) 1125

Strongly coupled pNRQCD

The LDMEs may be further factorized in pNRQCD.

We consider the case of strongly coupled pNRQCD:

$$mv^2 \ll \Lambda_{\rm QCD}$$

which may be suited to describe excited (non Coulombic) quarkonium states.

- Brambilla Pineda Soto Vairo NPB 566 (2000) 275

The pNRQCD factorization formula for the LDMEs reads at LO

$$\langle \Omega | \mathcal{O}^{\mathcal{Q}}(N) | \Omega \rangle = \frac{1}{\langle \boldsymbol{P} = \boldsymbol{0} | \boldsymbol{P} = \boldsymbol{0} \rangle} \int d^3x_1 d^3x_2 d^3x_1' d^3x_2' \, \phi^{(0)}_{\mathcal{Q}}(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$
$$\times \left[-V_{\mathcal{O}(N)}(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2) \delta^{(3)}(\boldsymbol{x}_1 - \boldsymbol{x}_1') \delta^{(3)}(\boldsymbol{x}_2 - \boldsymbol{x}_2') \right] \phi^{(0)\,*}_{\mathcal{Q}}(\boldsymbol{x}_1' - \boldsymbol{x}_2')$$

$\phi^{(0)}_{\mathcal{Q}}$ and $V_{\mathcal{O}(N)}$ have to be determined from matching the NRQCD LDMEs to pNRQCD.

The NRQCD energy eigenstates

The spectral decomposition of $H_{\rm NRQCD}$ in the $Q\bar{Q}$ sector of the Hilbert space reads

$$H_{\rm NRQCD}\big|_{Q\bar{Q}} = \sum_n \int d^3x_1\, d^3x_2\, |\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle\, E_n(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2)\, \langle \underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2|$$

$|\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle = \psi^\dagger(\boldsymbol{x}_1)\chi(\boldsymbol{x}_2)|n; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ are orthonormal states made of a heavy quark, ψ, a heavy antiquark, χ, and some light d.o.f. labeled by n.
E_n are operators in the coordinate, momentum and spin of the $Q\bar{Q}$.
In the static limit $E_n = E_n^{(0)}$ are the different energy excitations of a static $Q\bar{Q}$.
They may be computed in lattice QCD as a function of the $Q\bar{Q}$ distance.

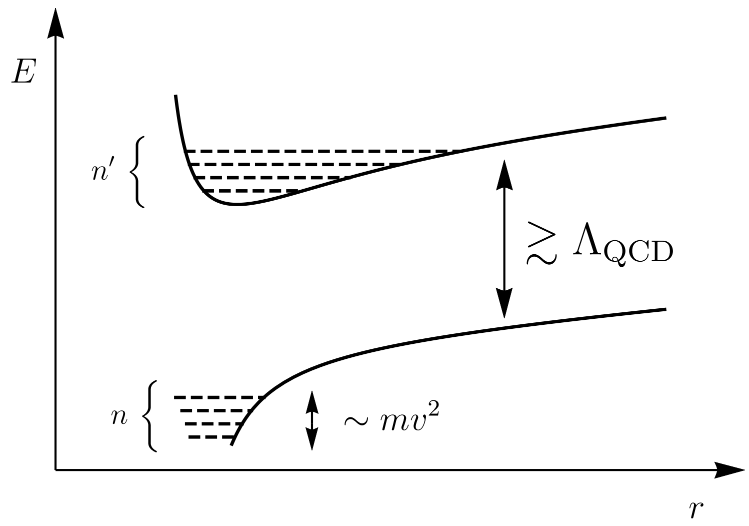
The eigenstates of the NRQCD Hamiltonian in the $Q\bar{Q}$ sector are

$$|\mathcal{Q}(n, \boldsymbol{P})\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}(n,\boldsymbol{P})}(\boldsymbol{x}_1, \boldsymbol{x}_2)\, |\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

The functions $\phi_{\mathcal{Q}(n,\boldsymbol{P})}(\boldsymbol{x}_1, \boldsymbol{x}_2)$ are eigenfunctions of $E_n(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2)$;
$\boldsymbol{P}$ is the center of mass momentum of the $Q\bar{Q}$ pair.

The NRQCD energy levels

In the strong coupling regime, we expect the different $E_n^{(0)}$ to develop an energy gap of order $\Lambda_{\rm QCD} \gg mv^2$, where mv^2 are the energies of the eigenstates of each single E_n.



- Bali et al PRD 62 (2000) 054503
  Juge Kuti Morningstar PRL 90 (2003) 161601

Matching the spectrum

In strongly coupled pNRQCD, the pNRQCD Hamiltonian is given by

$$H_{\text{pNRQCD}} = \int d^3x_1 \, d^3x_2 \; S_n^\dagger \, h_n(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2) \, S_n$$

S_n is a color singlet field containing a $Q\bar{Q}$;
h_n is obtained by matching the NRQCD energy E_n.

The matching may be performed order by order in $1/m$ by expanding the NRQCD Hamiltonian and the states $|\underline{\text{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ using quantum mechanical perturbation theory. At leading order in v we have

$$h_n(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2) = -\frac{\boldsymbol{\nabla}_1^2}{2m} - \frac{\boldsymbol{\nabla}_2^2}{2m} + V^{(0;n)}(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

The matching fixes the static potential $V^{(0;n)}$ to be the static energy $E_n^{(0)}$ of $H_{\text{NRQCD}}^{(0)}$.
As a consequence of the matching, the functions $\phi_{\mathcal{Q}(n,\boldsymbol{P})}$ are also eigenfunctions of h_n.

- Brambilla Pineda Soto Vairo PRD 63 (2001) 014023
  Pineda Vairo PRD 63 (2001) 054007

Hadroproduction LDMEs

Color singlet and octet operators for hadroproduction of quarkonia have the form

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color singlet}}) = \chi^\dagger \mathcal{K}_N \psi \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\mathbf{0})} \psi^\dagger \mathcal{K}'_N \chi$$

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color octet}}) = \chi^\dagger \mathcal{K}_N T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\mathbf{0})} \Phi_\ell^{bc}(0) \psi^\dagger \mathcal{K}'_N T^c \chi$$

$\Phi_\ell(x)$ is a Wilson line along the direction ℓ in the adjoint representation required to ensure the gauge invariance of the color octet LDME.

- Nayak Qiu Sterman PLB 613 (2005) 45

$\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})}$ projects onto a state containing a heavy quarkonium $\mathcal{Q}$ with momentum $\boldsymbol{P}$.
$\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})}$ commutes with the NRQCD Hamiltonian (the number of quarkonia is conserved) and is diagonalized by the same eigenstates of the NRQCD Hamiltonian:

$$\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})} = \sum_{n \in \mathbb{S}} |\mathcal{Q}(n, \boldsymbol{P})\rangle\langle \mathcal{Q}(n, \boldsymbol{P})|$$

The sum extends over $\mathbb{S}$, which are all states where the $Q\bar{Q}$ is in a color singlet at the origin in the static limit. This is a necessary condition to produce a quarkonium.

Matching the hadroproduction LDMEs

The matching condition for the hadroproduction contact terms $V_{\mathcal{O}(N)}$ is

$$\sum_{n\in\mathbb{S}} \int d^3x \, \langle\Omega| \left(\chi^\dagger \mathcal{K}_N \psi\right)(\boldsymbol{x})|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle\langle\underline{\mathrm{n}}; \boldsymbol{x}_1', \boldsymbol{x}_2'| \left(\psi^\dagger \mathcal{K}_N' \chi\right)(\boldsymbol{x})|\Omega\rangle$$
$$= -V_{\mathcal{O}(N)}(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2)\, \delta^{(3)}(\boldsymbol{x}_1 - \boldsymbol{x}_1')\delta^{(3)}(\boldsymbol{x}_2 - \boldsymbol{x}_2')$$

$$\sum_{n\in\mathbb{S}} \int d^3x \langle\Omega| \left(\chi^\dagger \mathcal{K}_N T^a \psi\right)(\boldsymbol{x})\Phi_\ell^{\dagger ab}(0, \boldsymbol{x})|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$
$$\times \langle\underline{\mathrm{n}}; \boldsymbol{x}_1', \boldsymbol{x}_2'|\Phi_\ell^{bc}(0, \boldsymbol{x}) \left(\psi^\dagger \mathcal{K}_N' T^c \chi\right)(\boldsymbol{x})|\Omega\rangle$$
$$= -V_{\mathcal{O}(N)}(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{\nabla}_1, \boldsymbol{\nabla}_2)\delta^{(3)}(\boldsymbol{x}_1 - \boldsymbol{x}_1')\delta^{(3)}(\boldsymbol{x}_2 - \boldsymbol{x}_2')$$

where we may expand the states $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ order by order in $1/m$ using quantum mechanical perturbation theory.

Matching the wavefunctions $\phi_{\mathcal{Q}(n,\boldsymbol{P})}$

The projector $\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})}$ depends on the wavefunction $\phi_{\mathcal{Q}(n,\boldsymbol{P})}$ with $n \in \mathbb{S}$.
$\phi_{\mathcal{Q}(n,\boldsymbol{P})}$ is a solution of the Schrödinger equation with static potential $V^{(0;n)}$.
$V^{(0;n)}$ is the energy of a static Wilson loop in the presence of disconnected gluon fields.

Lattice QCD determinations of $V^{(0;n)}$ for $n \in \mathbb{S}$ and $n \neq 0$ are not available yet.
One expects, however, disconnected gluon fields to produce mainly a constant shift to the potentials, e.g. in the form of a glueball mass. This is supported by the large N_c limit: the vacuum expectation value of a Wilson loop with additional disconnected gluon fields factorizes into the vacuum expectation value of the Wilson loop times the vacuum expectation value of the additional gluon fields up to corrections of order $1/N_c^2$.

If the slopes of the static potentials are the same for all $n \in \mathbb{S}$, then

$$\phi_{\mathcal{Q}(n,\boldsymbol{P})}(\boldsymbol{x}_1, \boldsymbol{x}_2) \approx e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\phi^{(0)}_{\mathcal{Q}}(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

$\phi^{(0)}_{\mathcal{Q}}$ is the leading order quarkonium wavefunction in the center of mass frame.

$$e^+e^- \to \chi_{\mathcal{Q}} + \gamma$$

LDMEs in NRQCD

We consider

$$e^+e^- \to \chi_{QJ}(nP) + \gamma$$

The NRQCD factorization formula up to relative order v^2 reads

$$\sigma_{\chi_{QJ}+\gamma} = \sigma^{\rm em}_{Q\bar{Q}(^3P_J^{[1]})} \langle\Omega|\mathcal{O}^{\chi_{QJ}}(^3P_J^{[1]};\text{em})|\Omega\rangle + \sigma^{\rm em\,\mathcal{T}}_{Q\bar{Q}(^3S_1^{[8]})} \langle\Omega|\mathcal{T}^{\chi_{QJ}}(^3P_J^{[8]};\text{em})|\Omega\rangle$$
$$+ \sigma^{\rm em\,\mathcal{P}}_{Q\bar{Q}(^3P_J^{[1]})} \langle\Omega|\mathcal{P}^{\chi_{QJ}}(^3P_J^{[1]};\text{em})|\Omega\rangle$$

The electromagnetic production matrix elements, $\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N;\text{em})|\Omega\rangle$, are related with the electromagnetic decay matrix elements, $\langle\mathcal{Q}|\mathcal{O}^{\text{em}}(N)|\mathcal{Q}\rangle$, through

$$\langle\Omega|\mathcal{O}^{\mathcal{Q}}(N;\text{em})|\Omega\rangle = (2J+1)\langle\mathcal{Q}|\mathcal{O}^{\text{em}}(N)|\mathcal{Q}\rangle$$

Matching the contact terms in pNRQCD

After matching with pNRQCD, the contact terms $V_{\mathcal{O}(N)}$ read

$$
\begin{aligned}
V_{\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]};\text{em})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) =&(2J+1)N_c T_{1J}^{ij}\,\nabla_{\boldsymbol{r}}^i\left(1+\frac{2}{3}\frac{i\mathcal{E}_2}{m}\right)\delta^{(3)}(\boldsymbol{r})\nabla_{\boldsymbol{r}}^j \\
V_{\mathcal{T}^{\chi_{QJ}}({}^3P_J^{[8]};\text{em})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) =&(2J+1)N_c T_{1J}^{ij}\,\nabla_{\boldsymbol{r}}^i\,\frac{4}{3}\frac{\mathcal{E}_1}{m}\,\delta^{(3)}(\boldsymbol{r})\nabla_{\boldsymbol{r}}^j \\
V_{\mathcal{P}^{\chi_{QJ}}({}^3P_J^{[1]};\text{em})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) =&(2J+1)N_c T_{1J}^{ij}\,\nabla_{\boldsymbol{r}}^i\delta^{(3)}(\boldsymbol{r})\left(-\boldsymbol{\nabla}_{\boldsymbol{r}}^2-\frac{5}{3}\mathcal{E}_1\right)\nabla_{\boldsymbol{r}}^j
\end{aligned}
$$

- $\boldsymbol{r}=\boldsymbol{x}_1-\boldsymbol{x}_2$.
- T_{1J}^{ij} are spin projectors: $T_{10}^{ij}=\sigma^i\otimes\sigma^j/3$, $T_{11}^{ij}=\epsilon_{kim}\epsilon_{kjn}\sigma^m\otimes\sigma^n/2$, $T_{12}^{ij}=\left((\delta_{im}\sigma^n+\delta_{in}\sigma^m)/2-\delta_{mn}\sigma^i/3\right)\otimes\left((\delta_{jm}\sigma^n+\delta_{jn}\sigma^m)/2-\delta_{mn}\sigma^j/3\right)$.
- $\mathcal{E}_n$ are correlators of two chromoelectric fields $\mathbf{E}$ (located at $\mathbf{0}$):

$$
\mathcal{E}_n=\frac{1}{2N_c}\int_0^\infty dt\,t^n\,\langle\Omega|gE^{i,a}(t)\Phi^{ab}(0,t)gE^{i,b}(0)|\Omega\rangle
$$

$\Phi^{ab}(0,t)$ is a Wilson line in the adjoint representation connecting $(t,\mathbf{0})$ with $(0,\mathbf{0})$.

LDMEs in pNRQCD

The pNRQCD factorization formulas for P-wave quarkonium em production are

$$\langle\Omega|\mathcal{O}^{\chi_{QJ}}(^3P_J^{[1]};\text{em})|\Omega\rangle = (2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\left[1+\frac{2}{3}\frac{i\mathcal{E}_2}{m}+O\left(v^2\right)\right]$$

$$\langle\Omega|\mathcal{T}^{\chi_{QJ}}(^3P_J^{[8]};\text{em})|\Omega\rangle = (2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\frac{4}{3}\frac{\mathcal{E}_1}{m}$$

$$\langle\Omega|\mathcal{P}^{\chi_{QJ}}(^3P_J^{[1]};\text{em})|\Omega\rangle = (2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\left[m\varepsilon-\frac{2}{3}\mathcal{E}_1+O\left(v^3\right)\right]$$

$R'(0)$ is the derivative of the radial wavefunction at the origin, and ε the binding energy.

Chromoelectric correlators for electromagnetic production

The wavefunctions at the origin may be computed solving the equation of motion of pNRQCD with potentials determined from lattice QCD or via phenomenological models.

The correlators can be fitted on data for $\chi_{c0}(1P) \to \gamma\gamma$, $\chi_{c2}(1P) \to \gamma\gamma$ and $\sigma(e^+e^- \to \chi_{c1}(1P) + \gamma)$ ($= 17.3^{+4.2}_{-3.9} \pm 1.7$ fb at $\sqrt{s}$ =10.6 GeV from Belle).

- `Belle coll PRD 98 (2018) 092015`

The correlators are universal: they do not depend neither on the flavor of the heavy quark nor on the quarkonium state:

$$\mathcal{E}_1 = -\,0.20^{+0.14}_{-0.14} \pm 0.90 \text{ GeV}^2$$

$$i\mathcal{E}_2 = 0.77^{+0.98}_{-0.86} \pm 0.85 \text{ GeV}$$

The universal nature of the correlators allows to use them to compute cross sections (and decay widths) for quarkonia with different principal quantum number and bottomonia.

$\sigma(e^+e^- \to \chi_{cJ}(1P) + \gamma)$

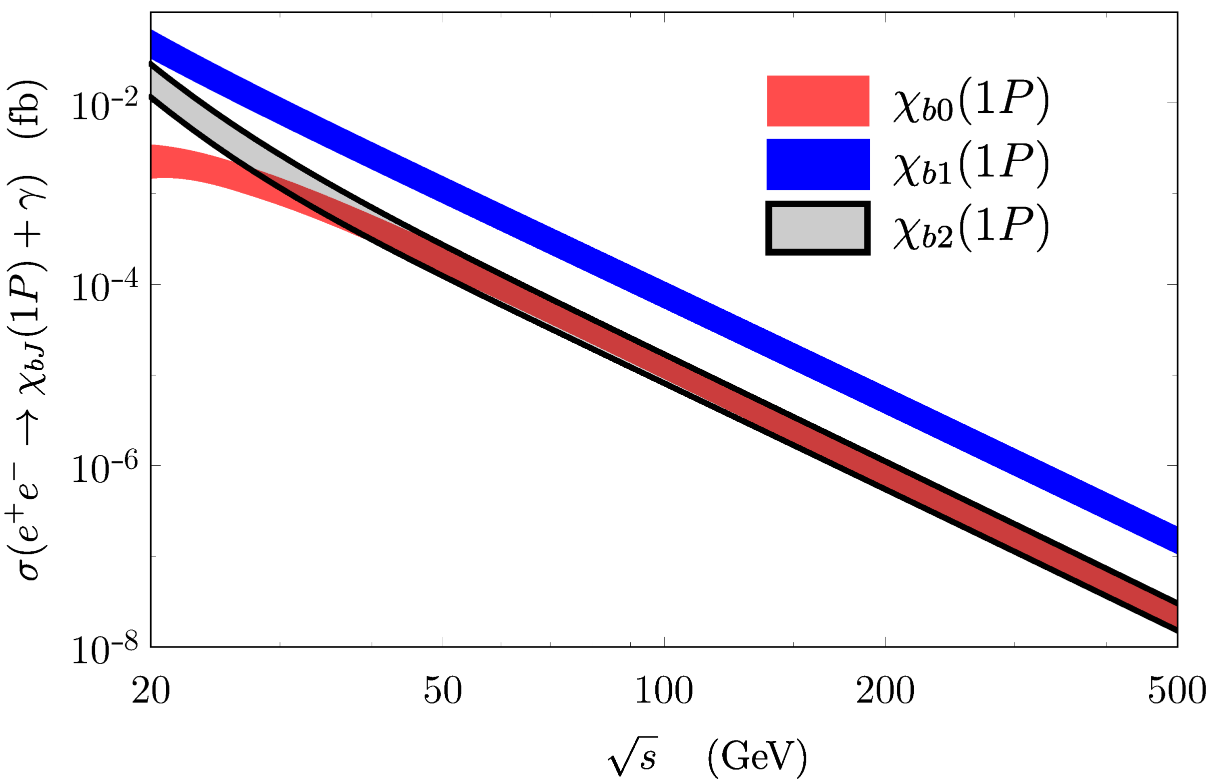
P-wave charmonium cross sections at the Belle center of mass energy $\sqrt{s}$ =10.6 GeV:

$$\sigma(e^+e^- \to \chi_{c0}(1P) + \gamma) = 1.84^{+0.25}_{-0.26} \pm 0.76 \text{ fb}$$

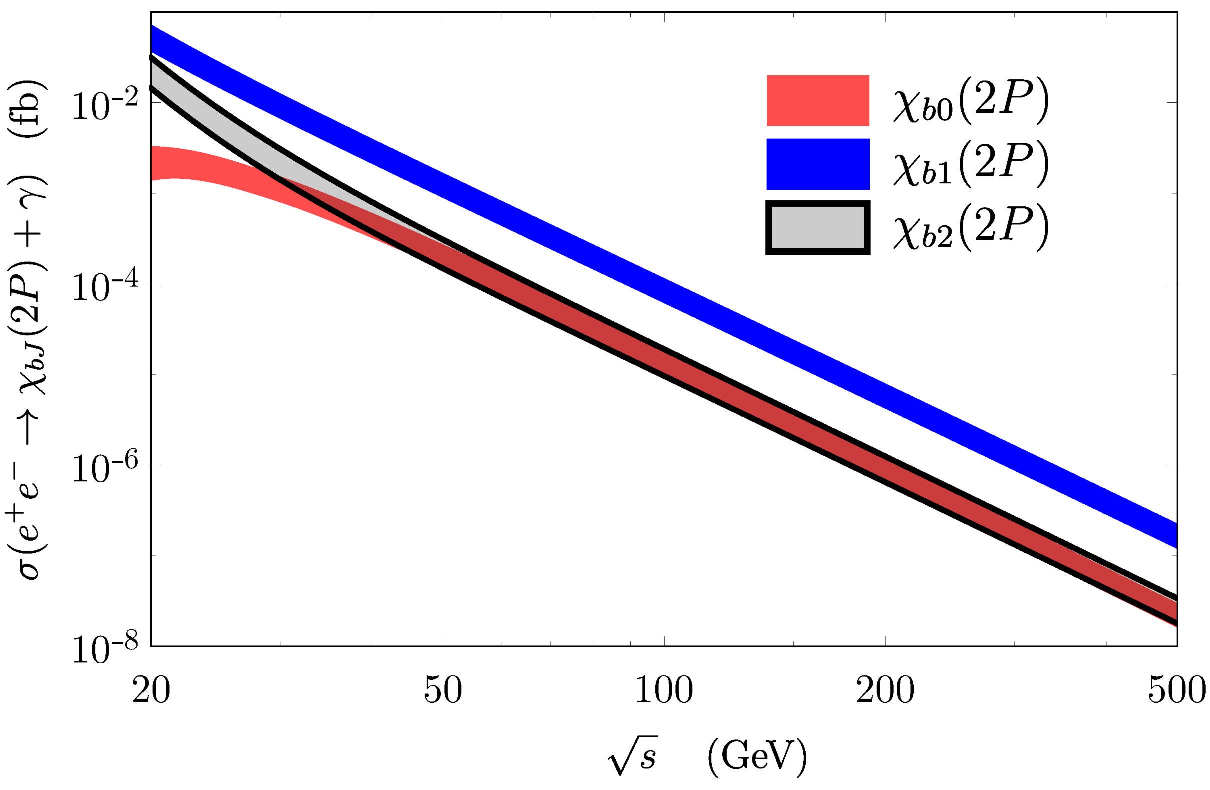
$$\sigma(e^+e^- \to \chi_{c1}(1P) + \gamma) = 16.4^{+0.2}_{-0.2} \pm 6.4 \text{ fb}$$

$$\sigma(e^+e^- \to \chi_{c2}(1P) + \gamma) = 3.75^{+0.67}_{-0.56} \pm 2.16 \text{ fb}$$

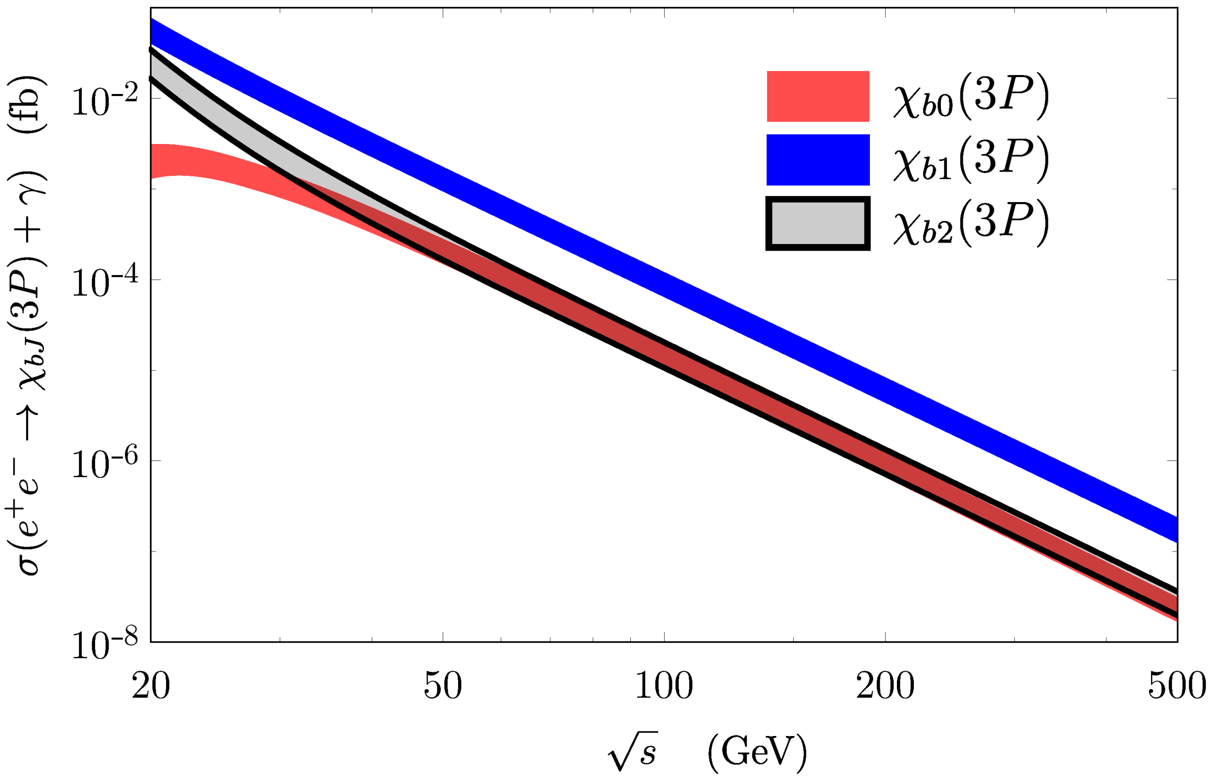
$\sigma(e^+e^- \to \chi_{bJ}(1P) + \gamma)$



$\sigma(e^+e^- \to \chi_{bJ}(2P) + \gamma)$



$\sigma(e^+e^- \to \chi_{bJ}(3P) + \gamma)$



$$pp \to \chi_{\mathcal{Q}} + X$$

LDMEs in NRQCD

We consider

$$pp \to h_Q(nP) + X \qquad \text{and} \qquad pp \to \chi_{QJ}(nP) + X$$

The NRQCD factorization formulas at leading order in v read

$$\sigma_{h_Q+X} = \sigma_{Q\bar{Q}(^1P_1^{[1]})} \langle \Omega | \mathcal{O}^{h_Q}(^1P_1^{[1]}) | \Omega \rangle + \sigma_{Q\bar{Q}(^1S_0^{[8]})} \langle \Omega | \mathcal{O}^{h_Q}(^1S_0^{[8]}) | \Omega \rangle$$

$$\sigma_{\chi_{QJ}+X} = \sigma_{Q\bar{Q}(^3P_J^{[1]})} \langle \Omega | \mathcal{O}^{\chi_{QJ}}(^3P_J^{[1]}) | \Omega \rangle + \sigma_{Q\bar{Q}(^3S_1^{[8]})} \langle \Omega | \mathcal{O}^{\chi_{QJ}}(^3S_1^{[8]}) | \Omega \rangle$$

Matching the contact terms in pNRQCD

After matching with pNRQCD, the contact terms read

$$V_{\mathcal{O}({}^1P_1^{[1]})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) = N_c \nabla^i_{\boldsymbol{r}} \delta^{(3)}(\boldsymbol{r}) \nabla^i_{\boldsymbol{r}}$$

$$V_{\mathcal{O}({}^1S_0^{[8]})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) = N_c \nabla^i_{\boldsymbol{r}} \delta^{(3)}(\boldsymbol{r}) \nabla^j_{\boldsymbol{r}} \frac{\mathcal{E}^{ij}}{N_c^2 m^2}$$

$$V_{\mathcal{O}({}^3P_J^{[1]})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) = T^{ij}_{1J} N_c \nabla^i_{\boldsymbol{r}} \delta^{(3)}(\boldsymbol{r}) \nabla^j_{\boldsymbol{r}}$$

$$V_{\mathcal{O}({}^3S_1^{[8]})}(\boldsymbol{r},\boldsymbol{\nabla}_{\boldsymbol{r}}) = \sigma^k \otimes \sigma^k N_c \nabla^i_{\boldsymbol{r}} \delta^{(3)}(\boldsymbol{r}) \nabla^j_{\boldsymbol{r}} \frac{\mathcal{E}^{ij}}{N_c^2 m^2}$$

$\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2$ and T^{ij}_{1J} are spin projectors.

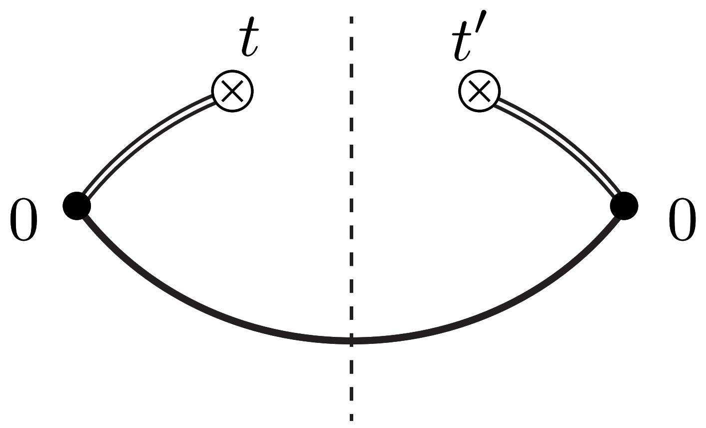
The tensor $\mathcal{E}^{ij}$ is defined by

$$\mathcal{E}^{ij} = \int_0^\infty dt\, t \int_0^\infty dt'\, t' \, \langle \Omega | \Phi_\ell^{\dagger ab} \Phi^{\dagger ad}(0;t) g E^{d,i}(t) g E^{e,j}(t') \Phi^{ec}(0;t') \Phi_\ell^{bc} | \Omega \rangle$$

$\Phi^{ab}(0,t)$ is a Wilson line in the adjoint representation connecting $(t,\mathbf{0})$ with $(0,\mathbf{0})$.

The chromoelectric correlators $\mathcal{E}^{ij}$ and $\mathcal{E}$

For a suitable choice of ℓ^0, the fields in $gE^{e,j}(t')\Phi^{ec}(0;t')\Phi_\ell^{bc}$ are time ordered ($\mathcal{T}$) and those in $\Phi_\ell^{\dagger ab}\Phi^{\dagger ad}(0;t)gE^{d,i}(t)$ are anti-time ordered ($\bar{\mathcal{T}}$).
Hence the correlator $\mathcal{E}^{ij}$ may be interpreted as a cut diagram:



For polarization-summed cross sections or for production of scalar states only the isotropic part of $\mathcal{E}^{ij}$ is relevant. This is the dimensionless gluonic correlator $\mathcal{E}$:

$$\mathcal{E} = \frac{3}{N_c}\int_0^\infty dt\, t \int_0^\infty dt'\, t' \, \langle\Omega|\Phi_\ell^{\dagger ab}\Phi^{\dagger ad}(0;t)gE^{d,i}(t)gE^{e,i}(t')\Phi^{ec}(0;t')\Phi_\ell^{bc}|\Omega\rangle$$

LDMEs in pNRQCD

The pNRQCD factorization formulas for P-wave quarkonium hadroproduction are

$$
\begin{aligned}
\langle\Omega|\mathcal{O}^{h_Q}({}^1P_1^{[1]})|\Omega\rangle =& 3\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\\
\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle =& 3\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{1}{9N_cm^2}\mathcal{E}\\
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle =& (2J+1)\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\\
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle =& (2J+1)\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{1}{9N_cm^2}\mathcal{E}
\end{aligned}
$$

$R^{(0)\prime}(0)$ is the derivative of the radial wavefunction at the origin at leading order in v.
LDMEs are polarization summed in the case of χ_{QJ} states.

The above expressions imply (at leading order in v) the universality of the ratios

$$
\frac{m^2\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle}{\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle}=\frac{m^2\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle}{\langle\Omega|\mathcal{O}^{h_Q}({}^1P_1^{[1]})|\Omega\rangle}=\frac{\mathcal{E}}{9N_c}
$$

Infrared divergences in NRQCD

For the pNRQCD expressions of the LDMEs to be consistent with perturbative QCD, they must reproduce the same infrared divergences. At two loop accuracy and at the lowest order in the relative momentum q of the Q and $\bar{Q}$, the infrared diverges in the NRQCD LDMEs can be cast in the infrared factor

$$\mathcal{I}_2(p,q) = \sum_N \int_0^\infty d\lambda'\, \lambda' \langle \Omega | \bar{\mathcal{T}} \left\{ \Phi_\ell^{\dagger c' b} \Phi_p^{\dagger a' c'}(\lambda') [p^\mu q^\nu F_{\nu\mu}^{a'}(\lambda' p)] \right\} | N \rangle$$
$$\times \int_0^\infty d\lambda\, \lambda \langle N | \mathcal{T} \left\{ \Phi_\ell^{bc} [p^\mu q^\nu F_{\nu\mu}^{a}(\lambda p)] \Phi_p^{ac}(\lambda) \right\} | \Omega \rangle$$

The sum over N contains all possible intermediate states, p is half the center-of-mass momentum of the $Q\bar{Q}$, and

$$\Phi_p(\lambda) = \mathcal{P} \exp \left[-ig \int_0^\lambda d\lambda'\, p \cdot A^{\rm adj}(\lambda' p) \right]$$

is an adjoint Wilson line along p.

- Nayak Qiu Sterman PLB 613 (2005) 45, PRD 72 (2005) 114012
  Nayak Qiu Sterman PRD 74 (2006) 074007

Consistency of pNRQCD with the NRQCD factorization

- Since in $\mathcal{I}_2(p,q)$ a momentum q comes from each side of the cut, the infrared factor contributes to the production of a color-singlet P-wave state.
- In the rest frame of the $Q\bar{Q}$: $\boldsymbol{p}=0$, $q^0=0$, $\Phi_p(\lambda)=\Phi(0,t)$ with $t=\sqrt{p^2}\lambda$, $p^\mu q^\nu F^a_{\nu\mu}(\lambda p)=-\sqrt{p^2}q^i E^{a\,i}(t)$ and $\mathcal{I}_2(p,q)$ can be written as

$$\mathcal{E}^{ij}\frac{q^i q^j}{p^2}$$

  Since this expression is proportional to the contact terms $V_{\mathcal{O}(^1S_0^{[8]})}$ and $V_{\mathcal{O}(^3S_1^{[8]})}$ in momentum space, the pNRQCD expressions for the color-octet LDMEs reproduce the same infrared divergences cast in the NRQCD infrared factor.
- The one-loop running of $\mathcal{E}$ ($C_F=(N_c^2-1)/(2N_c)$):

$$\frac{d}{d\log\Lambda}\mathcal{E}(\Lambda)=12C_F\frac{\alpha_s}{\pi}$$

  implies $\frac{d}{d\log\Lambda}\langle\mathcal{O}^{\chi_{QJ}}(^3S_1^{[8]})\rangle=\frac{4C_F\alpha_s}{3N_c\pi m^2}\langle\mathcal{O}^{\chi_{QJ}}(^3P_J^{[1]})\rangle$.
  This agrees with the one-loop evolution equation derived in perturbative NRQCD.

Chromoelectric correlator for hadroproduction

The correlator $\mathcal{E}$ can be fitted from the ratio

$$r_{21} = \frac{d\sigma_{\chi_{c2}(1P)}/dp_T}{d\sigma_{\chi_{c1}(1P)}/dp_T}$$

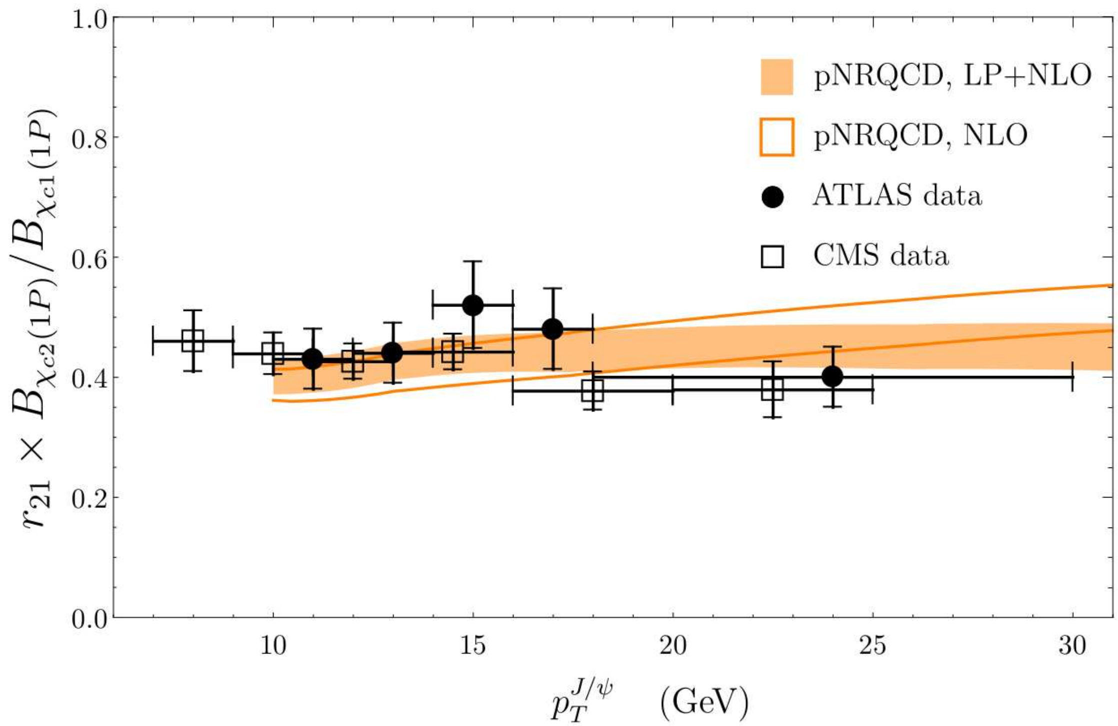
which does not depend (at leading order in v) on the wavefunction. One obtains

$$\mathcal{E}|_{\rm NLO}(\Lambda = 1.5\,\text{GeV}) = 1.17 \pm 0.05, \qquad \mathcal{E}|_{\rm LP+NLO}(\Lambda = 1.5\,\text{GeV}) = 4.48 \pm 0.14$$

combined into $\mathcal{E}(\Lambda = 1.5\,\text{GeV}) = 2.8 \pm 1.7$

The correlator is universal: it does not depend neither on the flavor of the heavy quark nor on the quarkonium state. The universal nature of the correlator allows to use it to compute cross sections for quarkonia with different principal quantum number and for bottomonia (once accounted for the running) without having to fit new octet LDMEs.

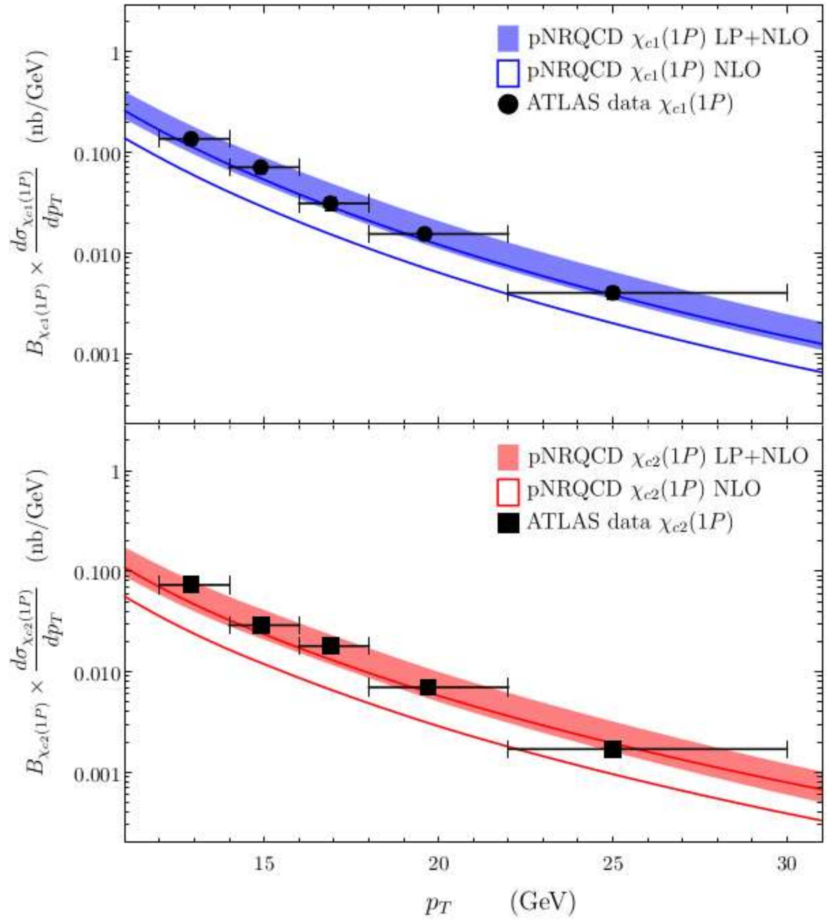
$(d\sigma_{\chi_{c2}(1P)}/dp_T)/(d\sigma_{\chi_{c1}(1P)}/dp_T)$



@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $|y| <$0.75.

- CMS coll EPJC 72 (2012) 2251
  ATLAS coll JHEP 07 (2014) 154

$\sigma(pp \to \chi_{cJ}(1P) + X)$



@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $|y| <$0.75.
Wavefunctions at the origin (at leading order in v) determined from $\Gamma(\chi_{c0,2}(1P) \to \gamma\gamma)$.

∘ ATLAS coll JHEP 07 (2014) 154

Polarized cross sections

For polarized cross sections, the non-isotropic part of $\mathcal{E}^{ij}$ can in principle contribute to the color-octet matrix elements, and, if such contribution is nonvanishing, the color-octet matrix elements can acquire a dependence on the direction of the gauge-completion Wilson lines. For the universality of the NRQCD matrix elements to be valid also for the case of polarized cross sections, such non-isotropic contributions should vanish in the NRQCD matrix elements. This has not been proved, but often assumed in the literature:

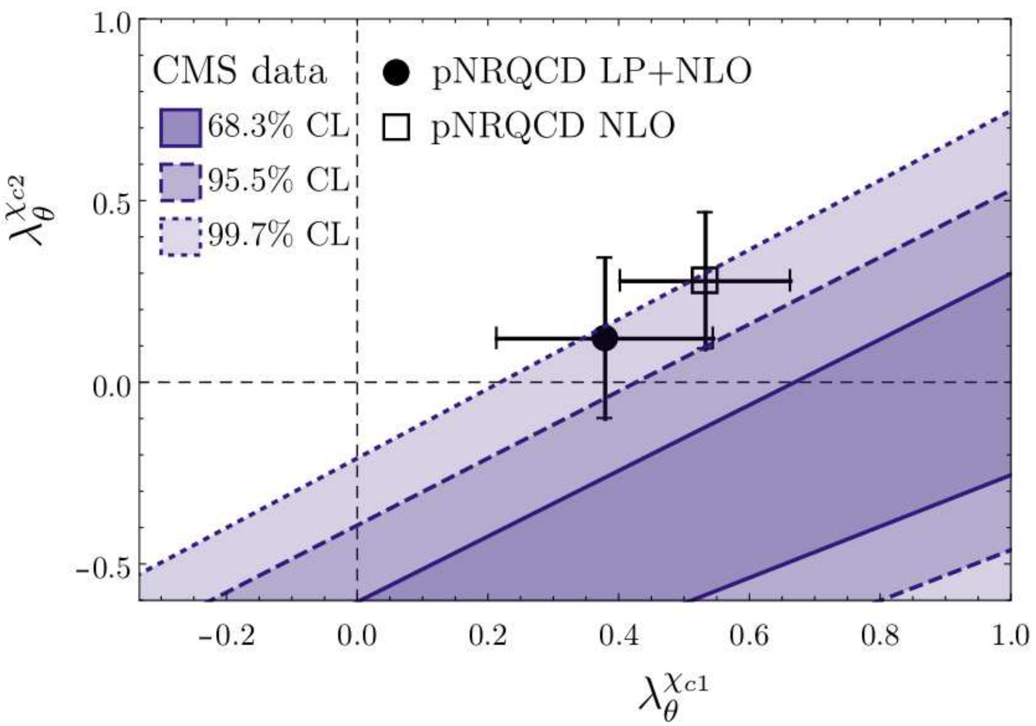
$$\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle = 3 \times \langle\Omega|\chi^\dagger T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{h_Q(\lambda,\boldsymbol{P}=\mathbf{0})} \Phi_\ell^{bc}(0) \psi^\dagger T^c \chi|\Omega\rangle$$
$$\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle = (2J+1) \times \langle\Omega|\chi^\dagger \sigma^i T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{\chi_{QJ}(\lambda,\boldsymbol{P}=\mathbf{0})} \Phi_\ell^{bc}(0) \psi^\dagger \sigma^i T^c \chi|\Omega\rangle$$

Under the universality assumption of the polarized color-octet LDMEs, one can compute the polarization parameters

$$\lambda_\theta^{\chi_{cJ}} = \frac{1 - 3\xi_{\chi_{cJ}}}{1 + \xi_{\chi_{cJ}}}$$

$\xi_{\chi_{cJ}}$ is the fraction of J/ψ produced with longitudinal polarization from decays of χ_{cJ}. We use the hadron helicity frame to define the spin quantization axis of the J/ψ.

Polarization of $\chi_{cJ}(1P)$

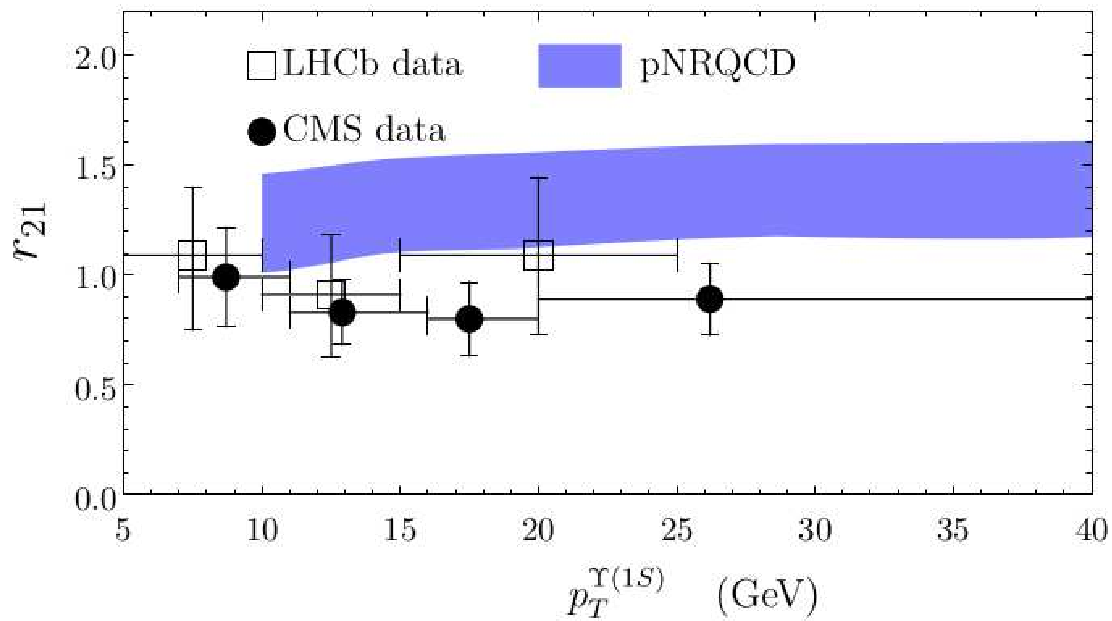


@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $|y| < 0.75$.

- CMS coll PRL 124 (2020) 16

$(d\sigma_{\chi_{b2}(1P)}/dp_T)/(d\sigma_{\chi_{b1}(1P)}/dp_T)$

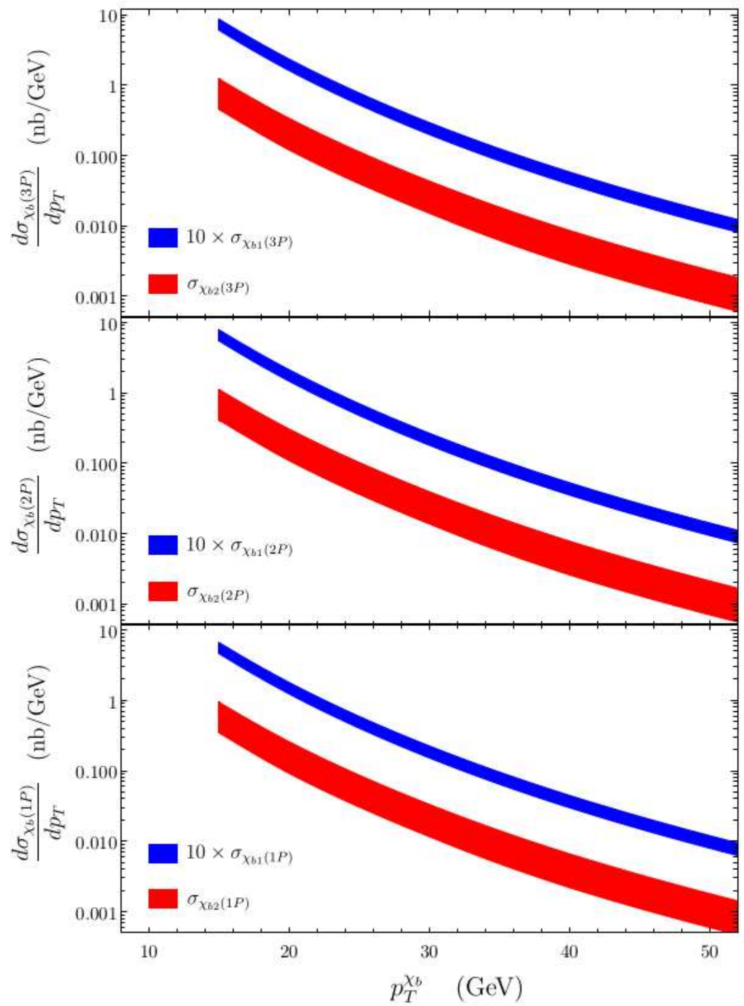
A test of the universality of the pNRQCD factorization is provided by the ratio $(d\sigma_{\chi_{b2}(1P)}/dp_T)/(d\sigma_{\chi_{b1}(1P)}/dp_T)$ that depends only of $\mathcal{E}$ (at the scale of the b mass) and therefore is expected to be the same also for $2P$ and $3P$ bottomonium states.



@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $2 < y < 4.5$.

- LHCb coll EPJC 74 (2014) 3092
  CMS coll PLB 743 (2015) 383

$\sigma(pp \to \chi_{bJ}(1P) + X)$

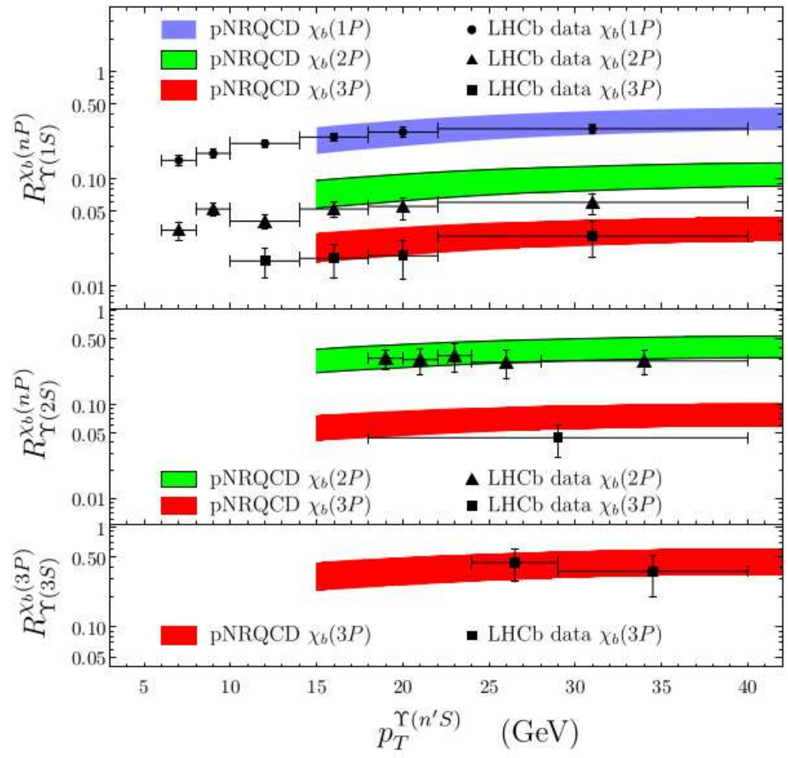


@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $2 < y < 4.5$.
Wavefunctions at the origin (at leading order in v) determined from models.

$\chi_{bJ}(nP)$ feeddown fractions

Feeddown fractions, $R^{\chi_b(nP)}_{\Upsilon(n'S)} = \frac{\sum_{J=1,2} \mathrm{Br}(\chi_{bJ}(nP) \to \Upsilon(n'S) + \gamma) \times \sigma_{\chi_{bJ}(nP)}}{\sigma_{\Upsilon(n'S)}}$,

are model dependent in the χ_{bJ} wavefunctions, $\sigma_{\Upsilon(nS)}$ and in some Br.



@ center of mass energy $\sqrt{s} = 7$ TeV and rapidity range $2 < y < 4.5$.

- LHCb coll EPJC 74 (2014) 3092

$$pp \to V_{\mathcal{Q}} + X$$

LDMEs in NRQCD

We consider

$$pp \to J/\psi + X, \quad pp \to \psi(2S) + X \qquad \text{and} \qquad pp \to \Upsilon(nS) + X$$

with $n \geq 2$.

The inclusive cross section of a spin-1 S-wave heavy quarkonium V is given in the NRQCD factorization formalism at relative order v^4 accuracy by

$$\sigma_{V+X} = \hat{\sigma}_{^3S_1^{[1]}} \langle \mathcal{O}^V(^3S_1^{[1]}) \rangle + \hat{\sigma}_{^3S_1^{[8]}} \langle \mathcal{O}^V(^3S_1^{[8]}) \rangle + \hat{\sigma}_{^1S_0^{[8]}} \langle \mathcal{O}^V(^1S_0^{[8]}) \rangle + \hat{\sigma}_{^3P_J^{[8]}} \langle \mathcal{O}^V(^3P_0^{[8]}) \rangle$$

Color octet LDMEs in pNRQCD

$$\langle \mathcal{O}^V(^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}\mathcal{E}_{10;10}(\mu)$$

$$\langle \mathcal{O}^V(^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R_V^{(0)}(0)|^2}{4\pi}c_F^2(\mu)\,\mathcal{B}_{00}(\mu)$$

$$\langle \mathcal{O}^V(^3P_0^{[8]})\rangle = \frac{1}{18N_c}\frac{3|R_V^{(0)}(0)|^2}{4\pi}\mathcal{E}_{00}$$

$R_V^{(0)}(0)$ is the vector quarkonium radial wavefunction at the origin.

$2c_F(\mu) = 2 + \frac{\alpha_s}{\pi}\left[\frac{N_c^2-1}{2N_c} + N_c\left(1+\log\frac{\mu}{m}\right)\right] + O(\alpha_s^2)$ is the quark magnetic moment.

$c_F^2(\mu)\,\mathcal{B}_{00}(\mu)$ is scale independent, whereas the scale dependence of $\mathcal{E}_{10;10}(\mu)$,

$$\frac{d}{d\log\mu}\mathcal{E}_{10;10}(\mu) = \mathcal{E}_{00}\times\frac{2\alpha_s}{3\pi}\frac{N_c^2-4}{N_c} + O(\alpha_s^2),$$

cancels against $\hat{\sigma}_{^3P_J^{[8]}}$.

Correlators

$$\mathcal{E}_{10;10} = \left| d^{dac} \int_0^\infty dt_1\, t_1 \int_{t_1}^\infty dt_2\, gE^{b,i}(t_2) \right.$$
$$\left. \times\, \Phi_0^{bc}(t_1;t_2) gE^{a,i}(t_1) \Phi_0^{df}(0;t_1) \Phi_\ell^{ef} |\Omega\rangle \right|^2$$

$$\mathcal{B}_{00} = \left| \int_0^\infty dt\, gB^{a,i}(t) \Phi_0^{ac}(0;t) \Phi_\ell^{bc} |\Omega\rangle \right|^2$$

$$\mathcal{E}_{00} = \left| \int_0^\infty dt\, gE^{a,i}(t) \Phi_0^{ac}(0;t) \Phi_\ell^{bc} |\Omega\rangle \right|^2$$

Fit results (from data for prompt J/ψ and $\psi(2S)$ production rates @ CMS and inclusive $\Upsilon(2S)$ and $\Upsilon(3S)$ cross sections @ ATLAS):

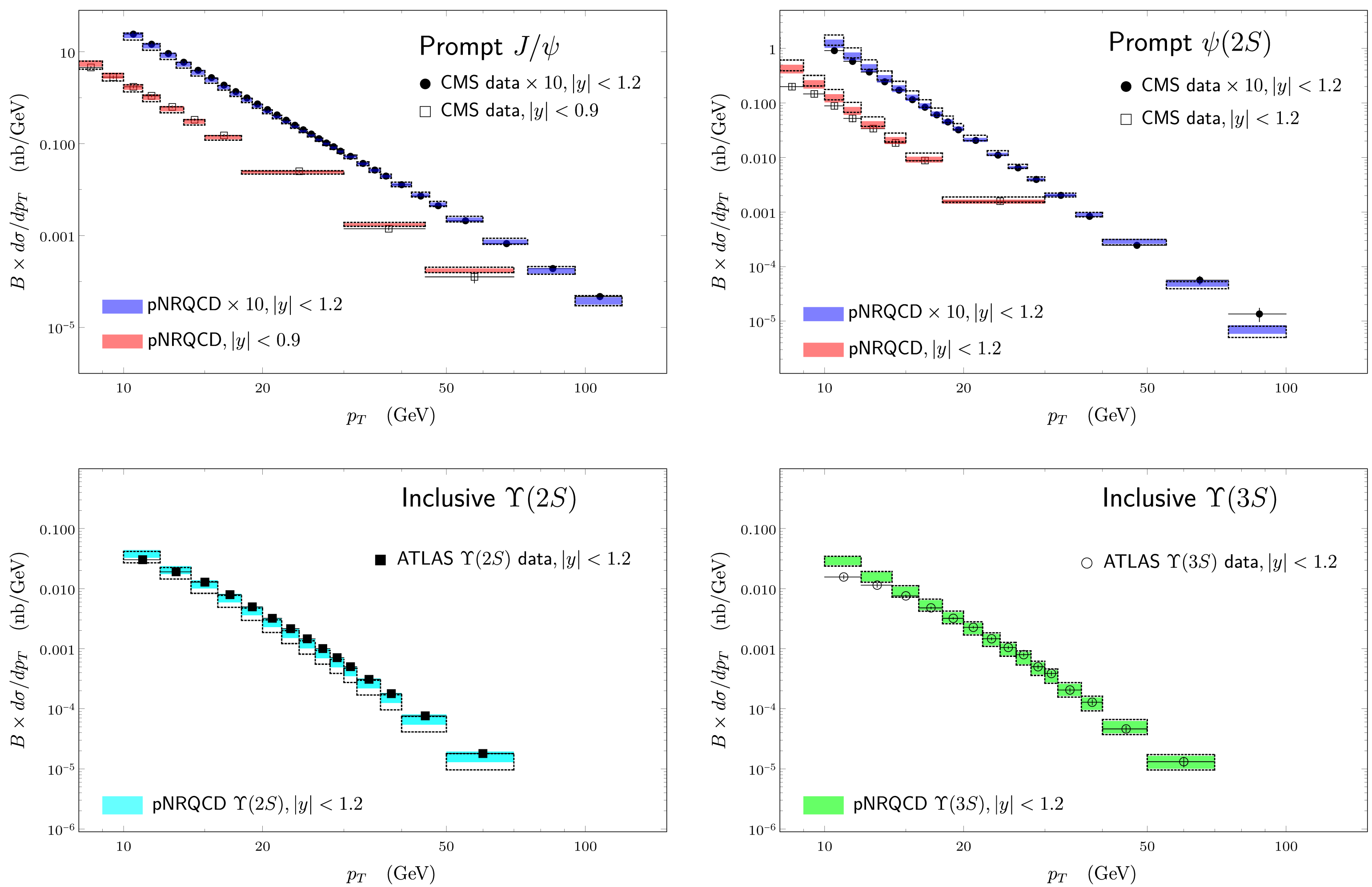
p_T cut	$\mathcal{E}_{10;10}$ (GeV^2)	$c_F^2 \mathcal{B}_{00}$ (GeV^2)	$\mathcal{E}_{00}$ (GeV^2)
$p_T/(2m) > 3$	1.14 ± 0.12	-7.13 ± 2.89	18.9 ± 2.16
$p_T/(2m) > 5$	0.96 ± 0.29	-1.29 ± 6.63	16.0 ± 5.11

LDMEs in NRQCD

V	p_T cut	$\langle \mathcal{O}^V(^3S_1^{[8]}) \rangle$	$\langle \mathcal{O}^V(^1S_0^{[8]}) \rangle$	$\langle \mathcal{O}^V(^3P_0^{[8]}) \rangle / m^2$
J/ψ	$p_T/(2m) > 3$	1.66 ± 0.18	-3.47 ± 1.41	3.07 ± 0.35
	$p_T/(2m) > 5$	1.40 ± 0.42	-0.63 ± 3.22	2.59 ± 0.83
$\psi(2S)$	$p_T/(2m) > 3$	0.99 ± 0.11	-2.07 ± 0.84	1.83 ± 0.21
	$p_T/(2m) > 5$	0.84 ± 0.25	-0.37 ± 1.92	1.55 ± 0.49
$\Upsilon(2S)$	$p_T/(2m) > 3$	1.79 ± 0.20	-1.12 ± 0.46	1.28 ± 0.15
	$p_T/(2m) > 5$	1.52 ± 0.47	-0.20 ± 1.04	1.08 ± 0.35
$\Upsilon(3S)$	$p_T/(2m) > 3$	1.39 ± 0.16	-0.87 ± 0.35	0.99 ± 0.11
	$p_T/(2m) > 5$	1.17 ± 0.37	-0.16 ± 0.81	0.84 ± 0.27

In units of 10^{-2} GeV^3.

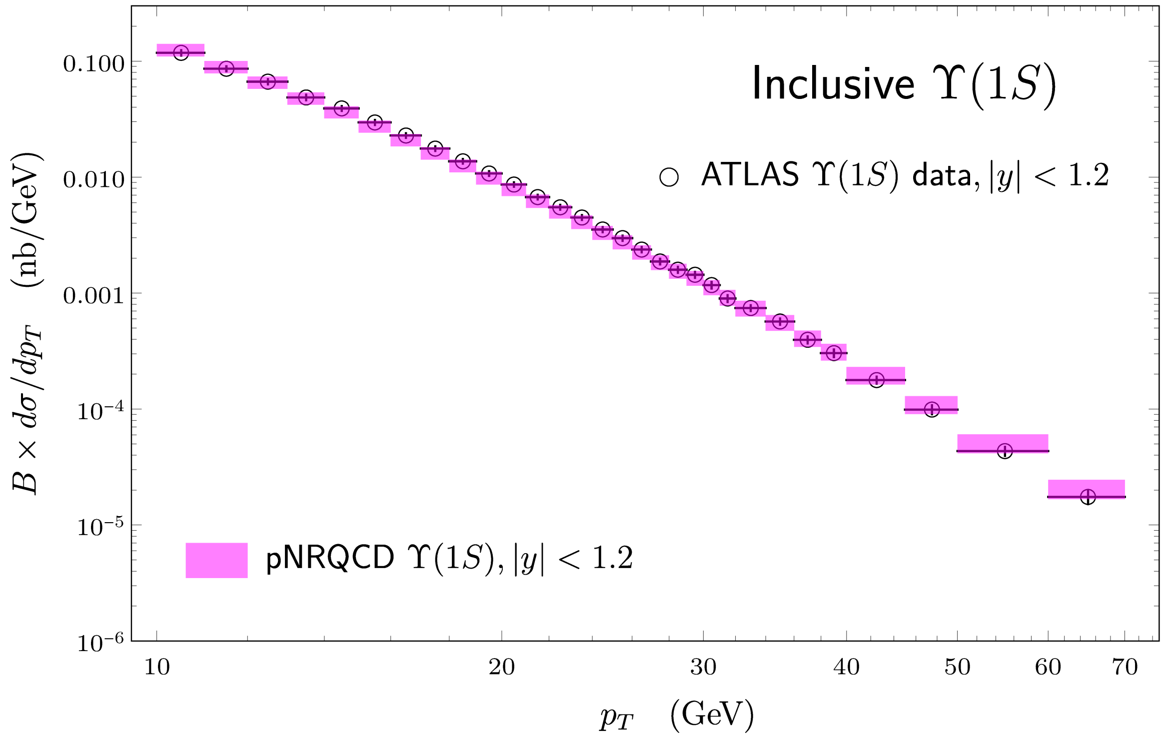
$\sigma(pp \to J/\psi + X)$, $\sigma(pp \to \psi(2S) + X)$ and $\sigma(pp \to \Upsilon(nS) + X)$



@ center of mass energy $\sqrt{s} = 7$ TeV.

- CMS coll JHEP 02 (2012) 011, PRL 114 (2015) 191802
  ATLAS coll PRD 87 (2013) 052004

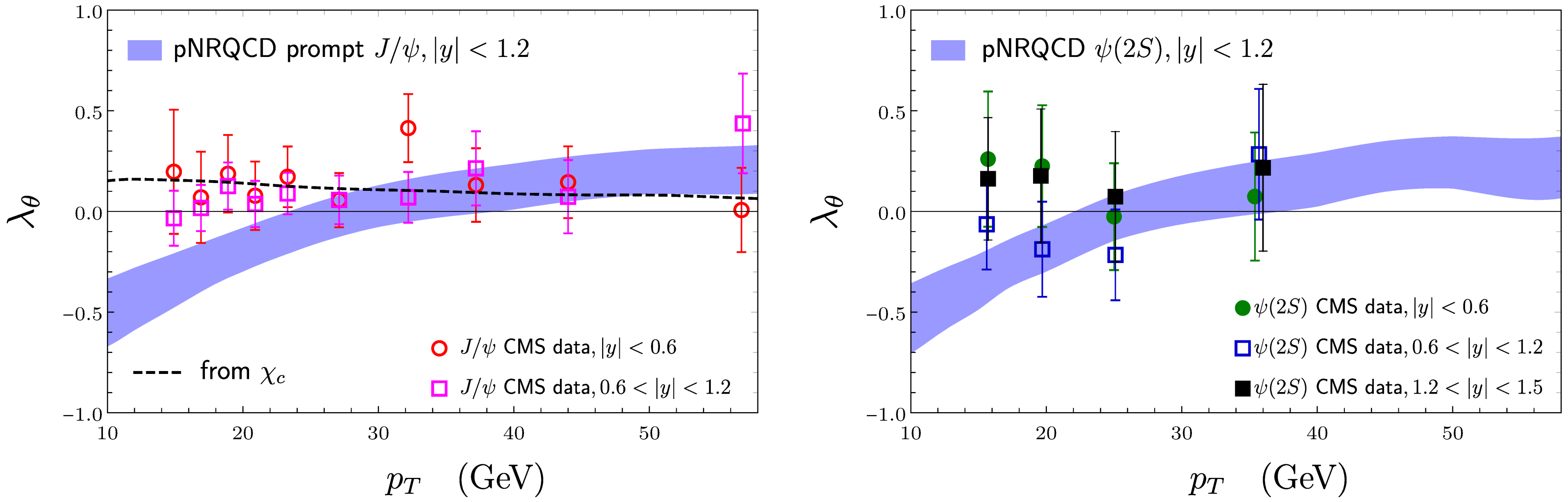
$\sigma(pp \to \Upsilon(1S) + X)$



@ center of mass energy $\sqrt{s} = 7$ TeV.

- ATLAS coll PRD 87 (2013) 052004

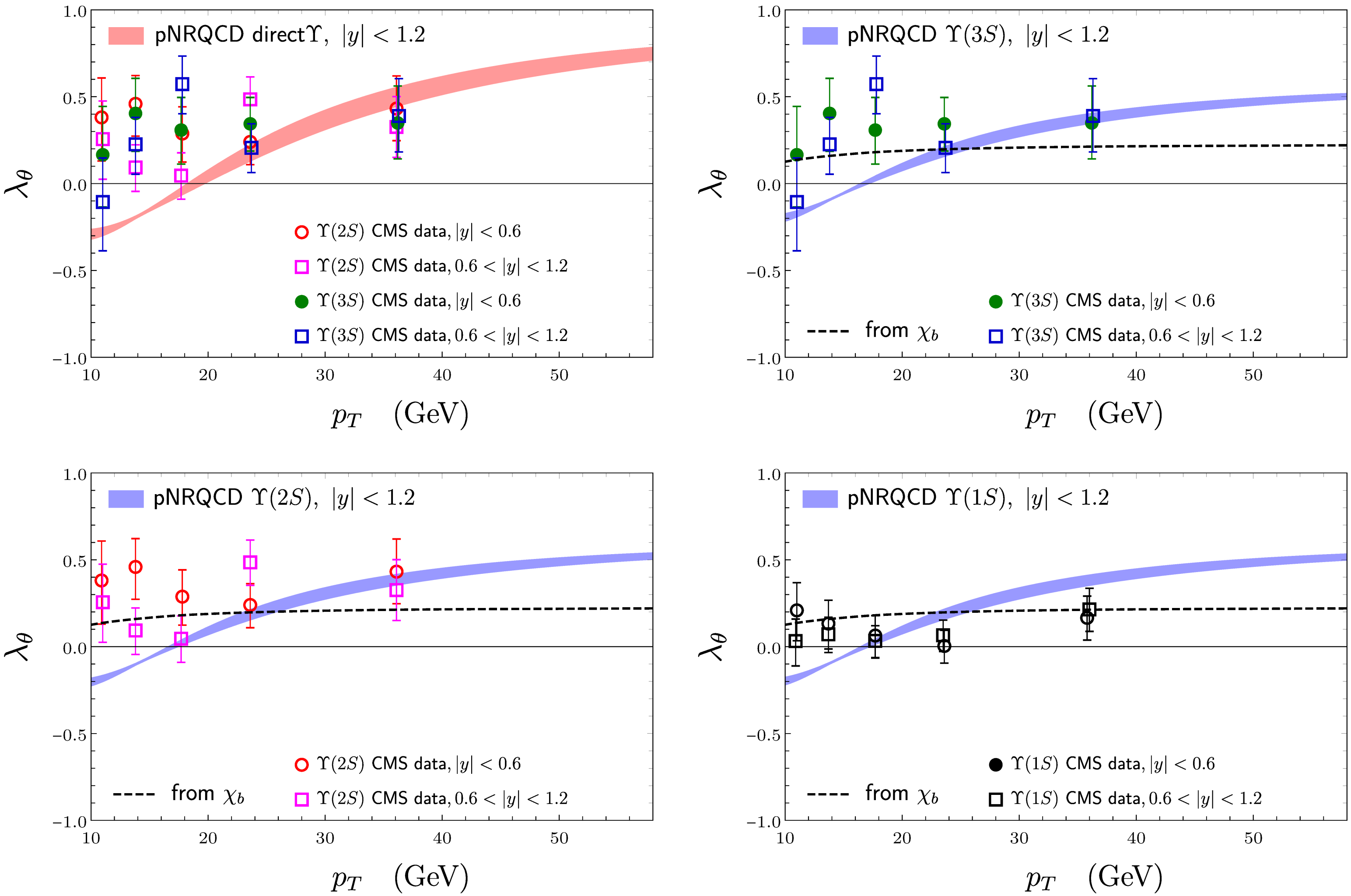
Polarization of J/ψ and $\psi(2S)$



@ center of mass energy $\sqrt{s} = 7$ TeV.

- CMS coll PRL 110 (2013) 081802, PLB 727 (2013) 381

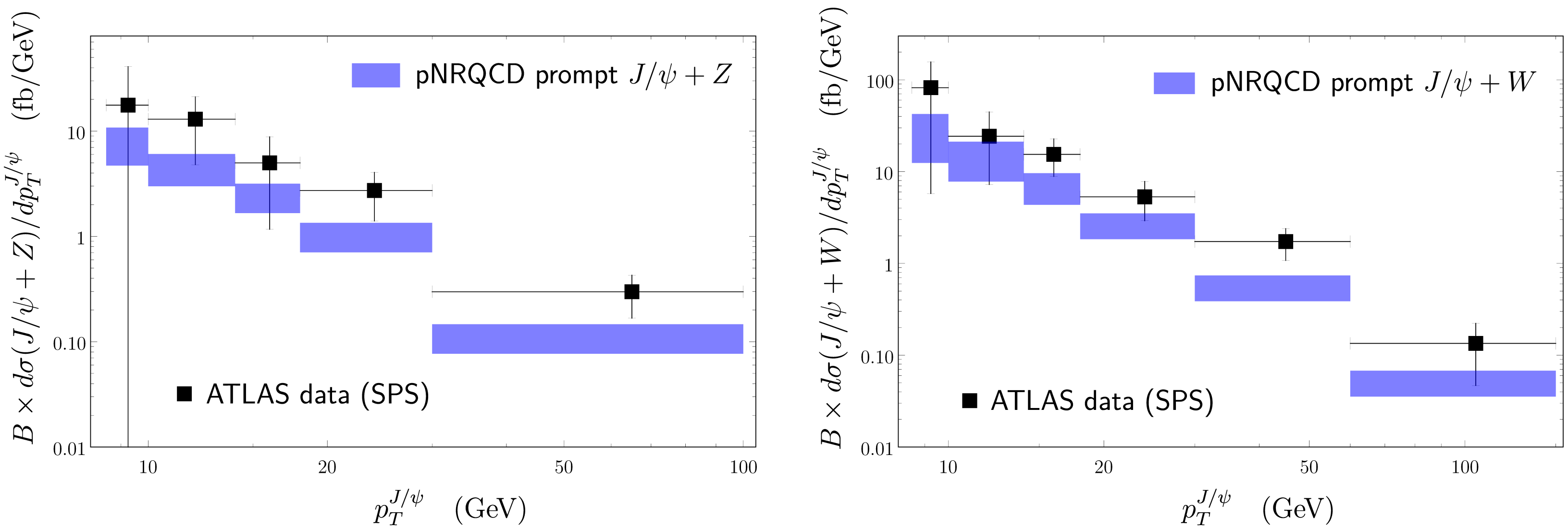
Polarization of $\Upsilon(nS)$



@ center of mass energy $\sqrt{s} = 7$ TeV.

- CMS coll PRL 110 (2013) 081802, PLB 727 (2013) 381

$\sigma(pp \to J/\psi + Z)$ and $\sigma(pp \to J/\psi + W)$



@ center of mass energy $\sqrt{s} = 8$ TeV and $|y^{J/\psi}| < 2.1$.

- ATLAS coll EPJC 75 (2015) 229, JHEP 01 (2020) 095

Conclusions

Outlook

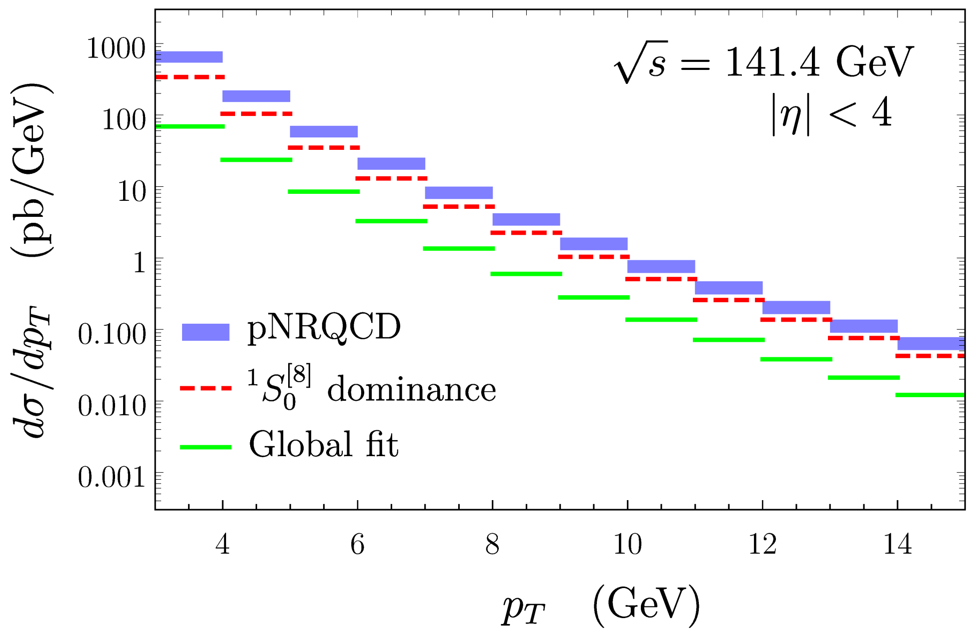
The present analysis could be significantly improved through

- computation of (integrals of) gauge field correlators in lattice QCD,
- model independent determinations of the bottomonium wavefunctions at the origin. This may require new data, e.g. for P-wave bottomonium decay widths.

Possible developments include:

- Computation of higher order corrections in the velocity expansion. They come from higher dimensional operators in the NRQCD factorization formula, from higher order corrections to the pNRQCD expansion of the NRQCD long-distance matrix elements, and from higher order corrections to the wavefunctions originating from higher order corrections to the pNRQCD potential.
- Extension of the formalism to quarkonium exotica (hybrids, tetraquarks) and to quarkonium production in electron-ion and heavy ion colliders.

$\sigma(eh \to J/\psi + X)$



@ center of mass energy $\sqrt{s} = 7$ TeV.

- Feng Gong Chang Wang PRD 99 (2019) 014044 [$1S_0^{[8]}$ dominance]
  Butenschoen Kniehl PRD 84 (2011) 051501 [Global fit]