B Physics

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TRR110 Final Colloquium

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EW/Higgs physics of Yukawa couplings, weakly coupled, flavour & CP violation

$$-\lambda^D_{IJ}\,\bar{Q}_I\phi D_J - \lambda^U_{IJ}\,\bar{Q}_I\tilde{\phi}U_J$$
$$\to -\frac{g}{\sqrt{2}}\sum_{I,J}\bar{u}_{LI}\gamma^\mu V^\dagger_{IJ,\mathrm{CKM}} d_{LJ}\,W^+_\mu + \ldots$$

QCD
strongly coupled
emergence of bound states

B physics
$m_W \gg m_b \gg \Lambda_{\mathrm{QCD}}$

EW/Higgs physics of Yukawa couplings, weakly coupled, flavour & CP violation

$$-\lambda^D_{IJ}\,\bar{Q}_I\phi D_J - \lambda^U_{IJ}\,\bar{Q}_I\tilde{\phi}U_J$$
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QCD
strongly coupled
emergence of bound states

B physics
$m_W \gg m_b \gg \Lambda_{\mathrm{QCD}}$

- Discrepancies between data and theoretical predictions ($B \to K\ell\ell$ (2013-2022), $B \to \pi K$ (2004–), $B \to DK$ (2020–), ...)

  New physics, strong interaction, experimental systematics

$$\mathcal{L} = \mathcal{L}_{\text{QCD+QED}}^{(n_f=5)} - \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \sum_i C_i(\mu)\, Q_i(\mu) + \text{h.c.}$$

Hadronic matrix elements $\langle f | Q_i(\mu) | \bar{B} \rangle$

$$\mathcal{L} = \mathcal{L}_{\rm QCD+QED}^{(n_f=5)} - \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \sum_i C_i(\mu)\, Q_i(\mu) + \text{h.c.}$$

Hadronic matrix elements $\langle f|Q_i(\mu)|\bar{B}\rangle$

	HQE/OPE, lattice, (QCD sum rules)		QCD factorization, SCET, flavour symmetries	
None – pure quantum interference	$\langle 0|\mathcal{O}|B\rangle$ $\langle B|\mathcal{O}|B\rangle$	$\langle M|\mathcal{O}|B\rangle$	$\langle M|\mathcal{O}|B\rangle$ $\langle M_1 M_2|\mathcal{O}|B\rangle$	

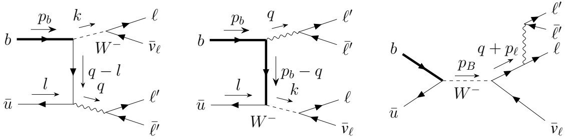
← Easier — Increasingly difficult ⇝

γ from $B \to DK$ [and related methods] 2β, $2\beta_{B_s}$	$B \to \tau\nu_\tau$ $B_s \to \mu^+\mu^-$ $\Delta M_{B_d,B_s}$ $\Delta\Gamma_{B_s}$	$B \to D\ell\nu_\tau$ $|V_{cb}|$, $|V_{ub}|$	$B \to \gamma\ell\nu$ $B \to \pi\ell\nu$ $B \to K^{(*)}\ell\ell$	Direct CP asym $B_{(s)} \to \pi K, KK, \ldots$ $B_s \to \pi\pi$ $B_s \to \phi\phi, K^{*0}\bar{K}^{*0}$ [all hadronic]

Basic methodolgy: heavy-quark and light-cone expansions

Example: radiative decays $\bar{B} \to W^* \gamma^*$

$$T^{\mu\nu} = \int d^4x\, e^{iqx} \langle 0 | \mathrm{T}\{ j^\mu_{\rm em}(x) (\bar{u}\gamma^\nu(1-\gamma_5)b)(0)\} | B^- \rangle$$



(a) (b) (c)

(a) Short-distance $x \sim 1/m_b$ heavy quark-W^* vertex

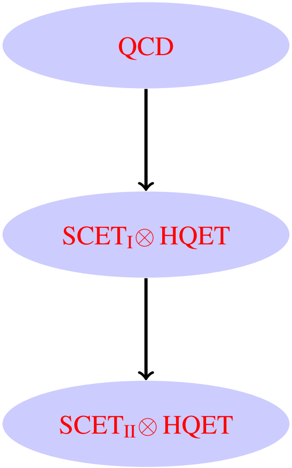
Light-cone expansion of $j_{\rm em}(x) j_{\rm weak}(0)$, $x^2 \sim 1/(n_+ q\Lambda) \ll 1/\Lambda^2$

Even when $q^2 = 0$, as long as $n_+ q \sim m_b$ (or $\gg \Lambda$) $[n_+ q = 2E_\gamma$ for $q^2 = 0]$.

$$T^{\mu\nu} \sim C(m_b) \int_0^\infty d\omega\, J(n_+ q\omega)\, \phi_B(\omega)$$

(b), (c) Always, short-distance, f_B

EFT for exclusive B decays into energetic hadrons



- Virtualities $E \sim m_b$, $\sqrt{E\Lambda_{\mathrm{QCD}}}$, Λ_{QCD}

  $$\langle f|Q_i(\mu)|\bar{B}\rangle$$

- Integrate out hard modes, virtuality m_b^2
  Virtualities $\sqrt{E\Lambda_{\mathrm{QCD}}}$, Λ_{QCD}

  (Generalized) form factors, collinear, light-like Wilson lines and convolutions

- Integrate out hard-collinear modes, virtuality $E\Lambda_{\mathrm{QCD}}$, leaving virtuality Λ_{QCD}

  (Generalized) Light-cone distribution amplitudes, soft Wilson lines and soft functions

Main B physics topics in this CRC

(I) Radiative decays and the B-meson LCDA (Munich, Beijing)

- $B \to \gamma\ell\nu,\ \gamma\ell\ell,\ \ell\ell\ell\nu,\ \ell\ell\ell\ell$
- Factorization, one-loop and power corrections
- B-meson LCDA determinations, parameterizations and QCD $\to$ bHQET matching

(II) Hadronic charmless decays (Munich, Beijing)

- NNLO calculation of QCD penguin modes and direct CP violation
- FAT – factorization-assisted topological amplitude approach
- Three-body final states

(III) Theory of hadronic structure-dependent QED effects (Munich)

- Power-enhanced effects in $B_s \to \mu^+\mu^-$
- QED factorization for hadronic decays
- Generalization of LCDAs to include QED

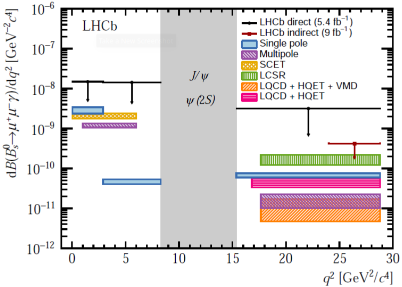
Radiative decays and the B-meson LCDA (Munich, Beijing)

Radiative decays, $B_s \to \mu^+\mu^-\gamma$

$B \to \gamma\ell\nu$, $\gamma\ell\ell$, $\ell\ell\ell\nu$, $\ell\ell\ell\ell$: Only upper bounds from BELLE (II) and LHCb experiments — true theoretical predictions.

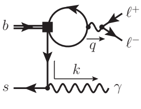
Example: $B_s \to \mu^+\mu^-\gamma$ FCNC decay

- Very rare, branching fraction $10^{-10} - 10^{-8}$ depending on the $q^2 = m^2_{\mu^+\mu^-}$ bin.
- Theoretically shares features with $B \to \ell\nu\gamma$ ($\to$ B-LCDA at LP) and $B \to K^{(*)}\ell\ell$

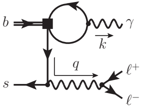


[LHCb, 2404.03375]

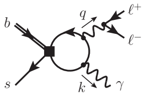
SCET computation of $B_s \to \mu^+\mu^-\gamma$



A-type

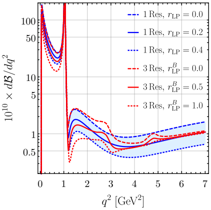


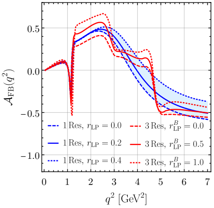
B-type



NLP

- Require an energetic photon, $E_\gamma > 1.5\,\text{GeV} \sim m_B/2$
- First calculation with systematic factorization methods.
  Accuracy: $\mathcal{O}(\alpha_s)$ at leading power, $\mathcal{O}(\alpha_s^0)$ at next-to-leading power Λ/E_γ
- Needs B-meson light-cone distribution amplitude (B-LCDA)





B-meson light-cone distribution amplitude

Key hadronic quantity for exclusive B decays with energetic final states particles

$$iF_{\rm stat}(\mu)\phi_{B+}(\omega,\mu) = \frac{1}{2\pi}\int dt\, e^{it\omega}\,\langle 0|(\bar{q}_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar{B}_v\rangle_\mu$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega,\mu), \qquad \sigma_n(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\,\ln^n\frac{\mu_0}{\omega}\,\phi_{B+}(\omega,\mu)$$

At LP, need only inverse, inverse-log moments, since $\omega \sim \Lambda_{\rm QCD}$.
Not related to local operators.

Several directions: new ideas to connect to lattice calculations, (sufficiently) general parameterisations, relation to QCD haevy meson LCDA, determination from (future) data on radiative decays

$$\Gamma(\gamma\ell\nu) \propto f_B^2\,\frac{\alpha_{\rm em}}{4\pi}\left(\frac{m_B}{\lambda_B}\right)^2$$

Factorization for $B \to \gamma^* \ell \nu$

QCD $\xrightarrow{\text{remove h}}$ SCET$_\mathrm{I}$ $\xrightarrow{\text{remove hc}}$ SCET$_\mathrm{II}$

Accuracy: $\mathcal{O}(\alpha_s)$ at LP, $\mathcal{O}(\alpha_s^0)$ at NLP Λ/n_+q

Factorization for $B \to \gamma^* \ell \nu$

$$\text{QCD} \overset{\text{remove h}}{\longrightarrow} \text{SCET}_{\text{I}} \overset{\text{remove hc}}{\longrightarrow} \text{SCET}_{\text{II}}$$

Accuracy: $\mathcal{O}(\alpha_s)$ at LP, $\mathcal{O}(\alpha_s^0)$ at NLP Λ/n_+q

Leading power (rigorous, all orders)

$$T^{\mu\nu}(p,q) = 2\, C_V^{(A0)} \int d^4x\, e^{iqx} \langle 0|T\left\{j^{\mu}_{q,\text{SCET}_\text{I}}(x), [\bar{q}_{\text{hc}}\gamma_\perp^\nu P_L h_v](0)\right\}|B_v^-\rangle$$

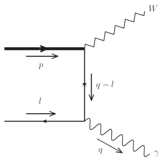
Only $F_L \neq 0$ at LP due to helicity conservation for $n_+q \gg \Lambda$. $F_R^{\text{LP}} = F_{A_\parallel}^{\text{LP}} = 0$.

$$F_L^{\text{LP}} = C_V^{(A0)}(\mu)\, \frac{Q_u F_B(\mu) m_B}{n_+q} \int_0^\infty d\omega\, \underbrace{\phi_+^B(\omega;\mu)}_{\text{B-LCDA}} \times \underbrace{\frac{J(n_+q, q^2, \omega; \mu)}{\omega - n_-q - i0^+}}_{\text{Generates rescattering phase}} \qquad [n_-q = q^2/n_+q]$$

Complex q^2-dependent inverse moment of the B-LCDA:

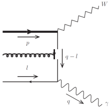
$$\frac{1}{\lambda_B^+(n_-q)} \equiv \int_0^\infty d\omega\, \frac{\phi_+^B(\omega)}{\omega - n_-q - i0^+}$$

Power corrections: Hard-collinear vs. soft



Intermediate light-quark propagator has hard-collinear virtuality $E_\gamma \Lambda$

- → Light-cone expansion in soft background field
- → Expressed in terms of moments of higher-twist and three-particle B-meson LCDAs.

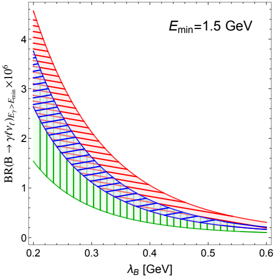


Intermediate light-quark propagator has soft virtuality $E_\gamma \omega \sim \Lambda^2$

- → Endpoint region of $\omega \sim \Lambda^2/E_\gamma$.
- → Evaluate through dispersion relation and light-cone QCD sum rule.

Still in progress: General factorization theorem and α_s corrections

Cut $E > E_{\gamma,\min}$ branching fraction

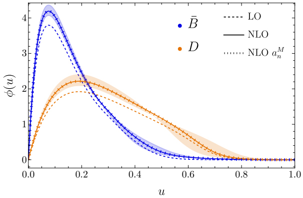


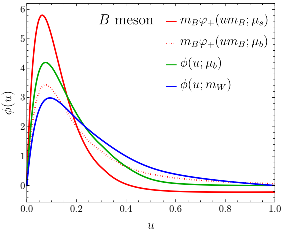
(The width of the band is not the uncertainty, since it contains the dependence on log-moments $\widehat{\sigma}_{1,2}$).

Matching the QCD B-meson LCDA to the universal HQET LCDA

$\varphi_+(\omega)$ is heavy-quark mass independent, infinite-mass limit (HQET). For $\mu \gg m_b$, the relevant quantity is defined in QCD and satifies the same ERBL evolution equation as light mesons.

$$\phi_{B,\mathrm{QCD}}(u) = \begin{cases} \frac{\tilde{f}_H}{f_H} \mathcal{J}_{\mathrm{peak}} m_B \varphi_+(u m_B)\,, & \text{for } u \sim \lambda\,, \\ \frac{\tilde{f}_H}{f_H} \mathcal{J}_{\mathrm{tail}}(u)\,, & \text{for } u \sim 1\,. \end{cases} \tag{1}$$





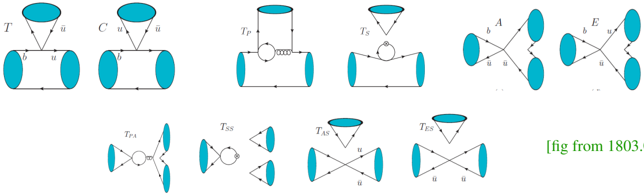
Hadronic B decays

(Munich, Beijing)

Hadronic quasi-two-body charmless B decays ($B \to \pi\pi, \pi K, \pi\rho, \ldots$)

Large number of different final states (130 $B_{u,d,s}$ to ground-state nonet).
Good place to look for direct CP violation.

$$\mathcal{A}(\bar{B} \to f) = V_{ub} V_{uD}^{*} A_f^u + V_{cb} V_{cD}^{*} A_f^c = \sum_i \left[\lambda_{\rm CKM} \times C \times \langle f | \mathcal{O} | \bar{B} \rangle_{\rm QCD+QED} \right]_i$$



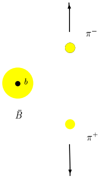
[fig from 1803.04227]

- Topological amplitudes (often with flavour SU(3) or SU(2))

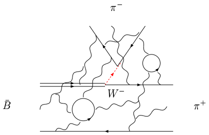
$$T, C, P, P_{\rm EW}, S, E, A, \ldots$$

- QCD factorization and factorization-assisted topological amplitude approach.

QCD theory



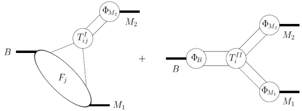
$$\langle \pi^- \pi^+ | \bar{B} \rangle_{\mathcal{L}_{\rm SM}} \quad =$$



"QCD factorization" , later understood and formulated as a SCET$_{\rm II}$ problem:

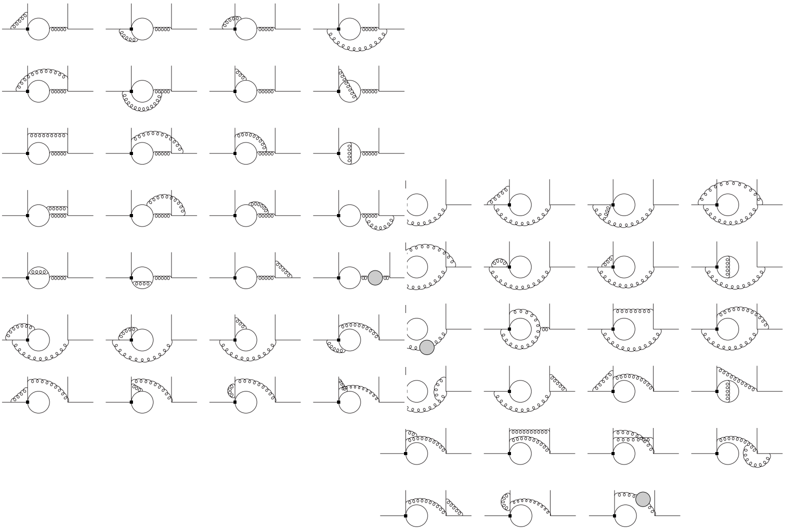
$$\text{QCD} \quad \overset{\text{remove h}}{\longrightarrow} \quad \text{SCET}_{\rm I} \quad \overset{\text{remove hc}}{\longrightarrow} \quad \text{SCET}_{\rm II}(c, \bar{c}, s)$$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = \underbrace{F^{BM_1}(0)}_{\text{form factor}} \int_0^1 du\, T_i^I(u) \Phi_{M_2}(u)$$



$$+ \int_0^1 dz du\, H_i^{\rm II}(z,u) \int_0^\infty d\omega \int_0^1 dv\, J(\omega, u, v)\ \underbrace{\Phi_B(\omega) \Phi_{M_1}(v) \Phi_{M_2}(u)}_{\text{LCDAs}}$$

Two-loop computation of $T_i^I(u)$ for the QCD penguin amplitude



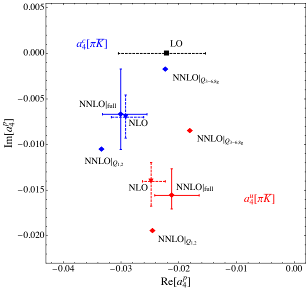
Numerical result

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.49 - 1.32i]_{P_1}$$
$$-[0.32 + 0.71i]_{P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{P_2,\,Q_{3-6,8g}}$$
$$+ \left[\frac{r_{\rm sp}}{0.434}\right]\Big\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} - [0.01 - 0.05i]_{\rm HP} + [0.07]_{\rm tw3}\Big\}$$
$$= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i$$

$$r_{\rm sp} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.05 - 0.62i]_{P_1}$$
$$-[0.77 + 0.50i]_{P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{P_2,\,Q_{3-6,8g}}$$
$$+ \left[\frac{r_{\rm sp}}{0.434}\right]\Big\{[0.13]_{\rm LO} + [0.14 + 0.12i]_{\rm HV} + [0.01 + 0.03i]_{\rm HP} + [0.07]_{\rm tw3}\Big\}$$
$$= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i$$

- $\mathcal{O}(\alpha_s^2)$ correction not large.
  Scale dependence reduced.



- Concludes the programme of computing perturbative corrections
- Data reanalysis postponed, focus on better understanding /modelling of power corrections
- Weak annihilation and the colour-suppressed tree amplitude C

Theory of hadronic structure-dependent QED effects (Munich)

Observables, virtual and radiation effects, scales

IR finite observable is

$$\Gamma_{\text{phys}} = \sum_{n=0}^{\infty} \Gamma(B \to f + n\gamma, \sum_{n} E_{\gamma,n} < \Delta E) \equiv \omega(\Delta E) \times \Gamma_{\text{non-rad.}}(B \to f)$$

Assume $\Delta E \ll \Lambda_{\text{QCD}} \sim$ size of hadrons

Observables, virtual and radiation effects, scales

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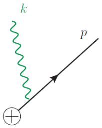
Universal soft radiative amplitude



$$A^{i\to f+\gamma}(p_j, k) = A^{i\to f}(p_j) \times \sum_{j=\text{legs}} \frac{-eQ_j p_j^\mu}{\eta_j p_j \cdot k + i\epsilon}$$

The amplitude implies that the charged particles (B-meson, pion, lepton, ...) are treated as point-like. Exponentiates for the decay rate, but the virtual correction is UV divergent in the soft limit. Cut-off Λ.

$$\Gamma = \Gamma_{\text{tree}}^{i\to f} \times \left(\frac{2\Delta E}{\Lambda}\right)^{-\frac{\alpha}{\pi}\sum_{i,j} Q_i Q_j f(\beta_{ij})}$$

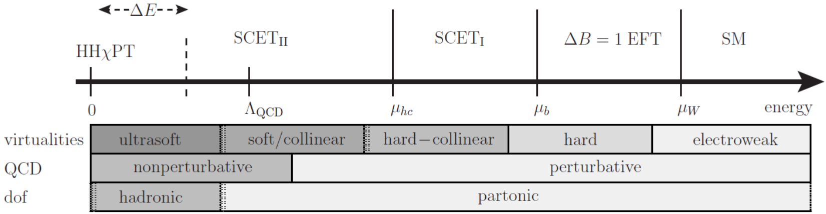
$\Lambda = m$ for point-like particle , size of hadron in QCD

Scales and Effective Field theories (EFTs)

↪ Structure-dependent virtual QED corrections between m_B and $\Lambda_{\rm QCD}$.

↪ Real radiation in point-like meson EFT.

Multiple scales: m_W, m_b, $\sqrt{m_b \Lambda_{\rm QCD}}$, $\Lambda_{\rm QCD}$, ΔE

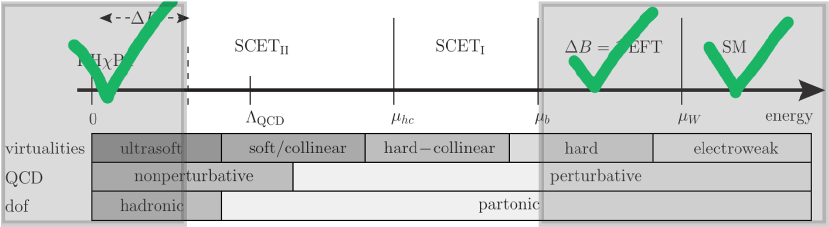


Scales and Effective Field theories (EFTs)

↪ Structure-dependent virtual QED corrections between m_B and Λ_{QCD}.

↪ Real radiation in point-like meson EFT.

Multiple scales: m_W, m_b, $\sqrt{m_b \Lambda_{\text{QCD}}}$, Λ_{QCD}, ΔE



Scales and Effective Field theories (EFTs)

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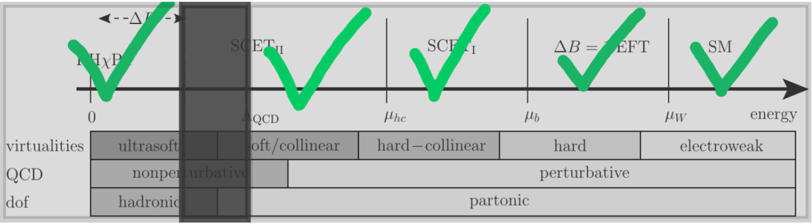
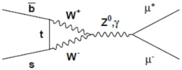
Multiple scales: $m_W, m_b, \sqrt{m_b\Lambda_{\rm QCD}}, \Lambda_{\rm QCD}, \Delta E$



Can sum leading logs, and calculate all QED effects between scale m_b and a few times $\Lambda_{\rm QCD}$, matching of $\text{SCET}_{\rm II}$ to the ultrasoft theory of point-like hadrons at a scale $\mu_c \sim \Lambda_{\rm QCD}$ is non-perturbative.

$B_s \to \mu^+\mu^-$

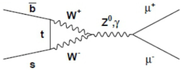
"Instantaneous", "non-radiative" branching fraction



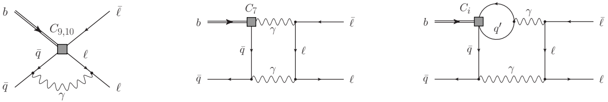
$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2 \alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \left| \frac{2m_\mu}{m_{B_s}} C_{10} \right|^2$$

$B_s \to \mu^+\mu^-$

"Instantaneous", "non-radiative" branching fraction



$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb}V_{ts}^*|^2 \left| \frac{2m_\mu}{m_{B_s}} C_{10} \right|^2$$



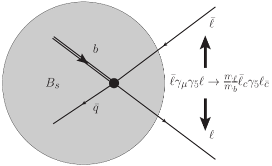
$$i\mathcal{A} = m_\ell f_{B_q}\mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5\ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell f_{B_q}\mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell$$

$$\times \left\{ \int_0^1 du\, (1-u)\, C_9^{\rm eff}(um_b^2)\, m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln\frac{m_b\omega}{m_\ell^2} + \ln\frac{u}{1-u} \right] \right.$$

$$\left. - Q_\ell C_7^{\rm eff} m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2\frac{m_b\omega}{m_\ell^2} - 2\ln\frac{m_b\omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \right\} + \ldots$$

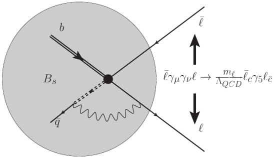
The virtual photon probes the B meson structure. B-meson LCDA and $1/\lambda_B$ enters.
m_B/Λ power-enhanced and (double) logarithmically enhanced, purely virtual correction

Interpretation of the m_B/Λ-enhanced QED correction



$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}_q(p)\rangle$$

Local annihilation and helicity flip.



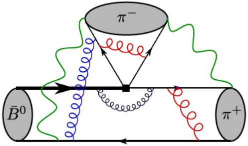
$$\langle 0|\int d^4x\, T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

Helicity-flip and annihilation delocalized by a hard-collinear distance

The virtual photon probes the B meson structure. Annihilation/helicity-suppression is “smeared out” over light-like distance $1/\sqrt{m_B\Lambda}$ [→ B-LCDA]. Still short-distance.

Logarithms are not the standard soft logarithms, but due to hard-collinear, collinear and soft regions, including final-state soft lepton exchange → Sudakov resummation.

Including virtual QED effects into the factorization theorem for hadronic two-body decays



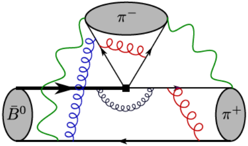
SCET$_\mathrm{I}$ operators

$$\mathcal{O}^{\mathrm{I}}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$\mathcal{O}^{\mathrm{II}}(t,s) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\not{\mathcal{A}}_{C,\perp}(sn_+)\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$S_{n_\pm}^{(q)} = \exp\left\{-iQ_q e\int_0^\infty ds\, n_\pm A_s(sn_\pm)\right\}$$

Including virtual QED effects into the factorization theorem for hadronic two-body decays



SCET$_\mathrm{I}$ operators

$$\mathcal{O}^\mathrm{I}(t) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$\mathcal{O}^\mathrm{II}(t,s) = [\bar{\chi}_{\bar{C}}(tn_-)\not{n}_-\gamma_5\chi_{\bar{C}}]\,[\bar{\chi}_C\not{\mathcal{A}}_{C,\perp}(sn_+)\,\mathbf{S}_{n_+}^{\dagger(Q_{M_2})}h_v]$$

$$S_{n_\pm}^{(q)} = \exp\left\{-iQ_q e\int_0^\infty ds\, n_\pm A_s(sn_\pm)\right\}$$

QCD + QED factorization formula

$$\langle M_1M_2|Q_i|\bar{B}\rangle_{|\mathrm{non-rad.}} = \mathcal{F}_{Q_2}^{BM_1}(0)\int_0^1 du\;\underbrace{T_{i,Q_2}^{\mathrm{I,QCD+QED}}(u)}_{\mathcal{O}(\alpha_\mathrm{em})\text{ corrected SD}}\;f_{M_2}\Phi_{M_2}(u)$$

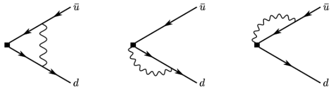
$$+\int_{-\infty}^{\infty}d\omega\int_0^1 dudv\,T_{i,\otimes}^{\mathrm{II,QCD+QED}}(z,u)\,f_B\Phi_{B,\otimes}(\omega)f_{M_1}\Phi_{M_1}(v)f_{M_2}\Phi_{M_2}(u)$$

Formula retains its form, but the hadronic matrix elements are generalized. They become process-dependent through the directions and charges of the *other* particles.

LCDA of a charged pion in QCD×QED

QCD definition

$$\langle \pi^-(p)|\, \bar{d}(tn_+)\frac{\not{n}_+}{2}\gamma_5[tn_+,0]u(0)|0\rangle = -\frac{in_+p}{2}\int_0^1 du\, e^{iu(n_+p)t} f_M \phi_M(u;\mu)$$



ERBL kernel [Efremov, Radyushkin, Brodsky, Lepage, 1979]

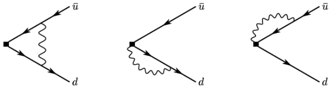
$$\Gamma(u,v;\mu) = -\left(\frac{\alpha_s C_F}{\pi}\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\,\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\,\theta(u-v)\right]_+$$

- LCDA symmetric in $u \leftrightarrow 1-u$
- One-loop kernel diagonalized by Gegenbauer polynomials, asymptotic behaviour $\Phi_\pi(u,\mu) \overset{\mu\to\infty}{\to} 6u(1-u)$.

LCDA of a charged pion in QCD×QED

QCD×QED

$$\langle \pi^-(p)| R_c^{(Q_M)}\,(\bar{d}W^{(d)})(tn_+)\frac{\not{n}_+}{2}\gamma_5[tn_+,0](W^{\dagger(u)}u)(0)|0\rangle = -\frac{in_+p}{2}\int_0^1 du\, e^{iu(n_+p)t} f_M \Phi_M(u;\mu)$$



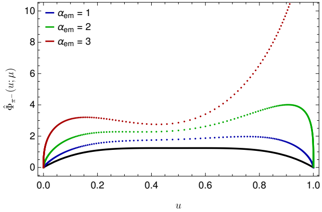
QCD×QED kernel

$$\Gamma(u,v;\mu) = -\frac{\alpha_{\rm em} Q_M}{\pi}\,\delta(u-v)\left(Q_M\left(\ln\frac{\mu}{2E}+\frac{3}{4}\right) - Q_d \ln u + Q_u \ln\bar{u}\right)$$
$$-\left(\frac{\alpha_s C_F}{\pi}+\frac{\alpha_{\rm em}}{\pi}Q_uQ_d\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\,\theta(v-u)+\left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\,\theta(u-v)\right]_+$$

- Cusp anomalous dimension, logarithms $\ln u$, $\ln(1-u)$ and energy dependence are a remnant of the soft physics and breaking of boost invariance.
- Gegenbauer polynomials are no longer eigenfunctions, asymptotic behaviour $\Phi_\pi(u,\mu) \overset{\mu\to\infty}{\to} 6u(1-u)$ no longer holds. QED evolution is asymmetric and endpoint behaviour changes from linear.

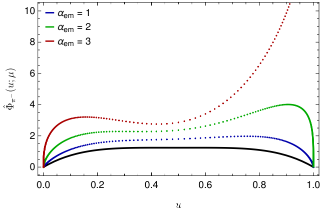
Endpoint behaviour and numerical QED effect

(Large $\alpha_{\rm em}$ for numerical illustration)



Endpoint behaviour and numerical QED effect

(Large $\alpha_{\rm em}$ for numerical illustration)



Example for the true $\alpha_{\rm em}$:

$$\left\langle \bar{u}^{-1} \right\rangle_{M^-}(\mu) = \int_0^1 \frac{du}{1-u} \Phi_{M^-}(u;\mu) = 3Z_\ell(\mu) \sum_{n=0}^{\infty} a_n^{M^-}(\mu)$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-}(5.3\,\text{GeV}) = 0.9997\Big|^{\text{QED}}_{\text{point charge}} (3.285^{+0.05}_{-0.05}\big|_{\text{LL}} - 0.020\big|_{\text{NLL}} + 0.017\Big|^{\text{QED}}_{\text{partonic}})$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-}(80.4\,\text{GeV}) = 0.985\Big|^{\text{QED}}_{\text{point charge}} (3.197^{+0.03}_{-0.03}\big|_{\text{LL}} - 0.022\big|_{\text{NLL}} + 0.042\Big|^{\text{QED}}_{\text{partonic}})$$

QED effects of similar size as NLL evolution for the inverse moments (Initial value: 3.42 at $\mu = 1$ GeV.)

QED effects for the B-LCDA generalise it to a soft function with support for negative ω.

QED corrections to $B \to D(\pi, K)$

The theoretically computed colour-allowed tree amplitude $T = a_1$ appears to be 15% smaller than the value inferred from data.

$$R_L^{(0),(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)(\Delta E)}{d\Gamma^{(0)}(\bar{B}_d \to D^{(*)+} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}}$$

$$= R_L^{(*)}|_{\rm QCD} \left(1 + \delta_{\rm QED}(D^{(*)}L) + \delta_{\rm U}^{(0)}(\Delta E)\right)$$

$$\delta_{\rm QED} \equiv \frac{2{\rm Re}(\delta a_1^{\rm WC} + \delta a_1^{\rm K} + \delta a_1^{\rm L}) a_1^{\rm tree}}{|a_1^{\rm QCD}|^2} \,.$$

$R_L^{(*)}$	LO	QCD NNLO	$+\delta_{\rm QED}$	$+\delta_{\rm U}$ ($\delta_{\rm U}^{(0)}$)
R_π	0.969 ± 0.021	$1.078^{+0.045}_{-0.042}$	$1.069^{+0.045}_{-0.041}$	$1.074^{+0.046}_{-0.043}$ $(1.003^{+0.042}_{-0.039})$
R_π^*	0.962 ± 0.021	$1.069^{+0.045}_{-0.041}$	$1.059^{+0.045}_{-0.041}$	$1.065^{+0.047}_{-0.042}$ $(0.996^{+0.043}_{-0.039})$
$R_K \cdot 10^2$	7.47 ± 0.07	$8.28^{+0.27}_{-0.26}$	$8.21^{+0.27}_{-0.26}$	$8.44^{+0.29}_{-0.28}$ $(7.88^{+0.26}_{-0.25})$
$R_K^* \cdot 10^2$	6.81 ± 0.16	$7.54^{+0.31}_{-0.29}$	$7.47^{+0.30}_{-0.29}$	$7.68^{+0.32}_{-0.30}$ $(7.19^{+0.29}_{-0.28})$

Table 3: Theoretical predictions for $R_L^{(*)}$ expressed in GeV^2 at LO, NNLO QCD and subsequently adding $\delta_{\rm QED}$ given in (82) and the ultrasoft effects $\delta_{\rm U}$ (or in brackets $\delta_{\rm U}^{(0)}$).

Other B physics topics in this CRC

- $B \to$ light meson form factors (Munich [van Dyk], Beijing), dispersive treatments and factorization + QCD sum rules

- Three-body decays $B \to 3\pi$ (Munich [Vos], Beijing)

- Dispersive treatment of the non-local hadronic amplitude in $B \to K^{(*)}\ell^+\ell^-$ decays (Munich)

- Baryon number violation (Munich, Beijing)

  Can there be observable baryon number violation in B decays, if BNV is confined to third-generation quarks? (cite LHCb) search. Excluded by proton decay

$$\langle \pi\ell | T(\mathcal{O}_{\rm BNV}(x)\mathcal{O}_{\Delta B=1}(0)|p\rangle$$

- BELLE II physics book

12. Charmless Hadronic B Decays and Direct CP Violation

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