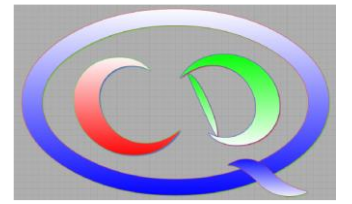


中国科学院高能物理研究所
Institute of High Energy Physics, CAS



中国科学院
CHINESE ACADEMY OF SCIENCES



Triangle singularity

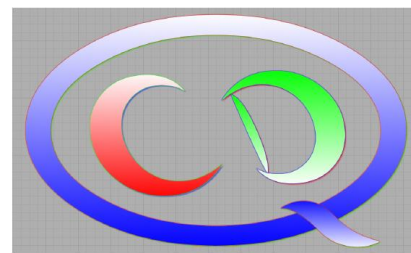
Qiang Zhao

Division of Theoretical Physics

Institute of High Energy Physics, CAS

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(On behalf of **B3** PIs)



Outline

- 1. B3: Hadronic molecules**
- 2. Open threshold phenomena and threshold dynamics**
- 3. Triangle singularity mechanism and its manifestations**
- 4. Brief summary**

1. B3: Hadron molecules with heavy meson loops

- **PIs of the 1st term:**
 - PD Dr. **Ch. Hanhart**, FZJ;
 - Prof. Dr. **Q. Zhao**, IHEP-CAS;
 - Dr. **F.-K. Guo**, HISKP-UB
- **PIs of the 2nd term:**
 - Prof. Dr. **Ch. Hanhart**, FZJ;
 - Dr. **Q. Wang**, HISKP-UB;
 - Prof. Dr. **Q. Zhao**, IHEP-CAS
- **PIs of the 3rd term:**
 - Prof. Dr. **Ch. Hanhart**, FZJ;
 - Prof. Dr. **Q. Wang**, IHEP-CAS/SCNU;
 - Prof. Dr. **Q. Zhao**, IHEP-CAS
- **In the 2nd term Feng-Kun** joined A1, A5 as a PI from ITP-CAS; **Qian**, who was a Postdoc in the 1st term, becomes a PI as a senior Postdoc from HISKP-UB
- **In the 3rd term Feng-Kun** continued as a PI in A1 and A5; **Qian** moved to SCNU as a faculty and held a joint position in IHEP-CAS

2. Open threshold phenomena and threshold dynamics

Hadrons beyond the conventional quark model

Exotics of Type-I:

J^{PC} are not allowed by $Q \bar{Q}$ configurations, e.g. $0^-, 1^+ \dots$

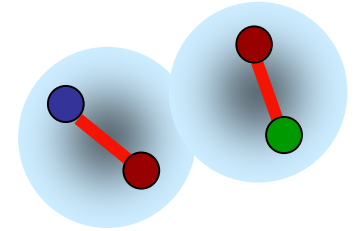
- Direct observation



Exotics of Type-II:

J^{PC} are the same as $Q \bar{Q}$ configurations

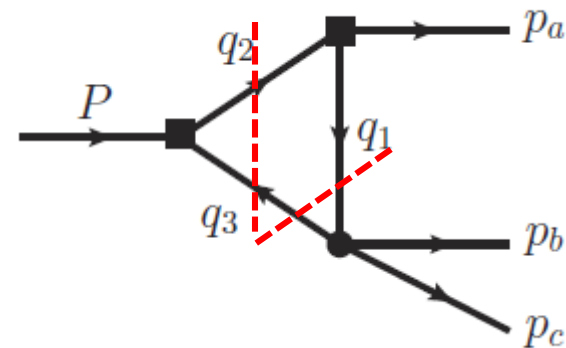
- Outnumbering of conventional QM states?
- Peculiar properties?



“Exotics” of Type-III:

Leading kinematic singularity can cause measurable effects, e.g. **the triangle singularity.**

- What's the impact?
- How to distinguish a genuine state from kinematic effects?



Production processes in experiment

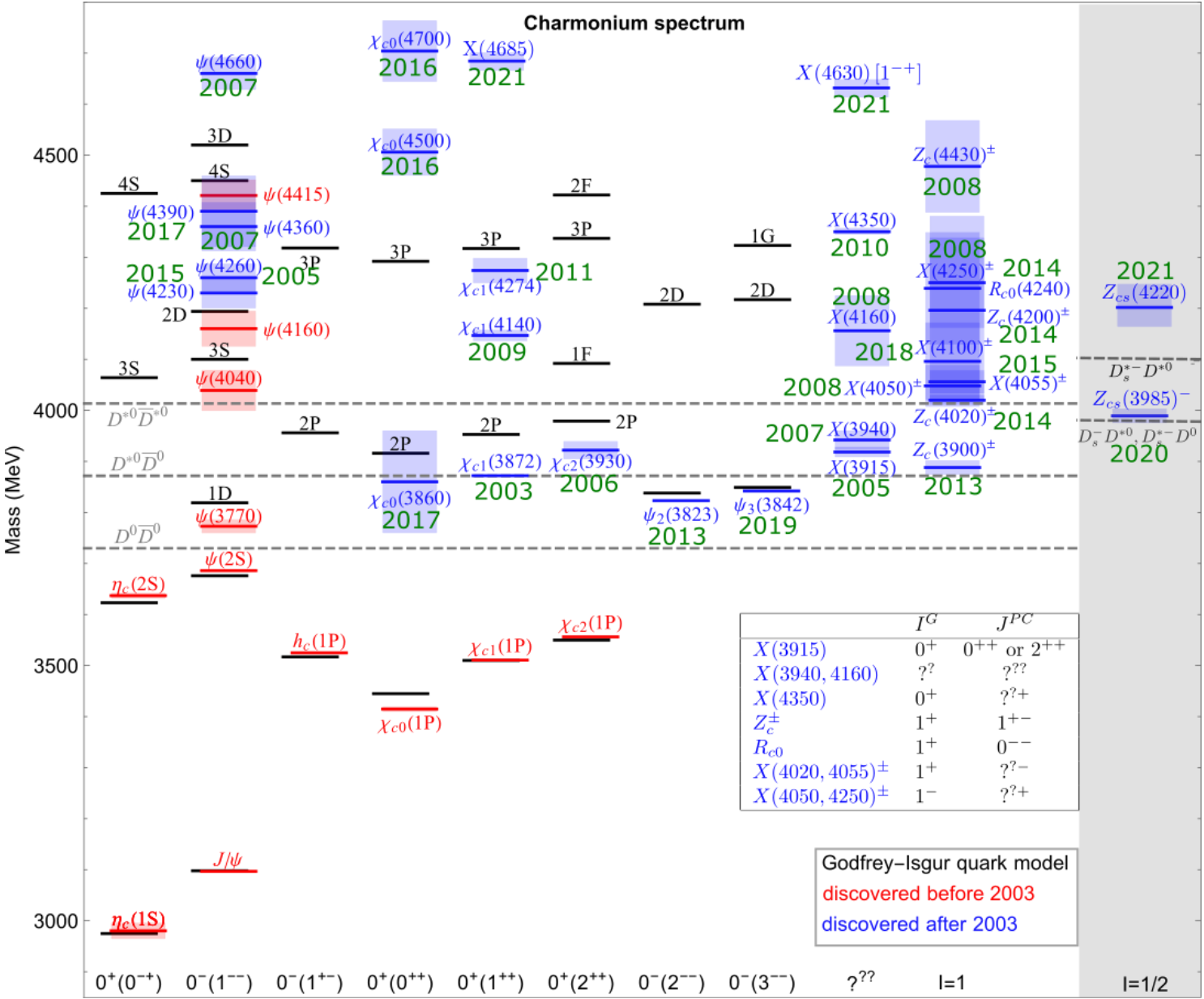
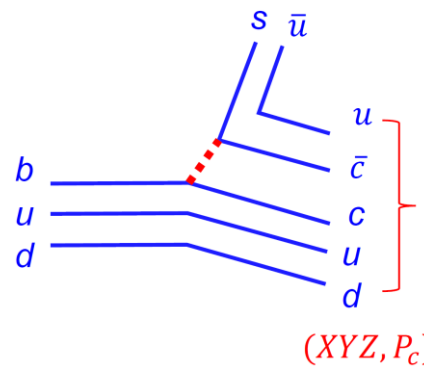
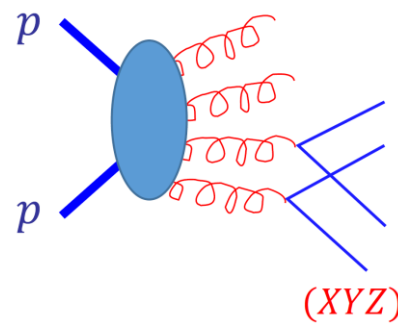
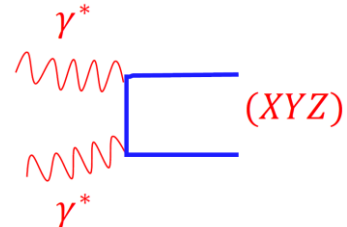
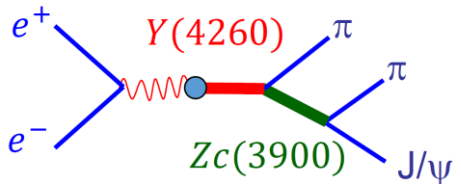
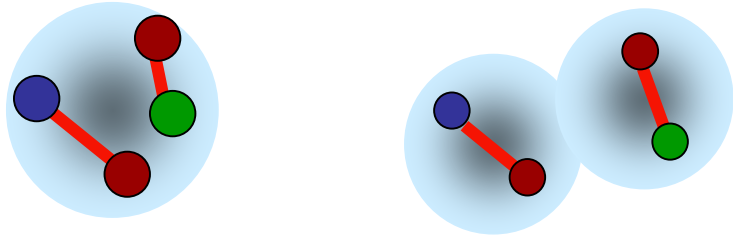


Chart plotted by Fengkun Guo

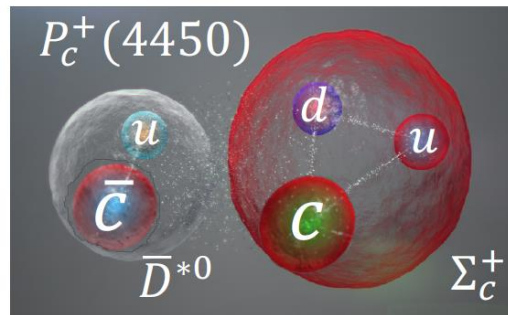
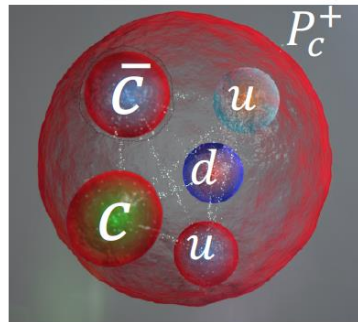


Genuine color-singlet multiquark states vs. hadron molecules

Tetraquark vs. hadronic molecule



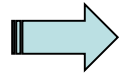
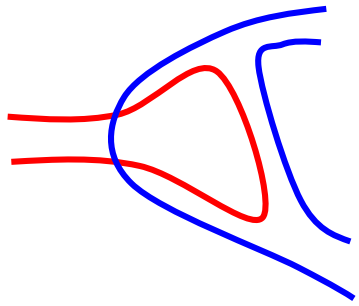
Pentaquark vs. hadronic molecule



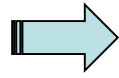
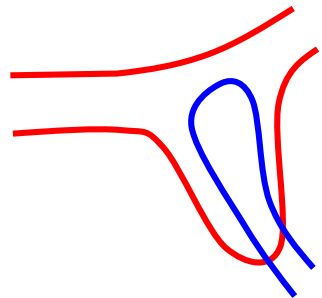
Status rating	
****	$X(3872), Z_c(3900)$
***	$Y(4260)/Y(4230), P_c(4440), P_c(4457), P_c(4312), T_{cc}(3876), X(6900)$
**	$\psi_2(3823), X(4140), X(4274), X(4500), X(4700), Z_c(4360), Z_c(4430), Z_{c1}(4050), Z_{c2}(4250), Z_c(4200), Z_c(4020), Z_b(10610), Z_b(10650), Y(4660), X(6200), X(7200) \dots$
*	$Y(4008), Z_{cs}(3985), Z_{cs}(4000) \dots$

- Why we do not see rich spectra arising from genuine color-singlet multiquark states?
- What is the manifestation of the threshold dynamics?
- What is the role played by the triangle singularity mechanism?
-

- Manifestations of the hadron loop effects and the TS mechanism in various threshold dynamics

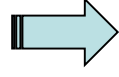
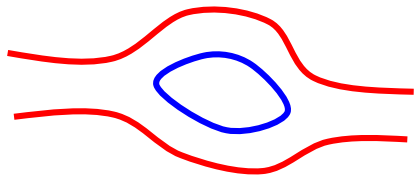


$\psi(3770) \rightarrow nonD\bar{D}$ Y.J. Zhang, G. Li, Q. Zhao, PRL(2009);
 “ $\rho\pi$ puzzle” X. Liu, B. Zhang, X.Q. Li, PLB(2009)
 Q. Wang et al. PRD(2012), PLB(2012)
 $\chi_{c1} \rightarrow VV, \chi_{c2} \rightarrow VP$ X.-H. Liu et al, PRD81, 014017(2010);
 X. Liu et al, PRD81, 074006(2010)
 $\eta_c(\eta'_c) \rightarrow VV$ Q. Wang et al, PRD2012



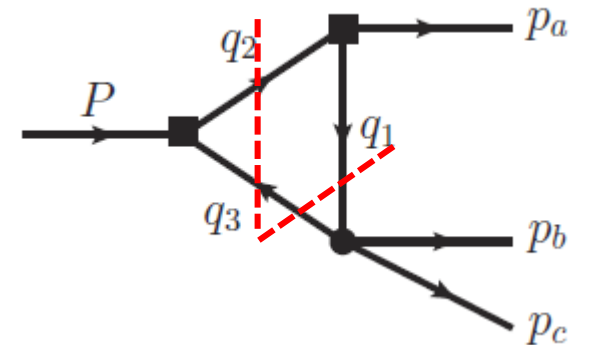
$\psi' \rightarrow J/\psi\pi^0, \psi' \rightarrow J/\psi\eta$
 $\psi' \rightarrow \gamma\eta_c, J/\psi \rightarrow \gamma\eta_c$

G. Li and Q. Zhao, PRD(2011)074005
 F.K. Guo, C. Hanhart, G. Li, U.-G. Meißner and Q. Zhao, PRD82, 034025 (2010); PRD83, 034013 (2011)
 F.K. Guo and Ulf-G Meißner, PRL108(2012)112002



$D_{s1}(2460) - D_{s1}(2536)$
 The mass shift in charmonia and charmed mesons, E.Eichten et al., PRD17(1987)3090
 X.-G. Wu and Q. Zhao, PRD85, 034040 (2012)

The open channel couplings introduce NOT ONLY additional dynamics (add. effective DOF) into the hadron structures, BUT ALSO novel kinematic effects, i.e. triangle singularity ...

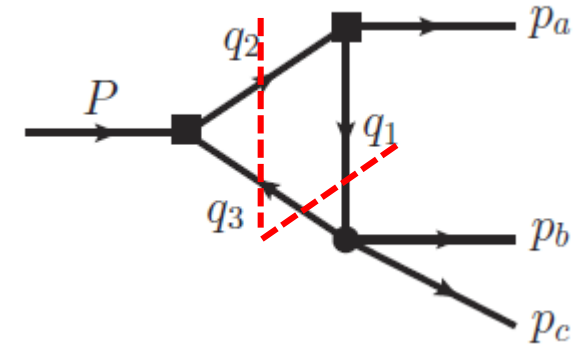


“Exotics” of Type-III:

Peak structures caused by kinematic effects, in particular, by triangle singularity.

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{i(2\pi)^4} \int \frac{d^4 q_1}{(q_1^2 - m_1^2 + i\epsilon)(q_2^2 - m_2^2 + i\epsilon)(q_3^2 - m_3^2 + i\epsilon)}$$
$$= \frac{-1}{16\pi^2} \int_0^1 \int_0^1 \int_0^1 da_1 da_2 da_3 \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon},$$

$$D \equiv \sum_{i,j=1}^3 a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2} [m_i^2 + m_j^2 - (q_i - q_j)^2]$$



The TS occurs when all the three internal particles can approach their on-shell condition simultaneously:

$$\partial D / \partial a_j = 0 \quad \text{for all } j = 1, 2, 3. \quad \Rightarrow \quad \det[Y_{ij}] = 0$$

L. D. Landau, Nucl. Phys. 13, 181 (1959);

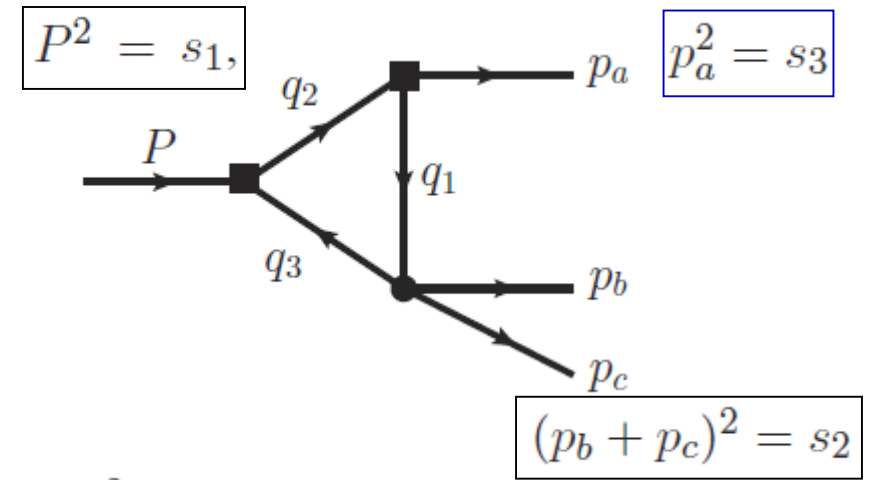
J.J. Wu, X.-H. Liu, Q. Zhao, B.-S. Zou, Phys. Rev. Lett. 108, 081003 (2012);

Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013); Phys. Lett. B 725, 106 (2013)

X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016);

F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, arXiv:1705.00141[hep-ph], Rev. Mod. Phys. 90, 015004 (2018); F.K. Guo, X.-H. Liu, S. Sakai, Prog.Part.Nucl.Phys. 112 (2020) 103757, arXiv:1912.07030[hep-ph]

Kinematics :



The ATS condition for **fixed** s_1 , m_j , and s_3 is:

$$s_2^\pm = (m_1 + m_3)^2 + \frac{1}{2m_2^2} [(m_1^2 + m_2^2 - s_3)(s_1 - m_2^2 - m_3^2) - 4m_2^2 m_1 m_3 \pm \lambda^{1/2}(s_1, m_2^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)],$$

Or for **fixed** s_2 , m_j , and s_3 :

$$s_1^\pm = (m_2 + m_3)^2 + \frac{1}{2m_1^2} [(m_1^2 + m_2^2 - s_3)(s_2 - m_1^2 - m_3^2) - 4m_1^2 m_2 m_3 \pm \lambda^{1/2}(s_2, m_1^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)].$$

$$\text{with } \lambda(x, y, z) \equiv (x - y - z)^2 - 4yz.$$

Single dispersion relation in s_2 :

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon} \sigma(s_1, s'_2, s_3)$$

For a typical scalar loop integral the spectral function $\sigma(s_1, s_2, s_3)$ can be obtained by either the Cutkosky's rules (absorptive part of the loop amplitude) or the following formula

$$\sigma(s_1, s_2, s_3) = \frac{-1}{16\pi} \int_0^1 \int_0^1 \int_0^1 da_1 da_2 da_3 \delta(1 - a_1 - a_2 - a_3) \delta(D).$$

which reads

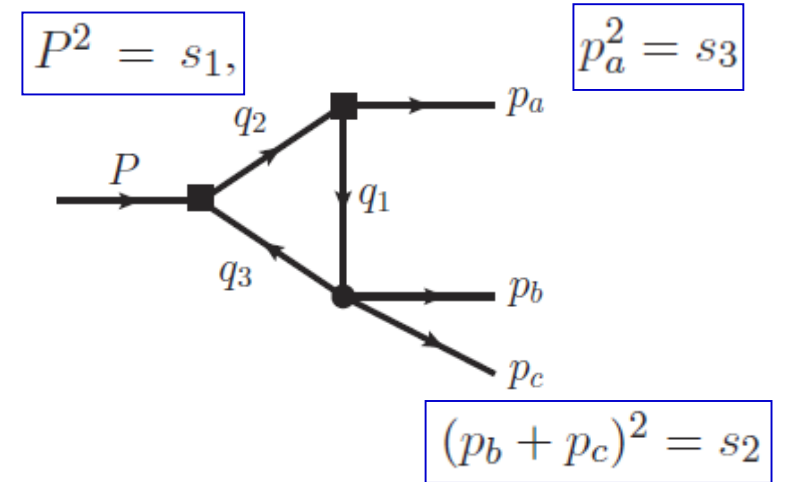
$$\begin{aligned} \sigma(s_1, s_2, s_3) &= \sigma_+ - \sigma_-, \\ \sigma_{\pm}(s_1, s_2, s_3) &= \frac{-1}{16\pi\lambda^{1/2}(s_1, s_2, s_3)} \log[-s_2(s_1 + s_3 - s_2 + m_1^2 + m_3^2 - 2m_2^2) \\ &\quad - (s_1 - s_3)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_1, s_2, s_3)\lambda^{1/2}(s_2, m_1^2, m_3^2)]. \end{aligned}$$

For fixed s_1, s_3 and m_i , the spectral function $\sigma(s_1, s_2, s_3)$ has logarithmic branch points s_2^{\pm} , which correspond to the anomalous thresholds by solving the Landau equation.

How the logarithmic branch points s_2^\pm move as s_1 increases from the threshold of $(m_2 + m_3)^2$, with s_3 and m_i fixed?

Substituting $s_1 \rightarrow s_1 + i\epsilon$, s_2^\pm in the s' -plane are then located at

$$s_2^\pm(s_1 + i\epsilon) = s_2^\pm(s_1) + i\epsilon \frac{\partial s_2^\pm}{\partial s_1},$$



The normal thresholds and critical values for s_1 and s_2 as follows,

$$s_{1N} = (m_2 + m_3)^2, \quad s_{1C} = (m_2 + m_3)^2 + \frac{m_3}{m_1} [(m_2 - m_1)^2 - s_3],$$

$$s_{2N} = (m_1 + m_3)^2, \quad s_{2C} = (m_1 + m_3)^2 + \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3],$$

With $\partial s_2^\pm / \partial s_1 = 0$ ($\partial s_1^\pm / \partial s_2 = 0$).

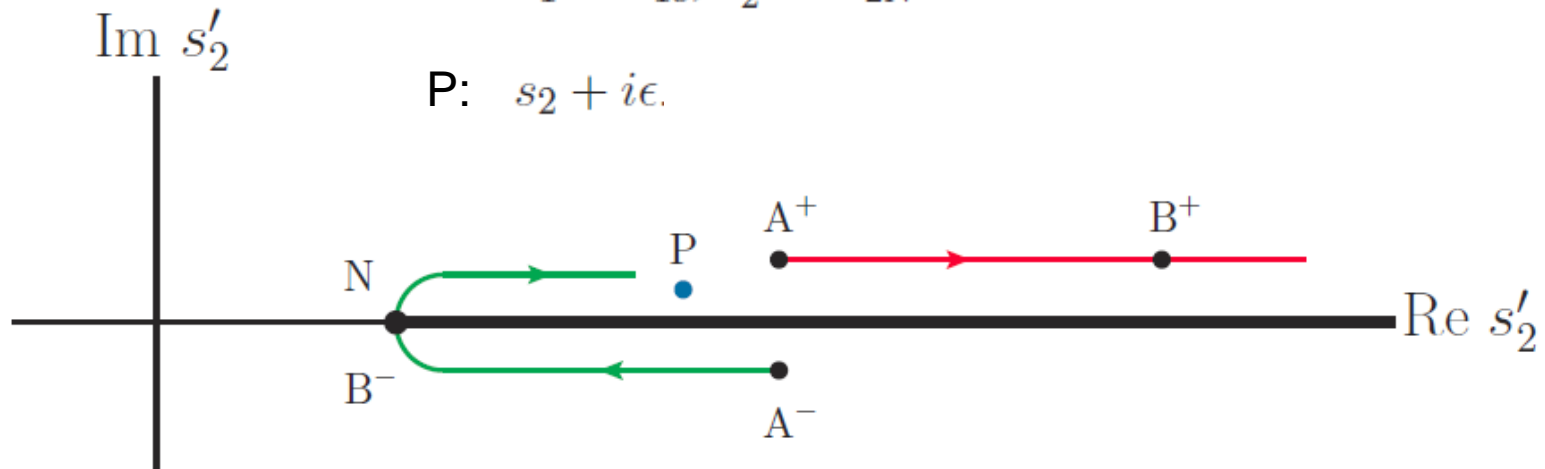
Trajectory of s_2^\pm in the complex s'_2 -plane with s_1 increasing from s_{1N} to ∞ .

$$A^\pm: s_1 = s_{1N}, s_2^\pm = s_{2C} \pm i\epsilon,$$

$$B^+: s_1 = s_{1c}, s_2^+ = s_{2N} + \frac{m_3}{m_1 m_2} \lambda(s_3, m_1^2, m_2^2) + i\epsilon$$

$$B^-: s_1 = s_{1c}, s_2^- = s_{2N}$$

$$P: s_2 + i\epsilon.$$

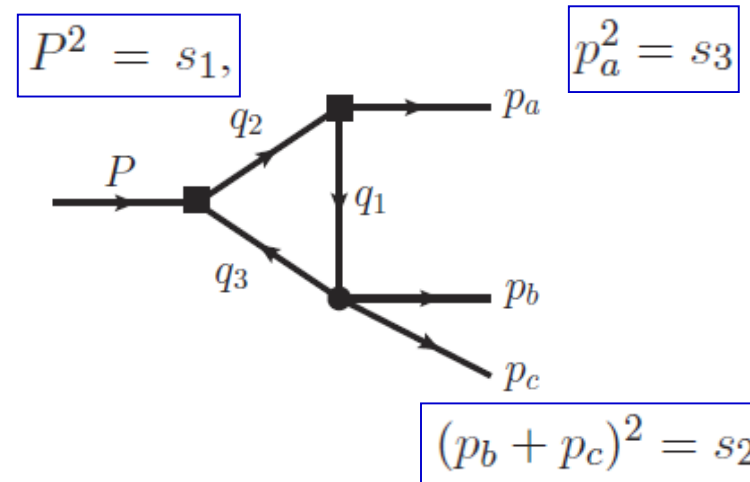


$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon} \sigma(s_1, s'_2, s_3)$$

The difference between the normal and anomalous thresholds:

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$



When $s_2 = s_{2N}$ ($s_1 = s_{1N}$), we will obtain the maximum value of Δs_1 (Δs_2),

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)} [(m_2 - m_1)^2 - s_3],$$

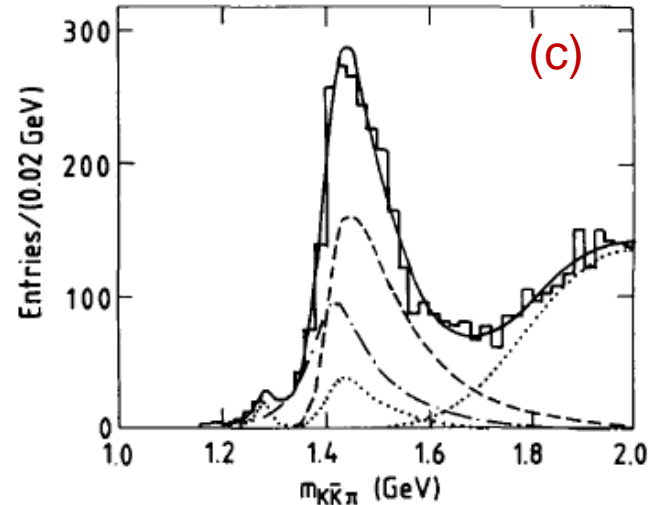
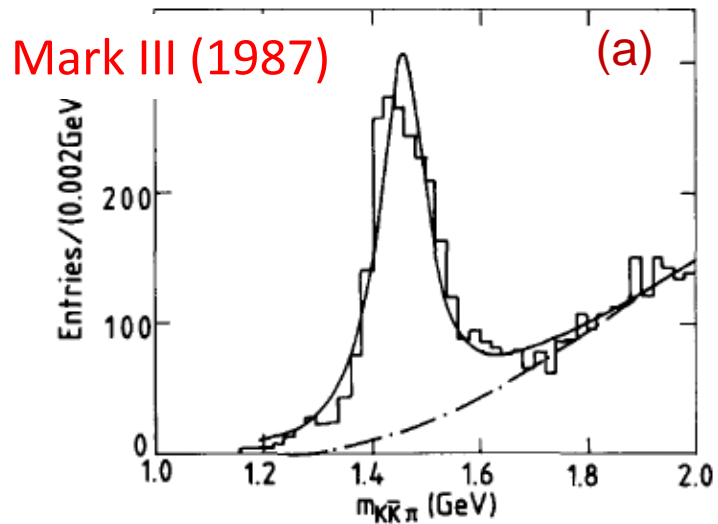
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1 + m_3)} [(m_2 - m_1)^2 - s_3].$$

- The larger Δs_1 (Δs_2), the more importance of the TS mechanism!
- For a specific transition process which fulfills the TS kinematic the detailed dynamics, such as the widths and coupling vertices, will affect the behavior of the TS mechanism.

3. Triangle singularity mechanism and its manifestations

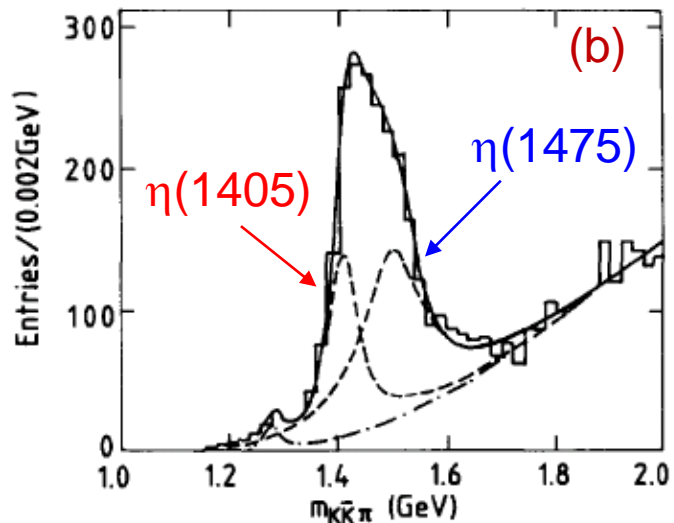
Case-I: The first evidence for the observable effects from the TS mechanism

Isoscalar pseudoscalar states $J^{PC} = 0^{-+}$ around 1.2~1.5 GeV found in experiment



- (a) A single Breit-Wigner fit
- (b) Two interfering B-W fit
- (c) Coupled channel B-W fit

Distorted lineshape?



$$M = 1416 \pm 8_{-5}^{+7}; \Gamma = 91_{-31-38}^{+67} {}^{+15} \text{ MeV}/c^2$$

$$M = 1490_{-8-6}^{+14+3}; \Gamma = 54_{-21-24}^{+37+13} \text{ MeV}/c^2$$

Also “confirmed” by Obelix collaboration

$\eta(1405)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$

See also the $\eta(1475)$.

$\eta(1405)$ MASS

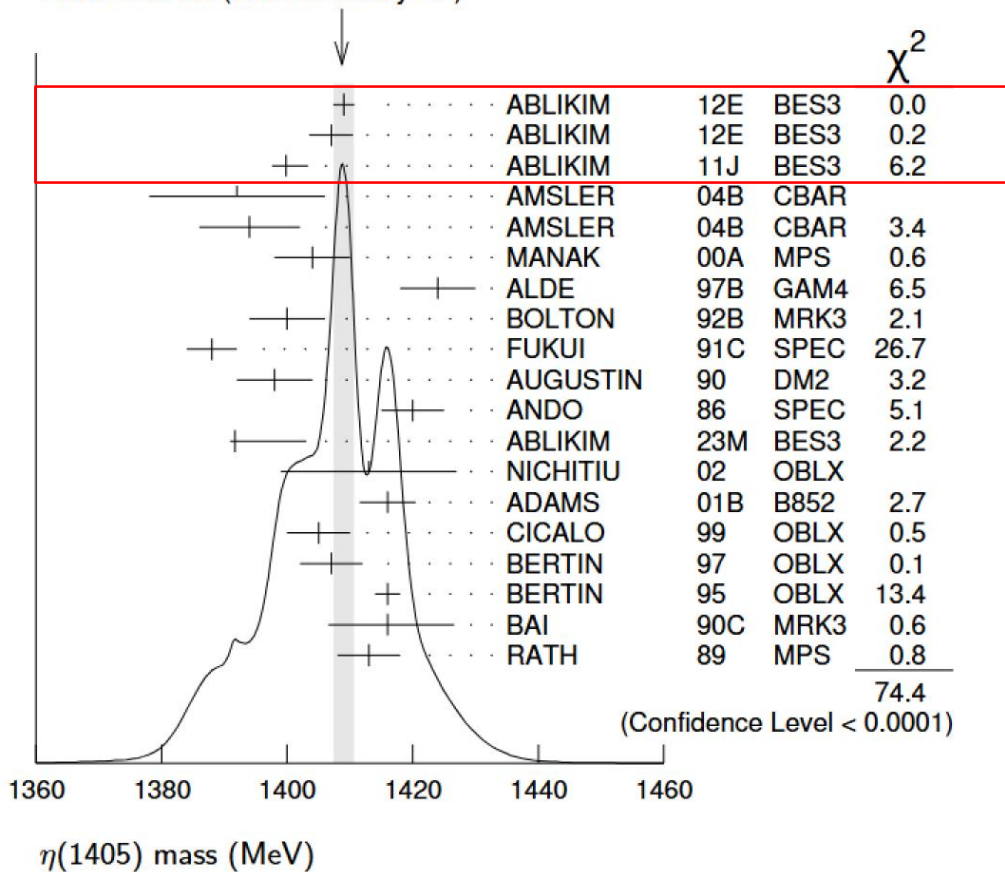
VALUE (MeV)

DOCUMENT ID

1408.7^{+2.0}_{-1.2} OUR AVERAGE Includes data from the 2 datablocks that follow this one.

Error includes scale factor of 2.2. See the ideogram below.

WEIGHTED AVERAGE
1408.7+2.0-1.2 (Error scaled by 2.2)



PDG2024

$\eta(1405)$ DECAY MODES

Mode	Fraction (Γ_i/Γ)
Γ_1 $K\bar{K}\pi$	seen
Γ_2 $\eta\pi\pi$	seen
Γ_3 $a_0(980)\pi$ CB in J/psi decays	seen
Γ_4 $\eta(\pi\pi)$ S-wave	seen
Γ_5 $f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$	not seen
Γ_6 $f_0(980)\eta$	seen
Γ_7 4π	seen
Γ_8 $\rho\rho$	<58 %
Γ_9 $\gamma\gamma$	
Γ_{10} $\rho^0\gamma$	seen
Γ_{11} $\phi\gamma$	
Γ_{12} $K^*(892)K$	seen



$$\Gamma(f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0) / \Gamma_{\text{total}} \quad \Gamma_5/\Gamma$$

VALUE	DOCUMENT ID	TECN	COMMENT
not seen	¹ ABLIKIM	17AJ BES3	$\psi(2S) \rightarrow \gamma\pi^+\pi^-\pi^0$
¹ ABLIKIM 17AJ reports $B(\psi(2S) \rightarrow \gamma\eta(1405) \rightarrow \gamma f_0(980)\pi^0 \rightarrow \gamma\pi^+\pi^-\pi^0) < 5.0 \times 10^{-7}$.			

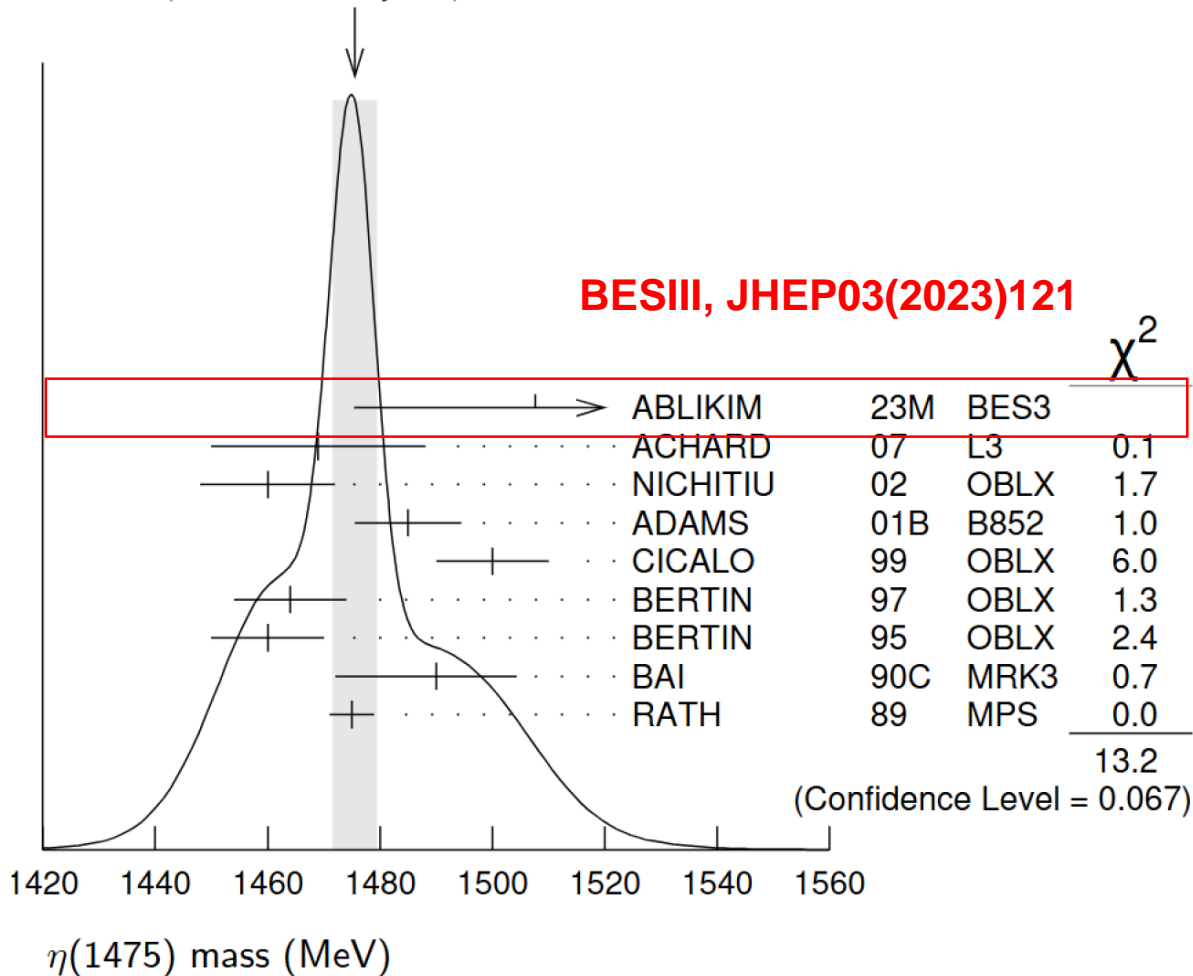
But seen in $J/\psi \rightarrow \gamma\pi\pi\pi$ at BESIII. Why not included?

$\eta(1475)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$

For the first time the BESIII data are accommodated in the category of $\eta(1475)$, but seems not to be in a self-consistent way.

WEIGHTED AVERAGE
1476±4 (Error scaled by 1.4)



BESIII, JHEP03(2023)121

$\eta(1475)$ DECAY MODES

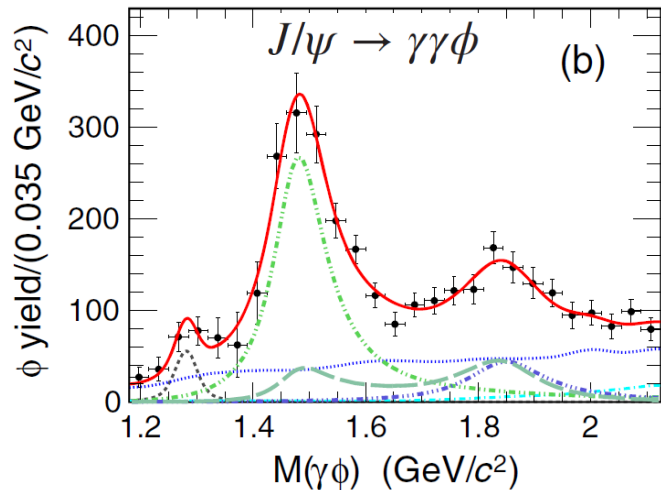
Mode	Fraction (Γ_i/Γ)
Γ_1 $K\bar{K}\pi$	seen
Γ_2 $K\bar{K}^*(892) + c.c.$	seen
Γ_3 $a_0(980)\pi$	seen
Γ_4 $\gamma\gamma$	seen
Γ_5 $K_S^0 K_S^0 \eta$	possibly seen
Γ_6 $\gamma\phi(1020)$	possibly seen

BESII

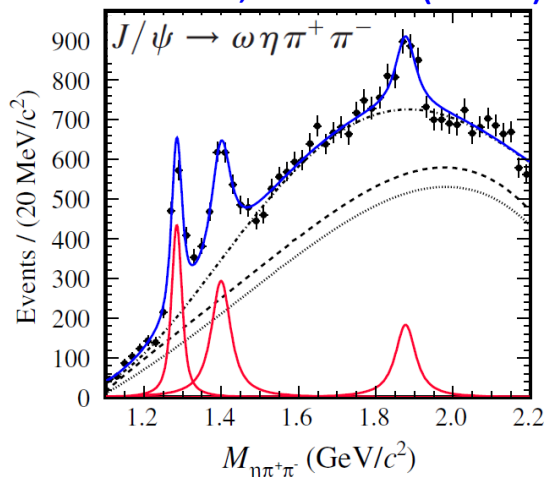
BESIII, Phys.Rev.D 97 (2018) 5, 051101

Only one state is observed at BES-III in various channels except for $K_S\bar{K}_S\pi^0$

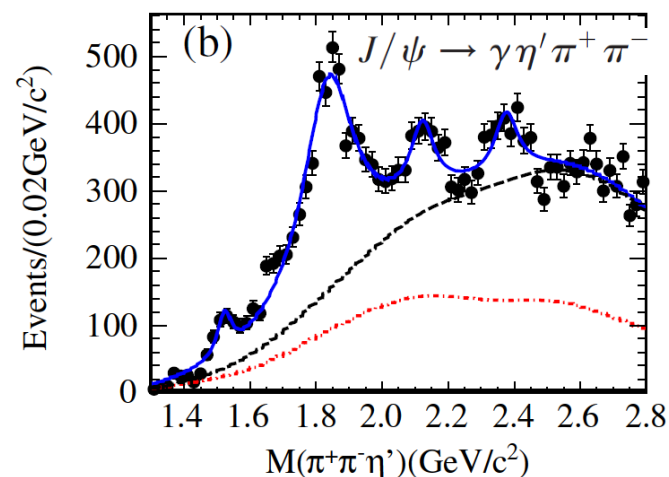
PRD97, 051101(R)(2018)



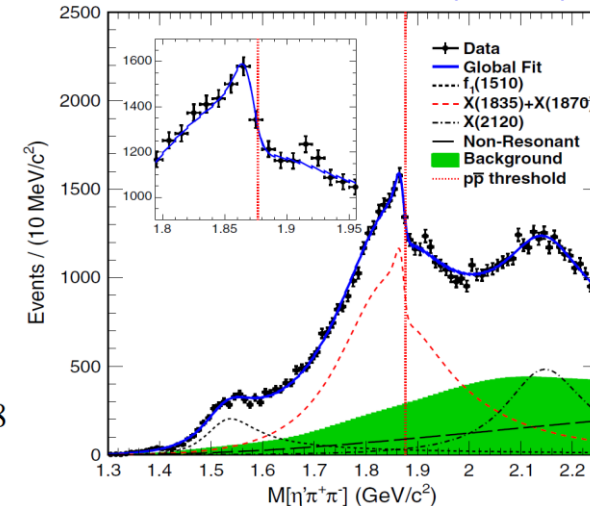
PRL 107, 182001 (2011)



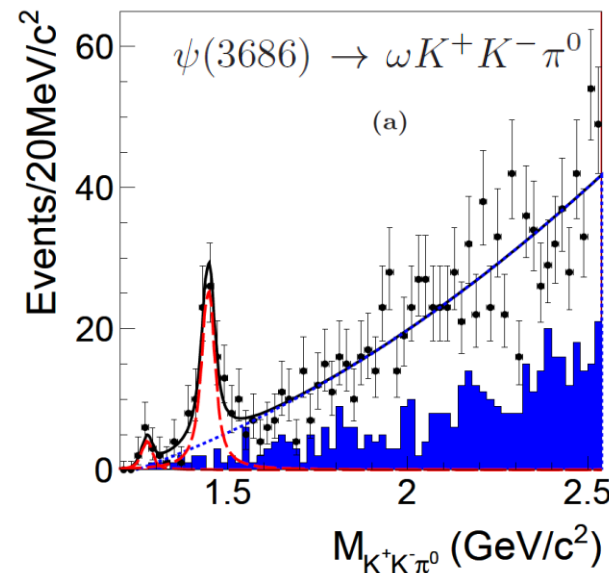
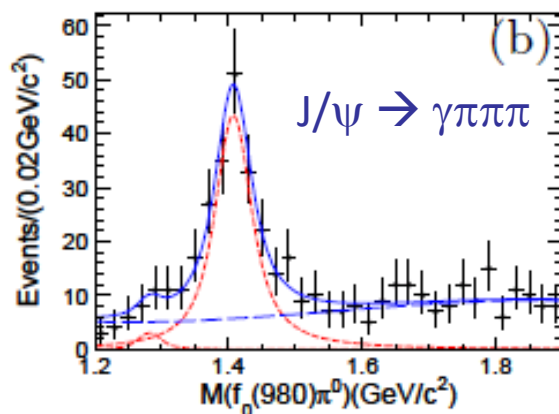
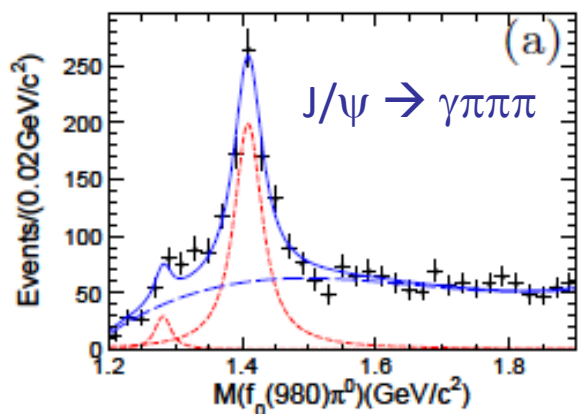
PRL 106, 072002 (2011)



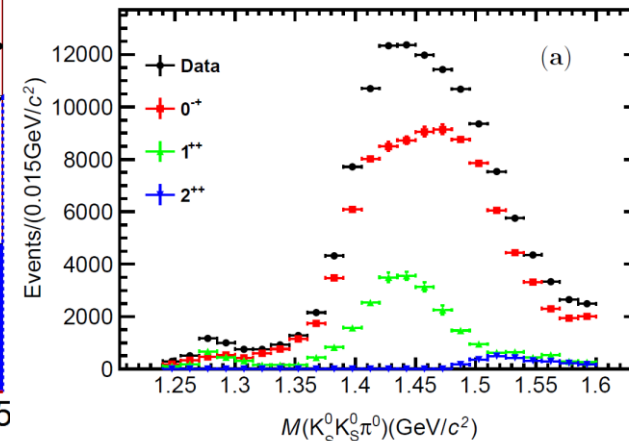
PRL 117, 042002 (2016)



PRL108, 182001 (2012)



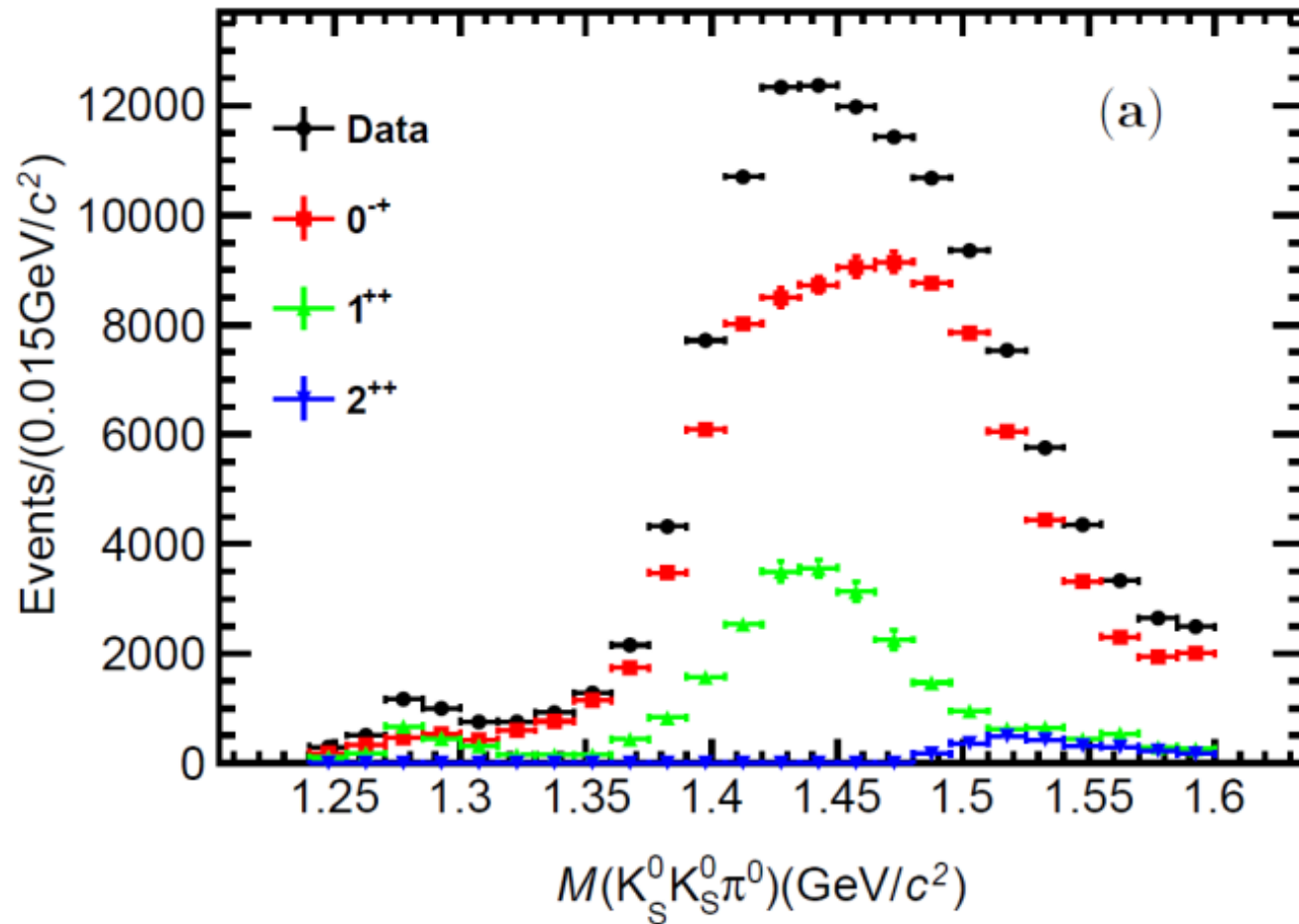
BESIII, JHEP03(2023)121



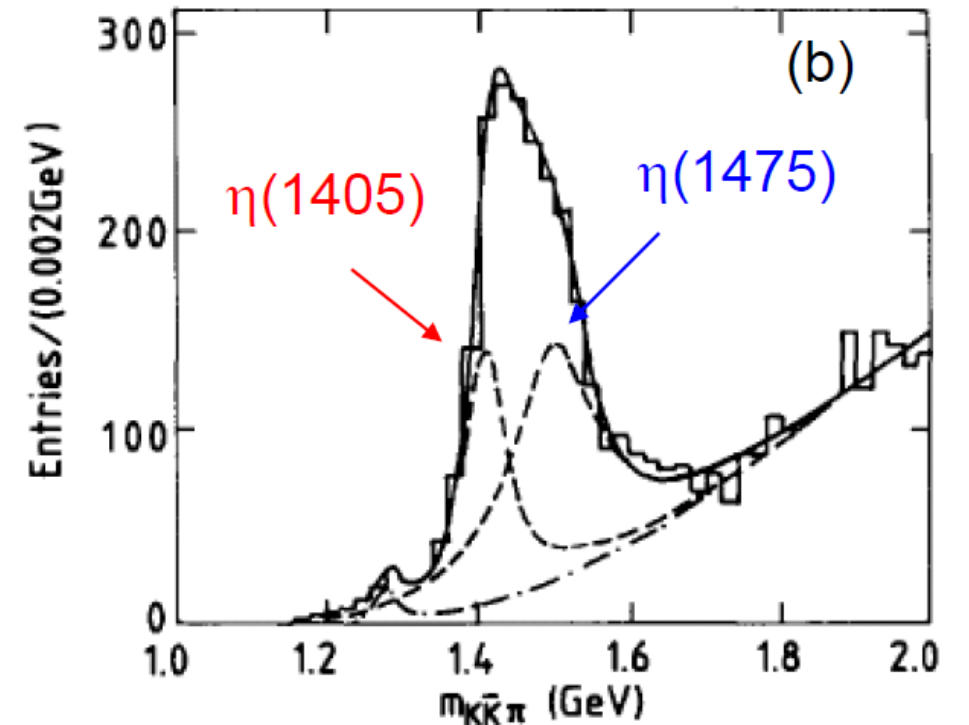
Anomalous lineshape in $J/\psi \rightarrow \gamma K_S \bar{K}_S \pi^0$

$J/\psi \rightarrow \gamma K_S \bar{K}_S \pi^0$

BESIII, JHEP03(2023)121



MarK-III (1986)



The abundance of 0^{-+} ($I=0$) states implies a glueball candidate?

Positive:

- Flux tube model favors $M_G \cong 1.4 \text{ GeV}$ [1]
- A dynamic model based on $U_A(1)$ anomaly gives a similar mass [2].

Disfavored:

- LQCD favors $M_G \cong 2.4 - 2.6 \text{ GeV}$ [3,4,5]
- Corrected version of [2] (see [6])
- Dynamic approach based on $U_A(1)$ [7]

How to understand the anomalous lineshape in the $K\bar{K}\pi$ invariant mass spectrum?

[1] Faddeev, Niemi, and Wiedner, PRD70, 114033 (2004)

[2] H. Y. Cheng, H. n. Li, and K. F. Liu, Phys. Rev. D 79, 014024 (2009)

[3] Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)

[4] Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)

[5] W. Sun et al. [CLQCD], arXiv:1702.08174[hep-lat]

[6] W. Qin, Q.Zhao and X.H.Zhong, PRD97, 096002 (2018)

[7] V. Mathieu and V. Vento, PRD81, 034004(2010)

The presence of the “triangle singularity”

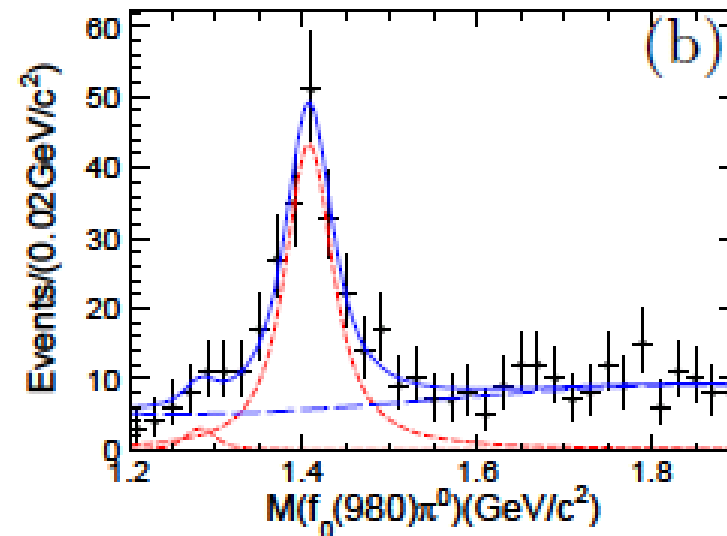
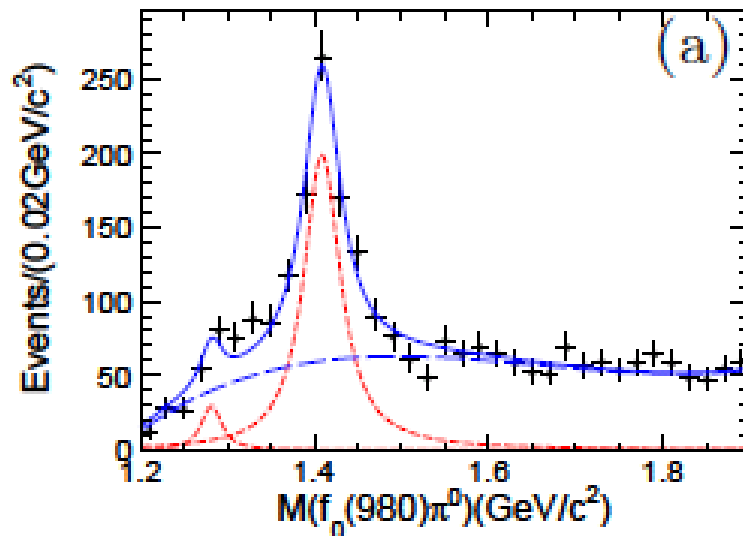
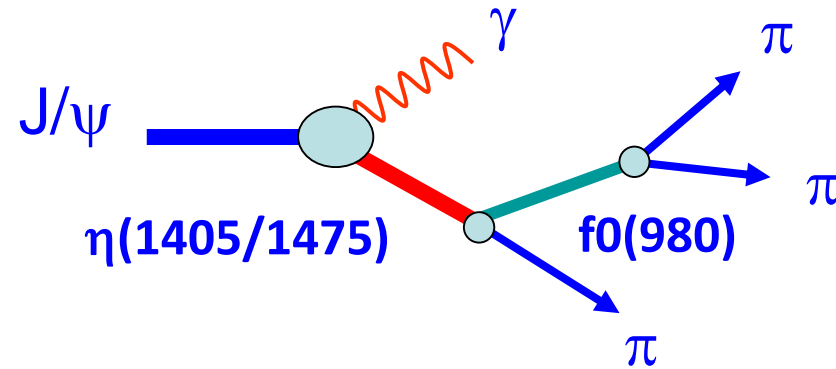
PRL 108, 182001 (2012)

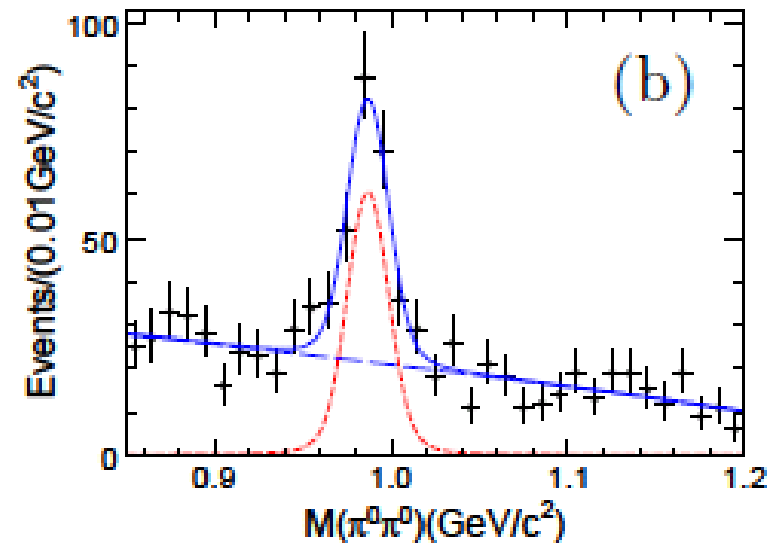
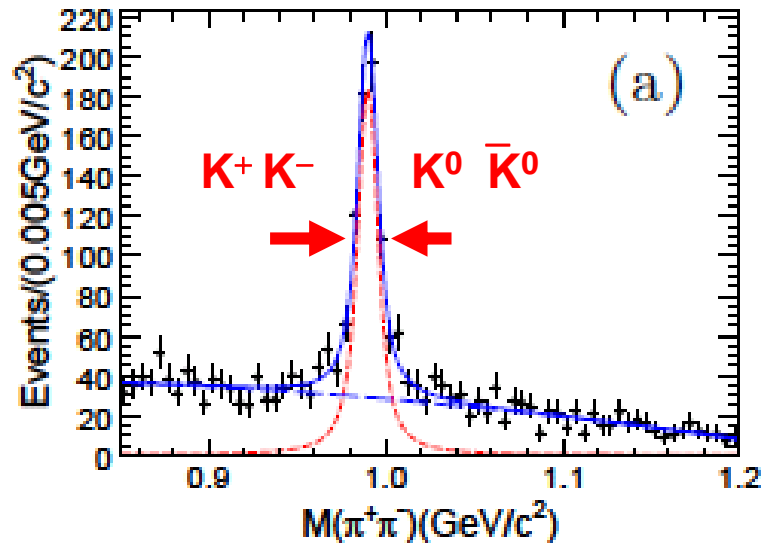
PHYSICAL REVIEW LETTERS

week ending
4 MAY 2012

First Observation of $\eta(1405)$ Decays into $f_0(980)\pi^0$

Isospin-violating decay
of $J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma \pi\pi\pi$





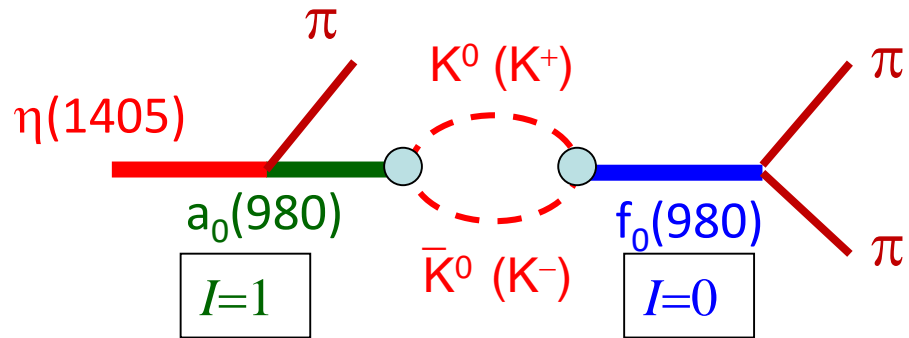
- $f_0(980)$ is extremely narrow: $\Gamma \cong 10$ MeV !

PDG: $\Gamma \cong 40 \sim 100$ MeV.

- Anomalously large isospin violation!

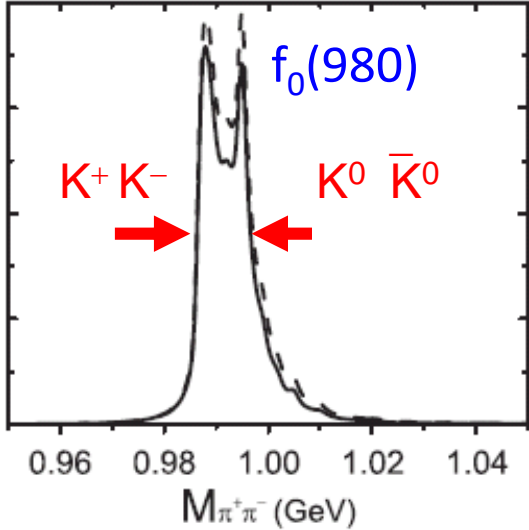
$$\frac{Br(\eta(1405) \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0)}{Br(\eta(1405) \rightarrow a_0^0(980)\pi^0 \rightarrow \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

“ $a_0(980)$ - $f_0(980)$ mixing” gives only $\sim 1\%$ isospin violation effects !



$$g(a_0 K^+ K^-) g(f_0 K^+ K^-) = -g(a_0 K^0 \bar{K}^0) g(f_0 K^0 \bar{K}^0)$$

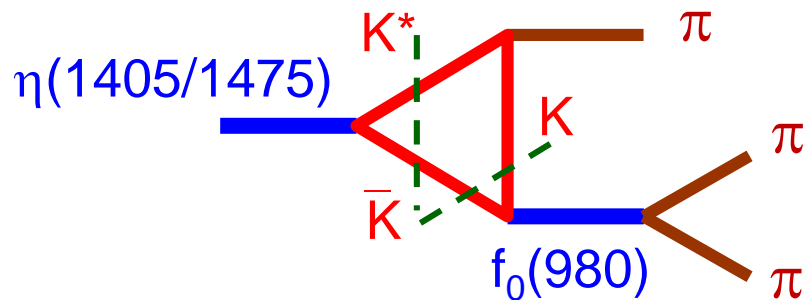
$$M(K^0) - M(K^\pm) = m_d - m_u$$



V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B586, 53-61 (2004)

V. Baru, J. Haidenbauer, C. Hanhart, A.E. Kudryavtsev and U.-G. Meissner, Eur. Phys. J. A23, 523-533 (2005)

“Triangle singularity”

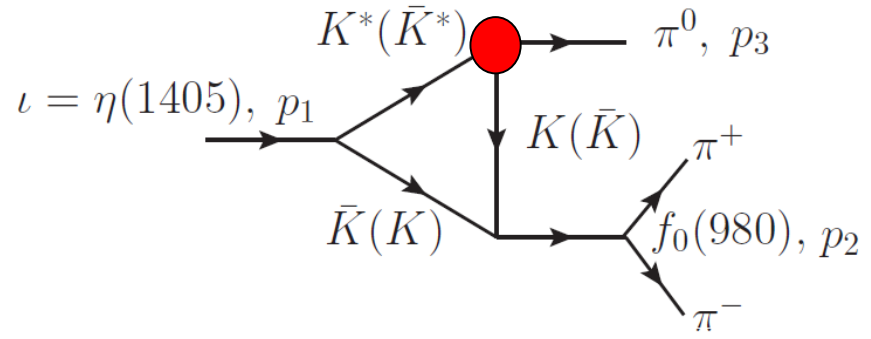


Internal $\bar{K} K^*(K)$ approach the on-shell condition simultaneously!

Manifestation of the TS singularity!

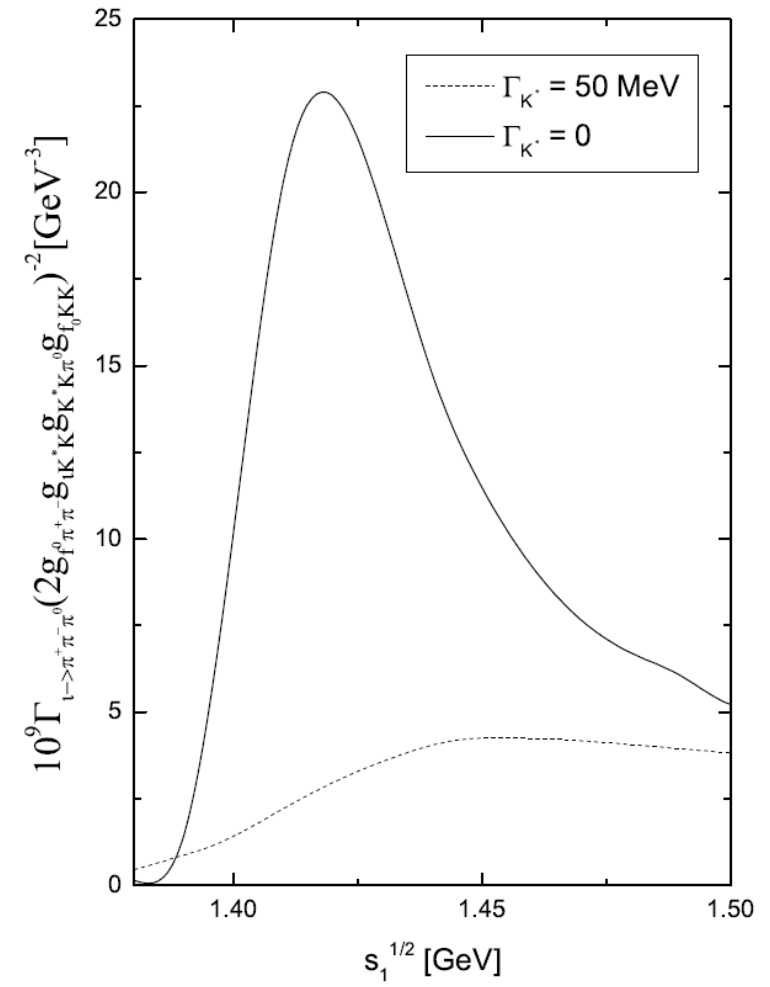
J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012);
 X.G. Wu, J.-J. Wu, Q. Z., and B.-S. Zou, PRD87, 014023 (2013)

Understanding the width effects from the intermediate K^* in $\eta(1405/1475) \rightarrow 3\pi, K \bar{\pi}\pi, \eta\pi\pi$

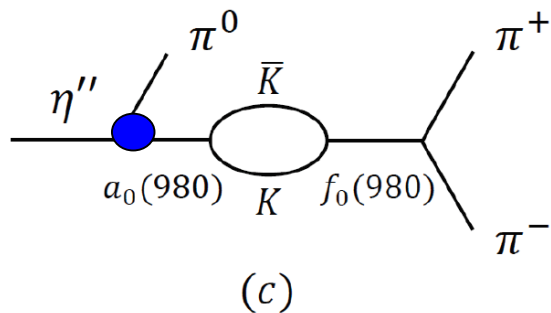
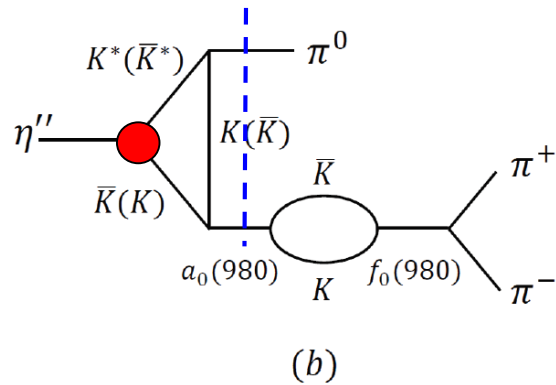
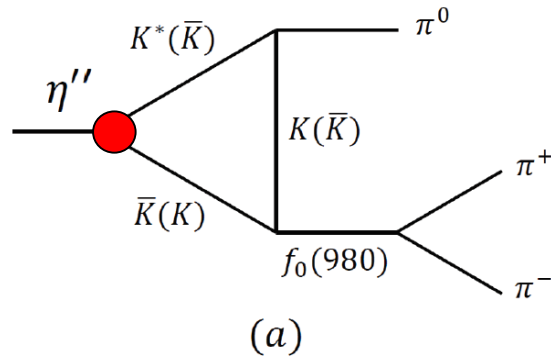


$a_0(980) - f_0(980)$ mixing is required to be enhanced.

However, experimental data do not support large b.r. for $\eta(1405)/\eta(1475) \rightarrow a_0(980)\pi$!



Updated study of $\eta(1405/1475) \rightarrow 3\pi, K \bar{K}\pi, \eta\pi\pi$ with width effects



- Direct isospin breaking via the TS mechanism

- **a0-f0 mixing enhanced by the TS mechanism**
 - Unitarized treatment for a0 and f0;
 - To separate (b) and (c) allows a self-contained evaluation of the TS and a0-f0 mixing contributions.

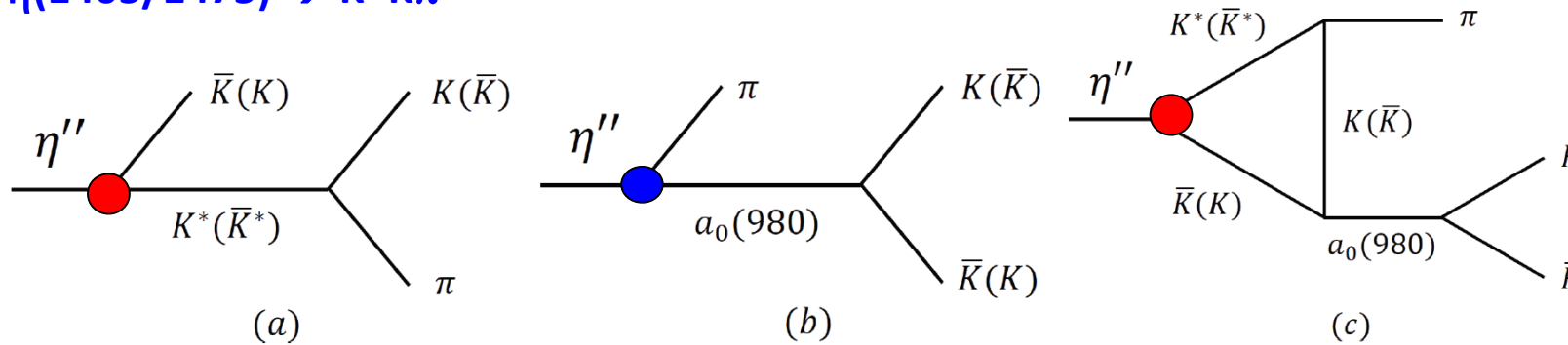
- **a0-f0 mixing at tree level**

M.C. Du and Q.Z., PRD100, 036005 (2019).

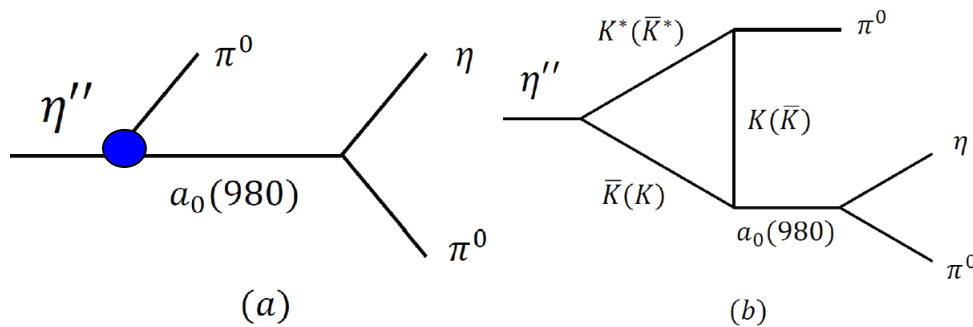
See also N.N. Achasov et al., PRD92, 036003 (2015)

Updated study of $\eta(1405/1475) \rightarrow 3\pi, K \bar{K}\pi, \eta\pi\pi$ with width effects

$\eta(1405/1475) \rightarrow K \bar{K}\pi$



$\eta(1405/1475) \rightarrow \eta\pi\pi$



Schmidt theorem is broken

M.C. Du and Q.Z., PRD100, 036005 (2019).

M.-C. Du, Y. Cheng, and Q. Zhao, Phys. Rev. D 106, no.5, 054019 (2022)

Y. Chen and Q.Z., Phys. Rev. D 105, 076023 (2022)

Dynamical features arising from the coupling vertices in association with the TS mechanism

$$I = -i \int \frac{d^4 q}{(2\pi)^4} \frac{(2p_1 - q)_\mu (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) (q - 2p_2)_\nu}{(q^2 - m_1^2 + im_1\Gamma)[(p_2 - q)^2 - m_2^2][(q - p_1)^2 - m_3^2]}$$

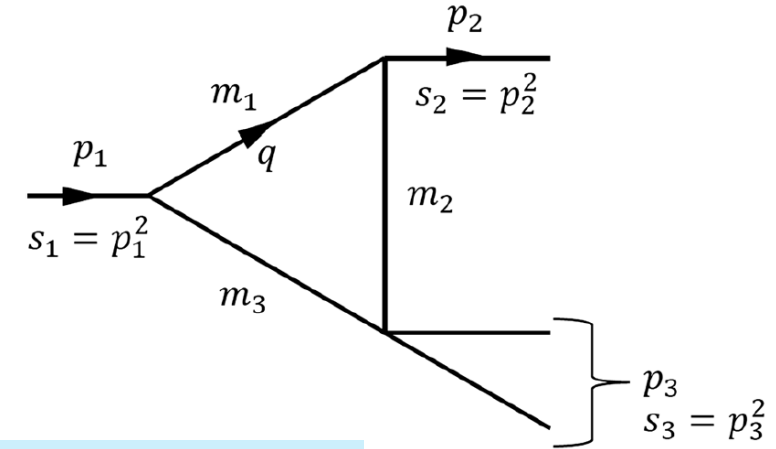
$$= -i \int \frac{d^4 q}{(2\pi)^4} \frac{(2p_1 - q)_\mu (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) (q - 2p_2)_\nu}{D_1 D_2 D_3}$$

$$= -i \left(s_1 - m_1^2 + im_1\Gamma_1 + s_2 - 2s_3 + 2m_K^2 - \frac{(s_1 - m_K^2)(s_2 - m_K^2)}{m_1^2 - im_1\Gamma_1} \right) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{D_1 D_2 D_3}$$

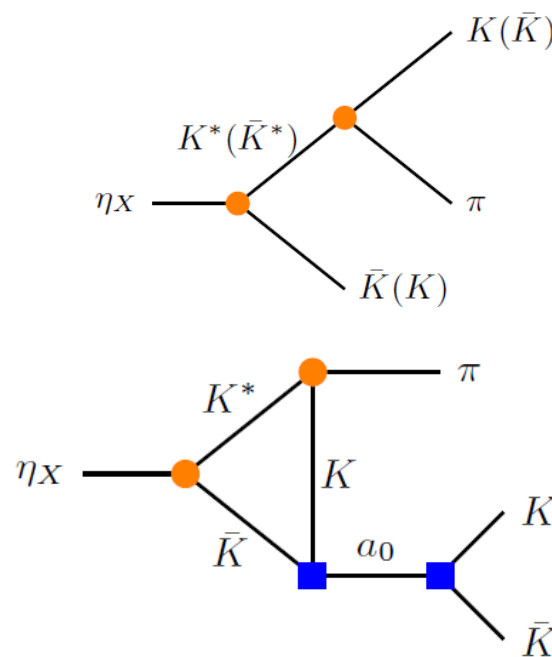
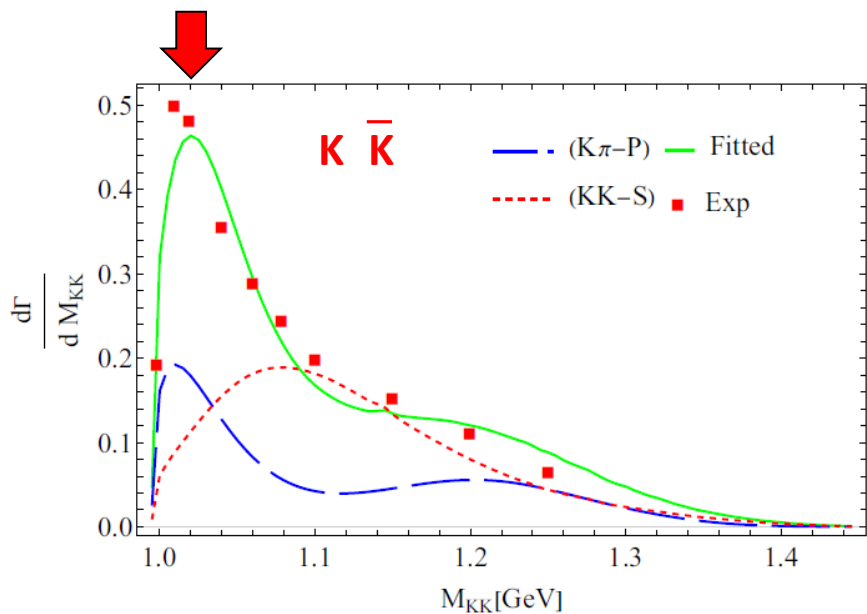
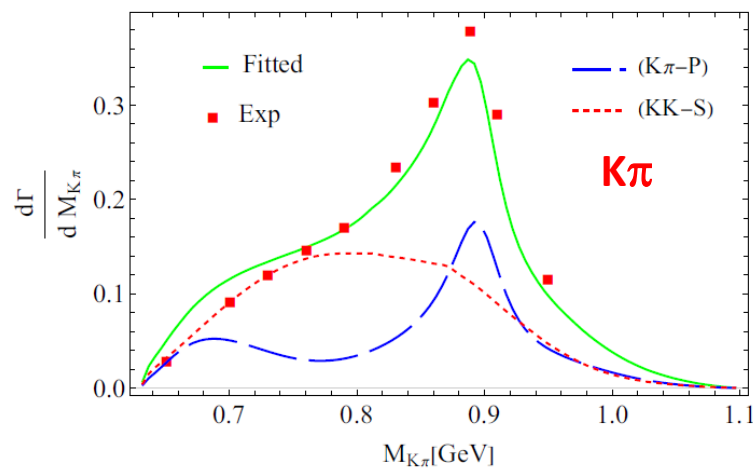
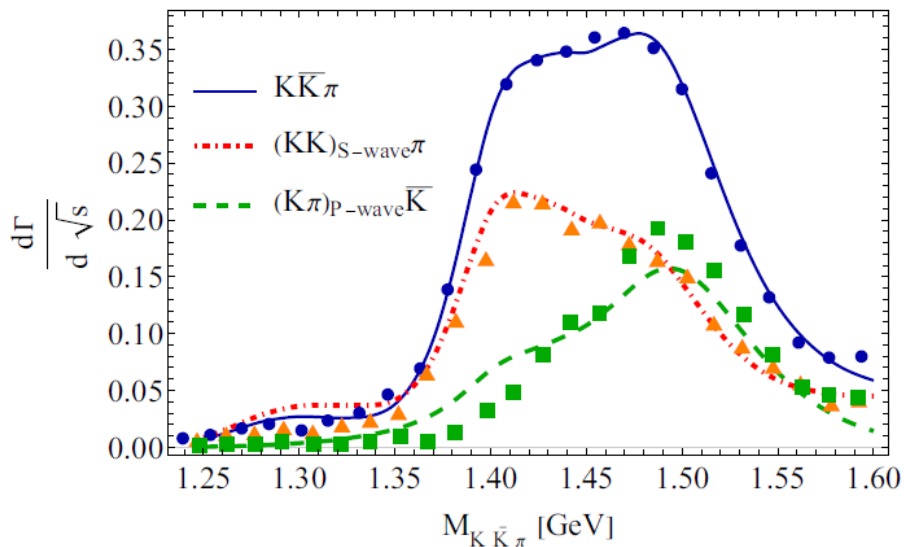
$$- i \left(\frac{(s_1 - m_K^2)(s_2 - m_K^2)}{m_1^2 - im_1\Gamma_1} \right) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 D_2 D_3} - i \left(1 + \frac{s_1 - m_K^2}{m_1^2 - im_1\Gamma_1} \right) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{D_1 D_3}$$

$$- i \left(1 + \frac{s_2 - m_K^2}{m_1^2 - im_1\Gamma_1} \right) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{D_1 D_2} + i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 D_1} - i \frac{m_K^2 - s_1}{m_1^2 - im_1\Gamma_1} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 D_3}$$

$$- i \frac{m_K^2 - s_2}{m_1^2 - im_1\Gamma_1} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 D_2} + i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{D_2 D_3}$$



$$J/\psi \rightarrow \gamma \eta(1405/1475) \rightarrow \gamma K \bar{K} \pi$$

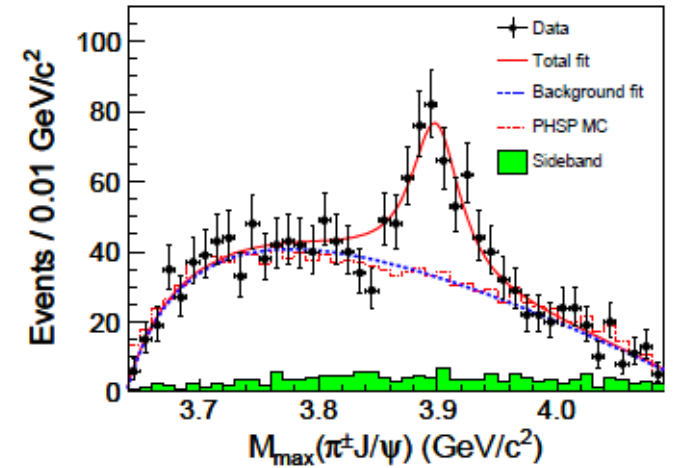
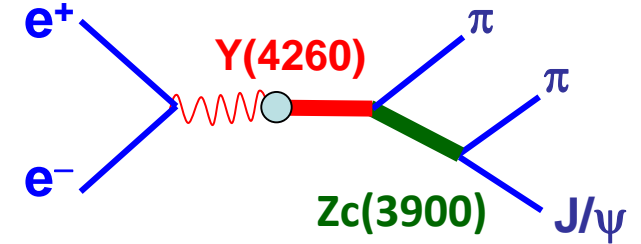
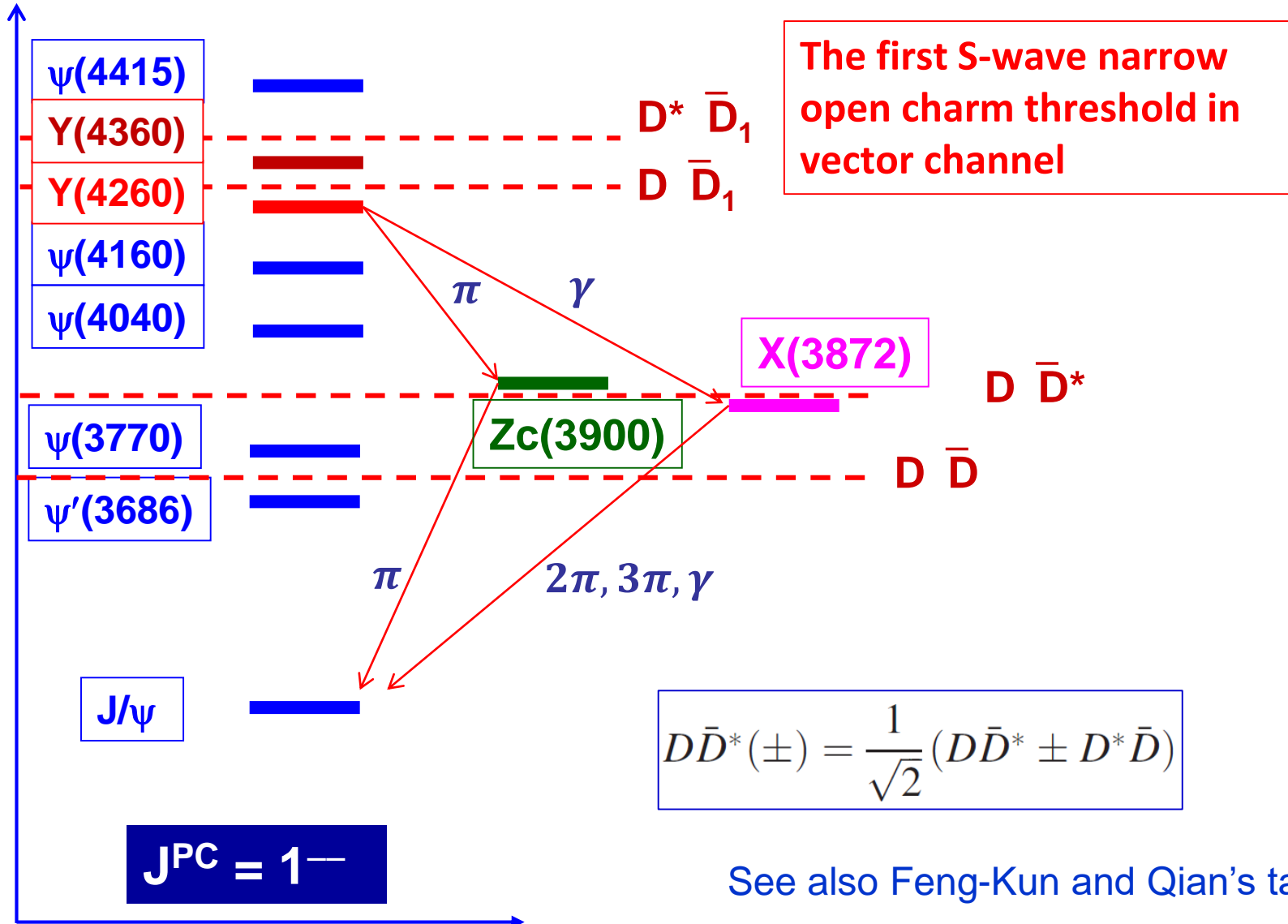


- Manifestation of the TS mechanism, which violates the Schmidt theorem
- Partial waves can be accounted for in an isobar model with two-body unitarity [Y. Cheng et al., paper to be submitted]
- Coupled-channel approach with three-body unitarity is in progress [L. Qiu, and Q. Zhao, in progress]
- The $\eta\pi\pi$ channel can also be accounted for

See also S.X. Nakamura et al., PRD109, 014021; PRD107, L091505

Case-II: The nature and property of Zc(3900)

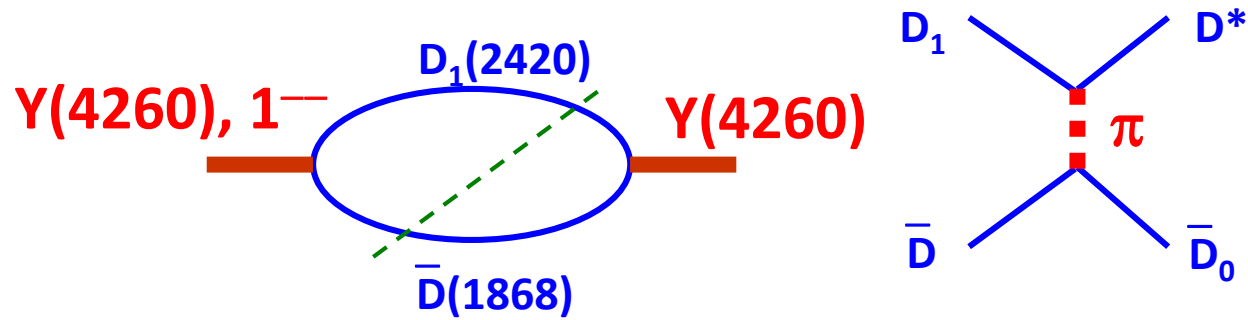
Correlations between Y(4260) and Zc(3900)/X(3872) via the TS mechanism



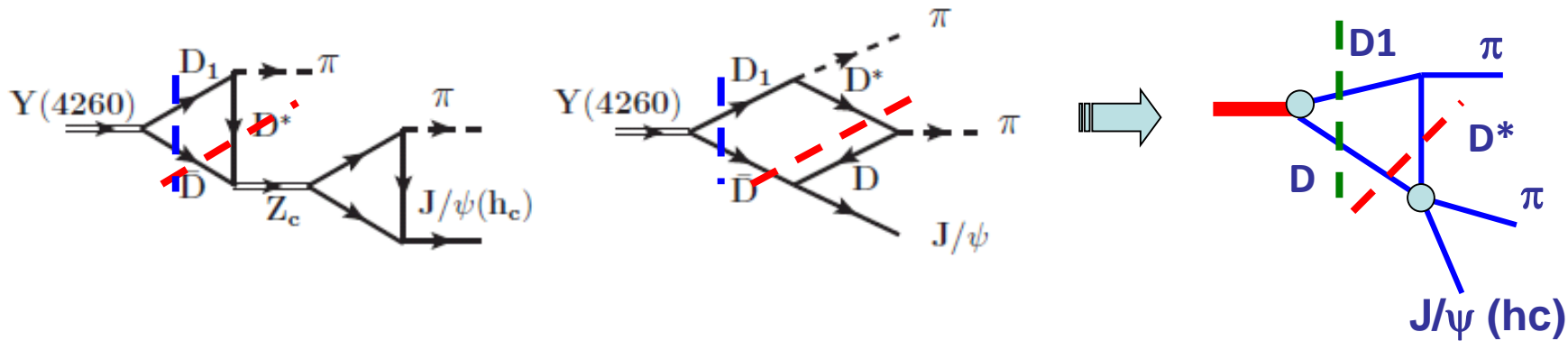
BESIII, PRL(2013)

See also Feng-Kun and Qian's talks

- $Y(4260)$ could be a hadronic molecule made of $DD_1(2420)$ with coupled channel effects.



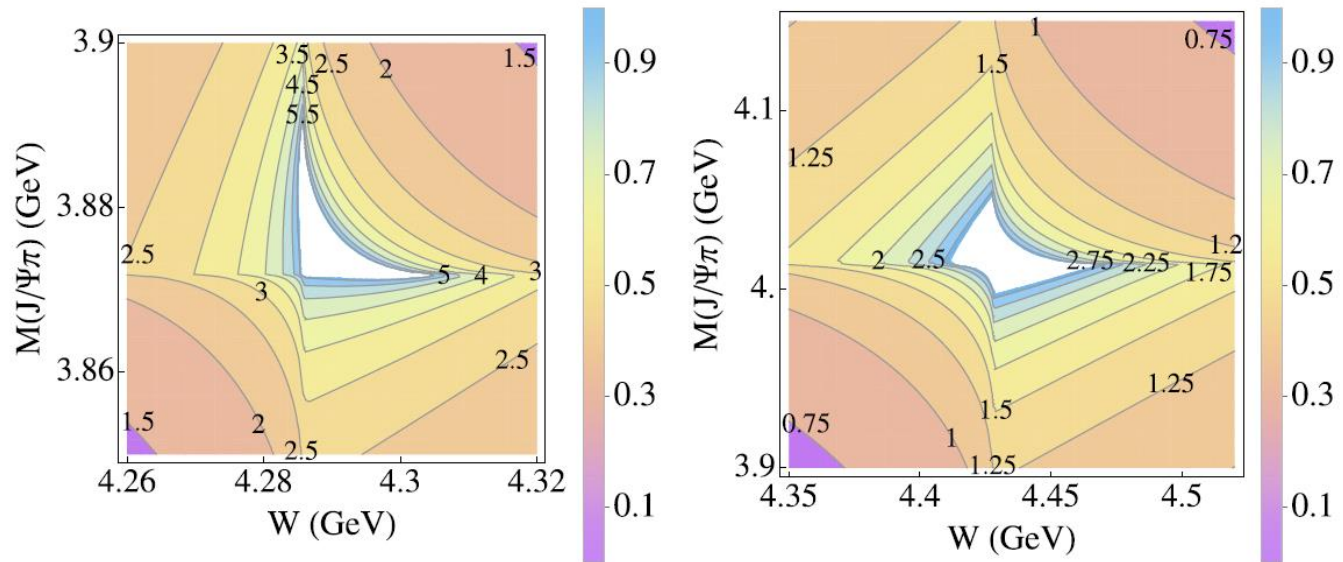
- The production of $Z_c(3900)$ is strongly correlated with $Y(4260)$ and enhanced by the triangle singularity kinematics.



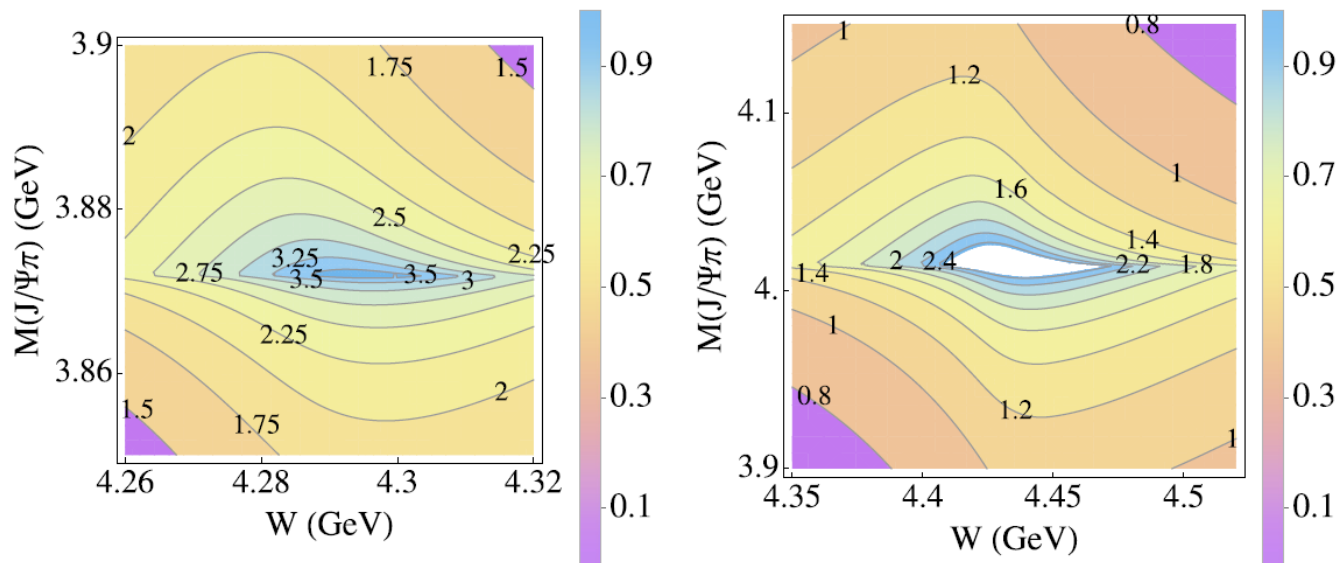
Q. Wang, C. Hanhart, Q. Zhao, PRL111, 132003 (2013)

Q. Wang, C. Hanhart, Q. Zhao, PLB725, 106 (2013)

- TS enhancements in $e^+e^- \rightarrow J/\psi\pi\pi$ for $Z_c(3900)$ and $Z_c(4020)$



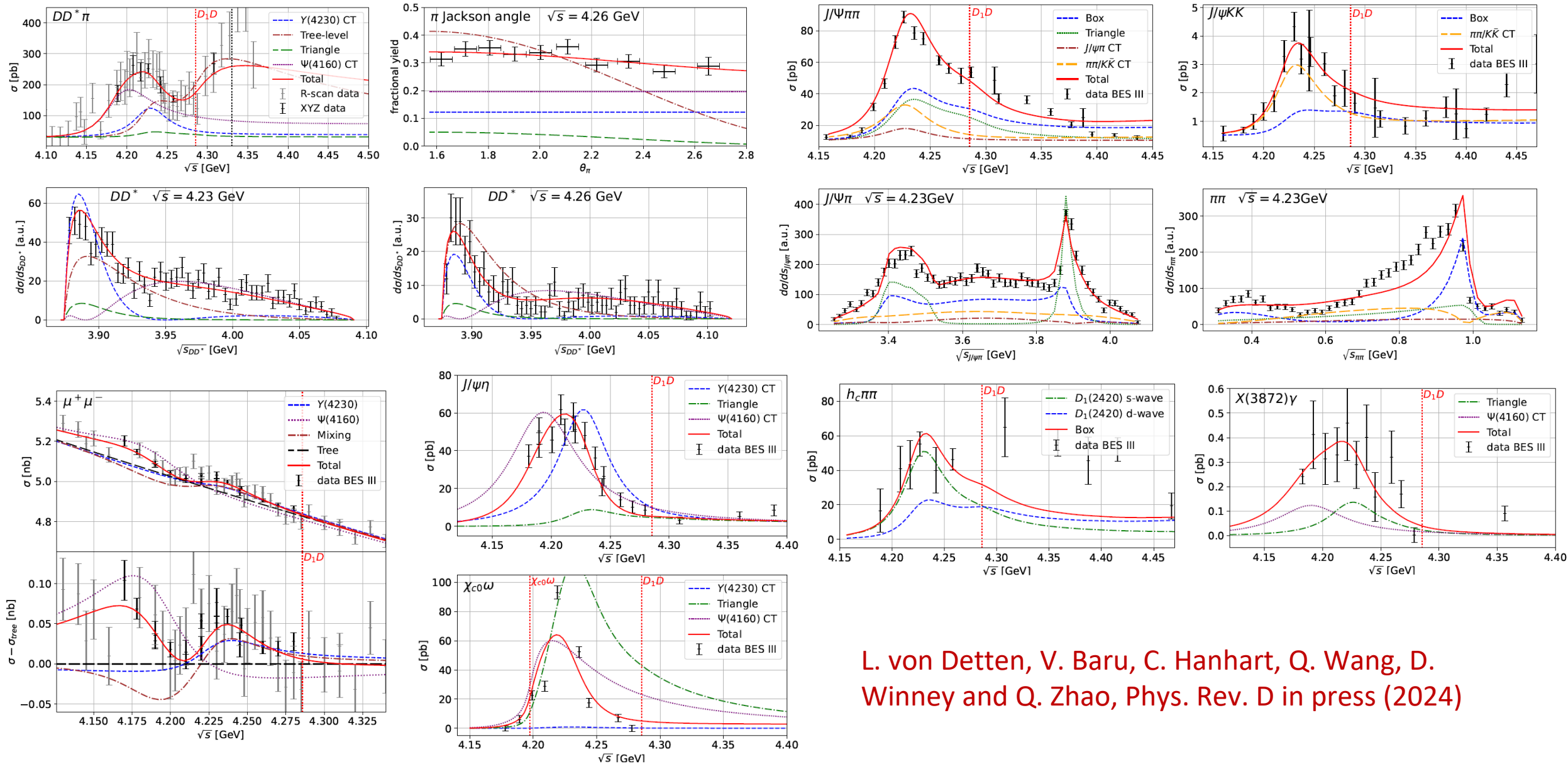
Zero width results



Non-zero width results:
 $\Gamma_{D_1} = 27 \text{ MeV}$
 $\Gamma_{D^*} = 190 \text{ keV}$

- TS enhancements are evident!
- Similar mechanism is present in the production of Z_b and Z_b'

Recent combined analysis of $e^+e^- \rightarrow J/\psi\pi\pi, h_c\pi\pi, DD^*\pi$ for $Z_c(3900)$ and $Z_c(4020)$



L. von Detten, V. Baru, C. Hanhart, Q. Wang, D. Winney and Q. Zhao, Phys. Rev. D in press (2024)

On the topic of Y(4260) and Zc(3900):

1. Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013) [**318** citations in INSPIRE]
2. Q. Wang, C. Hanhart, Q. Zhao, Phys. Lett. B725, 106 (2013)
3. F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang and Q. Zhao, Phys. Lett. B 725, 127 (2013)
4. Q. Wang, M. Cleven, F.K. Guo, C. Hanhart, U.-G. Meissner, X.G. Wu, and Q. Zhao, Phys. Rev. D89, 034001 (2014)
5. M. Cleven, Q. Wang, F.K. Guo, C. Hanhart, U.-G. Meissner, Q. Zhao, Phys. Rev. D90, 074039 (2014)
6. X.-G. Wu, C. Hanhart, Q. Wang and Q. Zhao, Phys. Rev. D 89, 054038 (2014)
7. W. Qin, S.R. Xue, Q. Zhao, Phys. Rev. D94, 054035 (2016)
8. X.-H. Liu, M. Oka and Q. Zhao, Phys. Lett. B 753, 297 (2016)
9. M. Cleven and Q. Zhao, Phys. Lett. B 768, 52 (2017)
10. L. von Detten, V. Baru, C. Hanhart, Q. Wang, D. Winney and Q. Zhao, Phys. Rev. D in press (2024)

- **A milestone publication:**

F.K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao and B.S. Zou, *Hadronic molecules*, **Rev. Mod. Phys.** **90**, no. 1, 015004 (2018)

1132 citations in INSPIRE

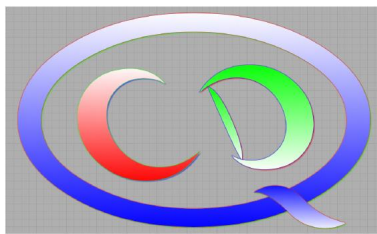
- **Tremendous papers in the literature on the TS mechanism since 2012**

4. Brief summary

- The CRC110-B3 has turned out to be a success in many aspects, i.e. physics studies, young researcher cultivations, etc.
- We made progresses on the threshold dynamics in association with the TS mechanism.
- We have also identified more focused questions to proceed: e.g. How the color force makes it possible to form hadrons beyond the simplest conventional mesons ($q\bar{q}$) and baryons (qqq)? (e.g. **multiquarks, hadronic molecules, hadroquarkonia** ...) And where to look for them?

Final remarks:

- Special thanks to **DFG** and **NSFC** for their support of the CRC110!
- Special thanks to **Ulf** and **Bing-Song** for their excellent leadership throughout the whole project!
- Special thanks to **Christoph, Feng-Kun, Qian, and other colleagues** for their selfless sharing of ideas and many enlightening discussions
- The CRC110 ends. But the efforts surely will continue.
- We anticipate that our collaborations will keep going and keep producing more high-quality joint works in the future!



FB23 Website

<http://fb23.ihep.ac.cn/>

FB23 THE 23rd INTERNATIONAL CONFERENCE ON FEW-BODY PROBLEMS IN PHYSICS (FB23) Sept. 22 -27, 2024 • Beijing, China

Host Institute of High Energy Physics, Chinese Academy of Sciences Tsinghua University University of Chinese Academy of Science
China Center of Advanced Science and Technology Institute of Theoretical Physics, Chinese Academy of Sciences South China Normal University
Co-host Chinese Physical Society (CPS) High Energy Physics Branch of CPS

Overview

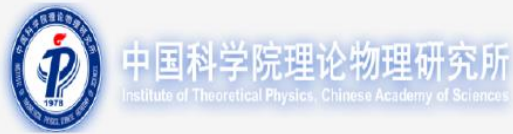
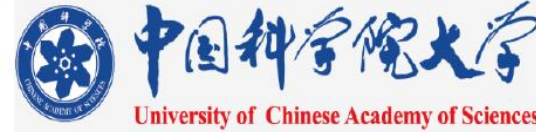
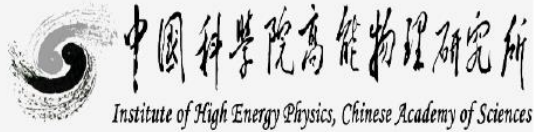
Circulars

Registration

Welcome Message

The 23rd International Conference on Few-Body Problems in Physics (FB23) will be held in Beijing, China on September 22-27, 2024, with Sept. 22 for registration. The registration website is <https://indico.ihep.ac.cn/event/21083/registrations/1689/>

Jointly organized by:



Thanks for your attention!
Welcome to Beijing!