Final Meeting (12th Anniversary) of CRC110

Hadronic molecules

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国家自然科学基金委员会 National Natural Science Foundation of China

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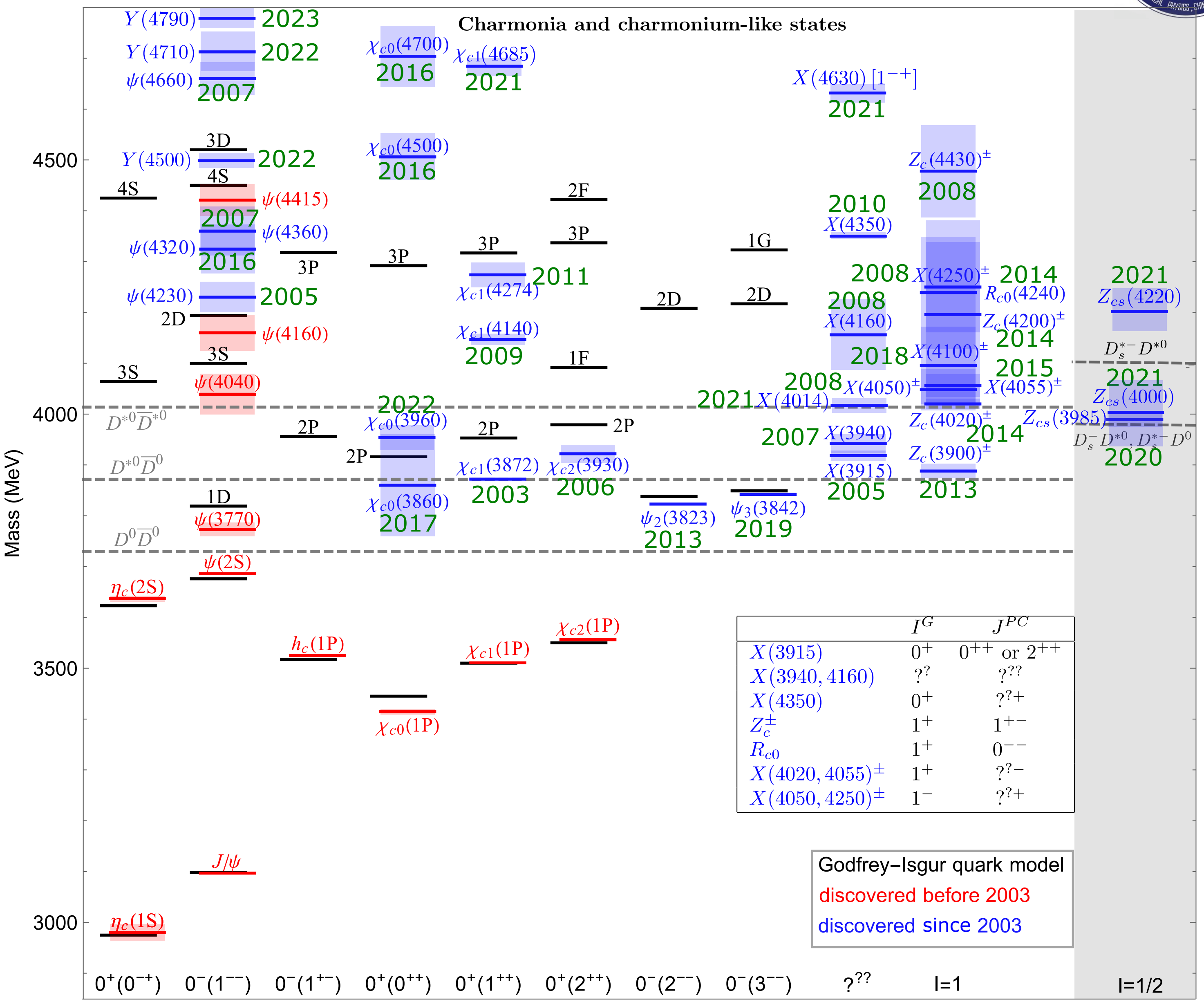
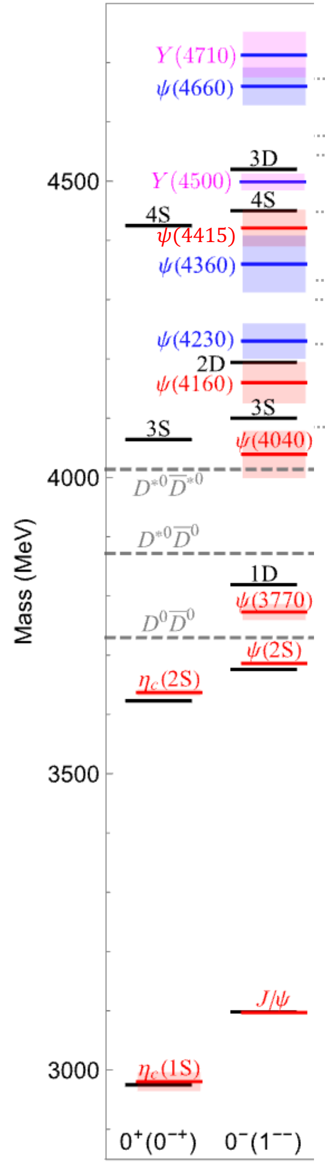
Charmonia and charmonium-like structures

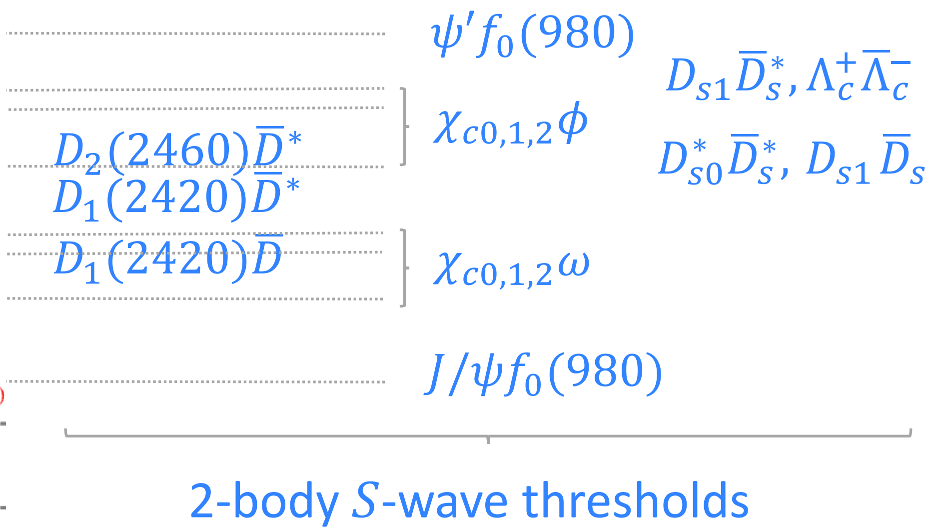
- Abundance of new states from peak hunting
  - ☐ b-hadron (B, Λ_b) decays
  - ☐ Hadron/heavy-ion collisions
  - ☐ $\gamma\gamma$ processes
  - ☐ e^+e^- collisions

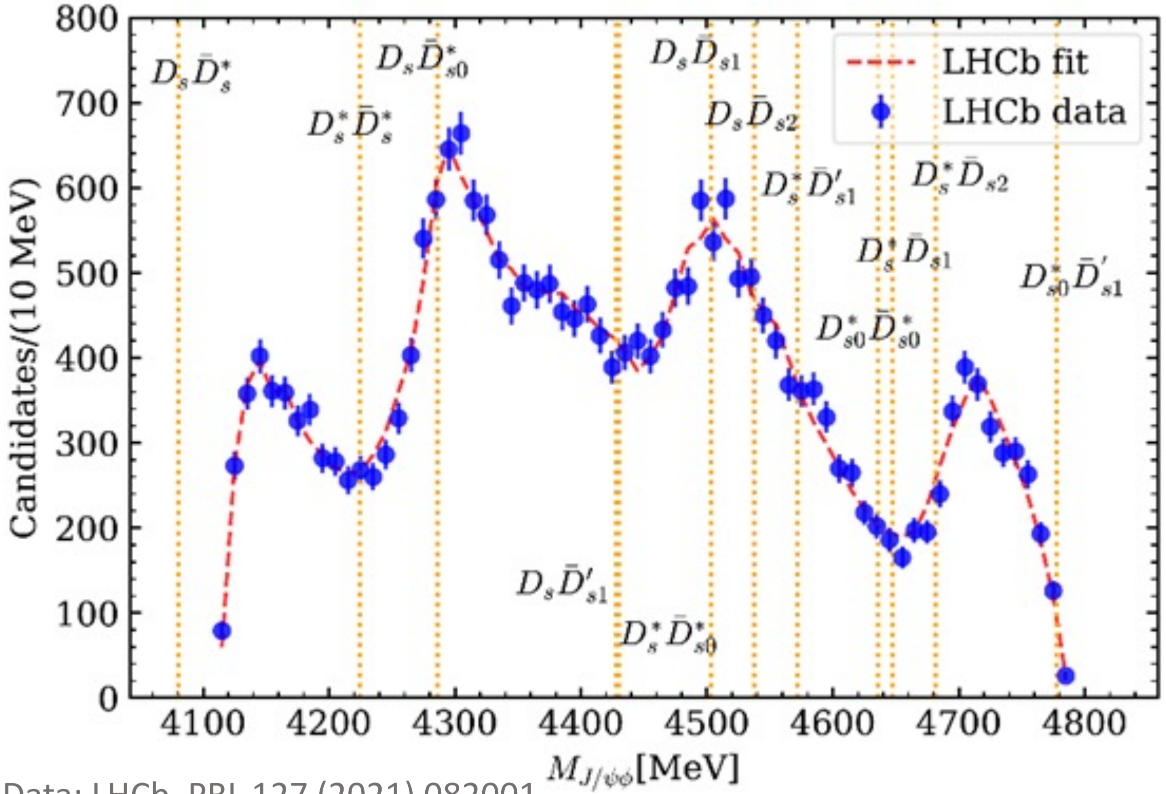
- What are they?
  - ☐ Nonperturbative QCD ⇒ difficult!



Many thresholds above 4 GeV







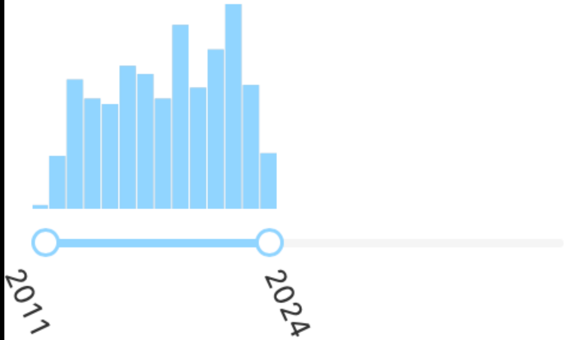
Data: LHCb, PRL 127 (2021) 082001
Plot: X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]

Thresholds play an essential role

- ➢ Complicates extraction of resonance properties!
  Measurements on various final states are important
- ➢ Hadronic molecules?

~1/3 CRC110 publications mentioning hadronic molecules

Date of paper



Number of authors

- Single author 42
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- article 511
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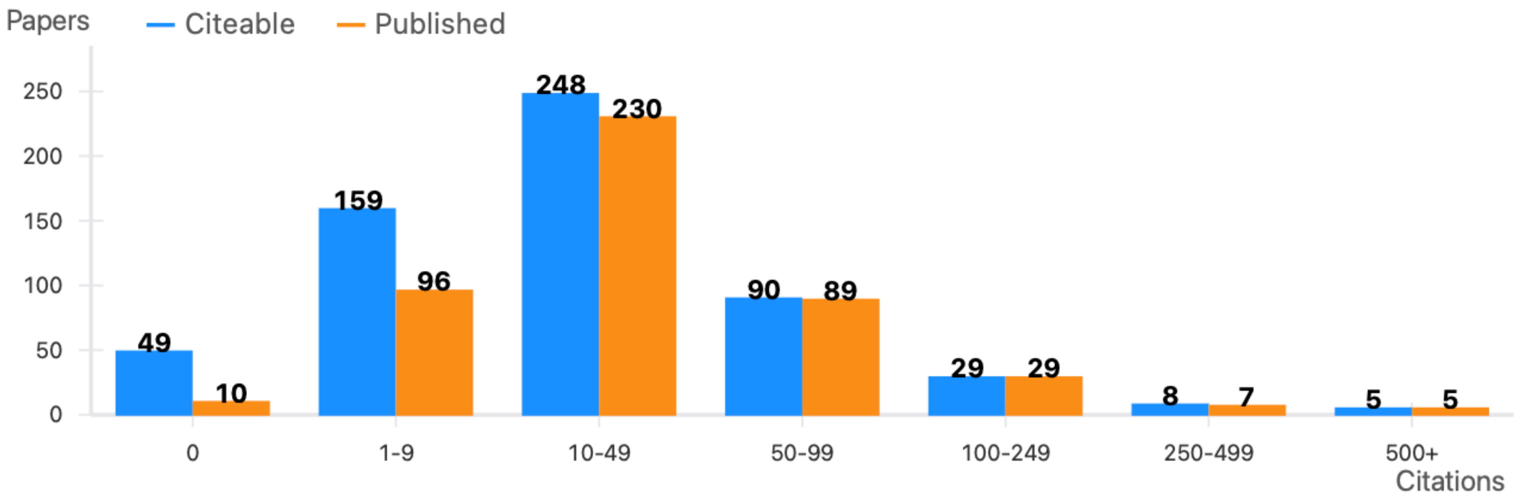
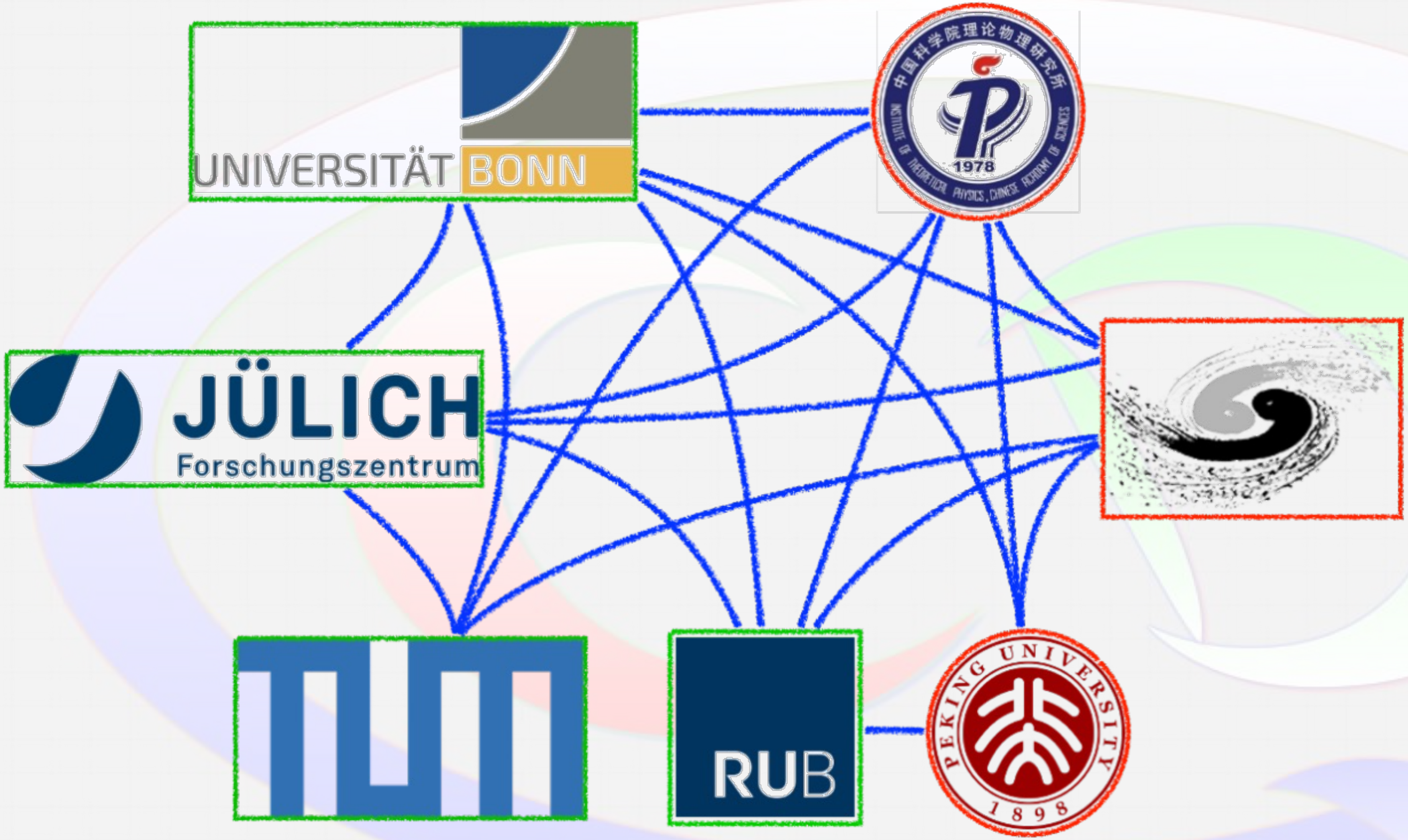


Collaborating network on hadronic molecules among CRC110 nodes



- Wide spectrum of theoretical methods
  - Effective field theories
  - Lattice QCD
  - One-boson exchange
  - QCD sum rules
  - Constituent quark model
  - Dispersive approach
  - Femtoscopic analysis
  - …
- Reviews

Reviews since the 2nd funding period

- >>10 review articles:
  - H.-X. Chen et al., *The hidden-charm pentaquark and tetraquark states*, Phys. Rept. 639 (2016) 1
  - A. Hosaka et al., *Exotic hadrons with heavy flavors: X, Y, Z, and related states*, PTEP 2016 (2016) 062C01
  - J.-M. Richard, *Exotic hadrons: review and perspectives*, Few Body Syst. 57 (2016) 1185
  - R. F. Lebed, R. E. Mitchell, E. Swanson, *Heavy-quark QCD exotica*, PPNP 93 (2017)143
  - A. Esposito, A. Pilloni, A. D. Polosa, *Multiquark resonances*, Phys. Rept. 668 (2017) 1
  - FKG, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, RMP 90 (2018) 015004
  - A. Ali, J. S. Lange, S. Stone, Exotics: *Heavy pentaquarks and tetraquarks*, PPNP 97 (2017) 123
  - S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, RMP 90 (2018) 015003
  - Y.-R. Liu et al., *Pentaquark and tetraquark states*, PPNP107 (2019) 237
  - N. Brambilla et al., *The XYZ states: experimental and theoretical status and perspectives*, Phys. Rept. 873 (2020) 154
  - Y. Yamaguchi et al., *Heavy hadronic molecules with pion exchange and quark core couplings: a guide for practitioners*, JPG 47 (2020) 053001
  - FKG, X.-H. Liu, S. Sakai, *Threshold cusps and triangle singularities in hadronic reactions*, PPNP 112 (2020) 103757
  - G. Yang, J. Ping, J. Segovia, *Tetra- and penta-quark structures in the constituent quark model*, Symmetry 12 (2020) 1869
  - C.-Z. Yuan, Charmonium and charmoniumlike states at the BESIII experiment, Natl. Sci. Rev. 8 (2021) nwab182
  - H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, S.-L. Zhu, *An updated review of the new hadron states*, RPP 86 (2023) 026201
  - L. Meng, B. Wang, G.-J. Wang, S.-L. Zhu, *Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules*, Phys. Rept. 1019 (2023) 2266;
  - ……
- + a book:
  - A. Ali, L. Maiani, A. D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)

Experiments Lattice EFT, models

Compositeness

- Composite systems of hadrons
  - analogues of the deuteron ($\approx pn$ bound state)
  - bound by the residual strong force, more extended than $1/\Lambda_{\rm QCD}$

Different confinement pictures

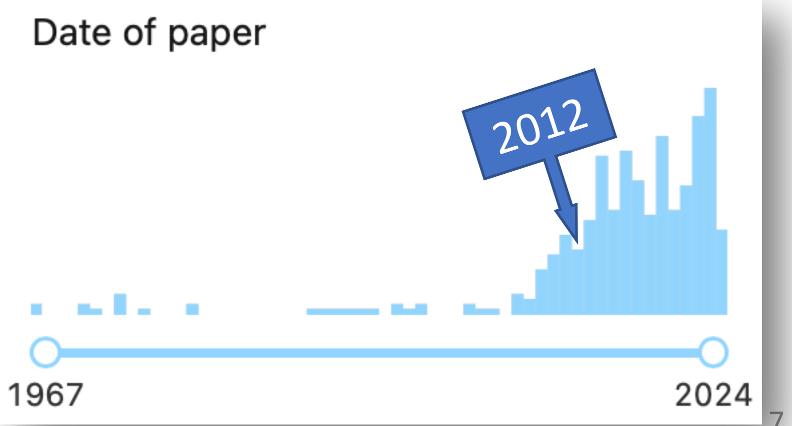
- Compositeness $1 - Z$

  S. Weinberg (1965); V. Baru et al. (2004); T. Hyodo et al. (2012); F. Aceti, E. Oset (2012); Z.-H. Guo, J. Oller (2016); I. Matuschek et al. (2021); J. Song et al. (2022); M. Albaladejo, J. Nieves (2022) ; ….
  for reviews, see T. Hyodo, IJMPA 28 (2013) 1330045; FKG, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, RMP 90 (2018) 015004

  - probability of finding the physical state in two-hadron component (S-wave loosely bound)
  - can be expressed in terms of low-energy observables
    - coupling constant $g_{\rm NR}^2 \approx (1 - Z)\dfrac{2\pi}{\mu^2}\sqrt{2\mu E_B}$ $\quad E_B$: binding energy; μ: reduced mass
    - ERE parameters (scattering length, effective range) S. Weinberg (1965)

$$a \approx -\frac{2(1-Z)}{(2-Z)\sqrt{2\mu E_B}}, \qquad r_e \approx -\frac{Z}{(1-Z)\sqrt{2\mu E_B}}$$

Problematic for $r_e > 0$ I. Matuschek, V. Baru, FKG, C. Hanhart, EPJA 57 (2021) 101



Compositeness: beyond Weinberg

- Weinberg's assumptions
  - Neglecting the non-pole term from the Low equation
  - Approximating the form factor $g(q) \equiv \langle q|\hat{V}|B\rangle$ by a constant

$$T_{p,k} = V_{p,k} + \frac{g(p)\, g^*(k)}{h_k - E_B} + \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{T_{p,q} T^*_{k,q}}{h_k + i\varepsilon - h_q}$$

w/ $h_k \equiv k^2/(2\mu)$

Question: for ERE up to $\mathcal{O}(p^2)$, is a constant $g(p)$ a consistent approximation?

- Improvement: replacing the constant form factor by a more general separable ansatz

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

$$T_{p,k} = t_k\, g(p)\, g^*(k)$$

Unitarity: $\mathrm{Im}\, t^{-1}(W) = \frac{k\mu}{8\pi^2} |g(k)|^2\, \theta(W) \Rightarrow$ twice-subtracted dispersion relation

$$t^{-1}(W) = (W - E_B) + (W - E_B)^2 \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{|g(q)|^2}{(h_q - E_B)^2 (h_q - W)}$$

Then, we get

$$t(W) = \frac{1}{1 - F(W)} \frac{1}{W - E_B}, \qquad F(W) \equiv (W - E_B) \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{|g(q)|^2}{(h_q - E_B)^2 (W - h_q)}$$

Compositeness: beyond Weinberg

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- Compositeness $X \equiv 1 - Z$ emerges

$$F(\infty) = \int_0^\infty \frac{q^2 dq}{(2\pi)^3} \frac{|\langle q|\hat{V}|B\rangle|^2}{(h_q - E_B)^2} = \int_0^\infty \frac{q^2 dq}{(2\pi)^3} |\langle q|B\rangle|^2 = X$$

- Phase shift δ_B with the nonpole term neglected (convention: $\delta_B(0) = 0$)

$$\delta_B(E = h_p) \equiv \arg T_{p,p} = -\arg\left(1 - F(E + i\varepsilon)\right)$$

$$F(0) \le 0, \quad \mathrm{Im}\, F(E + i\varepsilon) \le 0 \text{ for } E \ge 0$$

⇓

$$\delta_B \in [-\pi, 0]$$

Introducing

$$F_1(W) \equiv \frac{\ln[1 - F(W)]}{W - E_B}, \qquad \mathrm{Im}\, F_1(E + i\varepsilon) = -\frac{\delta_B(E)}{E - E_B}\theta(E)$$

- From the dispersion relation for $F_1(W)$, we obtain a solution:

$$F(W) = 1 - \exp\left(\frac{W - E_B}{\pi} \int_0^\infty dE \frac{-\delta_B(E)}{(E - W)(E - E_B)}\right)$$

and an expression for the compositeness

$$X = 1 - \exp\left(\frac{1}{\pi} \int_0^\infty dE \frac{\delta_B(E)}{E - E_B}\right) \in [0, 1]$$

Compositeness: beyond Weinberg

Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- Using $\mathrm{Im}\, F\left(h_p + i\epsilon\right) = -\frac{\pi p \mu}{(2\pi)^3}\frac{|g(p)|^2}{h_p - E_B}$, we get

$$|g(p)|^2 = -\frac{(2\pi)^3}{\pi\, p\mu}(h_p - E_B)\, \sin\delta_B(E) \exp\left[\frac{h_p - E_B}{\pi} \fint_0^\infty dE \frac{-\delta_B(E)}{(E-h_p)(E-E_B)}\right]$$

- Consider ERE $p\cot\delta_B \approx -\frac{8\pi^2}{\mu}\mathrm{Re}T^{-1}(h_p) = \frac{1}{a} + \frac{r}{2}p^2 + \mathcal{O}(p^4)$, we finally get

$$g^2(p) = \frac{8\pi^2}{\mu^2 R} \times \begin{cases} X_W + \mathcal{O}(p^4) & \text{for } a \in [-R, 0]\ \&\ r \le 0 \\ \frac{a^2}{R^2}\frac{1}{1+(a+R)^2p^2} + \mathcal{O}(p^4) & \text{for } a < -R\ \&\ r > 0 \end{cases}$$

constant

contains $\mathcal{O}(p^2)$ terms, thus not self-consistent if using a constant g^2 but still work up to $\mathcal{O}(p^2)$ in ERE. Weinberg's relations do not hold in this case

- Poles of the T-matrix with ERE up to $\mathcal{O}(p^2)$:

$$\frac{1}{a} + \frac{r}{2}p^2 - i\,p = \frac{r}{2}(p - p_+)(p - p_-)$$

➢ For $a \in [-R, 0]$, then $r < 0$, one bound state and one virtual state pole

$$g^2(p) = \frac{8\pi^2}{\mu^2 R}X_W + \mathcal{O}(p^4), \qquad X = X_W \simeq \sqrt{\frac{1}{1+2r/a}}$$

➢ For $a < -R$, then $r > 0$, two bound state poles (the remote one $\sim i/\beta$ is unphysical)

$$g^2(p) = \frac{8\pi^2}{\mu^2 R}\frac{a^2}{R^2}\frac{1}{1+(a+R)^2p^2} + \mathcal{O}(p^4), \qquad X \simeq 1 - e^{-\infty} = 1$$

For the deuteron, $R = 4.31$ fm, $a = -5.42$ fm, $a + R \sim \beta^{-1} \sim m_\pi^{-1}$

Uncertainty of the new relation

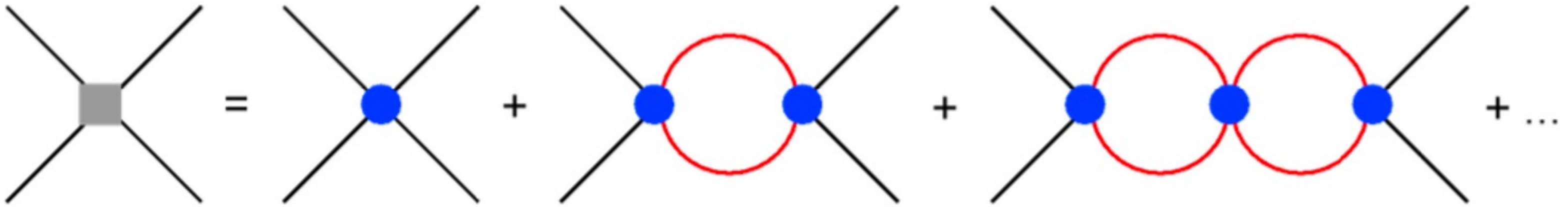
- The uncertainty was usually assumed to be $\mathcal{O}\left(\frac{\gamma}{\beta}\right)$, with $\gamma = \sqrt{2\mu|E_B|}$ the binding momentum. This comes from approximating the form factor by a constant $g(p^2) = 1 + \frac{p^2}{\Lambda^2} + \cdots, \Lambda \sim \beta$

$$\Delta X = \frac{1}{\Lambda^2}\int_0^\Lambda \frac{q^2 dq}{(2\pi)^3}\frac{q^2}{\left(h_q - E_B\right)^2} = \mathcal{O}\left(\frac{\gamma}{\Lambda}\right)$$

- This approximation has been lifted, the uncertainty should be of $\mathcal{O}\left(\frac{\gamma^2}{\beta^2}\right)$!

Hadronic molecules in a NREFT at leading order

- Consider two hadrons in S-wave, near-threshold region ⇒ nonrelativistic EFT



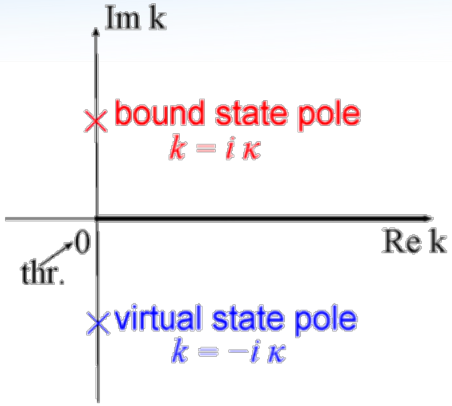
$$T_{\rm NR}(E) = C_0 + C_0\, G_{\rm NR}(E)\, C_0 + C_0\, G_{\rm NR}(E)\, C_0\, G_{\rm NR}(E)\, C_0 + \ldots$$
$$= \frac{1}{C_0{}^{-1} - G_{\rm NR}(E)} = \frac{2\pi/\mu}{2\pi/(\mu\, C_0^r) - \sqrt{-2\mu E - i\epsilon}}$$

☐ Effective coupling: $g_{\rm NR}^2 = \lim_{E\to -E_B} (E+E_B)\, T_{\rm NR}(E) = \frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$

☐ Recall $g_{\rm NR}^2 \approx (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$, the pole obtained at LO NREFT with a constant contact term is purely composite

- Range corrections: other components at shorter distances
  - ✧ coupling to additional states/channels
  - ✧ energy/momentum-dependent interactions: higher order

Molecular line shapes at LO



- Poles at LO NREFT: bound or virtual state
  - Bound and virtual state can hardly be distinguished above threshold $(E > 0)$

$$|T_{\rm NR}(E)|^2 \propto \left|\frac{1}{\pm\kappa + i\sqrt{2\mu E}}\right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

  - Different below threshold $(E < 0)$
    - bound state: peaked below threshold

$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

    - virtual state: sharp cusp at threshold

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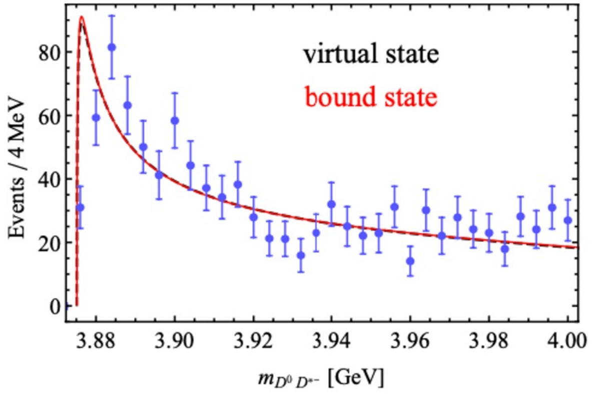

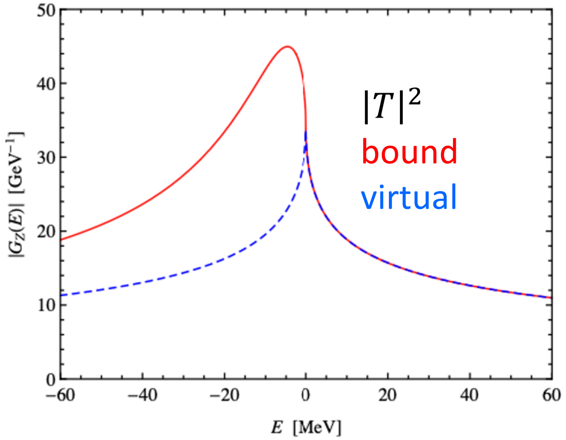

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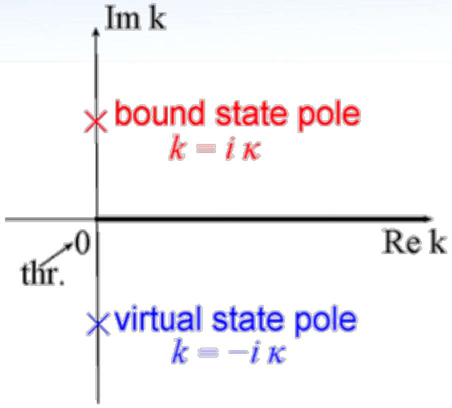
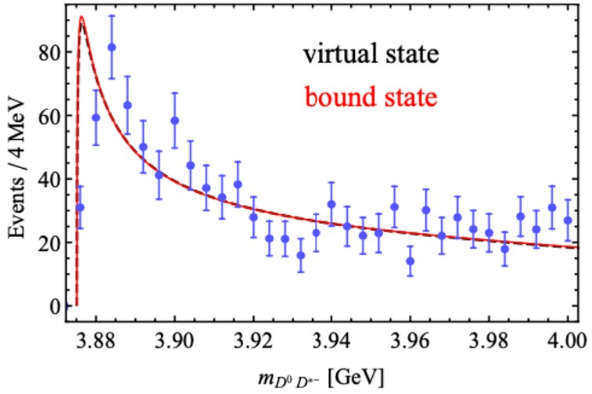
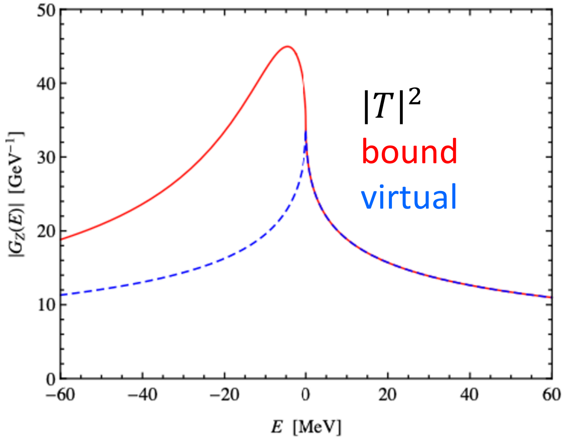
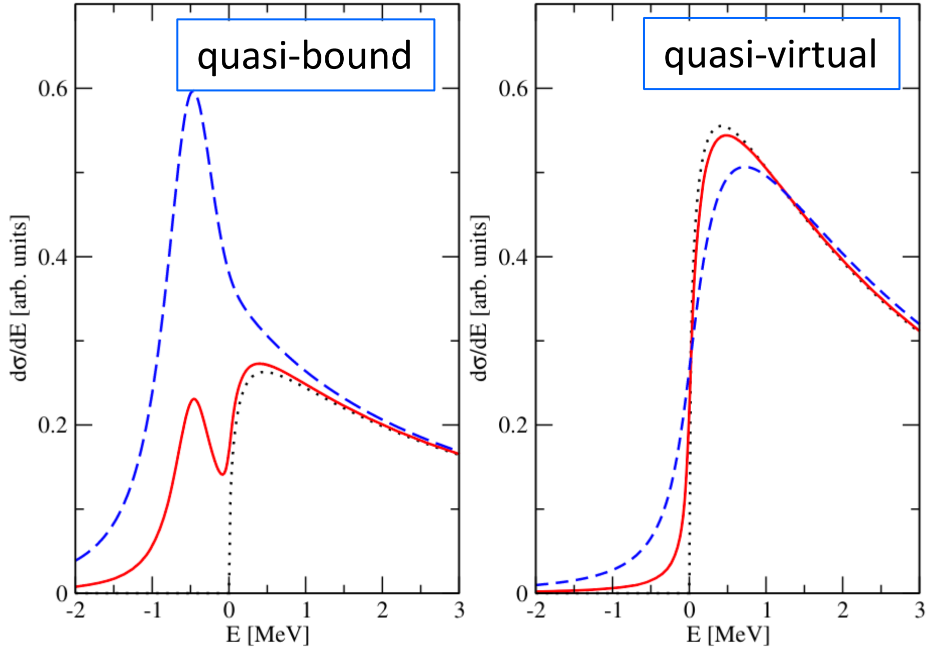






FKG, et al., RMP 90 (2018) 015004;
N. Brambilla et al., Phys.Rept. 873 (2020) 1

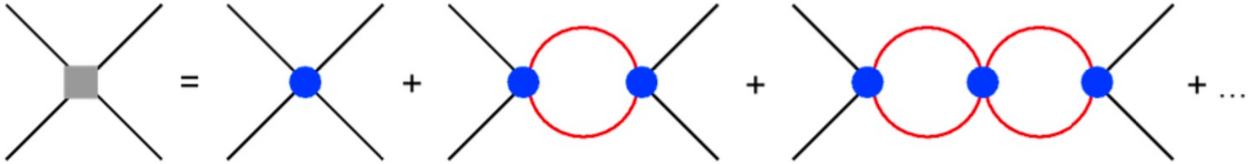
line shapes w/ phase space;
one unstable constituent:



$\Gamma = 0$	0.1 MeV	1 MeV
dotted	dashed	solid

NREFT at LO for coupled channels

- Full threshold structure needs to be measured in a lower channel (ch-1) $\Rightarrow$ coupled channels
- Consider a two-channel system, construct a "nonrelativistic" effective field theory (NREFT)
  - Energy region around the higher threshold (ch-2), Σ_2
  - Expansion in powers of $E = \sqrt{s} - \Sigma_2$
  - Momentum in the lower channel can also be expanded



$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1} = -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix}$$

$$\det = \left(\frac{1}{a_{11}} - ik_1\right)\left(\frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon}\right) - \frac{1}{a_{12}^2}$$

- a_{22}: single-ch. scattering length of ch-2
- a_{11}: single-ch. interaction strength of ch-1 at Σ_2

Effective scattering length with open-channel effects becomes complex, $\mathrm{Im}\,\frac{1}{a_{22,\mathrm{eff}}} \leq 0$

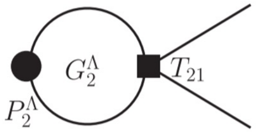
$$T_{22}(E) = -\frac{8\pi}{\Sigma_2}\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1} \qquad \frac{1}{a_{22,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1 + a_{11}^2 k_1^2)} - i\,\frac{a_{11}^2 k_1}{a_{12}^2(1 + a_{11}^2 k_1^2)}.$$

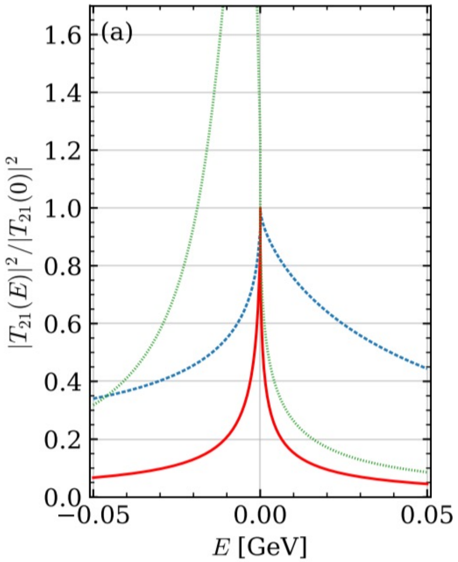
Distinct line shapes of the same pole

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

Line shapes of the same pole depend on the production mechanism. Consider production of particles in ch-1

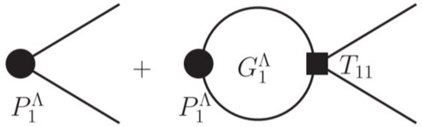
- Dominated by ch-2
  - ☐ Maximal at threshold for positive $\mathrm{Re}(a_{22,\mathrm{eff}})$ (attraction), FWHM $\propto 1/\mu$
    - ➢ more pronounced for heavier hadrons and stronger interactions
  - ☐ Peaking at pole for negative $\mathrm{Re}(a_{22,\mathrm{eff}})$

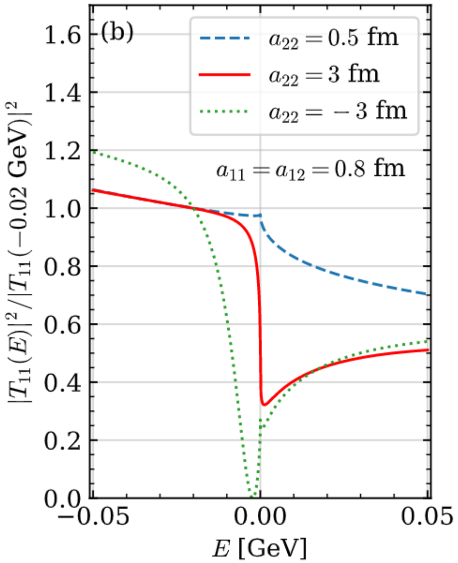




$$T_{21} \propto \frac{1}{a_{22,\mathrm{eff}}^{-1} - i\sqrt{2\mu_2 E}}$$

- Dominated by ch-1
  - ☐ One pole and one zero
  - ☐ Universality for large scattering length: for large $|a_{22}|$, there must be a dip around threshold (zero close to threshold)



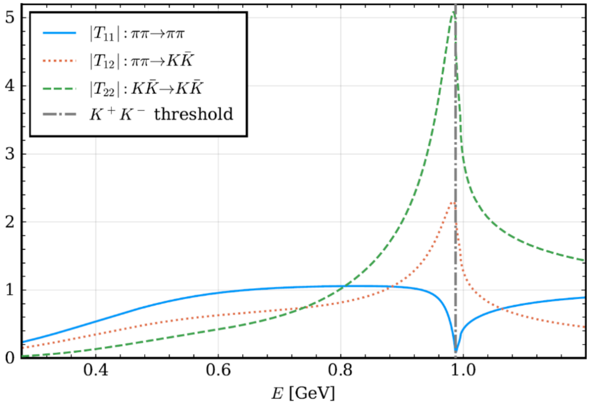


$$T_{11} \propto \frac{a_{22}^{-1} - i\sqrt{2\mu_2 E}}{a_{22,\mathrm{eff}}^{-1} - i\sqrt{2\mu_2 E}}$$

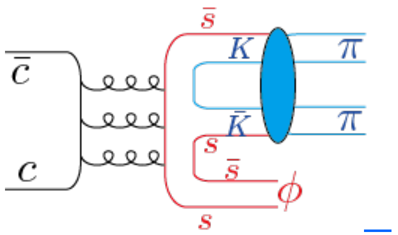
Distinct line shapes of the same pole

- Example-1: $f_0(980)$
  - T-matrix for $\pi\pi$ and $K\bar{K}$ coupled channels

    with the T-matrix from L.-Y. Dai, M. R. Pennington, PRD 90 (2014) 036004

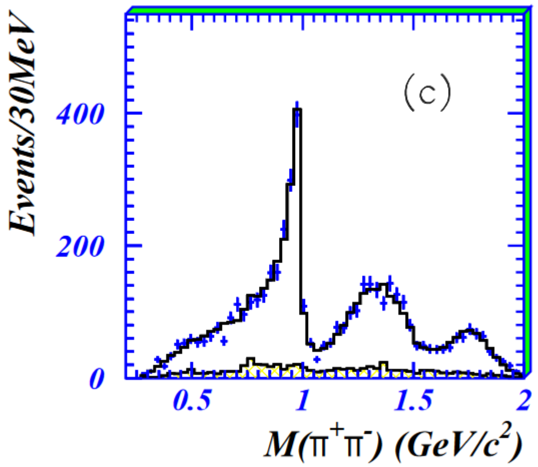


- $J/\psi \to \phi\pi^+\pi^-$



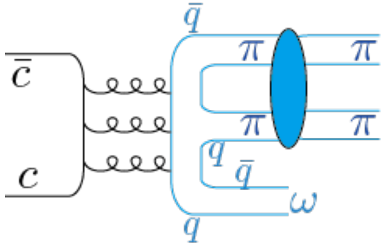
Driving channel: $K\bar{K}$

$J/\psi \to \phi K\bar{K} \to \phi\pi^+\pi^-$



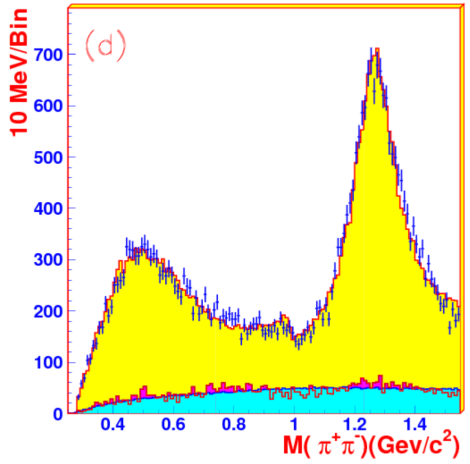
BES, PLB 607 (2005) 243

- $J/\psi \to \omega\pi^+\pi^-$



Driving channel: $\pi\pi$

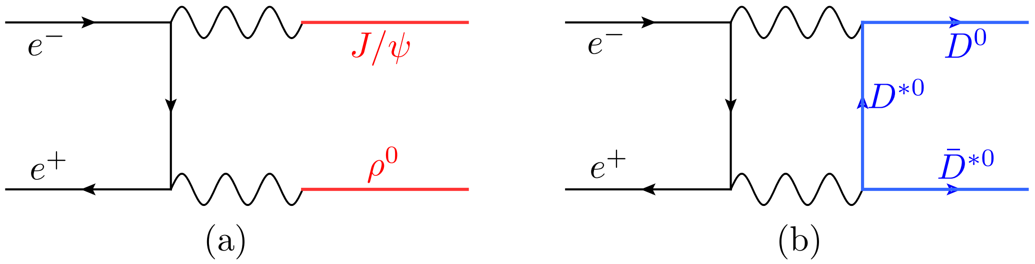
$J/\psi \to \omega\pi\pi \to \omega\pi^+\pi^-$



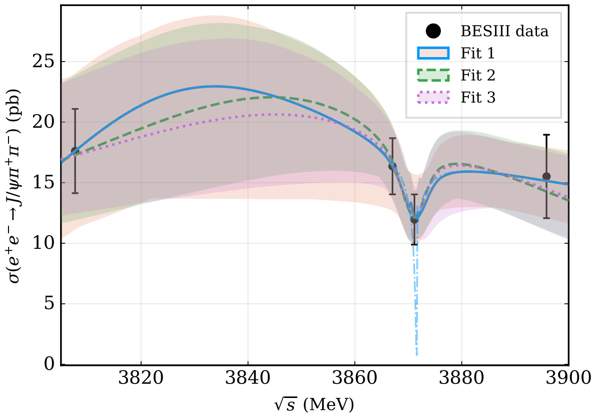
BES, PLB 598 (2004) 149

- Example-2: direct production of $X(3872)$ in e^+e^-

Baru, FKG, Hanhart, Nefediev, PRD (Letter), in print (2024) [2404.12003]



- Driving channel: J/ψ +light vector
- Prediction: dip around $D^*\bar{D}^*$ threshold



Binding mechanism

- One-boson exchange   Vector + scalar exchanges: M. Voloshin, L. Okun, JETP Lett. 23 (1976) 333

  - One-pion exchange
    N.A. Tönqvist, ZPC 61 (1994) 525; …
    - systems like $D\bar{D}, \Sigma_c\bar{D}$ unbound

Composite	J^{PC}	Deuson
$D\bar{D}^*$	0^{-+}	$\eta_c(\approx 3870)$
$D\bar{D}^*$	1^{++}	$\chi_{c1}(\approx 3870)$
$D^*\bar{D}^*$	0^{++}	$\chi_{c0}(\approx 4015)$
$D^*\bar{D}^*$	0^{-+}	$\eta_c(\approx 4015)$
$D^*\bar{D}^*$	1^{+-}	$h_{c0}(\approx 4015)$
$D^*\bar{D}^*$	2^{++}	$\chi_{c2}(\approx 4015)$
$B\bar{B}^*$	0^{-+}	$\eta_b(\approx 10545)$
$B\bar{B}^*$	1^{++}	$\chi_{b1}(\approx 10562)$
$B^*\bar{B}^*$	0^{++}	$\chi_{b0}(\approx 10582)$
$B^*\bar{B}^*$	0^{++}	$\eta_b(\approx 10590)$
$B^*\bar{B}^*$	1^{+-}	$h_b(\approx 10608)$
$B^*\bar{B}^*$	2^{++}	$\chi_{b2}(\approx 10602)$

  - One-vector exchange   S. Krewald, R. Lemmer, F. Sassen, PRD 69 (2004) 016003; …
    - $D\bar{D}$ bound state predicted
      Y.-J. Zhang, H.-C. Chiang, P.-N. Shen, B.-S. Zou, PRD 74 (2006) 014013;
      D. Gamermann et al., PRD 76 (2007) 074016; …
      - ✧ Lattice QCD   S. Prelovsek et al., JHEP06 (2021) 035
        Conflict: not in   D.J. Wilson et al., arXiv:2309.14070.   solution?
    - Hidden-charm pentaquarks >4 GeV (including $\Sigma_c\bar{D}$) predicted
      J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, PRL 105 (2010) 232001; …

- Soft-gluon exchanges: equivalent to OZI breaking $\pi\pi, K\bar{K}$, …
  X.-K. Dong et al., Sci. Bull. 66 (2021) 1577

☞ Survey of the molecular spectrum in a simple model
- light-vector-meson exchanges
- single channel
- neglecting mixing

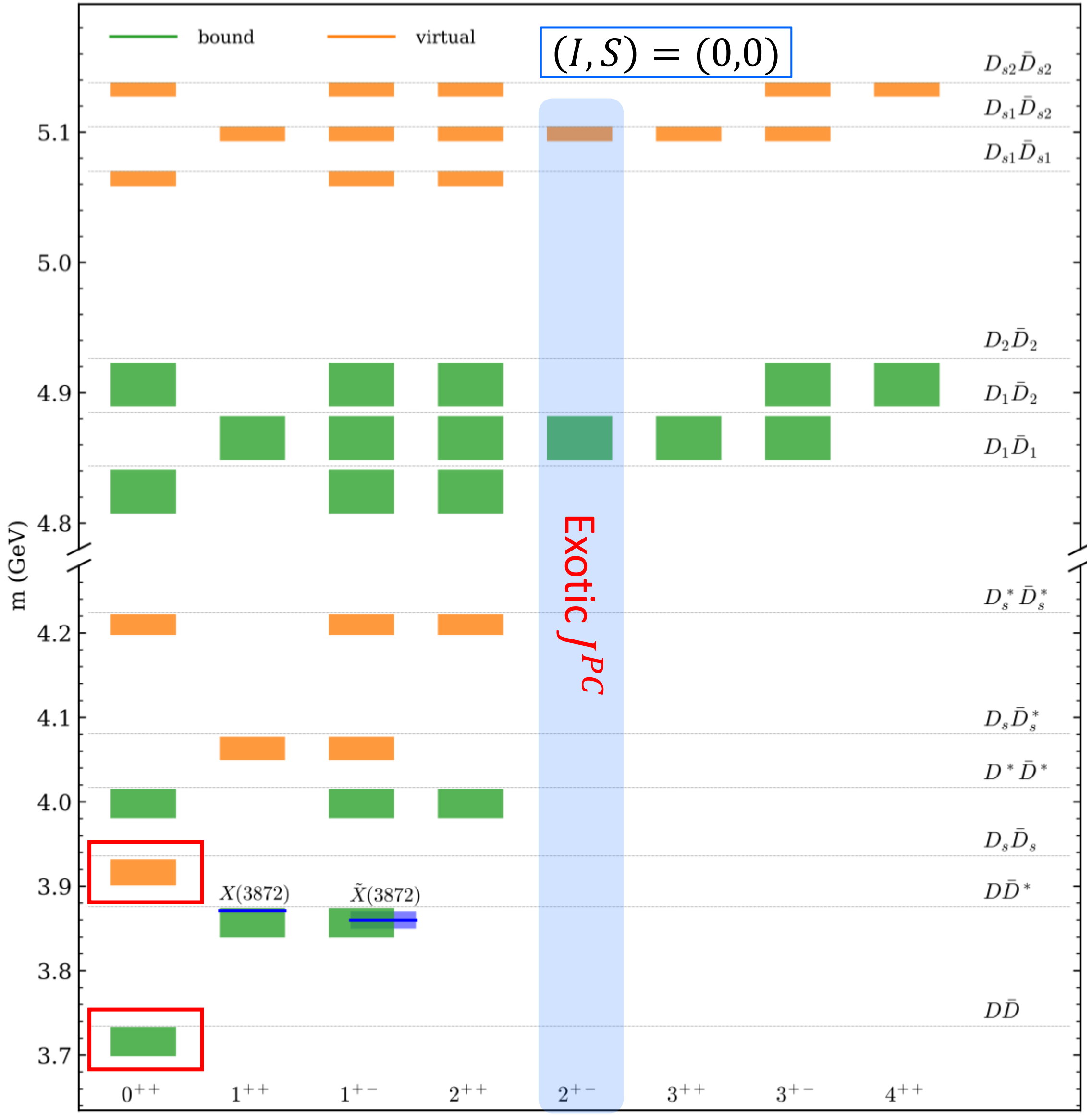
X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65; CTP 73 (2021) 015201

Extension of the survey including vector+scalar meson exchanges:
F.-Z. Peng, M. Sanchez-Sanchez, M.-J. Yan, M. Pavon Valderrama, PRD 105 (2022) 034028;
M.-J. Yan, F.-Z. Peng, M. Pavon Valderrama, PRD 109 (2024) 014023

For a list of the literature on one-boson exchange models, see, e.g., Y.-R. Liu et al., PPNP 107 (2019) 237

Survey of hadronic molecules: hidden-charm mesons w/ $P = +$

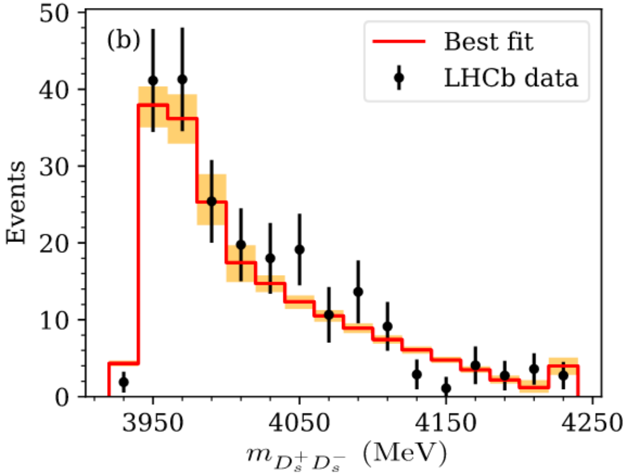
X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



- ✓ > 200 hidden-charm hadronic molecules
- ✓ $X(3872)$ as a $\bar{D}D^*$ bound state
- ✓ $\tilde{X}(3872)$ COMPASS, PLB 783 (2018) 334
- ✓ $\bar{D}D$ bound state from lattice S. Prelovsek et al., JHEP06 (2021) 035

  & other models C.-Y. Wong, PRC 69 (2004) 055202; Y.-J. Zhang et al., PRD 74 (2006) 014013; D. Gamermann et al., PRD 76 (2007) 074016; J. Nieves et al., PRD 86 (2012) 056004; ...

- ✓ $X(3960)$ in $B^+ \to D_s^+ D_s^- K^+$



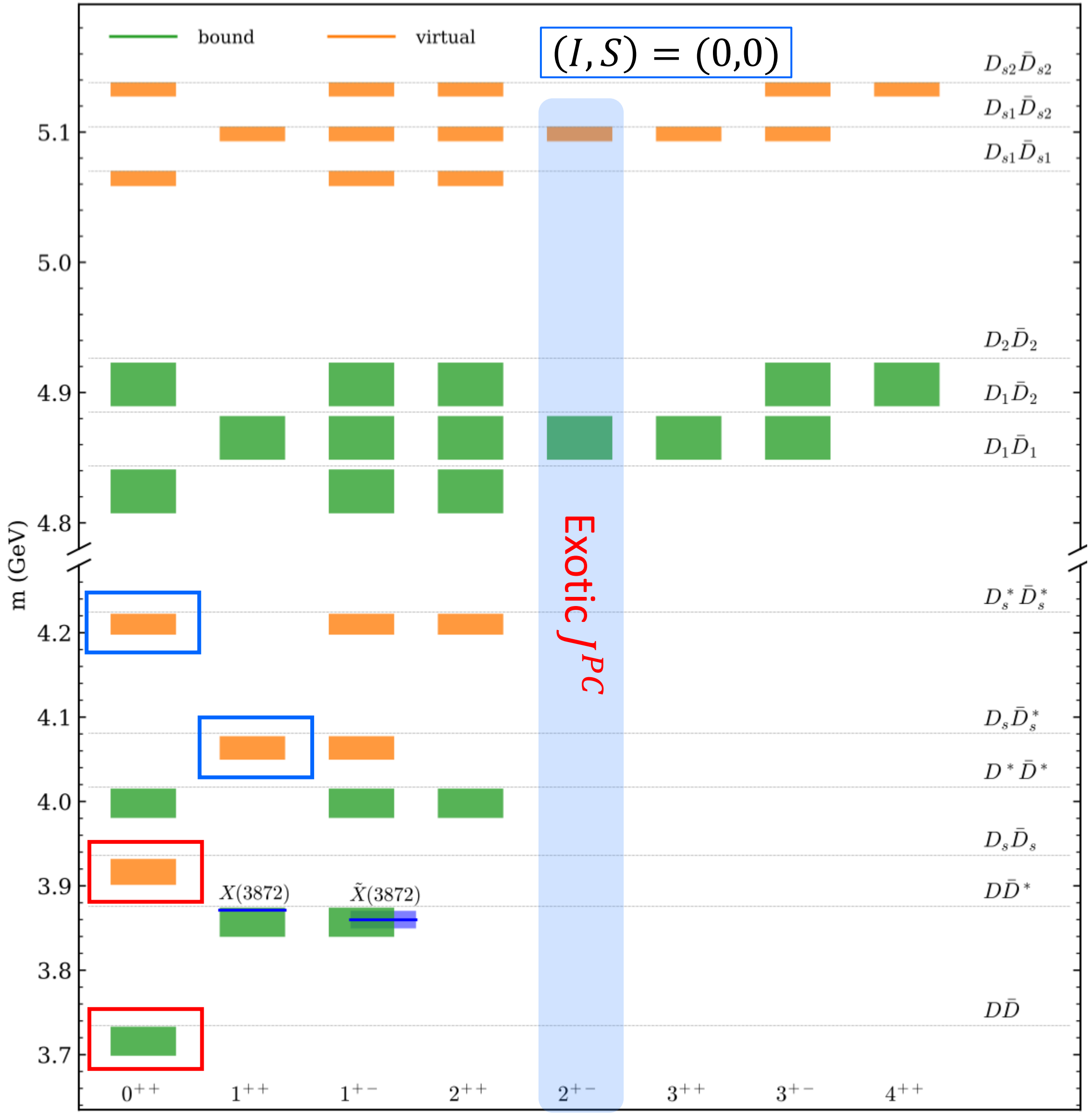
Data: LHCb, PRL 131 (2023) 071901

Fit in
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, FKG, J. Nieves, B.-S. Zou, Sci. Bull. 68 (2023) 2056

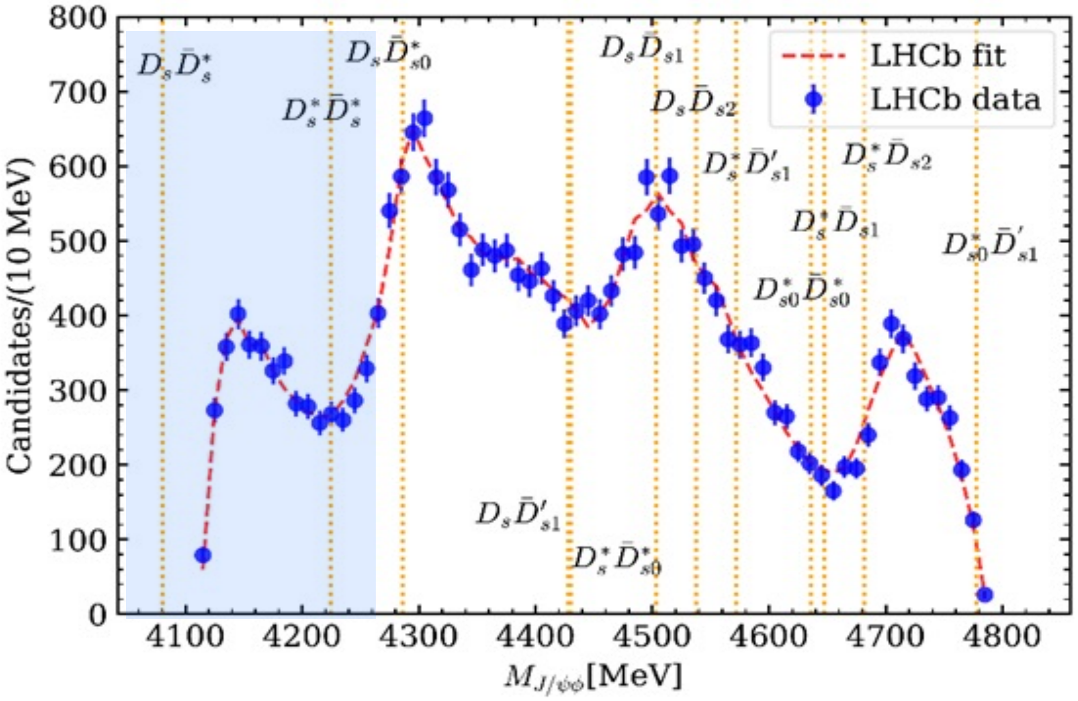
pole at $3936.5^{+0.4}_{-0.9} + i\left(16.1^{+4.2}_{-2.2}\right)$ MeV

Survey of hadronic molecules: hidden-charm mesons w/ $P = +$

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



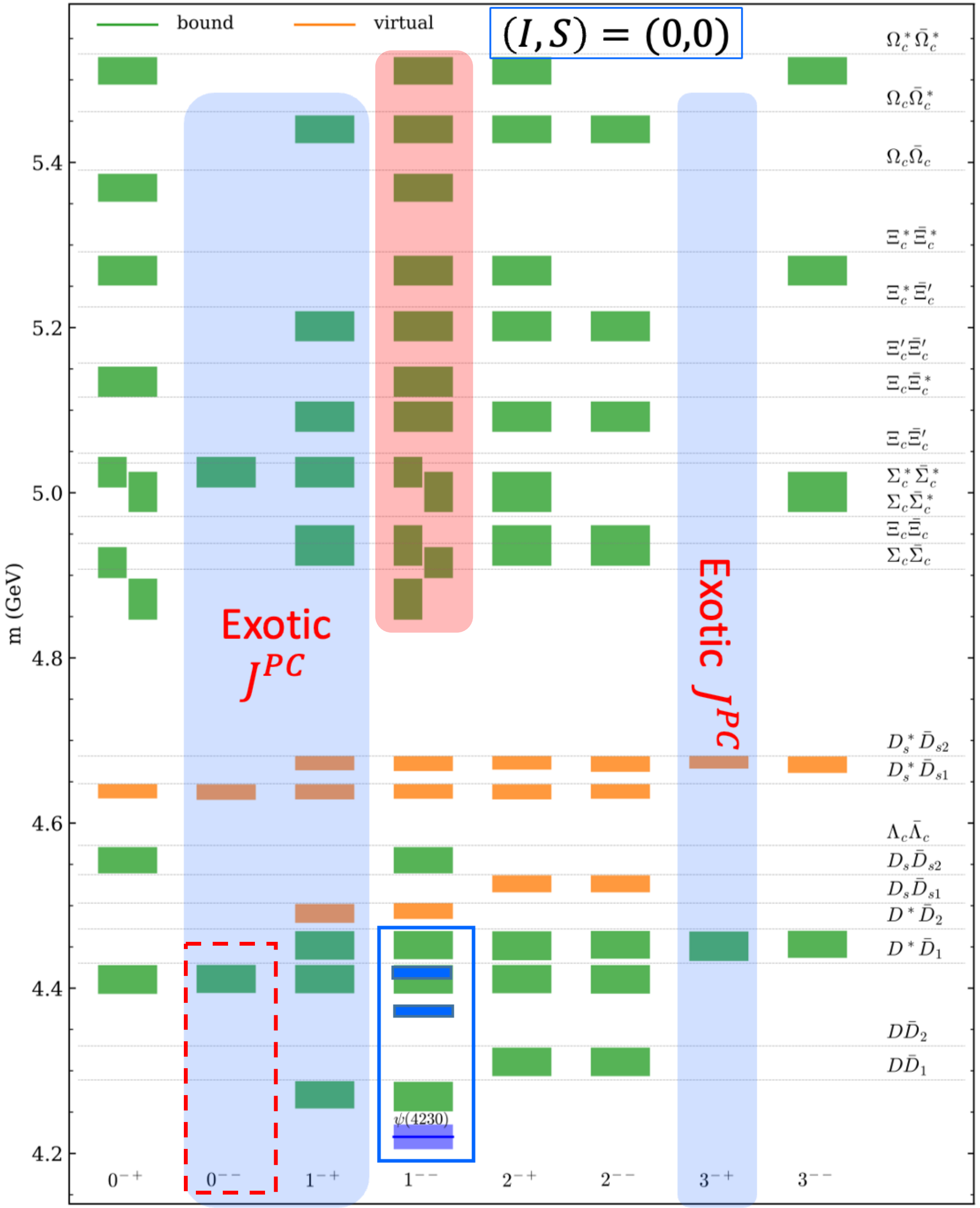
✓ $D_s\bar{D}_s^*$, $D_s^*\bar{D}_s^*$ virtual states?



Data: LHCb, PRL 127 (2021) 082001

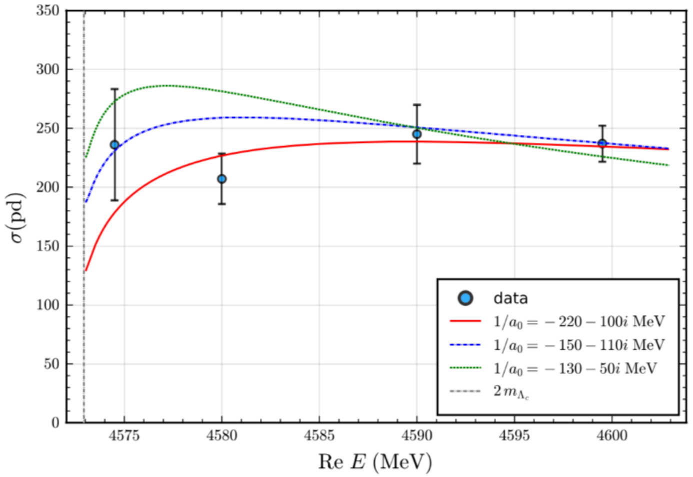
Virtual poles found from the fit in X. Luo, S.X. Nakamura, PRD 107 (2023) L011504

Hidden-charm mesons w/ $P = -$



- ✓ $Y(4260)/\psi(4230)$ as a $\bar{D}D_1$ bound state
- ✓ $\psi(4360), \psi(4415)$: $D^*\bar{D}_1, D^*\bar{D}_2$?
- ✓ Evidence for 1^{--} $\Lambda_c\bar{\Lambda}_c$ molecular state in BESIII data
  - Sommerfeld factor
  - near-threshold pole
  - different from $Y(4630)$

Data from BESIII, PRL 120 (2018) 132001;
see also Q.-F. Cao et al., PRD 100 (2019) 054040

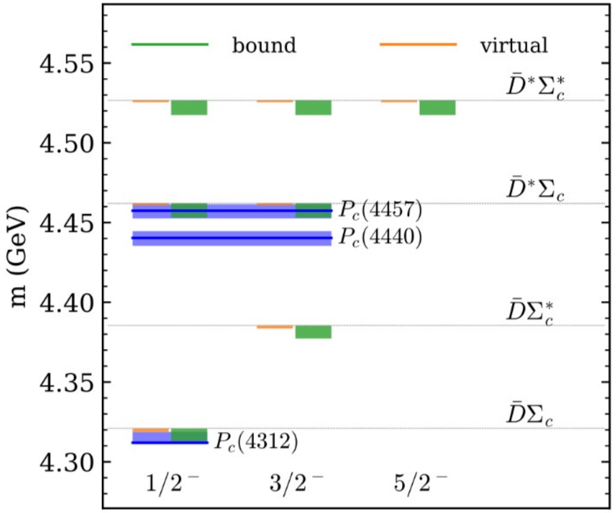


- ✓ Numerous states with exotic quantum numbers 0^{--} $[\psi_0]$, $1^{-+}[\eta_{c1}]$, $3^{-+}[\eta_{c3}]$
  e.g., $e^+e^- \to \gamma\eta_{c1,3}, \omega\eta_{c1,3}$; $\eta_{c1,3} \to D\bar{D}^*\pi, J/\psi\omega$, ...
- ✓ Many 1^{--} states in [4.8, 5.6] GeV: BEPC-II-Upgrade, Belle-II, LHCb, STCF, PANDA, ...

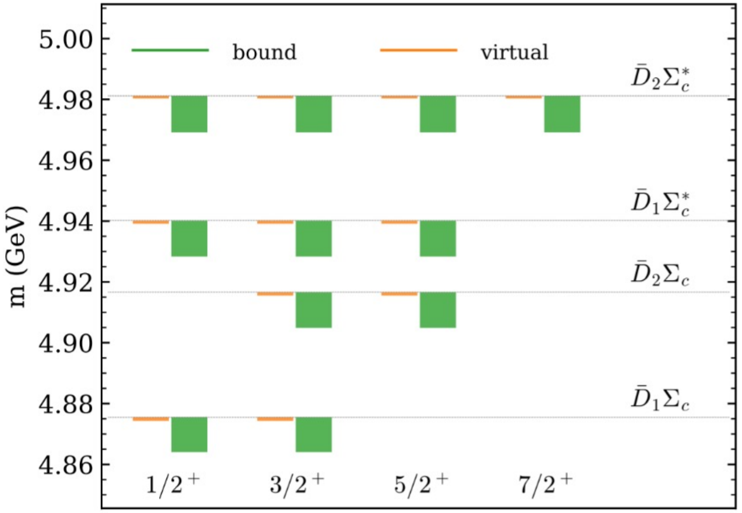
Hidden-charm pentaquarks

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65

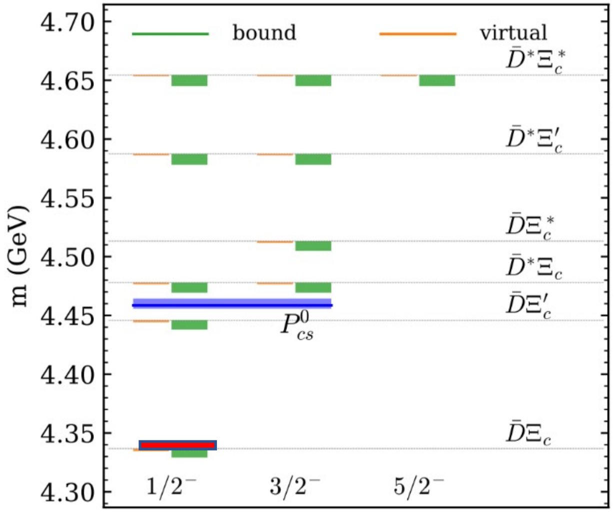
$(I, S) = (1/2, 0)$



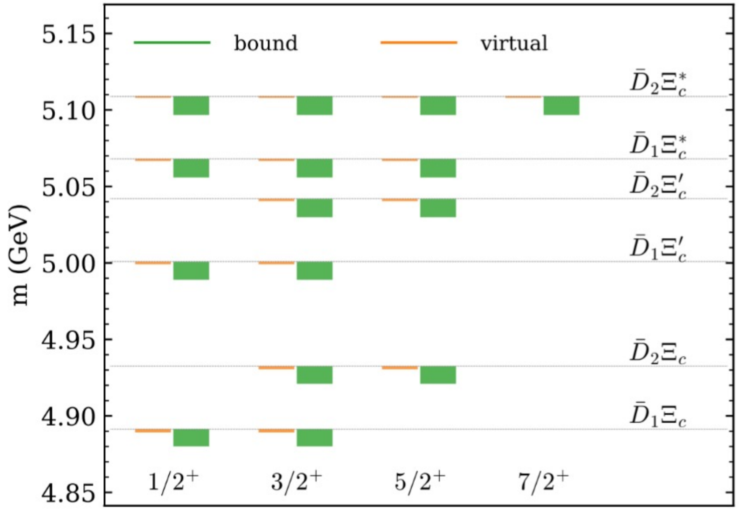
$(I, S) = (1/2, 0)$



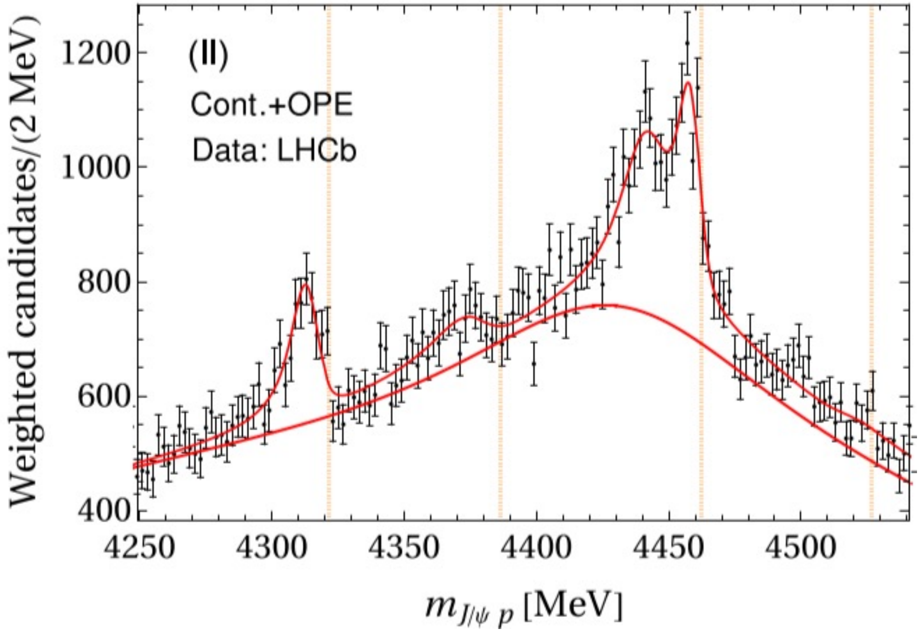
$(I, S) = (0, 1)$



$(I, S) = (0, 1)$



- ✓ P_c states as $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules
- ✓ The LHCb data can be well described in a pionful EFT



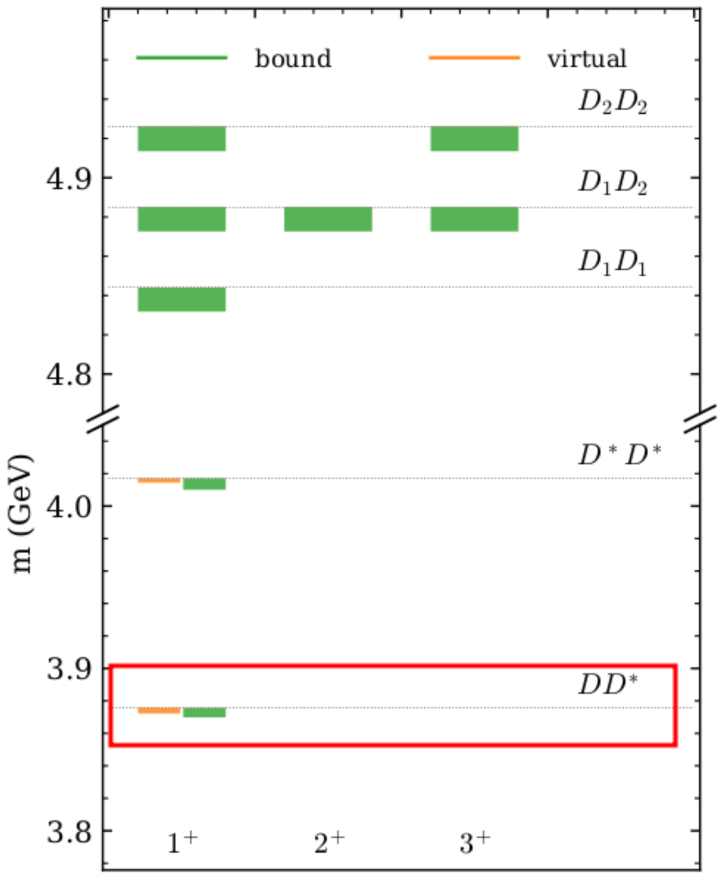
M.-L. Du et al., PRL 124 (2020) 072001; JHEP 08 (2021) 157

- ✓ $P_{cs}(4459)$: 2 $\bar{D}^*\Xi_c$ molecular states
- ✓ $P_{cs}(4338)$: $\bar{D}\Xi_c$ molecular state

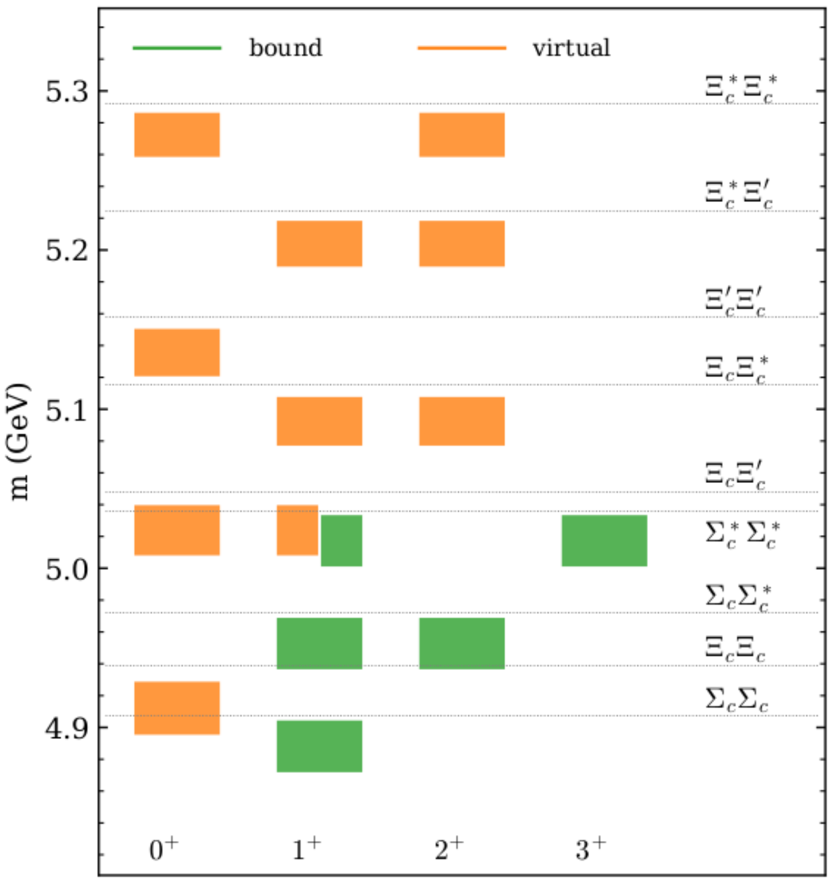
Double-charm tetraquarks and dibaryons

X.-K. Dong, FKG, B.-S. Zou, CTP 73 (2021) 125201

$(I, S, B) = (0,0,0)$

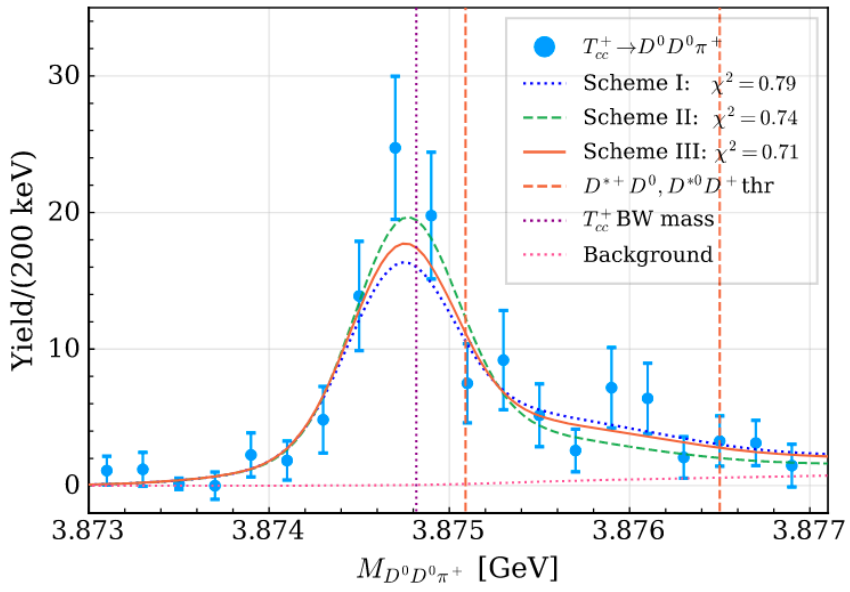


$(I, B) = (0,2)$



✓ $T_{cc}(3875)$ as D^*D molecule

✓ The LHCb data can be well described in a pionful EFT w/ 3-body effects



M.-L. Du et al., PRD 105 (2022) 014024

✓ isoscalar DD^* molecular state

✓ It has a spin partner 1^+ D^*D^* state

✓ Many (> 100) other similar double-charm molecular states

Closer look at the 0^{--} state

- $\psi(4230), \psi(4360), \psi(4415)$ as $D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_2$ hadronic molecules

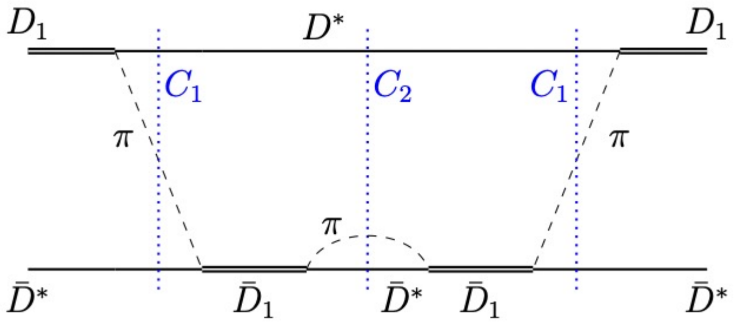
Q. Wang, C. Hanhart, Q. Zhao, PRL 111 (2013) 132002; Cleven et al. (2015); L. Ma et al. (2015); …

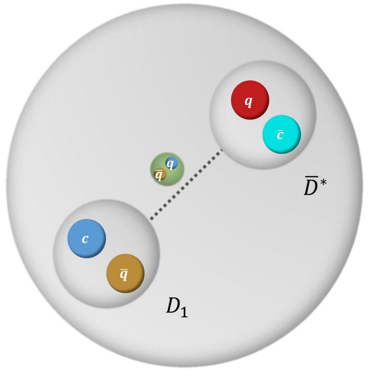
- 0^{--} spin partner $\psi_0(4360)$ $[D^*\bar{D}_1]$

T. Ji, X.-K. Dong, FKG, B.-S. Zou, PRL 129 (2022) 102002

- Robust against the inclusion of coupled channels and three-body effects

Molecule	Components	J^{PC}	Threshold	E_B
$\psi(4230)$	$\frac{1}{\sqrt{2}}(D\bar{D}_1 - \bar{D}D_1)$	1^{--}	4287	67 ± 15
$\psi(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	1^{--}	4429	62 ± 14
$\psi(4415)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_2 - \bar{D}^*D_2)$	1^{--}	4472	49 ± 4
ψ_0	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 + \bar{D}^*D_1)$	0^{--}	4429	63 ± 18

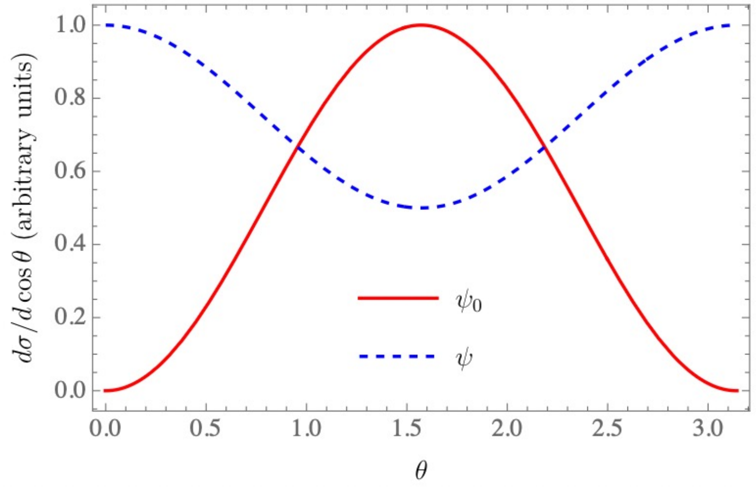




- May be searched for using $e^+e^- \to \psi_0\eta$, $\psi_0 \to J/\psi\eta, D\bar{D}^*, D^*\bar{D}^*\pi, \ldots$

$M = (4366 \pm 18)$ MeV,

$\Gamma < 10$ MeV



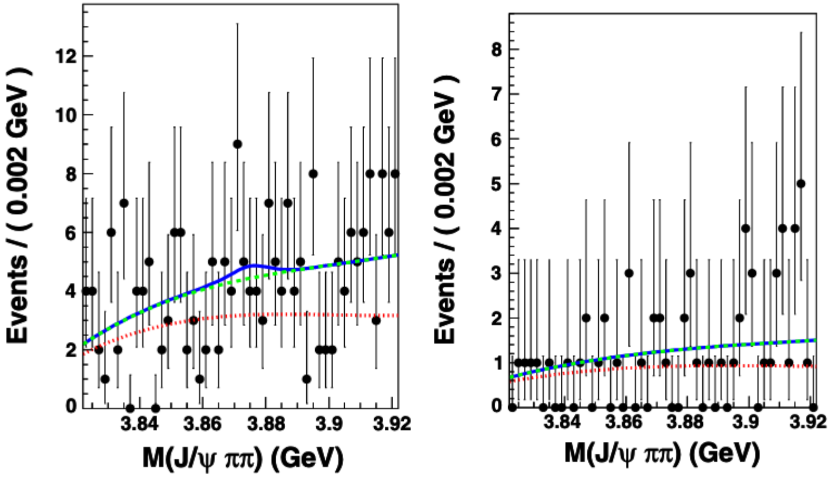
Prediction of an isospin vector partner of $X(3872)$

Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, C. Hanhart, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

- Isospin-1 partner of X(3872) was predicted in the compact tetraquark model

L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2004) 014028

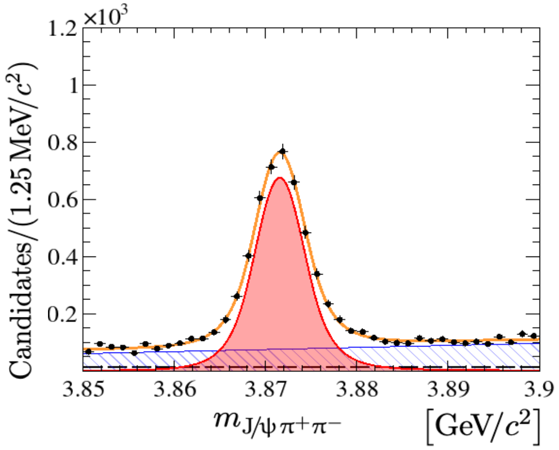
- No signal in the charged channel so far



$\bar{B}^0 \to K^- \pi^+ \pi^0 J/\psi$ $B^+ \to K^0 \pi^+ \pi^0 J/\psi$

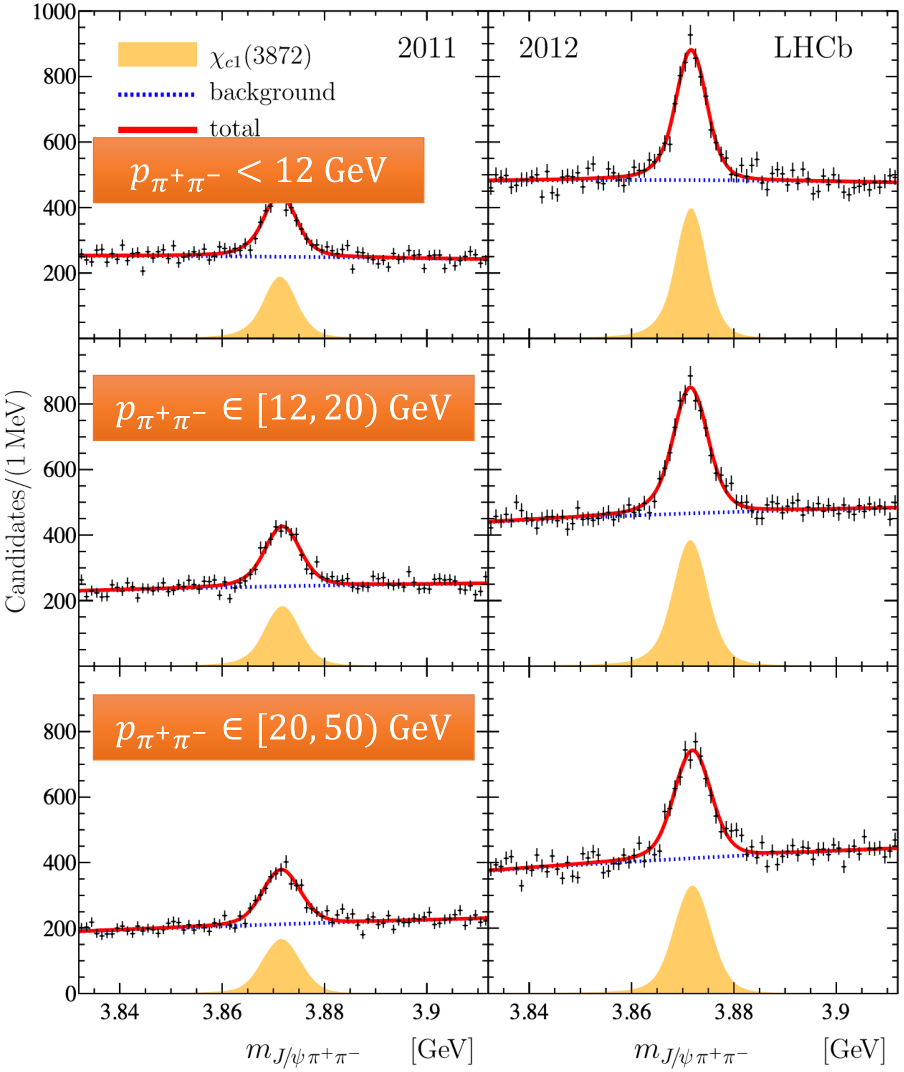
Belle, PRD 84 (2011) 052004

- No signal around the $D^+ D^{*-}$ threshold



$B^+ \to K^+ \pi^+ \pi^- J/\psi$

LHCb, JHEP 08 (2020) 123



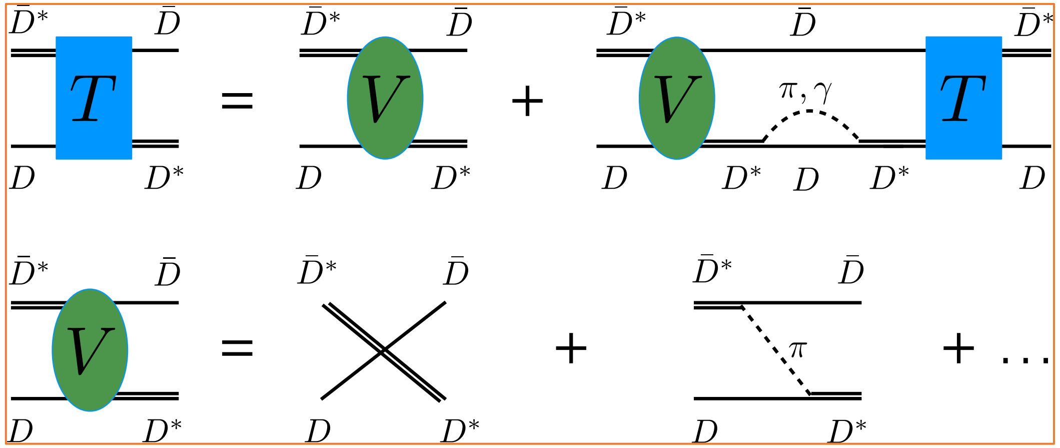
$\pi^+ \pi^- J/\psi$ from b-hadrons

LHCb, PRD 102 (2020) 092005

Prediction of an isospin vector partner of $X(3872)$

Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

- How about the $D\bar{D}^*$ hadronic molecular scenario?
- $D^0\bar{D}^{*0}, D^+D^{*-}$ coupled channels: $I = 0, 1$
  - Interactions at leading order: two LECs ($I = 0, 1$) C_{0X}, C_{1X}
  - Two inputs from $X(3872)$ properties



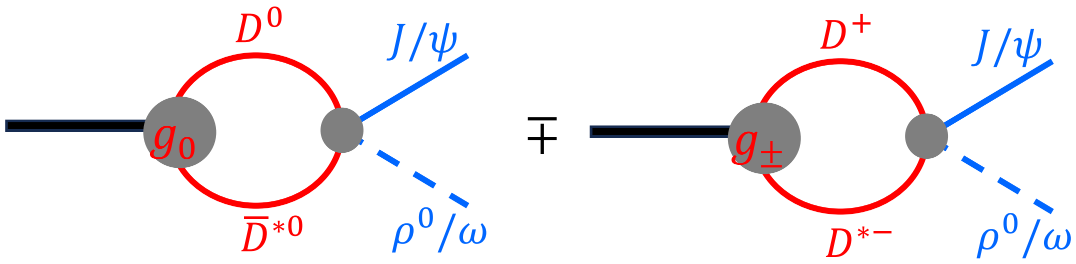
➤ **Mass**

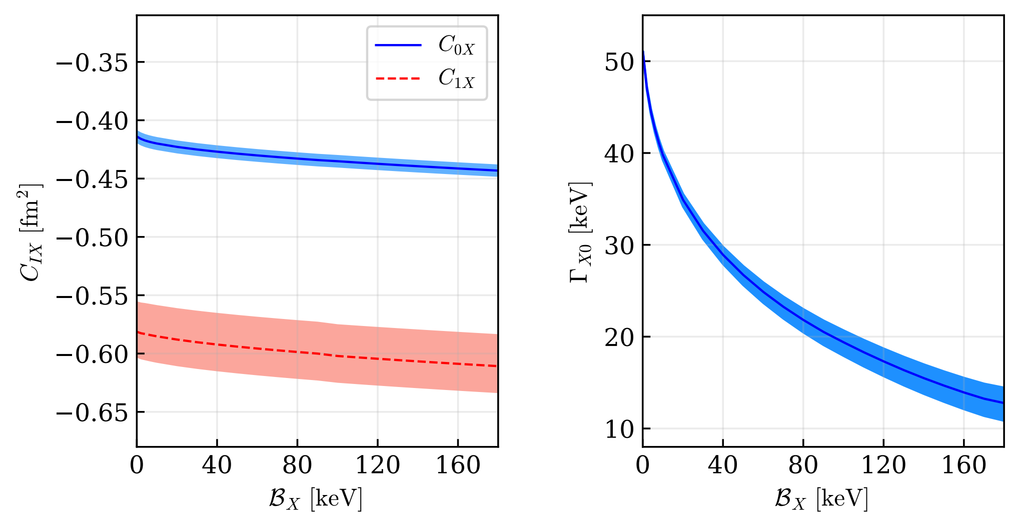
$$M_X = 3871.69^{+0.00+0.05}_{-0.04-0.13}\,\text{MeV}$$ LHCb, PRD 102 (2020) 092005

$$M_{D^0} + M_{D^{*0}} = 3871.69(7)\ \text{MeV}$$ PDG 2023

➤ **Isospin breaking** in decays LHCb, PRD 108 (2023) L011103

$$R_X = \left|\frac{\mathcal{M}_{X(3872)\to J/\psi\rho^0}}{\mathcal{M}_{X(3872)\to J/\psi\omega}}\right| = 0.29 \pm 0.04 = \left|\frac{g_0 - g_\pm}{g_0 + g_\pm}\right|$$





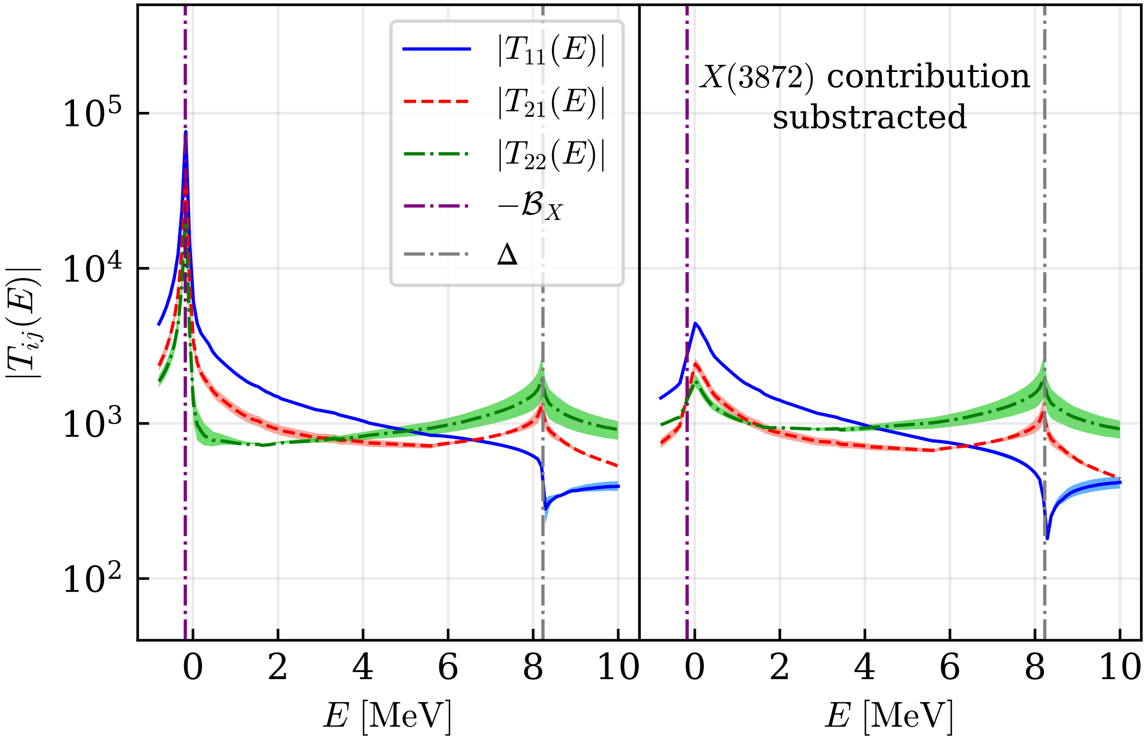
$$\mathcal{B}_X \equiv M_{D^0} + M_{D^{*0}} - M_{X(3872)}$$

Prediction of an isospin vector partner of $X(3872)$

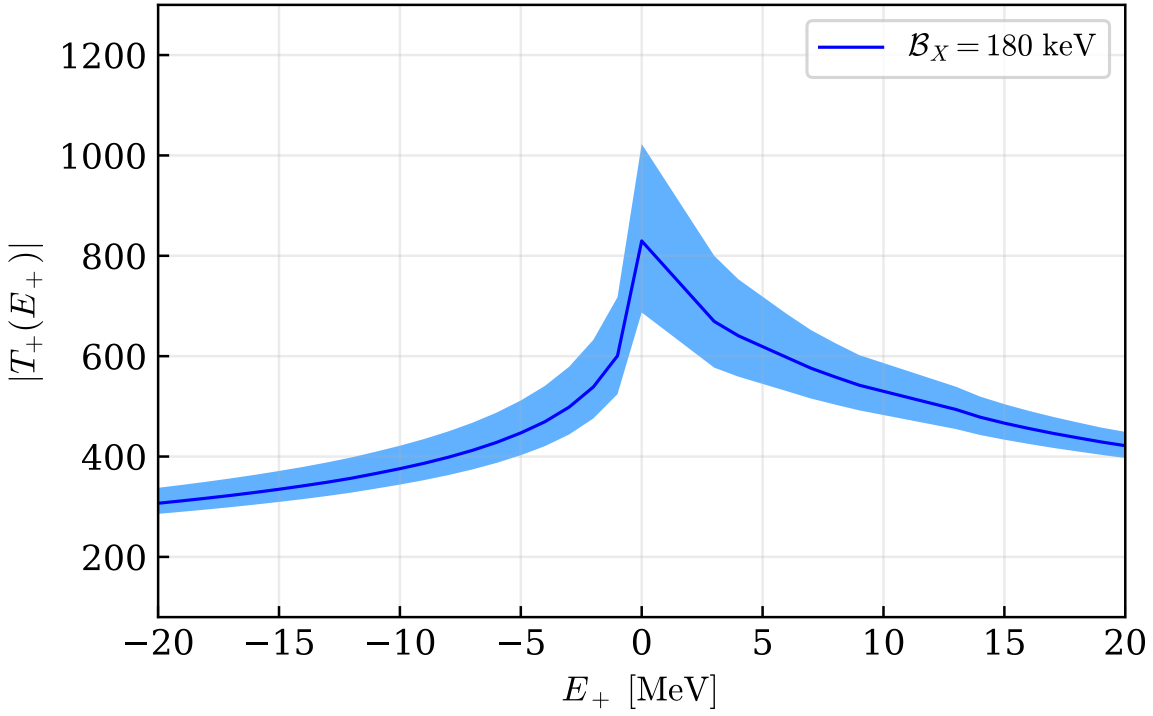
Z.-H. Zhang, T. Ji, X.-K. Dong, FKG, C.Hanhart, U.-G. Meißner, A. Rusetsky, arXiv:2404.11215

- There must be an isospin vector partner W_{c1}
  - Virtual state pole in the stable D^* limit $\Rightarrow$ explains why it has not observed so far!

➢ W_{c1}^0 in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes



➢ W_{c1}^+ in $D^+\bar{D}^{*0}$ scattering amplitude



W_{c1}^0: $3865.3^{+4.2}_{-7.4} - i0.15^{+0.04}_{-0.03}$ MeV

$W_{c1}^{\pm}$: $3866.9^{+4.6}_{-7.7} - i(0.07 \pm 0.01)$ MeV

➢ should be searched for in high-statistic $J/\psi\pi^+\pi^0$ data

➢ Compact tetraquark (Maiani et al. (2005)) cannot be virtual state!

Summary

CRC 110 has significantly advanced the knowledge of hadronic molecules

Selected works that I was involved:

- Generalization of Weinberg's compositeness relations
- A rich spectrum of hadronic molecules is expected
- General rules for (near-)threshold structures
  - S-wave attraction, more prominent for heavier particles and stronger attraction
  - Pole behavior: distinct line shapes depending on reaction mechanism
  - Universality: a dip (for large $|a_{22}|$) at the higher channel threshold in T_{11}
- Robust prediction of a ψ_0 with exotic $J^{PC} = 0^{--}$ as spin partner of $\psi(4230), \psi(4360), \psi(4415)$

  ⇒ extension to more spin partners ongoing
- Robust prediction of an isovector partner of $X(3872)$: $W_{c1}^{\pm,0}$

Thank you for your attention!