Collaborative research center CRC 110

“Symmetries and the emergence of structure in QCD”

Lattice QCD for Flavor Physics

Xu Feng (Peking U.)

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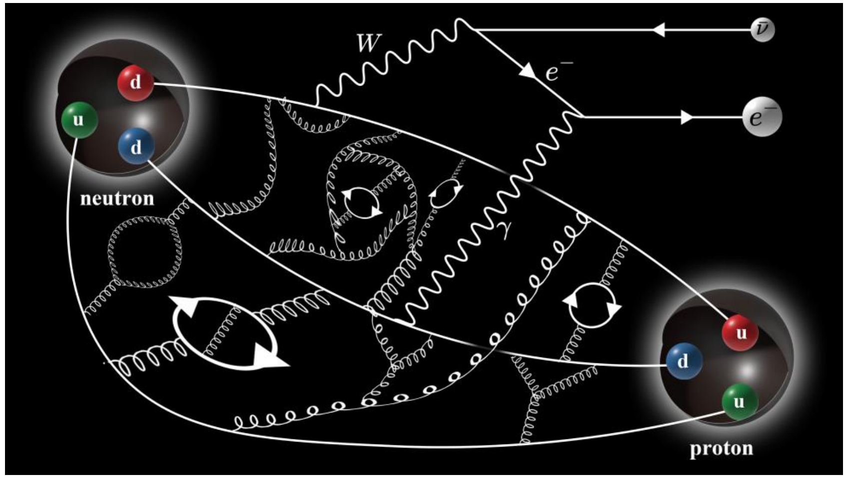
National Natural Science Foundation of China

Lattice Flavor Physics

- In flavor physics, LQCD and pQCD work together
  - pQCD used for short distance: Wilson coefficients
  - LQCD used for long distance: hadronic matrix elements



- The role played by lattice QCD is irreplaceable
  - High-order pQCD calculation is challenging: QED up to 5 loop (e.g. g-2); QCD up to NNNLO
  - More is different——P. W. Anderson

$$\underbrace{0+0+0+0+\cdots+0+0+0}\neq 0$$

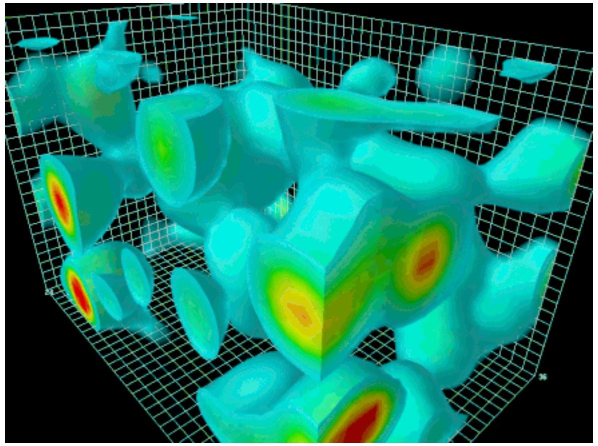
An infinite sum of zeros can be nonzero

Perturbative and nonperturbative regimes are intrinsically different

For example, QCD vacuum is nonperturbative and has chiral symmetry spontaneously breaking

  - Lattice QCD simulates QCD vacuum structure

——nontrivial topological charge density fluctation



Outline

- Test of first-row CKM unitarity

- Inclusion of isospin breaking effects

- Rare decays

Test of CKM unitarity

- In SM, CKM matrix is unitary, describing the strength of flavor-changing weak interaction

Cabibbo Kobayashi Maskawa

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

- Most stringent test of CKM unitarity is given by the first row condition

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

[PDG 2022]

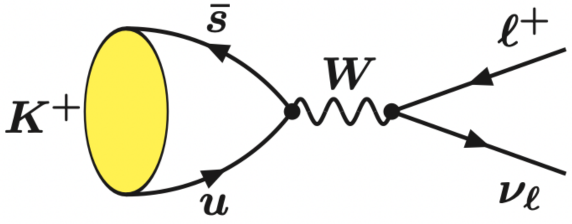
- $|V_{ub}|=3.82(24)\times 10^{-3}$, tiny contribution
- $|V_{ud}|=0.97373(31)$, most precise determination from superallowed nuclear beta decays
  (also from neutron & π beta decays, but uncertainties are 3 and 10 times larger)
- $|V_{us}|$, most precise determination from kaon decays (K_{l3} + $K_{\mu 2}/\pi_{\mu 2}$) ⟹ requires LQCD inputs
  (also from hyperon & tau decays, errors are about 3 and 2 times)

K/π systems provide idea laboratory for lattice QCD Study

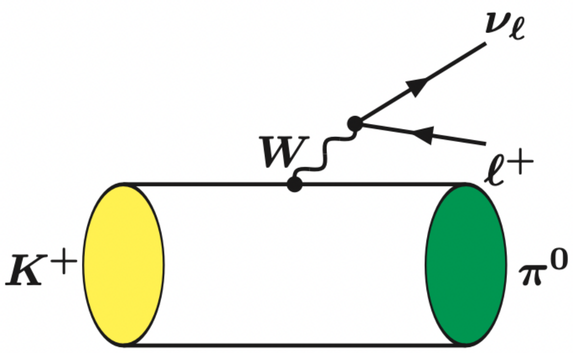
➢ Lattice QCD is powerful to study Kaon/pion decays

- Nearly no signal/noise problem
- Quark field contractions easily performed
- Simple final states: purely leptonic, 1 π， 2 π (K→ππ already very challenging!)
- Small recoil for hadronic particle in the final state
- Long-distance processes: much less low-lying intermediate states

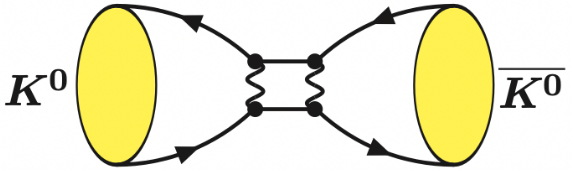
➢ Provide the hadronic matrix elements for precision SM tests



$K_{\ell 2} \to f_K$



$K_{\ell 3} \to f_+(0)$



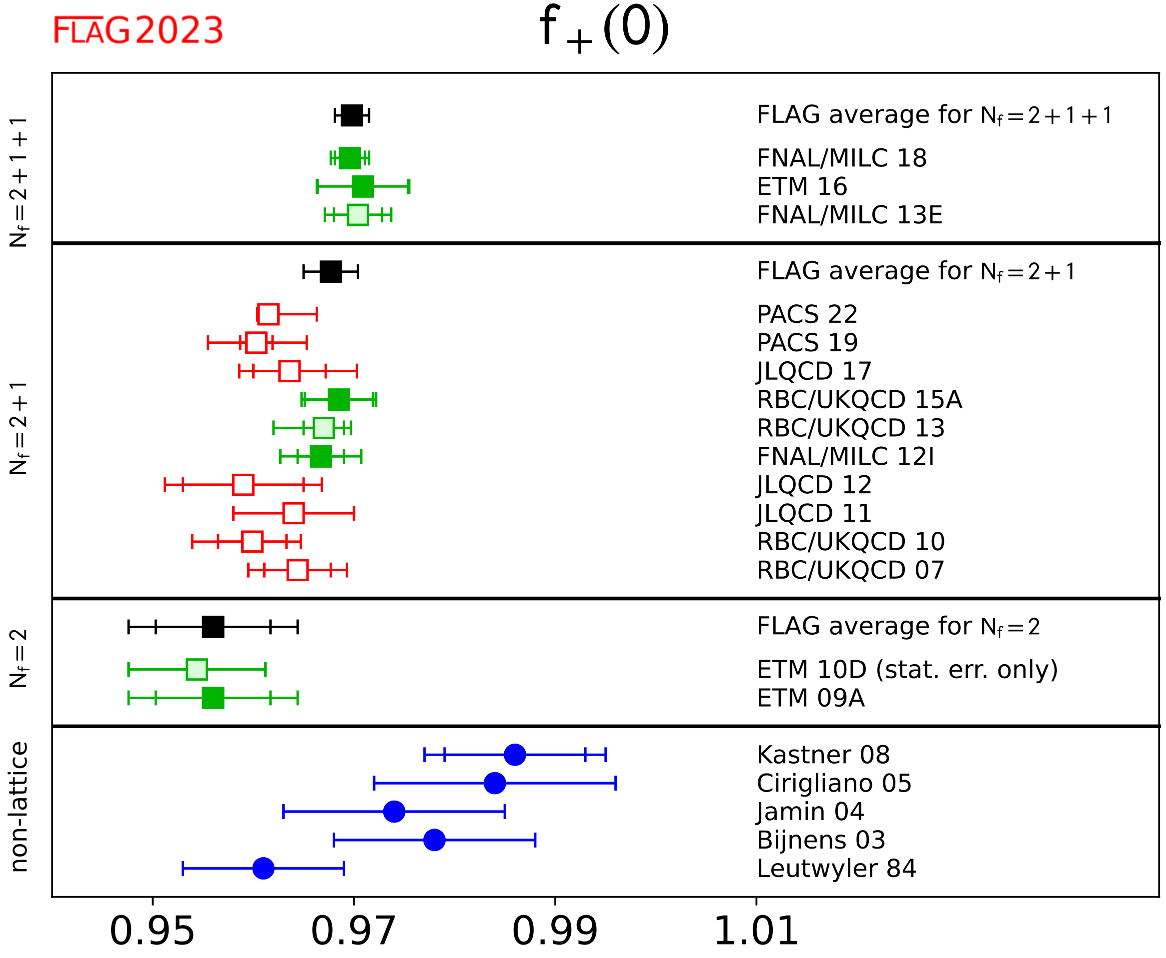
K^0-$\overline{K^0}$ mixing $\to B_K$

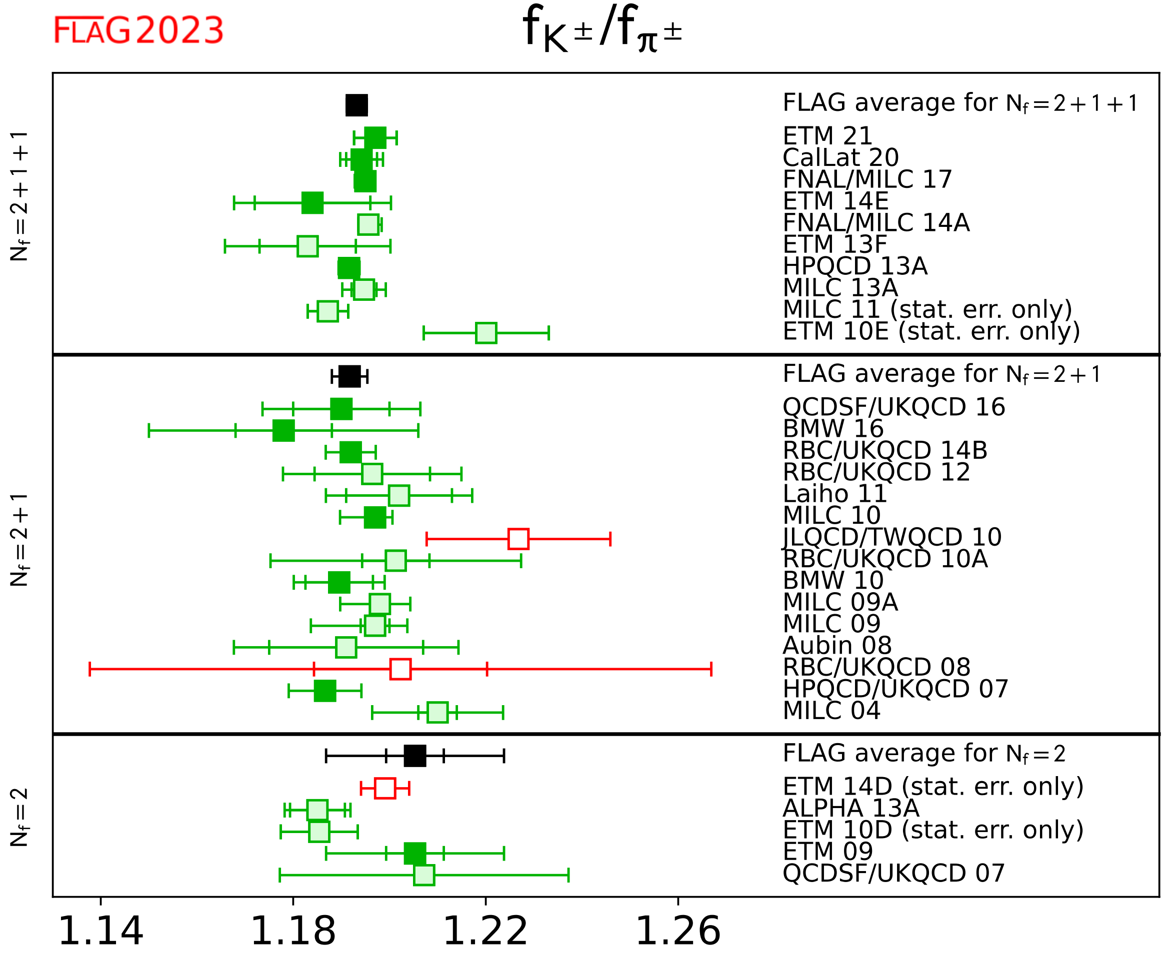
Leptonic and semileptonic decays

- Flavor Lattice Averaging Group (FLAG) average, updated on 2023

$$f_+^{K\pi}(0) = 0.9698(17) \quad \Rightarrow \quad 0.18\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1934(19) \quad \Rightarrow \quad 0.16\% \text{ error}$$



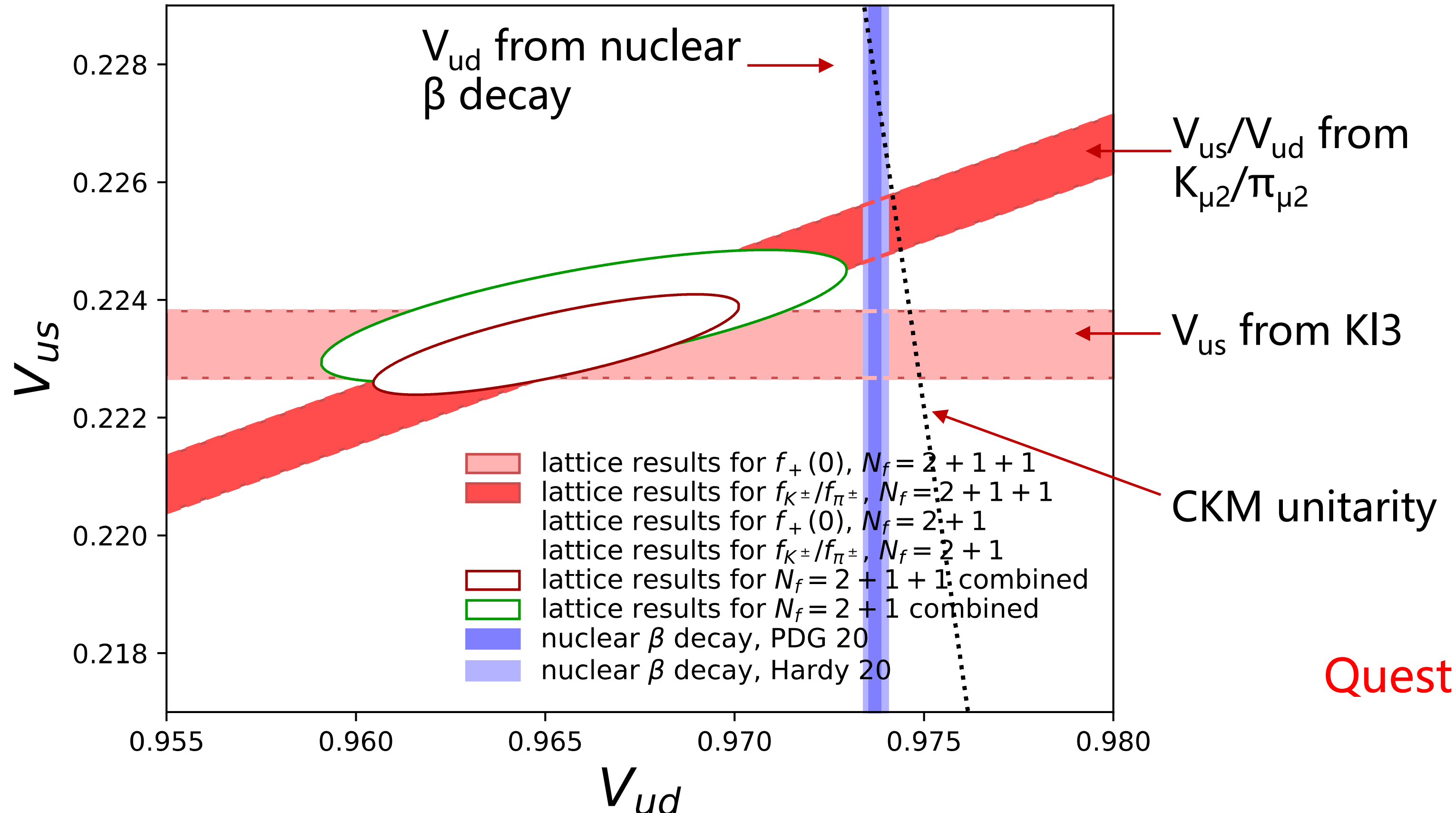


Extraction of V_{ud} and V_{us}

- Experimental information from kaon decays [arXiv:1411.5252, 1509.02220]

$$K_{\ell 3} \Rightarrow |V_{us}| f_+(0) = 0.2165(4) \quad \Rightarrow \quad |V_{us}| = 0.2232(6)$$

$$K_{\mu 2}/\pi_{\mu 2} \Rightarrow \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4) \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| = 0.2313(5)$$



- Use $|V_{us}|$ from K_{l3} + $|V_{us}/V_{ud}|$ from $K_{\mu2}/\pi_{\mu2}$ (more accurate results from N_f=2+1+1)

$$|V_u|^2 = 0.9816(64) \quad \Rightarrow \quad 2.9\ \sigma$$

- Use $|V_{us}|$ from K_{l3} + $|V_{ud}|$ from β decays

$$|V_u|^2 = 0.99800(65) \quad \Rightarrow \quad 3.1\ \sigma$$

- $|V_{us}/V_{ud}|$ from $K_{\mu2}/\pi_{\mu2}$ + $|V_{ud}|$ from β decay

$$|V_u|^2 = 0.99888(67) \quad \Rightarrow \quad 1.7\ \sigma$$

Question: Deviation due to $|V_{ud}|$ from β decays, $|V_{us}|$ from K_{l3} or new physics?

CKM matrix elements quoted by PDG 2022

- Use $|V_{us}/V_{ud}|$ from $K_{\mu2}/\pi_{\mu2}$ + $|V_{ud}|$ from β decay to determine $|V_{us}|$

$$|V_{us}| = 0.2255(8)\ (N_f = 2+1,\ K_{\mu 2}\text{ decays})$$
$$= 0.2252(5)\ (N_f = 2+1+1,\ K_{\mu 2}\text{ decays})$$

- Use $|V_{us}|$ from K_{l3}

$$|V_{us}| = 0.2236(4)_{\text{exp+RC}}(6)_{\text{lattice}}\ (N_f = 2+1,\ K_{\ell 3}\text{ decays})$$
$$= 0.2231(4)_{\text{exp+RC}}(4)_{\text{lattice}}\ (N_f = 2+1+1,\ K_{\ell 3}\text{ decays})$$

2.7 σ

- Average yields

$$|V_{us}| = 0.2244(5) \quad N_f = 2+1$$
$$|V_{us}| = 0.2243(4) \quad N_f = 2+1+1$$

- Enlarge the error by a scale factor of 2.7 and average N_f=2+1 and N_f=2+1+1 values

$$|V_{us}| = 0.2243(8) \quad \Rightarrow \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)(4).$$

Conservative estimate of $|V_{us}|$ due to the deviation between K_{l3} and $K_{\mu2}$ ⇒ 2.1 σ deviation

Role played by V_{ud}

- Interesting to review the deviation from CKM unitarity changes within recent years

$$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

- PDG 2019 → PDG 2020 → PDG 2022

	PDG 2019	PDG 2020	PDG 2022
$\lvert V_{ud}\rvert$	0.97420(21)	0.97370(14)	0.97373(31)
$\lvert V_{us}\rvert$	0.2243(5)	0.2245(8)	0.2243(8)
$\lvert V_{ub}\rvert$	0.00394(36)	0.00382(24)	0.00382(20)
Δ_{CKM}	−0.00061(47)	−0.00149(45)	−0.00152(70)

- 2020 update: 3.3 σ deviation from CKM unitarity due to the update of EWR corrections
- 2022 update: 2.1 σ deviation only

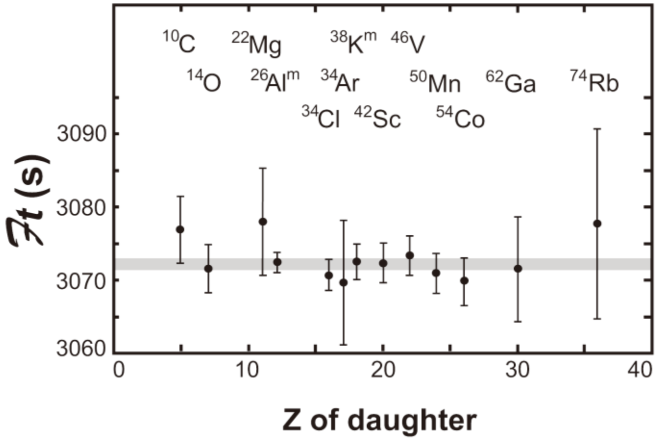
For V_{ud}, central value nearly unchanged, but uncertainty becomes twice larger

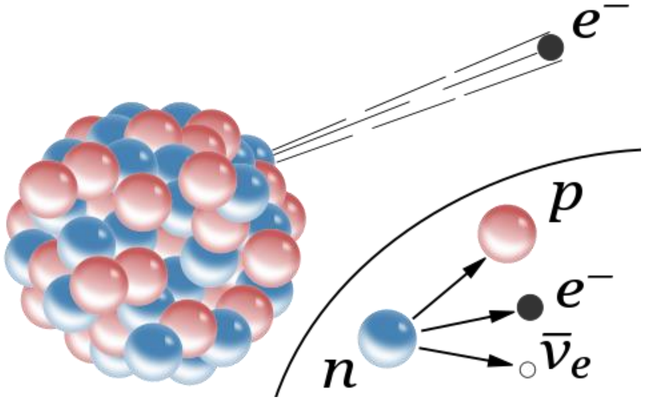
➡ A more conservative estimate of nuclear structure uncertainties

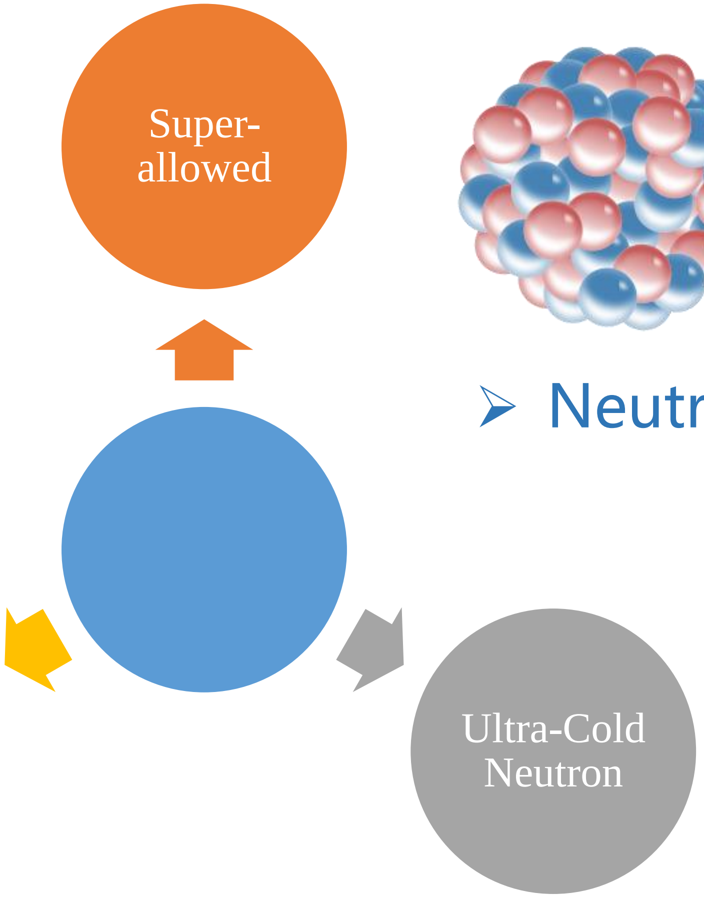
[M. Gorchtein, PRL123 (2019) 042503]

V_{ud} from different measurements

➢ Superallowed nuclear β decays

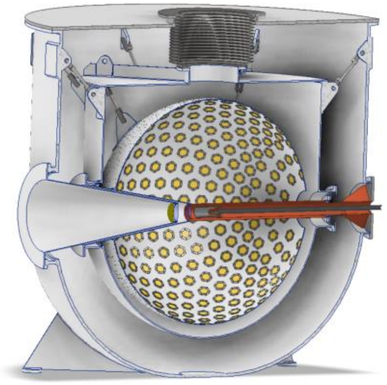






➢ Neutron β decays

➢ Pion β decays



Important uncertainty from γW box diagram

Superallowed nuclear β decays

$$|V_{ud}|^2 = 0.97154(22)(54)_{\mathrm{NS}}/(1+\Delta_{\mathrm{R}}^{\mathrm{V}})$$

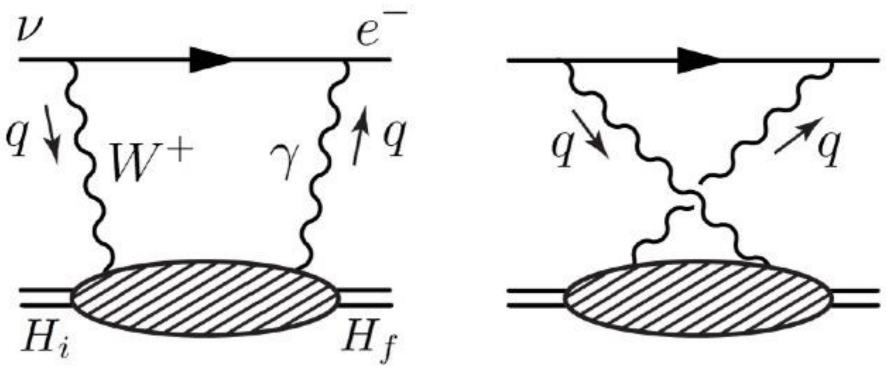
Neutron β decays

$$|V_{ud}|^2 = \frac{5024.7\ \mathrm{s}}{\tau_n(1+3g_A^2)(1+\Delta_R^V)}$$

Universal electroweak radiative corrections (EWR)

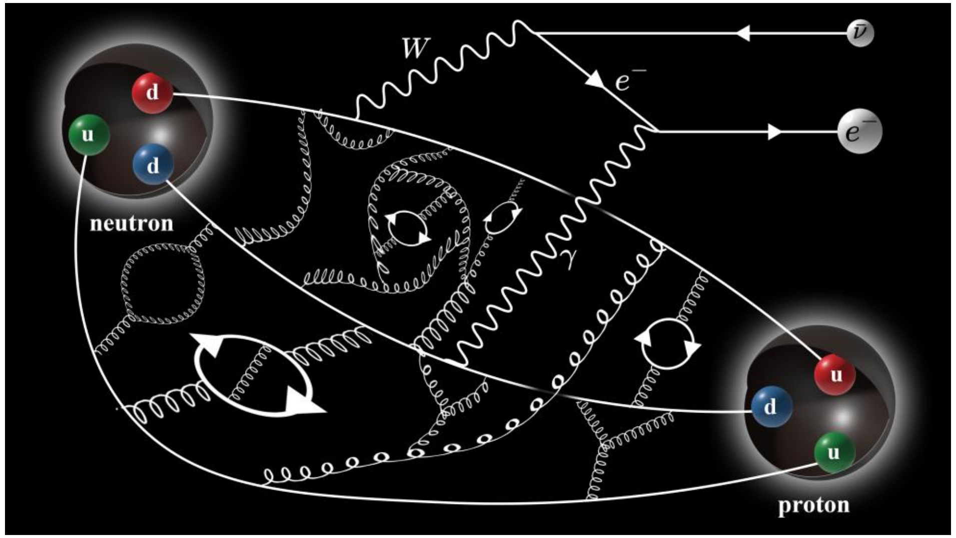
- Based current algebra, only axial γW box diagram is sensitive to hadronic scale

[A. Sirlin, Rev. Mod. Phys. 07 (1978) 573



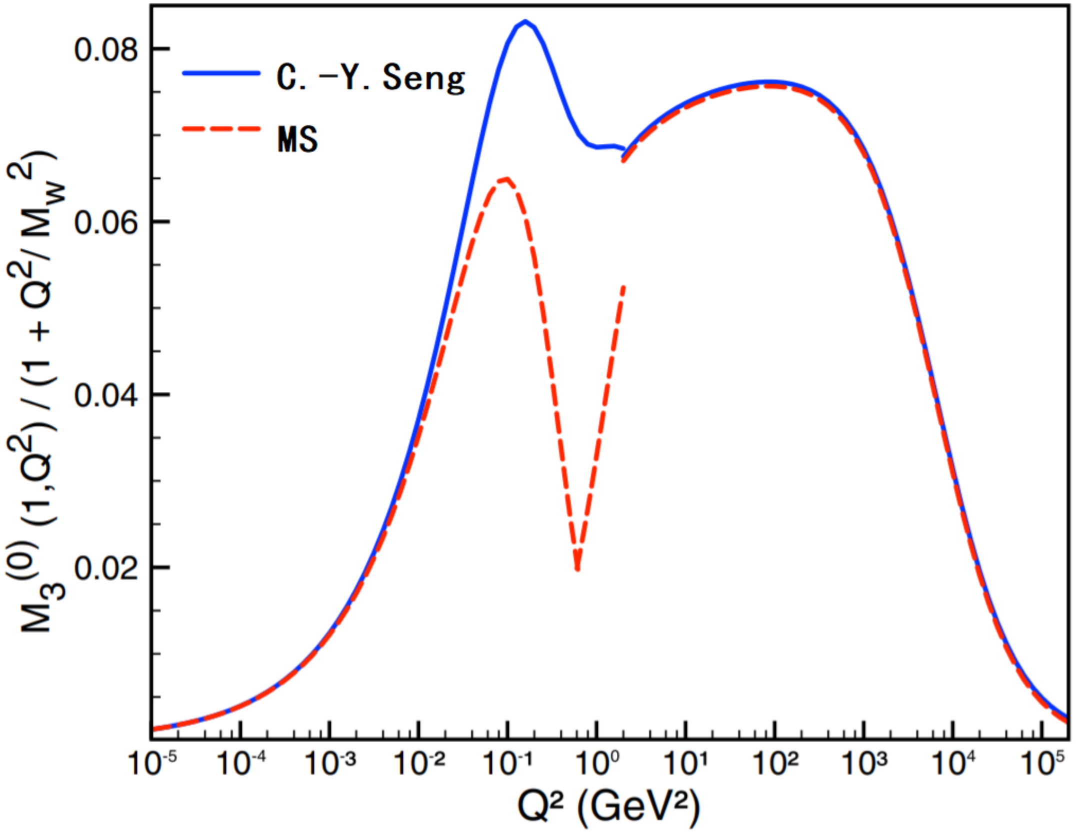
$$\square_{\gamma W}^{VA} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_n(Q^2).$$

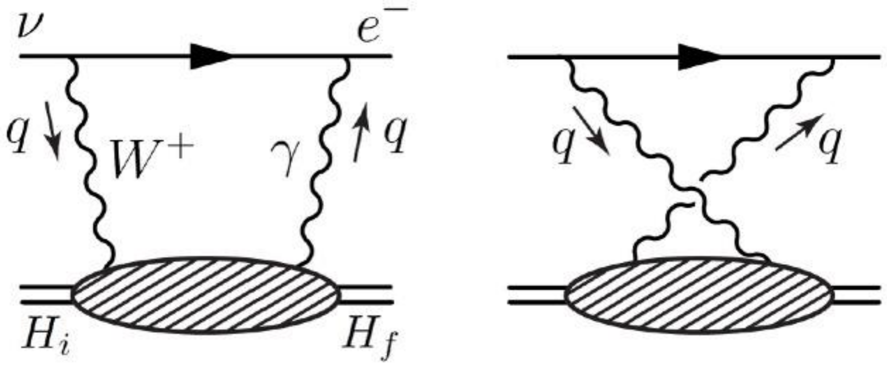
It dominates the uncertainties in EWR



Important uncertainty from γW box diagram

$$\Box^{VA}_{\gamma W} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_n(Q^2).$$





- PDG 2019 → PDG 2020

	PDG 2019	PDG 2020
$\lvert V_{ud}\rvert$	0.97420(21)	0.97370(14)
$\lvert V_{us}\rvert$	0.2243(5)	0.2245(8)
$\lvert V_{ub}\rvert$	0.00394(36)	0.00382(24)
$\Delta_{\rm CKM}$	−0.00061(47)	−0.00149(45)

It is responsible for the update of PDG and 3.3 σ deviation in CKM unitarity

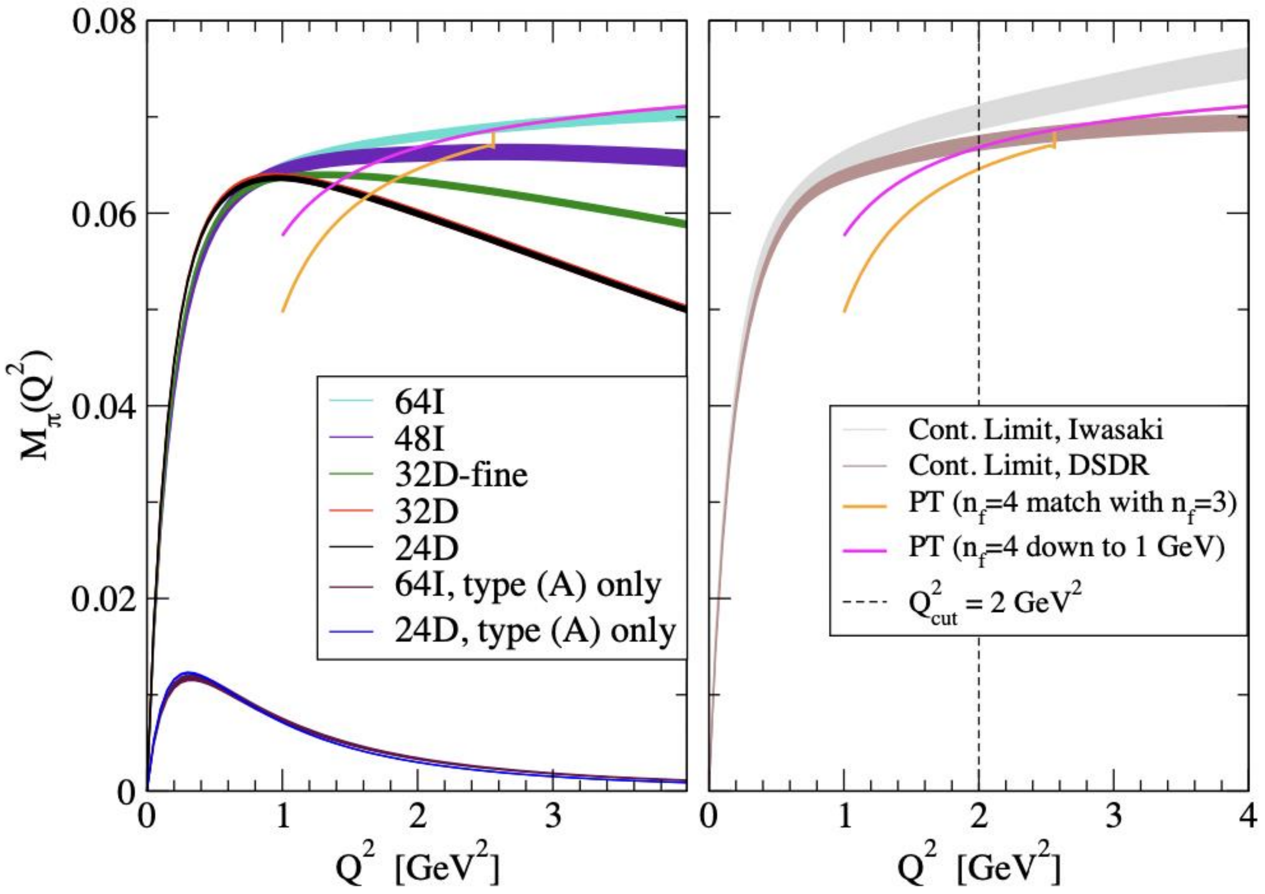
[1] Marciano & Sirlin, PRL96, 032002 (2006)
[2] Seng et.al. PRL 121, 241804 (2018)

Calculation of γW box diagram from lattice QCD

➢ Use pion β decay to design the calculation strategy

5 DWF ensembles @ physical pion mass

ensemble	M_π/MeV	$L^3 \times T$	a/fm
24D	141.2(4)	$24^3 \times 64$	0.1944
32D	141.4(3)	$32^3 \times 64$	0.1944
32D-fine	143.0(3)	$32^3 \times 64$	0.1432
48I	135.5(4)	$48^3 \times 96$	0.1140
64I	135.3(2)	$64^3 \times 128$	0.0836



- For pion decay, originally use EFT with LECs

Reduce the hadronic uncertainty by a facor of 10

XF, M. Gorchtein, L. Jin, et.al. PRL124 (2020) 19, 192002

Interplay between theory and experiment

- V_{ud} from π β decay

$$|V_{ud}| = 0.9740(28)_{\rm exp}(1)_{\rm th}$$

XF, M. Gorchtein, L. Jin, et.al.
PRL124 (2020) 19, 192002

- Main uncertainty arises from exp. measurements

which is normalized using the very precisely measured $BR(\pi^+ \to e^+\nu_e(\gamma)) = 1.2325(23) \times 10^{-4}$ [7], rather than the theoretical branching ratio of $1.2350(2) \times 10^{-4}$, which if used, would increase $|V_{ud}|$ to 0.9749(27). Theoretical uncertainties in pion beta decay are very small [21], leaving open more than an order of magnitude improvement of its experimental branching ratio before theory uncertainties become a problem. Although challenging, improved measurements of pion beta decay currently under discussion would allow this decay mode to compete with superallowed beta decays and future neutron decay efforts for the most precise direct $|V_{ud}|$ determination.

PDG 2022, reviewed by E. Blucher & W. J. Marciano

- Past Experiment - PIBETA

D. Pocanic et.al. PRL 93 (2004) 181803

- Precision 0.6%

- New Experiment - PIONEER

M. Hoferichter, arXiv:2403.18889

Phase I： π leptonic decays

Phase II+III： π β decays

- Ultimate precision 3×10^{-4} , 20 times better than PIBETA

Future exp. uncertainty comparable to theoretical one!

Status for V_{ud}

- Superallowed β decays $|V_{ud}|=0.9737(3)$
  - $0^+ \rightarrow 0^+$ nuclear beta decays, which are pure vector transition at leading order
  - Estimate of nuclear structure uncertainties is important
- Neutron β decays $|V_{ud}|=0.9737(9)$
  - Free from nuclear structure uncertainties
  - Nuclear-structure independent radiative correction (RC) is same as superallowed nuclear β decay
- Pion semileptonic β decays $|V_{ud}|=0.9739(29)$
  - More difficult to measure pion decays
  - Theoretically simpler, especially for lattice QCD

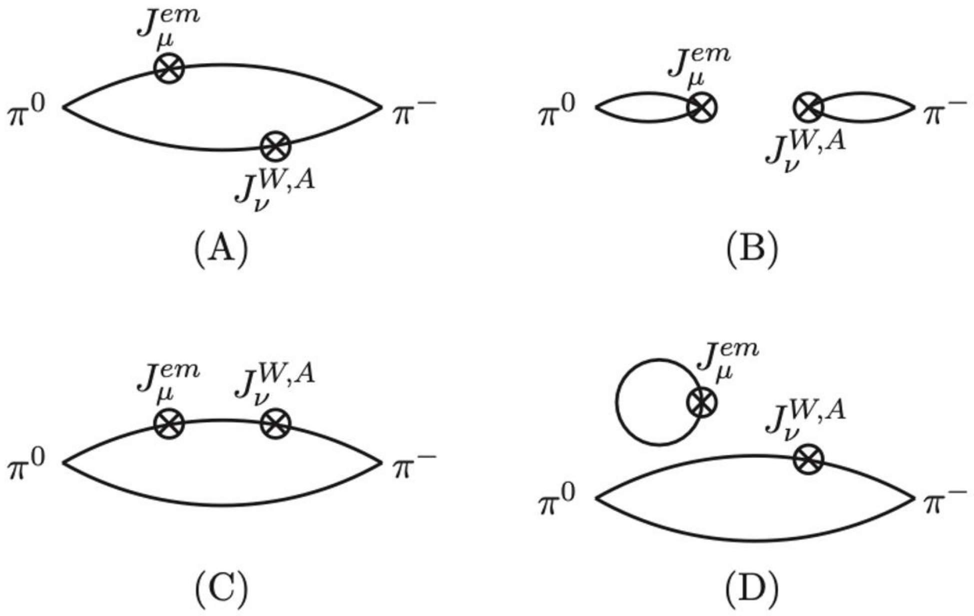
◆ Summary

- To extract V_{ud} from superallowed decay or neutron β decay

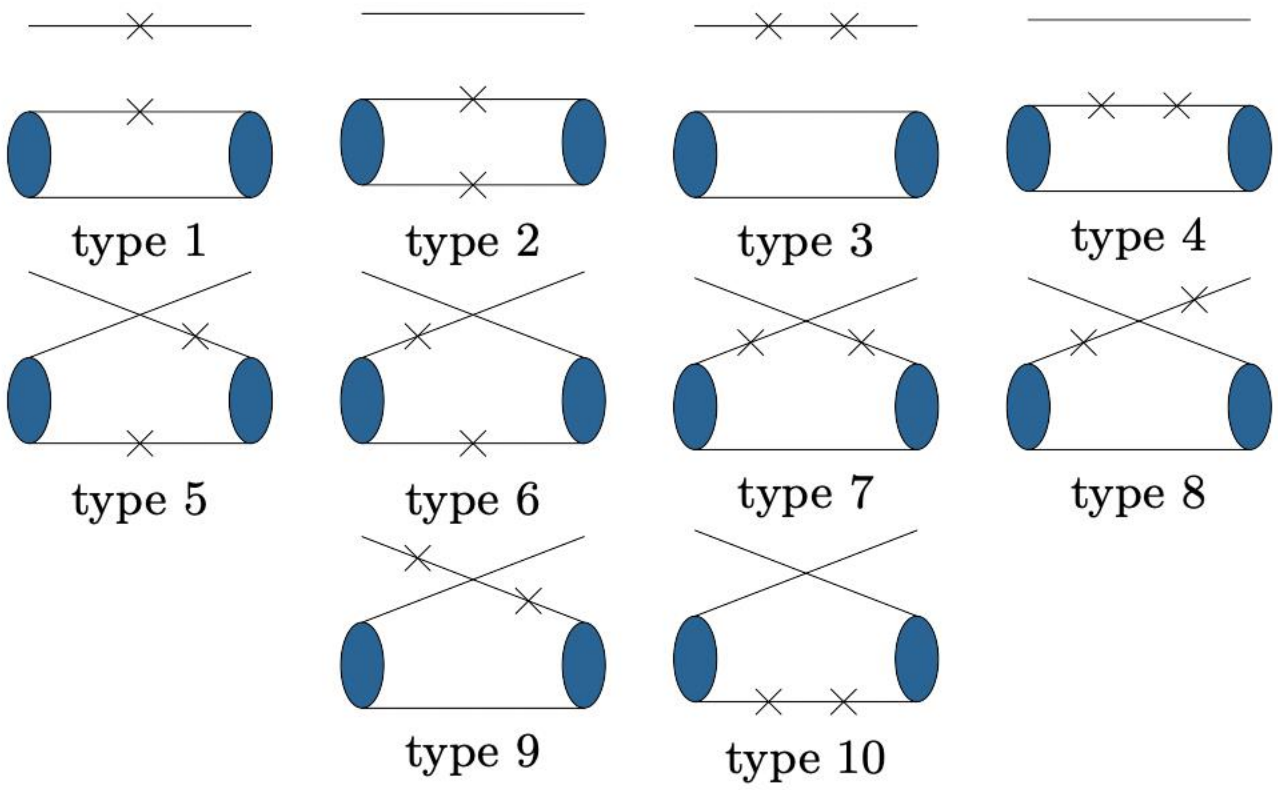
  ⇒ Need a well determined EW radiative corrections

From π to nucleon sector

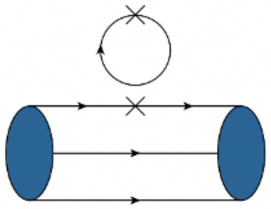
- π γW box diagram



- Nucleon γW box diagram
  - Connected diagram (8 of 10)



  - Disconnected diagram



Vanish in flavor SU(3) limit, a O(1%) effect

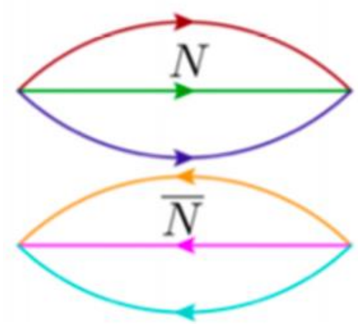
From π to nucleon sector

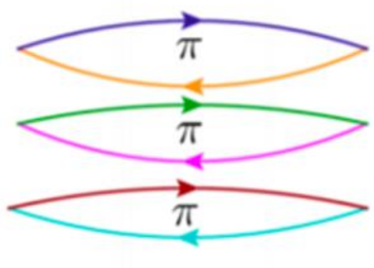
- Nucleon system – severe signal/noise (S/N) problem

  - Statistics tells us that variance is given by $\langle O^2\rangle - \langle O\rangle^2$

Square of signal

Variance is dominated by $\langle O^2\rangle$

$\langle O\rangle^2 =$ 

$\langle O^2\rangle =$ 

- S/N is $\exp\left[-(M_N - \frac{3}{2}M_\pi)t\right]$ → γW box diagram requires 4-pt correlation function and thus large *t* separation

It is essentially a sign problem!

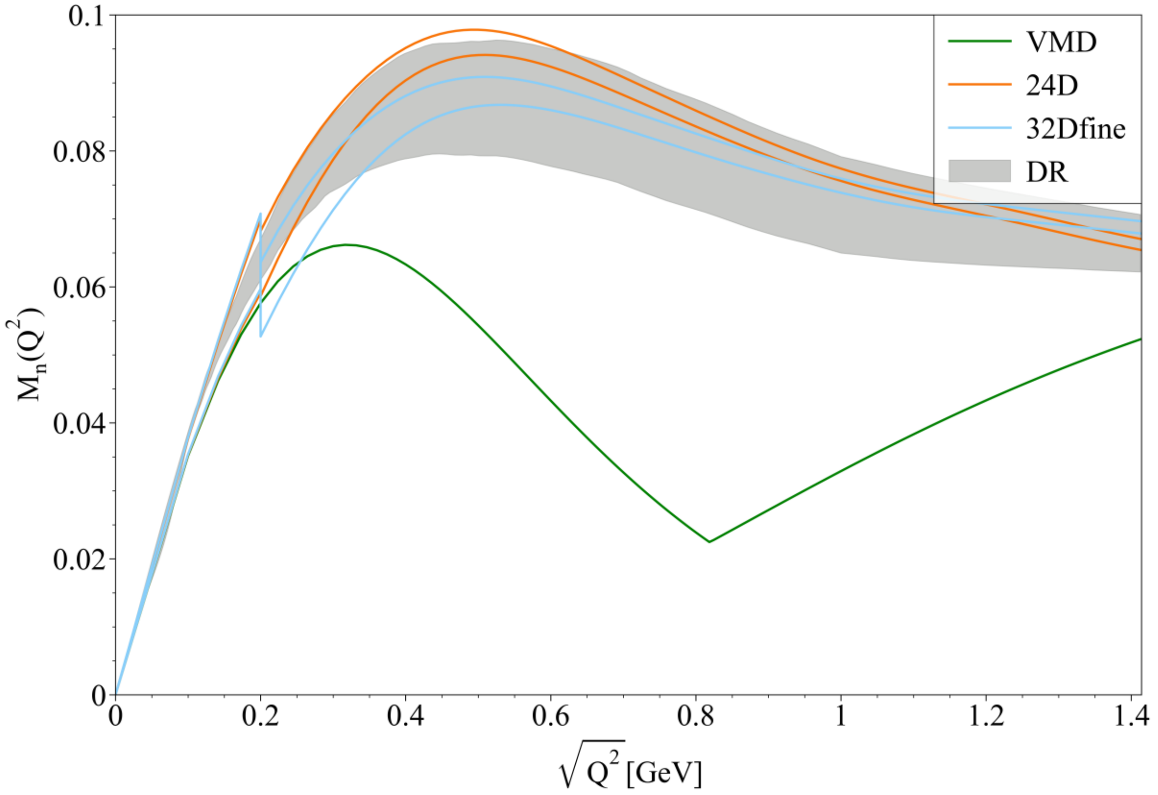
γW box diagram in neutron β decay

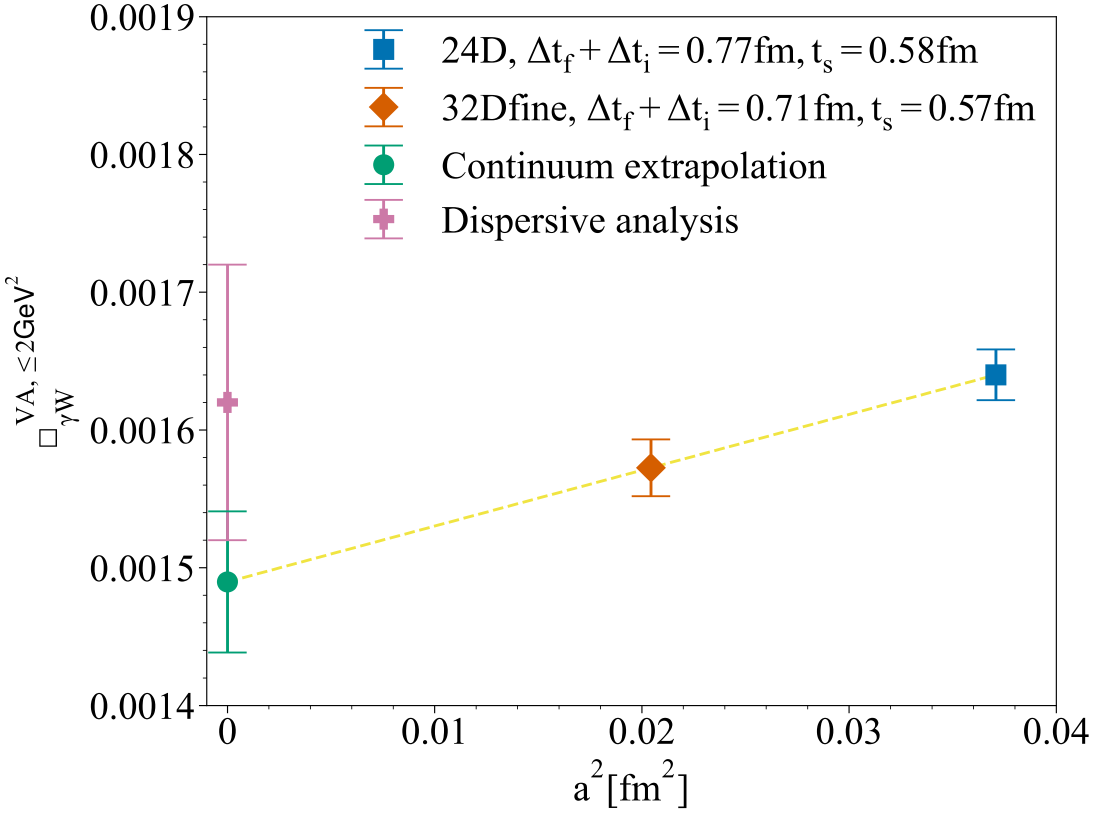
- Ensemble information

Ensemble	m_π [MeV]	L	T	a^{-1} [GeV]	N_{conf}
24D	142.6(3)	24	64	1.023(2)	207
32D-fine	143.6(9)	32	64	1.378(5)	69

- Numerical lattice results

P. Ma, XF, M. Gorchtein, L. Jin, C. Seng, Z. Zhang, PRL132 (2024) 191901





Using lattice input, deviation from CKM unitarity: 2.1 σ → 1.8 σ

From π to K sector

- For π and neutron β decays， initial/final-state hadron has nearly the same mass

  ➡ only axial γW box diagram is sensitive to hadronic scale

- For K_{l3} decays, LQCD needs to calculate all the diagrams, not only just γW box diagram!

- Idea is to combine LQCD with ChPT [C. Seng, XF, M. Gorchtein, L. Jin, U.-G. Meißner, JHEP 10 (2020) 179

  - Use ChPT to determine EWR correction

$$\delta_{\rm em}^{K^\pm} = 2e^2\left[-\frac{8}{3}X_1 - \frac{1}{2}\tilde{X}_6^{\rm phys}(M_\rho) - 2K_3^r(M_\rho) + K_4^r(M_\rho) + \frac{2}{3}K_5^r(M_\rho) + \frac{2}{3}K_6^r(M_\rho)\right]$$

$$\delta_{\rm em}^{K^0} = 2e^2\left[\frac{4}{3}X_1 - \frac{1}{2}\tilde{X}_6^{\rm phys}(M_\rho)\right] + \cdots$$

  ➡ still requires LECs X_1 and $\tilde{X}_6^{\rm phys}$

  - Use LQCD to calculate EWR at flavor SU(3) limit with m_s=m_u=m_d

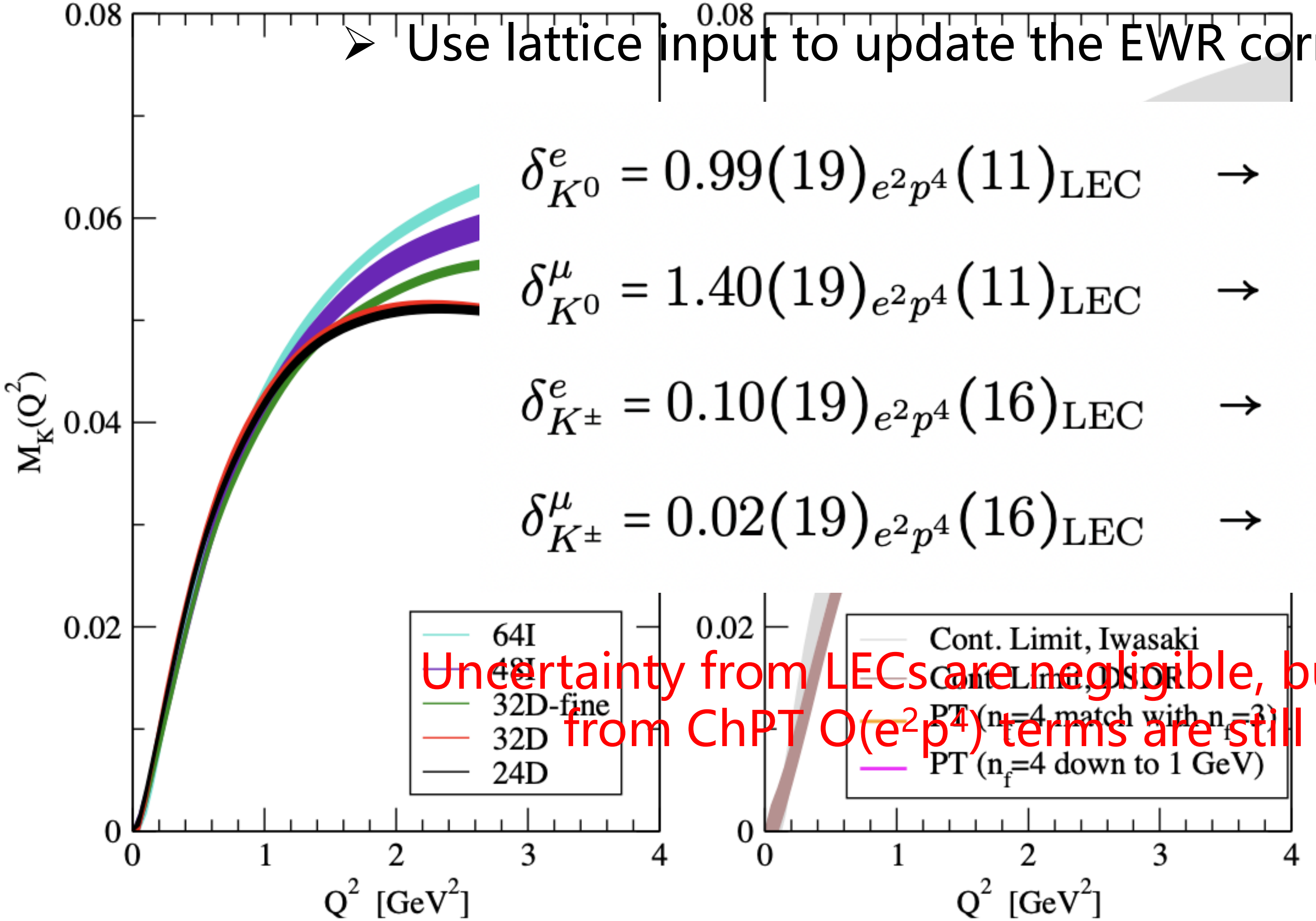
  ➡ provide LECs, which are independent of quark masses

Axial γW-box diagram contribution to $K^0 \to \pi^+$ decays

$$\Box^{VA}_{\gamma W}\big|_H = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_H(Q^2)$$

[P. Ma, XF, M. Gorchtein, L. Jin, C. Seng, PRD103 (2021) 114503]

Calculation is performed in the flavor $SU(3)$ limit with $m_K = m_\pi$



- SU(3) K^0 decay

$-\frac{8}{3}X_1 + \tilde{X}_6^{\text{phys}} = 0.0197(10)$ for $K^0 \to \pi^+$

ay

hys $= 0.0110(6)$ for $\pi^- \to \pi^0$

ne LECs

$\times 10^{-3}$, $\tilde{X}_6^{\text{phys}} = 13.9(7) \times 10^{-3}$

imal resonance model

$X_1 = -3.7(3.7) \times 10^{-3}$ $\tilde{X}_6^{\text{phys}} = 10.4(10.4) \times 10^{-3}$

LECs are consistent, but error from lattice is much smaller

➢ Use lattice input to update the EWR correction

$$\delta^e_{K^0} = 0.99(19)_{e^2p^4}(11)_{\text{LEC}} \quad \to \quad 1.00(19)$$

$$\delta^\mu_{K^0} = 1.40(19)_{e^2p^4}(11)_{\text{LEC}} \quad \to \quad 1.41(19)$$

$$\delta^e_{K^\pm} = 0.10(19)_{e^2p^4}(16)_{\text{LEC}} \quad \to \quad -0.01(19)$$

$$\delta^\mu_{K^\pm} = 0.02(19)_{e^2p^4}(16)_{\text{LEC}} \quad \to \quad -0.09(19)$$

Uncertainty from LECs are negligible, but uncertainty from ChPT $O(e^2p^4)$ terms are still large

Outline

- Test of first-row CKM unitarity
- Inclusion of isospin breaking effects
- Rare decays

Inclusion of IB effects becomes important

- Flavor Lattice Averaging Group (FLAG) average, updated on 2023

$$f_+^{K\pi}(0) = 0.9698(17) \quad \Rightarrow \quad 0.18\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1934(19) \quad \Rightarrow \quad 0.16\% \text{ error}$$

- FLAG average results

  - Error < 1%

	N_f	FLAG average	Frac. Err.
f_K/f_π	$2+1+1$	1.1934(19)	0.16%
$f_+(0)$	$2+1+1$	0.9698(17)	0.18%
f_D	$2+1+1$	212.0(7) MeV	0.33%
f_{D_s}	$2+1+1$	249.9(5) MeV	0.20%
f_{D_s}/f_D	$2+1+1$	1.1783(16)	0.13%
$f_+^{DK}(0)$	$2+1+1$	0.7385(44)	0.60%
f_B	$2+1+1$	190.0(1.3) MeV	0.68%
f_{B_s}	$2+1+1$	230.3(1.3) MeV	0.56%
f_{B_s}/f_B	$2+1+1$	1.209(5)	0.41%

  - Error < 5%

	N_f	FLAG average	Frac. Err.
$\hat{B}_K$	$2+1$	0.7625(97)	1.3%
$f_+^{D\pi}(0)$	$2+1$	0.666(29)	4.4%
$\hat{B}_{B_s}$	$2+1$	1.35(6)	4.4%
B_{B_s}/B_{B_d}	$2+1$	1.032(38)	3.7%
$\cdots$			

Important to study the IB effects

Inclusion of IB effects becomes important

- Flavor Lattice Averaging Group (FLAG) average, updated on 2023

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$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1+\delta_K^\ell + \delta_{SU2}) C^2 \left|V_{us}\right|^2 f_+^2(0) I_K^\ell$$

$$\frac{\Gamma(K_{\mu 2})}{\Gamma(\pi_{\mu 2})} = \left|\frac{V_{us}}{V_{ud}} \frac{f_K^{(0)}}{f_\pi^{(0)}}\right|^2 \frac{M_\pi^3}{M_K^3} \left(\frac{M_K^2 - m_\mu^2}{M_\pi^2 - m_\mu^2}\right)^2 (1+\delta R_{K\pi})$$

Long-distance IB effects, ChPT provides a useful tool

Frontier for lattice QCD – inclusion of IB

- For K_{l3} decays [P. Ma, XF, M. Gorchtein, L. Jin, C. Seng, PRD103 (2021) 114503]
  - So far only a combined analysis with LQCD and ChPT
- For $K_{\mu 2}/\pi_{\mu 2}$ decays
  - 1^{st} calculation by RM123-SOTON collaboration @$m_\pi \approx 220$ MeV

LQCD		ChPT
$\delta R_{K\pi} = -1.26(14)\%$	vs	$\delta R_{K\pi} = -1.12(21)\%$
[PRL 2018, PRD 2019]		[Cirigliano & Neufeld, PLB 2011]

  - 2^{nd} calculation @m_π=139 MeV, $m_\pi L$=3.863

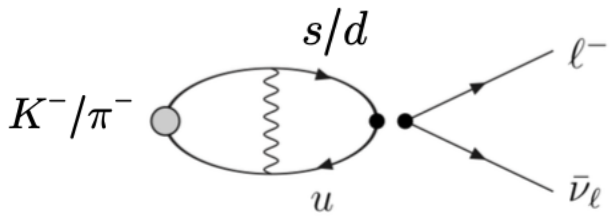
$$\delta R_{K\pi} = -0.0086\,(3)_{\text{stat.}}\,(^{+11}_{-4})_{\text{fit}}\,(5)_{\text{disc.}}\,(5)_{\text{quench.}}\,(39)_{\text{vol.}}$$

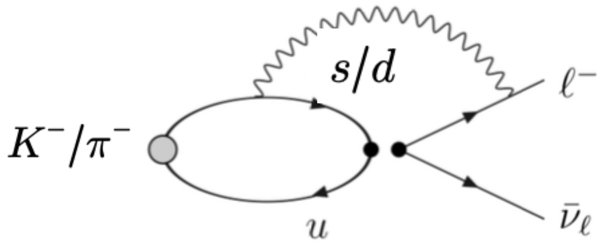
[P. Boyle et. al., JHEP 02 (2023) 242]

indicating large finite-volume effects

- O(1/L): universal and analytical known
- $O(1/L^2)$: structure dependent, found to be small
- $O(1/L^3)$: structure dependent, potentially large

Difficulties to include E&M effects





$m_\gamma=0$ ⟹ Long-range propagator enclosed in the lattice box

⟹ Power-law finite-volume effects

➢ Various methods proposed to treat photon on the lattice

- QED_L and QED_{TL} [Hayakawa & Uno, 2008, S. Borsany et. al., 2015]
- Massive photon [M. Endres et. al., 2016]
- C^* boundary condition [B. Lucini et. al., 2016]

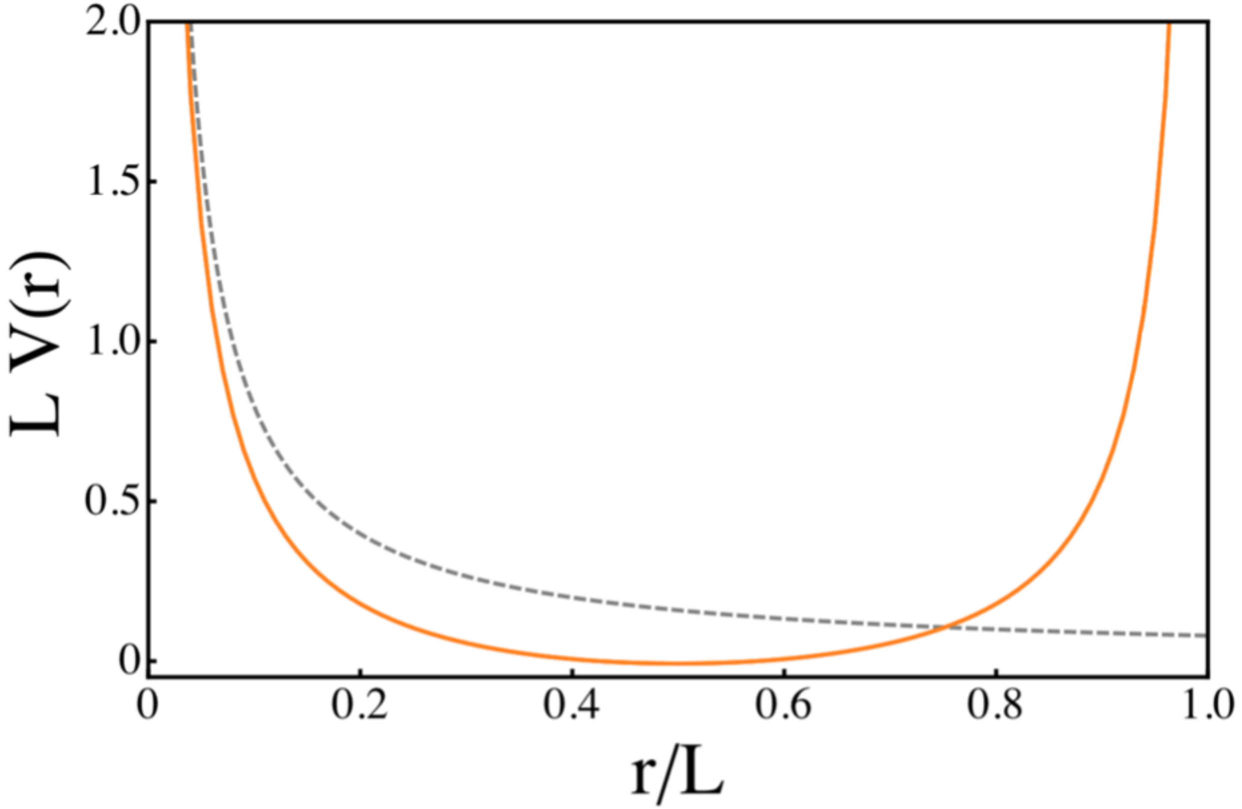
⟸ For leptonic decay, first two calculations use QED_L

Change photon propagator to make it suitable for lattice

Remove zero mode - QED_L

Infinite-volume propagator → Finite-volume propagator

$$S_\infty(x) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ipx}}{p^2} = \frac{1}{4\pi^2x^2} \quad \rightarrow \quad S_L(x) = \frac{1}{VT}\sum_p{}' \frac{e^{ipx}}{p^2}, \quad p = \frac{2\pi}{L}n \neq 0$$



Z. Davoudi, M.Savage
PRD90 (2014) 054503

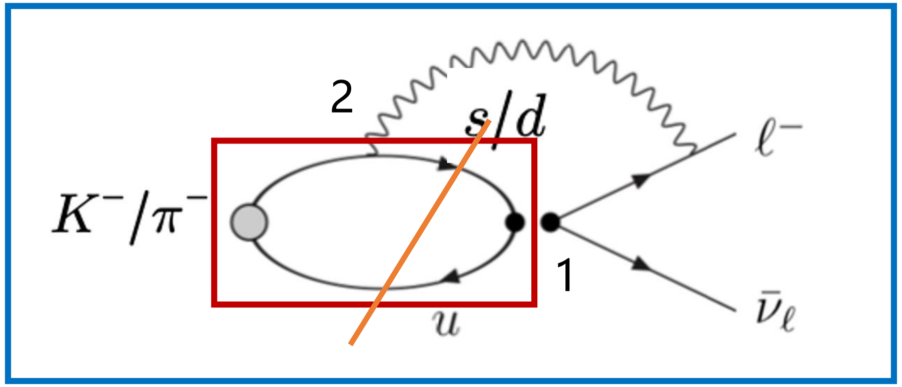
Power-law ($1/L^n$) finite-volume effect as lattice size L increases

Infinite-volume reconstruction

A new method proposed

➡ Exp. suppressed FV effects

XF, L. Jin, PRD100 (2019) 094509



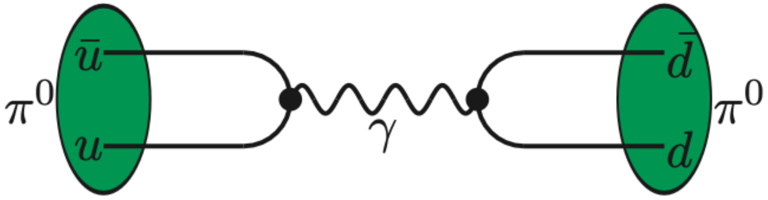
- QCD part is localized in a finite volume
- QED part is included analytically in the infinite volume
- Problem: QCD and QED parts do not match?

➡ Solution:

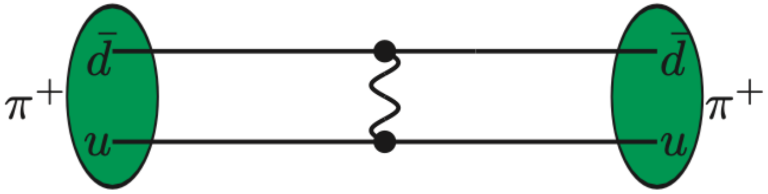
- Only when points 1 & 2 are separated with long distance, finite-volume effects become important
- At long distance, single-particle propagation between 1 & 2
- Reconstruct the infinite-volume single-particle propagation using the finite-volume one as input

Use QED self energy – pion mass splitting as an example

$m_{\pi^+} - m_{\pi^0}$:



type 1



type 2

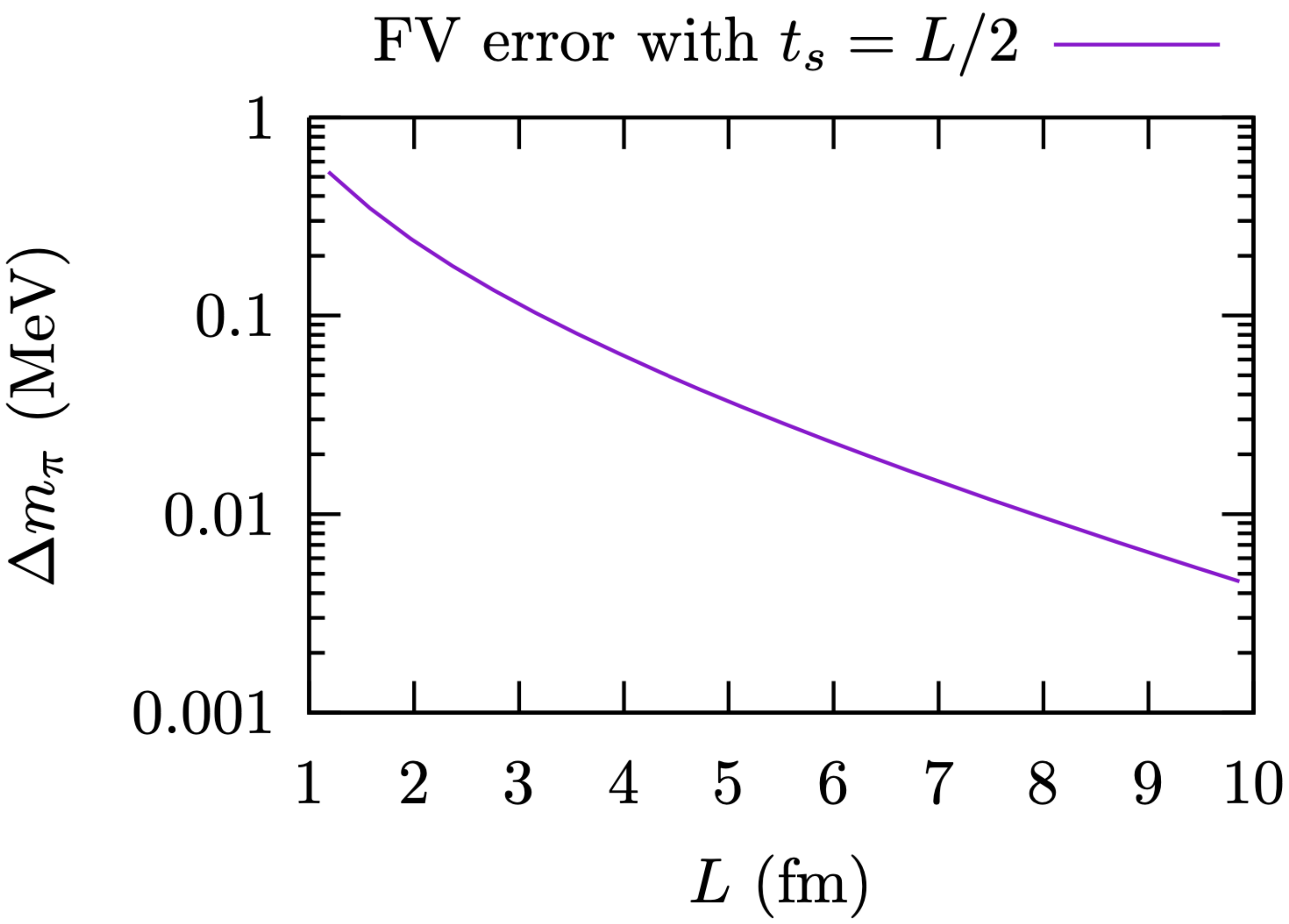
Isospin breaking effects: EM (α_e) + strong $\left(\frac{m_u - m_d}{\Lambda_{\mathrm{QCD}}}\right)$ contributions

Strong IB appear at $O\left(\left(\frac{m_u - m_d}{\Lambda_{\mathrm{QCD}}}\right)^2\right)$ ➡ Dominated by EM effects

Ideal testing ground to isolate the QED effects

Use QED self energy – pion mass splitting as an example

- Finite-volume effects mimicking by scalar QED



FV error exponentially suppressed

Use QED self energy – pion mass splitting as an example

➢ Numerical calculation XF, L. Jin, M. Riberdy, PRL128 (2022) 052003

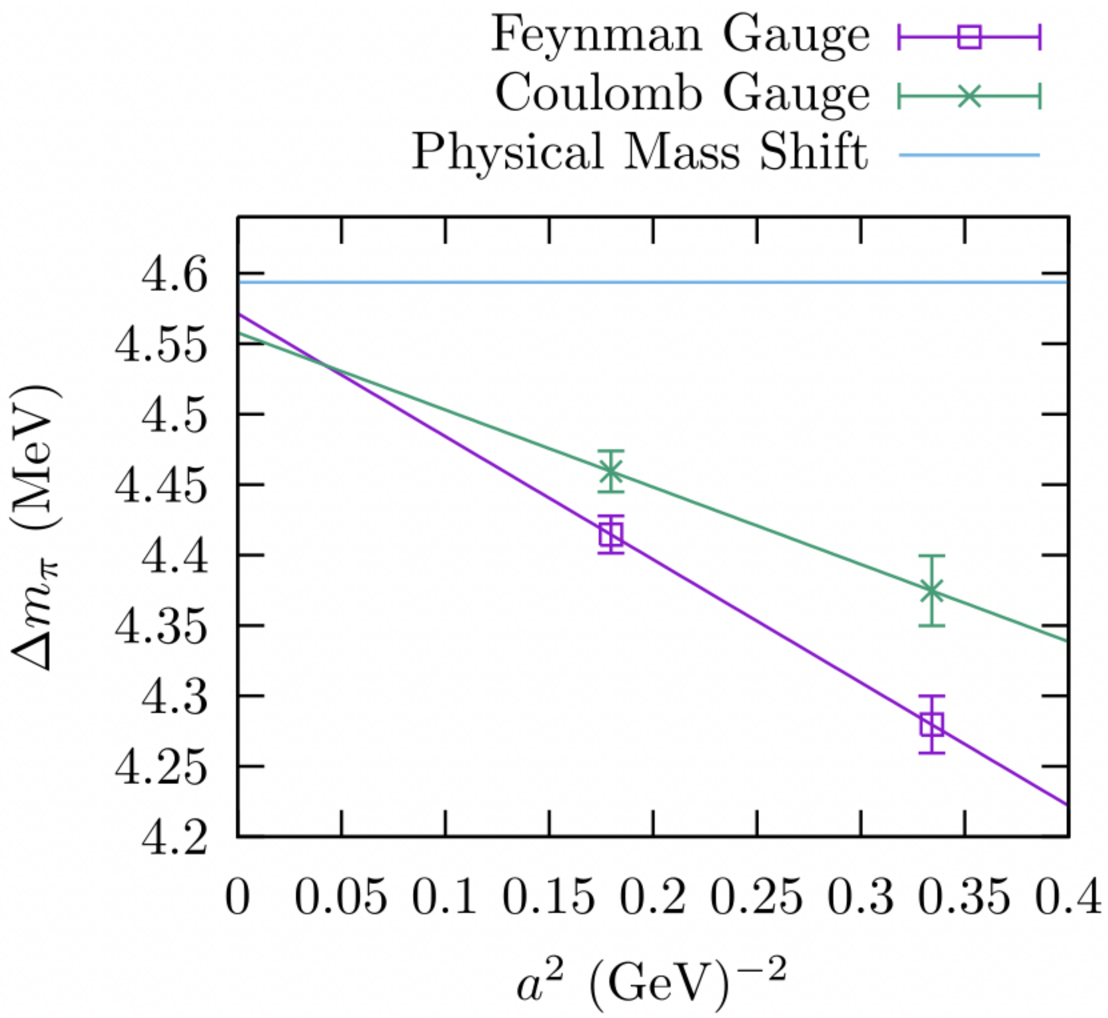


TABLE I. Previous lattice calculations of $m_{\pi^\pm} - m_{\pi^0}$ are compared to this Letter. Note $m_{\pi^\pm}$ is the charged pion mass m_{π^0} is the neutral pion mass

Reference	$m_{\pi^\pm} - m_{\pi^0}$ (MeV)
RM123 2013 [5]	$5.33(48)_{\rm stat}(59)_{\rm sys}$ [a]
R. Horsley *et al.* 2016 [7]	$4.60(20)_{\rm stat}$
RM123 2017 [9]	$4.21(23)_{\rm stat}(13)_{\rm sys}$
This Letter	$4.534(42)_{\rm stat}(43)_{\rm sys}$

Compared to previous studies, precision is 5-10 times improved

➢ Method extended from mass splitting to leptonic decay

N. Christ, XF, L. Jin, C. Sachrajda, T. Wang, PRD108 (2023) 014501

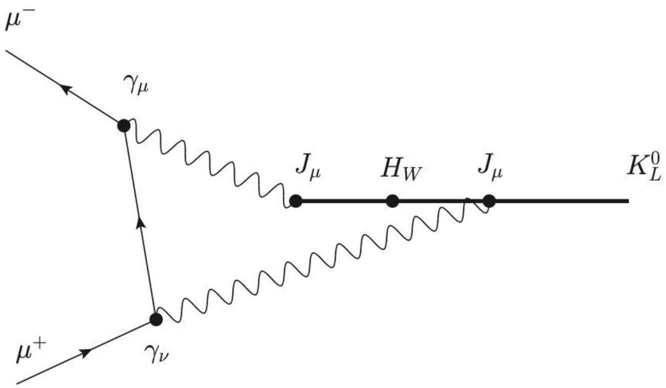
Numerical work is under going

Outline

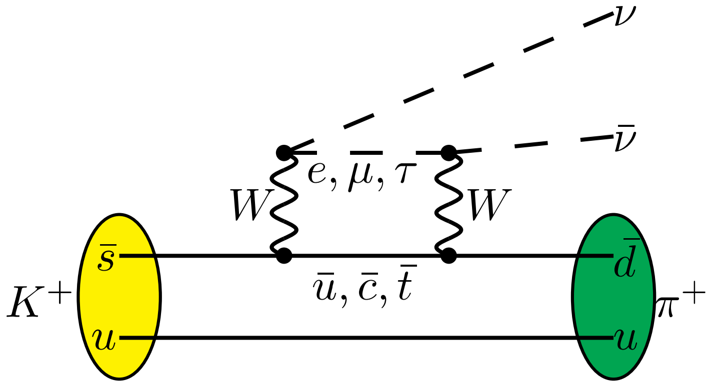
- Test of first-row CKM unitarity
- Inclusion of isospin breaking effects
- Rare decays

Interesting rare processes

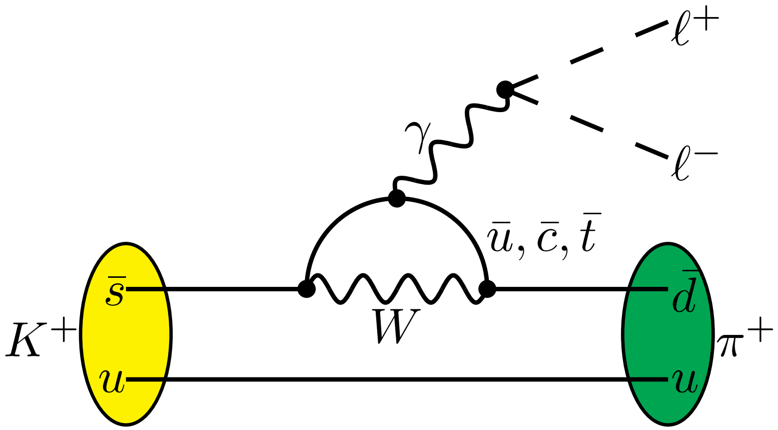
$K_L \to \mu^+\mu^-$: BR $= 6.84(11) \times 10^{-9}$



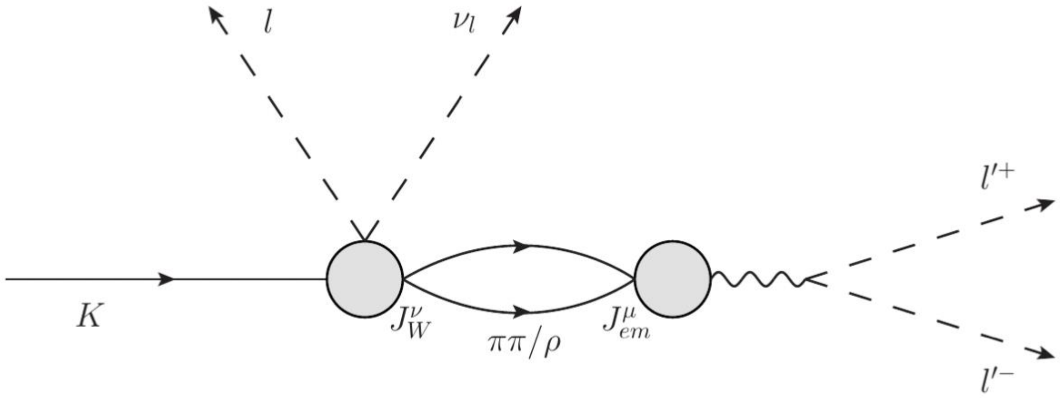
$K^+ \to \pi^+ \nu\bar{\nu}$: BR$=1.73^{+1.15}_{-1.05} \times 10^{-10}$



$K^+ \to \pi^+ e^+ e^-$: BR$=3.14(10) \times 10^{-7}$



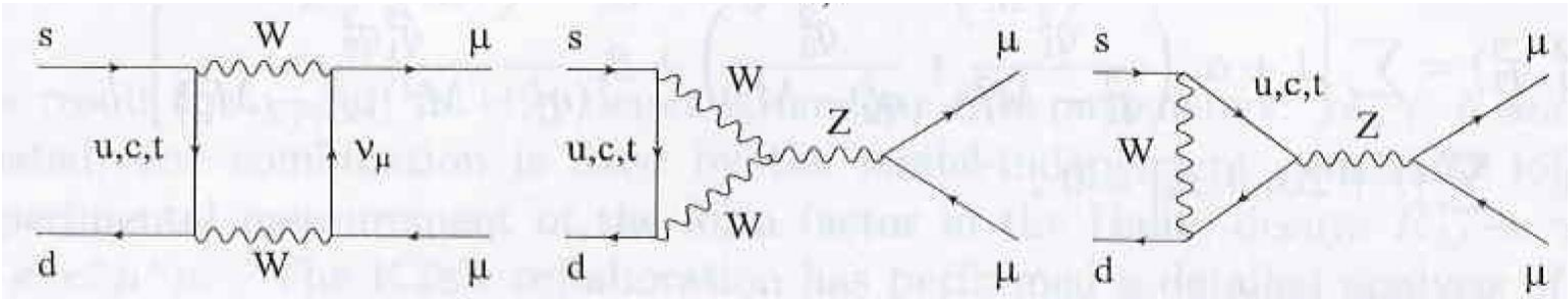
$K \to \ell \nu_\ell \ell'^+ \ell'^-$: BR $= O(10^{-8})$



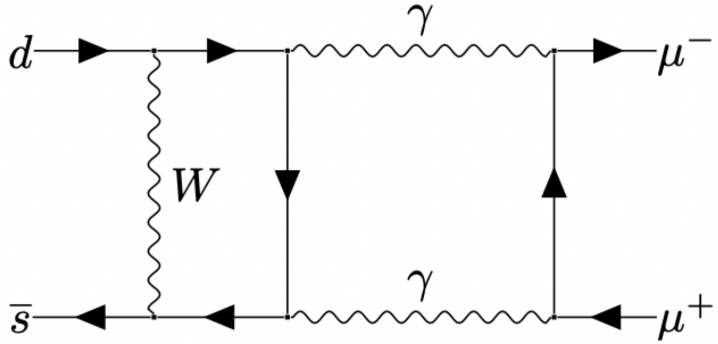
Interesting rare processes (1)

- In SM, $K_L \to \mu^+\mu^-$ is a FCNC process
  - SD contribution via W & Z boson exchange, contributes ~12% to BR

M. Gorbahn & U. Haisch, PRL97 (2006) 122002



  - LD contribution via two-photon exchange is nonperturbative



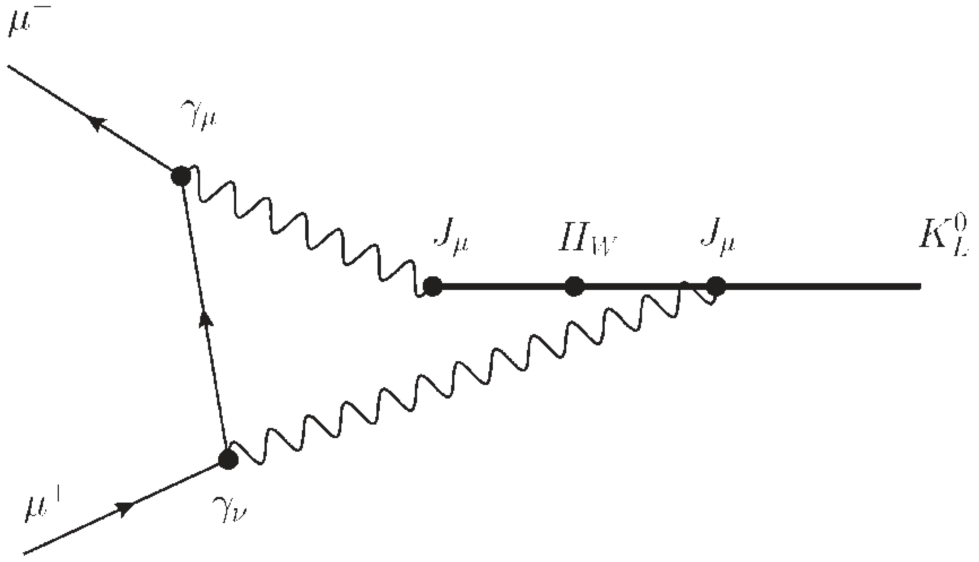
- Imaginary part known from optical theorem and $K_L \to \gamma\gamma$ decay rate
- Real part is not well understood → largest uncertainty

Cirigliano, Ecker, Neufeld, Pich, Portoles, Rev.Mod.Phys. 84 (2012) 399

Decay process involves photon and lepton loop

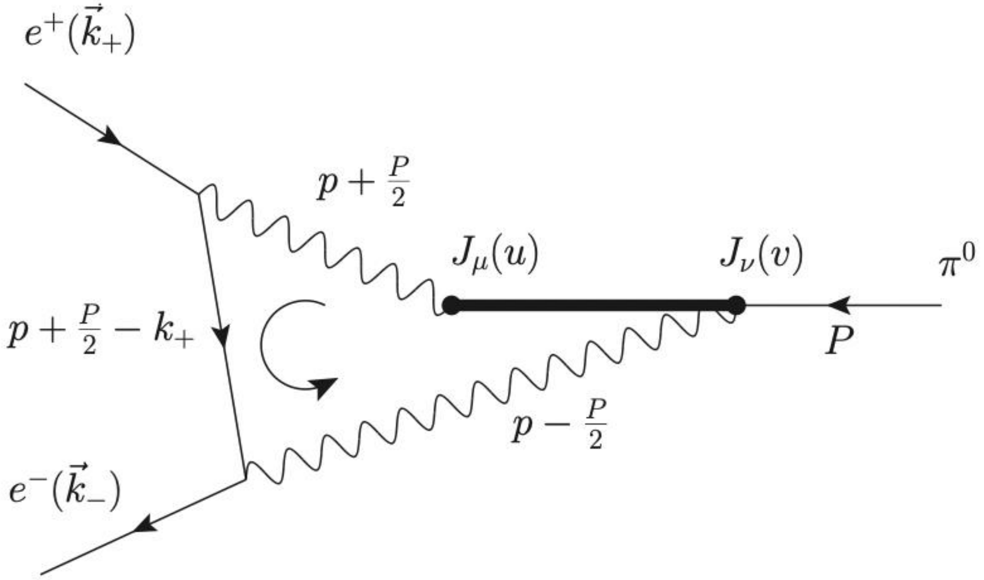
➢ Lattice QCD calculation

- $K_L \to \mu^+\mu^-$



- 5 vertices, 60 different time ordering
- Many intermediate states with $E<M_K$
- Hadronic part involves 4pt function

- $\pi^0 \to \mu^+\mu^-$

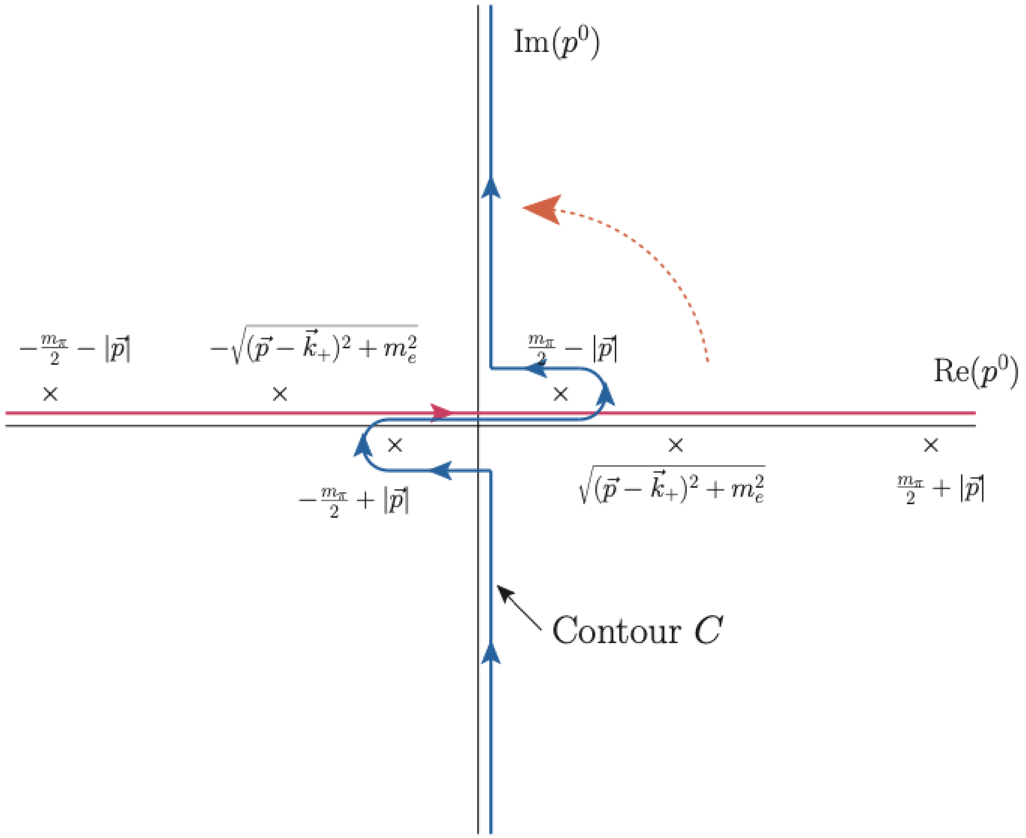


- 4 vertices, 12 different time ordering
- Only two-photon state with $E<M_\pi$
- Used to develop methodology

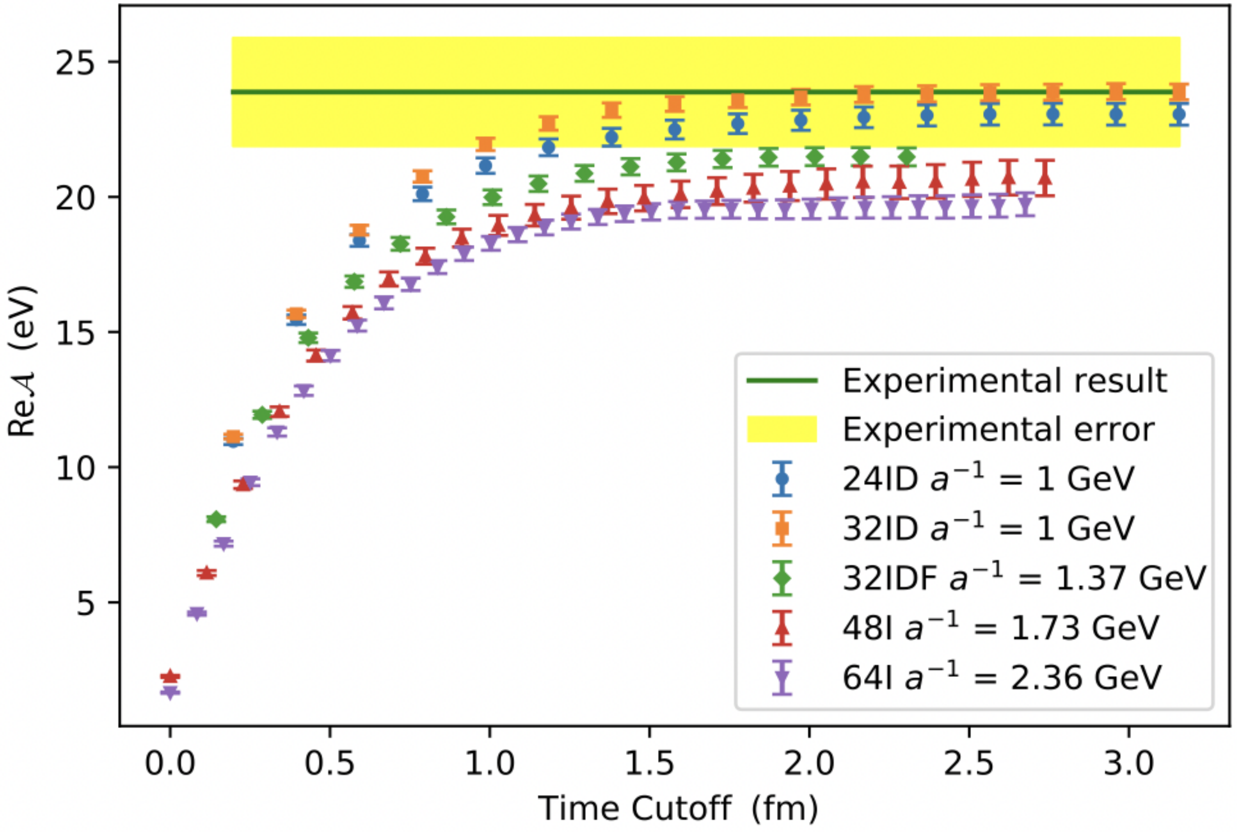
Decay process involves photon and lepton loop

➢ Lattice methodology

- Calculate non-QCD part in Minkowski spacetime
- Then Wick rotate it to Euclidean spacetime



➢ Re[A(π→e⁺e⁻)]@m_π=140 MeV, RBC-UKQCD



N. Christ, XF, L. Jin et.al, PRL 130 (2023) 191901

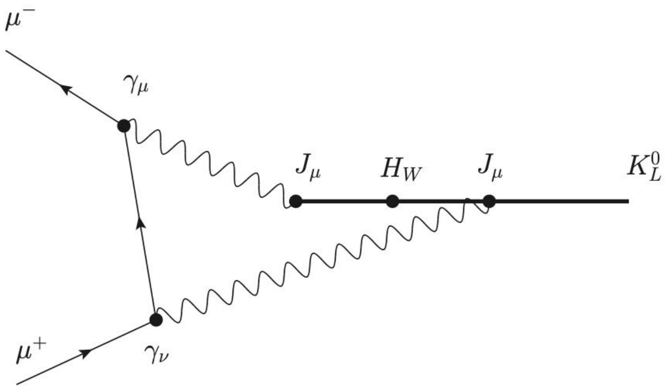
- Precision 6-7 times better than exp. measurement
- 1.8 σ deviation is obtained

➢ Methodology extended to $K_L\to\mu^+\mu^-$ and exploratory numerical calculation undertaken

E. Chao, N. Christ, XF, L. Jin, PoS Lattice2023, 250

Interesting rare processes

$K_L \to \mu^+\mu^-$: $\mathrm{BR} = 6.84(11) \times 10^{-9}$



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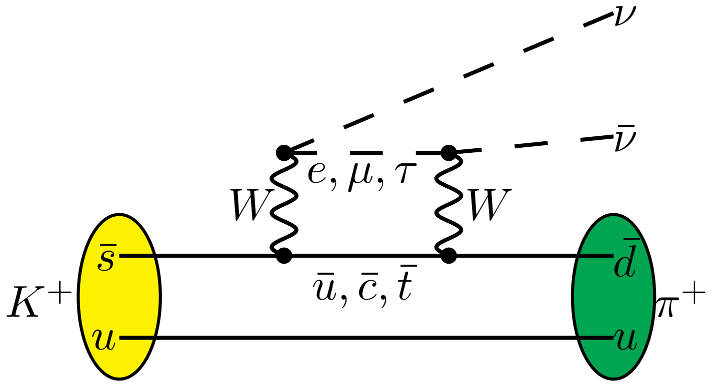

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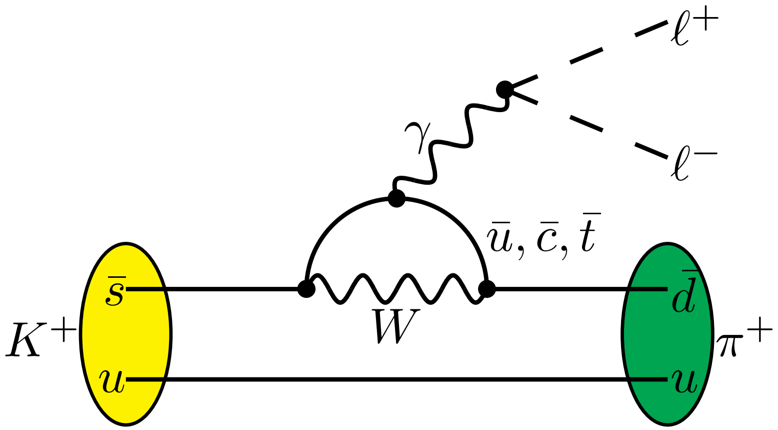

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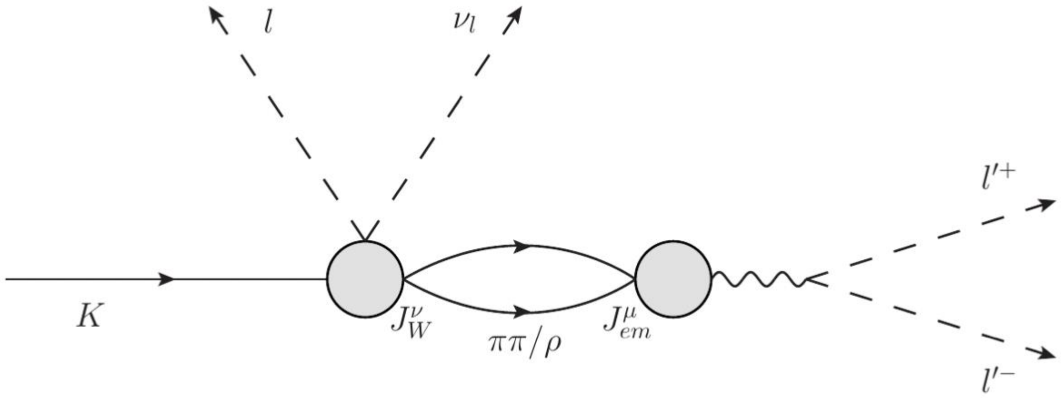
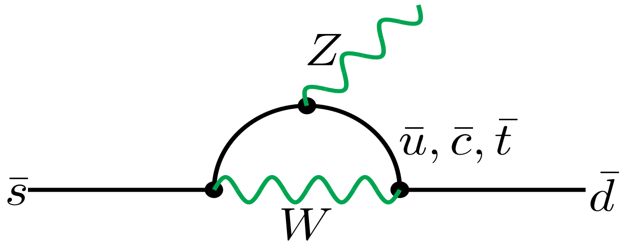


Comparison between two rare decay channels

Factors of $1/M_W^4$ or $1/(M_W^2 M_Z^2)$ implies quadratic GIM mechanism

uadratic GIM mechanism

$K^+ \to \pi^+ \nu \bar{\nu}$



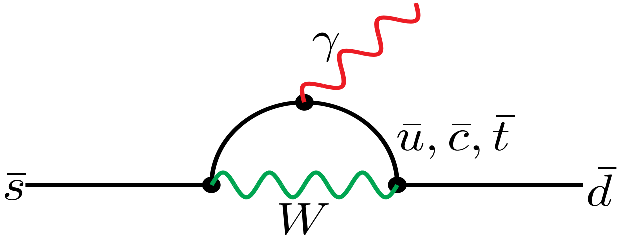
top (SD): $\lambda_t \frac{m_t^2}{M_W^2}$

charm (SD): $\lambda_c \frac{m_c^2}{M_W^2} \ln \frac{m_c^2}{M_W^2}$

charm (LD): $\lambda_c \frac{m_c^2}{M_W^2}$

Logarithmic GIM mechanism

$K^+ \to \pi^+ \ell^+ \ell^-$

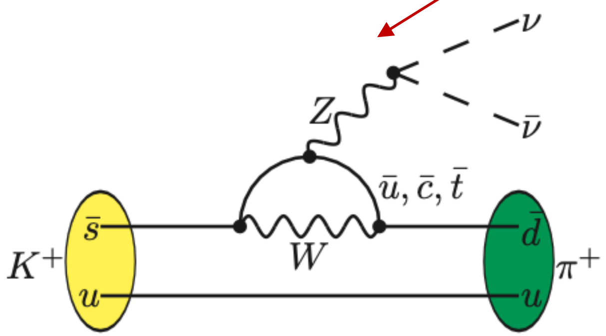


top (SD): $\lambda_t \ln \frac{m_t^2}{\Lambda_{\rm QCD}^2}$

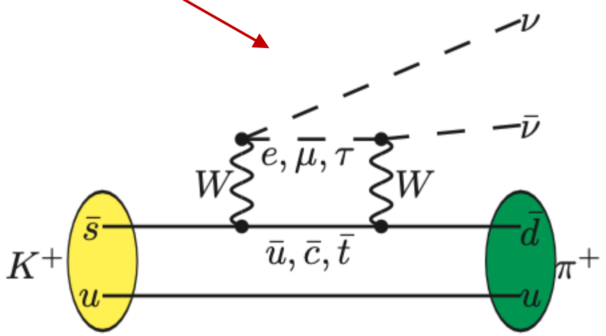
charm & light (LD): $\lambda_c \ln \frac{m_c^2}{\Lambda_{\rm QCD}^2}$

Comparison between two rare decay channels

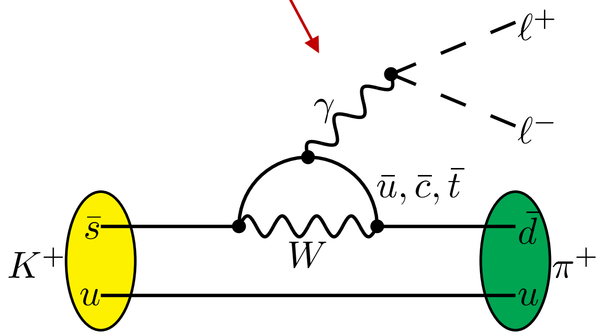
➢ Calculation of $K^+ \to \pi^+ \nu \bar{\nu}$ is more challenging than $K^+ \to \pi^+ \ell^+ \ell^-$



Z-exchange



W-*W*

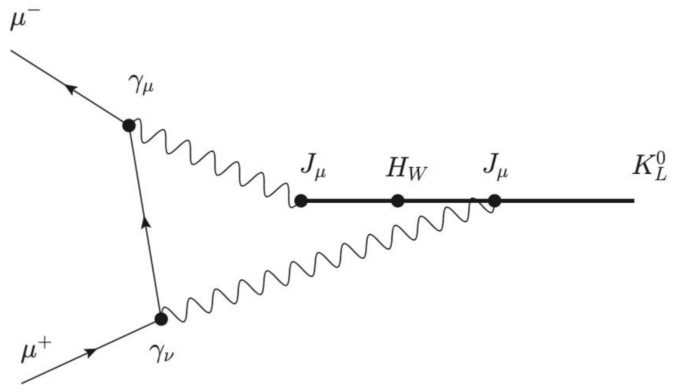


γ-exchange

- Z-exchange diagram involves both vector and axial vector current insertions
- In W-W diagram, neutrinos are not connected at 1 point → Dalitz study of the amplitude
- SD divergent, requires UV subtraction

Interesting rare processes

$K_L \to \mu^+\mu^-$: BR $= 6.84(11) \times 10^{-9}$



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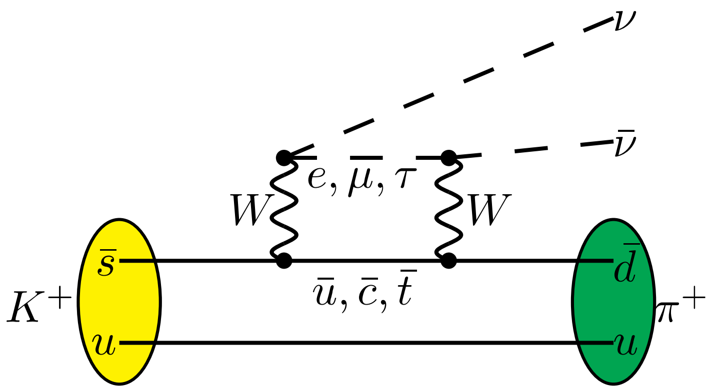
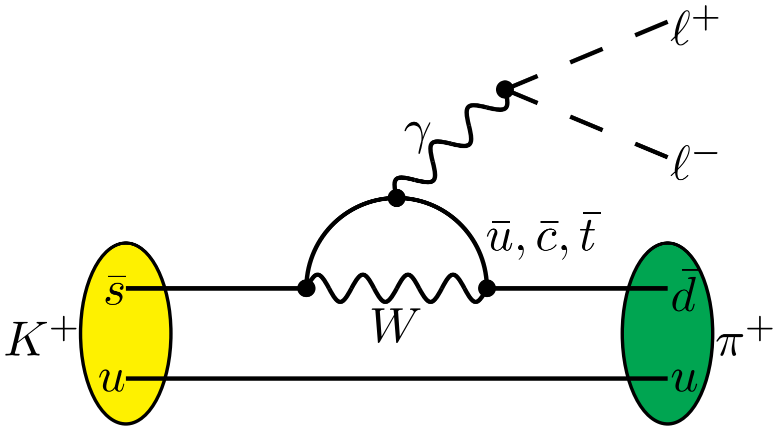
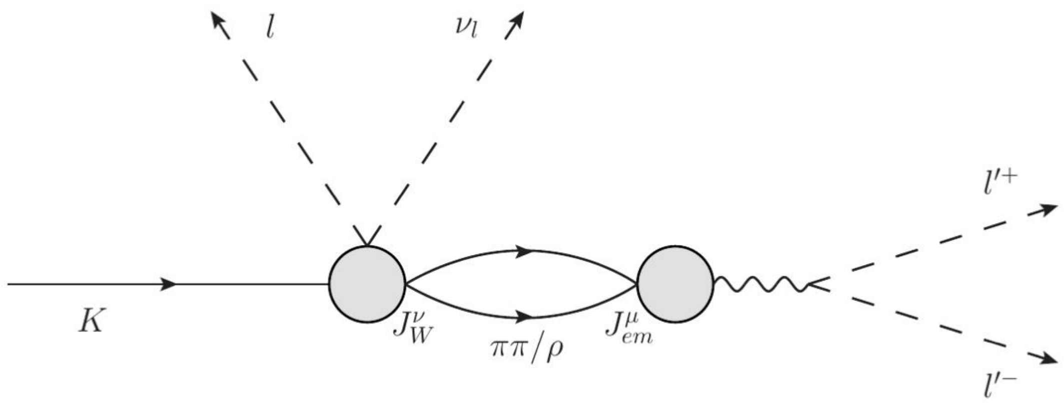


$K^+ \to \pi^+e^+e^-$: BR$=3.14(10) \times 10^{-7}$



$K \to \ell\nu_\ell\ell'^+\ell'^-$: BR $= O(10^{-8})$



Form factor relevant for $K^+ \to \pi^+ \ell^+ \ell^-$

- Experimental measurement

$$\mathrm{Br}(K^+ \to \pi^+ e^+ e^-) = 3.00(9) \times 10^{-7} \qquad \mathrm{Br}(K^+ \to \pi^+ \mu^+ \mu^-) = 9.4(6) \times 10^{-8}$$

New results from NA62 [NA62, JHEP 11 (2022) 011]

$$\mathrm{Br}(K^+ \to \pi^+ \mu^+ \mu^-) = 9.15(8) \times 10^{-8}$$

- Hadronic amplitude is described by a form factor

$$A_+^\mu(p_K, p_\pi) = \int d^4x\, e^{iqx} \langle \pi(p_\pi)| T\{J^\mu_{em}(x) \mathcal{H}^{\Delta S=1}(0)\} |K^+/K_S(p_K)\rangle$$
$$= \frac{G_F M_K^2}{(4\pi)^2} V_+(z) \left[z(k+p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

with $q = p_K - p_\pi,\ z = q^2/M_K^2,\ r_\pi = M_\pi/M_K$

- Form factor is parameterized as

$$V_+(z) = a_+ + b_+ z + V^{\pi\pi}(z)$$

Measurement	a_+	b_+
E865 - $K_{\pi ee}$	-0.587 ± 0.010	-0.655 ± 0.044
NA48/2 - $K_{\pi ee}$	-0.578 ± 0.016	-0.779 ± 0.066
NA48/2 - $K_{\pi\mu\mu}$	-0.575 ± 0.039	-0.813 ± 0.145
NA62 - $K_{\pi\mu\mu}$	-0.575 ± 0.013	-0.722 ± 0.043

Exploratory lattice calculation

Use $24^3 \times 64$ ensemble, $N_{\mathrm{conf}} = 128$

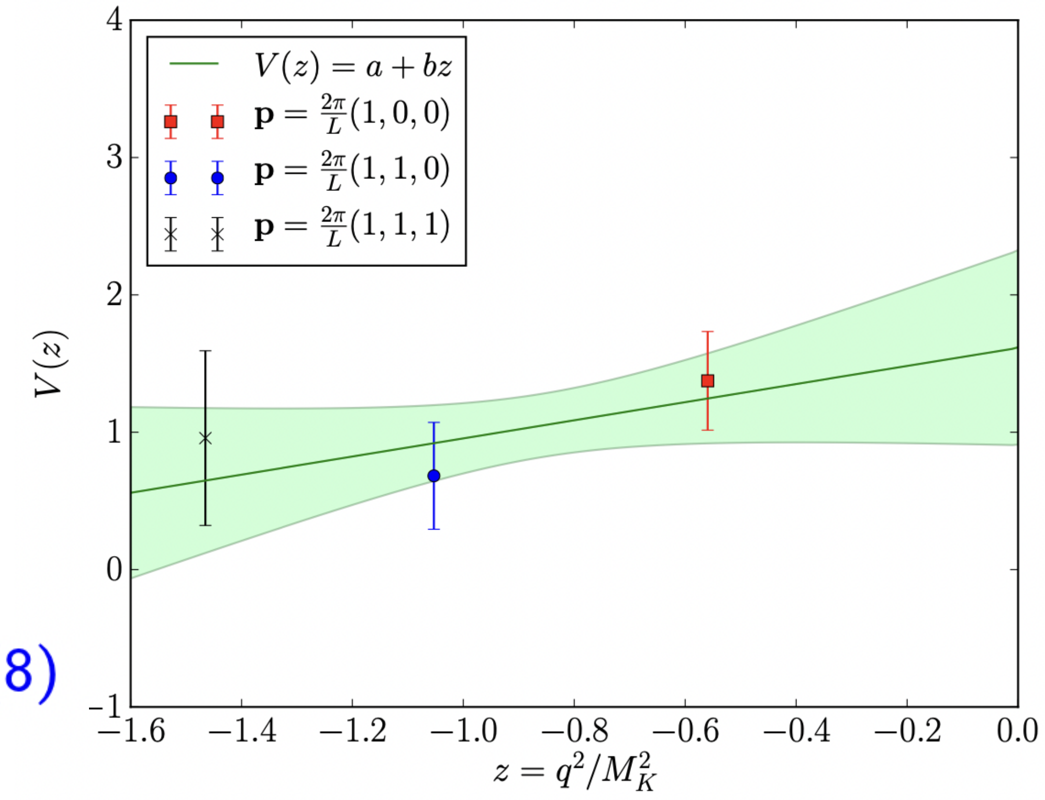
[N. Christ, XF, A. Lawson, et.al. PRD94 (2016) 114516]

$a^{-1} = 1.78$ GeV, $m_\pi = 430$ MeV

$m_K = 625$ MeV, $m_c = 530$ MeV

Momentum dependence of $V_+(z)$

$V_+(z) = a_+ + b_+ z \quad \Rightarrow \quad a_+ = 1.6(7),\ b_+ = 0.7(8)$



Experimental data + phenomenological analysis yields $a_+<0$ and $b_+<0$

$$V_j(z) = a_j + b_j z + \underbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z - z_0)}{G_F M_K^2 r_\pi^4}}_{K\to\pi\pi\pi} \underbrace{\left[1 + \frac{z}{r_V^2}\right]}_{F_V(z)} \underbrace{\left[\phi(z/r_\pi^2) + \frac{1}{6}\right]}_{\text{loop}}, \quad j = +, S$$

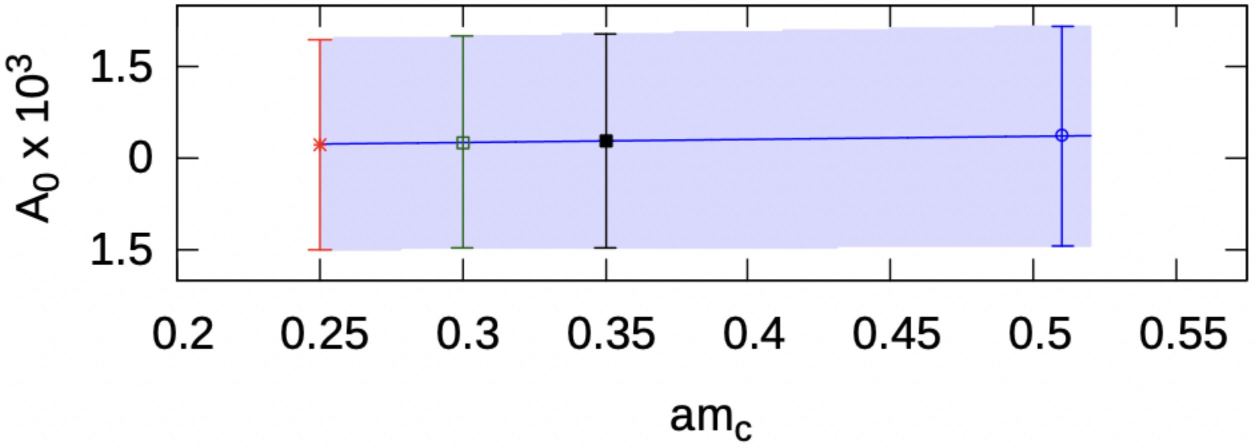
- Experimental data only provide $\frac{d\Gamma}{dz} \Rightarrow$ square of form factor $|V_+(z)|^2$
- Need phenomenological knowledge to determine the sign for a_+, b_+

Calculation at physical pion mass

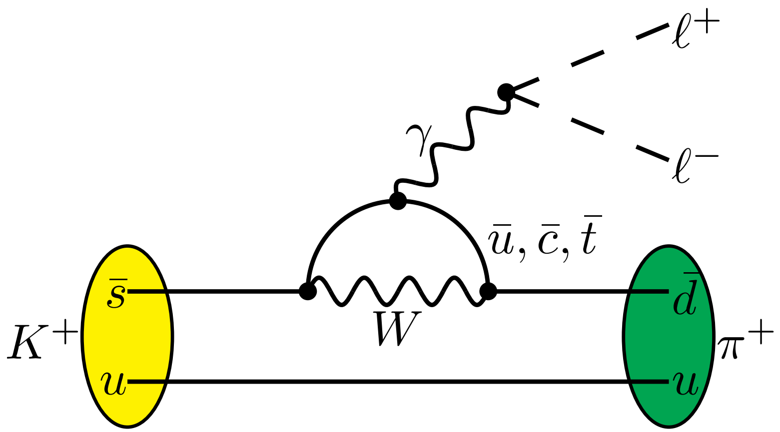
- 2+1 flavor DWF with $a^{-1} = 1.730(4)\text{GeV}$

- Physical pion mass

- Three charm quark masses used for extrapolation to physical point



- Large statistical error from stochastic estimated quark loops

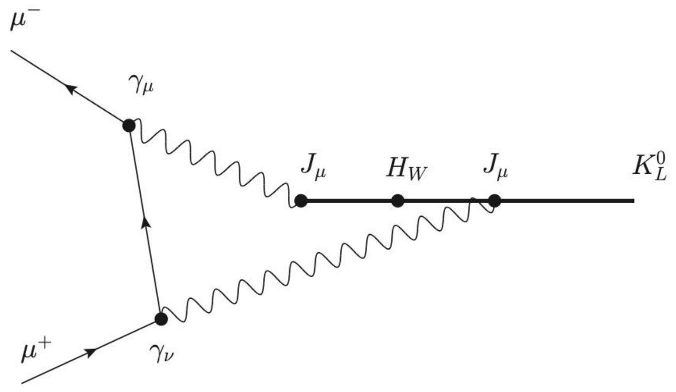


$$V(z = 0.013(2)) = -0.87(4.44)$$

[P Boyle et.al. PRD107 (2023) L011503]

Interesting rare processes

$K_L \to \mu^+\mu^-$: $\mathrm{BR} = 6.84(11) \times 10^{-9}$



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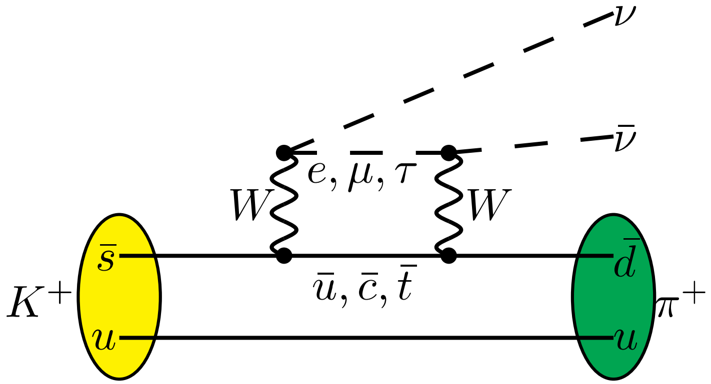

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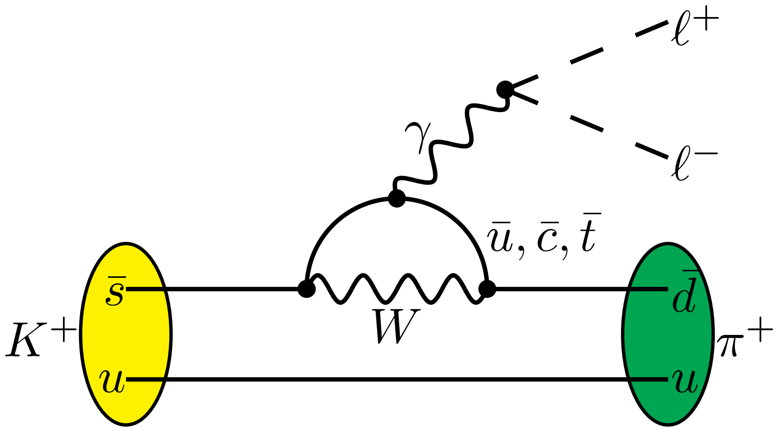

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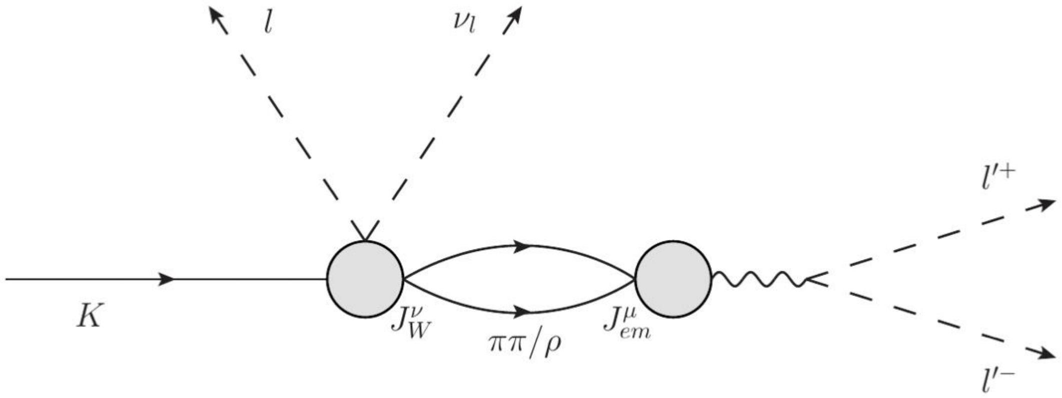


$K^+ \to \pi^+ \nu\bar{\nu}$: in the Standard Model prediction

Branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ [Buras, Buttazzo, Girrbach-Noe, Knegjens, '15]

$$\text{Br} = \kappa_+ (1 + \Delta_{\text{EM}}) \cdot \Big[\big(\underbrace{\frac{\text{Im}\,\lambda_t}{\lambda^5} X(x_t)}_{0.270\times 1.481(9)} \big)^2 + \big(\underbrace{\frac{\text{Re}\,\lambda_c}{\lambda} P_c}_{-0.974\times 0.405(23)} + \underbrace{\frac{\text{Re}\,\lambda_t}{\lambda^5} X(x_t)}_{-0.533\times 1.481(9)} \big)^2 \Big]$$

- $X(x_t)$: top quark contribution; P_c: charm and LD contribution

> Without P_c, branching ratio is 50% smaller

Uncertainty budget

- dominant uncertainty from CKM factor λ_t
- once fixing CKM factor, then P_c dominates the uncertainty
  - P_c's uncertainty mainly come from LD

> Important to determine the LD contribution to P_c accurately

Results for charm quark contribution

Charm quark contribution

$$P_c = P_c^{\mathrm{SD}} + \delta P_{c,u}$$

NNLO QCD [A. Buras, M. Gorbahn, U. Haisch, U. Nierste, JHEP 11 (2006) 002]

$$P_c^{\mathrm{SD}} = 0.365(12)$$

Chiral perturbation theory [G. Isidori, F. Mescia, C. Smith, NPB 718 (2005) 319]

$$\delta P_{c,u} = 0.040(20)$$

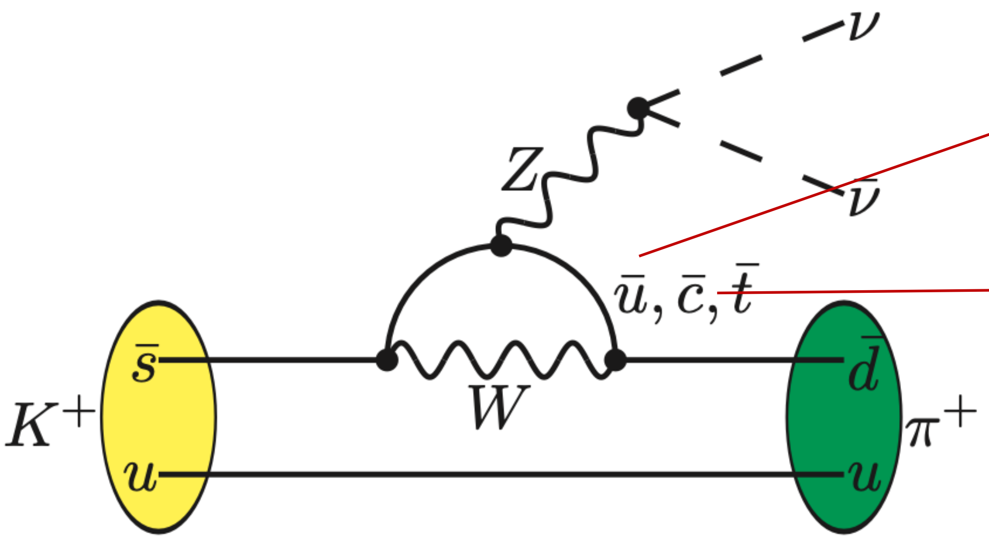
First lattice results @ m_π=420 MeV, m_c=860 MeV [Z. Bai, N. Christ, XF, et.al. PRL118 (2017) 252001]

$$P_c = 0.2529(\pm 13)_{\mathrm{stat}}(\pm 32)_{\mathrm{scale}}(-45)_{\mathrm{FV}}$$
$$P_c - P_c^{\mathrm{SD}} = 0.0040(\pm 13)_{\mathrm{stat}}(\pm 32)_{\mathrm{scale}}(-45)_{\mathrm{FV}}$$

- As a smaller m_c is used, P_c is also smaller
- Cancellation in W-W and Z-exchange diag. leads to small $P_c - P_c^{\mathrm{SD}}$
- Important to perform the calculation at physical m_π and m_c

Short summary

- At physical kinematics, calculation is very challenging



- Involve light-quark loop → Physical pion mass

  Large volume to control FV effects from π

- Involve charm-quark loop → Physical charm mass

  Fine lattice spacing to control lattice artifacts from charm quark

→ Need a very large lattice (or new idea?)

- From Kaon to hyperon

Measurement of the Absolute Branching Fraction and Decay Asymmetry of Λ→nγ

BESIII Collaboration • M. Ablikim (Beijing, Inst. High Energy Phys.) et al. (Jun 21, 2022)

Published in: *Phys.Rev.Lett.* 129 (2022) 21, 212002 • e-Print: 2206.10791 [hep-ex]

→ 5.6σ deviation from past experiments

Precision Measurement of the Decay $\Sigma^+ \to p\gamma$ in the Process $J/\psi \to \Sigma^+ \bar{\Sigma}^-$

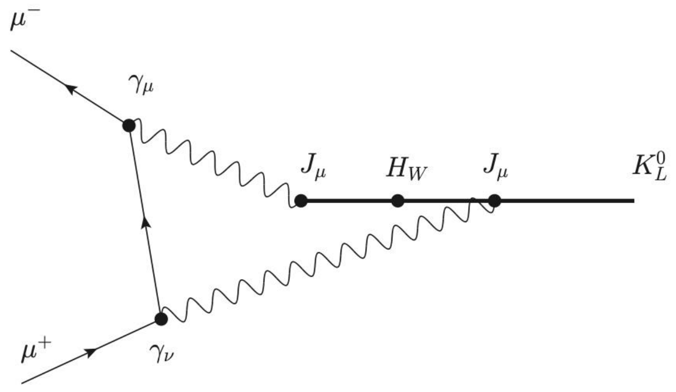
BESIII Collaboration • M. Ablikim (Beijing, Inst. High Energy Phys.) et al. (Feb 27, 2023)

Published in: *Phys.Rev.Lett.* 130 (2023) 21, 211901 • e-Print: 2302.13568 [hep-ex]

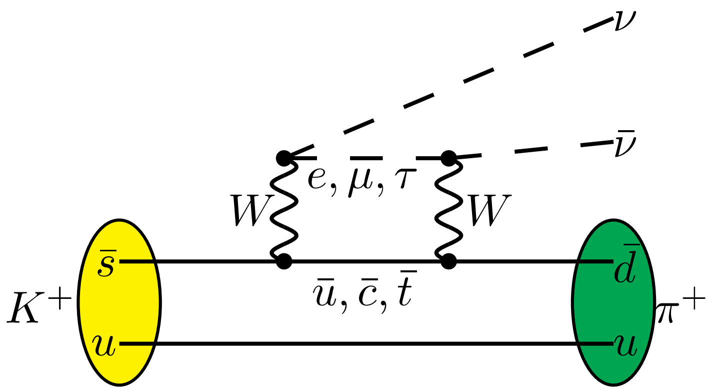
→ 4.2σ deviation from past experiments

Interesting rare processes

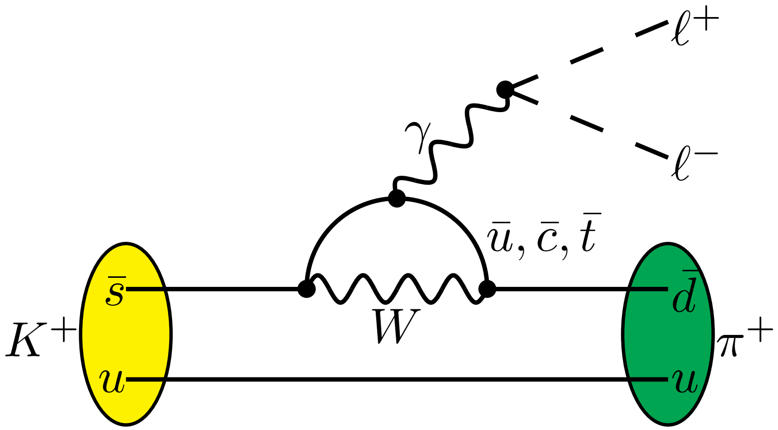
$K_L \to \mu^+\mu^-$: BR $= 6.84(11) \times 10^{-9}$



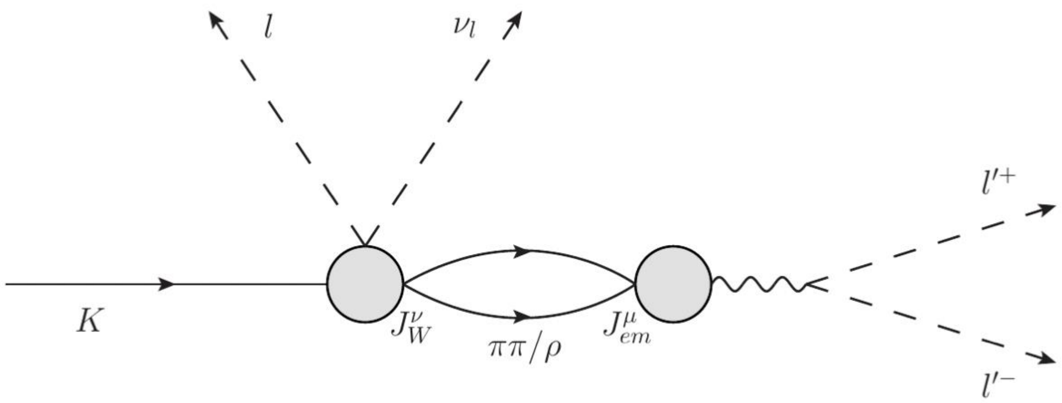
$K^+ \to \pi^+\nu\bar{\nu}$: BR$=1.73^{+1.15}_{-1.05} \times 10^{-10}$



$K^+ \to \pi^+e^+e^-$: BR$=3.14(10) \times 10^{-7}$



$K \to \ell\nu_\ell\ell'^+\ell'^-$: BR $= O(10^{-8})$



Conclusion

- Test of first-row CKM unitarity
  - $|V_{ud}|$ Theory: EWR, Nuclear structure
  - $f_+(0)$: More lattice calculations for average
- Inclusion of isospin breaking effects
  - An interesting frontier
  - More studies + new method
- Rare decays
  - Ideal place to search for BSM physics
  - For example:

$K \to \pi \ell^+ \ell^-$ We believe that over the next 5-10 years, lattice QCD will be in a position to produce predictions of a_s, a_+, b_s, b_+ with uncertainties below the 10 % level

[Snowmass 2021, T. Blum et.al., arXiv:2203.10998]