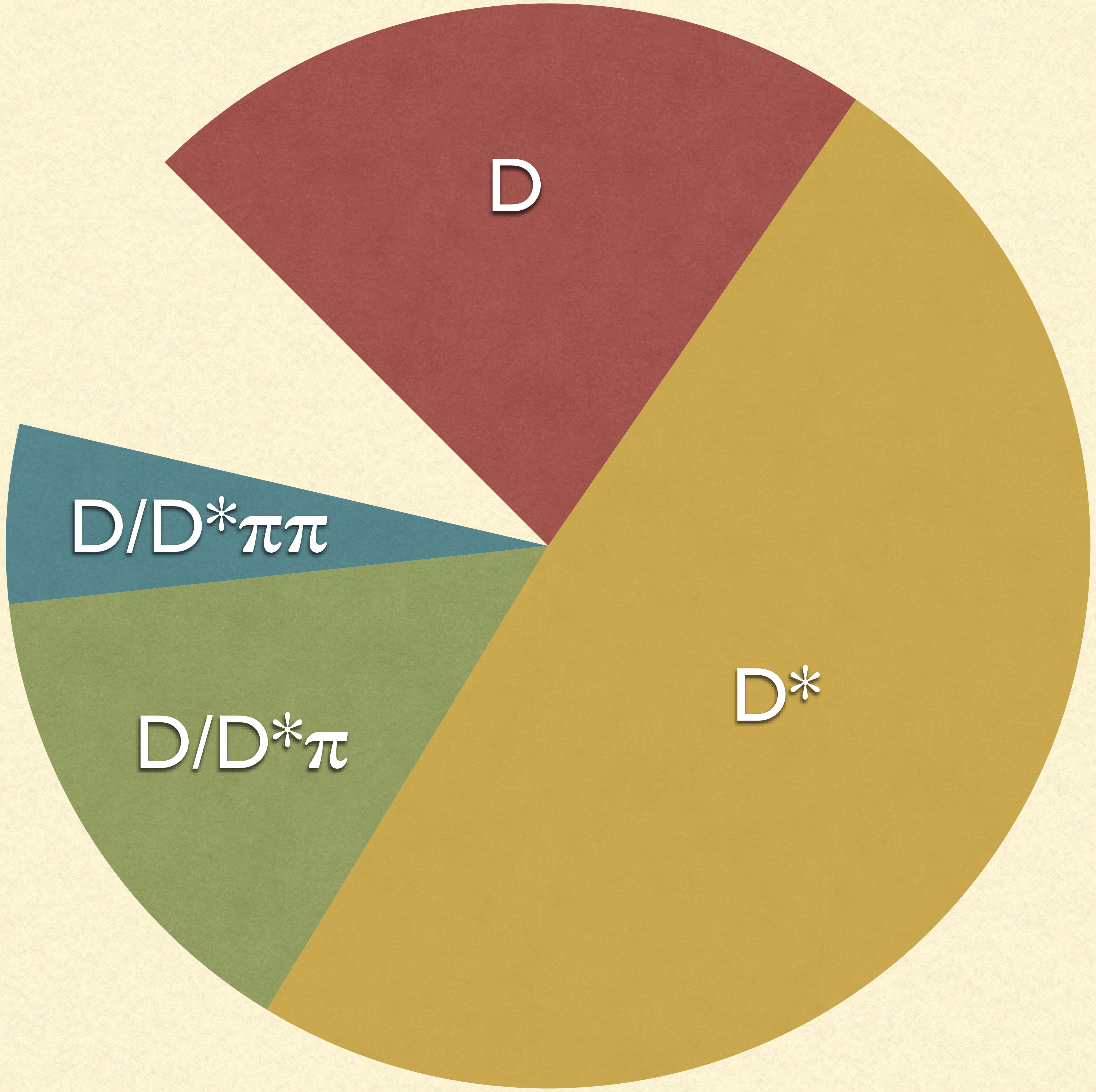
A fresh look at $B \to D\pi\ell\nu$

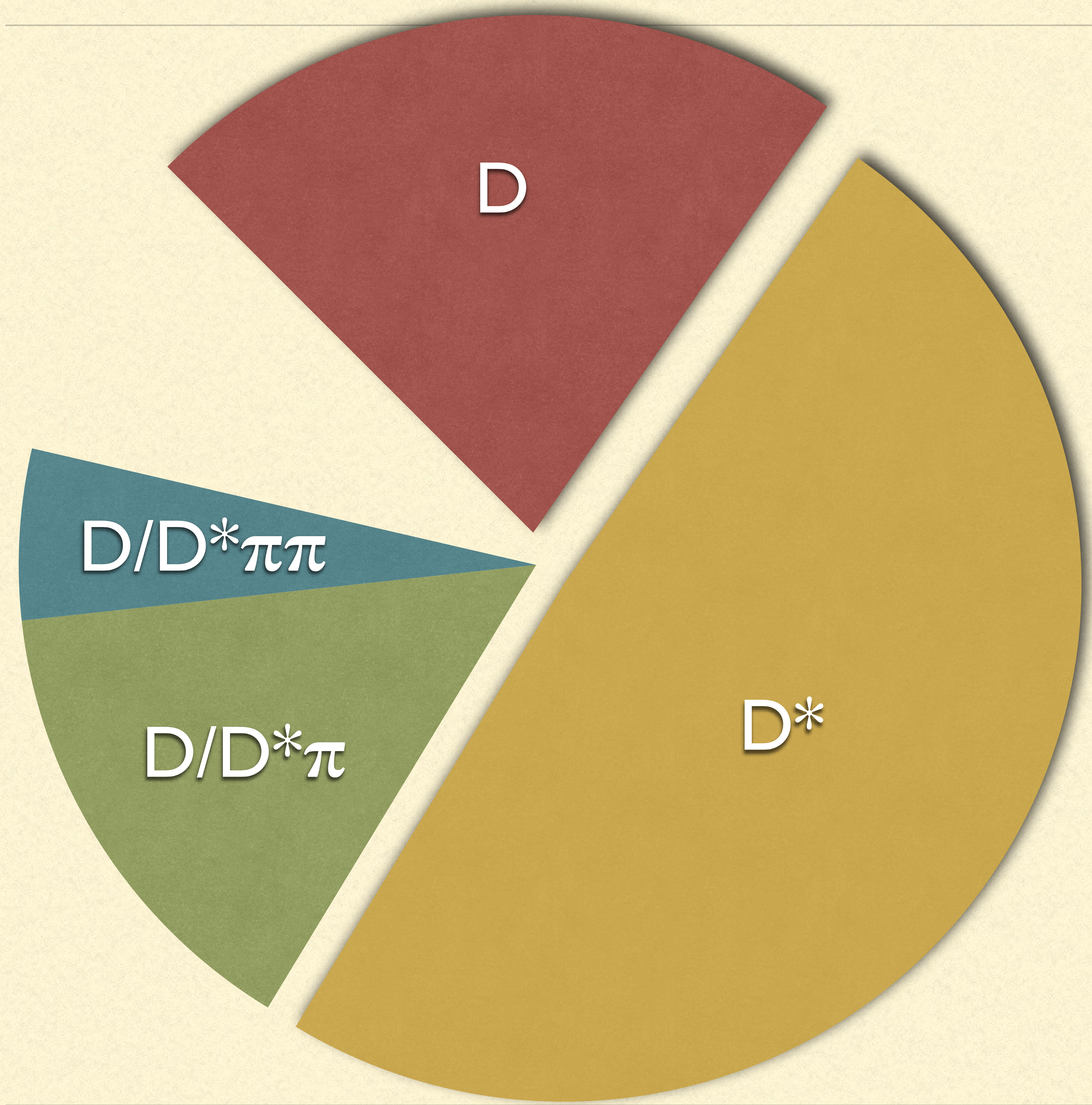
Based on 2311.00864, in collaboration with Erik Gustafson, Ruth Van de Water, Raynette van Tonder & Michael Wagman

Semileptonic $B \to X_c \ell \nu$ decays



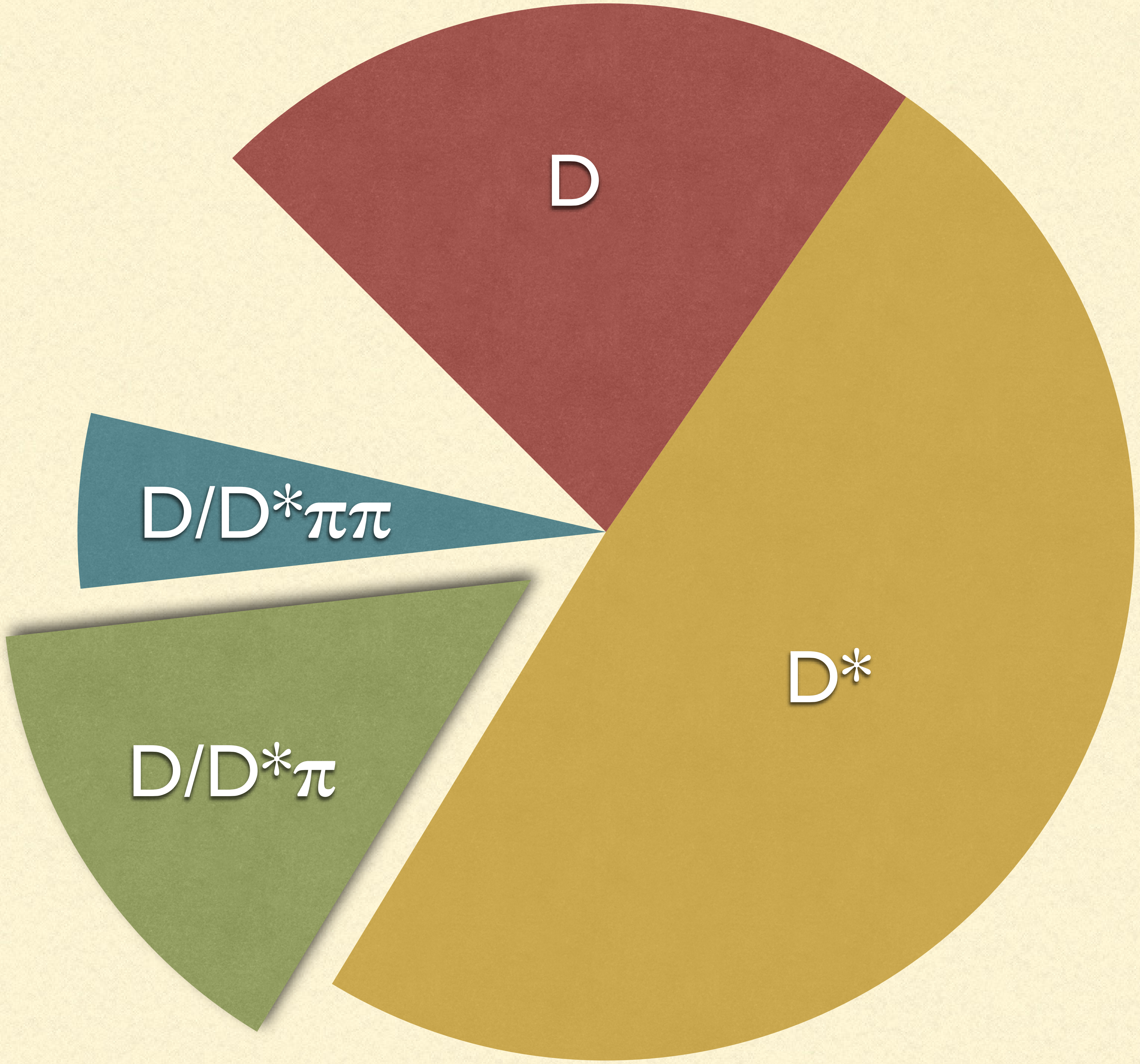
- Semileptonic decays comprise more than 10% of all B-meson decays
- Ideal laboratory to determine $|V_{cb}|$ with multiple complementary approaches
- Allows for precise tests of light lepton flavour universality
- $R\left(D^{(*)}\right)$ anomalies
- Important background for $B \to X_u \ell \nu$ decays and other rare processes

Semileptonic $B \to D^{(*)}\ell\nu$ decays



- 75% made up by (quasi-)three-body modes
- Branching ratios known at few percent level
- Form factors from lattice QCD collaborations: Fermilab/MILC, HPQCD, JLQCD
- Model-independent (BGL) & HQET-based (Bernlochner et al.) FF parameterizations used
- D^* has narrow width and decays to $D\pi$ & $D\gamma$

Semileptonic $B \to D^{(*)}\pi\ell\nu$ decays



- 10% made up by (quasi)-four-body modes
- Thought to mostly proceed through the four 1P D-Meson excitations: D_0^*, D_1', D_1, D_2^*
- Only a few, sometimes conflicting, BF measurements
- Even less measurements of differential spectra
- HQET-based FF parameterization (LLSW)
- Some recent LCSR computations of D^{**} FFs
- "Less hot topic than $D^{(*)}$" (Nico Gubernari, this morning)

"The heavier 1P charmed mesons, collectively known as D^{**}, are a leading background in this measurement and their description in the simulation is thus a critical component "

– *Belle II $R(D^*)$ preprint*

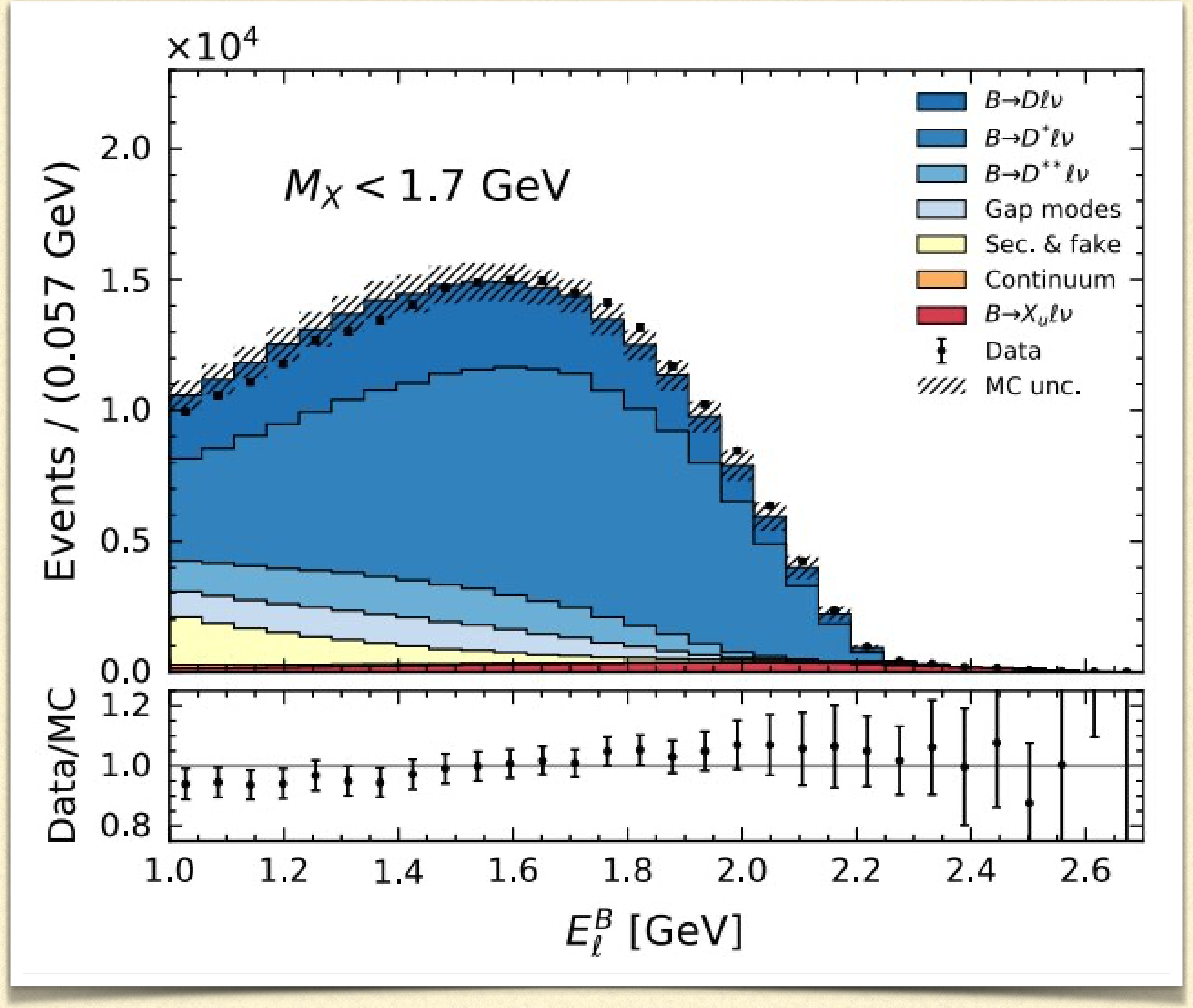
Where are $B \to D^{(*)}\pi\ell\nu$ decays relevant?

Internal fit uncertainties	$\sigma_{\mathcal{R}(D^*)}(\times 10^{-2})$	$\sigma_{\mathcal{R}(D^0)}(\times 10^{-2})$
Statistical uncertainty	1.8	6.0
Simulated sample size	1.5	4.5
$B \to D^{(*)}DX$ template shape	0.8	3.2
$\overline{B} \to D^{(*)}\ell^-\overline{\nu}_\ell$ form-factors	0.7	2.1
$\overline{B} \to D^{**}\mu^-\overline{\nu}_\mu$ form-factors	0.8	1.2
$\mathcal{B}(\overline{B} \to D^* D_s^-(\to \tau^-\overline{\nu}_\tau)X)$	0.3	1.2
MisID template	0.1	0.8
$\mathcal{B}(\overline{B} \to D^{**}\tau^-\overline{\nu}_\tau)$	0.5	0.5
Combinatorial	< 0.1	0.1
Resolution	< 0.1	0.1
Additional model uncertainty	$\sigma_{\mathcal{R}(D^*)}(\times 10^{-2})$	$\sigma_{\mathcal{R}(D^0)}(\times 10^{-2})$
$B \to D^{(*)}DX$ model uncertainty	0.6	0.7
$\overline{B}_s^0 \to D_s^{**}\mu^-\overline{\nu}_\mu$ model uncertainty	0.6	2.4
Baryonic backgrounds	0.7	1.2
Coulomb correction to $\mathcal{R}(D^{*+})/\mathcal{R}(D^{*0})$	0.2	0.3
Data/simulation corrections	0.4	0.8
MisID template unfolding	0.7	1.2
Normalization uncertainties	$\sigma_{\mathcal{R}(D^*)}(\times 10^{-2})$	$\sigma_{\mathcal{R}(D^0)}(\times 10^{-2})$
Data/simulation corrections	$0.4 \times \mathcal{R}(D^*)$	$0.6 \times \mathcal{R}(D^0)$
$\tau^- \to \mu^-\nu\overline{\nu}$ branching fraction	$0.2 \times \mathcal{R}(D^*)$	$0.2 \times \mathcal{R}(D^0)$
Total systematic uncertainty	2.4	6.6
Total uncertainty	3.0	8.9

LHCb Collaboration, PRL 131 111802 (2023)

- Background in $R\left(D^{(*)}\right)$ measurements
- Background in inclusive $|V_{ub}|$ measurements
- Signal component in inclusive $B \to X_c\ell\nu$ & $B \to X\tau\nu$ measurements
- FEI calibration in Belle II
- …

Where are $B \to D^{(*)}\pi\ell\nu$ decays relevant?



L. Cao et al. (Belle), PRD 104, 012008 (2021)

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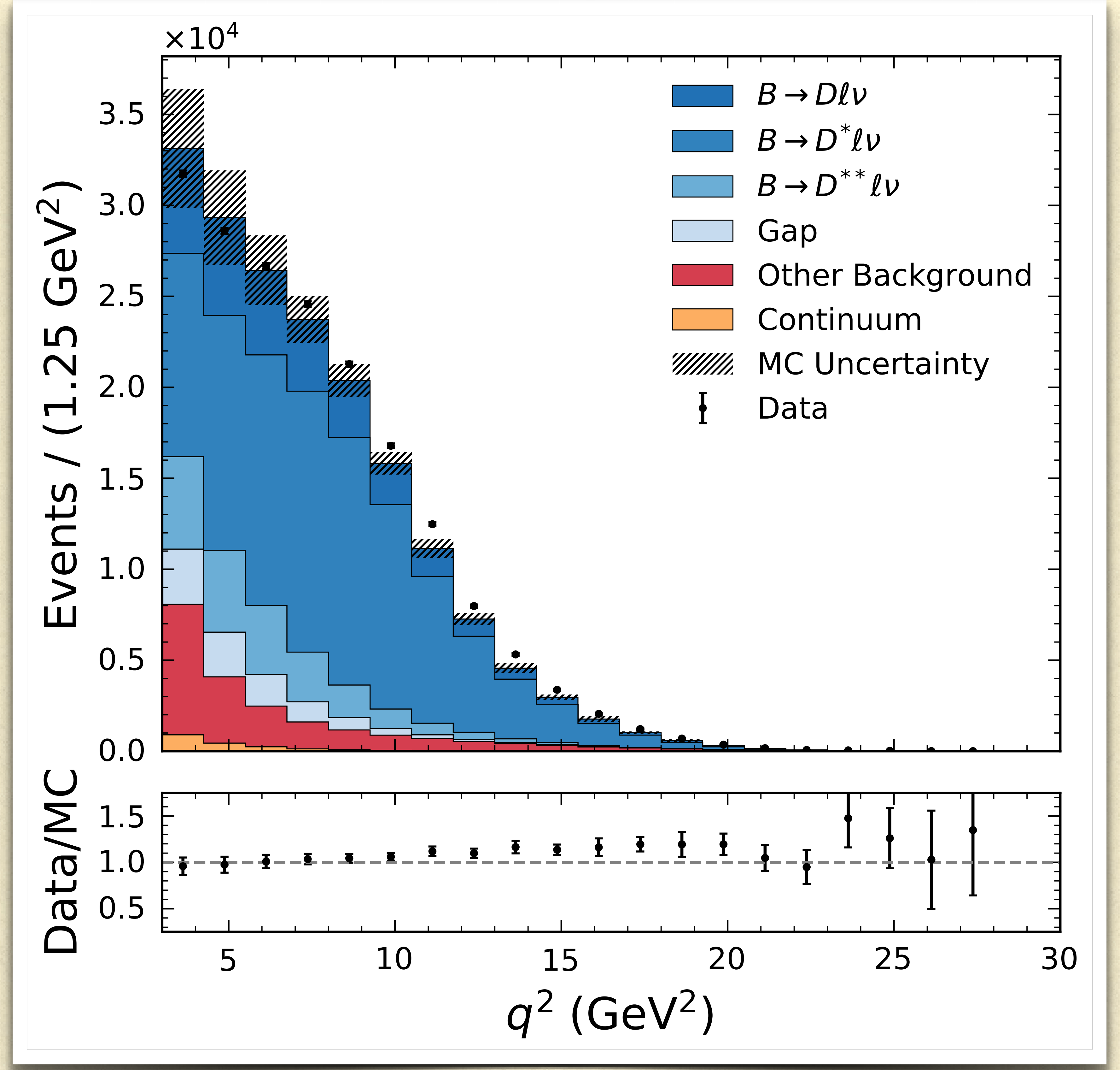

R. van Tonder et al. (Belle), PRD 104, 112011 (2021)

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Where are $B \to D^{(*)}\pi\ell\nu$ decays relevant?

Source	Uncertainty [%] e	μ	ℓ
Experimental sample size	8.8	12.0	7.1
Simulation sample size	6.7	10.6	5.7
Tracking efficiency	2.9	3.3	3.0
Lepton identification	2.8	5.2	2.4
$X_c\ell\nu$ M_X shape	7.3	6.8	7.1
Background (p_ℓ, M_X) shape	5.8	11.5	5.7
$X\ell\nu$ branching fractions	7.0	10.0	7.7
$X\tau\nu$ branching fractions	1.0	1.0	1.0
$X_c\tau(\ell)\nu$ form factors	7.4	8.9	7.8
Total	18.1	25.6	17.3

Belle II Collaboration, 2311.07248

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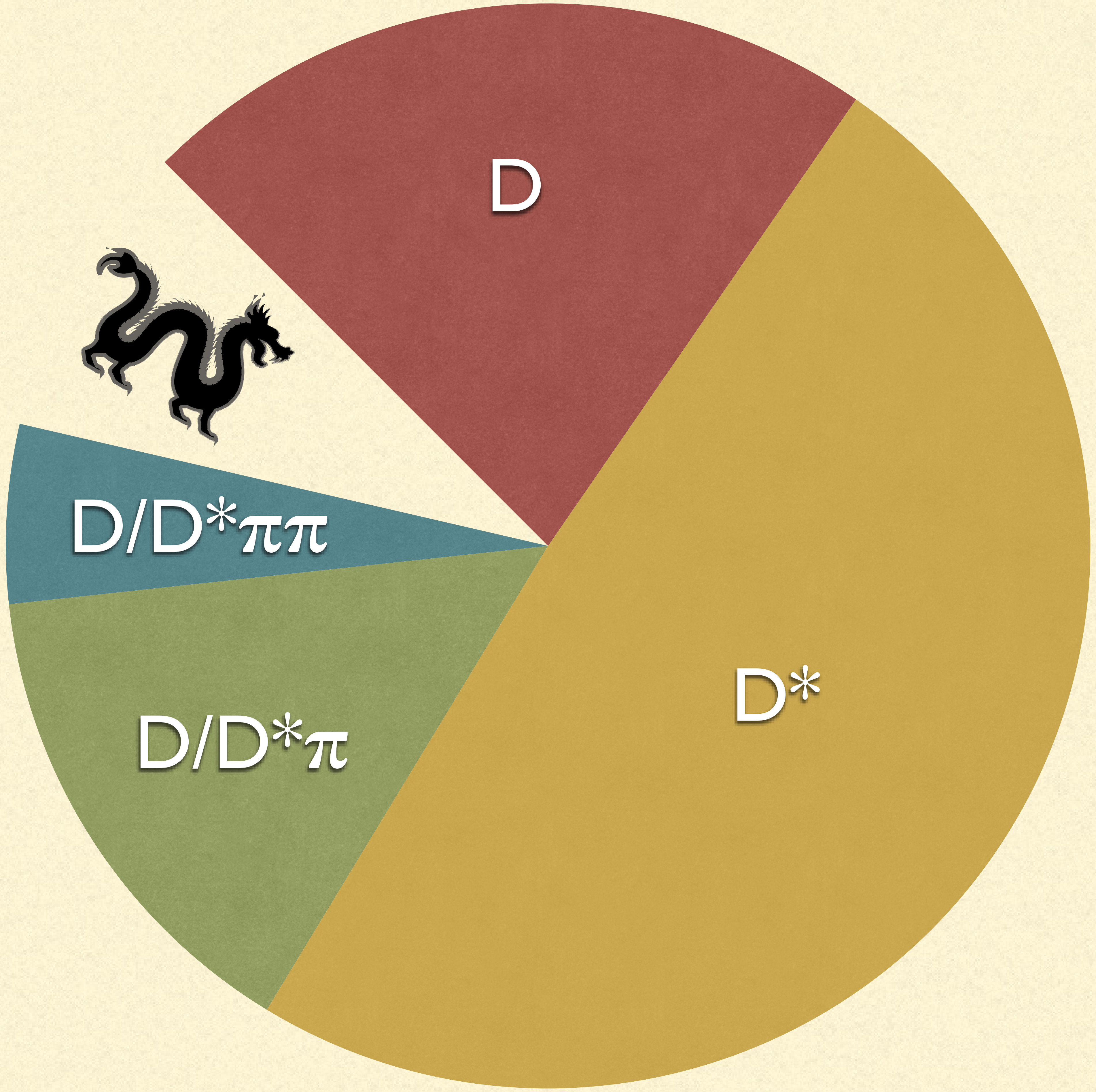
Why are they interesting in their own right?



- They might shed light on the semileptonic gap, i.e. through $B \to D^{**}(\to D^{(*)}\gamma)\ell\nu$ decays
- Different J^P than D & D^*, thus could be affected differently by new physics
- The 1/2 vs. 3/2 puzzle: ratio between broad and narrow contributions does not match theory expectation [Bigi et al. EPJC 52 (2007) 975-985]
- Tensions in measurements
- Do we understand the broad states?

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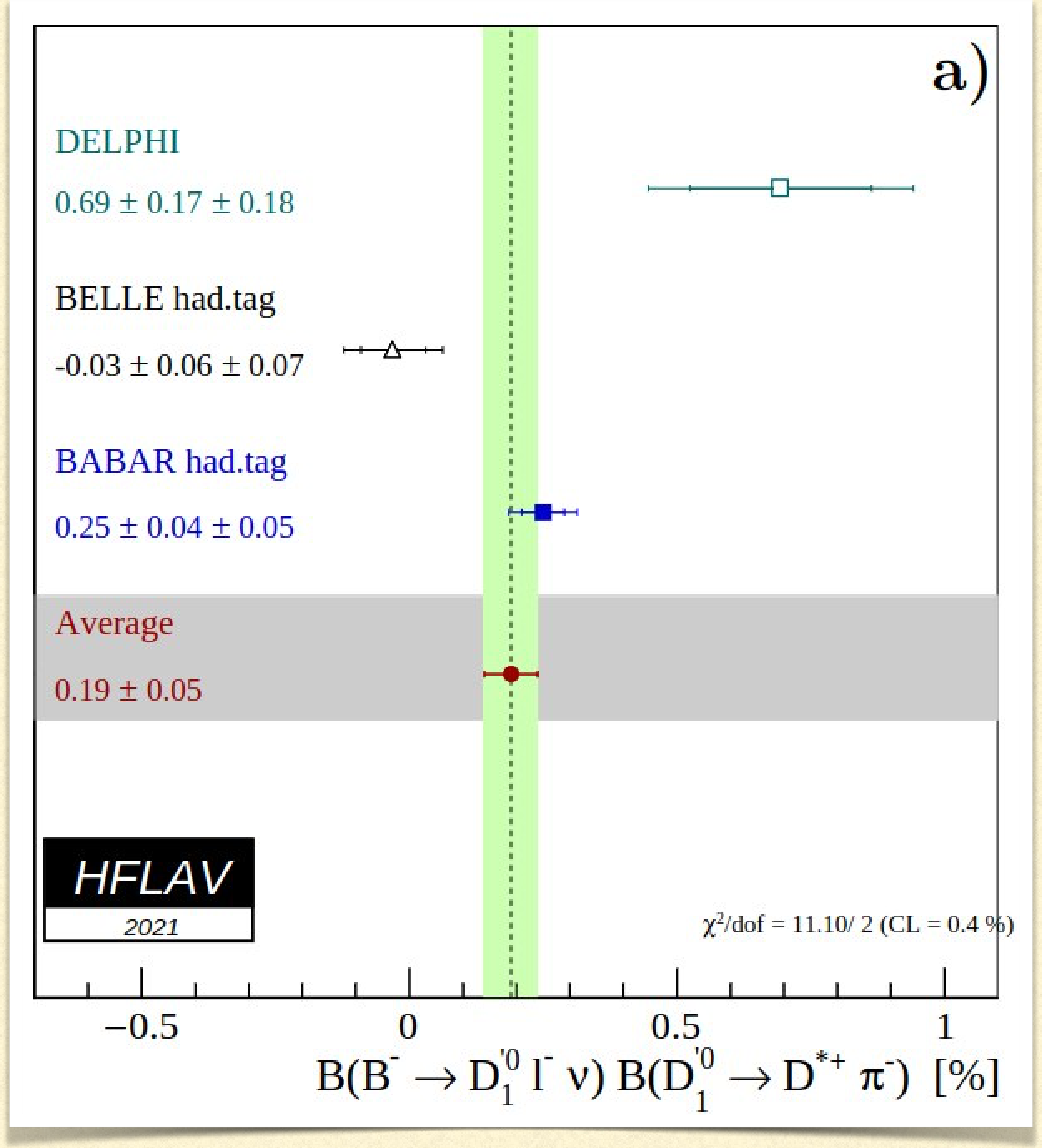

HFLAV collaboration, PRD 107, 052008 (2023)

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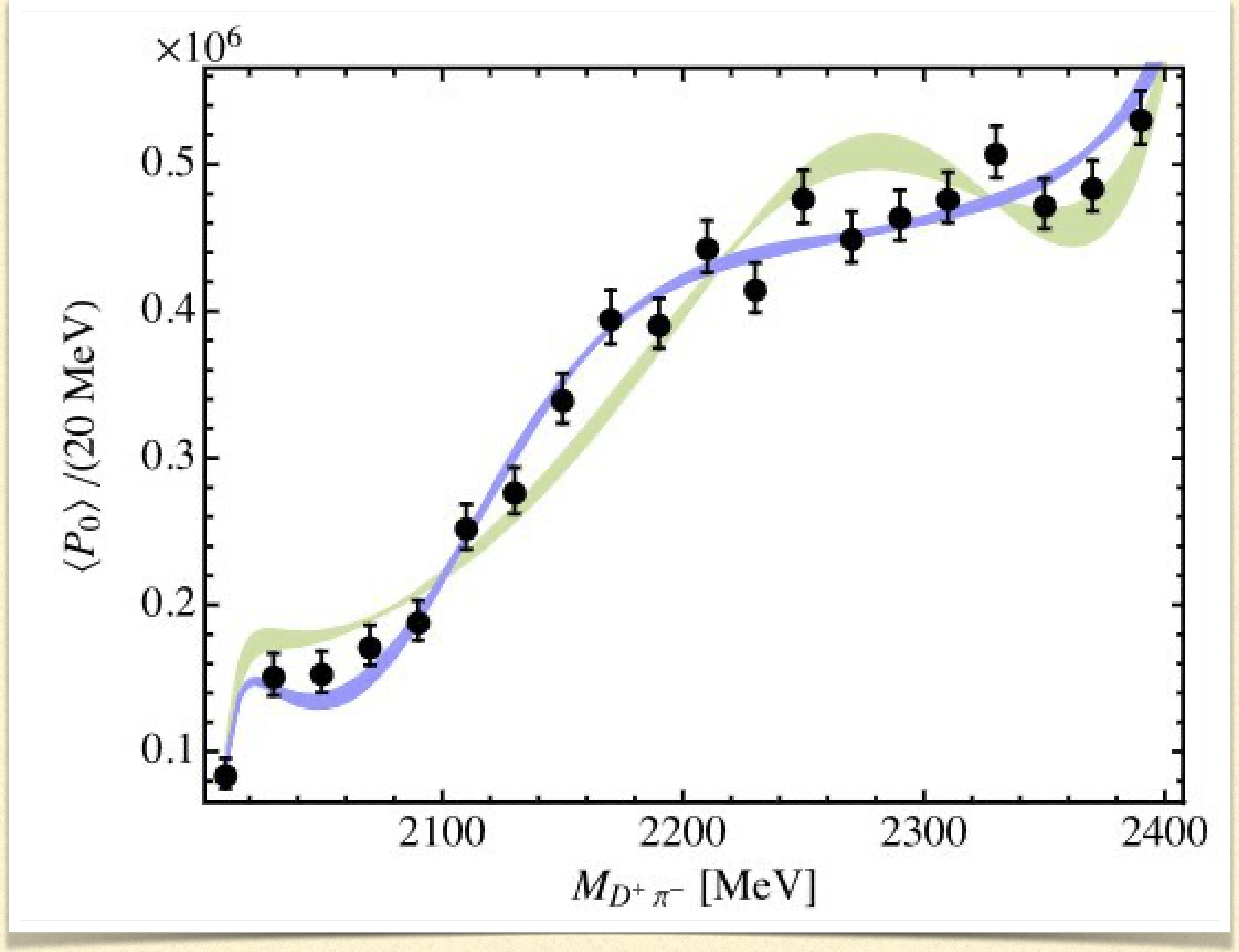

Du, Guo, Hanhart, Kubis, Meissner, PRL 126 192001 (2021)

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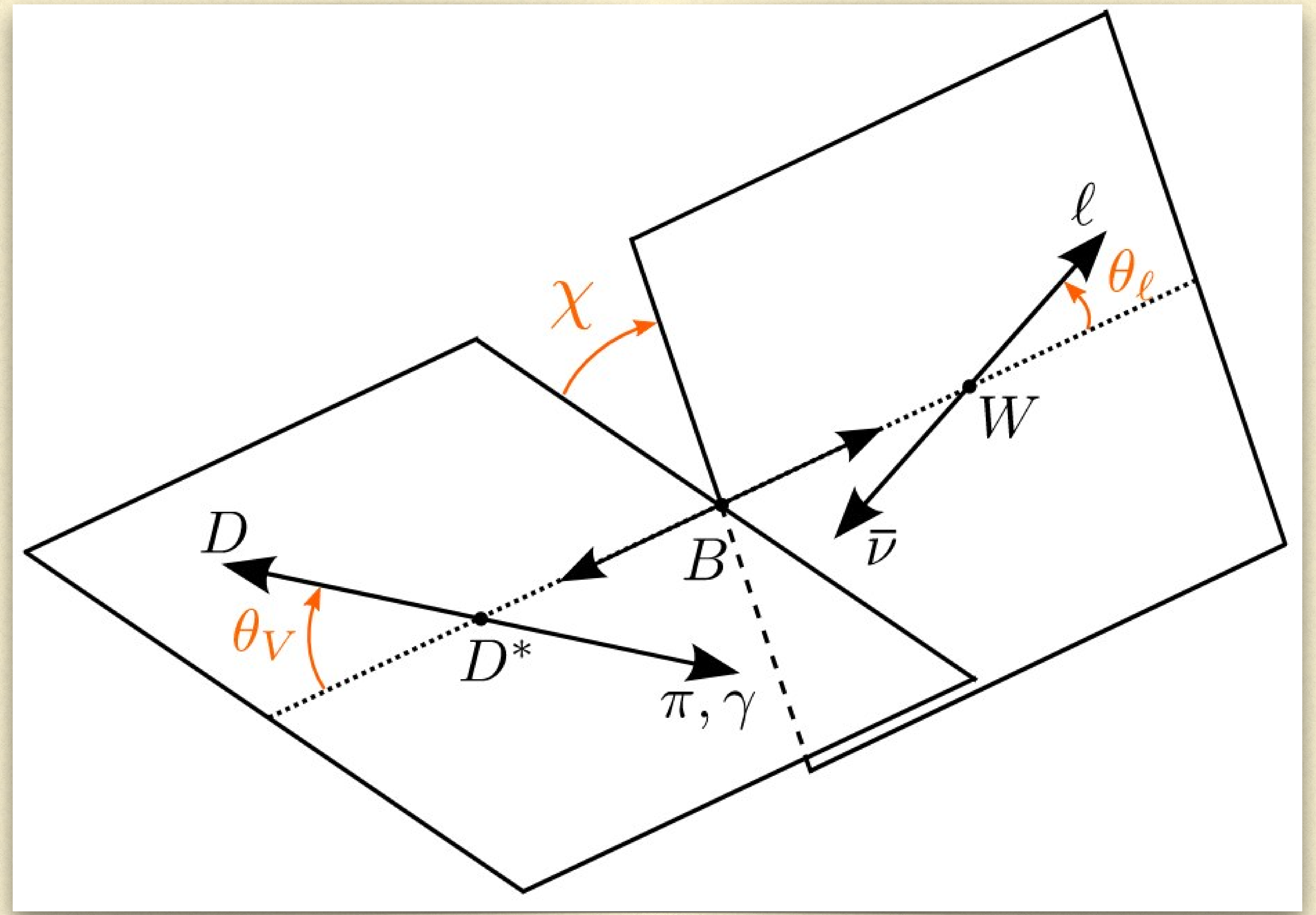
Form-factor decompositions: $B \to D\ell\nu$

$$\left\langle D(p_D) \middle| V^\mu \middle| \overline{B}(p_B) \right\rangle = \left(p_B^\mu + p_D^\mu - \frac{\Delta M^2}{q^2} q^\mu \right) f_+(q^2) + \frac{\Delta M^2}{q^2} f_0(q^2)$$

- Only vector current component of the weak current contributes
- Each tensor structure only couples to one component of the current
- Momentum dependence of terms in the decay rate due to J^P of the current components

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2 |V_{cb}|^2}{384\pi^3 M_B^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \left[\left(2 + \frac{m_l^2}{q^2}\right) \lambda^{3/2} |f_+|^2 + 3\frac{m_l^2}{q^2} \lambda^{1/2} |f_0|^2 \right]$$

Form-factor decompositions: $B \to D\pi\ell\nu$



M. Prim et al. (Belle), PRD 108, 012002 (2023)

- Similar to $B \to D^* \ell\nu$ form factors
- Each structure of the axial current only couples to one polarization of the current
- Vectors $L_\mu^{(l)}$ constructed to lead to correct angular dependence of a given partial wave
- Interference between partial waves if not integrating over angles

Form-factor decompositions: $B \to D\pi\ell\nu$

$$\langle D(p_D)\pi(p_\pi)|V^\mu|B(p_B)\rangle = i\epsilon^\mu_{\nu\rho\sigma}p^\rho_{D\pi}p^\sigma_B \sum_{l>0} L^{(l),\nu} g_l(q^2, M^2_{D\pi})$$

$$\langle D(p_D)\pi(p_\pi)|A^\mu|B(p_B)\rangle =$$
$$\frac{1}{2}\sum_{l>0}\left(L^{(l),\mu} + \frac{4}{\lambda_B}\left[(p_B\cdot p_{D\pi})q^\mu - (p_{D\pi}\cdot q)p^\mu_B\right]L^{(l),\nu}q_\nu\right) f_l(q^2, M^2_{D\pi})$$
$$+\frac{M_{D\pi}(M^2_B - M^2_{D\pi})}{\lambda_B}\left[(p_B + p_{D\pi})^\mu - \frac{M^2_B - M^2_{D\pi}}{q^2}q^\mu\right]\sum_{l>0} L^{(l),\nu}q_\nu \mathcal{F}_{1,l}(q^2, M^2_{D\pi})$$
$$+M_{D\pi}\frac{q^\mu}{q^2}\sum_{l>0} L^{(l),\nu}q_\nu \mathcal{F}_{2,l}(q^2, M^2_{D\pi})$$

$$L^{(l)}_\mu q^\mu \propto P_l(\cos\theta) \quad L^{(l)}_\mu p^\mu_{D\pi} = 0$$

$$\frac{d^2\Gamma}{dM^2_{D\pi}dq^2} = \frac{G_F^2|V_{cb}|^2}{(4\pi)^5}M_B M^2_{D\pi}\sum_{l>0} W^{2l+1}\left[\frac{4(M^2_B - M^2_{D\pi})^2}{3(2l+1)}\frac{|\mathcal{F}_{1,l}|^2}{\lambda_B} + \frac{l(l+1)}{(2l+1)}q^2\left(|g_l|^2 + \frac{|f_l|^2}{\lambda_B}\right)\right]$$

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Connecting form factors to perturbative quantities

$$\Pi^{L/T}_{(J)}(q) \equiv i \int d^4x\, e^{iq\cdot x} \left\langle 0 \right| J^{L/T}(x)\, J^{L/T}(0) \left| 0 \right\rangle$$

$$\chi^{L}_{(J)}(Q^2) \equiv \left.\frac{\partial \Pi^{L}_{(J)}}{\partial q^2}\right|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty dq^2 \frac{\mathrm{Im}\Pi^{L}_{(J)}(q^2)}{(q^2-Q^2)^2}$$

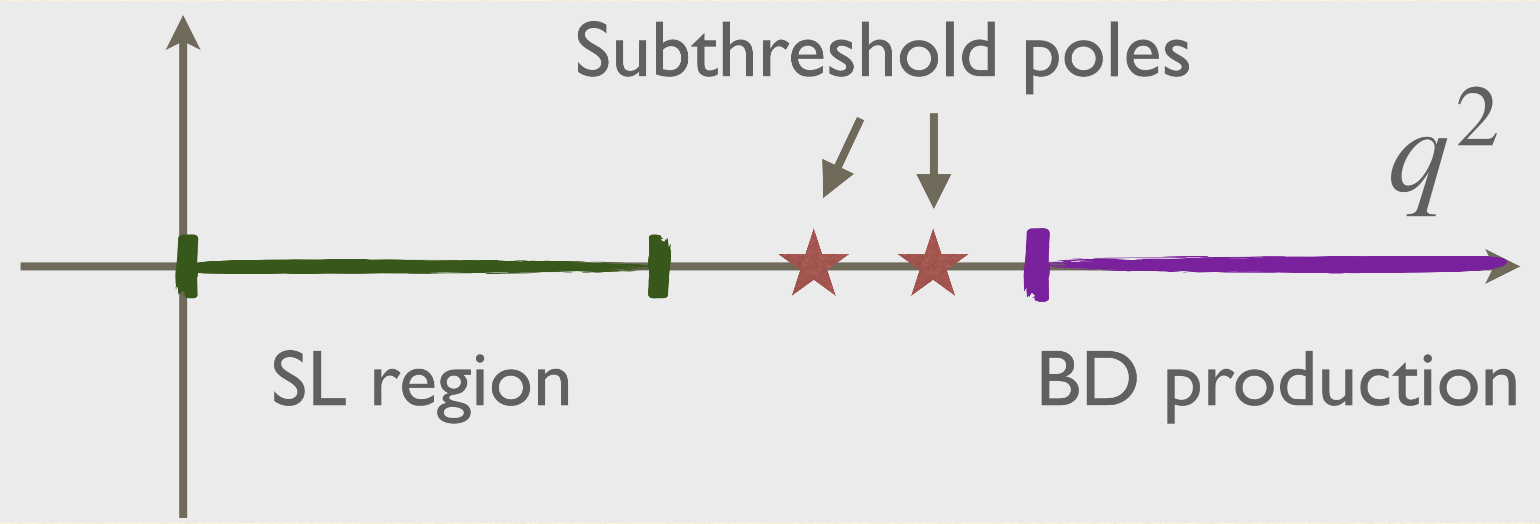
$$\chi^{T}_{(J)}(Q^2) \equiv \left.\frac{1}{2}\frac{\partial^2 \Pi^{T}_{(J)}}{\partial (q^2)^2}\right|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty dq^2 \frac{\mathrm{Im}\Pi^{T}_{(J)}(q^2)}{(q^2-Q^2)^3}$$

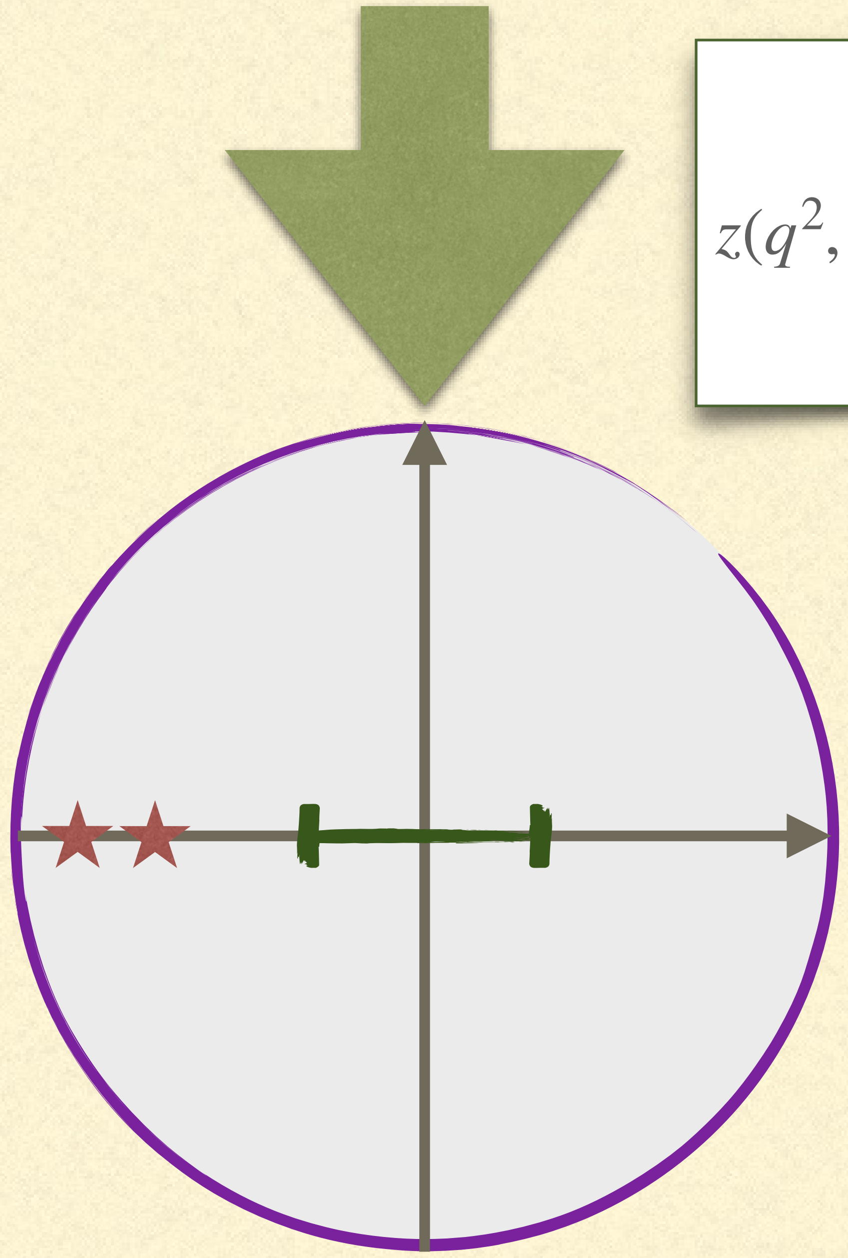
- Starting point: once and twice subtracted dispersion relations [Boyd, Grinstein, Lebed]
- Susceptibilities perturbatively computable for large space-like Q^2 or at $Q^2 = 0$ if heavy quarks involved
- Optical theorem allows to write the imaginary part as sum over all possible final states
- Neglecting a final state leads to an inequality

$$\mathrm{Im}\Pi^{T/L}_{(J)} = \frac{1}{2}\sum_X \int \mathrm{dPS}\, P^{\mu\nu}_{T/L} \left\langle 0 \left| J_\mu \right| X \right\rangle \left\langle X \left| J_\nu \right| 0 \right\rangle \delta^{(4)}(q - p_X)$$

$$\mathrm{Im}\Pi^{T}_{(V)}\,|_{BD} = K(q^2)\, |f_+(q^2)|^2$$

Conformal mapping, outer functions and all that





$$z(q^2, q_0^2) = \frac{\sqrt{q_+^2 - q^2} - \sqrt{q_+^2 - q_0^2}}{\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2}}$$

$$1 \geq \frac{1}{2\pi i} \oint \frac{\mathrm{d}z}{z} \left| B(z)\Phi(z)f(z) \right|^2$$

$$f(z) = \frac{1}{\Phi(z)B(z)} \sum_{i=0}^{\infty} a_i z^i \qquad 1 \geq \sum_{i=0}^{\infty} |a_i|^2$$

- Mapping q^2 to the dimensionless variable z transforms integration region to unit circle
- In this form it is evident that our FFs live in the Hardy space H^2
- Insert Blaschke products to get rid of subthreshold poles and zeroes in kinematic factors
- Series expand product
- Semileptonic region: $|z| < 1$

Unitarity bounds on $B \to D\pi\ell\nu$ form factors

$$\mathrm{Im}\Pi_V^T\Big|_{D\pi} = \frac{1}{192\pi^3}\frac{M_B^4}{q^2}\int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} \mathrm{d}M_{D\pi}^2\, M_{D\pi}^2 \sum_{l>0} W^{2l+1}\frac{l+1}{l(2l+1)}|g_l|^2$$

$$\mathrm{Im}\Pi_A^T\Big|_{D\pi} = \frac{1}{192\pi^3}\frac{M_B^4}{q^2}\int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} \mathrm{d}M_{D\pi}^2 \left(\frac{M_{D\pi}^2}{\lambda_B}\sum_{l>0}\frac{W^{2l+1}}{2l+1}\left(\frac{|\mathcal{F}_{1,l}|^2}{q^2}+\frac{l+1}{l}|f_l|^2\right)+W\lambda_B\frac{|f_+|^2}{q^2M_B^2}\right)$$

$$\mathrm{Im}\Pi_A^L\Big|_{D\pi} = \frac{1}{64\pi^3}\frac{M_B^4}{q^4}\int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} \mathrm{d}M_{D\pi}^2 \left(M_{D\pi}^2\sum_{l>0}\frac{W^{2l+1}}{2l+1}|\mathcal{F}_{2,l}|^2+W\frac{(M_B^2-M_{D\pi}^2)^2}{M_B^2}|f_0|^2\right)$$

$$f^{(l)}(q^2, M_{D\pi}^2) = \hat{f}^{(l)}(q^2, M_{D\pi}^2)g^{(l)}(M_{D\pi}^2) \approx \tilde{f}^{(l)}(q^2)g^{(l)}(M_{D\pi}^2)$$

$$1 \geq \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{M_B^4}{192\pi^3\chi}\frac{C^{(l)}}{(2l+1)(q^2)^a}\mathcal{I}^{(l)}_{(b,c,d)}(q^2)\,|\tilde{f}^{(l)}(q^2)|^2$$

- Two integrals involved
- If $M_{D\pi}^2$ integration can be carried out, we are back to standard case
- Watson-Migdal theorem allows factorization of final-state interactions from weak decay
- Remaining dependence on hadronic invariant mass often found to be small

The simple case: Breit-Wigner x Blatt-Weisskopf

$$f_l(q^2, M_{D\pi}^2) \approx \frac{\hat{f}_l(q^2) X^{(l)}(|\vec{p}_D| r_{BW}, |\vec{p}_{D,0}| r_{BW})}{(M_{D\pi}^2 - M_R^2) + i M_R \Gamma(M_{D\pi}^2)}$$

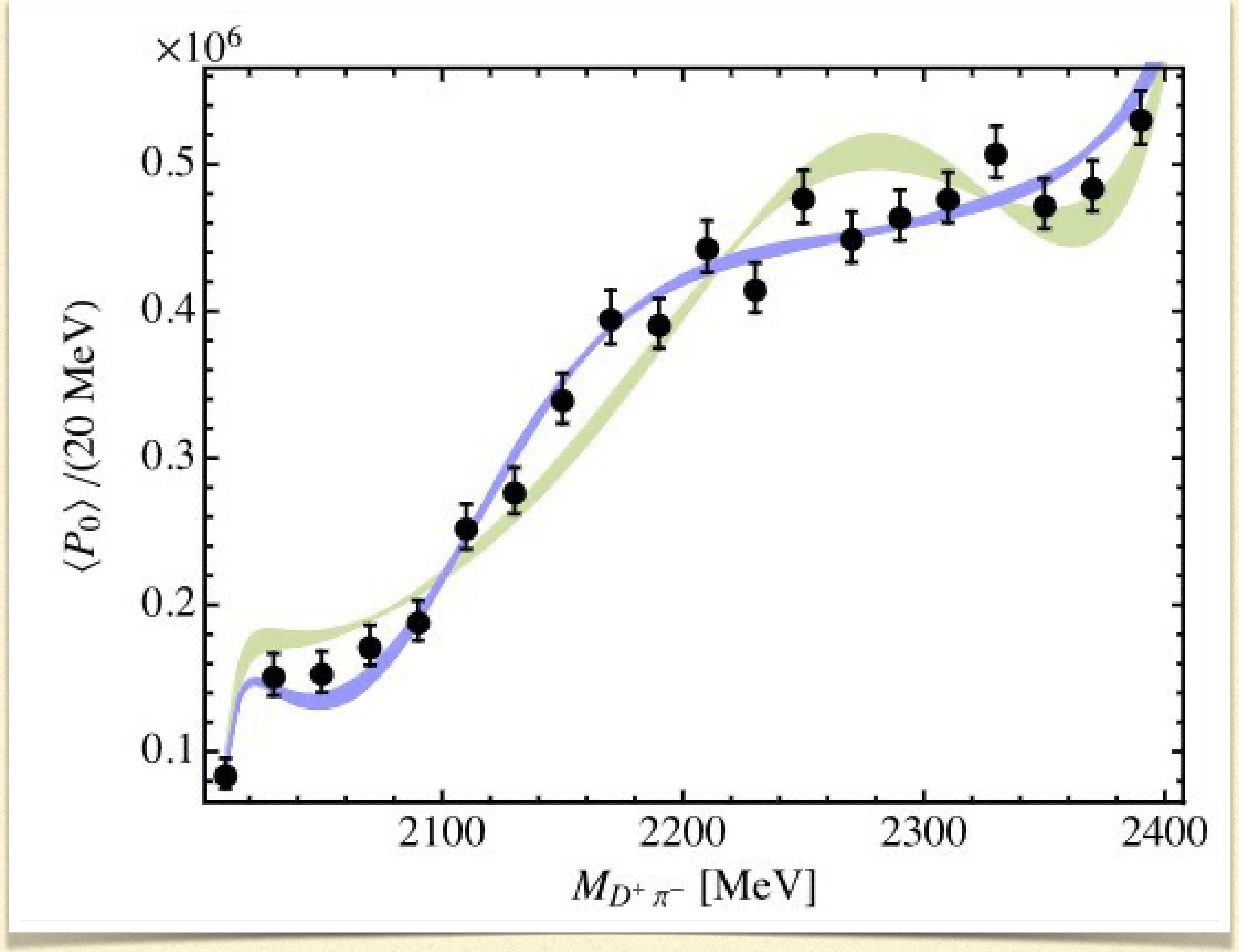
$$X^{(0)}(z, z_0) = 1$$

$$X^{(1)}(z, z_0) = \sqrt{(1 + z_0)/(1 + z)}$$

$$X^{(2)}(z, z_0) = \sqrt{(9 + 3z_0^2 + z_0^4)/(9 + 3z^2 + z^4)}$$

- In the simplest cases we can assume a relativistic Breit-Wigner function
- Tails can be too long
- Angular-momentum dependent Blatt-Weiskopf damping factors
- Free parameter r_{BW}
- Common practice for not too broad resonances in the literature

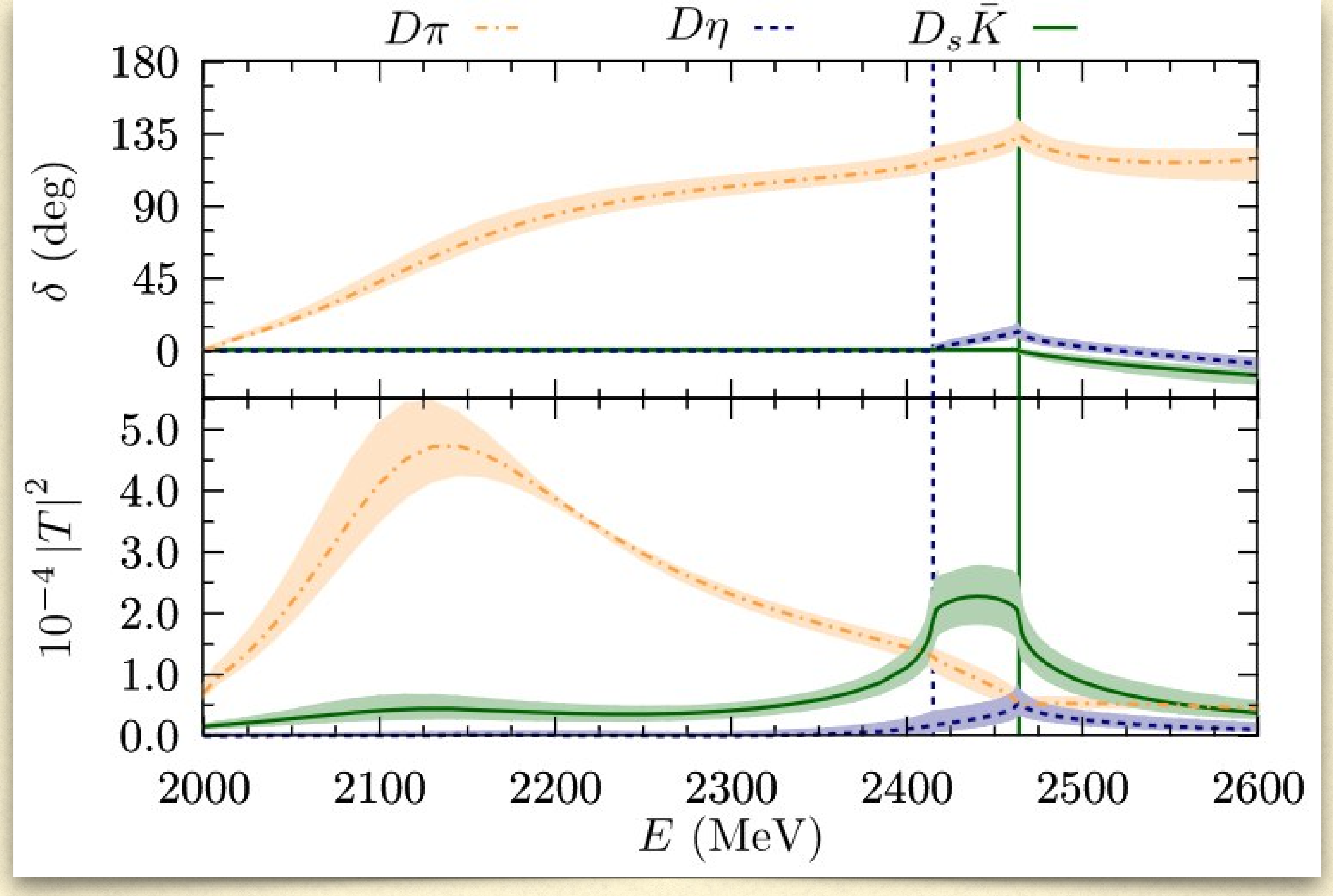
Where is the lightest charmed scalar meson?



Du, Guo, Hanhart, Kubis, Meissner, PRL 126 192001 (2021)

- Not all resonances are well described by Breit-Wigner functions
- Calculations within unitarized chiral perturbation theory suggest that the D_0^* is one of them
- Recent analyses point to two poles, one with low mass, $D_0^*(2100)$, and one at higher mass: $D_0^*(2450)$
- Nonleptonic B decays strongly favour this picture over the standard one

Omnès factors and matrices



Albaladejo, Fernandez-Soler, Guo, Nieves, PLB 767 (2017) 465-469
Input from: Liu, Orginos, Guo, Hanhart, Meissner, PRD 87, 014508 (2013)

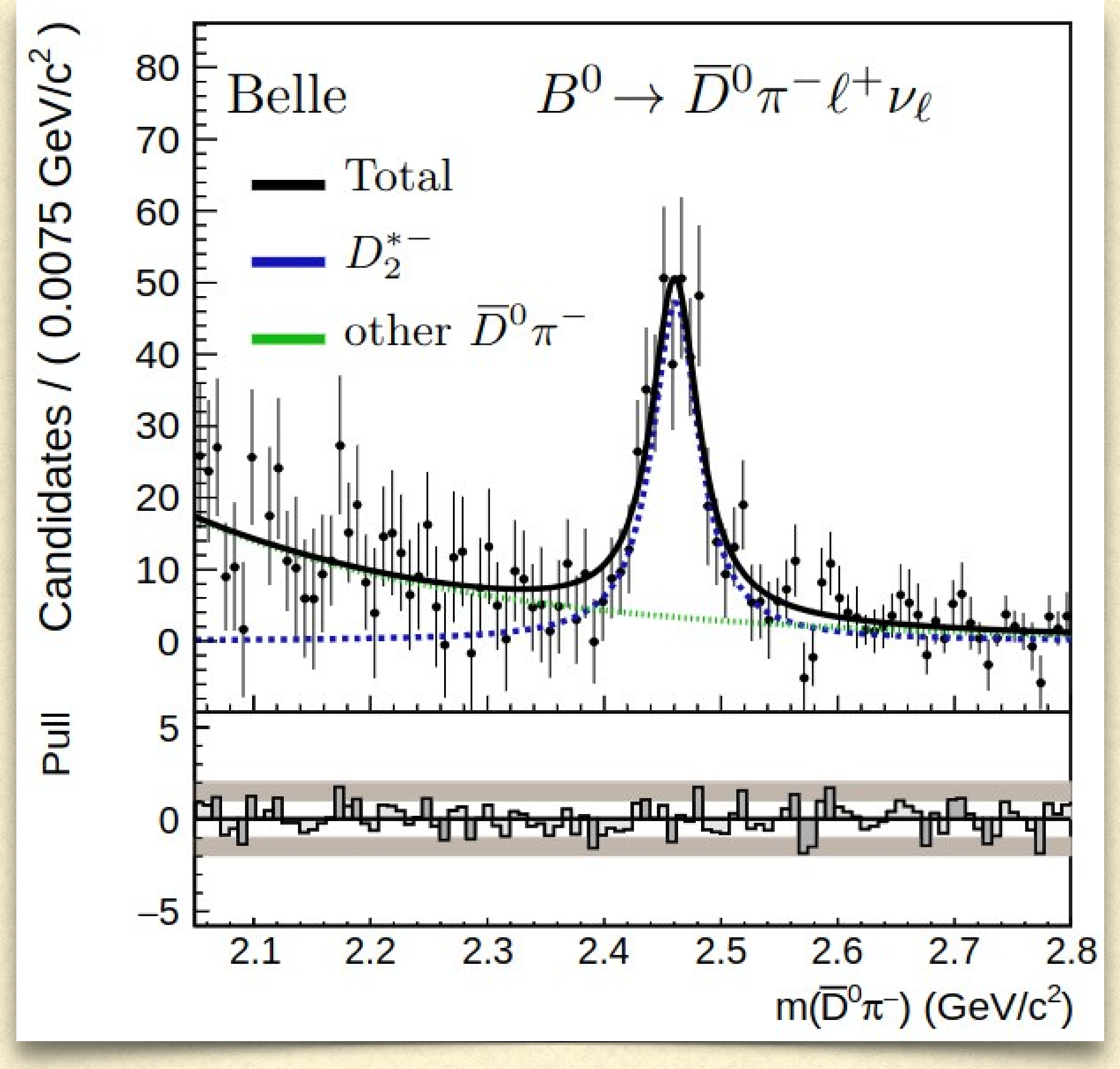
$$\mathrm{Im}\vec{f}(q^2, M_{D\pi}^2 + i\epsilon) = T^*(M_{D\pi}^2 + i\epsilon)\Sigma(M_{D\pi}^2)\vec{f}(q^2, M_{D\pi}^2 + i\epsilon)$$

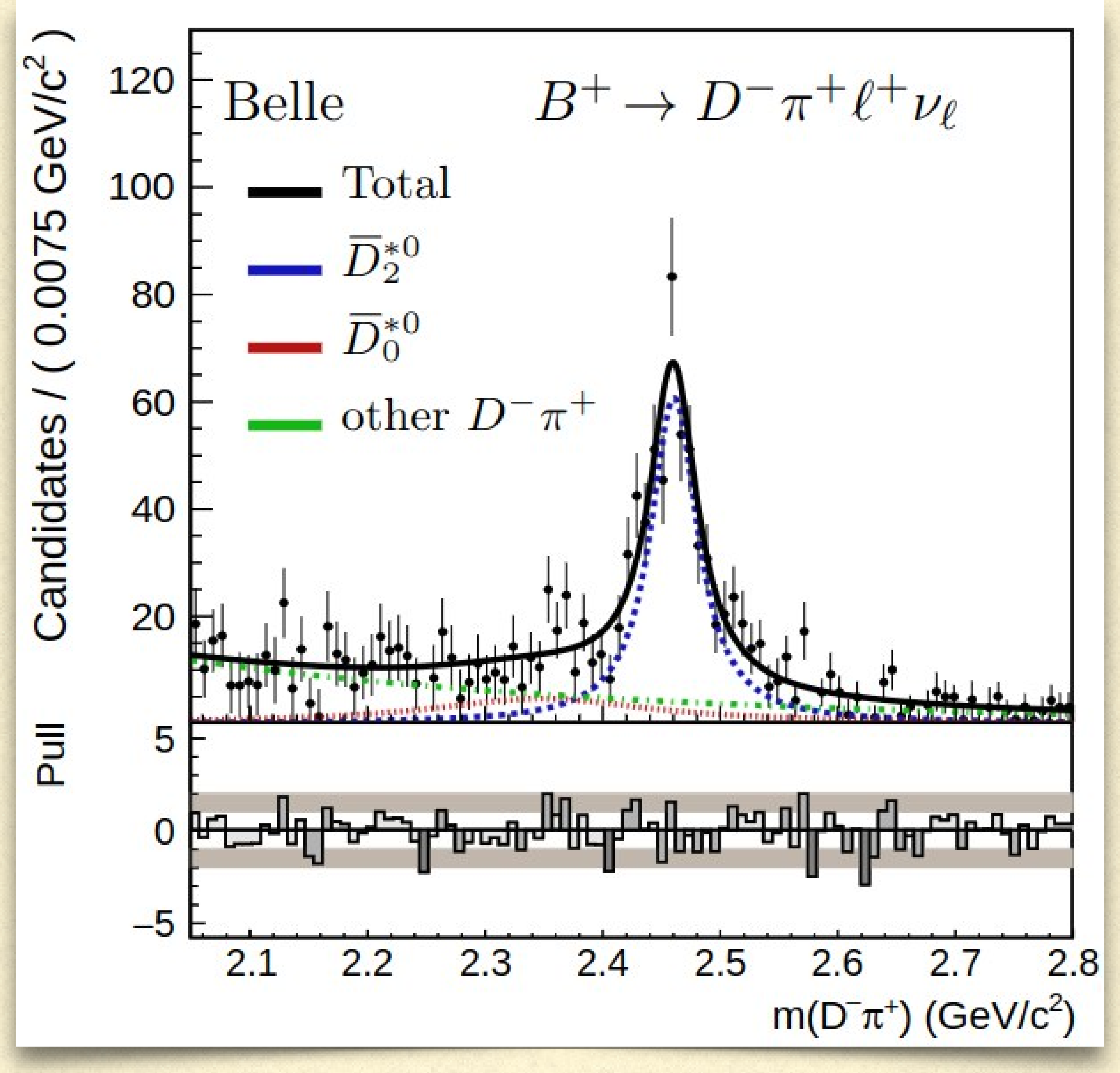
$$\vec{f}(q^2, M_{D\pi}^2) = \Omega(M_{D\pi}^2)\vec{P}(q^2, M_{D\pi}^2)$$

$$\mathrm{Im}\Omega(s + i\epsilon) = \frac{1}{\pi}\int_{s_{\mathrm{thr}}}^{\infty} \frac{T^*(s')\Sigma(s')\Omega(s')}{s' - s - i\epsilon}\mathrm{d}s'$$

- One can relate the $M_{D\pi}^2$-dependence to the Omnès-Matrix
- The Omnès-Matrix is related to the Scattering-Matrix
- In this case: Lattice + Unitarized ChiPT

The available data





F. Meier et al. (Belle), PRD 107, 092003 (2023)

- Measurements by Belle and Babar of the invariant mass distribution
- Model-dependent measurement of the q^2 and $|cos\theta|$ distribution for $B \to D_2^*\ell\nu$
- Masses and widths from a plethora of inclusive measurements or $B \to D\pi\pi$ decays

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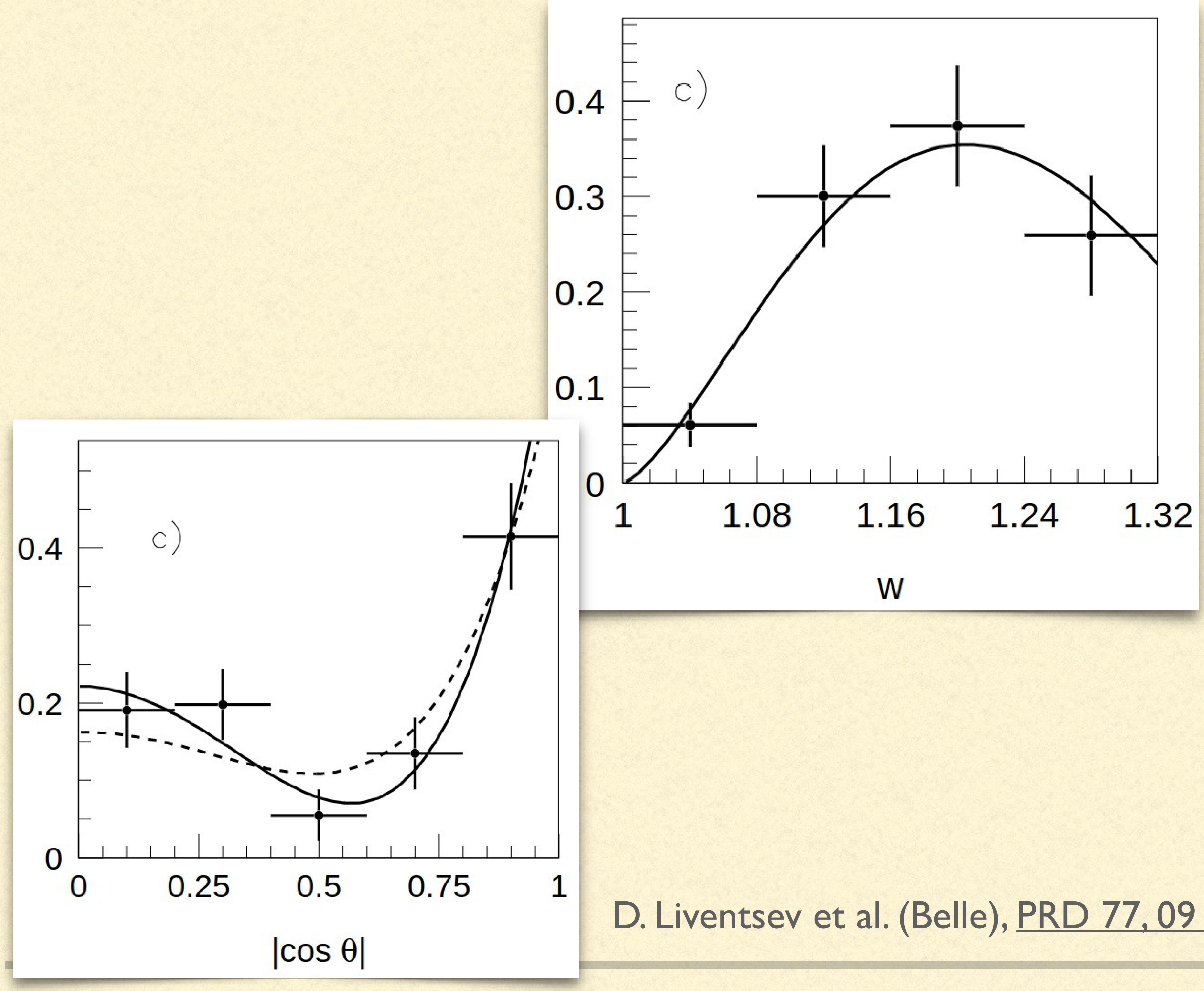

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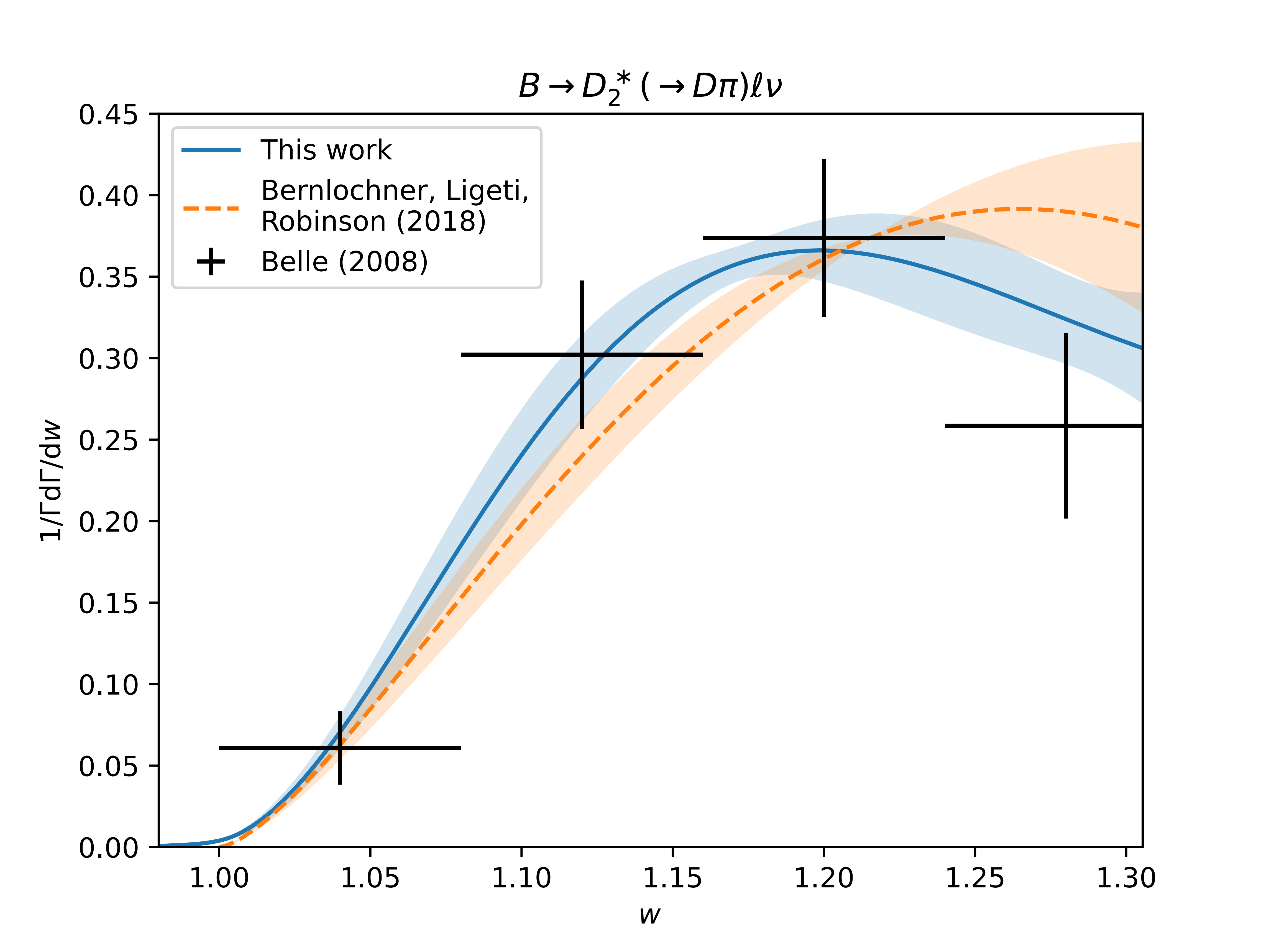
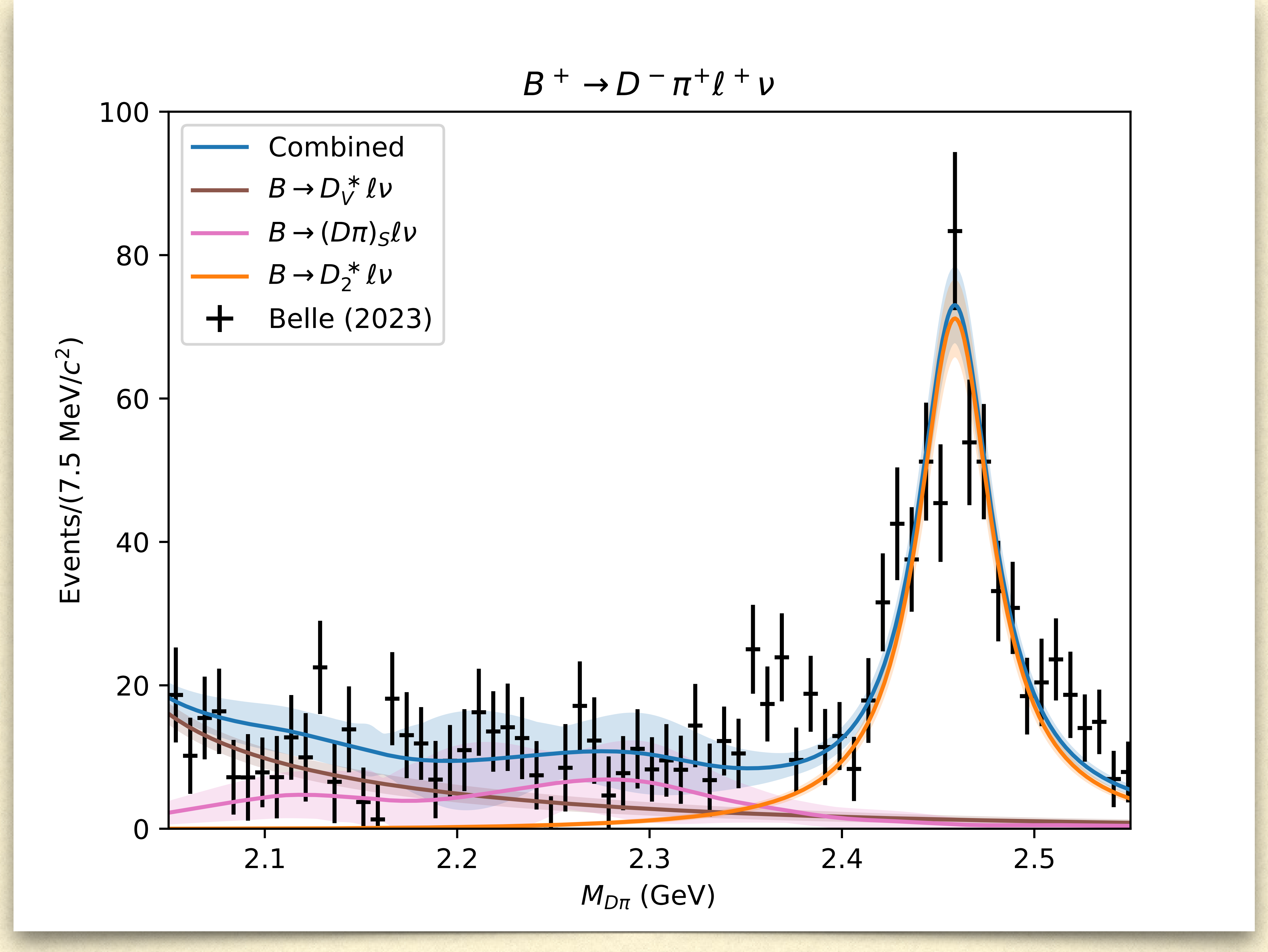
D. Liventsev et al. (Belle), PRD 77, 091503 (2008)

Fitting the D_2^* spectra



- Breit-Wigner x Blatt-Weisskopf to describe lineshape
- Fit the three relevant FFs up to linear order in z
- Angular spectrum fixes relative size of $\mathscr{F}_1$ to a linear combination of f and g
- Since no information on $\cos\theta_l$ spectrum f and g are mostly degenerate
- Some tension w.r.t. to shape of q^2 distribution with Bernlochner, Ligeti, Robinson

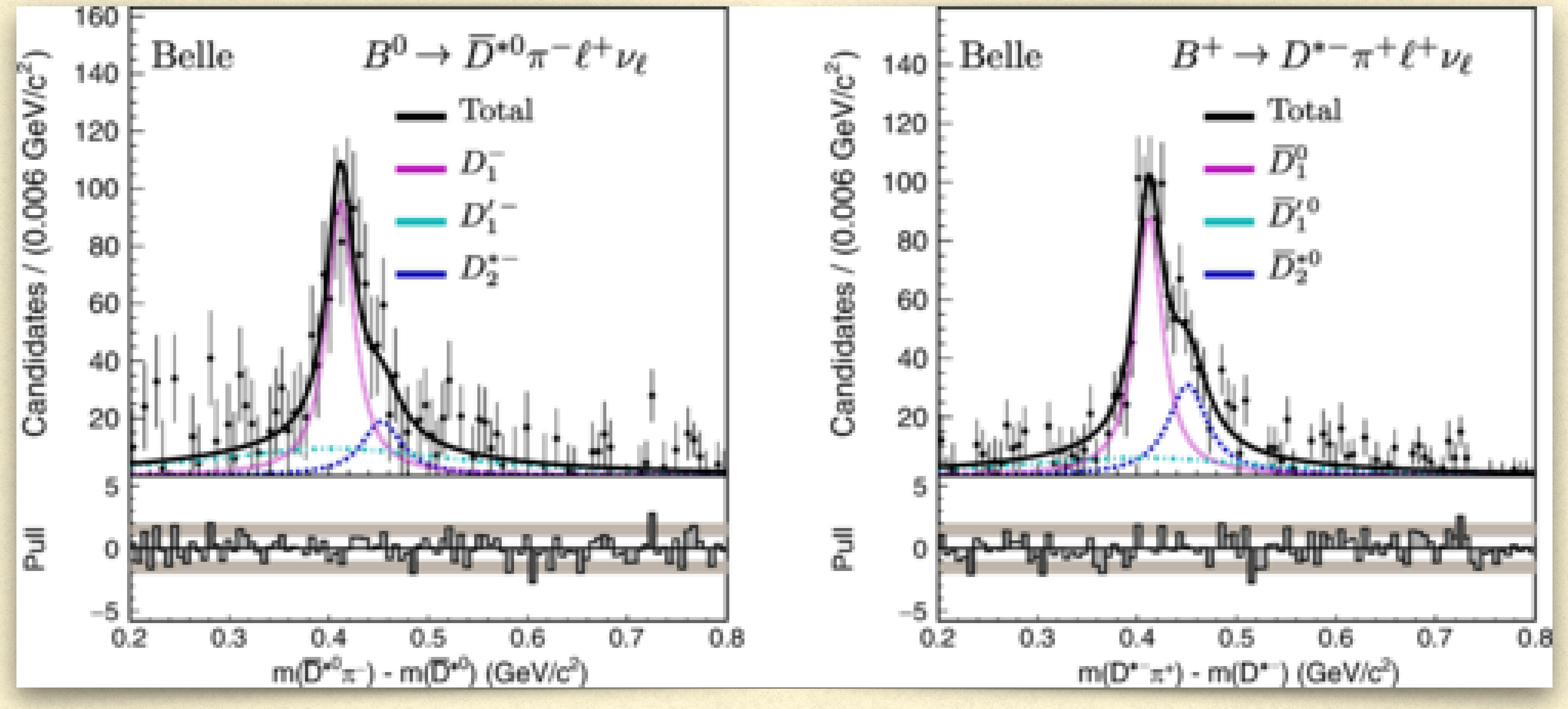
Fitting the mass spectra



- Simultaneously fit both charge modes measured by Belle
- χ^2_{aug}/dof $= 124.4/133$
- Fit with Breit-Wigner for $D_0^*(2300)$ only slightly worse, at the cost of longer D^* tail
- In both cases smaller S-wave BF than assumed by PDG
- The resulting branching fractions for $B \to D_s K \ell \nu$ and $B \to D \eta \ell \nu$ are $\mathcal{O}\left(10^{-5}\right)$

How can we do better?

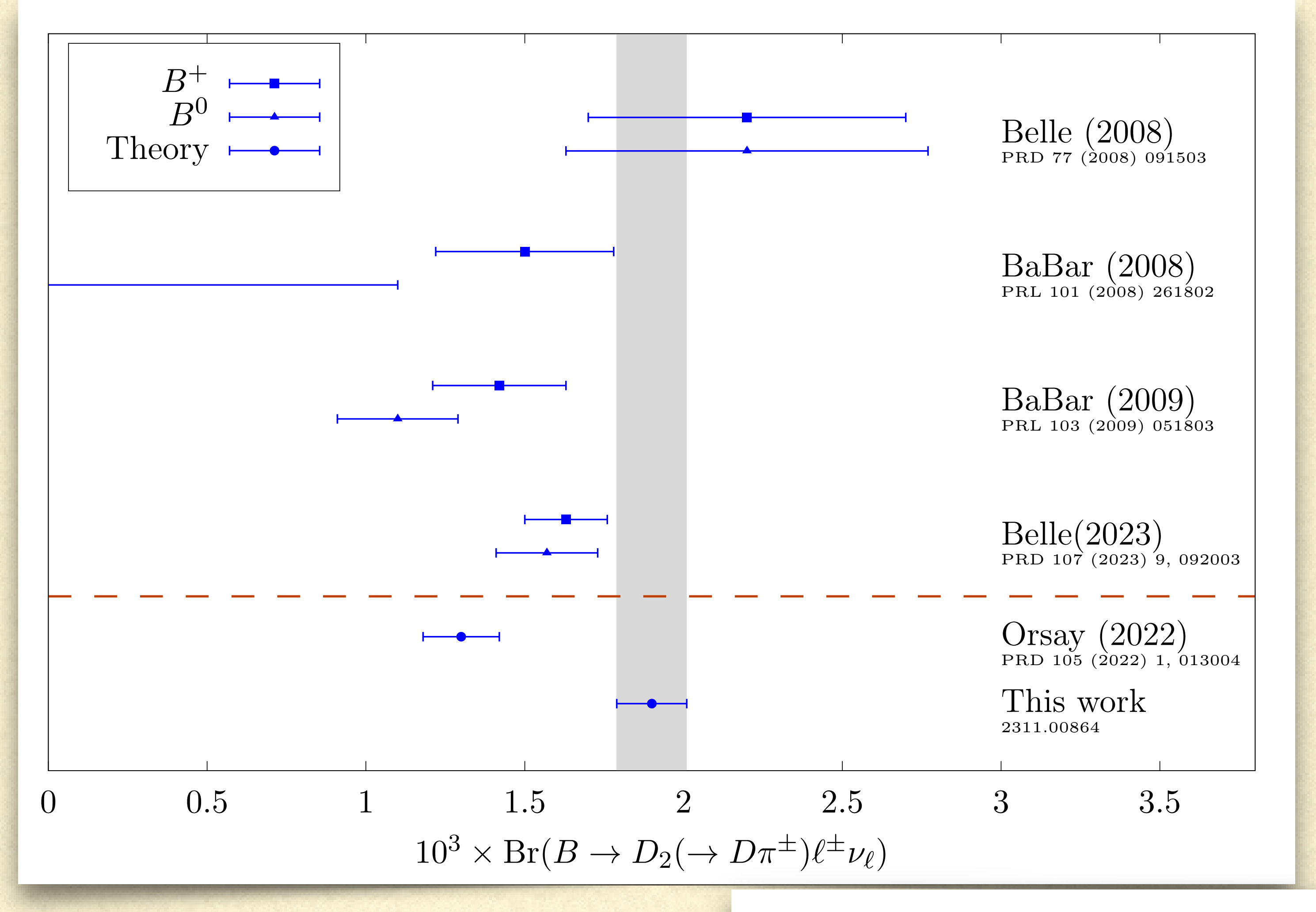
$$\mathscr{A}_{FB}^{D} = \frac{1}{\mathrm{d}\Gamma/\mathrm{d}M^2_{D\pi}}\left(\int_0^1 \mathrm{d}\cos\theta \frac{\mathrm{d}^2\Gamma}{\mathrm{d}M^2_{D\pi}\mathrm{d}\cos\theta} - \int_{-1}^0 \mathrm{d}\cos\theta \frac{\mathrm{d}^2\Gamma}{\mathrm{d}M^2_{D\pi}\mathrm{d}\cos\theta}\right)$$

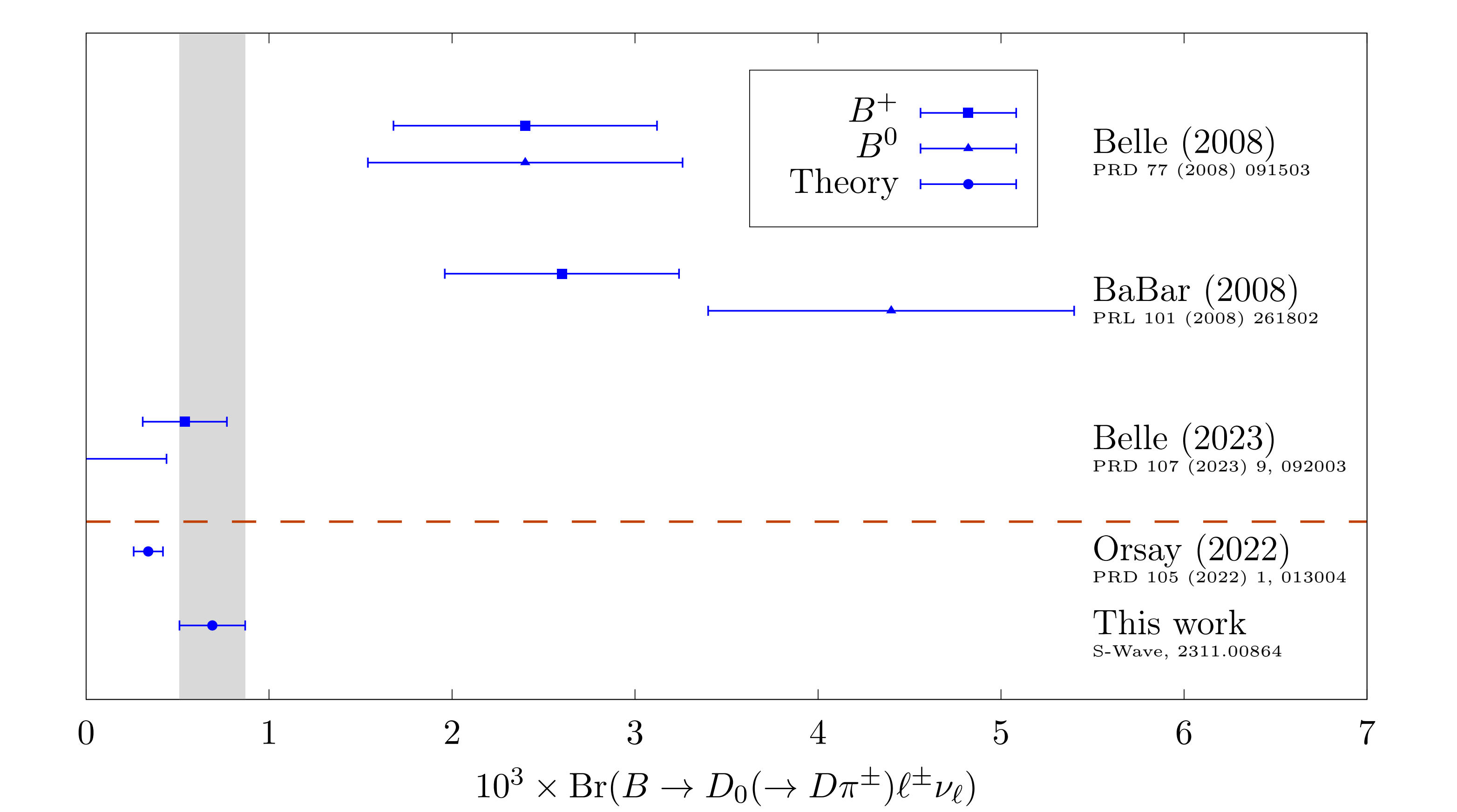


F. Meier et al. (Belle), PRD 107, 092003 (2023)

- Provide z-expansion coefficients for $B \to D_2^*\ell\nu$ FFs and implementation in EvtGen
- Study the forward-backward asymmetry of the D-meson to extract the $D\pi$ S-wave phase from experiment
- Extend to $B \to D^*\pi\ell\nu$
- Include LCSR results and HQET constraints in fits
- Find a better handle on neglected terms

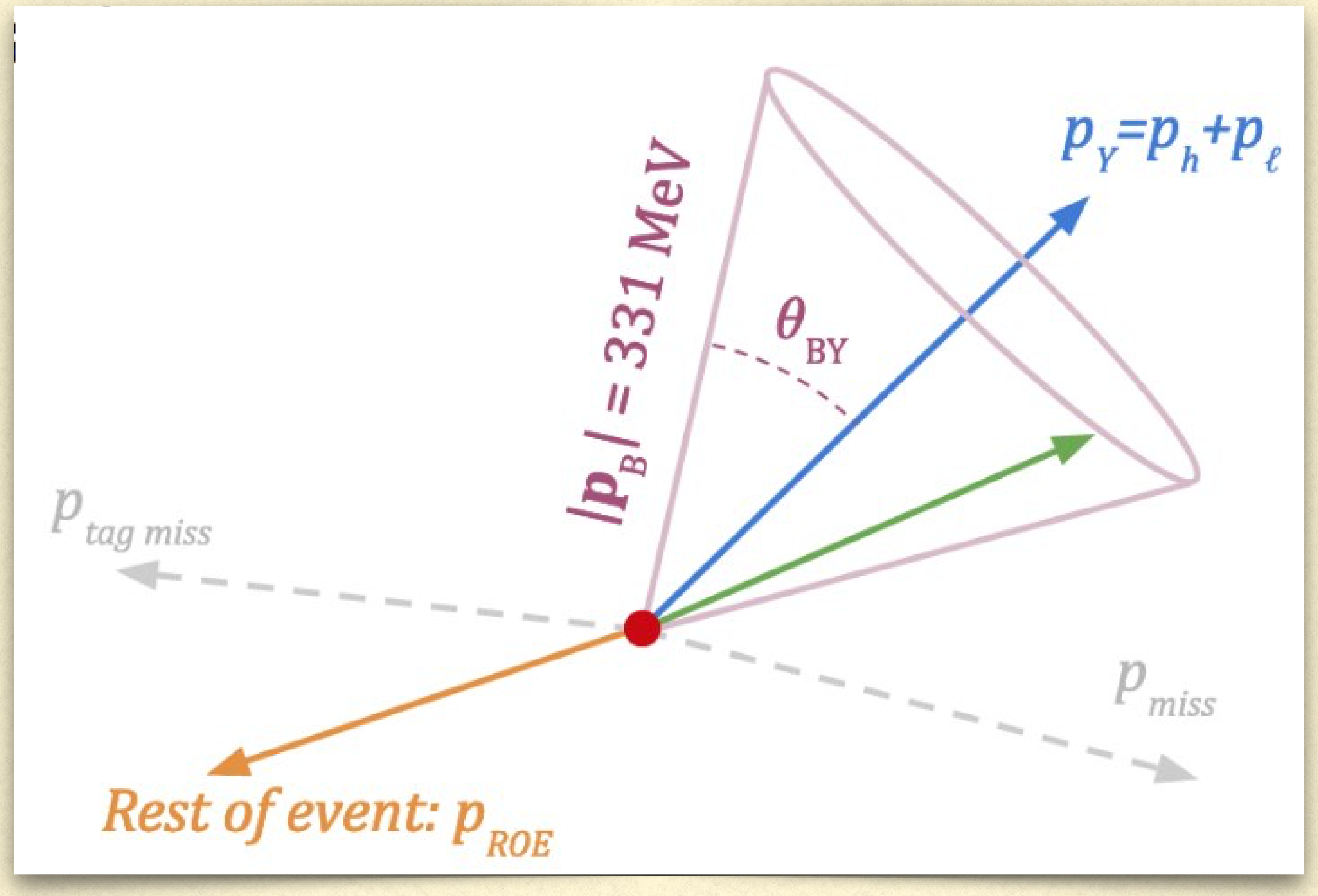
What can the experiments do?





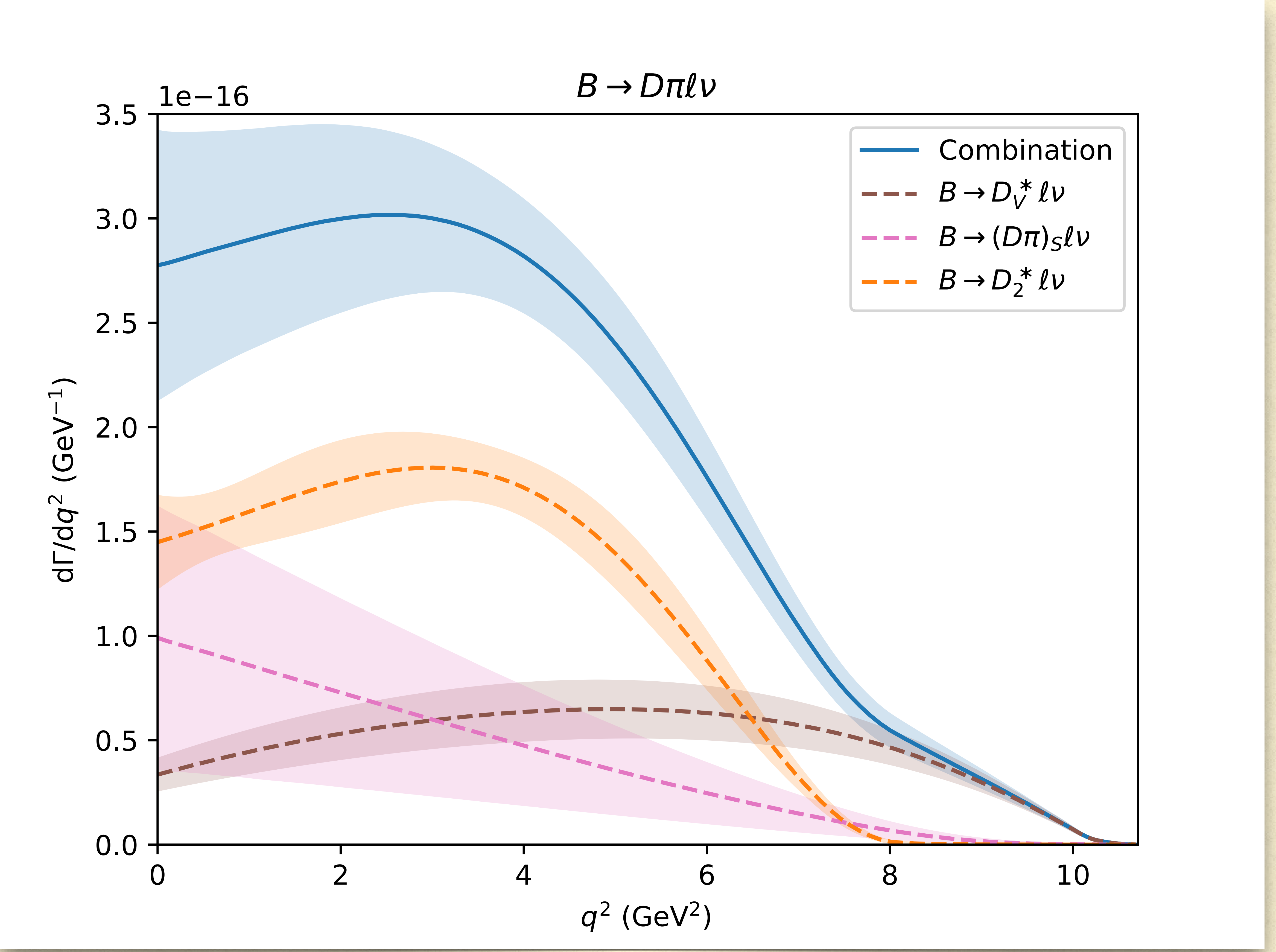
- Provide more model-independent measurements, i.e. spectra, not just branching fractions
- Differential measurements, especially in $\cos\theta$
- A measurement of the forward-backward asymmetry of the D-meson in the low $M_{D\pi}^2$ region
- A study of $B \to D_s K\pi$ could help to determine which other resonances contribute to $B \to D_s K \ell \nu$

What can the experiments do?



- Provide more model-independent measurements, i.e. spectra, not just branching fractions
- Differential measurements, especially in $\cos\theta$: Can this be done with inclusive tagging at Belle II?
- A measurement of the forward-backward asymmetry of the D-meson in the low $M^2_{D\pi}$ region
- A study of $B \to D_s K\pi$ could help to determine which other resonances contribute to $B \to D_s K \ell \nu$

Summary



- There are interesting semileptonic decays beyond $B \to D\ell\nu$ and $B \to D^*\ell\nu$
- To maximise what we can get from measurements of $R\left(D^{(*)}\right)$ or inclusive measurements we need to understand them
- Interesting connection to hadron spectroscopy
- A lot remains to be done, both in experiment and theory
- Some developments might prove useful to the study of semileptonic D-meson decays