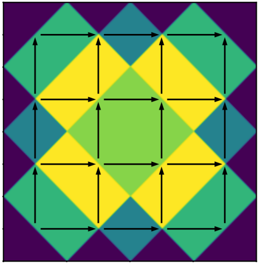
Hadron Resonances from Lattice QCD

Status and Prospects



Carsten Urbach

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

⟨NuMeriQS⟩

UNIVERSITÄT BONN

Quantum Chromodynamics

$$S[A_\mu, \bar{\psi}, \psi] = \int d^4x \ \left\{ \frac{1}{4} G_{\mu\nu}^2 + \bar{\psi}_q \left(\mathrm{i}\gamma_\mu D_\mu + m_q \right) \psi_q \right\}$$

- astonishingly simple action, intriguingly complex dynamics
- running coupling: QCD is non-perturbative at low energies

$\Rightarrow$ hadron spectrum requires non-perturbative methods

Lattice QCD Regularisation

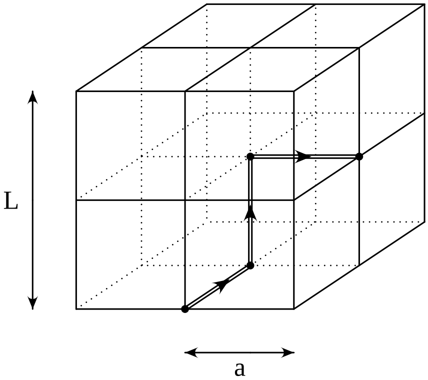
- quantum field theory requires regularisation

- lattice regularisation:

$\Rightarrow$ discretise space-time

  - hyper-cubic $L^3 \times T$-lattice with lattice spacing a

  $\Rightarrow$ momentum cut-off: $k_{\text{max}} \propto 1/a$
  - derivatives $\Rightarrow$ finite differences
  - integrals $\Rightarrow$ sums
  - gauge potentials A_μ in $G_{\mu\nu} \Rightarrow$ link matrices U_μ ('$\bullet\!\!\longrightarrow\!\!\bullet$')



- work in Euclidean space-time $\Rightarrow$ **use Monte Carlo**

Lattice QCD Regularisation

- Monte Carlo: access to equilibrium, vacuum properties
- fundamental observables: **Euclidean correlation functions**

$$\langle \mathcal{O}_i^\dagger(p,t)\,\mathcal{O}_j(p,t')\rangle \;\propto\; \sum_n c_{i,n}c_{j,n}e^{-E_n t}$$

- with interpolating operators $\mathcal{O}_i$ with certain quantum numbers
- simulations at bare parameters need to renormalise

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- **continuum limit**:

$$\lim_{a\to 0}$$

(i.e. at least 3 lattice spacing values)

- **infinite volume limit**:

$$\lim_{L\to\infty}$$

- **physical mass limit**:

$$\lim_{m_\ell\to m_\ell^{\text{phys}}} \quad \text{or} \quad M_\pi^2 \to (M_\pi^{\text{phys}})^2$$

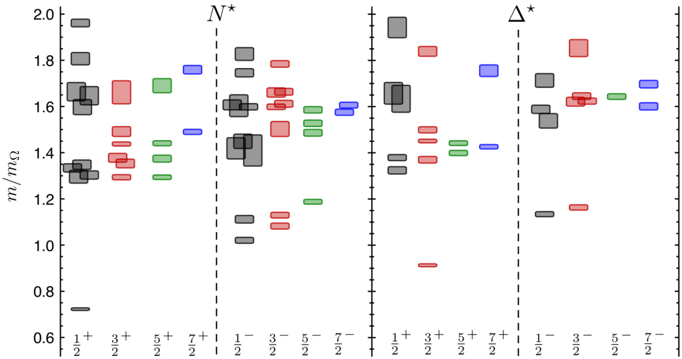
And then: Compute the Spectrum!

- excited baryon spectrum (2011)

  [Edwards et al., Phys.Rev.D 84 (2011) 074508]

- spin identified, 3 M_π values
- both parities up to $J = 7/2$

at $M_\pi \approx 400$ MeV



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  "... a counting of levels that is consistent with the non-relativistic qqq constituent quark model."

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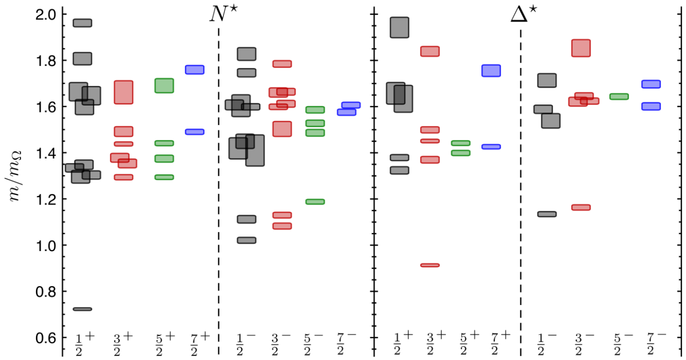

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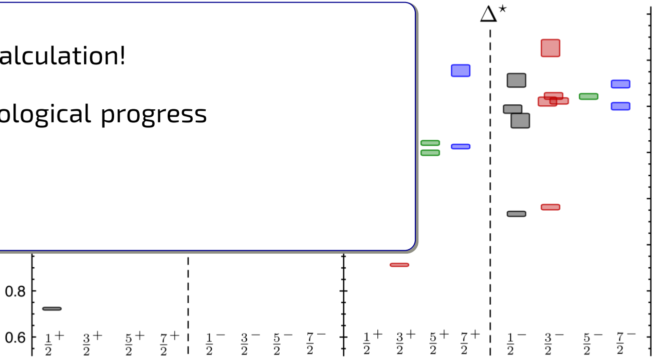
- significant methodological progress



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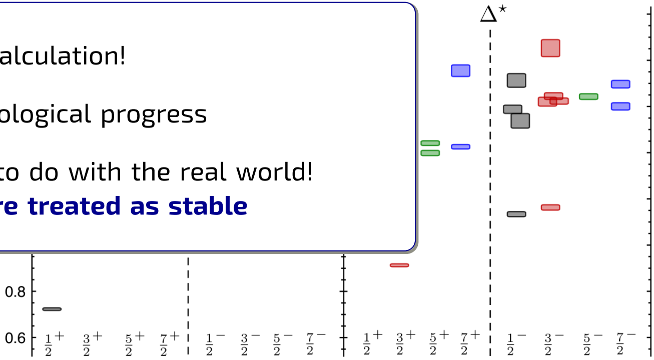
at $M_\pi \approx 400$ MeV

- really impressive calculation!
- significant methodological progress
- but has not much to do with the real world!
  unstable states are treated as stable



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Particle Interactions from Lattice QCD

- lattice stochastic methods:
  work in finite volume / Euclidean space-time

$\Rightarrow$ real valued, quantised eigenvalues of the lattice Hamiltonian
no continuum of states

- Maiani and Testa:
  interactions properties cannot be studied directly

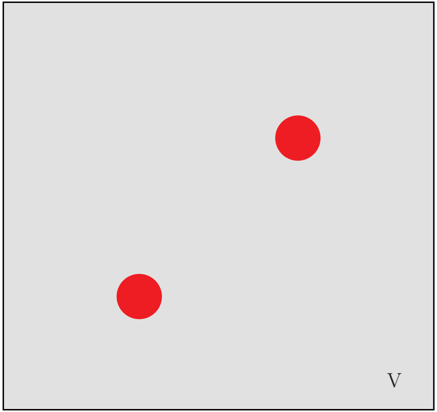
  [Maiani and Testa, (1990)]

$\Rightarrow$ there is no one-to-one correspondence of an energy level to a resonance state

- the connection is only provided by the Lüscher method!

Lüscher Method

finite volume: boon and bane!



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$\Rightarrow$ interaction probability very low

$\Rightarrow$ $E_{2p}(p=0) = 2E_{1p}$

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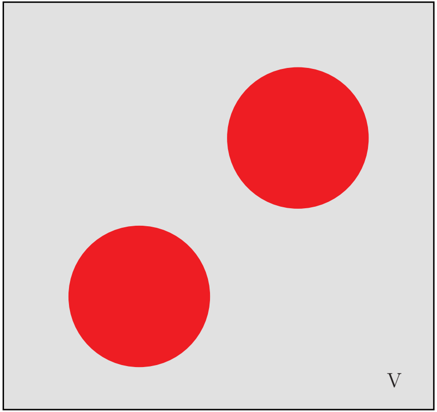

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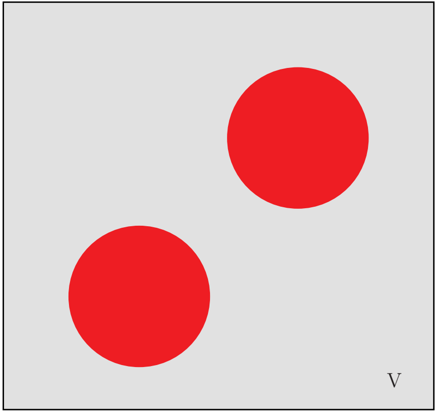

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- Lüscher: correction in $1/V$ related to scattering properties!

[Lüscher, 1986]

The $1+1$-dimensional Analog

- plane wave acquires phase shift $\delta(k)$
- finite extend L, periodic BC

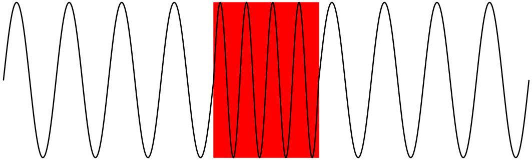
$$e^{\mathrm{i}kL+2\mathrm{i}\delta(k)} = e^{ik0} = 1$$

- quantisation condition

$$k_n L + 2\delta(k_n) = 2n\pi$$

- momenta k_n from dispersion relation

$$W_n = 2\sqrt{m^2 + k_n^2}$$

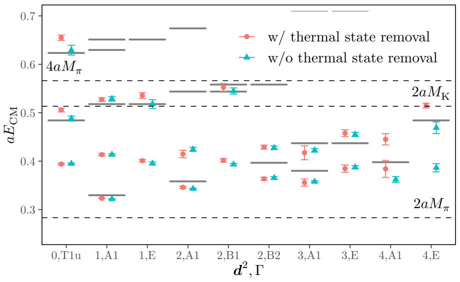


Procedure

1. determine non-interacting m
2. determine energies W_n
3. $W_n \quad \to \quad k_n$
4. $k_n \quad \to \quad \delta(k_n)$

The General Case

Lattice Energy Levels E



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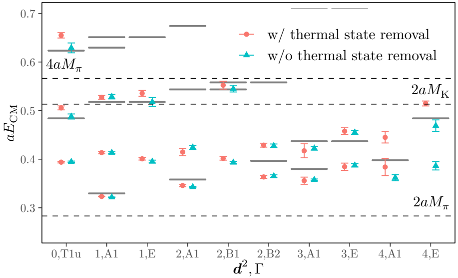

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$$\det\left[\mathcal{M}^{\Gamma,\mathbf{d}}(E) - \cot(\delta)\right] = 0$$

($\mathcal{M}$ Lüscher function)

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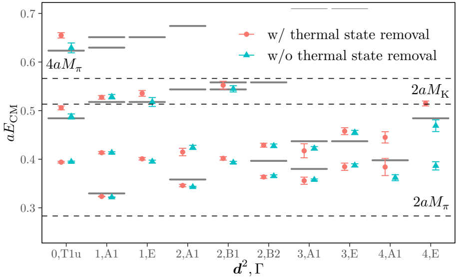

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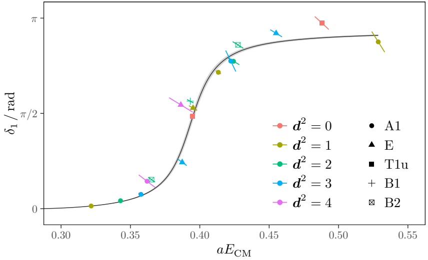

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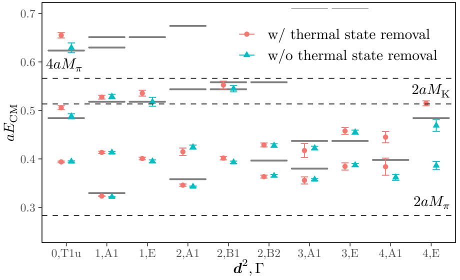

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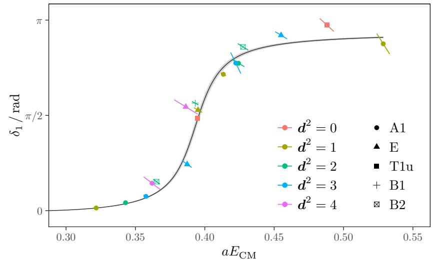

Parametrise Energy Dependence

e.g. Breit-Wigner or better

Lüscher Method: The Simplest Case

- consider only S-wave, no mixing, and assume finite range expansion

$$\Delta E \;=\; -\frac{2\pi a_0}{\mu L^3}\left(1 + c_1\frac{a_0}{L} + c_2\frac{a_0^2}{L^2} + c_3\frac{a_0^3}{L^3}\ldots\right)$$

- The scattering length a_0 can be determined by inverting this equation!
- c_i known, L the box extent, μ the reduced two particle non-interacting mass
- works excellent e.g. for $\pi\pi$ scattering with $I = 0, 2$

Example: Complex ϕ^4 Theory

- how well does this work?

  [Romero-Lopez, Rusetsky, CU, EPJC (2018)]

$\Rightarrow$ complex φ^4 theory as toy model

- lattice action

$$S = \sum_x \Big(- \kappa \sum_\mu (\varphi_x^\star \varphi_{x+\mu} + cc) + \lambda(|\varphi_x|^2 - 1)^2 + |\varphi_x|^2 \Big)$$

- big advantage: fast to simulate

$\Rightarrow$ can simulate basically arbitrary volumes

- and study the interaction of two scalar particles

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⇒ can simulate basically arbitrary volumes

- and study the interaction of two scalar particles

  (or three, four, five, ... scalar particles)

Example: Complex ϕ^4 Theory

- need to compute $\Delta E = E_2 - 2M_1$
- single particle energy from

$$C_1(t) = \sum_{t'} \sum_{x,y} \langle \hat{\mathcal{O}}_\varphi(\mathbf{x}, t') \hat{\mathcal{O}}_\varphi^\dagger(\mathbf{y}, t + t') \rangle \quad \overset{t\to\infty}{\propto} \quad e^{-M_1 t}$$

- n-particle energy from

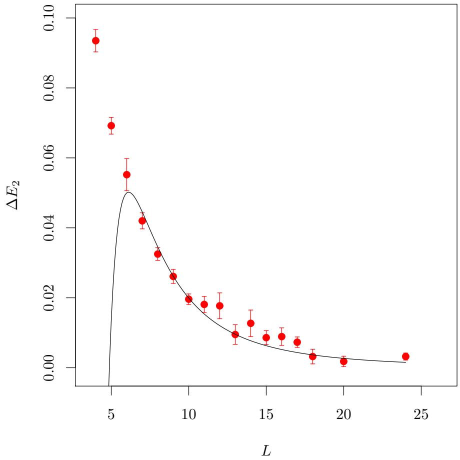
$$C_1(t) = \sum_{t'} \sum_{x,y} \langle \hat{\mathcal{O}}_{2\varphi}(\mathbf{x}, t') \hat{\mathcal{O}}_{2\varphi}^\dagger(\mathbf{y}, t + t') \rangle \quad \overset{t\to\infty}{\propto} \quad e^{-E_2 t} + \text{thermal pollutions}$$

- thermal pollutions due to finite time extend T and periodic BCs

$\Rightarrow$ have to be taken care of

Example: Complex ϕ^4 Theory

- compute ΔE as function of L
- for chosen bare parameters: repulsive interaction
- depending on fit range sensitive to a_0 or r
- for too small L description breaks down



$\Rightarrow$ ΔE_2 gives access to a_0 and r

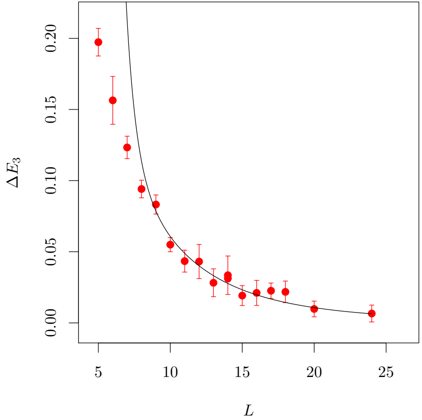
Example: Complex ϕ^4 Theory

- three particle formula (zero total momentum)

$$\Delta E_3 = E_3 - 3M_1 = -\frac{12\pi a_0}{M_1 L^3}(1+\ldots) - \frac{D}{48 M_1^3 L^6}$$

[see e.g. Sharpe 2017]

- D encodes three body interaction
- data well described
- a_0, r input from ΔE_2
- clear evidence for non-zero three particle interaction!



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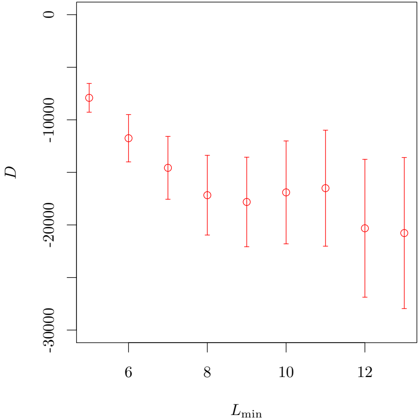


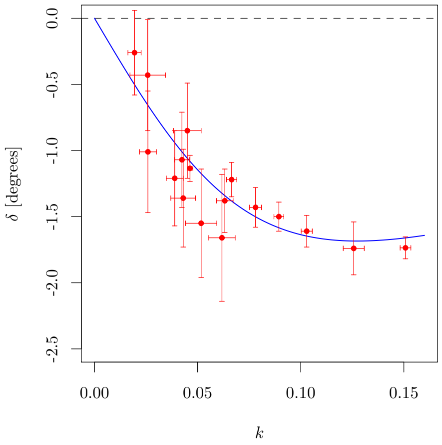
Example: Complex ϕ^4 Theory

- phase-shift reconstruction
- for the example: consider S-wave only
- determinant equation reduces to

$$\Rightarrow \quad \pi^{3/2}\,\mathrm{cot}(\delta_0) = Z_{00}(1, q^2)/q$$

Lüscher function Z

$\Rightarrow$ every energy level $\quad\rightarrow\quad$ one pair $(\delta_0(k), k)$



- blue line reconstructed from

$$\mathrm{cot}\,\delta \;=\; \frac{1}{a_0} + \frac{r}{2}k^2$$

Results: Well Separated Resonances

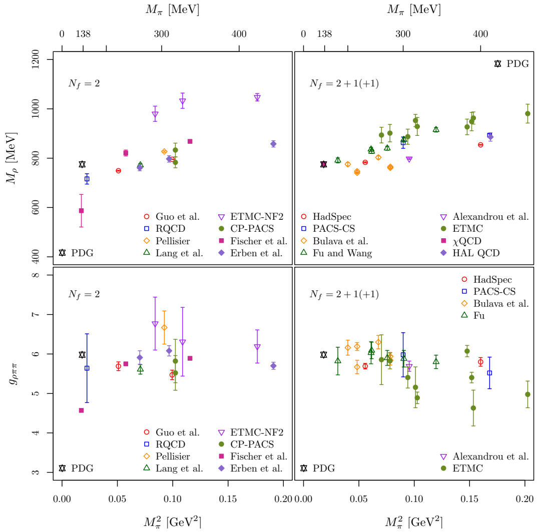
- status of LQCD results for hadron resonances
- comparisons compiled in a review with M. Mai and Ulf-G. Meißner

  [Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66]
- here: my personal choice
- mostly states where more than one lattice study available
- first focus on the “easy” cases, well separated resonances

The $\rho(770)$

- ρ -resonance a poster Breit-Wigner resonance
- best studied resonance from Lattice QCD
- summary of 16 Lattice studies

  [Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66]
- bare lattice results for $N_f = 2$ and $N_f = 2 + 1(+1)$
- systematics clearly visible



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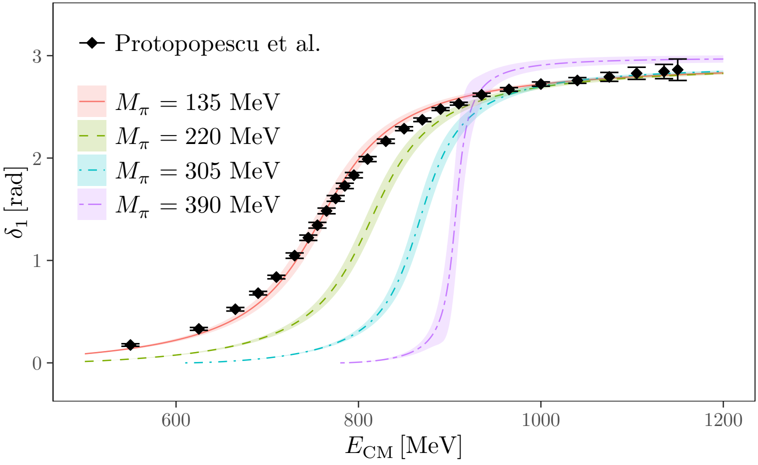
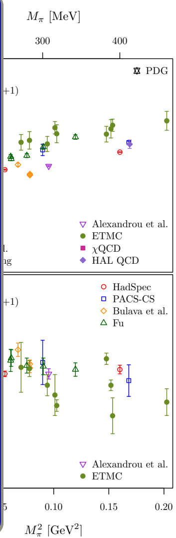


[Werner et al., Phys.Lett.B 819 (2021) 136449]

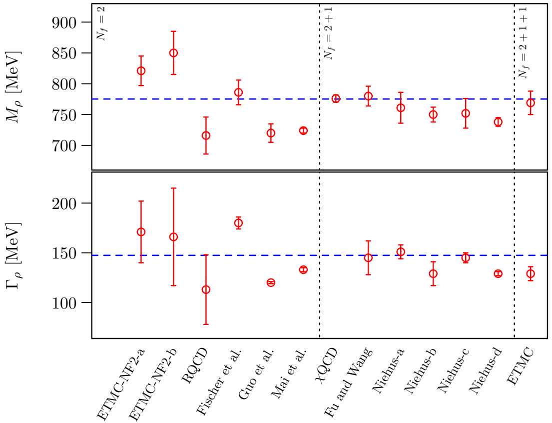


ρ -Meson Summary

- focus complex mass value at physical point

  [Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66]

- uncertainties shrink over time
- but there are still discrepancies different chiral extrapolations!
- N_f dependence?



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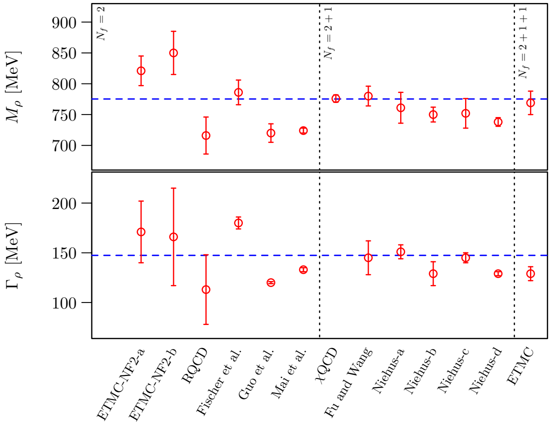
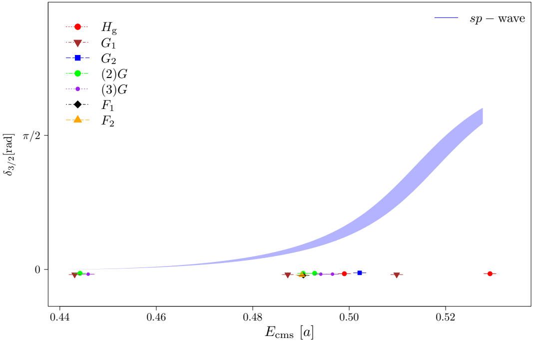
- **calculation at physical pion mass with several lattice spacings needed!**



Elastic πN-scattering and the Δ Resonance

- Δ: lowest lying baryon resonance
- significantly more challenging
  - proliferation of noise
  - elastic window small
  - S- and P-wave mixing
- 6(7) LQCD studies
- little control on systematics

Example $\delta_{3/2}$ phase shift



[Alexandrou et al., Phys.Rev.D 109 (2024) 3, 3]

Elastic πN-scattering and the Δ Resonance

- lattice overview for Δ

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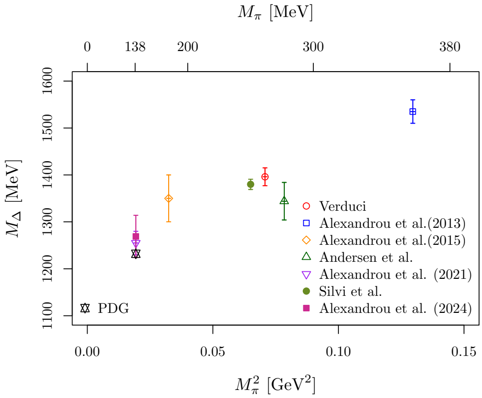
- resonance mass in reasonable agreement with experiment
- width (or coupling) more problematic
- scattering lengths still await precise determinations
- note: Alexandrou 2021 and 2024 are different analysis stages on the same data



[Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66; plus update]

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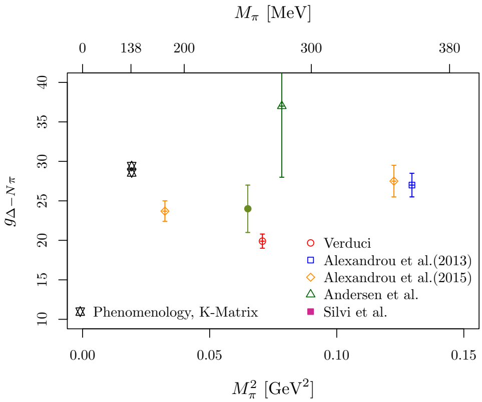


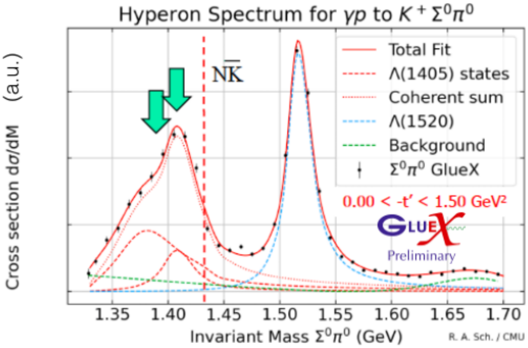
[Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66; plus update]

Results: Coupled Channels/Thresholds

- many exotic states found experimentally recently
- non-standard nature
- threshold effects and coupled channels relevant
- however: LQCD treatment significantly more challenging

$\Lambda(1405)$: The Mother of Complicated Pole Structures

- $\frac{1}{2}^{-}$ $\Lambda(1405)$ just below $N\bar{K}$ threshold
- decays predominantly to $\Sigma\pi$
- two pole structure?
- phenomenologically still undecided
- chiral EFTs $+$ unitarity: two pole structure
- Lattice QCD: requires coupled channel analysis!



[GlueX, Wickramaarachchi et al, EPJ Web Conf. 271 (2022) 07005]

The $\Lambda(1405)$ from LQCD

- a single LQCD calculation available

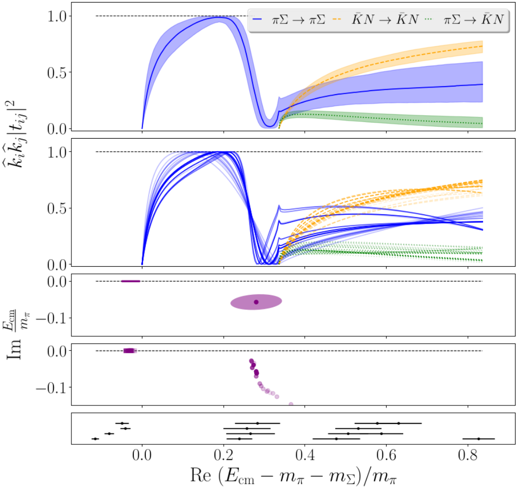
  [Bulava et al., Phys.Rev.Lett. 132 (2024) 5, 051901 & PRD]

- based on a single ensemble
  $M_\pi \approx 200$ MeV , $M_K \approx 487$ MeV

- coupled channel Lüscher approach

- sophisticated analysis procedure

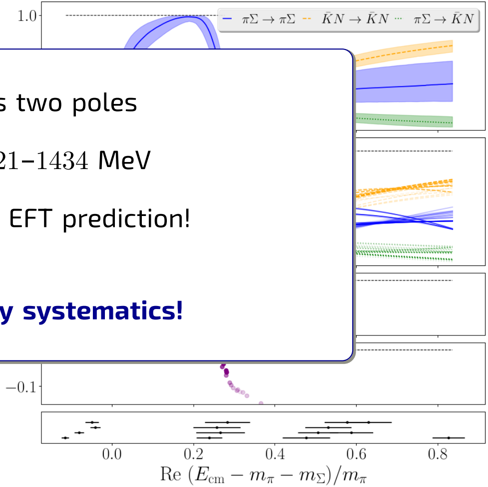
- model averaging to avoid bias



The $\Lambda(1405)$ from LQCD

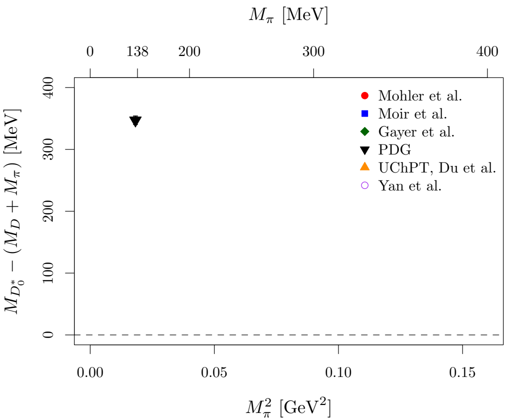
- a si
  [Bulava
- base
  M_π
- cou
- sopl
- model averaging to avoid bias

- this one lattice investigation favours two poles
- Re $E_1 = 1325$–1380 MeV, Re $E_2 = 1421$–1434 MeV
- first indication confirming the Chiral EFT prediction!
- however, please remember:
  one ensemble doesn't control many systematics!



$D\pi$-scattering and the $D_0^*(2300)$

- open charm, $J^P = 0^+$ and $I = 1/2$, decay to $D\pi$
- slightly above $D\pi$ threshold



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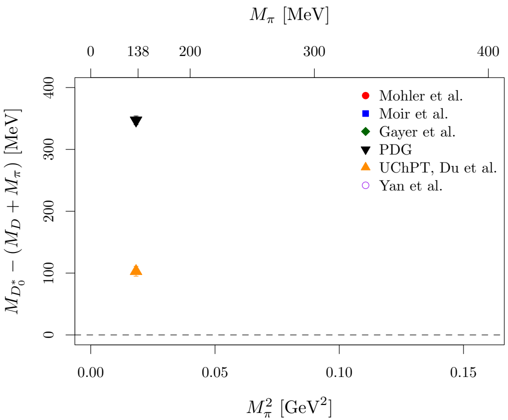
[Du et al., PRD 98 (9) (2018), 094018]



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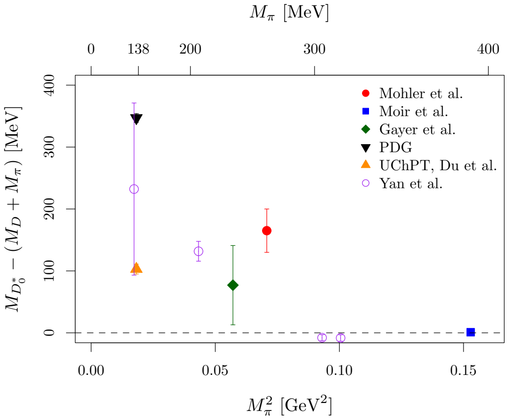
  [Du et al., PRD 98 (9) (2018), 094018]
- a bunch of LQCD results
- virtual state below $M_\pi \approx 300$ MeV
- inconclusive at M_π^{phys}!



[Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66; plus update]

$D\pi$-scattering and the $D_0^*(2300)$

- open charm, $J^P = 0^+$ and $I = 1/2$,
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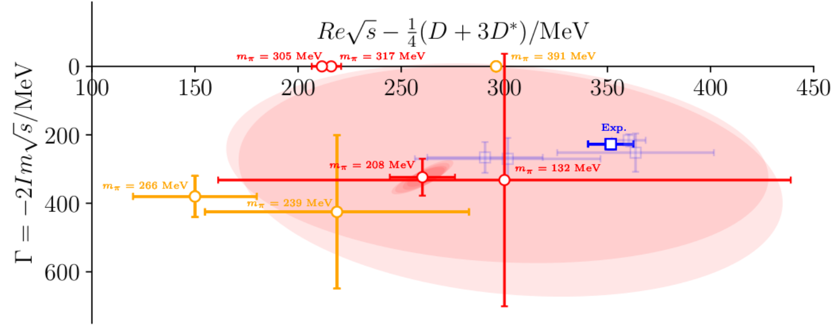
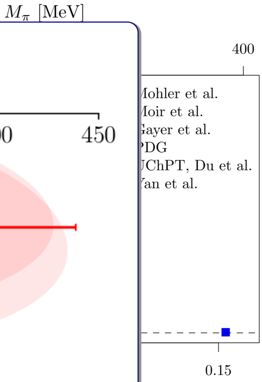


[Yan et al, arXiv:2405.13479]



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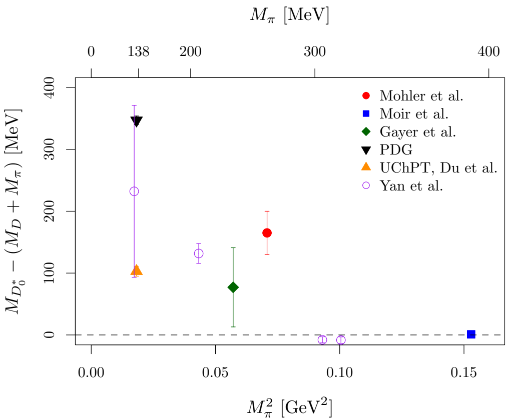
$D\pi$-scattering and the $D_0^*(2300)$

- open charm, $J^P = 0^+$ and $I = 1/2$, decay to $D\pi$
- slightly above $D\pi$ threshold
- expect two pole structure analysis of $B \to \phi\phi$ data

  [Du et al., PRD 98 (9) (2018), 094018]
- a bunch of LQCD results
- virtual state below $M_\pi \approx 300$ MeV
- inconclusive at M_π^{phys}!
- PDG revising their conclusions.

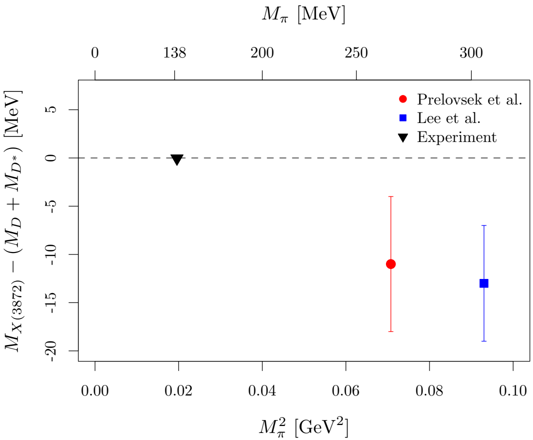


[Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66; plus update]

The $X(3872)$ (now called $\chi_{c1}(3872)$)

- one of the first exotic states
- narrow, $J^{PC} = 1^{++}$ charmonium like
- two lattice studies available, one $N_f = 2$, one $N_f = 2+1+1$

  [Prelovsek et al., Lee et al.,]
- both exploratory, single ensemble investigations
- candidate $I = 0$ states very close to threshold



The elusive $Z_c(3900)$ (now called $T_{c\bar{c}1}(3900)$)

- closed charm, $I^G = 1^+$, $J^{PC} = 1^{+-}$
- rather narrow $\Gamma \approx 30$ MeV
- decay channels
  $J/\psi\pi, \eta_c\pi\pi, D\bar{D}^*, DD^*$
- number of LQCD calculations of
  $Z_c(3900)$
- LQCD conclusion so far:
  no evidence for the Z_c was found
- only HAL QCD finds a state at threshold

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We have to control systematics!

Need to:

- investigate pion mass dependence
- study lattice spacing dependence
- perform a full coupled channel investigation

A Third Particle Enters the Game

- Three particle decays highly relevant
- Three-pion decays of $K, \eta, \omega, a_1(1260), a_1(1420)$
- Decays of exotica, e.g.: $X(3872) \to \bar{D}^* D \to \bar{D} D\pi$, $Y(4260) \to J/\psi\,\pi\pi$
- Roper resonance $\to \pi N$ and $\pi\pi N$
- Few-body physics: reactions with light nuclei

Lattice Energy Levels E
Finite Volume, discrete, real

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Lattice Energy Levels E
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↓ EFTs

Interaction Properties
Infinite Volume, possibly complex

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Three equivalent EFTs

- RFT (Relativistic Field Theory)
  [Hansen, Sharpe, 2014]
- NREFT (Non-relativistic EFT)
  [Hammer, Pang, Rusetsky, 2017]
- FVU (Finite Volume Unitarity)
  [Mai, Döring, 2017]

Understanding Systematics of 3pt Quantisation in ϕ^4-Theory

- two scalar fields ϕ_0, ϕ_1

  [Garofalo, Mai, Romero-López, Rusetsky, Urbach, 2023]

- include interaction term

$$S_{\text{int}} \;=\; +\frac{g}{2}\phi_0\phi_1^3$$

- allows decay $\phi_0 \to 3\phi_1$

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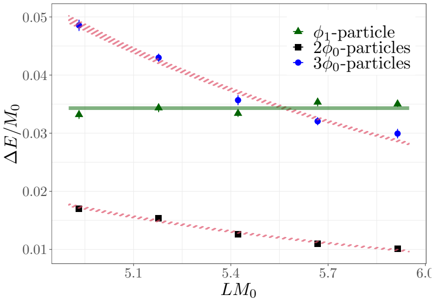

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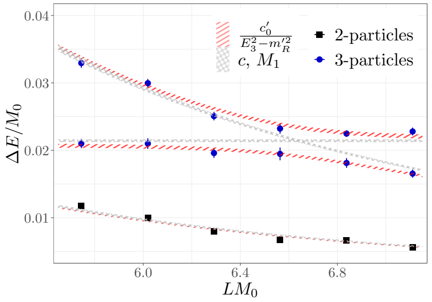
- at $g = 0$ no coupling
- at $g > 0$ avoided level crossing

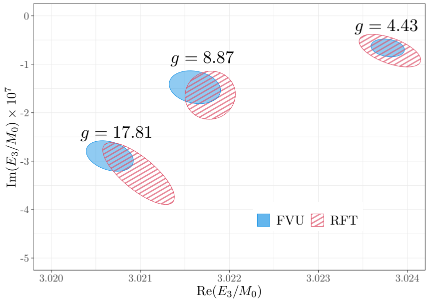


First observation in
a Lattice calculation

Understanding Systematics of 3pt Quantisation in ϕ^4-Theory

- can go further in this model

  [Garofalo, Mai, Romero-López, Rusetsky, Urbach, 2023]
- reconstruct complex mass
- compare FVU and RFT approaches
- we find good agreement!
- but systematics visible



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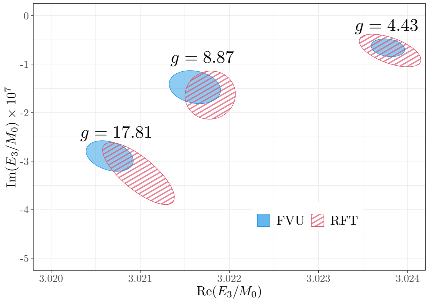
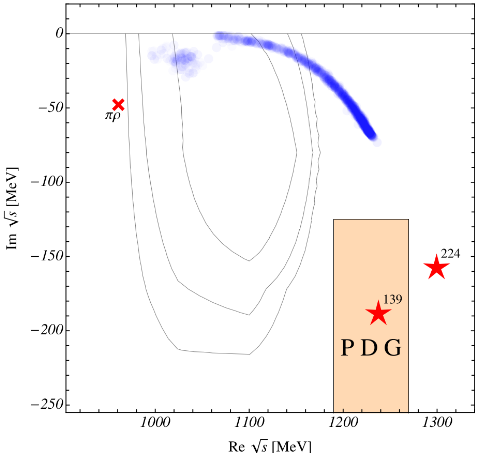


Equivalence of FVU and RFT shown in practice in controlled model

The Axial $a_1(1260)$ Meson

- $I^G(J^{PC} = 1^-(1^{++})\ a_1$ axial meson
- decays to three pions exclusively
- the only LQCD calculation of 3-body effects
- single ensemble, $M_\pi = 224$ MeV
- proof of feasibility



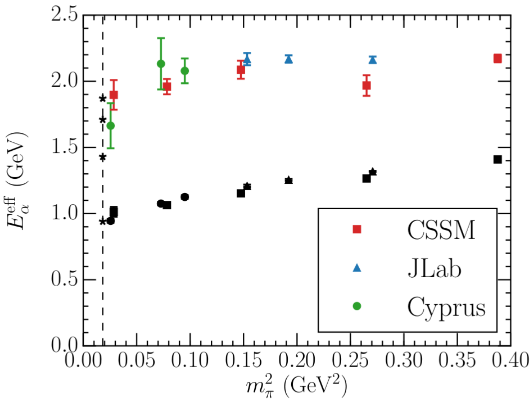
[Mai et al., Phys.Rev.Lett. 127 (2021) 22, 222001]

The Roper Resonance

- long standing puzzle from quark model viewpoint
- lighter than parity partner of N
- $N\pi$ and $N\pi\pi$ important decay channels
- many LQCD investigations
  references see review
  - no true Lüscher analysis so far
    one utilising Hamiltonian EFT
  - no LQCD calculation including 3-body dynamics

The Roper Resonance

- one LQCD conclusion:
  qqq state ruled out
- Leinweber et al. claim Roper partially seen...
- I think: nothing more definite to conclude yet!
- need to include $N\pi\pi$ operators
- and coupled channel Lüscher analysis



[Leinweber et al., JPS Conf.Proc. 10 (2016) 010011]

...and Glueballs?

- famous quenched calculation from Morningstar and Peardon (1999)
- in dynamical QCD particularly hard problem
- mixing with many lower lying states
- quenched not a good approximation
- novel ideas with
  - gradient flow

    [Sakai and Sasaki, Phys.Rev.D 107 (2023) 3, 034510]
  - dynamical simulations

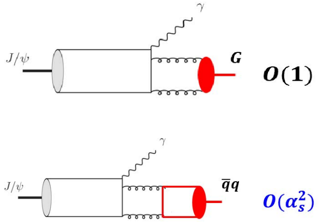
    [Bulava et al., AIP Conf.Proc. 2249 (2020) 1, 030032]

Glueballs from Radiative J/ψ Decays

- can one identify one of the ten scalar mesons as predominantly a glueball?
- highly non-trivial question (and maybe not well defined)
- radiative decays of J/ψ might be a good place to look for scalar glueballs
- decay into $\bar{q}q$ naïvly suppressed by α_s^2
- here: exploratory quenched study, two lattice spacings

  [L.Gui, et al. (CLQCD), Phys. Rev. Lett. 110, 021601 (2013)]
- for tensor glueballs see

  [Y.B. Yang et al .(CLQCD), Phys. Rev. Lett. 111, 091601 (2013)]



- **not** based on Lüscher method
- matrix element

$$\propto \langle G|j^\mu|J/\psi\rangle$$

estimated from three-point function

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[L.Gui, et al. (CLQCD),

$\mathbf{1})$

$\alpha_s^2)$

d

Main conclusion

$\Gamma(J/\psi \to \gamma \to G_{0^+})/\Gamma_{\text{tot}} = 3.8(9) \times 10^{-3}$ from LQCD

consistent with $f_0(1710)$ PDG production rate

$\text{BR}(J/\psi \to \gamma \to \gamma f_0(1710)) = 1.9 \times 10^{-3}$

and inconsistent with rates of other scalar mesons.

Quenched, so systematics are uncontrolled!

- for tensor glueballs see

[Y.B. Yang et al .(CLQCD), Phys. Rev. Lett. 111, 091601 (2013)]

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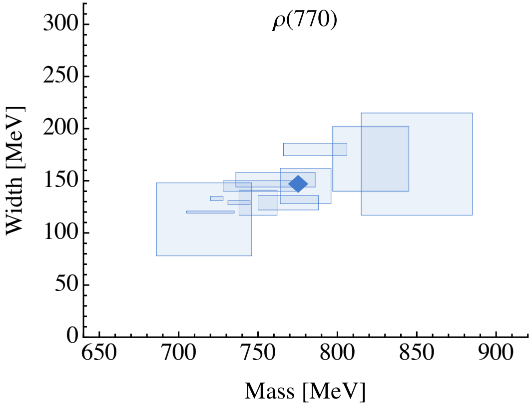
Summary

- resonances in LQCD challenging problem
- well separated resonances on a good way
- coupled channels / threshold phenomena promising results emerge
- can expect more in the future!



[Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66]

Summary

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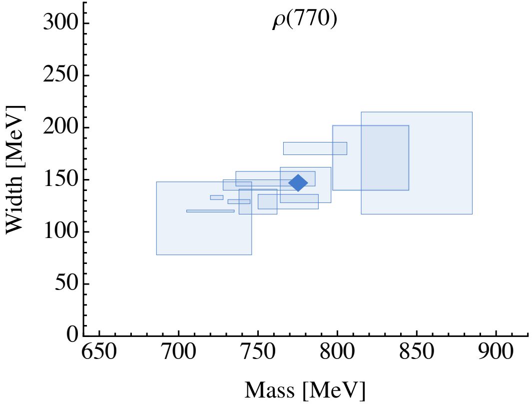
- can expect more in the future!
- **... thank you for you attention!**



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