Two-pole structures in QCD

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Details in: UGM, *Symmetry* **12** (2020) 981 [2005.06909 [hep-ph]]
Mai, UGM, Urbach, *Phys. Rept.* **1001** (2023)1 [2206.01477 [hep-ph]]

Short introduction: Bound states in QCD

Bound states in QCD

- Long time a playground of the Quark Model (QM):

  $\hookrightarrow$ mesons ($\bar{q}q$) and baryons (qqq)

- Exotics w.r.t. the QM (already mentioned by Gell-Mann in 1964): Phys.Lett. 8 (1964) 214

  $\hookrightarrow$ tetraquarks, pentaquarks, hybrids,..., glueballs (truely exotic)

- Even more structures:

  $\hookrightarrow$ dynamically generated states, hadronic molecules, ..., nuclei $\to$ next slide

- Revival of hadron spectroscopy started around 2003:

  $\hookrightarrow D^\star_{s0}(2317), D_{s1}(2460), \chi_{c1}(3872)$ aka $X(3872), \ldots$

$\Rightarrow$ The hadron spectrum is arguably the least understood part of the Standard Model

$\Rightarrow$ Discuss one new feature here, the two-pole structures

Dynamically generated states / hadronic molecules

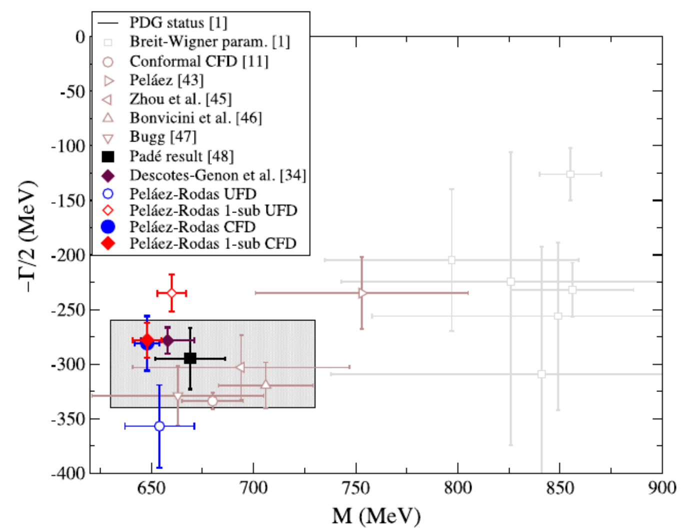
- Hadron-hadron (or three-hadron) interactions can dynamically generate resonances
- Hadronic molecules: a subclass of these (shallow binding, close to the real axis)
- Prime example: The light scalar mesons $\underbrace{f_0(500)}_{\sigma}, \underbrace{f_0(700)}_{\kappa}, f_0(980)$

$$M_{f_0(500)} = 441^{+16}_{-8}\ \text{MeV}$$
$$\Gamma_{f_0(500)} = 544^{+18}_{-25}\ \text{MeV}$$

Caprini, Colangelo, Leutwyler (2005)

$$M_{f_0(700)} = 648 \pm 7\ \text{MeV}$$
$$\Gamma_{f_0(700)} = 280 \pm 16\ \text{MeV}$$

Pelaez, Rodas (2020)



$$\left.\begin{aligned} M_{f_0(980)} &= 990 \pm 20\ \text{MeV} \\ \Gamma_{f_0(980)} &= 10 - 100\ \text{MeV} \end{aligned}\right\}$$ in between the K^+K^- and $K^0\bar{K}^0$ thresholds

$\hookrightarrow$ it is a molecule!

Two-pole structures

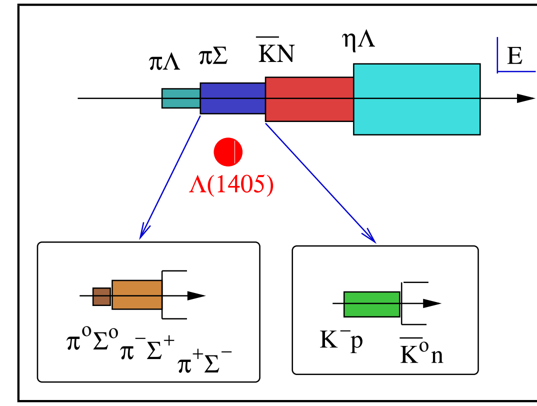
- What is a two-pole structure ?

> The term two-pole structure refers to the fact that particular single states in the hadron spectrum as listed in the PDG tables are indeed two states.

- Basic ingredients:
  - coupled channels
  - molecular states / dynamically generated states

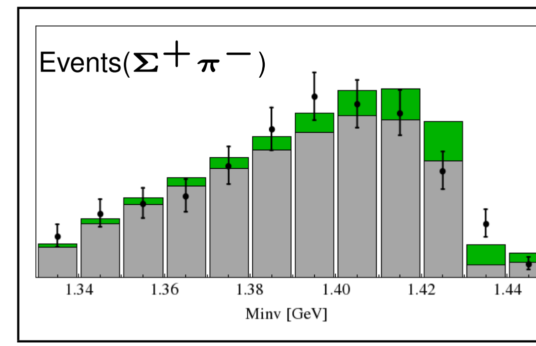
A tale of the two Λ(1405) states

The first exotic – the story of the two $\Lambda(1405)$



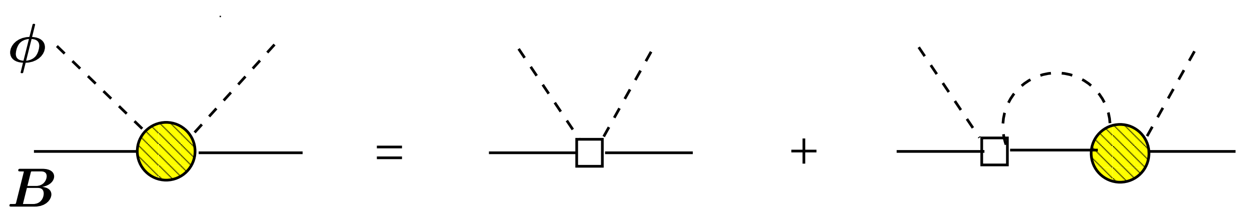
- Quark model: uds excitation with $J^P = \frac{1}{2}^-$ CLAS (2014)
  a few hundred MeV above the $\Lambda(1116)$
  $m = 1405.1^{+1.3}_{-1.0}\,\text{MeV}\,,\ \Gamma = 50.5 \pm 2.0\,\text{MeV}$ [PDG 2015]

- Prediction as early as 1959 by Dalitz and Tuan:
  Resonance between the coupled $\pi\Sigma$ and $\bar{K}N$ channels
  Dalitz, Tuan, Phys. Rev. Lett. **2** (1959) 425; J.K. Kim, PRL **14** (1965) 29



- Clearly seen in $K^-p \to \Sigma 3\pi$ reactions at 4.2 GeV at CERN
  Hemingway, Nucl.Phys. B **253** (1985) 742

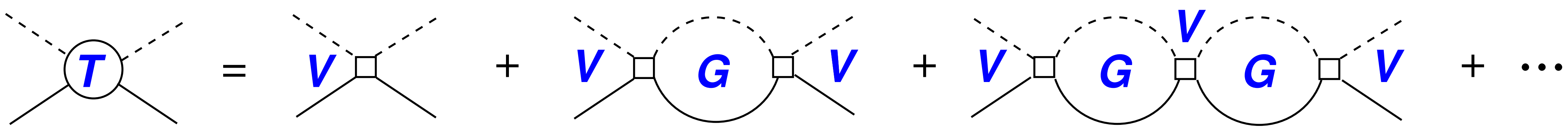
- An enigma: Too low in mass for the quark model,
  but well described in unitarized chiral perturbation theory: $\phi B \to \phi B$



Kaiser, Siegel, Weise, Ramos, Oset, Oller, UGM, Lutz, ...

Enter chiral dynamics

- Great idea:
  Combine (leading-order) chiral SU(3) Lagrangian with coupled-channel dynamics
  Kaiser, Siegel, Weise, Nucl. Phys. A **594** (1995) 325



$\hookrightarrow$ Dominance of the Weinberg-Tomozawa term, excellent description of K^-p data and $\pi\Sigma$ mass distribution, also inclusion of NLO terms with constrained fits

$\hookrightarrow$ The $\Lambda(1405)$ appears as a dynamically generated state (MB molecule)

$\hookrightarrow$ Highly cited follow-ups from TUM group plus other groups, esp. "Spanish Mafia"
Oset, Ramos, Nucl. Phys. A **635** (1998) 99, . . .

- But: unpleasant regulator dependence (Yukawa-type, momentum cut-off)
  gauge invariance in photo-reactions?

A new twist

- Re-analysis of coupled-channel $K^- p$ scattering and the $\Lambda(1405)$

Oller, UGM Phys. Lett. B **500** (2001) 263

- Technical improvements:
  - Subtracted meson-baryon loop with dim reg $\hookrightarrow$ **standard method**
  - Coupled-channel approach to the $\pi\Sigma$ mass distribution
  - Matching formulas to any order in chiral perturbation theory established

- Most significant finding:

> "Note that the $\Lambda(1405)$ resonance is described by **two poles** on sheets II and III with rather different imaginary parts indicating a clear departure from the Breit-Wigner situation..."

[pole 1: (1379.2 -i 27.6) MeV, pole 2: (1433.7 -i 11.0) MeV on RS II]

$\hookrightarrow$ Chiral dynamics generates **two** poles, but: how?

Jido, Oller, Oset, Ramos, UGM, Nucl. Phys. A **725** (2003) 181

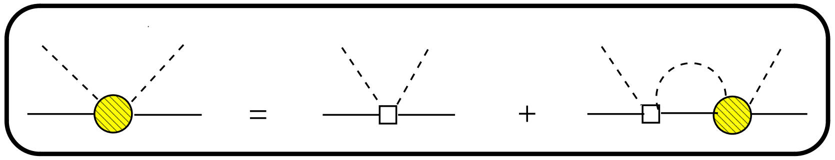
Some formalism

- Coupled channels with $S = -1$:

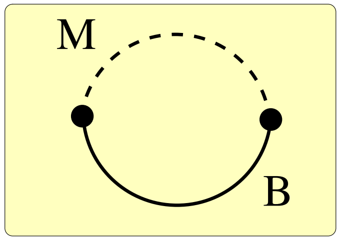
$$K^- p \to K^- p,\ \bar{K}^0 n,\ \pi^0\Sigma^0,\ \pi^+\Sigma^-,\ \pi^-\Sigma^+,\ \pi^0\Lambda,\ \eta\Lambda,\ \eta\Sigma^0,\ K^+\Xi^-,\ K^0\Xi^0$$

- Lippmann-Schwinger eq. in matrix space:

$$T(W) = [\mathcal{I} + \mathcal{V}(W)\cdot g(s)]^{-1}\cdot \mathcal{V}(W)$$



$$g(s)_i = \frac{1}{16\pi^2}\left\{a_i(\mu) + \log\frac{m_i^2}{\mu^2}\frac{M_i^2 - m_i^2 + s}{2s}\log\frac{M_i^2}{m_i^2} + \frac{q_i}{\sqrt{s}}\log\frac{m_i^2 + M_i^2 - s - 2\sqrt{s}q_i}{m_i^2 + M_i^2 - s + 2\sqrt{s}q_i}\right\}$$



- Matching to chiral perturbation theory, say to orders $\mathcal{O}(p)$, $\mathcal{O}(p^2)$, $\mathcal{O}(p^3)$:

$$T_1 = \mathcal{V}_1\ , \qquad T_1 + T_2 = \mathcal{V}_1 + \mathcal{V}_2$$

$$T_1 + T_2 + T_3 = \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{V}_3 - \mathcal{V}_1\cdot g\cdot \mathcal{V}_1$$

The two-pole scenario explained

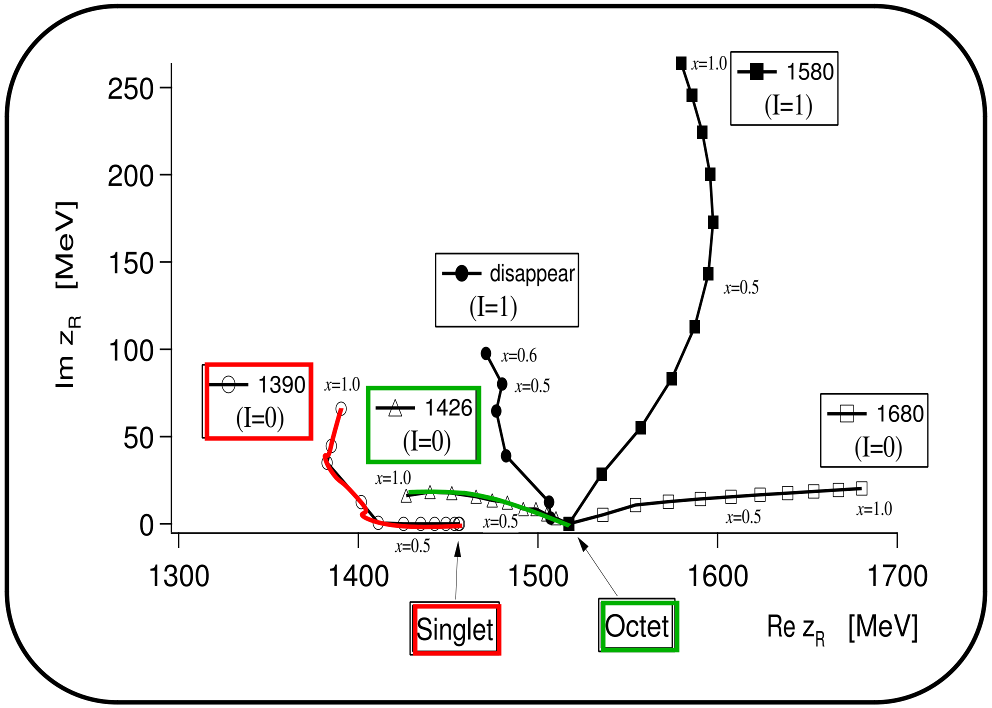
- Detailed analysis found **two** poles in the complex energy plane

$\hookrightarrow$ generated by chiral dynamics, but can we understand this in more detail?

- Group theory:

$$\mathbf{8} \otimes \mathbf{8} = \underbrace{\mathbf{1} \oplus \mathbf{8}_s \oplus \mathbf{8}_a}_{\text{binding at LO}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

- Follow the pole movement from the SU(3) limit to the physical masses:
  Jido, Oller, Oset, Ramos, UGM, Nucl. Phys. A **725** (2003) 181



- Verified by various groups world-wide

- However: scattering and kaonic atom data alone do not lead to a unique solution (two poles, but spread in the complex plane)

- Photoproduction to the rescue: $\gamma p \to K^+ \Sigma \pi$ CLAS, Phys. Rev. C **87**, 035206 (2013)

SU(3) symmetry considerations - details

Jido, Oller, Oset, Ramos, UGM, Nucl. Phys. A **725** (2003) 181

- SU(3) limit: $m_u = m_d = m_s \neq 0$

$\hookrightarrow$ all GB mesons have equal mass M_0 , all octet baryons have equal mass m_0

$\Rightarrow$ from the SU(3) limit at $x = 0$
to the physical world w/ $x = 1$

$$m_i(x) = m_0 + x(m_i - m_0)$$

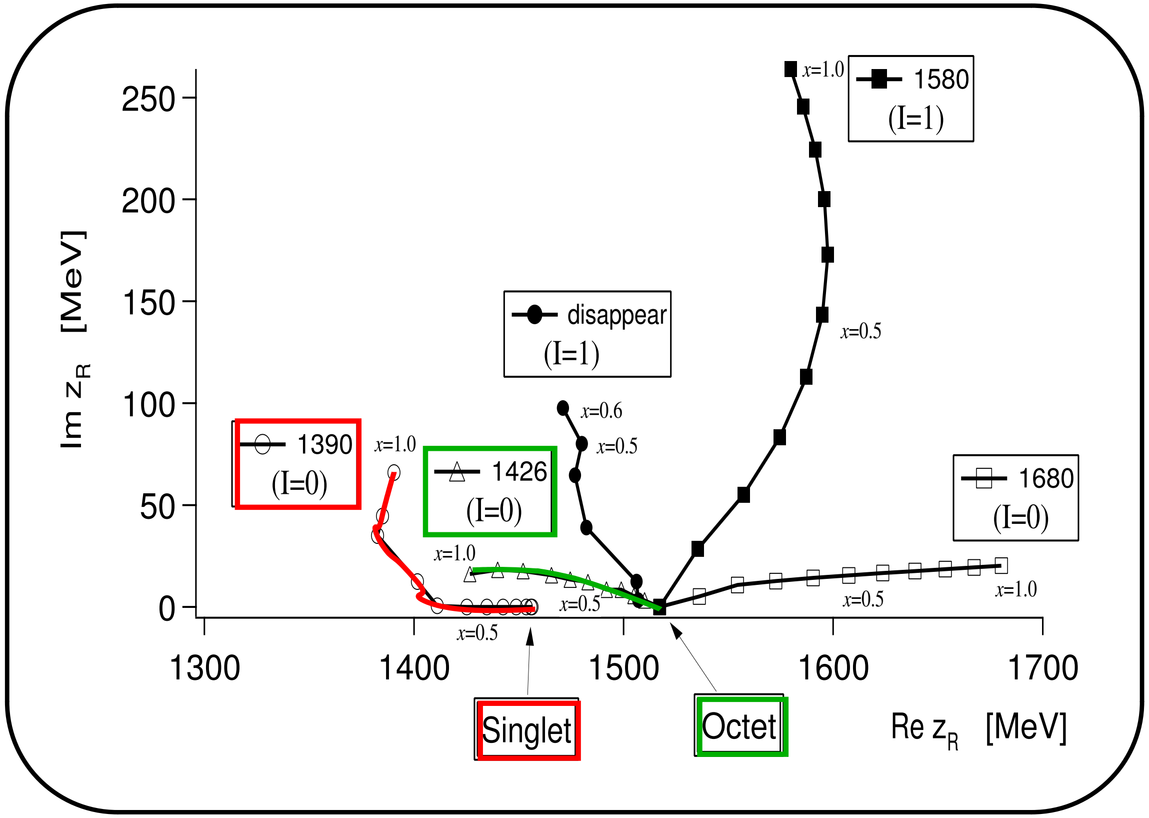
$$M_i^2(x) = M_0^2 + x(M_i^2 - M_0^2)$$

$$a_i(x) = a_0 + x(a_i - a_0)$$

$m_0 = 1151$ MeV

$M_0 = 368$ MeV

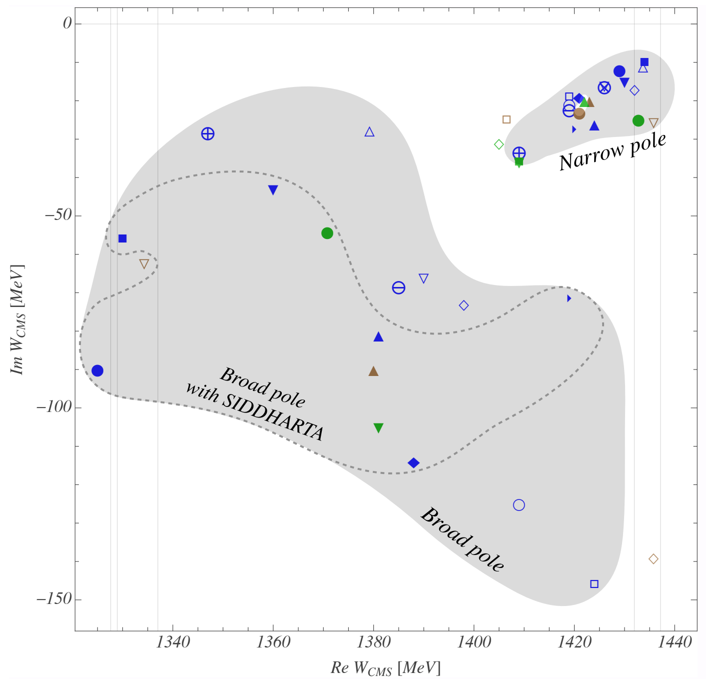
$a_0 = -2.148$

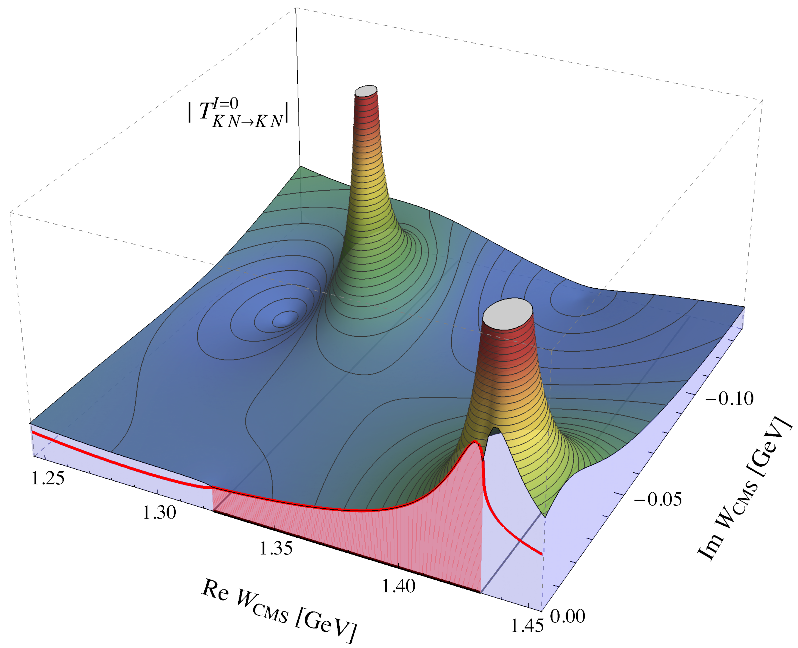


Present status of the two-pole scenario

- Two poles from scattering + SIDDHARTA data (one well, the other not-so-well fixed):

for details, see Mai, Eur. Phys. J. ST **230** (2021) 1593 [arXiv:2010.00056 [nucl-th]]





Figures courtesy Maxim Mai

Resonances are poles in the complex plane!

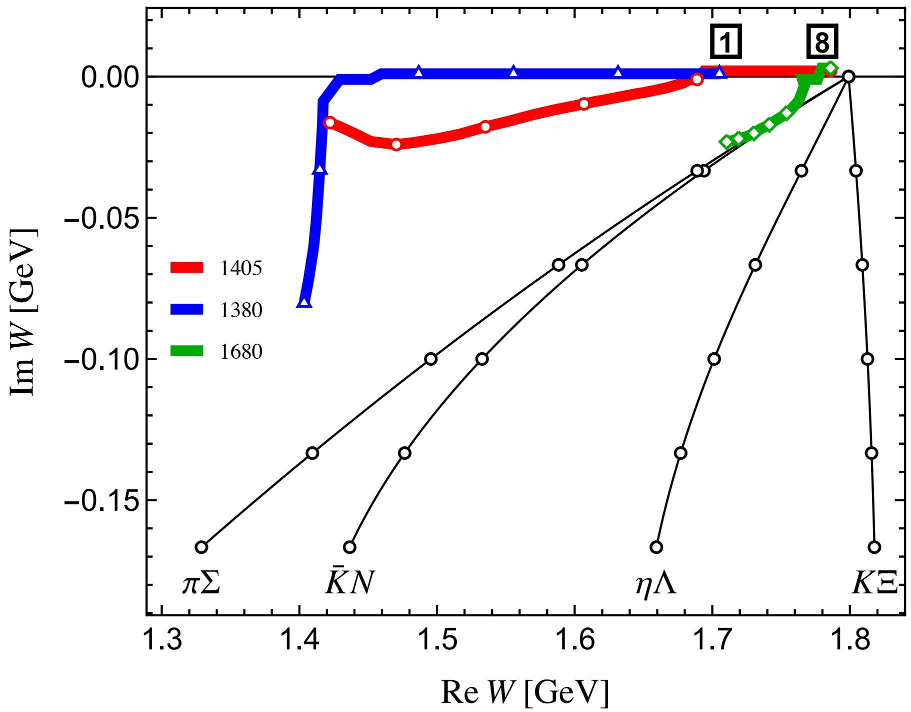
→ PDG 2016: **http://pdg.lbl.gov/2015/reviews/rpp2015-rev-lam-1405-pole-struct.pdf**

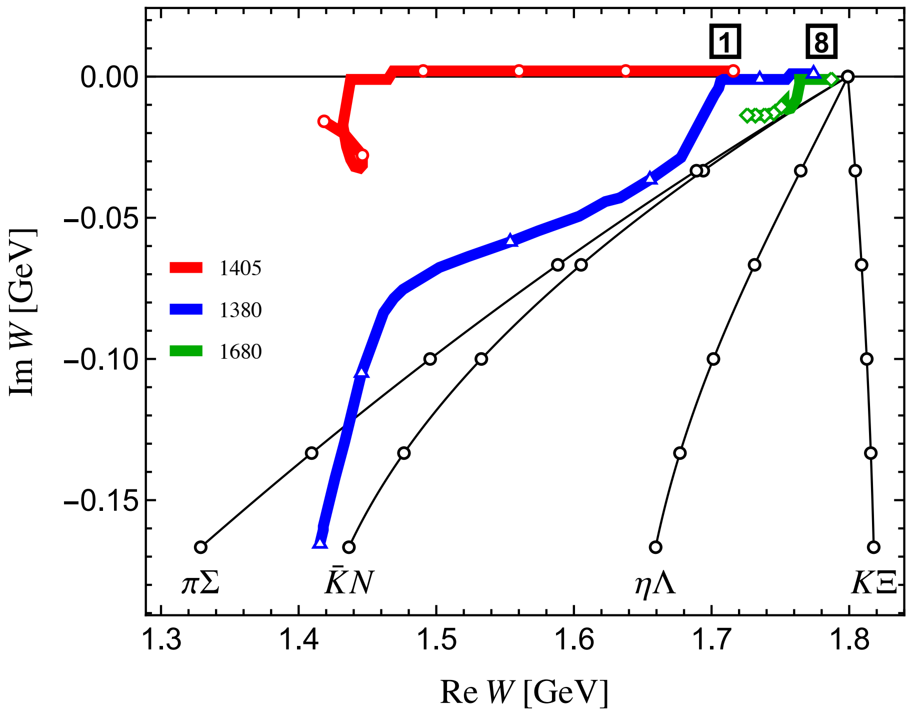
POLE STRUCTURE OF THE $\Lambda(1405)$ REGION
Written first November 2015 by Ulf-G. Meißner and Tetsuo Hyodo

SU(3) symmetry considerations - a new twist

Guo, Kamiya, Mai, UGM, PLB **846** (2023) 138264

- Interesting interchange of trajectories from LO to NLO





$\hookrightarrow$ can be tested on the lattice

$\hookrightarrow$ different findings in Zhuang, Molina, Lu, Geng, [2405.07686 [hep-ph]] ?

Status in the Review of Particle Physics

- Two excited Λ states listed in the 2020 RPP edition:

P. A. Zyla *et al.* [Particle Data Group], PTEP **2020** (2020) 083C01

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1380)\ 1/2^-$ $J^P = \frac{1}{2}^-$ Status: $**$

OMITTED FROM SUMMARY TABLE

See the related review on "Pole Structure of the $\Lambda(1405)$ Region."

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\Lambda(1405)\ 1/2^-$ $I(J^P) = 0(\frac{1}{2}^-)$ Status: $****$

In the 1998 Note on the $\Lambda(1405)$ in PDG 98, R.H. Dalitz discussed the S-shaped cusp behavior of the intensity at the N-$\overline{K}$ threshold observed in THOMAS 73 and HEMINGWAY 85. He commented that this behavior "is characteristic of S-wave coupling; the other below threshold hyperon, the $\Sigma(1385)$, has no such threshold distortion because its N-$\overline{K}$ coupling is P-wave. For $\Lambda(1405)$ this asymmetry is the sole direct evidence that $J^P = 1/2^-$."

A recent measurement by the CLAS collaboration, MORIYA 14, definitively established the long-assumed $J^P = 1/2^-$ spin-parity assignment of the $\Lambda(1405)$. The experiment produced the $\Lambda(1405)$ spin-polarized in the photoproduction process $\gamma p \to K^+ \Lambda(1405)$ and measured the decay of the $\Lambda(1405)$ (polarized) $\to \Sigma^+$(polarized) π^-. The observed isotropic decay of $\Lambda(1405)$ is consistent with spin $J = 1/2$. The polarization transfer to the Σ^+(polarized) direction revealed negative parity, and thus established $J^P = 1/2^-$.

See the related review(s):
Pole Structure of the $\Lambda(1405)$ Region

Hyodo, UGM

- a new two-star resonance at 1380 MeV
- still not in the summary table
- there are more such two-pole states!
- this is a fascinating phenomenon
  intimately tied to molecular structures
- Two Λ's: recenty confirmed by lattice QCD Bulava et al., PRL **132** (2024) 051901
  $\hookrightarrow$ nature of the lower pole not really pinned down
- for a review, see UGM, *Symmetry* **12** (2020) 981

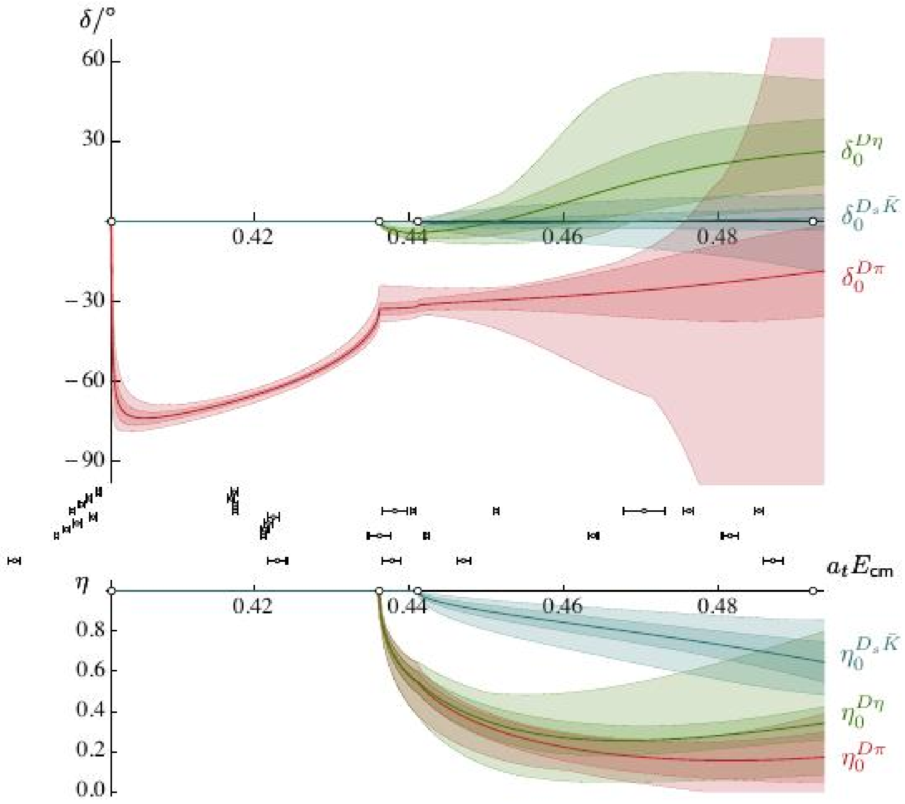
Two-pole structure of the $D_0^*(2300)$

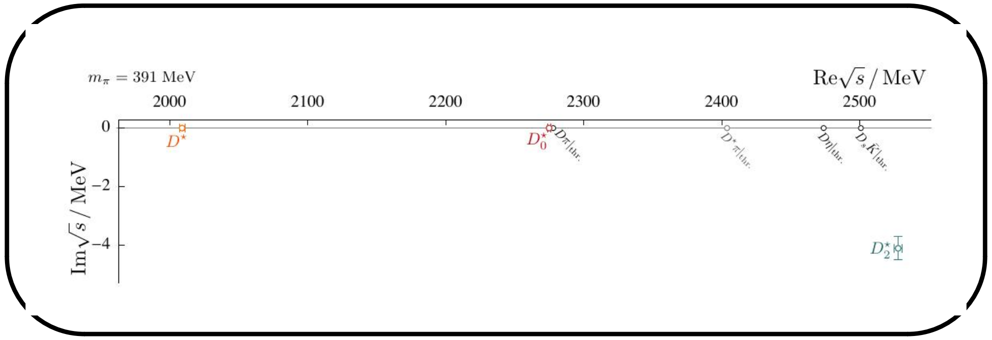
Coupled channel scattering on the lattice

Moir, Peardon, Ryan, Thomas, Wilson [HadSpec], JHEP **1610** (2016) 011

- $D\pi$, $D\eta$, $D_s\bar{K}$ scattering with $I = 1/2$:

- 3 volumes, one a_s, one a_t, $M_\pi \simeq 390$ MeV, various K-matrix type extrapolations





- S-wave pole at (2275.9 ± 0.9) MeV

- close to the $D\pi$ threshold

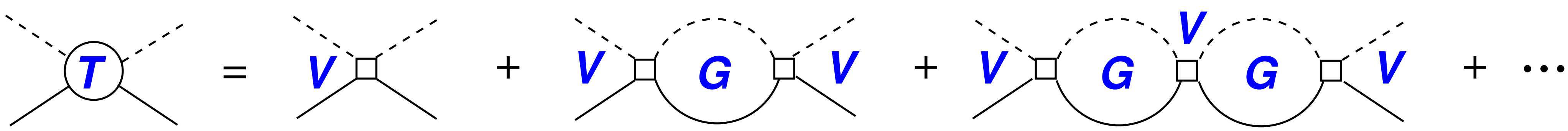
- consistent w/ $D_0^\star(2300)$ of PDG

- BUT: symmetries ignored... :-(

Coupled channel dynamics

Kaiser, Weise, Siegel (1995), Oset, Ramos (1998), Oller, UGM (2001), Kolomeitsev, Lutz (2002),Jido et al. (2003), Guo et al. (2006), . . .

- $D\phi$ bound states: Poles of the T-matrix (potential from CHPT and unitarization)



- Unitarized CHPT as a non-perturbative tool:

$$T^{-1}(s) = \mathcal{V}^{-1}(s) - G(s)$$

- $\mathcal{V}(s)$: derived from the SU(3) heavy-light chiral Lagrangian, 6 LECs up to NLO

$\longrightarrow$ next slide

- $G(s)$: 2-point scalar loop function, regularized w/ a subtraction constant $a(\mu)$

- $T, \mathcal{V}, G$: all these are matrices, channel indices suppressed

Coupled channel dynamics cont'd

Barnes et al. (2003), van Beveren, Rupp (2003), Kolomeitsev, Lutz (2004), Guo et al. (2006), . . .

- NLO effective chiral Lagrangian for coupled channel dynamics

Guo, Hanhart, Krewald, UGM, Phys. Lett. B **666** (2008) 251

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$$

$$\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - M_D^2 D D^\dagger \;, \quad D = (D^0, D^+, D_s^+)$$

$$\mathcal{L}^{(2)} = D\left[-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right] D$$

$$+\mathcal{D}_\mu D\left[h_4\langle u^\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right]\mathcal{D}_\nu D$$

with $u_\mu \sim \partial_\mu\phi\;,\;\; \chi_+ \sim \mathcal{M}_{\rm quark}\;,\;\; \ldots$

- LECs:

$\hookrightarrow$ h_0 absorbed in masses

$\hookrightarrow$ $h_1 = 0.42$ from the D_s-D splitting

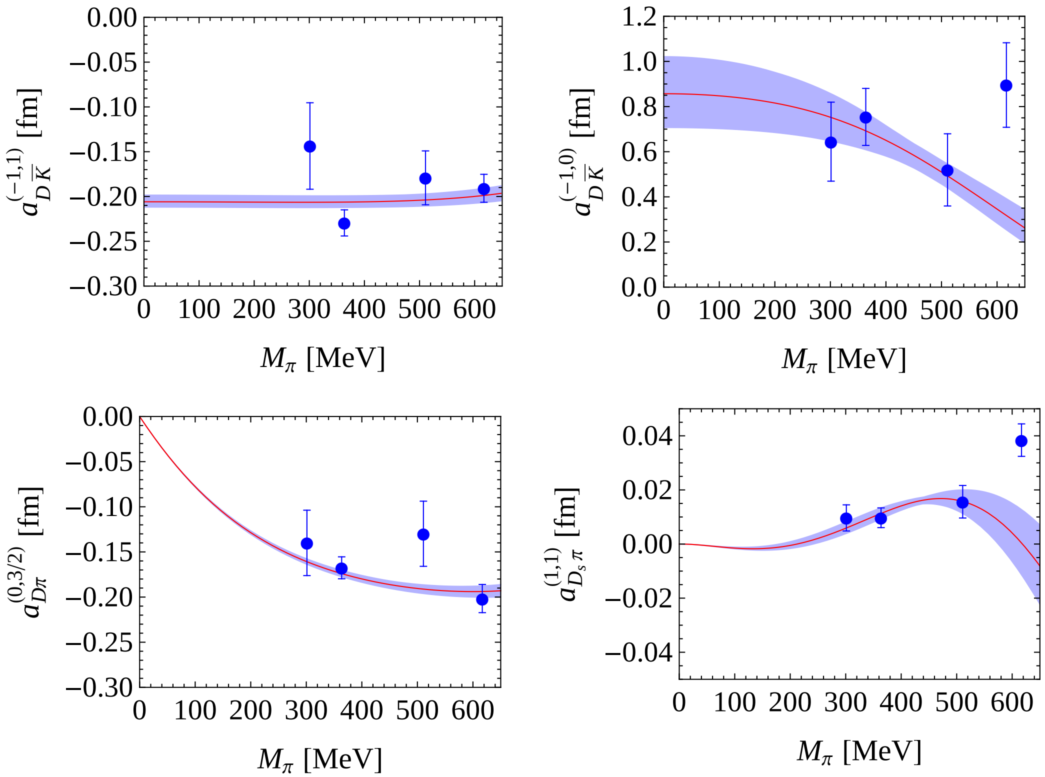
$\hookrightarrow$ $h_{2,3,4,5}$ from a fit to lattice data ($D\pi \to D\pi, D\bar{K} \to D\bar{K}, \ldots$)

Liu, Orginos, Guo, Hanhart, UGM, Phys. Rev. D **87** (2013) 014508

Fit to lattice data

Liu, Orginos, Guo, Hanhart, UGM, PRD **87** (2013) 014508

- Fit to lattice data in 5 “simple” channels: no disconnected diagrams



$[M_\pi = 600$ MeV not fitted]

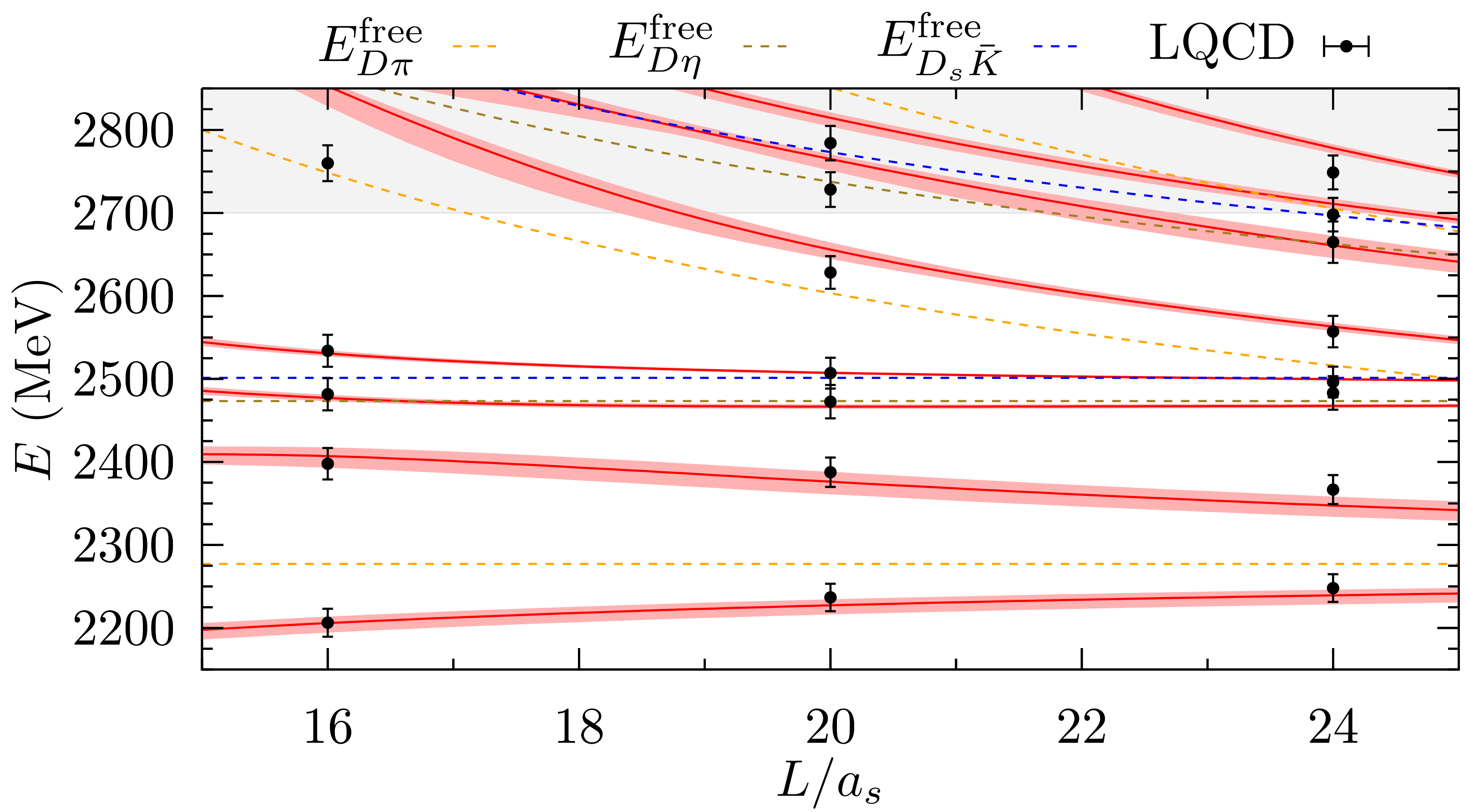
- Prediction: Pole in the $(S, I) = (1, 0)$ channel: 2315^{+18}_{-28} MeV

Experiment: $M_{D^\star_{s0}(2317)} = (2317.8 \pm 0.5)$ MeV PDG2021

What about the $D_0^\star(2300)$?

- Calculate the finite volume energy levels for $I = 1/2$, compare w/ the LQCD results

Albaladejo, Fernandez-Soler, Guo, Nieves, Phys. Lett. B **767** (2017) 465



- this is NOT a fit!
- all LECs taken from the earlier study of Liu et al. (discussed before)

What about the $D_0^\star(2300)$? – cont'd

Albaladejo, Fernandez-Soler, Guo, Nieves (2017)

- reveals a two-pole scenario! [cf. $\Lambda(1405)$]
- understood from group theory

$$\bar{\mathbf{3}} \otimes \mathbf{8} = \underbrace{\bar{\mathbf{3}} \oplus \mathbf{6}}_{\text{attractive}} \oplus \overline{\mathbf{15}}$$

- this was seen earlier in various calc's

  Kolomeitsev, Lutz (2004), F. Guo, Shen, Chiang, Ping, Zou (2006), F. Guo, Hanhart, UGM (2009), Z. Guo, UGM, Yao (2009)

- Again: important role of **chiral symmetry**
- Lattice QCD test: sextet pole becomes a b.s.

  for $M_\phi > 575$ MeV in the SU(3) limit

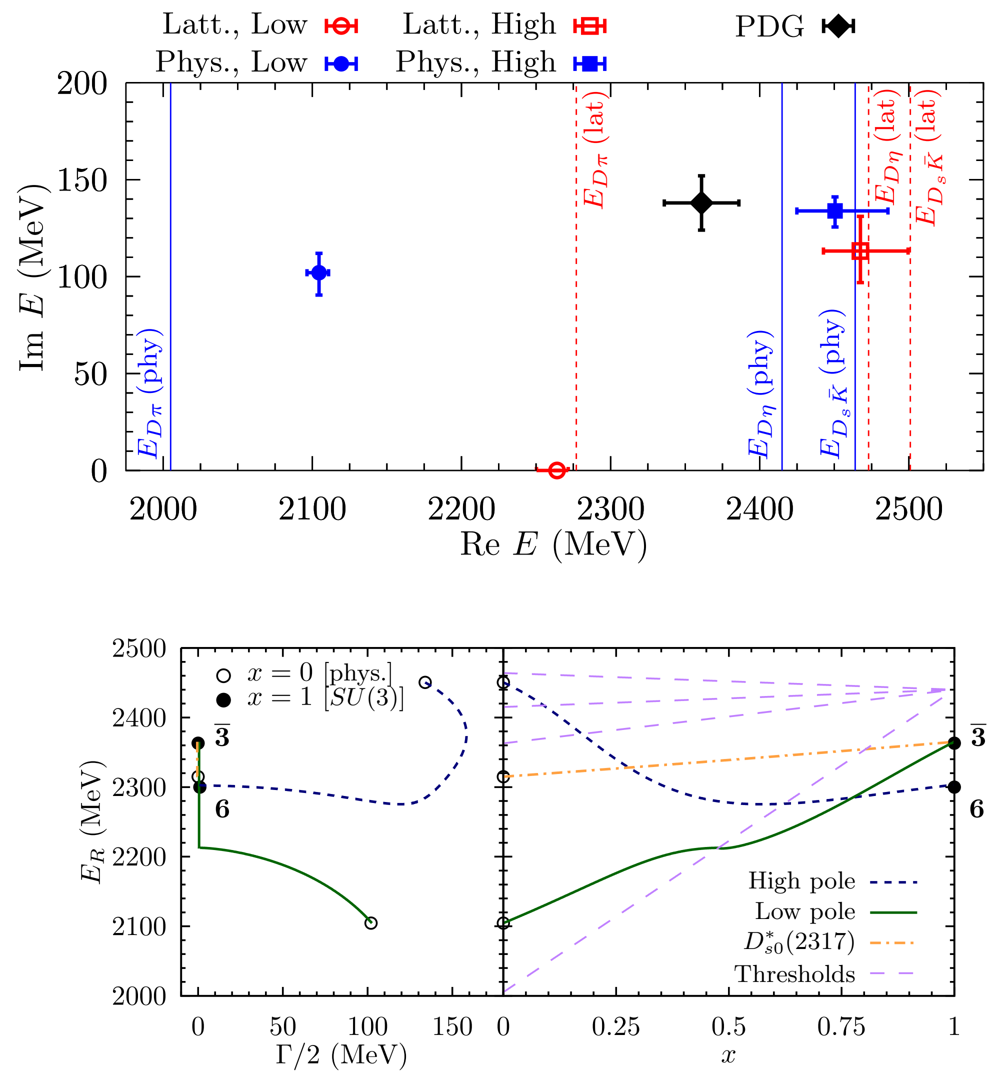
  Du et al., Phys.Rev. D **98** (2018) 094018

- FZJ LQCD finds a b.s. for $M_\pi = 600$ MeV

  Gregory et al., 2106.15391 [hep-ph]

- HadSpec finds a virtual state ($M_\pi = 700$ MeV)

  Yeo et al., 2403.10498 [hep-lat]



Two-pole structure consistent with the lattice data?

Ashokan, Tang, Guo, Hanhart, Kamiya, UGM, EPJ **C 83** (2023) 850

- Can we understand why HadSpec only reported one pole?

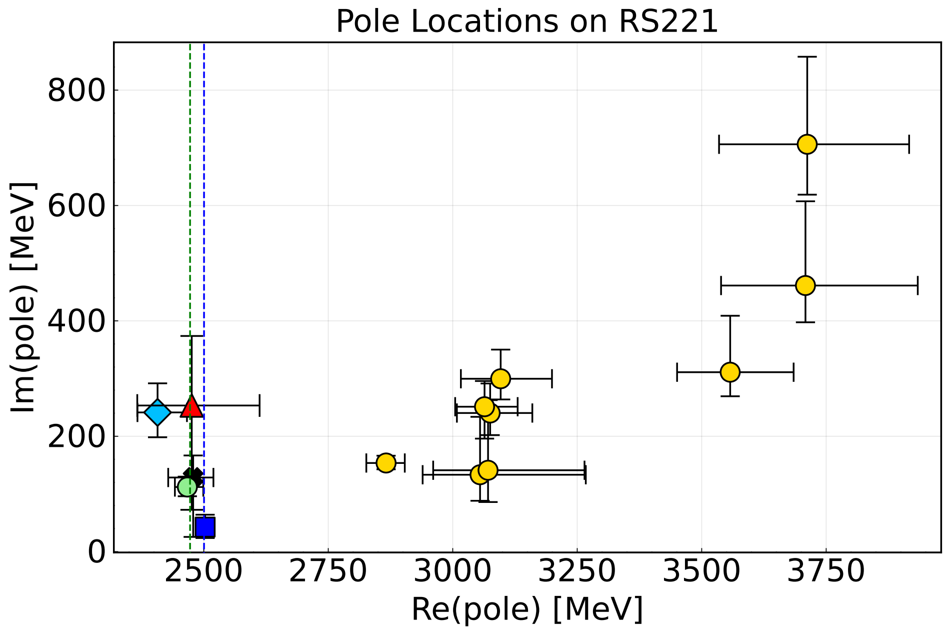
- Impose SU(3) symmetry on the K-matrix to fit the FV energy levels $\rightarrow$ less parmeters!

$$\boldsymbol{K} = \left(\frac{g_{\bar{3}}^2}{m_{\bar{3}}^2 - s} + c_{\bar{3}} \right) C_{\bar{3}} + \left(\frac{g_6^2}{m_6^2 - s} + c_6 \right) C_6 + c_{\overline{15}} \, C_{\overline{15}}.$$

- perform various fits
  (switch off various terms)

$\hookrightarrow$ Poles are consistent w/ UChPT !

$\hookrightarrow$ never ignore symmetries!



Two-pole scenario in the heavy-light sector

- Invoke HQSS and HQFS:

$\hookrightarrow$ Two states in various $I = 1/2$ states in the heavy meson sector $(M, \Gamma/2)$

	Lower [MeV]	Higher [MeV]	PDG2024 [MeV]
$D_0^\star$	$\left(2105^{+6}_{-8}, 102^{+10}_{-11}\right)$	$\left(2451^{+36}_{-26}, 134^{+7}_{-8}\right)$	$(2343 \pm 10, 115 \pm 8)$
D_1	$\left(2247^{+5}_{-6}, 107^{+11}_{-10}\right)$	$\left(2555^{+47}_{-30}, 203^{+8}_{-9}\right)$	$(2412 \pm 9, 157 \pm 15)$
$B_0^\star$	$\left(5535^{+9}_{-11}, 113^{+15}_{-17}\right)$	$\left(5852^{+16}_{-19}, 36 \pm 5\right)$	—
B_1	$\left(5584^{+9}_{-11}, 119^{+14}_{-17}\right)$	$\left(5912^{+15}_{-18}, 42^{+5}_{-4}\right)$	—

$\rightarrow$ but is there further experimental support for this?

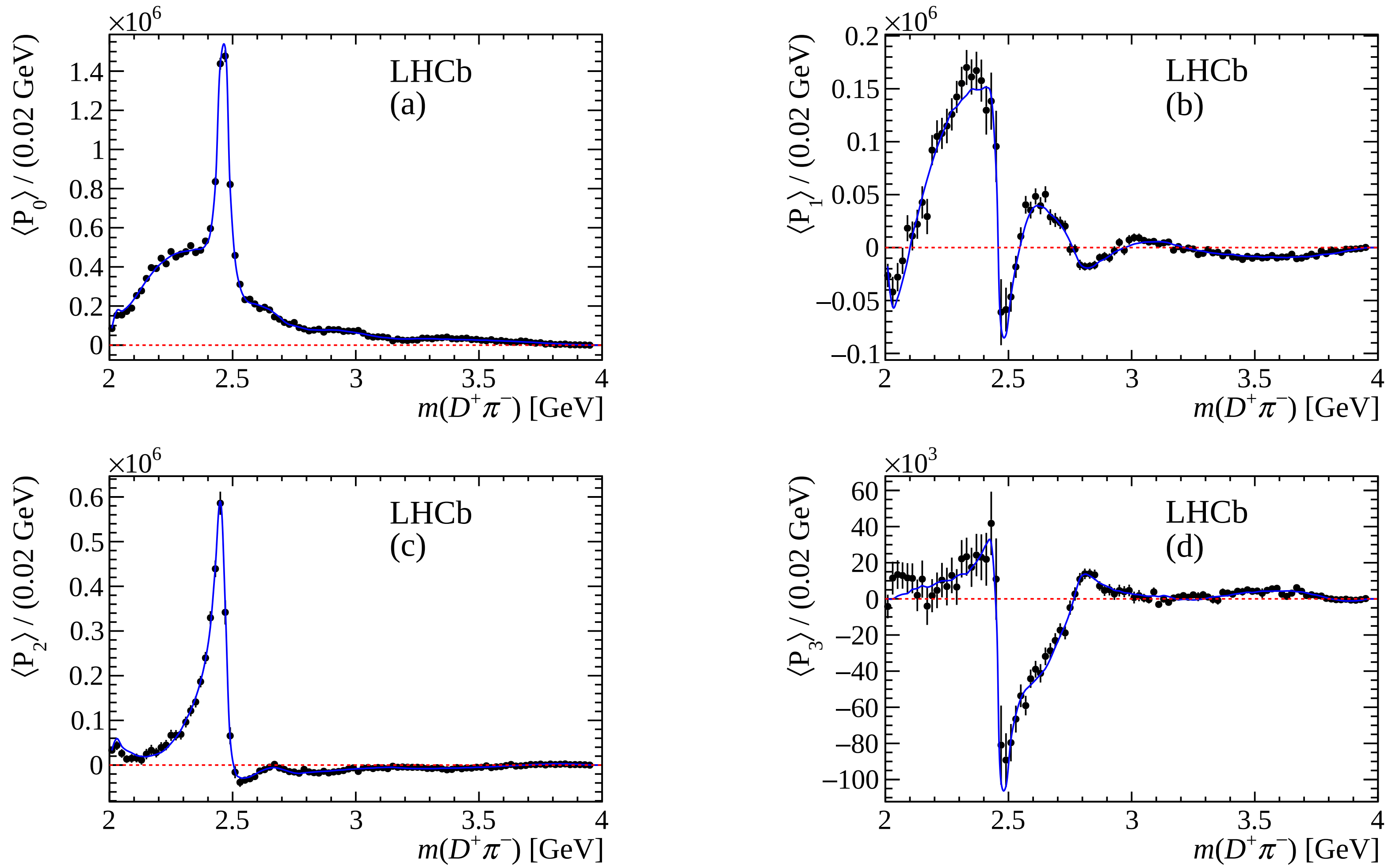
Amplitude Analysis of $B \to D\pi\pi$

Data for $B \to D\pi\pi$

- Recent high precision results for $B \to D\pi\pi$ from LHCb

Aaji et al. [LHCb], Phys. Rev. D **94** (2016) 072001, . . .

- Spectroscopic information in the angular moments ($D\pi$ FSI):



Theory of $B \to D\pi\pi$

Du, Albadajedo, Fernandez-Soler, Guo, Hanhart, UGM, Nieves, Phys. Rev. D **98** (2018) 094018

- Effective Lagrangian for $B \to D$ transitions w/ one fast & one slow pseudoscalar

  Savage, Wise, Phys. Rev. D **39** (1989) 3346

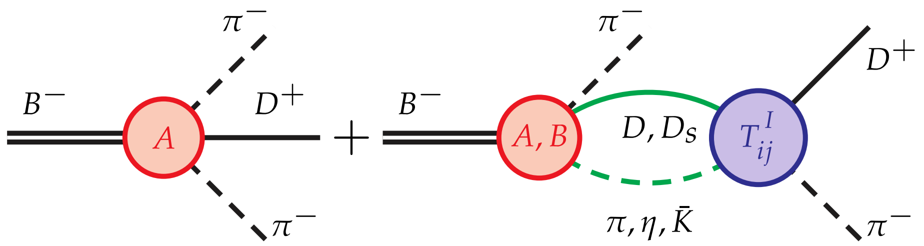
- $B^- \to D^+\pi^-\pi^-$ contains coupled-channel $D\pi$ FSI

- Consider S, P, D waves: $\mathcal{A}(B^- \to D^+\pi^-\pi^-) = \mathcal{A}_0(s) + \mathcal{A}_1(s) + \mathcal{A}_2(s)$

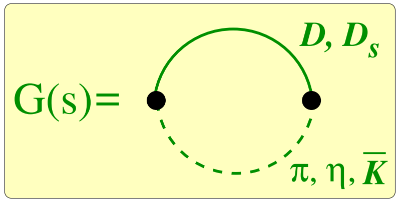
  $\rightarrow$ P-wave: $D^\star, D^\star(2680)$; D-wave: $D_2(2460)$ as by LHCb

  $\rightarrow$ S-wave: use coupled channel $(D\pi, D\eta, D_s\bar{K})$ amplitudes with all parameters fixed before

  $\rightarrow$ only two parameters in the S-wave (one combination of the LECs c_i and one subtraction constant in the G_{ij})



$$\mathcal{A}_0(s) \propto E_\pi \left[2 + G_{D\pi}(s)\left(\tfrac{5}{3}T^{1/2}_{11}(s) + \tfrac{1}{3}T^{3/2}_{11}(s)\right)\right]$$
$$+\tfrac{1}{3}E_\eta G_{D\eta}(s)T^{1/2}_{21}(s) + \sqrt{\tfrac{2}{3}}E_{\bar{K}}G_{D_s\bar{K}}(s)T^{1/2}_{31}(s) + \ldots$$



Analysis of $B \to D\pi\pi$

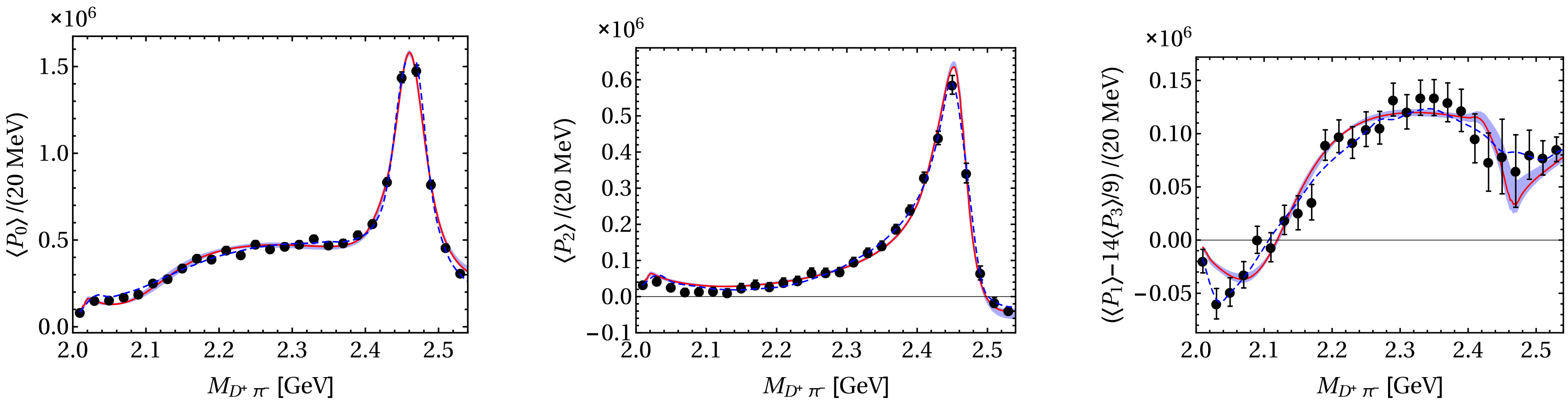
Du, Albadajedo, Fernandez-Soler, Guo, Hanhart, UGM, Nieves, Yao, Phys. Rev. D **98** (2018) 094018

- More appropriate combinations of the angular moments:

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2$$

$$\langle P_2 \rangle \propto \tfrac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2| \cos(\delta_2 - \delta_0)$$

$$\langle P_{13} \rangle = \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1| \cos(\delta_1 - \delta_0)$$

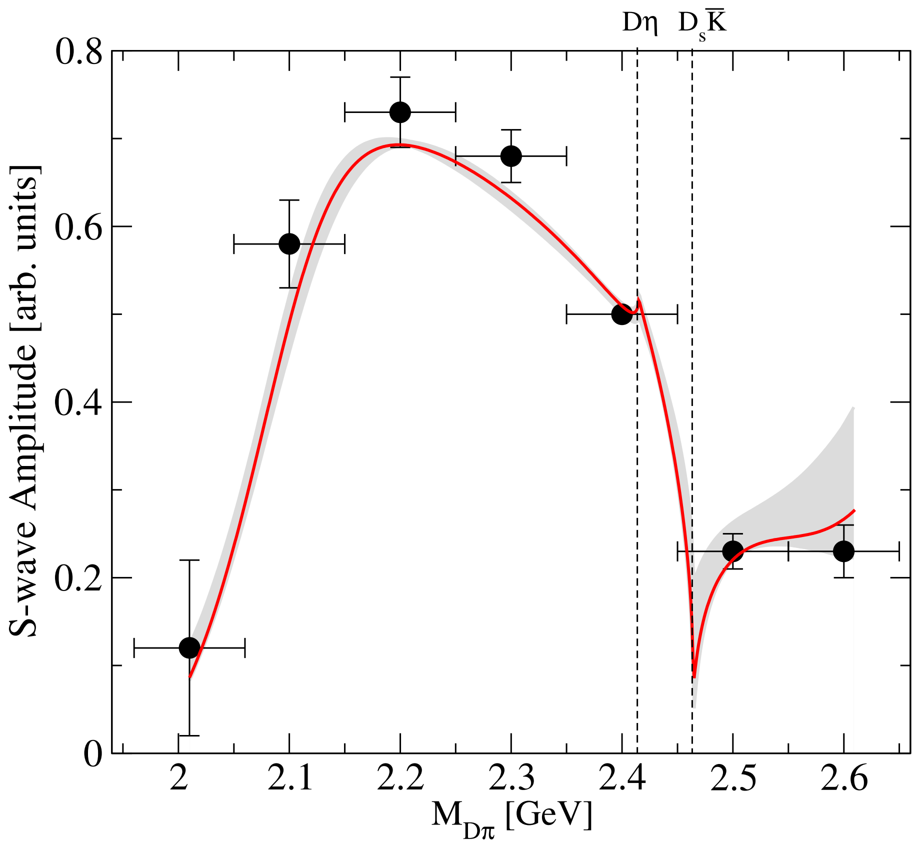


- The **S-wave** $D\pi$ can be very well described using pre-fixed amplitudes
- Fast variation in [2.4,2.5] GeV in $\langle P_{13} \rangle$: cusps at the $D\eta$ and $D_s\bar{K}$ thresholds

$\hookrightarrow$ should be tested experimentally

A closer look at the S–wave

- LHCb provides anchor points, where the strength and the phase of the S-wave were extracted from the data and connected by cubic spline

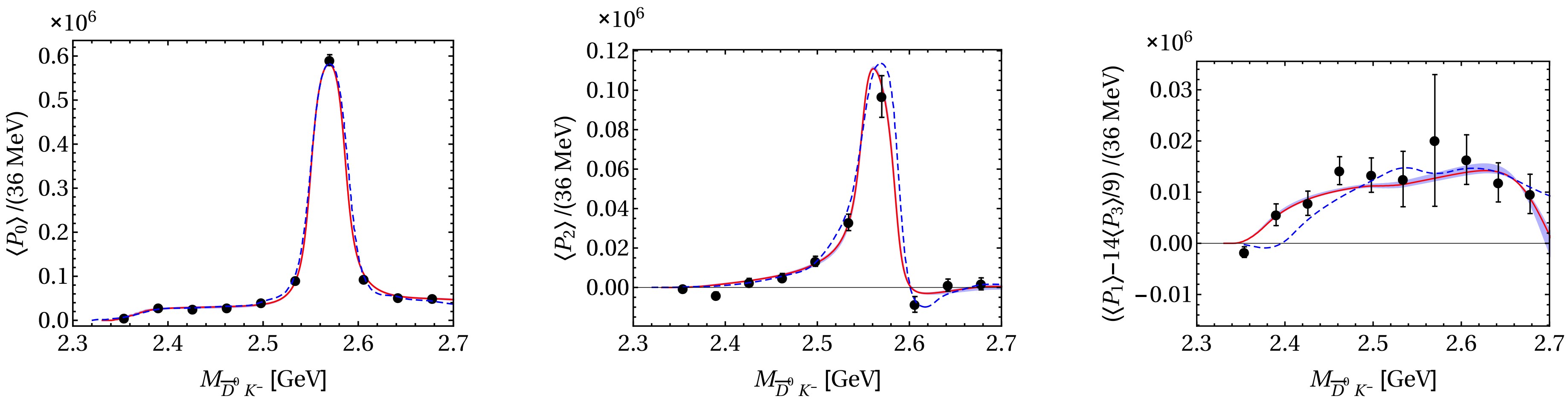


- Higher mass pole at 2.46 GeV clearly amplifies the cusps predicted in our amplitude

Theory of $B_s^0 \to \bar{D}^0 K^- \pi^+$

Du, Albadajedo, Fernandez-Soler, Guo, Hanhart, UGM, Nieves, Yao, Phys. Rev. **D98** (2018) 094018

- LHCb has also data on $B_s^0 \to \bar{D}^0 K^- \pi^+$, but less precise
- Same formalism as before, one different combination of the LECs c_i
- same resonances in the P- and D-wave as LHCb $\hookrightarrow$ one parameter fit!



$\Rightarrow$ these data are also well described

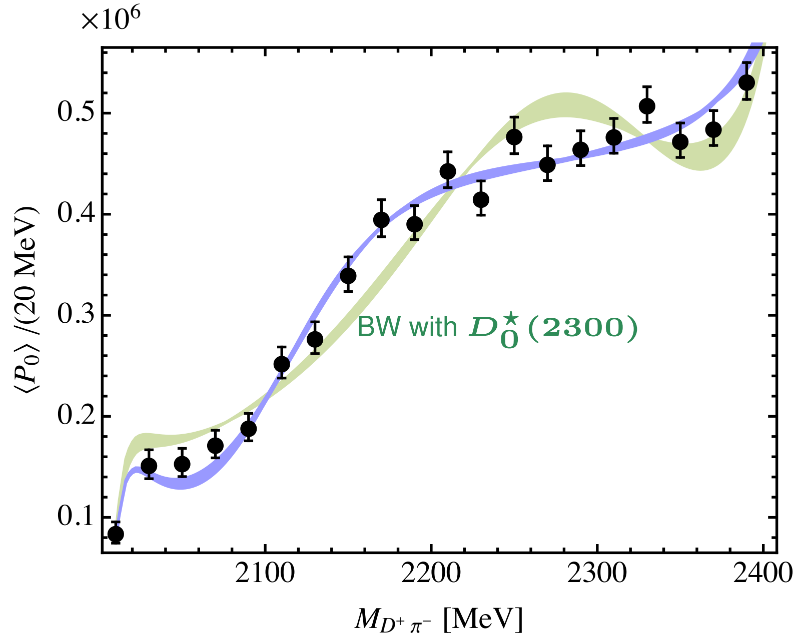
$\Rightarrow$ better data for $\langle P_{13} \rangle$ would be welcome

$\Rightarrow$ even more channels, see Du, Guo, UGM, Phys. Rev. D **99** (2019) 114002

Where is the lowest charm-strange meson?

Du, Guo, Hanhart, Kubis, UGM, Phys. Rev. Lett. **126** (2021) 192001 [2012.04599]

- Precise analysis of the LHCb data on $B^- \to D^+\pi^-\pi^-$ using UChPT and Khuri-Treiman eq's (3-body unit.)
  Aaji et al. [LHCb], Phys. Rev. D **94** (2016) 072001



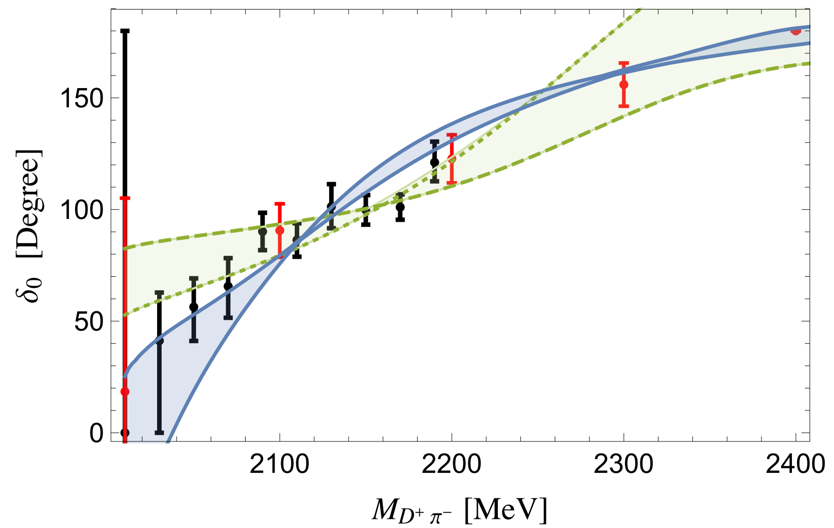
- Breit-Wigner description not appropriate for the S-wave but UChPT and the dispersive analysis are!

- First determination of the $D\pi$ phase shift

- The lowest charm-strange meson is located at:

$$\left(2105^{+6}_{-8} - i\,102^{+10}_{-11}\right) \text{MeV}$$

- Recently confirmed by Lattice QCD!
  Cheung et al. [HadSpec], JHEP **02** (2021) 100 [2008.06432]



PDG update

- The PDG group is like a heavy tanker, still there is motion:

$D_0^*(2300)$ $\qquad I(J^P) = \frac{1}{2}(0^+)$

was $D_0^*(2400)$

There is a strong evidence that recent data on $B \to D\pi\pi$ (AAIJ 15Y, AAIJ 16AH) and $B \to D\pi K$ (AAIJ 14BH, AAIJ 15V, AAIJ 15X) call for two poles in the scalar $I = 1/2$ πD amplitude in this mass range. The data are consistent with a lower pole at $(2105^{+6}_{-8}) - i(102^{+10}_{-11})$ MeV and a higher pole at $(2451^{+35}_{-26}) - i(134^{+7}_{-8})$ MeV (DU 18A, DU 19, DU 21). For details see review on "Heavy Non-$q\bar{q}$ Mesons."

$D_0^*(2300)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
2343±10 OUR AVERAGE	Error includes scale factor of 1.5. See the ideogram below.				

RPP 2024: 79. Heavy Non-$q\bar{q}$ Mesons, Hanhart, Gutsche, Mitchell

$\Rightarrow$ stay tuned!

Summary

Summary & discussion

- Chiral coupled-channel dynamics of QCD generates two-pole structures
  Oller, UGM (2001), Jido et al. (2005)
- Further two-pole structures beyond the $\mathbf{\Lambda(1405)}$ and $\boldsymbol{D_0^\star(2300)}$
  ↪ $\boldsymbol{K_1(1270)}$ meson Roca et al., PRD **72** (2005) 014002, Geng et al., PRD **75** (2007) 014017
  ↪ $\mathbf{\Xi(1820)}$ baryon Sarkar et al., Nucl. Phys. A **750** (2005) 294, ...
  ↪ $\boldsymbol{Y(4260)}$ meson? Ablikim et al. [BESIII], Phys. Rev. D **102** (2020) 031101
  ↪ more to be found ... (interplay of lattice QCD / EFT/ disp. rel./ data)
- All this is not properly reflected in the PDG tables
  ↪ summary tables e.g. only lists one pole for the $\mathbf{\Lambda(1405)}$
  ↪ many states analyzed using BW parametrization :-(
  ↪ exp. collaborations must stop committing sins like
  using BW parametrization close to threshold (BESIII, LHCb, ...)
  ↪ PDG needs a more serious approach to the hadron spectrum!

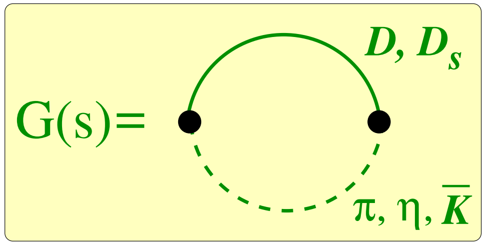
SPARES

Finite volume formalism

- Goal: postdict the finite volume (FV) energy levels for $I = 1/2$ and compare with the recent LQCD results from Moir et al. using the already fixed LECs
  $\to$ parameter-free insights into the $D_0^\star(2300)$

- In a FV, momenta are quantized: $\vec{q} = \frac{2\pi}{L}\vec{n}\,, \quad \vec{n} \in \mathbb{Z}^3$

$\Rightarrow$ Loop function $G(s)$ gets modified: $\int d^3\vec{q} \to \frac{1}{L^3}\sum_{\vec{q}}$



$$\tilde{G}(s,L) = G(s) = \lim_{\Lambda\to\infty}\left[\frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}|<\Lambda} I(\vec{q}) - \int_0^\Lambda \frac{q^2 dq}{2\pi^2} I(\vec{q})\right]$$

Döring, UGM, Rusetsky, Oset, Eur. Phys. J. A**47** (2011) 139

- FV energy levels from the poles of $\tilde{T}(s,L)$:

$$\tilde{T}^{-1}(s,L) = \mathcal{V}^{-1}(s) - \tilde{G}(s,L)$$

Chiral Lagrangian for $B \to D$ transitions

Savage, Wise, Phys. Rev. D39 (1989) 3346

- Consider $\bar{B} \to D$ transition with the emission of two light pseudoscalars (pions)
  $\hookrightarrow$ chiral symmetry puts constraints on one of the two pions
  $\hookrightarrow$ the other pion moves fast and does not participate in the final-state interactions

- Chiral effective Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\rm eff} = \bar{B}\big[& c_1\,(u_\mu t M + M t u_\mu) + c_2\,(u_\mu M + M u_\mu)\,t \\
& + c_3\,t\,(u_\mu M + M u_\mu) + c_4\,(u_\mu \langle M t\rangle + M\langle u_\mu t\rangle) \\
& + c_5\,t\langle M u_\mu\rangle + c_6\langle (M u_\mu + u_\mu M)\,t\rangle\big]\partial^\mu D^\dagger
\end{aligned}
$$

with

$\bar{B} = (B^-, \bar{B}^0, \bar{B}_s^0)\,, \quad D = (D^0, D^+, D_s^+)$

M is the matter field for the fast-moving pion

$t = uHu$ is a spurion field for Cabbibo-allowed decays

$\to$ only some combinations of the LECs c_i appear

$$
H = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

Some formalism

- Exact three-body unitarity via Khuri-Treiman equations: Khuri, Treiman (1960)

$\hookrightarrow$ write $\mathcal{A}_{+--}(B^- \to D^+\pi^-\pi^-)$ and $\mathcal{A}_{00-}(B^- \to D^0\pi^0\pi^-)$ as [reconstruction theorem]

$$\mathcal{A}_{+--}(s,t,u) = \mathcal{F}_0^{1/2}(s) + \frac{\kappa(s)}{4} z_s \mathcal{F}_1^{1/2}(s) + \frac{\kappa(s)^2}{16}(3z_s^2-1)\mathcal{F}_2^{1/2}(s) + (t \leftrightarrow s)$$

$$\mathcal{A}_{00-}(s,t,u) = -\frac{1}{\sqrt{2}}\mathcal{F}_0^{1/2}(s) - \frac{\kappa(s)}{4\sqrt{2}} z_s \mathcal{F}_1^{1/2}(s) - \frac{\kappa(s)^2}{16\sqrt{2}}(3z_s^2-1)\mathcal{F}_2^{1/2}(s) + \frac{\kappa_u(u)}{4} z_u \mathcal{F}_1^1(u)$$

$$z_s = \cos\theta_s = \frac{s(t-u)-\Delta}{\kappa(s)},\ z_u = \cos\theta_u = \frac{t-s}{\kappa_u(u)}\ , \quad \Delta = (M_B^2 - M_\pi^2)(M_D^2 - M_\pi^2)$$

$$\kappa(s) = \lambda^{1/2}(s, M_D^2, M_\pi^2)\lambda^{1/2}(s, M_B^2, M_\pi^2),\ \kappa_u(u) = \lambda^{1/2}(u, M_B^2, M_D^2)\sqrt{1-4M_\pi^2/u}$$

$\mathcal{F}_\ell^I$: angular momentum $\ell \leq 2$, isospin $I < 3/2$

- Solve via the Omnès ansatz:

$$\mathcal{F}_\ell^I(s) = \Omega_\ell^I(s)\left\{Q_\ell^I(s) + \frac{s^n}{\pi}\int_{s_{\rm th}}^\infty \frac{ds'}{s'^n}\frac{\sin\delta_\ell^I(s')\hat{\mathcal{F}}_\ell^I(s')}{|\Omega_\ell^I(s')|(s'-s)}\right\},$$

$Q_\ell^I(s)$ = polynom of degree zero (one subtraction suffices)

$$\Omega_\ell^I(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\rm th}}^\infty ds' \frac{\delta_\ell^I(s')}{s'(s'-s)}\right\}$$