Pion Form Factor Analysis

Topic : Pion form factor Calculation

ETMC Meeting : 25/09/19

C. Urbach, B. Kostrzewa, F. Pittler, Y. Lee

universitätbonn
Rheinische
Friedrich-Wilhelms-

HISKP

Extended Twisted Mass
Collaboration

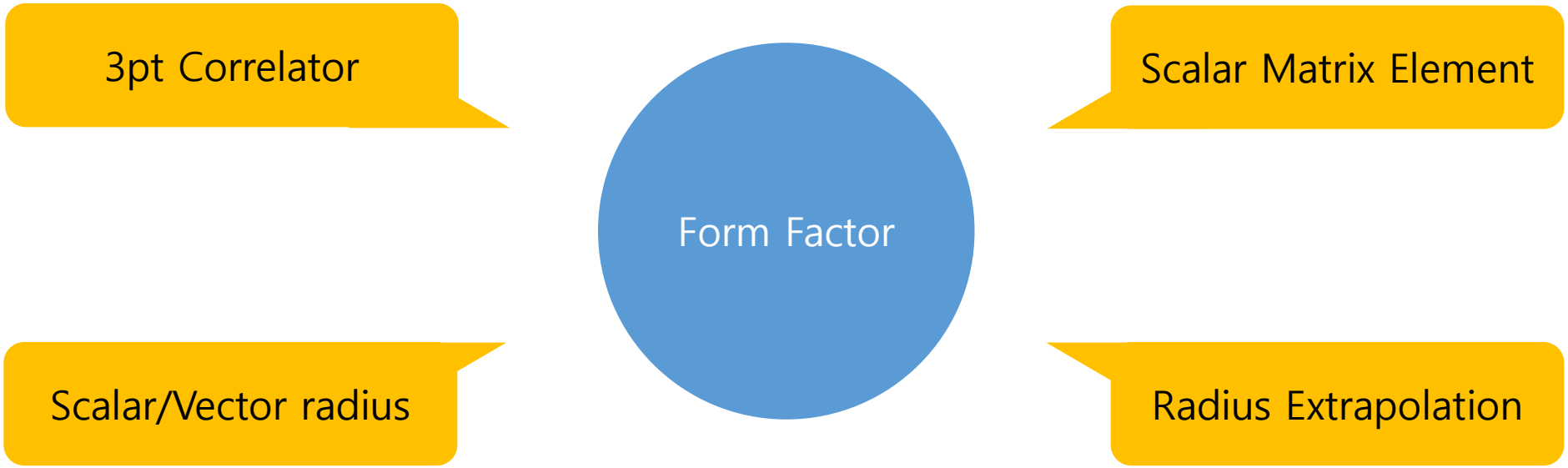
Contents

Extended Twisted Mass Collaboration

Introduction

Extended Twisted Mass Collaboration

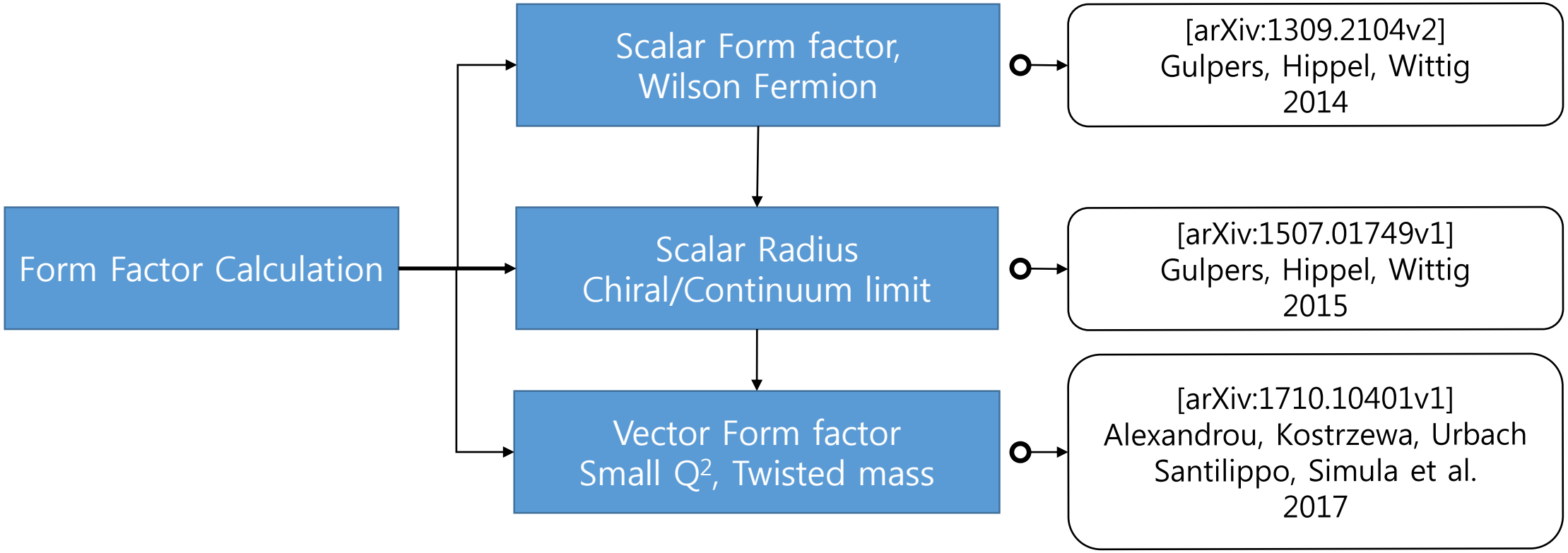
Why Form Factor?



Introduction

Extended Twisted Mass Collaboration

Approach



Introduction

Extended Twisted Mass Collaboration

Project Purpose

1. Calculate Scalar/Vetor Form factor behavior by Q^2 from 3pt Correlator
2. Calculate Scalar/Vector radius behavior by M_π^2 (*Preliminary Frame work finish)
3. Using twisted mass, one-end-trick, and measuring disconnected part contribution
4. Reduce and understand Systematic Errors for further improvement
5. Testing the necessity for momentum shell average during analysis

*Q^2 : $-(p_f-p_i)^2$

Introduction

Extended Twisted Mass Collaboration

Test Environment

1. **Ensemble :**

Test Sample

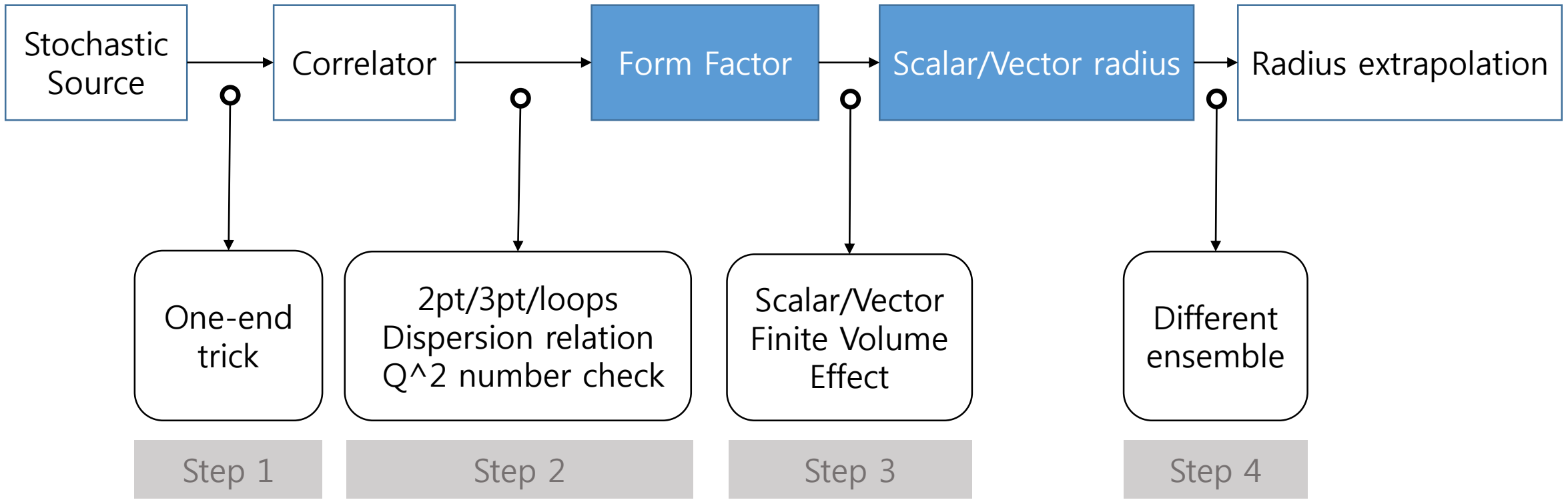
name	beta	a	L	aM_π
cA211.53.24	1.726	0.098	24	0.1667
cA211.30.32	1.726	0.098	24	0.1256
cA211.40.24	1.726	0.098	24	0.1452
cB211.25.48	1.778	0.081	48	0.1042
cB211.072.64 (Physical point)	1.778	0.081	64	0.0566

2. **Correlator : Pion 2pt, 3pt function with insertion, loop**
3. **Source-Sink separation (Delta t) : 12, 16, 20, 24, 28, 32, 36**
4. **Stochastic source :** $\xi_r^{a,\alpha}(\vec{x},t)\,|\,r\in[1,\ldots,N_R]\quad \mathbb{Z}(2)+i\mathbb{Z}(2)$

Introduction

Extended Twisted Mass Collaboration

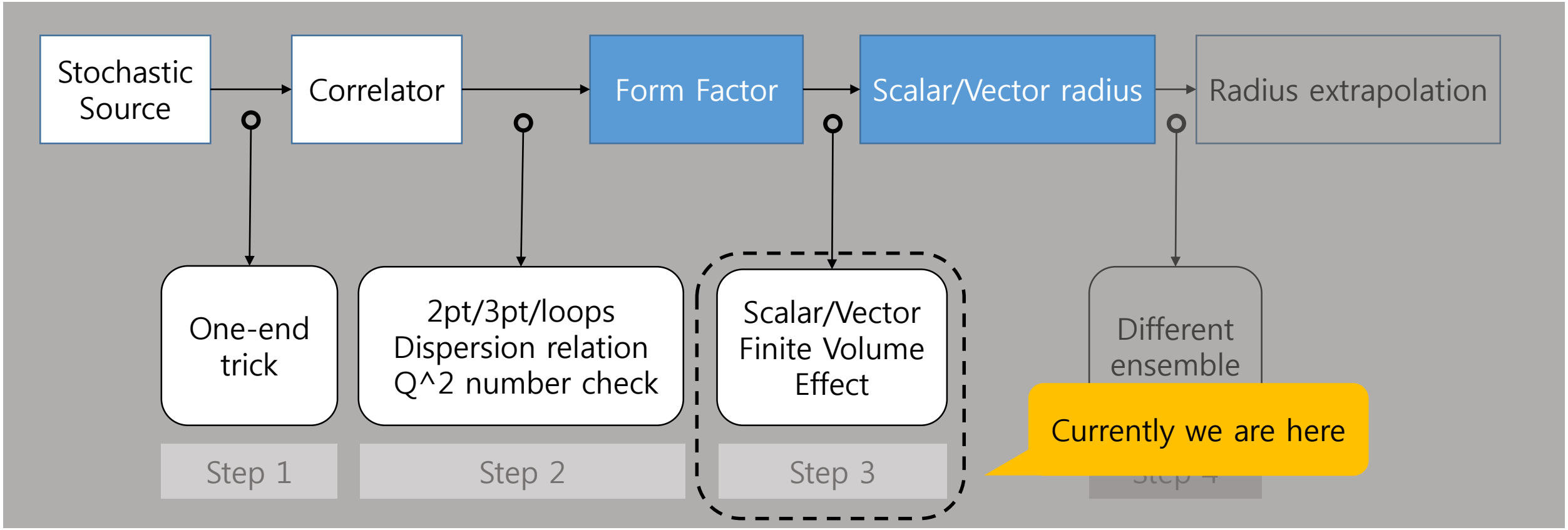
Analysis Framework for one Ensemble



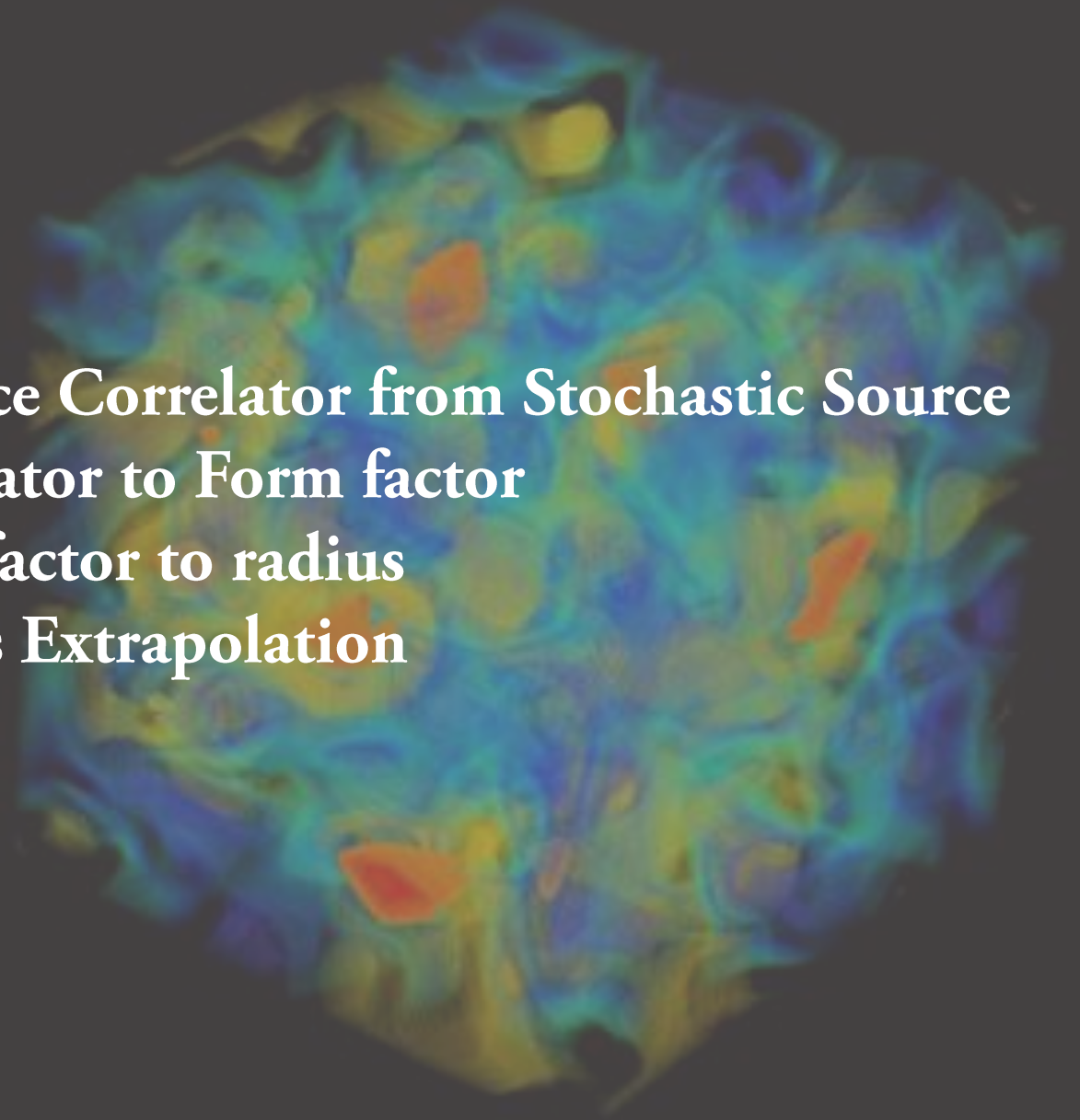
Introduction

Extended Twisted Mass Collaboration

Analysis Framework for one Ensemble



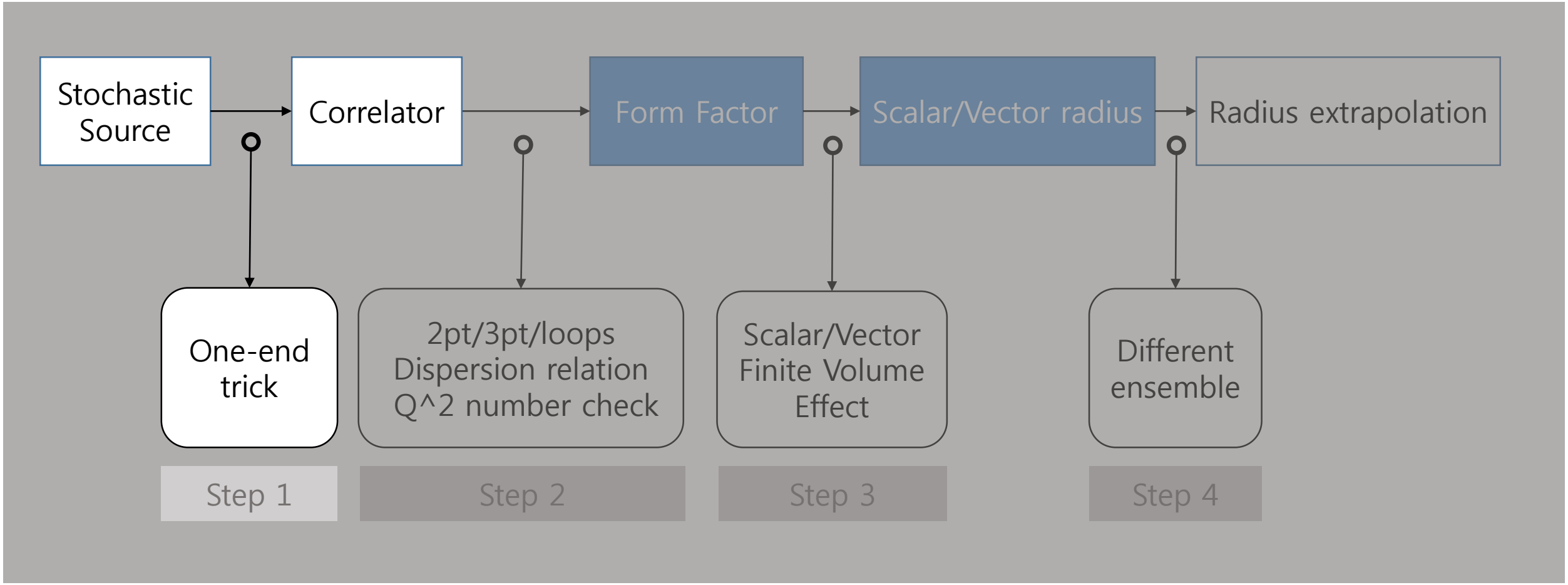
Theory



1. Step1 : Produce Correlator from Stochastic Source
2. Step2 : Correlator to Form factor
3. Step3 : From factor to radius
4. Step4 : Radius Extrapolation

Theory_ Correlator Produce

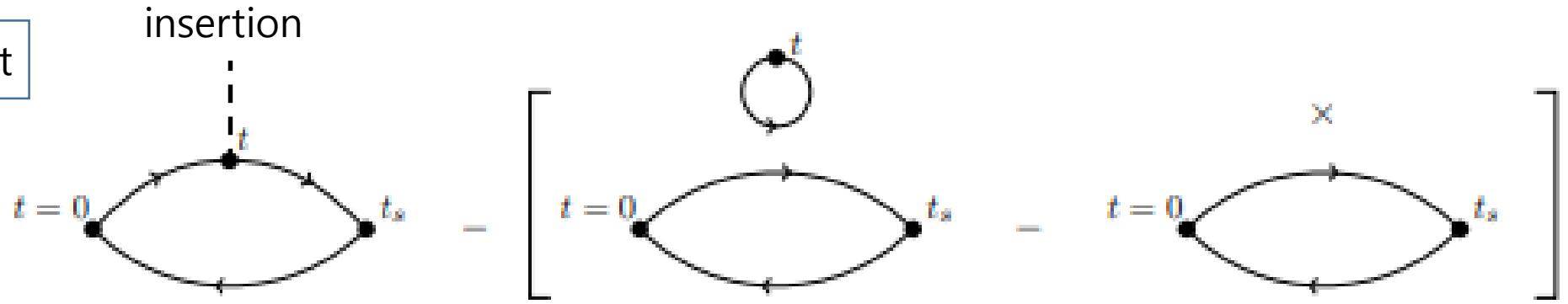
Extended Twisted Mass Collaboration



Theory_ Correlator Produce

Extended Twisted Mass Collaboration

3-point



C_π^{3pt} = ① (Full Connected 3pt) – ② (2pt with Loop) + ③ (2pt with Vacuum)

Theory_ Correlator Produce

Extended Twisted Mass Collaboration

Total Disconnected Contribution

$$C^{\pi\pi}_{S,f}(t,t',\vec{p}_i,\vec{p}_f) = \sum_{x,y,z} \langle \bar{d}(y)\gamma_5 u(y)\, \bar{O}_f(x)\, \left(\bar{d}(z)\gamma_5 u(z)\right)^\dagger \rangle\, \delta_{t,t_x-t_z}\delta_{t',t_y-t_z} e^{-i\vec{p}_i\cdot(\vec{x}-\vec{z})+i\vec{p}_f\cdot(\vec{x}-\vec{y})}$$

$$\mathcal{D}^{\pi\pi}_{S,f}(t,t',\vec{p}_i,\vec{p}_f) = \sum_{x,y,z} \left\langle \left[\bar{d}(y)\gamma_5 u(y)\, \left(\bar{d}(z)\gamma_5 u(z)\right)^\dagger\right]\, \left[\bar{O}_f(x)\right] \right\rangle\, \delta_{t,t_x-t_z}\delta_{t',t_y-t_z} e^{-i\vec{p}_i\cdot(\vec{x}-\vec{z})+i\vec{p}_f\cdot(\vec{x}-\vec{y})}$$

$$\mathcal{D}^{\pi\pi}_{S,f,\mathrm{vac}}(t,t',\vec{p},\vec{p}) = \sum_{x,y,z} \langle \bar{d}(y)\gamma_5 u(y)\, \left(\bar{d}(z)\gamma_5 u(z)\right)^\dagger \rangle\, \langle \bar{O}_f(x) \rangle\, \delta_{t,t_x-t_z}\delta_{t',t_y-t_z} e^{-i\vec{p}\cdot(\vec{y}-\vec{z})}$$

Total Contribution

$$C^{\pi\pi}_{S,\ell}(t,t',\vec{p}_i,\vec{p}_f) - \mathcal{D}^{\pi\pi}_{S,f}(t_x-t_{\mathrm{src}},t_y-t_{\mathrm{src}},\vec{p}_i,\vec{p}_f) + \delta(\vec{p}_f-\vec{p}_i)\mathcal{D}^{\pi\pi}_{S,f,\mathrm{vac}}(t_x-t_{\mathrm{src}},t_y-t_{\mathrm{src}},\vec{p}_i,\vec{p}_f)$$

Theory_ Correlator Produce

Extended Twisted Mass Collaboration

Stochastic Source

$$\xi_r^{a,\alpha}(\vec{x},t)\,|\,r\in[1,\ldots,N_R] \qquad \lim_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N}\xi_r^{a,\alpha}(\vec{x})\cdot[\xi_r^{b,\beta}(\vec{y})]^* = \delta_{a,b}\,\delta_{\alpha,\beta}\,\delta_{\vec{x},\vec{y}} \qquad \lim_{N\to\infty}\frac{1}{N}\sum_{r=1}^{N}\xi_r^{a,\alpha}(\vec{x}) = 0$$

$$\mathbb{Z}(2)+i\mathbb{Z}(2)$$

One-end trick, Sequential Propagator & 2pt function

$$\eta_{r,\ell}^{\vec{\theta}}(\vec{x},t,t_{\rm src}) = \sum_{\vec{y}}(D_\ell^{\vec{\theta}})^{-1}(\vec{x},t\leftarrow\vec{y},t_{\rm src})\cdot\xi_r(\vec{y},t_{\rm src}) \qquad \ell = u,d,s^+,s^- \,, \qquad \vec{\theta}=\frac{\vec{p}L}{2\pi}$$

$$\sum_{\vec{x},\alpha,a}\eta_{r,\ell}^{a,\alpha,\vec{\theta}}(\vec{x},t,t_{\rm src})\cdot\left[\eta_{r,\ell}^{a,\alpha,\vec{\theta}}(\vec{x},t,t_{\rm src})\right]^* = C_\pi(t-t_{\rm src},\vec{p})+\text{noise}, \qquad \gamma_5 D_u\gamma_5 = D_d^\dagger$$

Theory_ Correlator Produce

Extended Twisted Mass Collaboration

Sequential propagator & Three point function

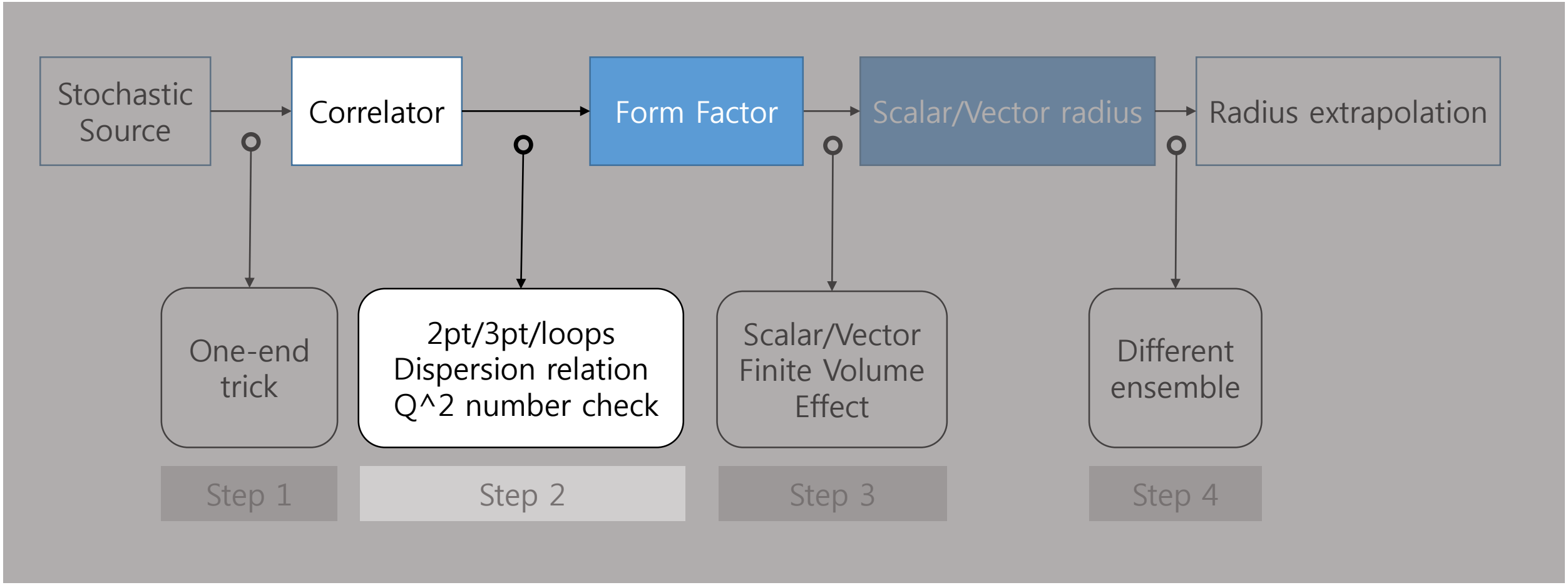
$$\eta^{\vec{\theta}}_{r,\ell}(\vec{x},t,t_{\rm src}) = \sum_{\vec{y}} (D^{\vec{\theta}}_{\ell})^{-1}(\vec{x},t \leftarrow \vec{y},t_{\rm src}) \cdot \xi_r(\vec{y},t_{\rm src}) \quad \ell = u,d,s^+,s^- \ ,$$

$$\phi^{\vec{\theta},\vec{\theta}'}_{r,\ell,\ell'} = \sum_{\vec{y}} (D^{\vec{\theta}'}_{\ell'})^{-1}(\vec{x},t \leftarrow \vec{y},t') \cdot \left[\gamma_5 \eta^{\vec{\theta}}_{r,\ell}(\vec{y},t',t_{\rm src})\right]$$

$$\sum_{\vec{x},a,\alpha,\beta} \eta^{a,\alpha,\vec{\theta}}_{r,u}(\vec{x},t) \cdot \left[\phi^{a,\beta,\vec{0},-\vec{\theta}}_{r,d,u}\right]^* (\gamma_5\gamma_0)_{\beta\alpha} \propto C_0^{\pi\pi}(t,t',\vec{p},-\vec{p}) + \text{noise}$$

Theory_ Correlator to Form factor

Extended Twisted Mass Collaboration



Theory_ Correlator to Form factor

Extended Twisted Mass Collaboration

2-point

$$C^{\pi}(t,\vec{p_f}) = \sum_{x,z} < \bar{d}(x)\gamma_5 u(x)(\bar{d}(z)\gamma_5 u(z))^{\dagger})\delta_{t,t_x-t_z} e^{-ip_f(x-z)} >$$

$$C^{\pi}(t,\vec{p_f}) \rightarrow \frac{G^2(p_f)}{2E(p_f)}[e^{-E(p_f)t} + e^{E(p_f)(T-t)}]$$

$$|G(\vec{p})| = \left|< 0|\bar{d}(0)\gamma_5 u(0)|\pi^+(\vec{p}) >\right|$$

Theory_ Correlator to Form factor

3-point

$$C^{\pi}_{3pt} = < \pi^{\dagger}(\vec{p_f}, t)|O(\tau)|\pi^{\dagger}(\vec{p_i}, 0) >$$

$$= < 0|\pi^{\dagger}(\vec{p_f})|\pi(\vec{p_f}) > \frac{e^{-E_{p_f}(t-\tau)}}{2E_{p_f}} < \pi^{\dagger}(\vec{p_f})|O(\tau)|\pi^{\dagger}(\vec{p_i}) > \frac{e^{-E_{p_i}(\tau-0)}}{2E_{p_i}} < \pi(\vec{p_i})|\pi^{\dagger}(\vec{p_i})|0 >$$

$$\rightarrow \frac{G(\vec{p_f})G(\vec{p_i})}{4E(\vec{p_f})E(\vec{p_i})} < \pi^{\dagger}(p_f)|O(\tau)|\pi^{\dagger}(p_i) > [e^{-E_{p_f}(T-\tau)} + e^{-E_{p_i}(\tau)}]$$

Theory_ Correlator to Form factor

Extended Twisted Mass Collaboration

Correlator to Form factor

$$< \pi^\dagger(p_f)|O(\tau)|\pi^\dagger(p_i) >= C^{\pi}_{3pt}\left(\frac{G(\vec{p_f})G(\vec{p_i})}{4E(\vec{p_f})E(\vec{p_i})}[e^{-E_{p_f}(T-\tau)}+e^{-E_{p_i}(\tau)}]\right)^{-1}$$

$$O_S = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s \qquad O_V = \frac{2}{3}\bar{u}\gamma_0 u - \frac{1}{3}\bar{d}\gamma_0 d \approx \bar{u}\gamma_0 u$$

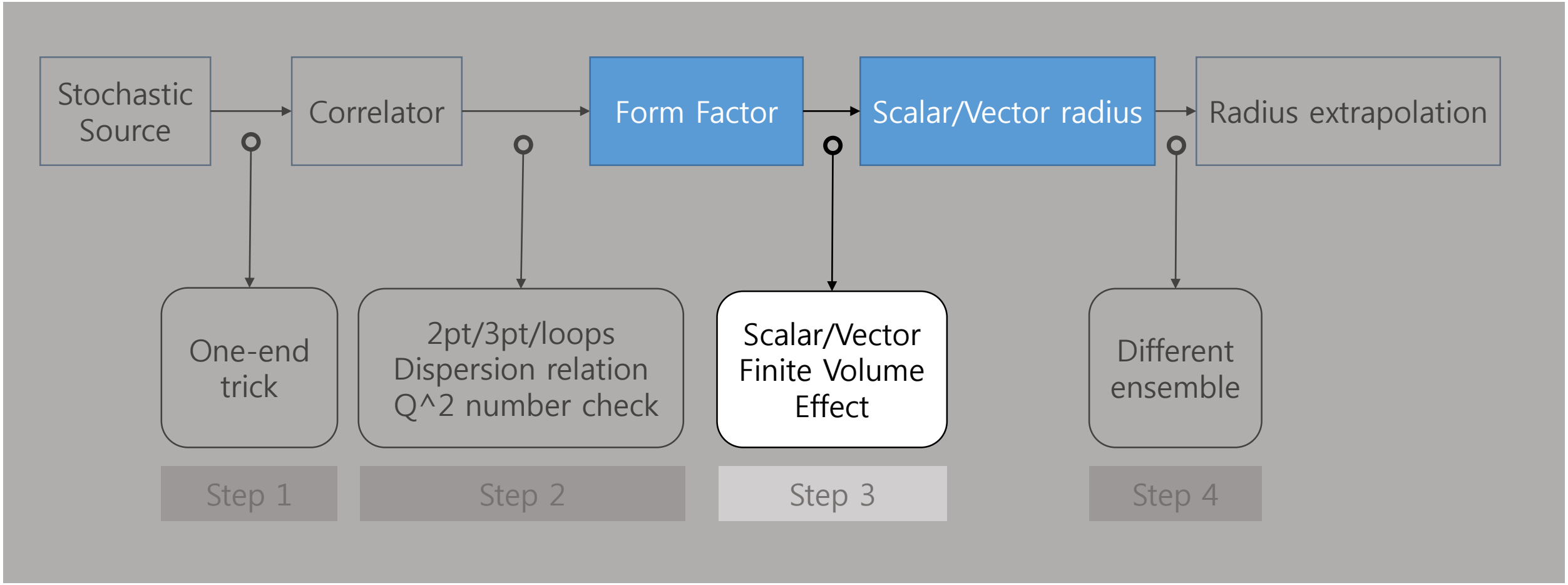
$$F^{\pi}_S(Q^2) =< \pi^\dagger(p_f)|O_S|\pi^\dagger(p_i) >$$

$$F^{\pi}_V(Q^2) =< \pi^\dagger(p_f)|O_V|\pi^\dagger(p_i) > /(E(p_f)+E(p_i))$$

u and d terms are equal magnitude and opposite sign

Theory_ Form factor to radius

Extended Twisted Mass Collaboration



Theory_ Form factor to radius

Form factor to pion radius

$$< r^2 >^{\pi}_{S,V} = -\frac{6}{F^{\pi}_{S,V}(0)^2}\frac{\partial F^{\pi}_{S,V}(Q^2)}{\partial Q^2}\Big|_{Q^2=0}$$

Vera Guelpers et al. 2015 [arXiv:1507.01749v1]

$$F^{\pi}_{S,V}(Q^2) = F^{\pi}_{S,V}(0)(1 - \frac{1}{6} < r^2 >^{\pi}_{S,V} Q^2 + O(Q^4))$$

$$F^{\pi}_{S,V}(Q^2)/F^{\pi}_{S,V}(0) = (1 - \frac{1}{6} M_{\pi}^2 < r^2 >^{\pi}_{S,V} (Q/M_{\pi})^2 + O(Q/M_{\pi})^4)$$

$$1/F^{\pi}_{S,V}(Q^2)_{normalized} = 1 + \frac{1}{6} M_{\pi}^2 < r^2 >^{\pi}_{S,V} (Q/M_{\pi})^2 + O(Q/M_{\pi})^4$$

Theory_ Form factor to radius

Form factor to pion radius

$$< r^2 >^{\pi}_{S,V} = -\frac{6}{F^{\pi}_{S,V}(0)^2}\frac{\partial F^{\pi}_{S,V}(Q^2)}{\partial Q^2}\Big|_{Q^2=0}$$

Vera Guelpers et al. 2015 [arXiv:1507.01749v1]

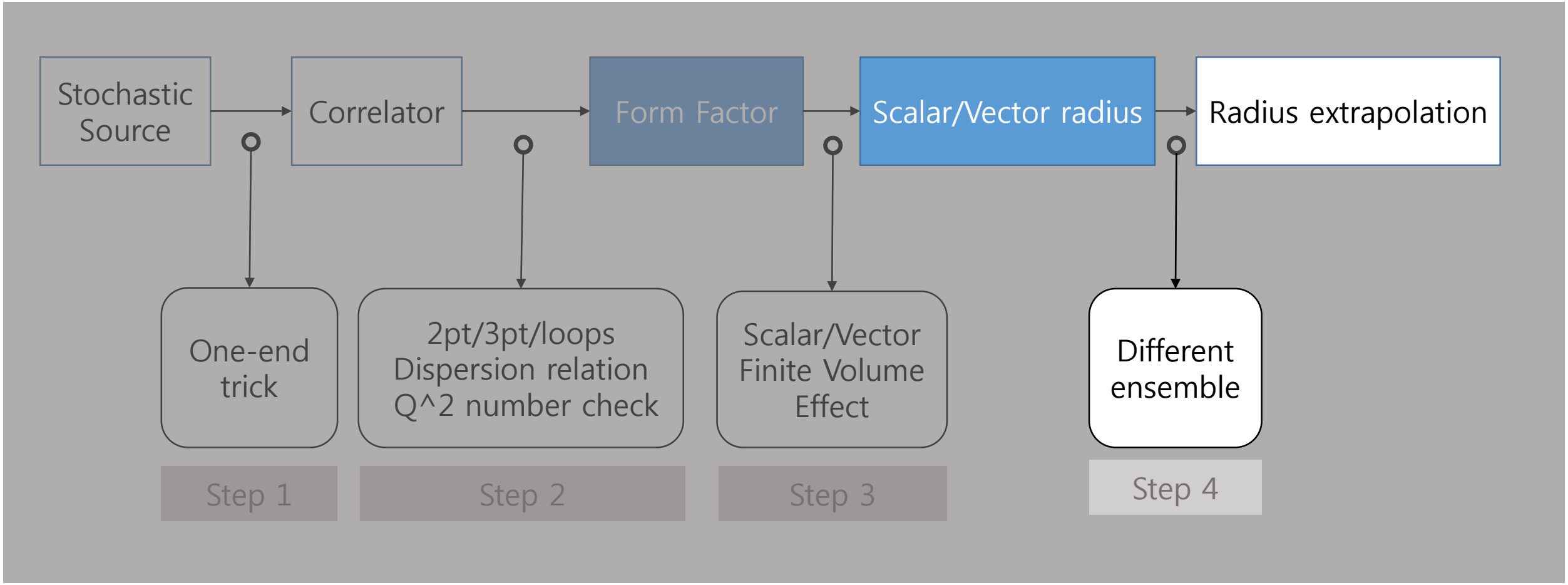
$$F^{\pi}_{S,V}(Q^2) = F^{\pi}_{S,V}(0)(1 - \frac{1}{6} < r^2 >^{\pi}_{S,V} Q^2 + O(Q^4))$$

$$F^{\pi}_{S,V}(Q^2)/F^{\pi}_{S,V}(0) = (1 - \frac{1}{6} M_{\pi}^2 < r^2 >^{\pi}_{S,V} (Q/M_{\pi})^2 + O(Q/M_{\pi})^4)$$

$$1/F^{\pi}_{S,V}(Q^2)_{normalized} = 1 + \frac{1}{6} M_{\pi}^2 < r^2 >^{\pi}_{S,V} (Q/M_{\pi})^2 + O(Q/M_{\pi})^4$$

Theory_ Radius Extrapolation

Extended Twisted Mass Collaboration



Theory_ Radius Extrapolation

Extended Twisted Mass Collaboration

General Pion Radius Form as function of m_π

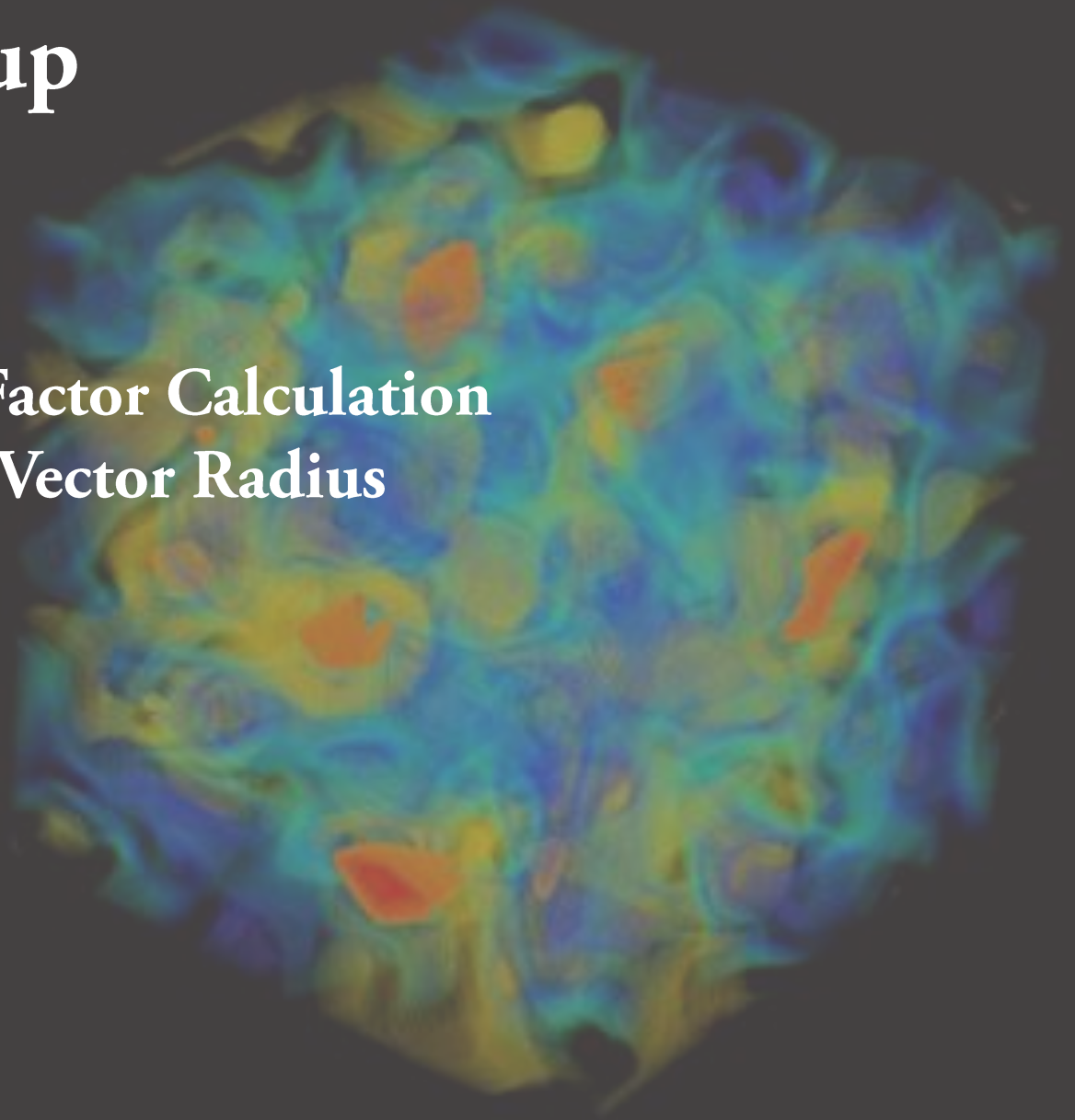
$$< r^2 >_S^\pi = \frac{1}{(4\pi f_\pi)^2}\frac{13}{2} + \frac{6}{(4\pi f_\pi)^2}\left[\bar{l}_4 + \log(m_{\pi,phy}^2/m_\pi^2)\right]$$

Vera Guelpers et al. 2015 [arXiv:1507.01749v1]

$$< r^2 >_V^\pi = \frac{1}{(4\pi f_\pi)^2}\left[2(\bar{l}_6 - 1) + \frac{m_\pi^2}{(4\pi f_\pi)^2} b_1\right]$$

C. Alexandrou et al. 2017 [arXiv:1710.10401v1]

Analysis Setup



1. Step2 : Form Factor Calculation
2. Step3 : Scalar/Vector Radius

Setup. #1_Form Factor Calculation

Extended Twisted Mass Collaboration

Step 2

Correlator

Form Factor

2pt, 3pt Correlator → Matrix Element → Form Factor

$$< \pi^\dagger(p_f)|O(\tau)|\pi^\dagger(p_i) >= C^{\pi}_{3pt}\left(\frac{G(\vec{p_f})G(\vec{p_i})}{4E(\vec{p_f})E(\vec{p_i})}[e^{-E_{p_f}(T-\tau)} + e^{-E_{p_i}(\tau)}]\right)^{-1}$$

$$O'_S = \bar{u}u + \bar{d}d \qquad O'_V = \bar{u}\gamma_0 u$$

$$F^{\pi}_S(Q^2) = m_u < \pi^\dagger(p_f)|O'_S|\pi^\dagger(p_i) >$$

$$F^{\pi}_V(Q^2) =< \pi^\dagger(p_f)|O'_V|\pi^\dagger(p_i) > /(E(p_f) + E(p_i))$$

Setup. #2_Scalar/Vector radius

Extended Twisted Mass Collaboration

Form Factor

Scalar/Vector radius

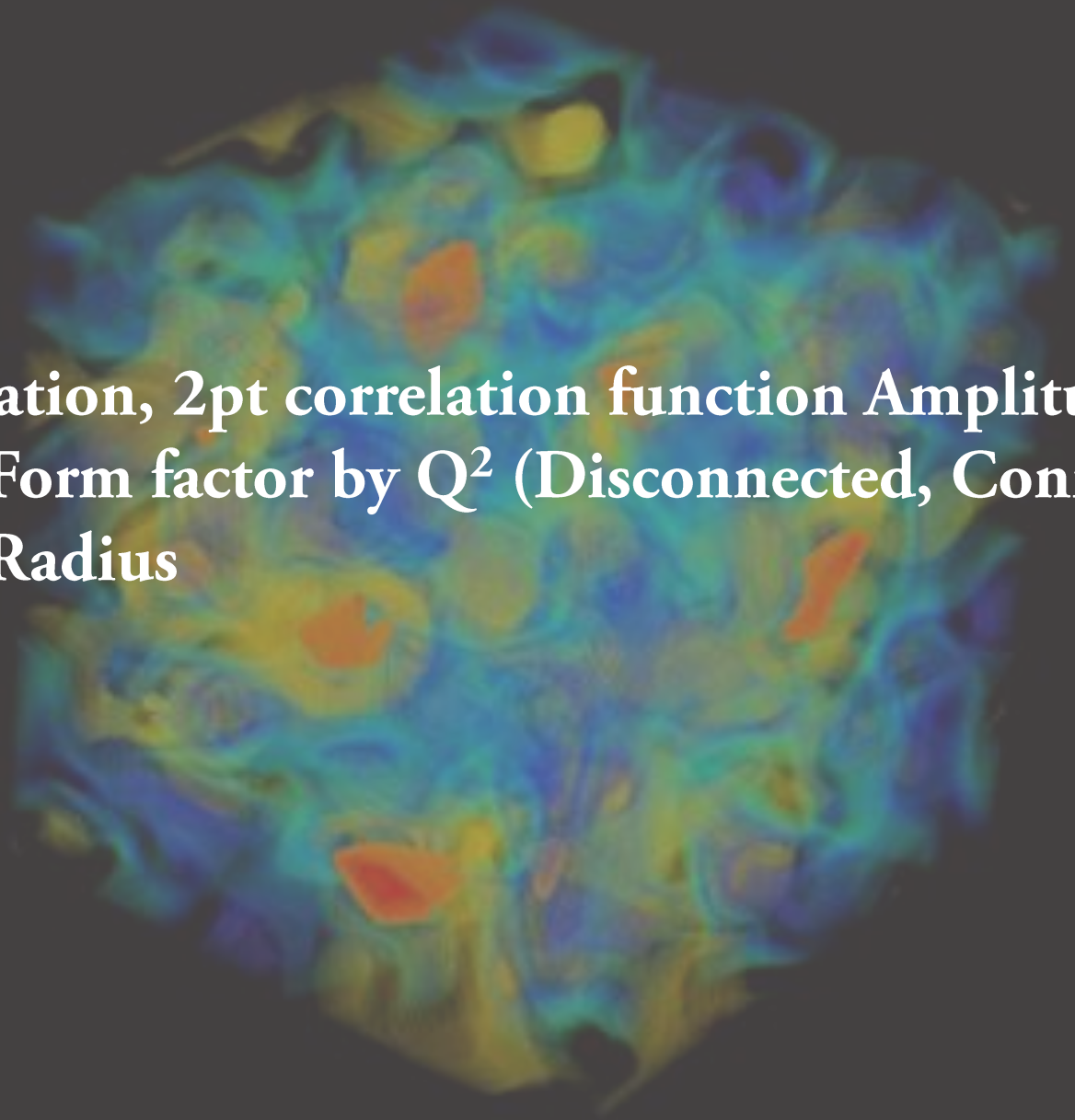
Step 3

Form Factor linear fitting with Q^2 → $< r^2 >$

$$1/F^{\pi}_{S,V}(Q^2)_{normalized} = 1 + \frac{1}{6} M_{\pi}^2 < r^2 >^{\pi}_{S,V} (Q/M_{\pi})^2 + O(Q/M_{\pi})^4$$

Linear Fitting : $y=1+Ax$, $< r^2 >=6A/M_{\pi}^2$

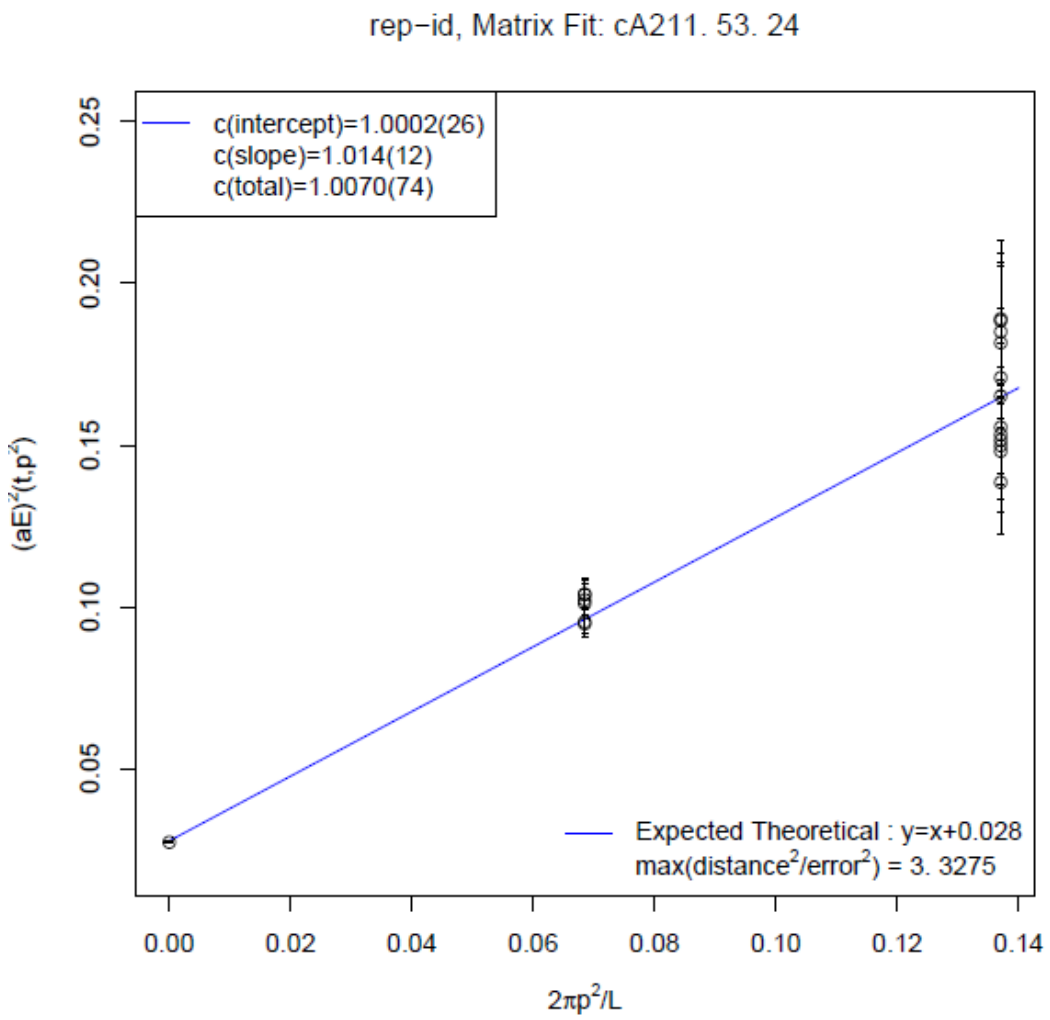
Result

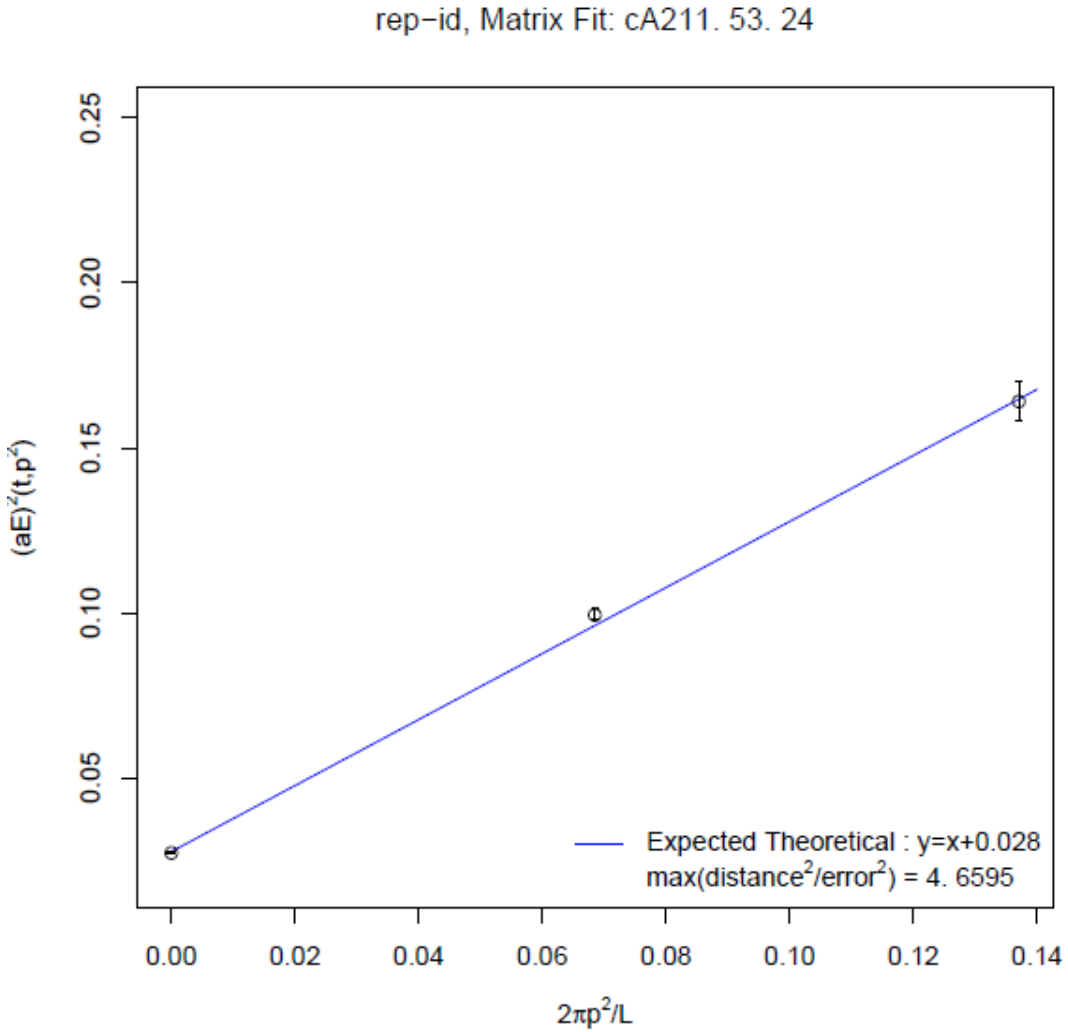


1. Dispersion relation, 2pt correlation function Amplitude $G_\pi(t, p^2)$
2. Scalar/Vector Form factor by Q^2 (Disconnected, Connected, Full)
3. Scalar/Vector Radius

Result. #1_ Kinematic Configuration & Average values

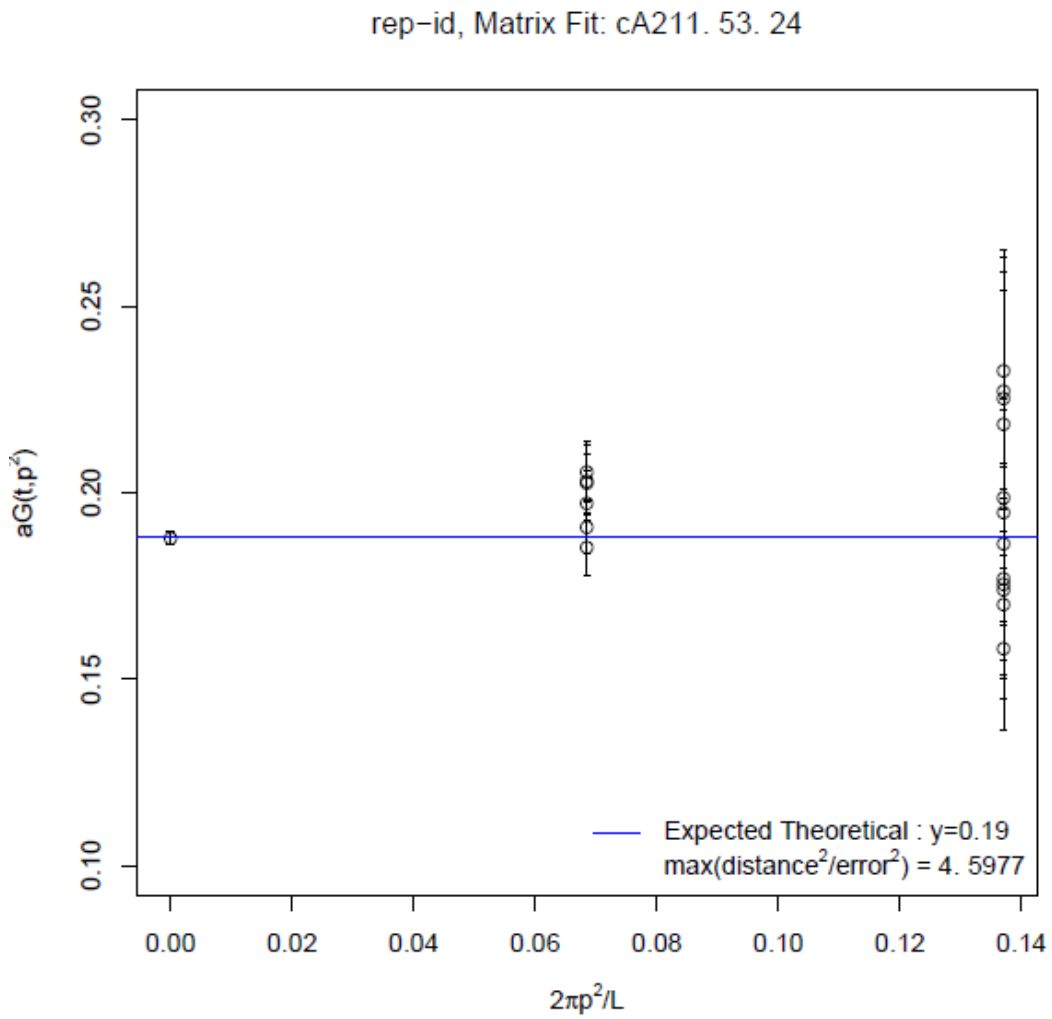
Extended Twisted Mass Collaboration

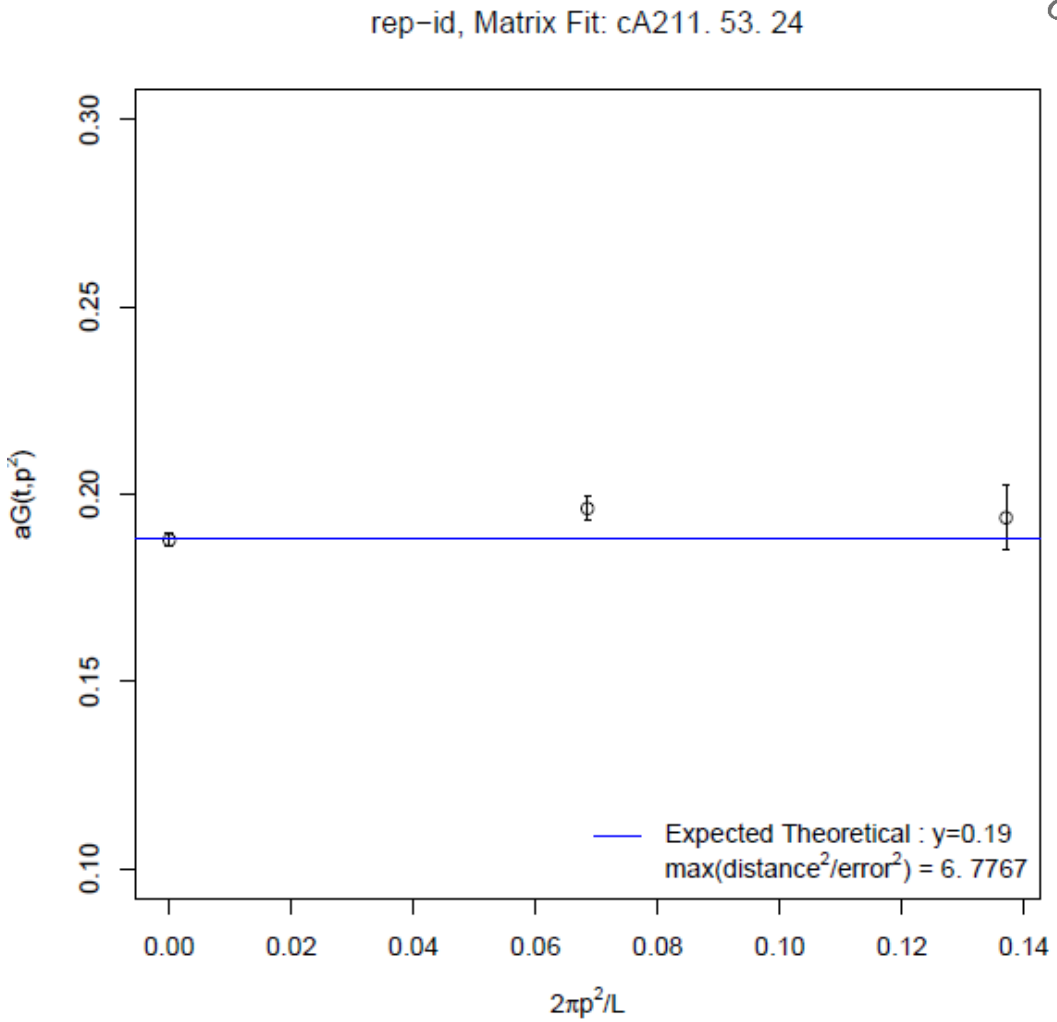




Result. #1_ Kinematic Configuration & Average values

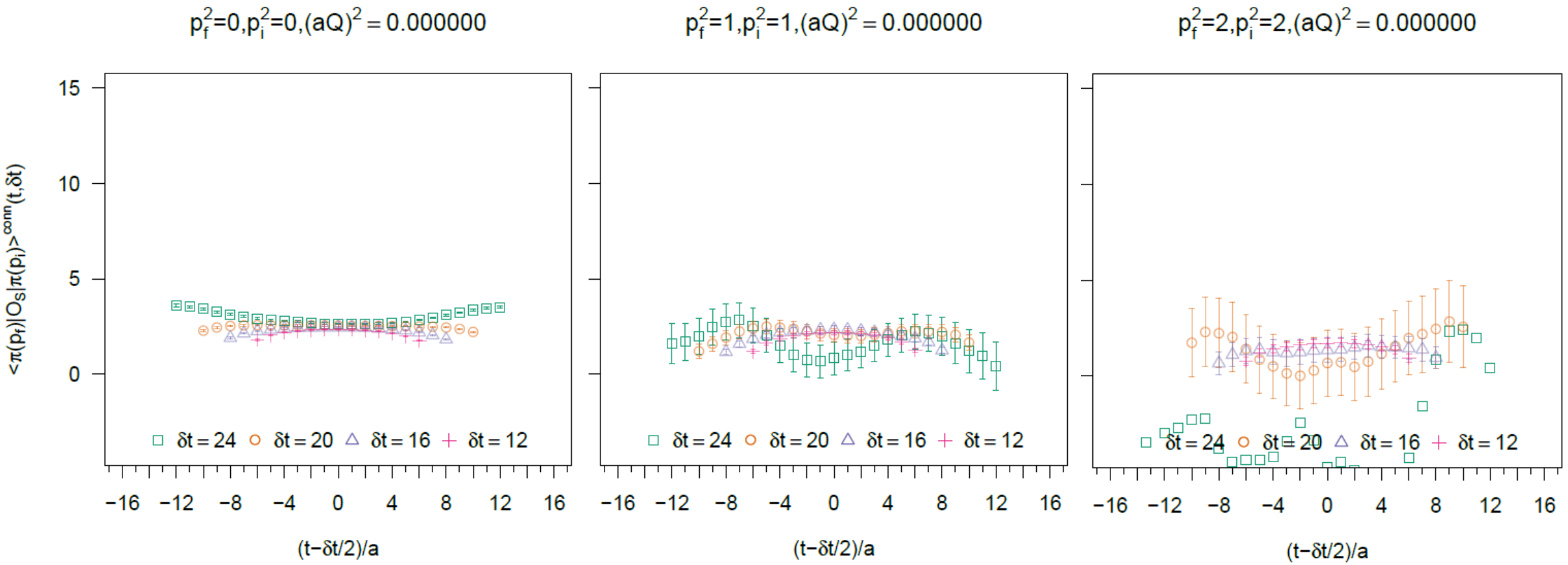
Extended Twisted Mass Collaboration





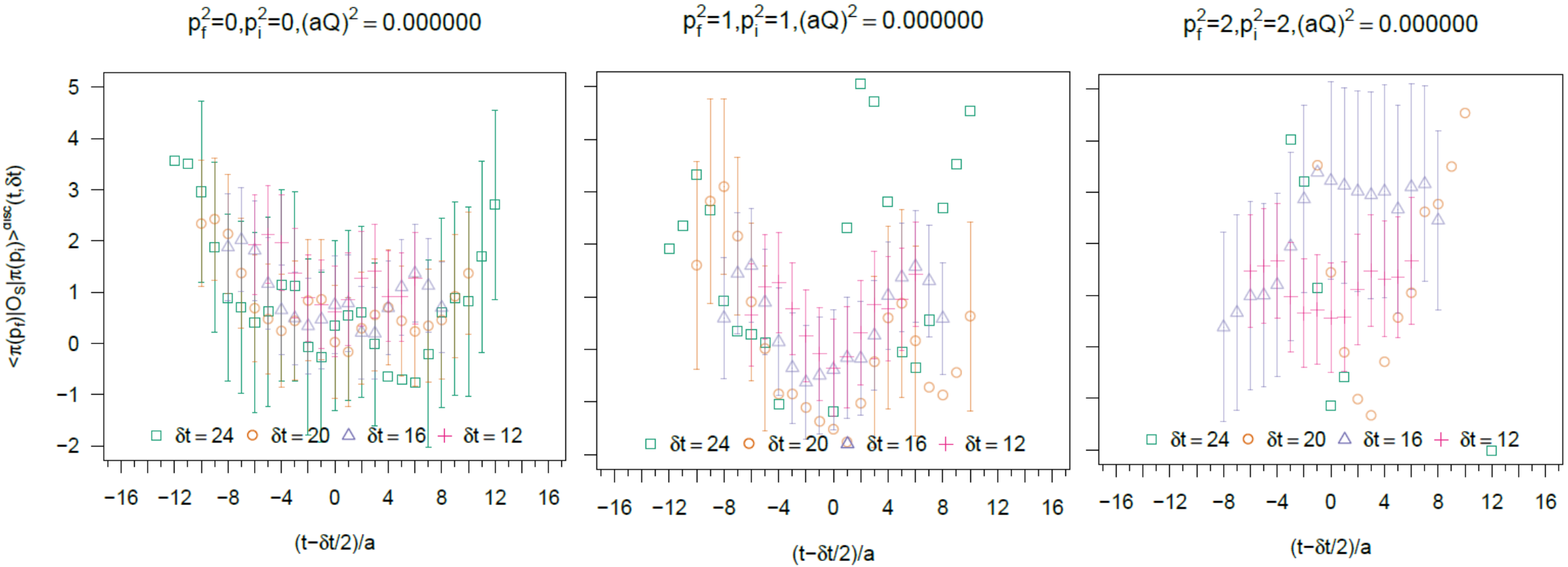
Result. #2_ Scalar Matrix Element, Conn

Extended Twisted Mass Collaboration



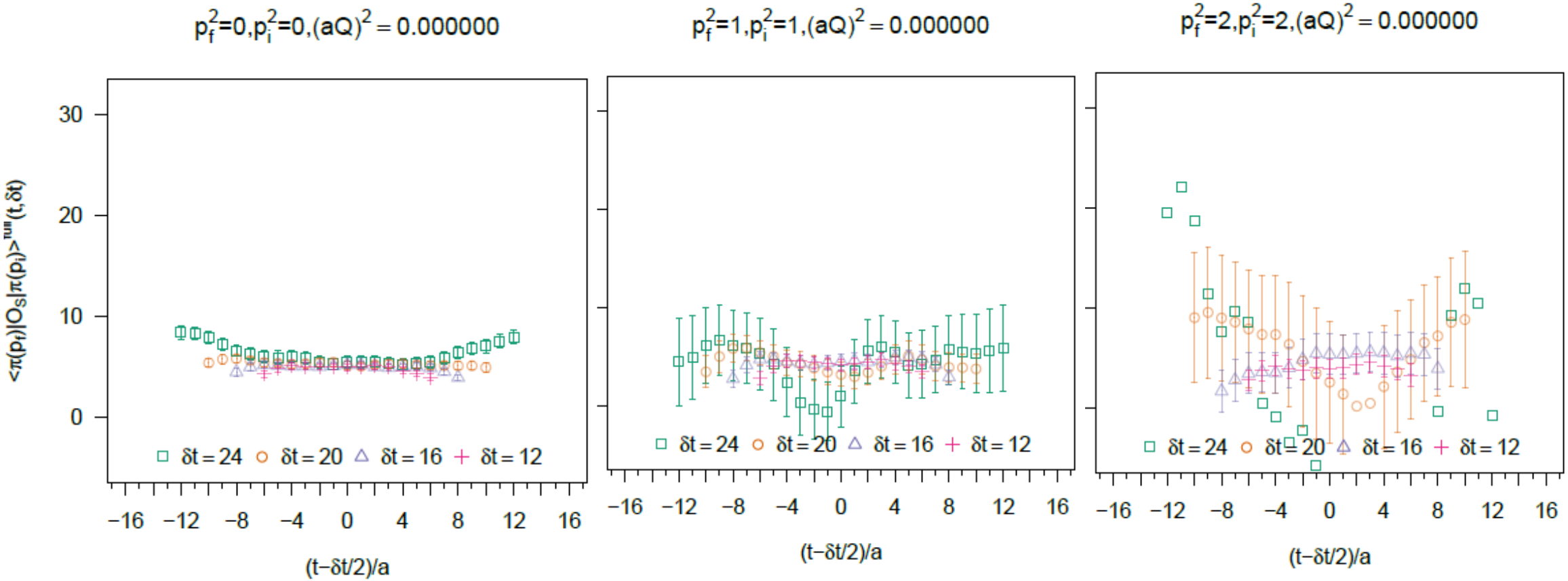
Result. #2_ Scalar Matrix Element, Disc

Extended Twisted Mass Collaboration



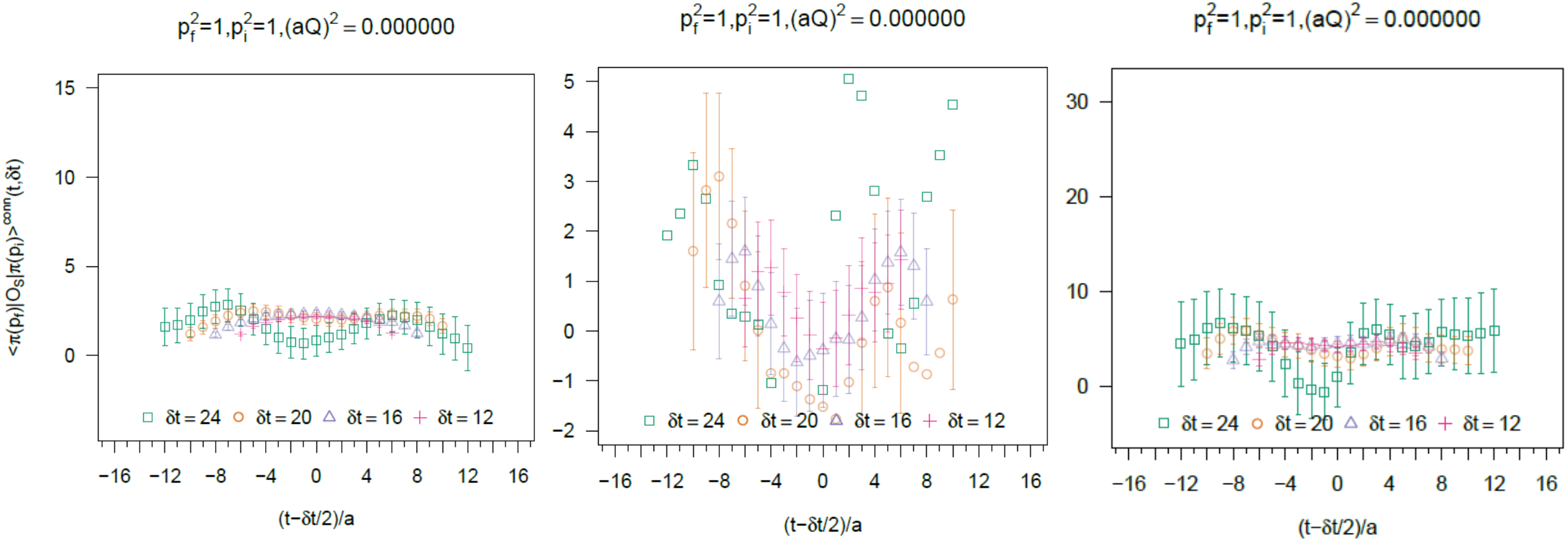
Result. #2_ Scalar Matrix Element, Full

Extended Twisted Mass Collaboration



Result. #2_ Scalar Matrix Element

Extended Twisted Mass Collaboration

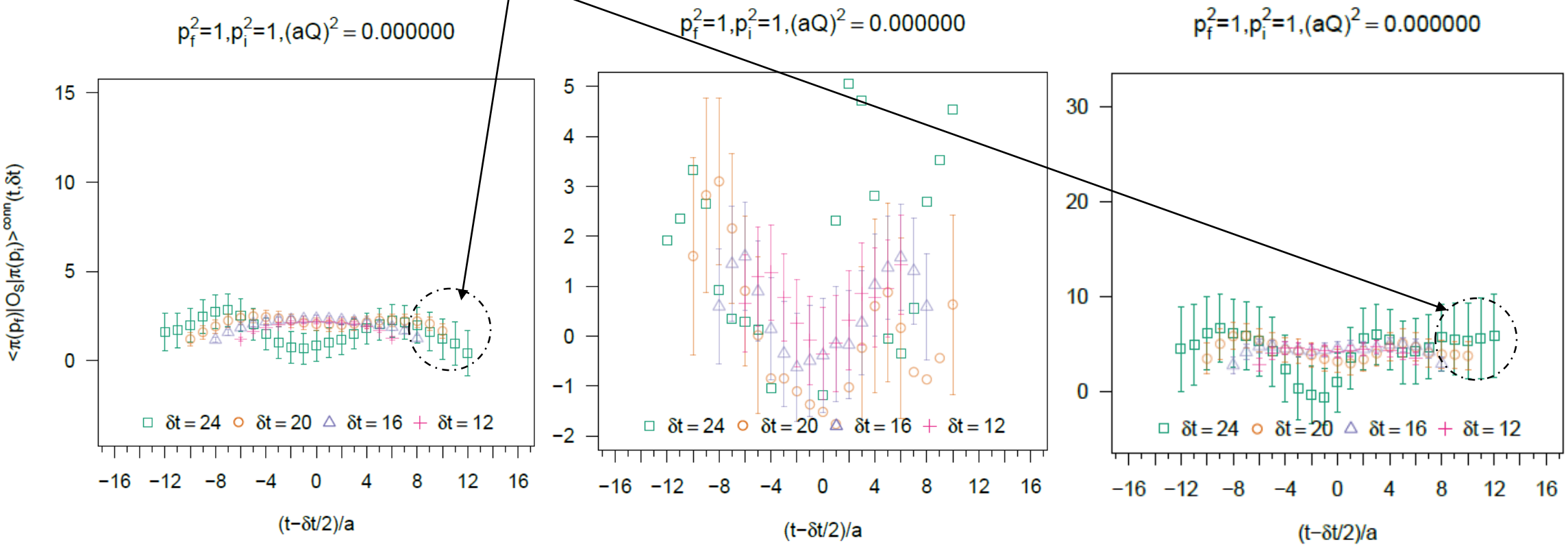


Conn **Disconnect** **Full**

Result. #2_ Scalar Matrix Element

Extended Twisted Mass Collaboration

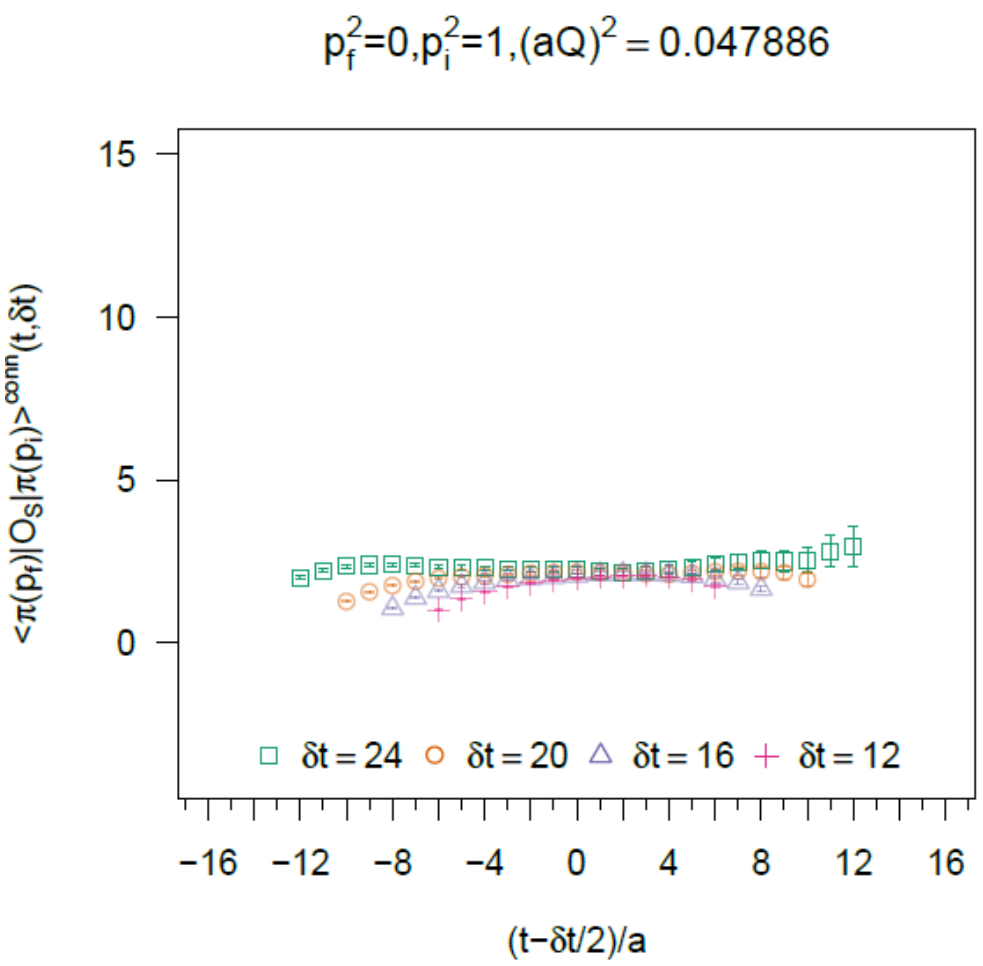
Disconnected contribution

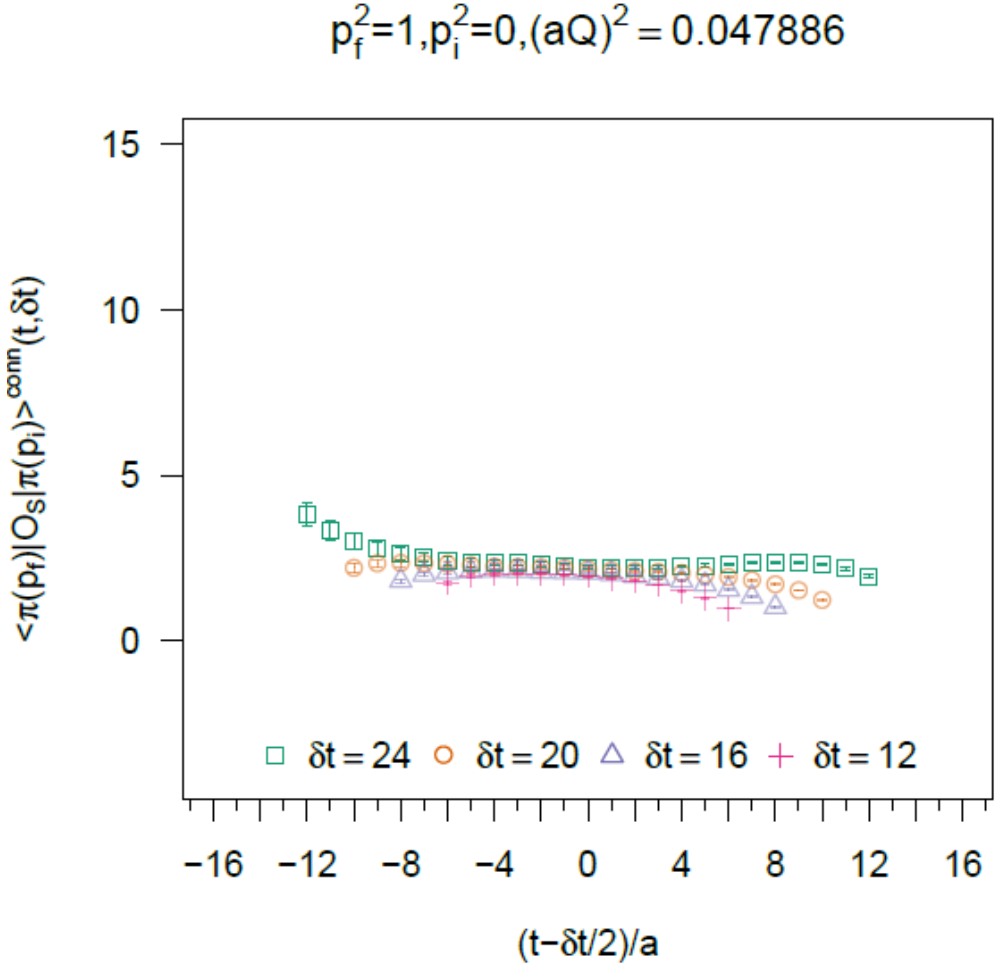


Conn **Disconnect** **Full**

Result. #2_ Scalar Matrix Element, Conn

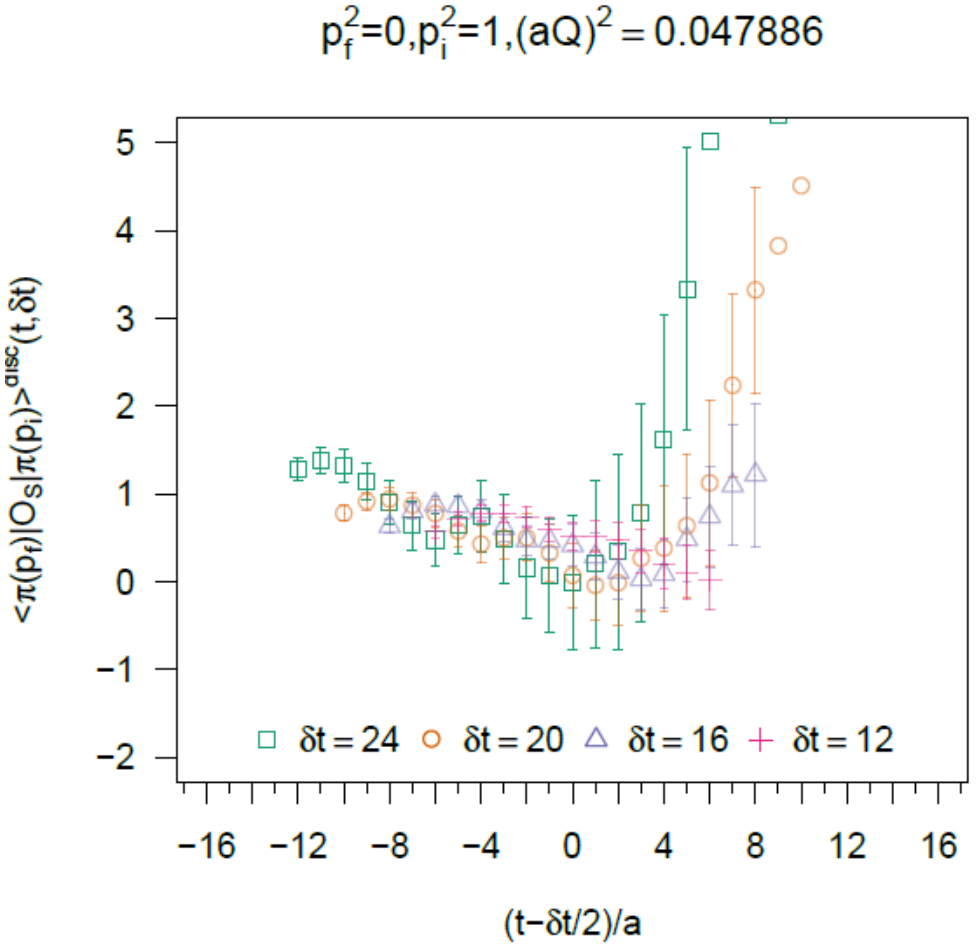
Extended Twisted Mass Collaboration

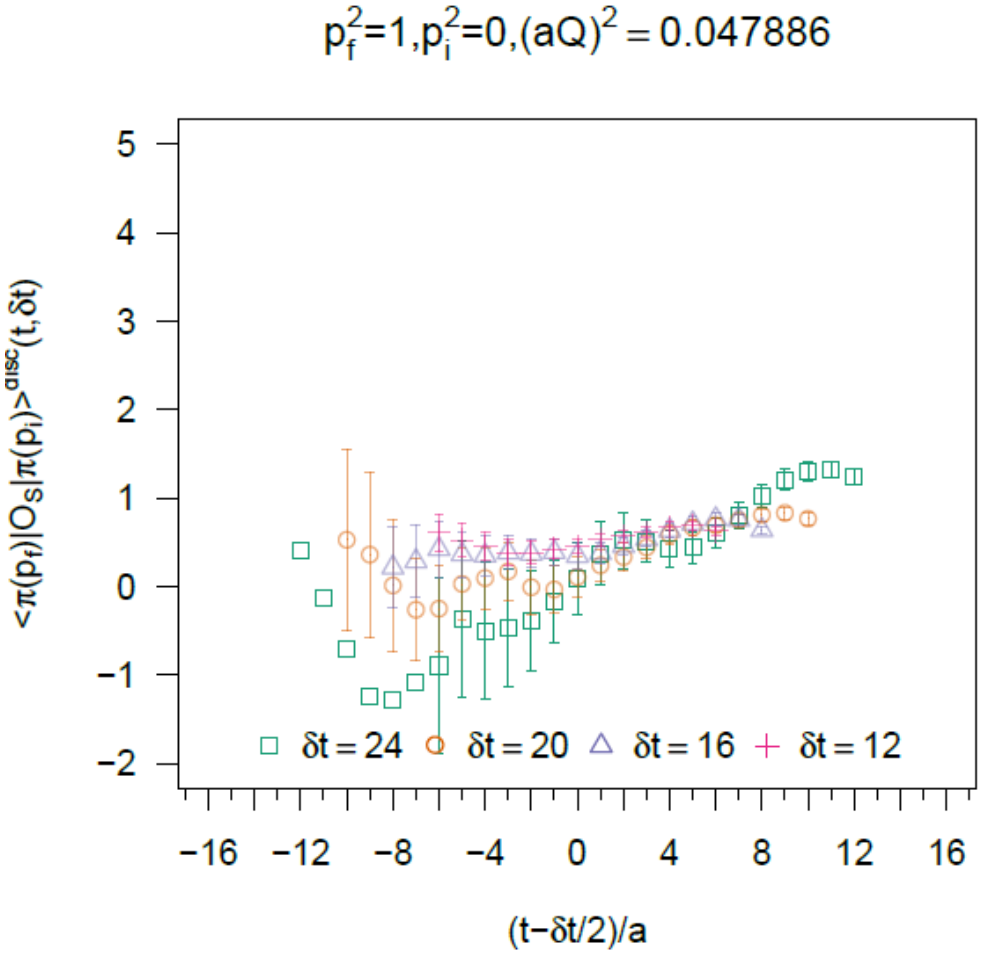




Result. #2_ Scalar Matrix Element, Disc

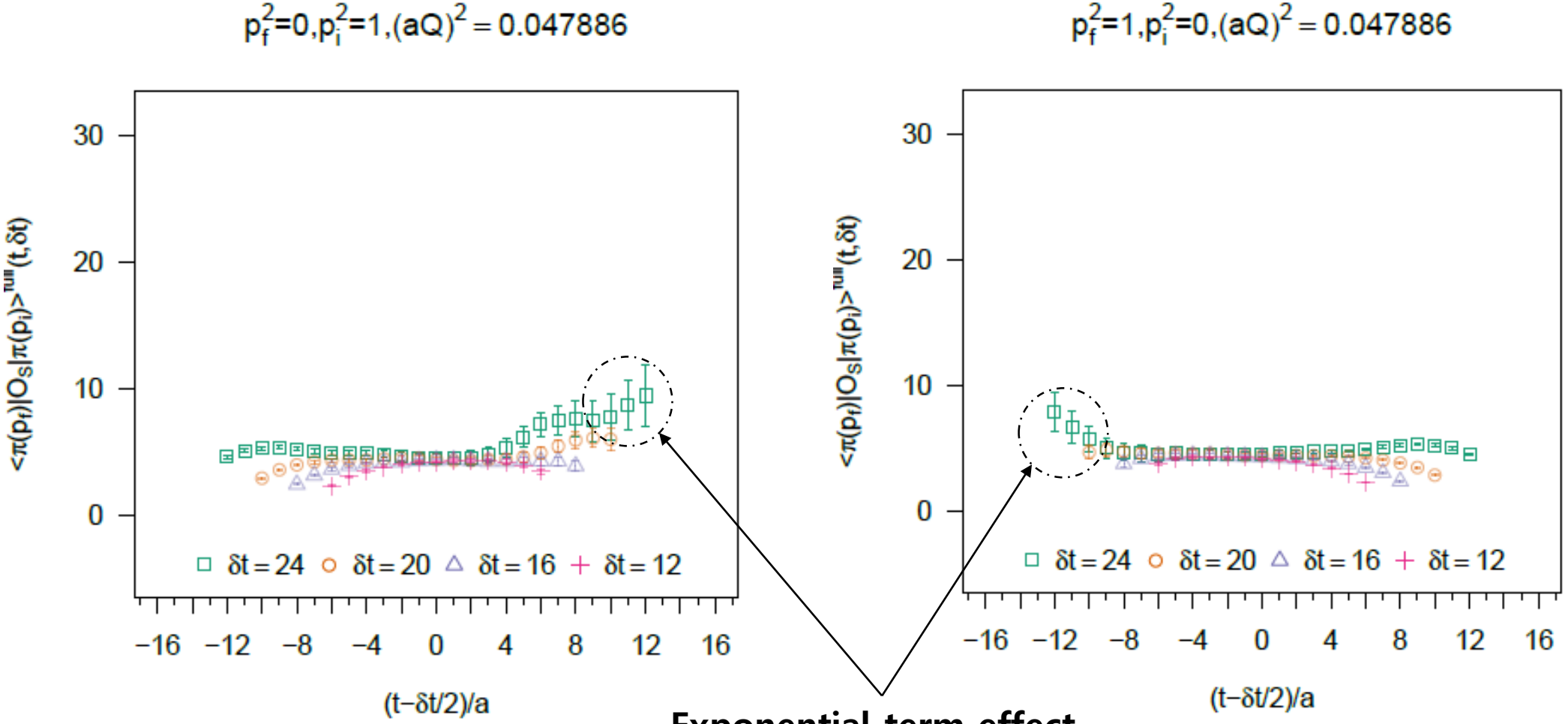
Extended Twisted Mass Collaboration





Result. #2_ Scalar Matrix Element, Full

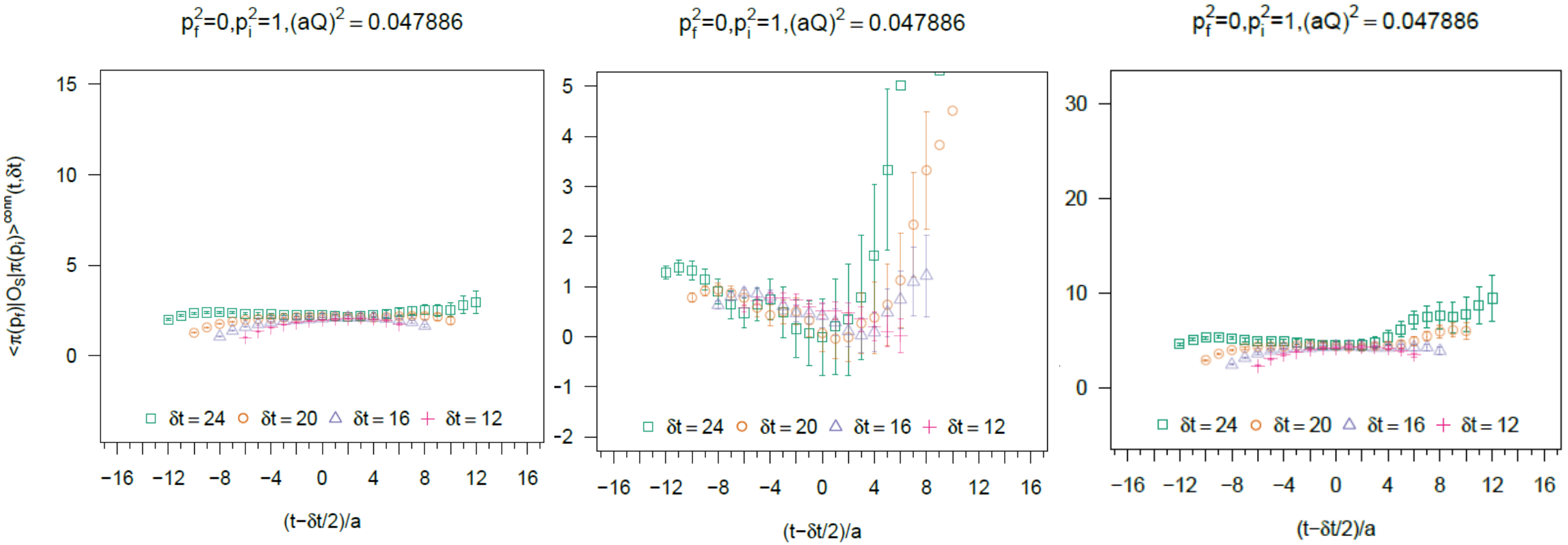
Extended Twisted Mass Collaboration



Exponential term effect

Result. #2_ Scalar Matrix Element

Extended Twisted Mass Collaboration

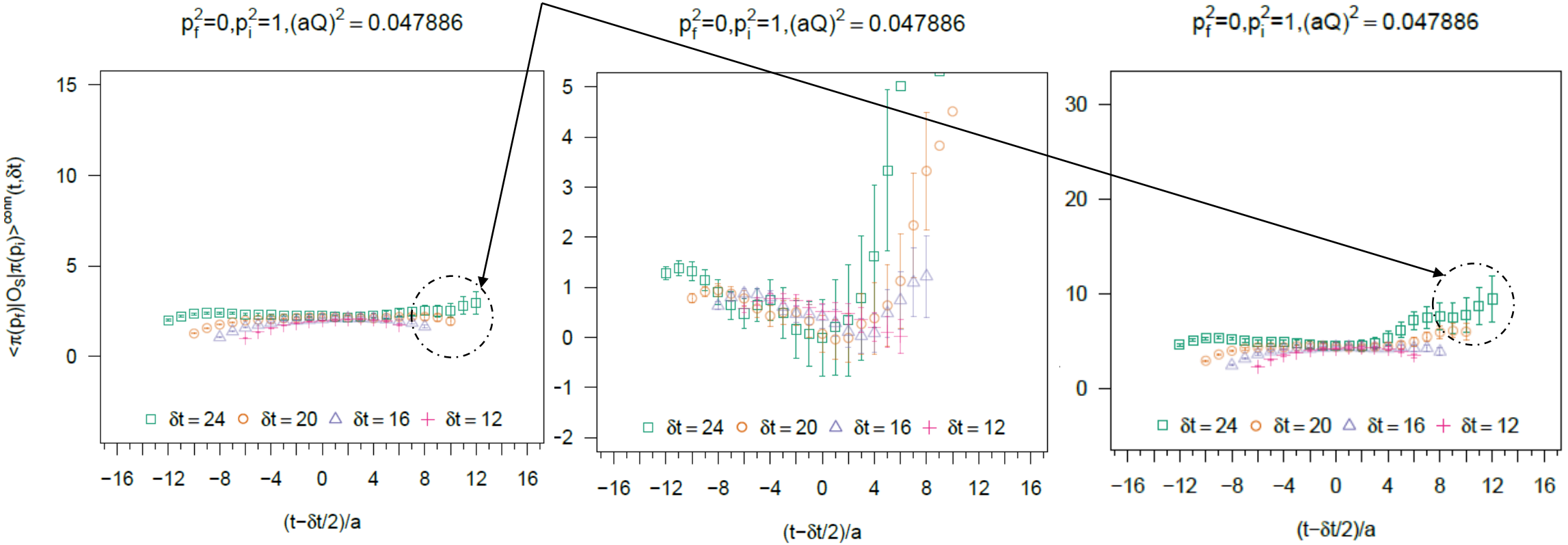


Conn **Disconnect** **Full**

Result. #2_ Scalar Matrix Element

Extended Twisted Mass Collaboration

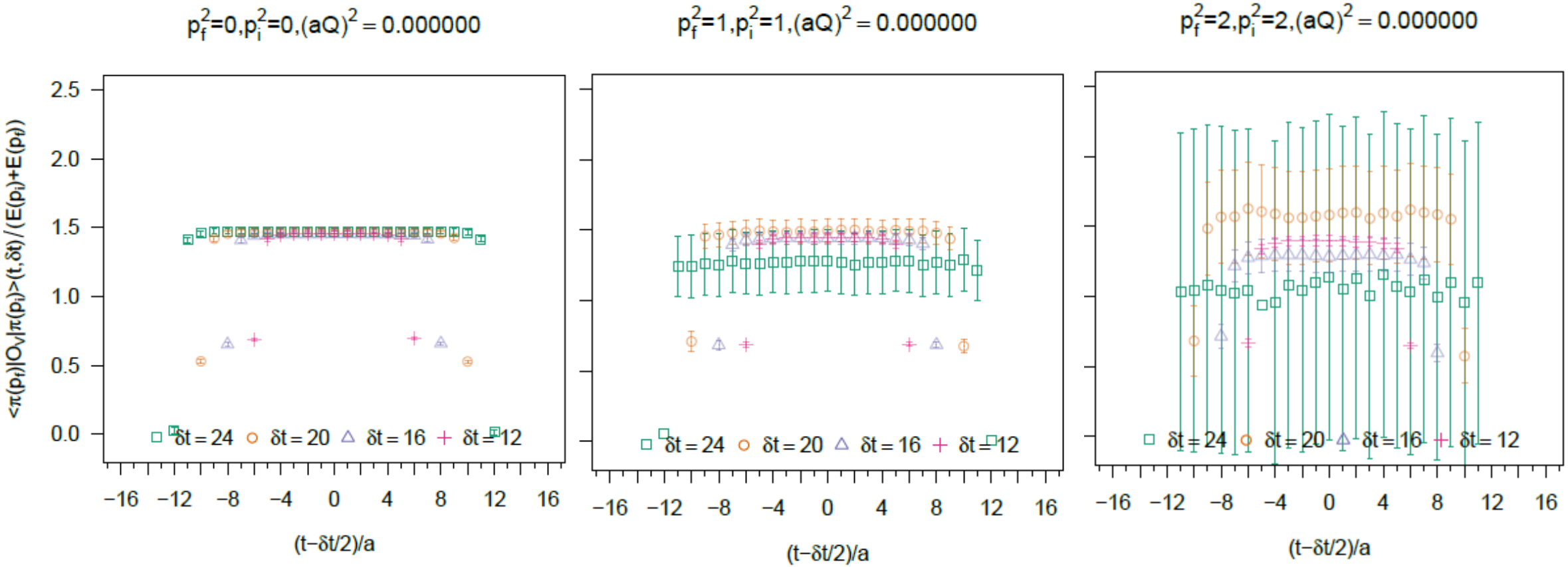
Disconnected contribution



Conn **Disconnect** **Full**

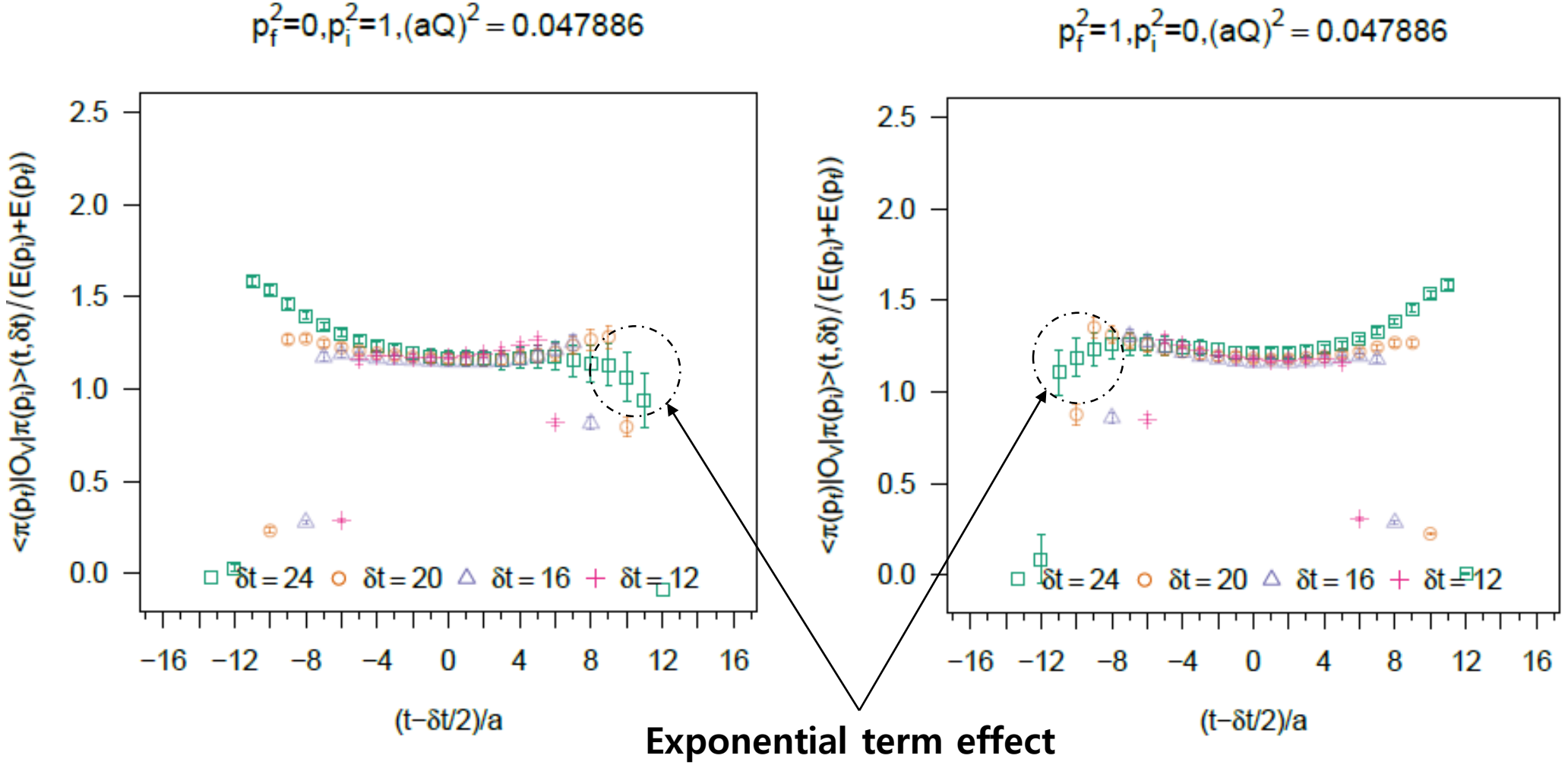
Result. #2_Vector Matrix Element

Extended Twisted Mass Collaboration



Result. #2_Vector Matrix Element

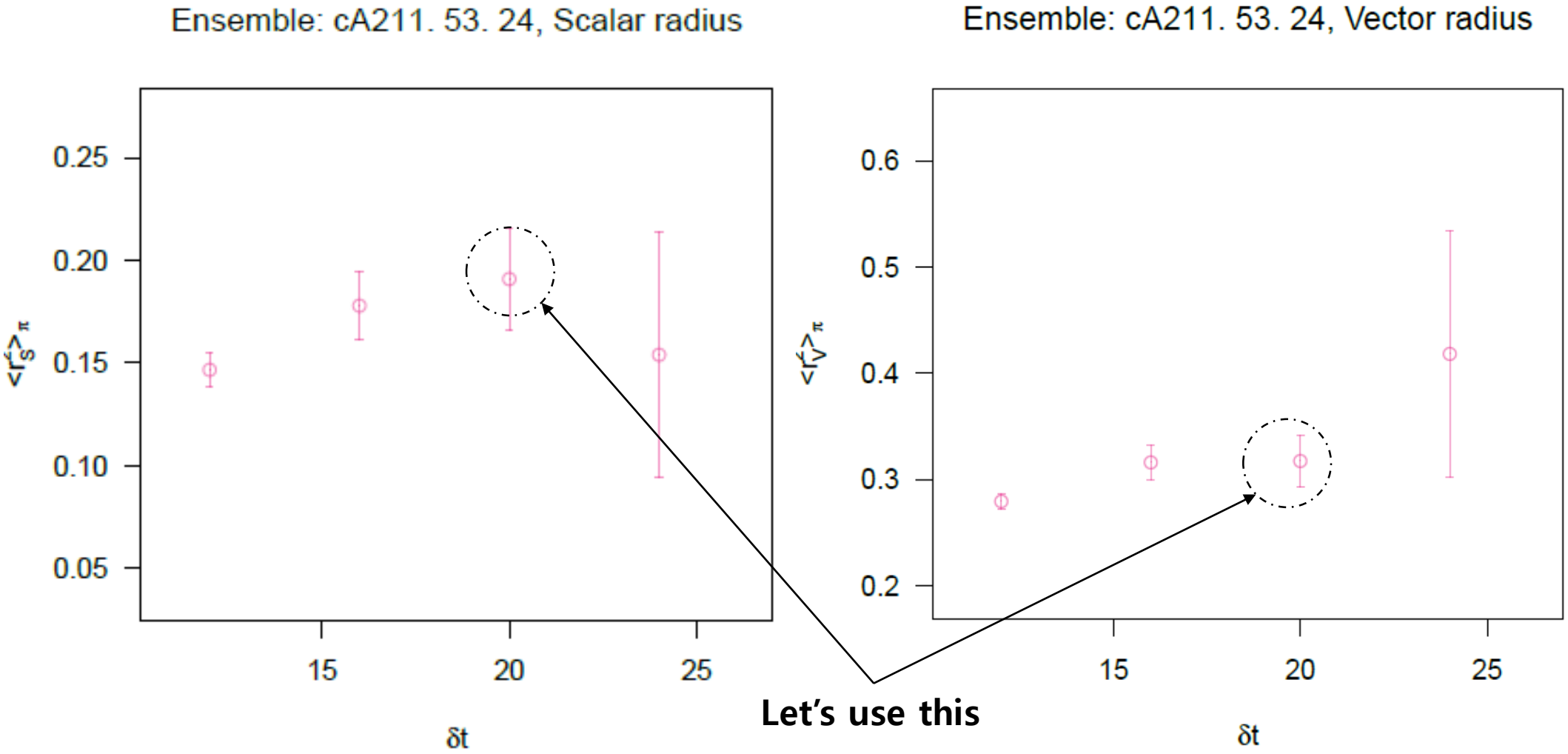
Extended Twisted Mass Collaboration



Exponential term effect

Result. #3_ Pion radius by source-sink separation

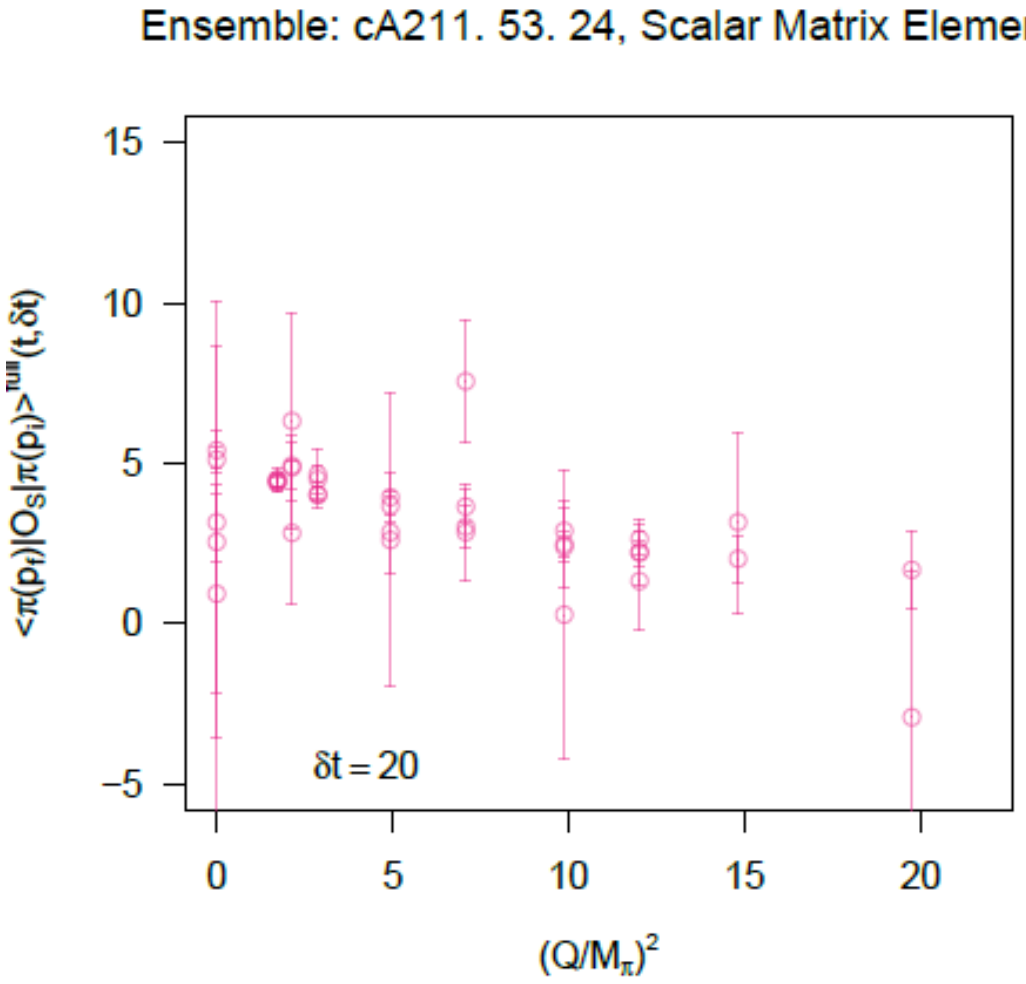
Extended Twisted Mass Collaboration

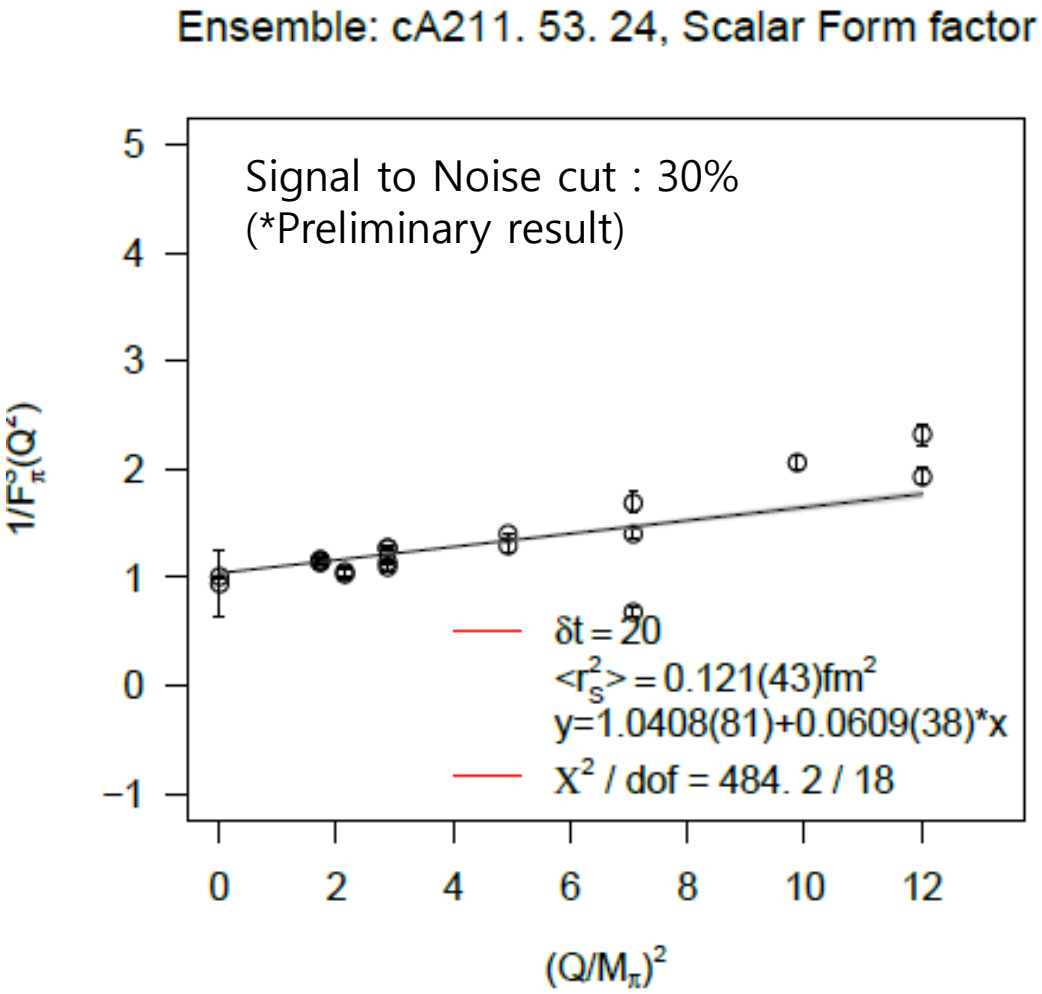


Let's use this

Result. #3_ Scalar Form factor, Scalar radius

Extended Twisted Mass Collaboration

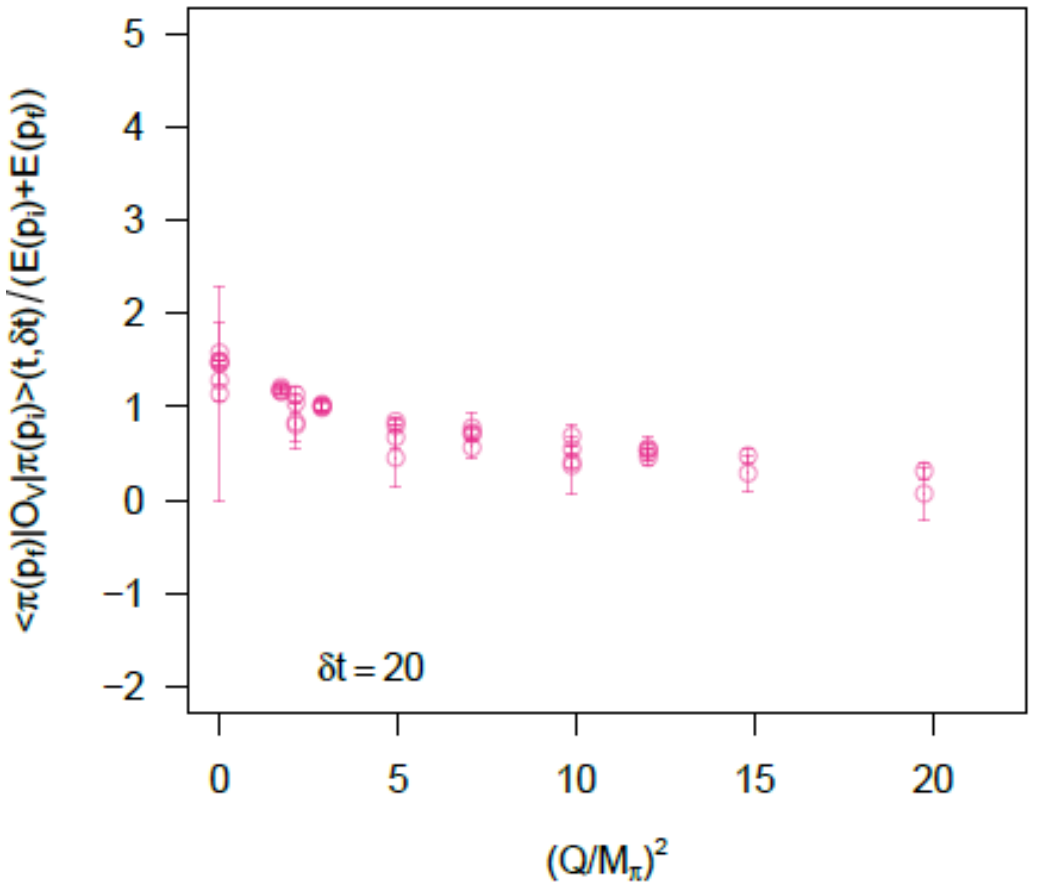




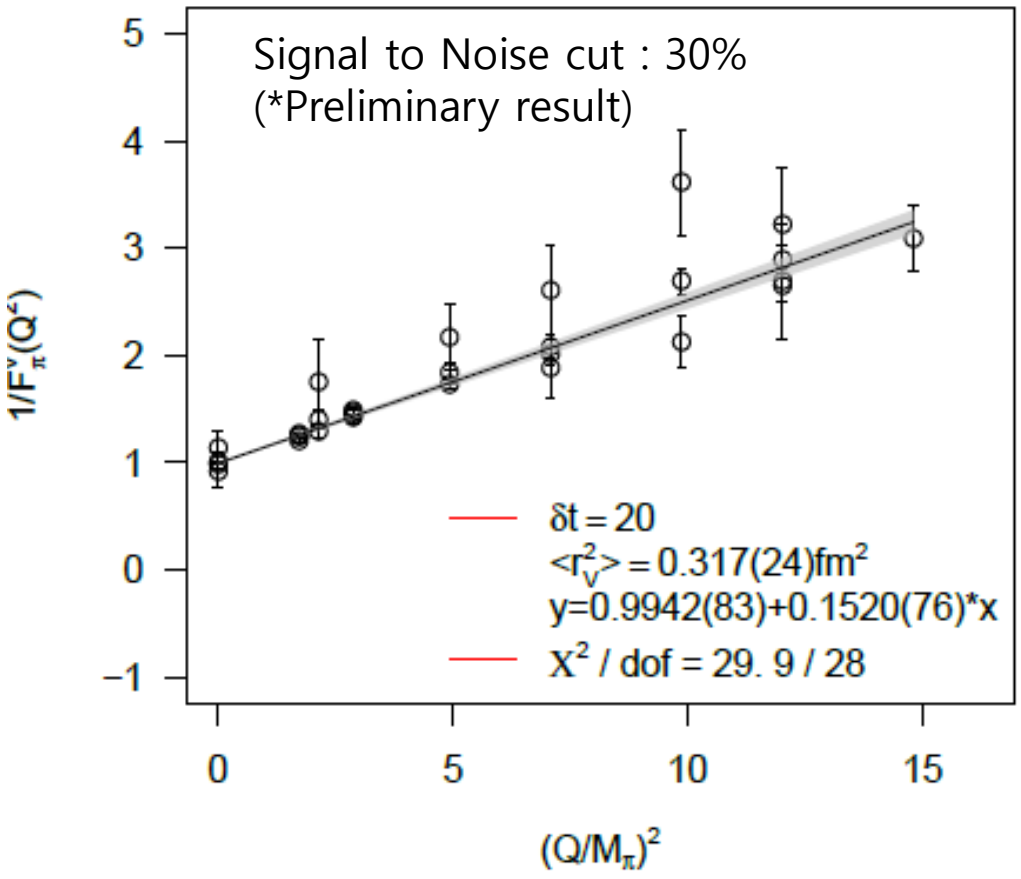
Result. #3_ Vector Form factor, Vector radius

Extended Twisted Mass Collaboration

Ensemble: cA211. 53. 24, Vector Matrix Element

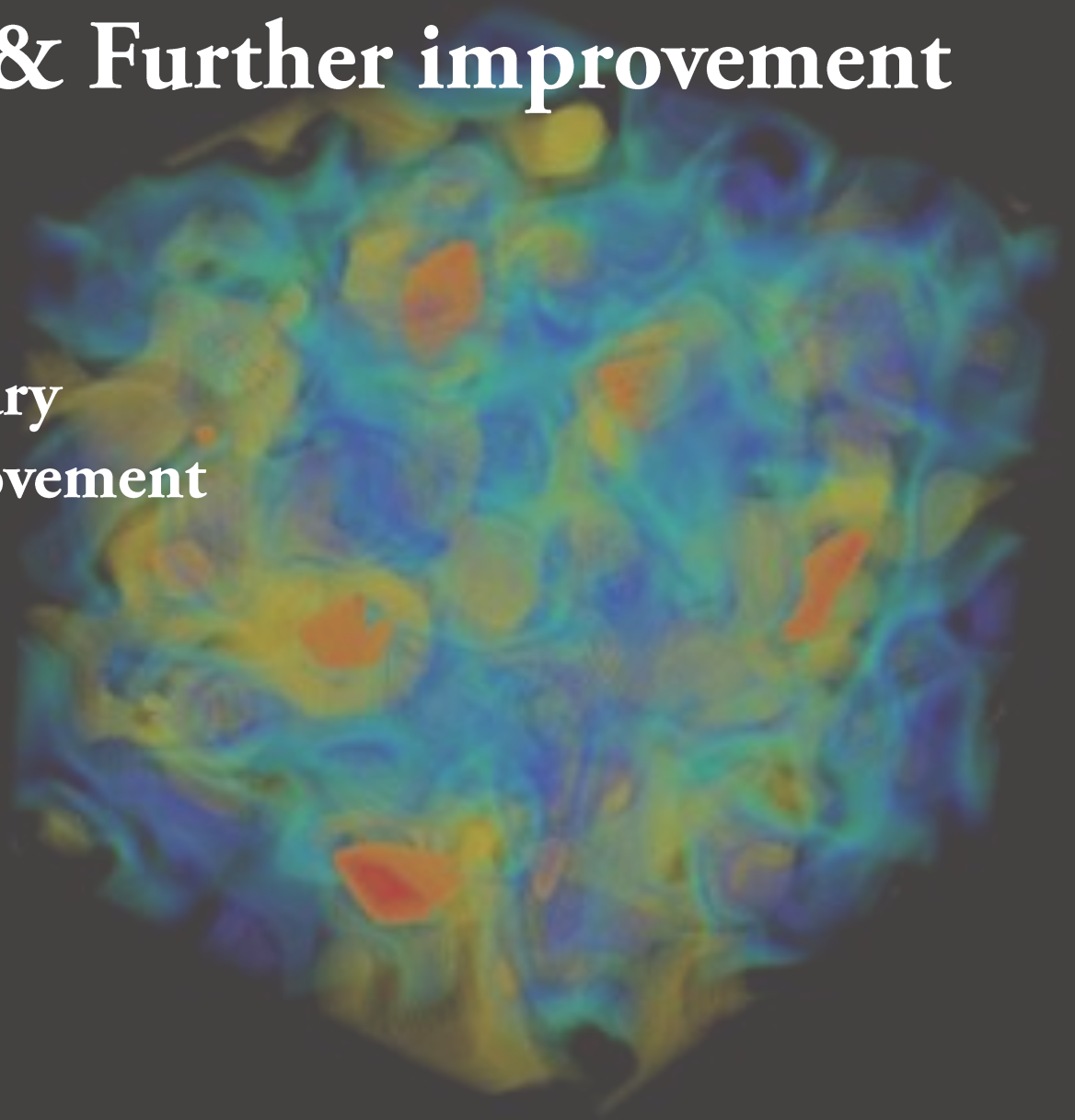


Ensemble: cA211. 53. 24, Vector Form factor



Conclusion & Further improvement

1. Result Summary
2. Further Improvement



Conclusion

- **Momentum Shell average is necessary for reducing errors**
- **Scalar/Vector Matrix Element from 2pt, 3pt Correlator Calculation is usable**
- **Disconnected contribution is measured relatively smaller than conncected contribution**
- **In high momentum shell result for matrix element, the noise also get higher.**

Further Improvement

- **Calculating Form factors, Finite Volume Effect need to be considered.**
- **$\langle r^2 \rangle$ extrapolation with pion mass is needed with other ensemble analysis**
- **Using G(0,t) value at different momentum class to reduce errors**
- **Using higher momentum shell to gain more Q^2 values in smaller range.**
- **Strange quark contribution in Scalar Form factor calculation**
- **Understand/Quantifies the Systematic Errors**

Reference

Extended Twisted Mass Collaboration

1. Analysis_flavor_singlet, Bartek, Github
2. Advanced R, Wickham
3. The pion vector form factor from Lattice QCD at the physical point, C.Alexandrou, S. Bacchio, P,Dimopoulos, Kostrzewa, Sanfilippo, Urbach et al. [arXiv:1710.10401v1]
4. The scalar pion form factor in two-flavor lattice QCD, Vera Guelpers , Georg von Hippel, Hartmut Wittig [arXiv:1309.2104v2]
5. The scalar radius of the pion from lattice QCD in the continuum limit, Vera Guelpers, Georg von Hippel, Hartmut Wittig [arXiv:1507.01749v1]