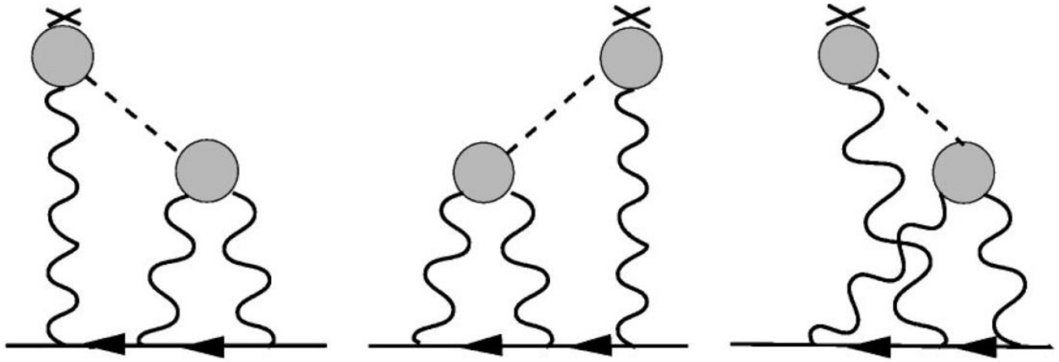
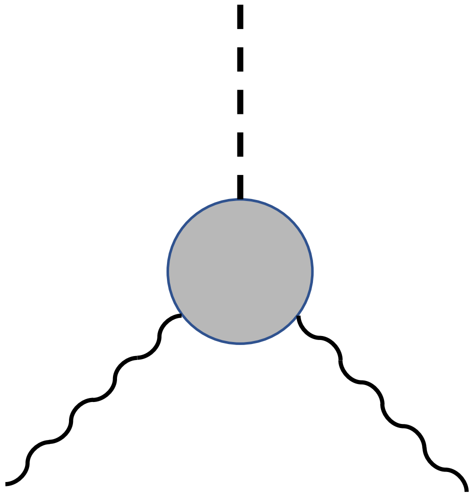
Pseudoscalar Transition Form Factor





Andrew Gasbarro

Universität Bern

ETMC Meeting

September 24, 2019

HISKP & BCTP, Universität Bonn

With Sebastian Burri (Bern), Marcus Petschlies (Bonn), Urs Wenger (Bern)

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Flavour Singlet Physics from Lattice QCD in Fully Physical Conditions

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Abstract

In this proposal for a production project we ask for computer time on Piz Daint in order to calculate within lattice Quantum Chromodynamics (QCD) some flavour singlet quantities in fully physical conditions, that is, at physical quark masses and in the continuum limit. The project concentrates on properties of η and η' mesons, the scalar form factor of the pion, and the first gluon moment of the pion. These quantities are of high phenomenological interest, but are difficult or even impossible to obtain from experiment. In contrast, the proposed lattice QCD calculation allows to determine them from the underlying quantum field theory from first principles. The calculation involves the computation of so-called disconnected quark contributions which are obtained from appropriate contractions of all-to-all quark propagators. The calculation of the latter requires a significant computational effort and computer resources which can only be provided through a production project on Piz Daint.

Gauge Ensembles

- Action: Nf=2+1+1, Iwasaki Gauge, Clover Improvement, Maximal Twist [1]
  - Twisted basis

$$S = S_{Iwasaki}[U] + \bar{\chi}_l D_l[U]\chi_l + \bar{\chi}_h D_h[U]\chi_h$$

$$D_l = D_W[U] + i\mu_l\gamma_5\tau_3 \qquad D_h = D_W[U] + i\mu_\sigma\gamma_5\tau_1 + \mu_\delta\tau_3$$

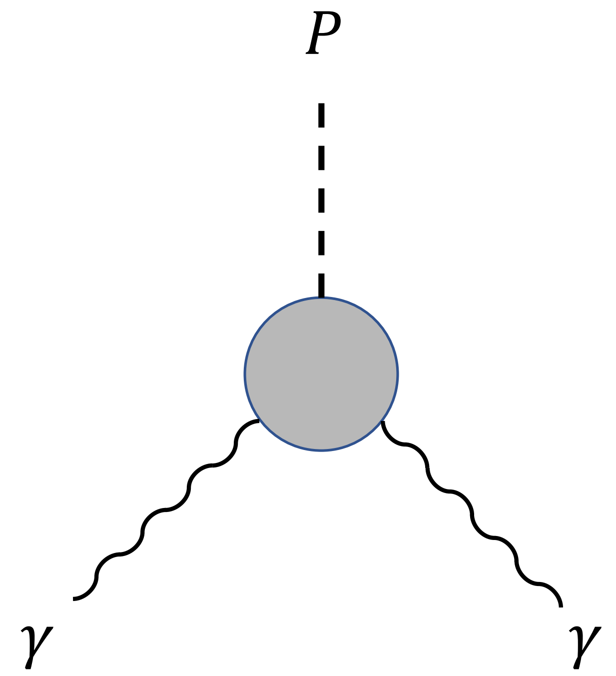
ensemble	$L^3 \cdot T/a^4$	β	c_{SW}	M_π [MeV]	a [fm]	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	κ_{crit}
cA53.24	$24^3 \cdot 48$	1.726	1.74	350	0.097	0.0053	0.1408	0.1521	0.1400645
cA40.24	$24^3 \cdot 48$	1.726	1.74	301	0.097	0.0040	0.1408	0.1521	0.1400645
cA30.32	$32^3 \cdot 64$	1.726	1.74	260	0.097	0.0030	0.1408	0.1521	0.1400645
cA12.48	$48^3 \cdot 96$	1.726	1.74	165	0.097	0.0012	0.1408	0.1521	0.1400645
cB07.64	$64^3 \cdot 128$	1.778	1.69	135	0.079	0.00072	0.1247	0.1315	0.1394265
cB25.48	$48^3 \cdot 96$	1.778	1.69	255	0.079	0.0025	0.1247	0.1315	0.1394267
cC06.80	$80^3 \cdot 160$	1.836	1.645	135	0.069	0.00062	0.1060	0.1135	0.1387510

1.) Finkenrath, Jacob, et al. "Simulation of an ensemble of Nf= 2+ 1+ 1 twisted mass clover improved fermions at physical quark masses." EPJ Web of Conferences. Vol. 175. EDP Sciences, 2018. Arxiv: 1712.09579

Outline

- Pseudoscalar transistion form factor
  - Definition
  - π_0 decay
  - Role in hadronic light-by-light
- Lattice Calculation
  - Kinematics & Analytic Continuation
  - Connected & Disconnected
- Some early looks at the data
- Looking forward

Pseudoscalar Transition Form Factor



Minkowski space amplitude:

$$M_{\mu\nu}(q_1, q_2) = i\int d^4x\, e^{iq_1x}\langle\Omega|T\{J_\mu(x)J_\nu(0)\}|P(p)\rangle = \epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F_{P\gamma\gamma}(q_1^2, q_2^2)$$

q_1 and q_2 photon momenta, off shell in general;
$p = q_1 + q_2$ pseudoscalar momentum, on shell.

For $q_{1,2}^2 < \mathrm{m_V^2} = \min\left(m_\rho^2, (2m_\pi)^2\right)$, continue to Euclidean space[1]

$$M_{\mu\nu}^E(q_1, q_2) = -\int d\tau e^{\omega_1\tau}\int d^3\vec{x}\, e^{i\vec{x}\vec{q_1}}\langle 0|T\{J_\mu(x)J_\nu(0)\}|P(p)\rangle$$

Note at large τ

$$\langle 0|T\{J_\mu(x)J_\nu(0)\}|P(p)\rangle \sim e^{-\tau E_V}$$

And integral does not converge for $\omega_1 > E_V$
Nonetheless, can reach on-shell photons and even slightly time-like

1.) arxiv:hep-ph/0111058v2

Pseudoscalar Transition Form Factor

Kinematics, $\vec{p} = \vec{0}$

$$q_1^2 = \omega_1^2 - \vec{q}_1^2$$

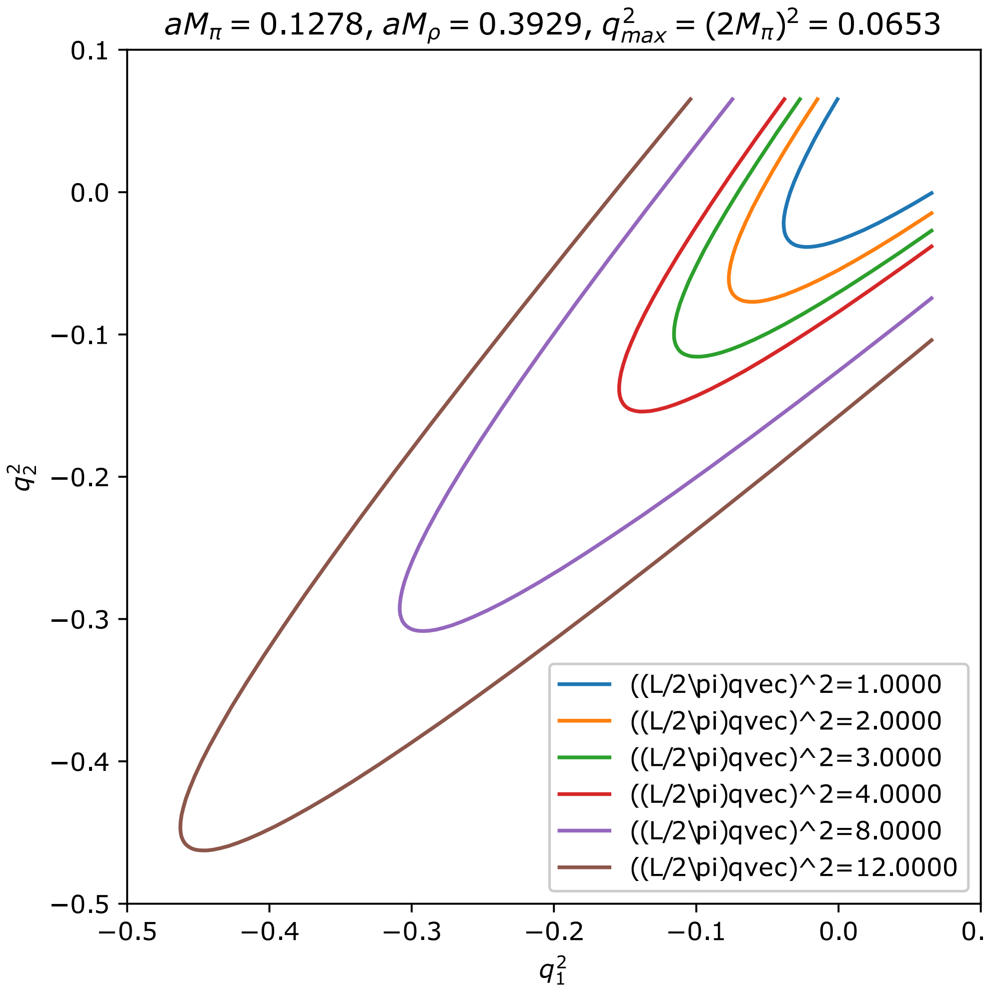
$$q_2^2 = (m_\pi - \omega_1)^2 - \vec{q}_1^2$$

Analytic continuation bound

$$m_V = \min(m_\rho, 2m_\pi)$$

$$q_{1,2}^2 < m_V^2$$

$$-\sqrt{m_V^2 + \vec{q}_1^2} + m_\pi < \omega_1 < \sqrt{m_V^2 + \vec{q}_1^2}$$



Pseudoscalar Transition Form Factor

Neutral pion decay rate

In chiral limit, and at low energies, ABJ dictates

$$F_{\pi_0\gamma\gamma}(q_1^2 = 0, q_2^2 = 0)\Big|_{m_{\pi=0}} = \frac{1}{4\pi^2 f_\pi} \qquad f_\pi = F_\pi(m_\pi = 0)$$

To good approximation away from chiral limit [(1)]

$$F_{\pi_0\gamma\gamma}(q_1^2 = 0, q_2^2 = 0)\Big|_{m_\pi} = \frac{1}{4\pi^2 F_\pi}$$

Model to extrapolate to double on-shell point

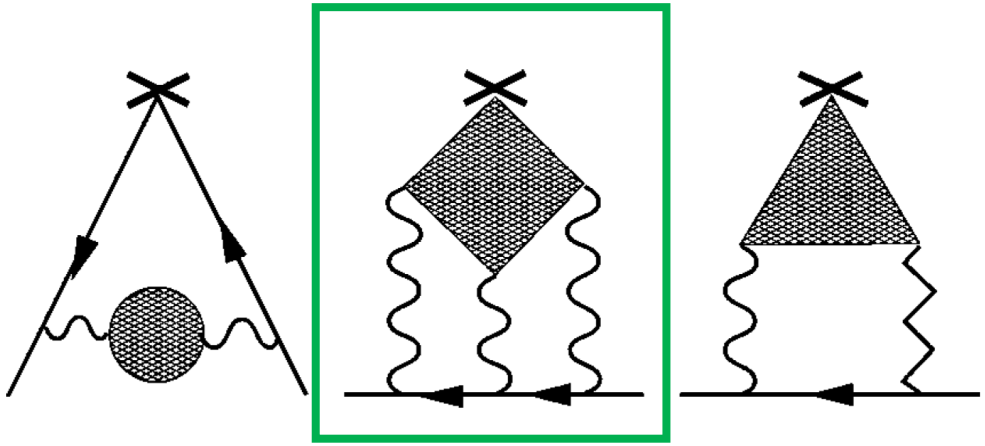
Related to $\pi_0 \to \gamma\gamma$ decay rate by [(2)]

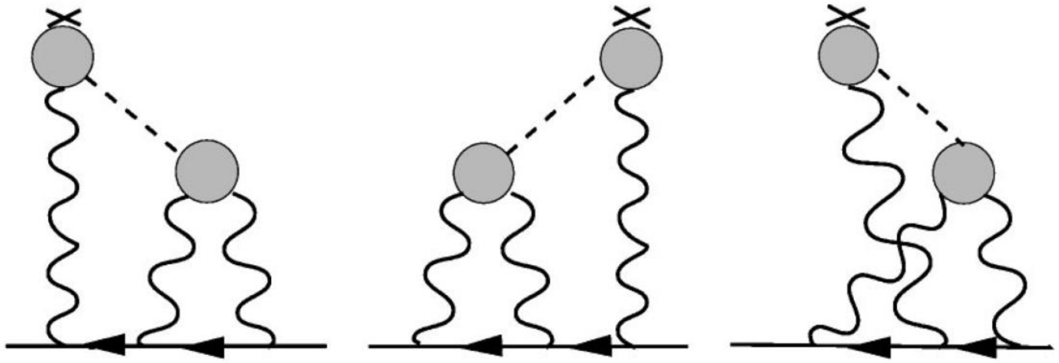
$$\Gamma_{\pi_0\gamma\gamma} = \frac{\pi\alpha_e^2 m_\pi^3}{4} F_{\pi_0\gamma\gamma}(q_1^2 = 0, q_2^2 = 0)$$

Previous Study (2)

1.) arxiv:1607.08174v2 2.) arxiv:1206.1375

Eta, Eta' contribution to hadronic light by light





$$(-ie)\overline{\mathrm{u}}(p')\hat{\Gamma}_{\rho}(p',p)\mathrm{u}(p) \equiv \langle \mu^-(p')|(ie)j_{\rho}(0)|\mu^-(p)\rangle$$

$$= \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{(-i)^3}{q_1^2 q_2^2 (q_1+q_2-k)^2} \frac{i}{(p'-q_1)^2-m^2} \frac{i}{(p'-q_1-q_2)^2-m^2}$$

$$\times (-ie)^3 \overline{\mathrm{u}}(p')\gamma^{\mu}(\not{p}'-\not{q}_1+m)\gamma^{\nu}(\not{p}'-\not{q}_1-\not{q}_2+m)\gamma^{\lambda}\mathrm{u}(p)(ie)^4 \Pi_{\mu\nu\lambda\rho}(q_1,q_2,k-q_1-q_2)$$

$$\Pi^{(\pi^0)}_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = i\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_3^2,(q_1+q_2+q_3)^2)}{(q_1+q_2)^2-M_\pi^2}\varepsilon_{\mu\nu\alpha\beta}q_1^{\alpha}q_2^{\beta}\varepsilon_{\lambda\rho\sigma\tau}q_3^{\sigma}(q_1+q_2)^{\tau}$$

$$+i\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,(q_1+q_2+q_3)^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_2^2,q_3^2)}{(q_2+q_3)^2-M_\pi^2}\varepsilon_{\mu\rho\alpha\beta}q_1^{\alpha}(q_2+q_3)^{\beta}\varepsilon_{\nu\lambda\sigma\tau}q_2^{\sigma}q_3^{\tau}$$

$$+i\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_3^2)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_2^2,(q_1+q_2+q_3)^2)}{(q_1+q_3)^2-M_\pi^2}\varepsilon_{\mu\lambda\alpha\beta}q_1^{\alpha}q_3^{\beta}\varepsilon_{\nu\rho\sigma\tau}q_2^{\sigma}(q_1+q_3)^{\tau}.$$

1.) Knecht, Marc, and Andreas Nyffeler. "Hadronic light-by-light corrections to the muon g– 2: the pion-pole contribution." Physical Review D 65.7 (2002): 073034.

Eta, Eta’ contribution to hadronic light by light

- Neutral pion transition form factor
  - Descriptions from a variety of models [1]
    - WZW, VMD, LMD, LMD+V
  - Recent lattice calculations by Mainz group [2,3]
    - Find good fit to LMD+V model
    - Determine new value for $a_\mu^{Hlbl;\pi_0}$
      - Using non-pert input & LMD+V model

$$\mathcal{F}^{WZW}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_C}{12\pi^2 F_\pi},$$

$$\mathcal{F}^{VMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_C}{12\pi^2 F_\pi}\frac{M_V^2}{(q_1^2-M_V^2)}\frac{M_V^2}{(q_2^2-M_V^2)},$$

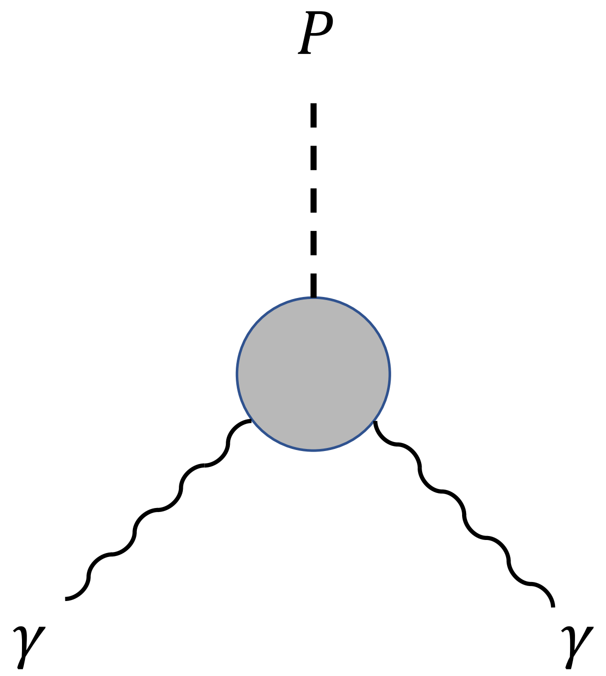
$$\mathcal{F}^{LMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{F_\pi}{3}\frac{q_1^2+q_2^2-c_V}{(q_1^2-M_V^2)(q_2^2-M_V^2)},$$

$$\mathcal{F}^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{F_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

- Eta,eta’ transition form factor
  - Originally described by simple VMD model [1]
  - Recently more sophisticated description by Canterbury Approximants [4]
  - No lattice determination yet

1.) Knecht, Marc, and Andreas Nyffeler. "Hadronic light-by-light corrections to the muon g− 2: the pion-pole contribution." Physical Review D 65.7 (2002): 073034.
2.) Gérardin, Antoine, Harvey B. Meyer, and Andreas Nyffeler. "Lattice calculation of the pion transition form factor π 0→ γ* γ." Physical Review D 94.7 (2016): 074507
3.) 1903.09471
4.) Masjuan et al. Phys. Rev. D 95 (2017) 054026

Computing the form factor



3-pt amplitude

$$C^3_{\mu\nu}(t_i, t_f, t) = \sum_{x_1, x} \langle T\{J_\mu(0, t_2) J_\nu(x_1, t_1) P^\dagger(x, t)\} \rangle e^{i(q_1 x_1 - px)}$$

Large $t_P = \min(t - t_2, t - t_1)$,
project onto pseudoscalar gnd state

$$C^3_{\mu\nu}(t_i, t_f, t) \xrightarrow{t_P \to \infty, t_f > t_i} \propto e^{-E_P t_P} \sum_{x_1} \langle 0 | J_\mu(0, t_2) J_\nu(x_1, t_1) | P(p) \rangle e^{i(q_1 x_1)}$$

$$A_{\mu\nu}(t_f - t_i) = \lim_{t_P \to \infty} e^{E_P t_P} C^3_{\mu\nu}(t_i, t_f, t)$$

Finally transform back from $\tau = t_f - t_i$ to ω_1 [1]

$$M^{\mathrm{E}}_{\mu\nu} = \frac{2E_\pi}{Z_\pi} \left(\int_{-\infty}^{0} \mathrm{d}\tau \, e^{\omega_1 \tau} A_{\mu\nu}(\tau) \, e^{-E_\pi \tau} + \int_0^\infty \mathrm{d}\tau \, e^{\omega_1 \tau} A_{\mu\nu}(\tau) \right) = \frac{2E_\pi}{Z_\pi} \int_{-\infty}^{\infty} \mathrm{d}\tau \, e^{\omega_1 \tau} \widetilde{A}_{\mu\nu}(\tau)$$

$$\widetilde{A}_{\mu\nu}(\tau) = \lim_{t_\pi \to +\infty} e^{E_\pi (t_f - t_0)} C^{(3)}_{\mu\nu}(\tau, t_\pi) = \begin{cases} A_{\mu\nu}(\tau) & \tau > 0 \\ A_{\mu\nu}(\tau) \, e^{-E_\pi \tau} & \tau < 0 \end{cases} .$$

$$A_{kl}(\tau) = -i q_{kl} \, A(\tau)$$

$$q_{kl} \equiv \epsilon_{kl\alpha\beta} \, q_1^\alpha \, q_2^\beta = m_\pi \, \epsilon_{kli} \, q_1^i$$

1.) arxiv:1607.08174

Computing the form factor (π_0)

Operators and symmetries

$$P^1 = i\bar{\chi}\chi \to \bar{\chi}\chi \qquad P^1 = \bar{\psi}\gamma_5\tau_3\psi$$

$$J_\mu^{\text{loc}} = \bar{\chi}\gamma_\mu Q\chi \qquad J_\mu^{\text{loc}} = \bar{\psi}\gamma_\mu Q\psi$$

$$J_\mu^{\text{cvc}}(x) = \frac{1}{2}\bar{\chi}(x)U_\mu(x)(\gamma_\mu - r)Q\chi(x+\mu) + \frac{1}{2}\bar{\chi}(x+\mu)U_\mu^\dagger(x)(\gamma_\mu + r)Q\chi(x)$$

$$J_\mu^Q = \frac{1}{2}J_\mu^{(0)} + \frac{1}{6}J_\mu^{(1)}$$

Operation	$P^{(0)}(t,\vec{p})$	$P^{(1)}(t,\vec{p})$	$J_k^{(0)}(t,\vec{p})$	$J_k^{(1)}(t,\vec{p})$
$\mathcal{P}\ \tau^1$	$-P^{(0)}(t,-\vec{p})$	$+P^{(1)}(t,-\vec{p})$	$-J_k^{(0)}(t,-\vec{p})$	$+J_k^{(1)}(t,-\vec{p})$
$\mathcal{C}$	$+P^{(0)}(t,\vec{p})$	$+P^{(1)}(t,\vec{p})$	$-J_k^{(0)}(t,\vec{p})$	$-J_k^{(1)}(t,\vec{p})$
$\mathcal{P}\ \tau^1 \times \mathcal{C}$	$-P^{(0)}(t,-\vec{p})$	$+P^{(1)}(t,-\vec{p})$	$+J_k^{(0)}(t,-\vec{p})$	$-J_k^{(1)}(t,-\vec{p})$

Computing the form factor (π_0)

Connected contractions, symmetry considerations

$$\left\langle J_\mu^Q J_\nu^Q P \right\rangle_{conn} = \frac{1}{4}\left\langle J_\mu^{(0)} J_\nu^{(0)} P \right\rangle_{conn} + \frac{1}{36}\left\langle J_\mu^{(1)} J_\nu^{(1)} P \right\rangle_{conn} + \frac{1}{12}\left(\left\langle J_\mu^{(0)} J_\nu^{(1)} P \right\rangle_{conn} + \left\langle J_\mu^{(1)} J_\nu^{(0)} P \right\rangle_{conn} \right)$$

After averaging over exact discrete lattice symmetries

Wrong isospin combinations

$$\left\langle J_\mu^{(0/1)} J_\nu^{(0/1)} P \right\rangle_{conn} \sim \; Re(C_u(\{p\}) + C_u(\{-p\}))$$

$$P\; C_u(\{p\}) = -C_u(\{-p\})$$

Correct isospin combinations

$$\left\langle J_\mu^{(1/0)} J_\nu^{(0/1)} P \right\rangle_{conn} \sim \; Re(C_u(\{p\}) - C_u(\{-p\}))$$

$$C_{u,d} = \left\langle 0 \left| \bar{\chi}_{u,d}(x_2)\Gamma_\mu^2 \chi_{u,d}(x_2) \bar{\chi}_{u,d}(x_1)\Gamma_\nu^1 \chi_{u,d}(x_1) \bar{\chi}_{u,d}(x)\Gamma^P \chi_{u,d}(x) \right| 0 \right\rangle_{\text{conn}}$$

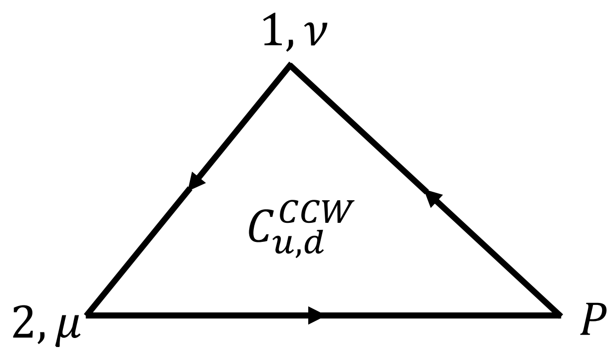
Computing the form factor (π_0)

Connected contractions

$$\left\langle J_\mu^Q J_\nu^Q P \right\rangle_{conn} = \frac{1}{12}\left(\left\langle J_\mu^{(0)} J_\nu^{(1)} P \right\rangle_{conn} + \left\langle J_\mu^{(1)} J_\nu^{(0)} P \right\rangle_{conn}\right) = \frac{1}{6}\left(C_u(\{p\}) - C_d(\{p\})\right)$$

After averaging over exact discrete lattice symmetries

$$= \frac{1}{6} Re\left(C_u^{CCW}(\{p\}) - C_d^{CCW}(\{p\}) - C_u^{CCW}(\{-p\}) + C_d^{CCW}(\{-p\})\right)$$

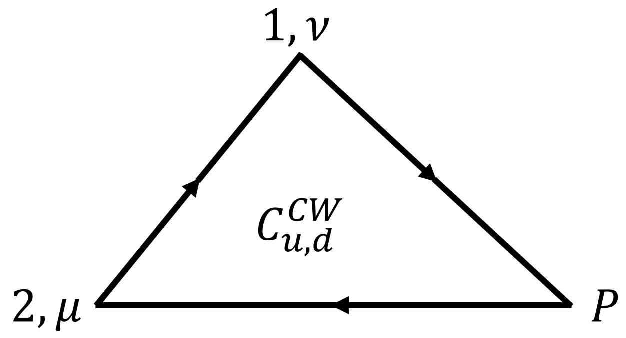


Having utilized the gamma-5 hermiticity relation

$$\gamma_5 C_{u,d}^{CW} \gamma_5 = \left(C_{d,u}^{CCW}\right)^*$$

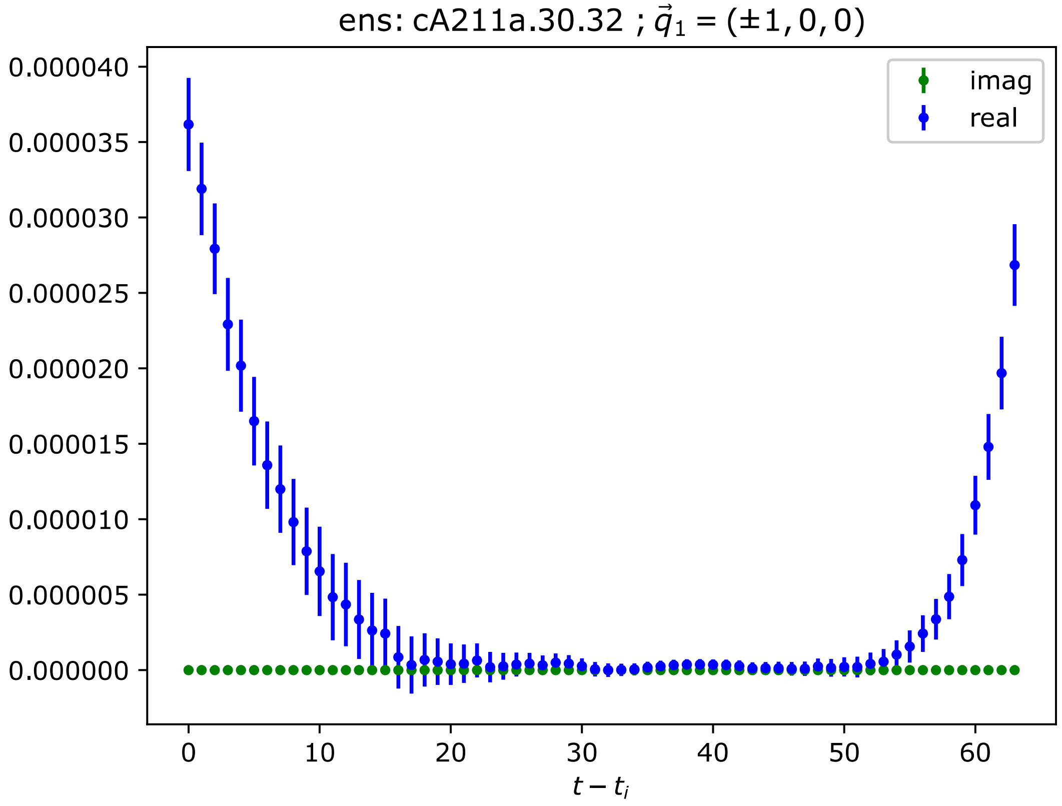
And individual Wick contractions look like

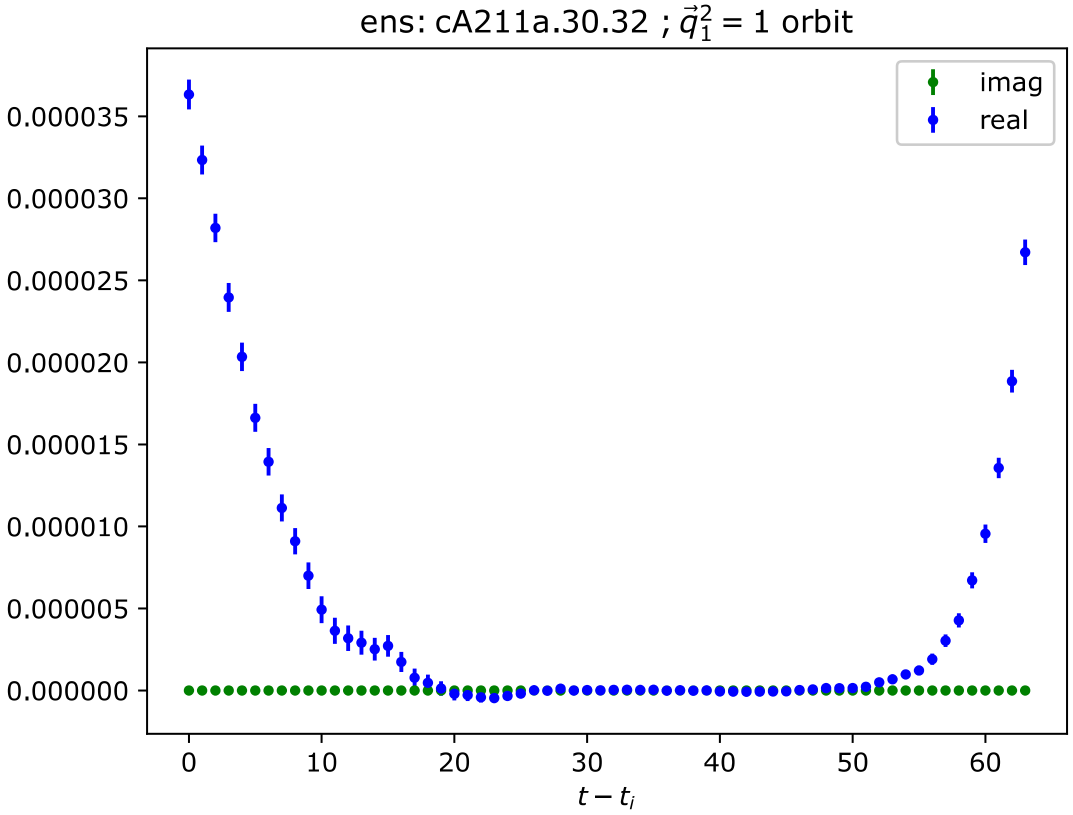
$$C_u^{CCW} = \mathrm{Tr}\left[G_{1P}^u \Gamma_P G_{P2}^u \Gamma_2 G_{21}^u \Gamma_1\right]$$
$$C_u^{CW} = \mathrm{Tr}\left[G_{2P}^u \Gamma_P G_{P1}^u \Gamma_1 G_{12}^u \Gamma_2\right]$$



Connected Form Factor (π_0)

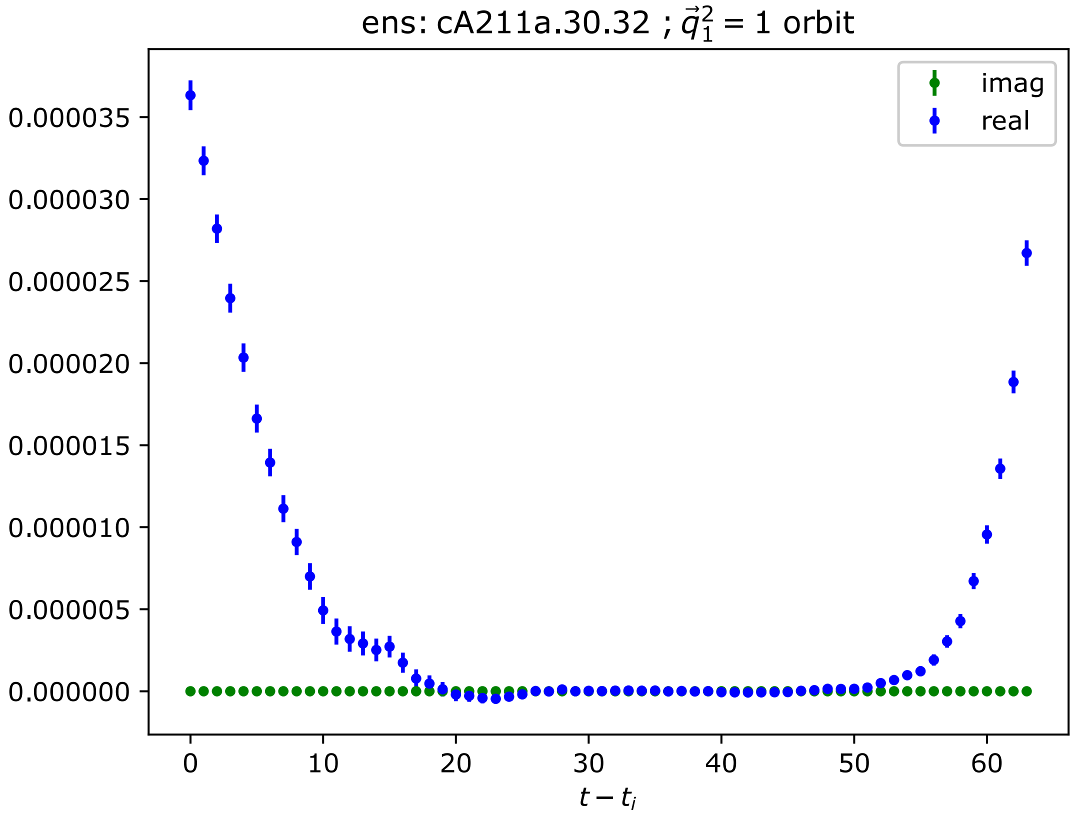
Single momentum component vs avg over orbit

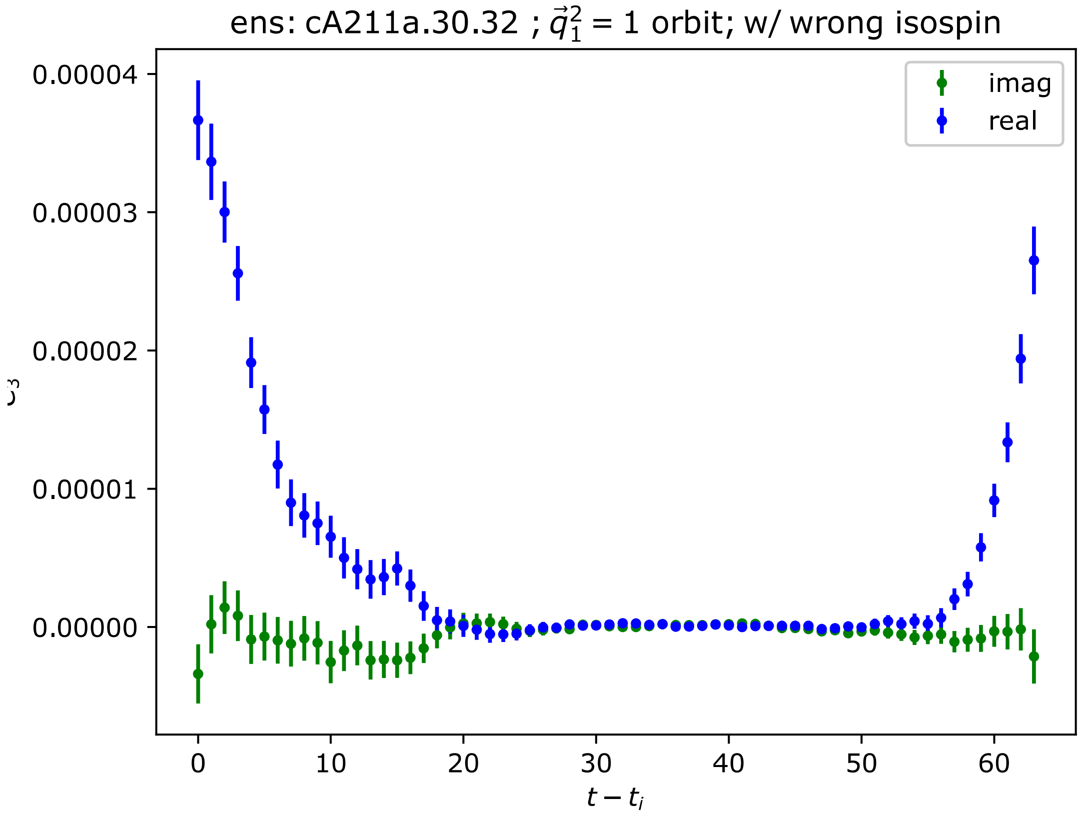




Connected Form Factor (π_0)

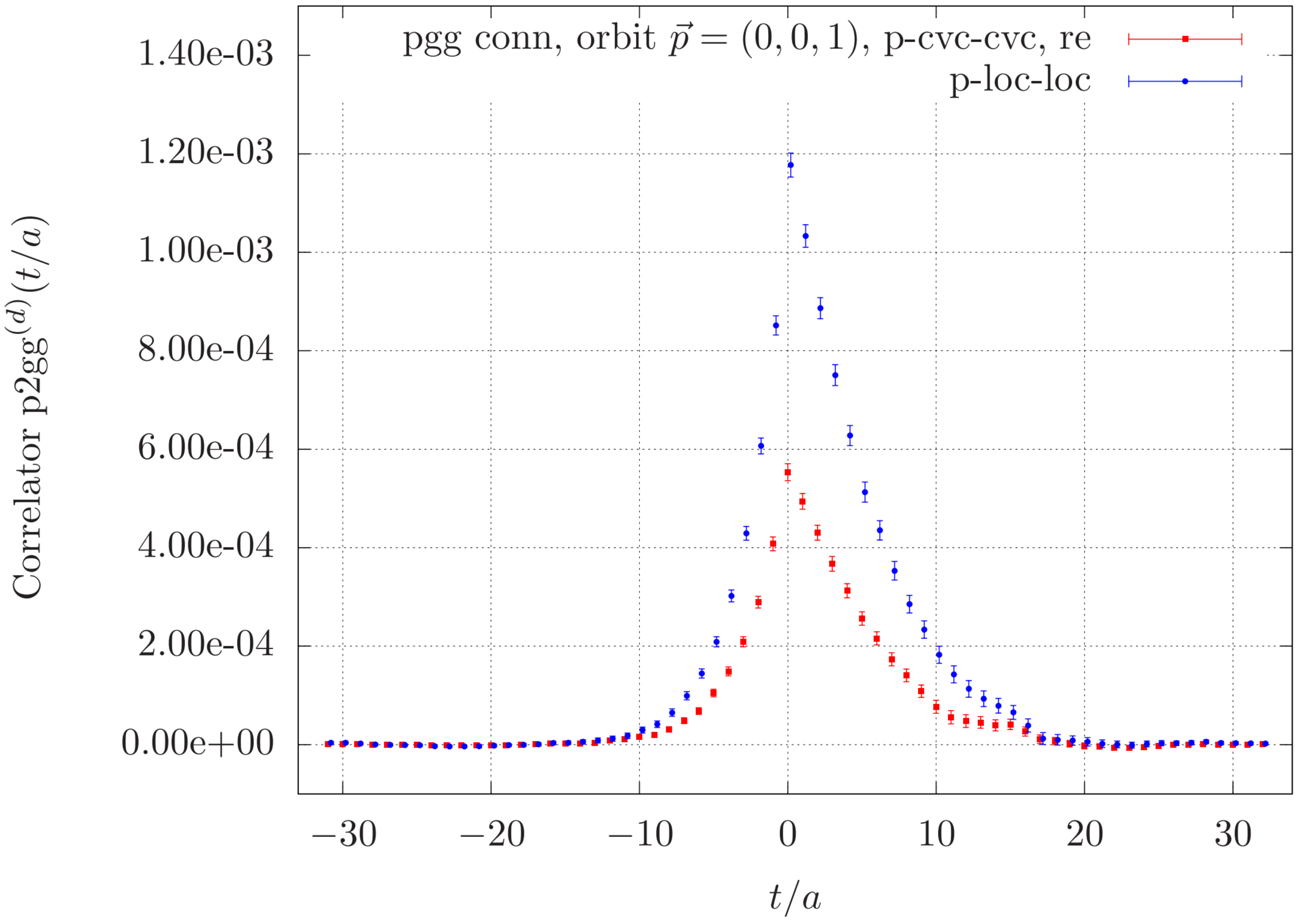
Correct isospin combination vs all isospin combinations



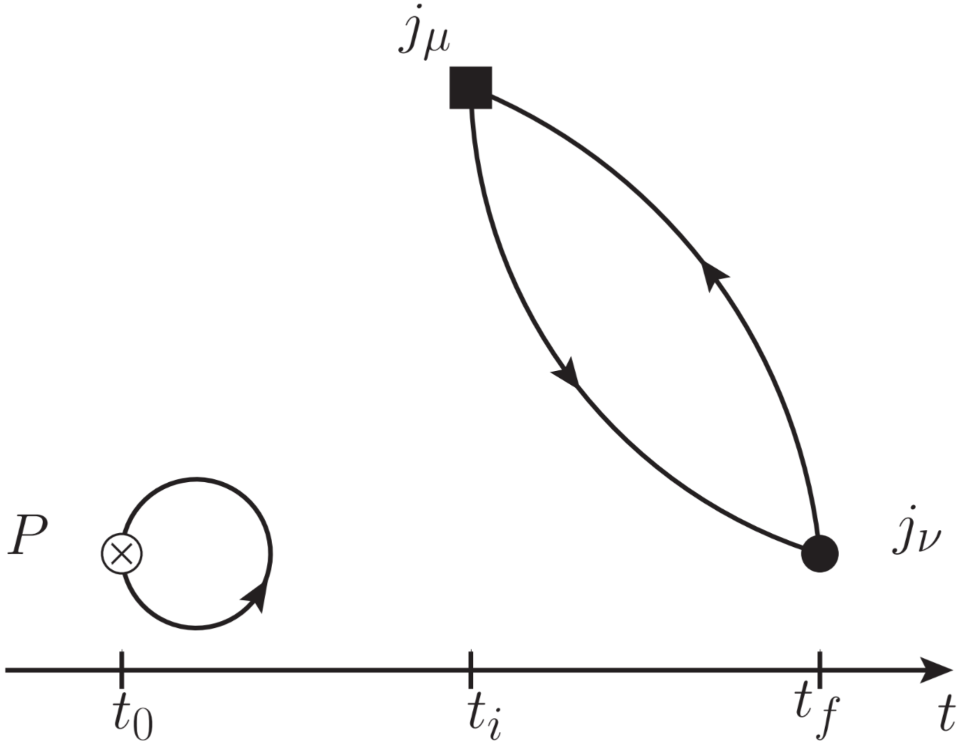


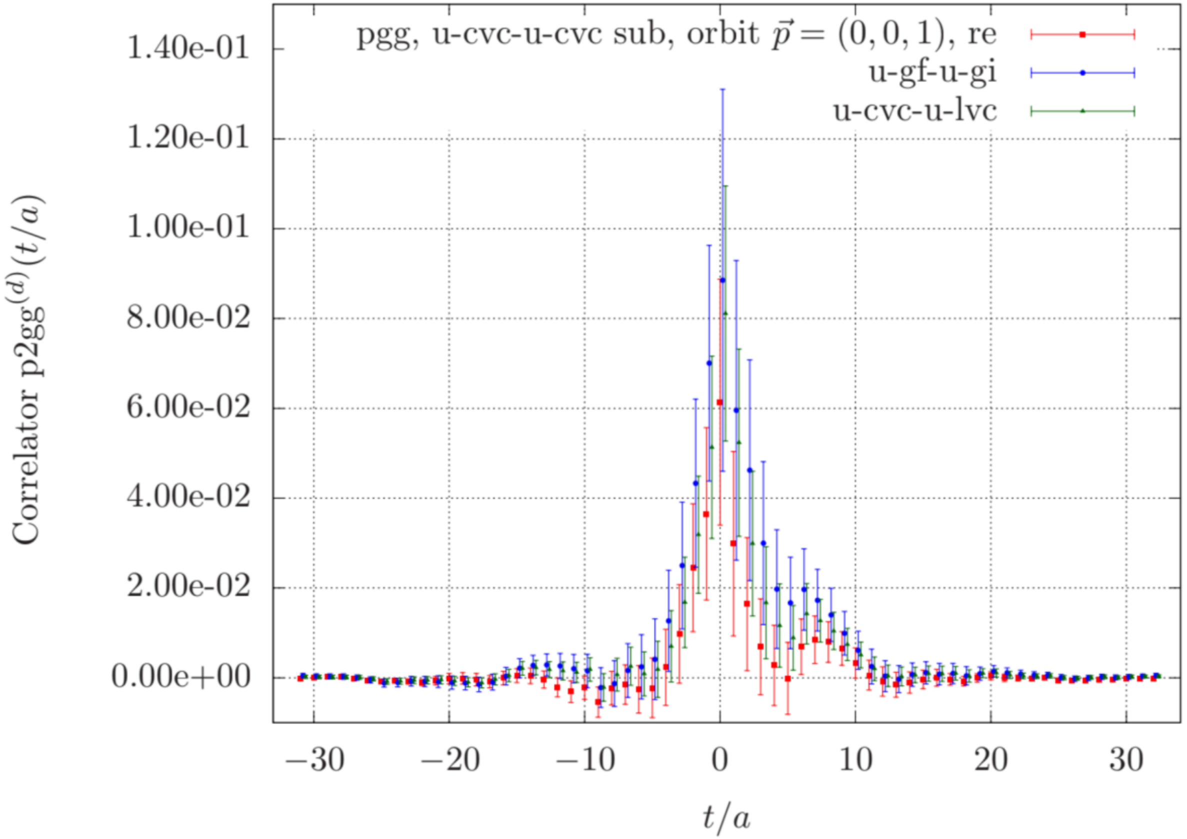
Connected Form Factor (π_0)

Cross checking with M. Petschlies



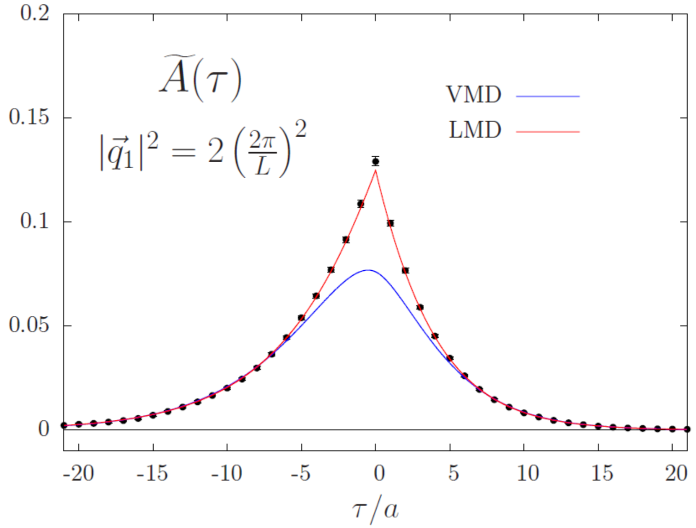
First look, disconnected Form Factor (π_0) [1]

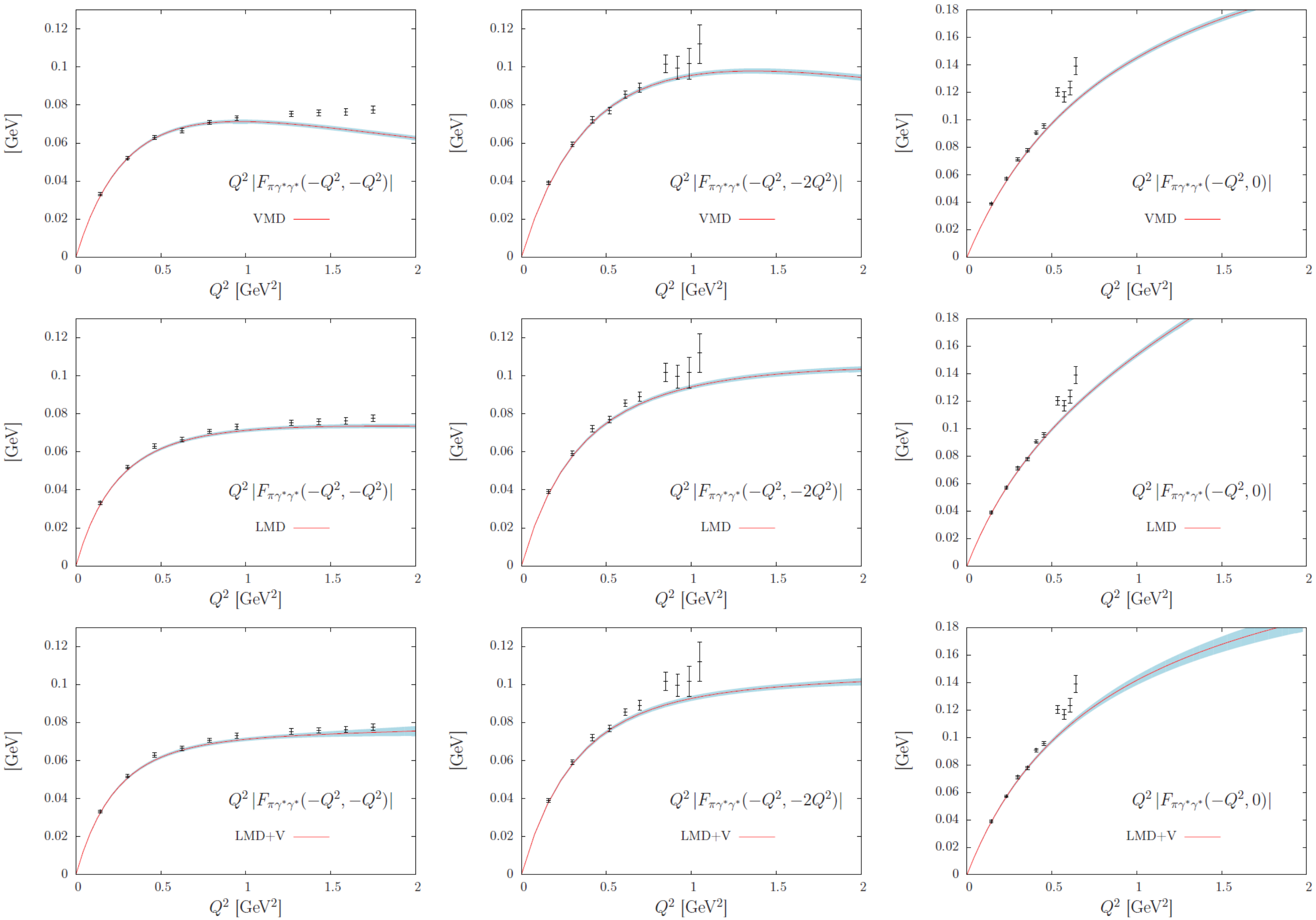




1.) from Marcus Petschlies

Looking forward: Modeling the form factor, from arxiv:1607.08174





$$\mathcal{F}^{WZW}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_C}{12\pi^2 F_\pi},$$

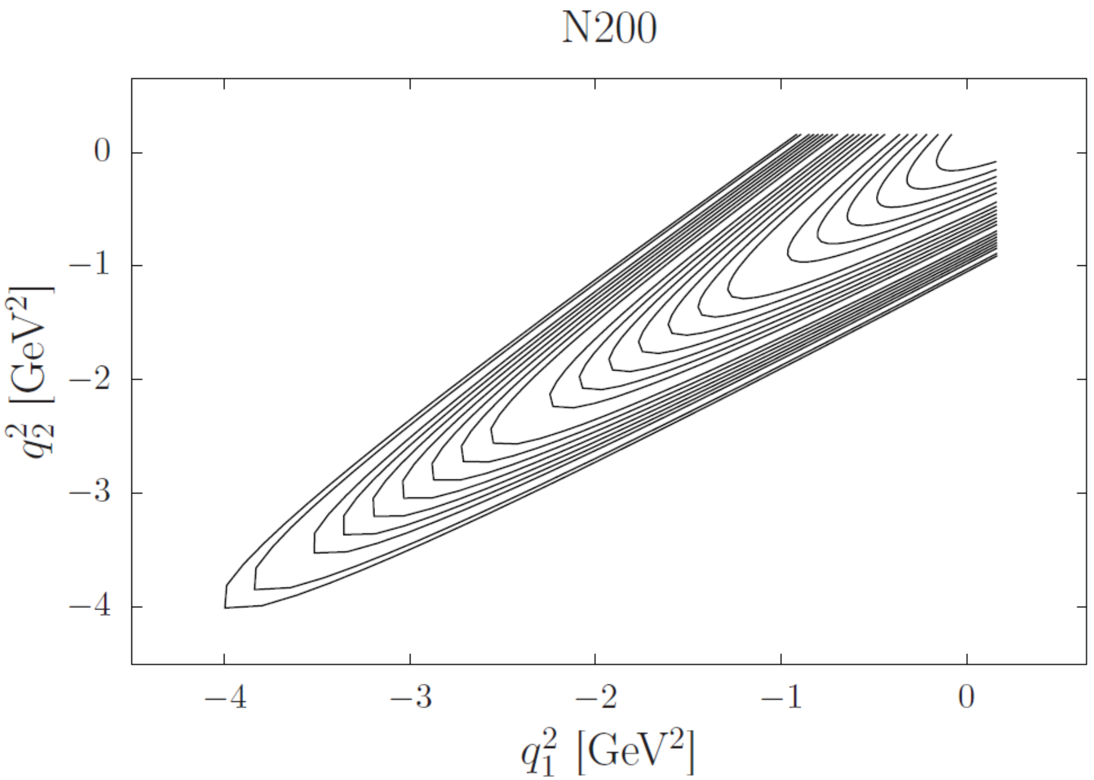
$$\mathcal{F}^{VMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_C}{12\pi^2 F_\pi}\frac{M_V^2}{(q_1^2-M_V^2)}\frac{M_V^2}{(q_2^2-M_V^2)},$$

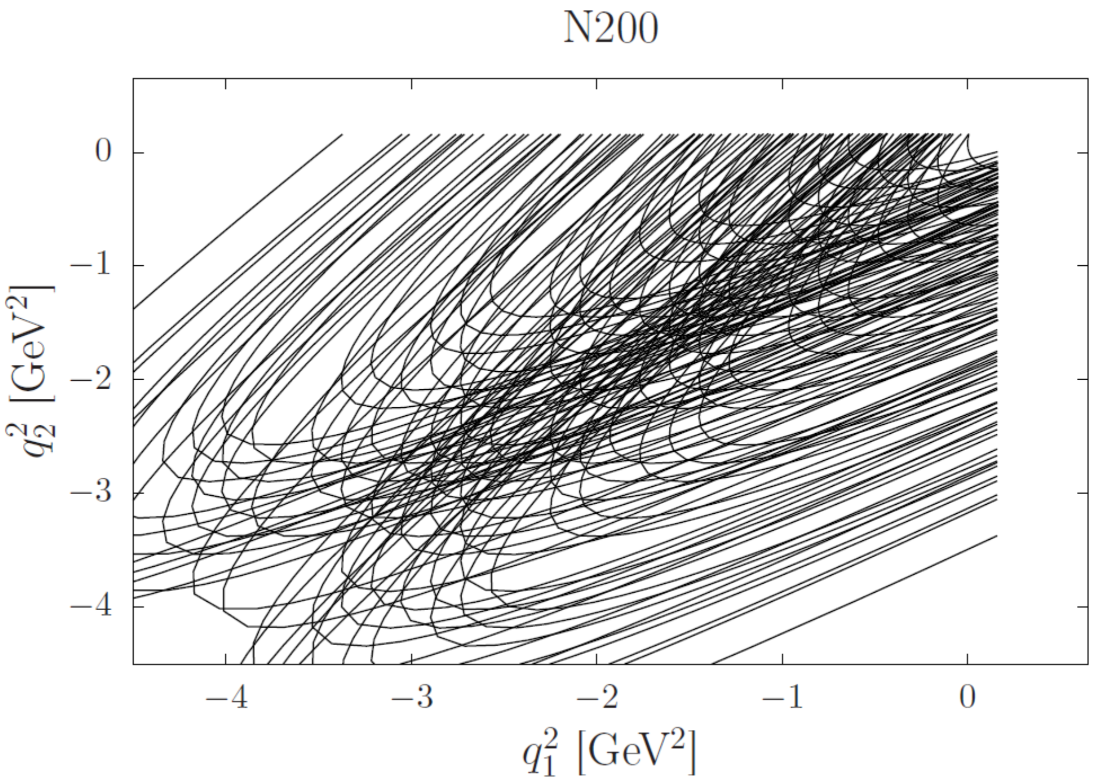
$$\mathcal{F}^{LMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{F_\pi}{3}\frac{q_1^2+q_2^2-c_V}{(q_1^2-M_V^2)(q_2^2-M_V^2)},$$

$$\mathcal{F}^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{F_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Looking forward

Moving frames [1]





1.) arxiv:1903.09471

Status of transition form factor project

- Working towards first physical point computation of neutral pion transition form factor, and first determination of eta,eta’ transition form factors
  - Dominant and subdominant in hadronic LbL for $(g-2)_\mu$
- Converging on cross checks for neutral pion connected piece
- Already significant work on pseudoscalar disconnected pieces by M. Petschlies
  - Improvements at large t from low-mode deflation (not shown)
- Vector current disconnected pieces seen to be small in Mainz studies