Non-perturbative generation of elementrary particle masses: lattice evidence for fermion mass generation

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Motivation

Incompleteness of the Standard Model

- SM renormalizable and successful, but no explanation of
  - fine-tuning of scalar VEV (no extra symmetry when v takes its "physical" value)
  - fermion mass hierarchy
  - neutrino masses and mixings, not enough CP violation for baryogenesis, dark matter, dark energy & quantum gravity
- Low energy Lagrangian of a BSM model cannot differ much from SM one
- Hierarchy problems for the EW scale and fermion masses are entangled

Alternative: Comprehensive NP elementary particle mass generation

- R. Frezzotti and G.C. Rossi [Phys. Rev. D92 (2015) 054505] (fermion mass) *and* [arXiv:1811.10326] (towards realistic models with weak interactions)
- R. Frezzotti and M. Garofalo and G.C. Rossi [Phys. Rev. D93 (2016) 1050030] (possible GUT unification)
- Rome/Bonn "bsmsimul" group [Phys. Rev. Lett. 123 (2019) 061802] (lattice demonstration)

$\Rightarrow$ "all" elementary masses related to some energy scale Λ_T, many other nice features

Non-perturbative mass generation in a toy model

Toy model field content

- A_μ – SU(3) gauge field with renormalised coupling g_s
- $Q = (u, d)^T$ – Dirac doublet, triplet under SU(3)
- $\varphi = (\varphi_0 + i\varphi_3, -\varphi_2 + i\varphi_1)^T$ – complex scalar doublet (SU(3) singlet)
  - conventient matrix form $\Phi = [\varphi | -i\tau^2\varphi^*] = \begin{bmatrix} \varphi_0 + i\varphi_3 & \varphi_2 + i\varphi_1 \\ -\varphi_2 + i\varphi_1 & \varphi_0 - i\varphi_3 \end{bmatrix}$

$$\mathcal{L}_{\text{toy}}(Q, A, \Phi) = \mathcal{L}_{\text{kin}}(Q, A, \Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{\text{Yuk}}(Q, \Phi) + \mathcal{L}_{\text{Wil}}(Q, A, \Phi) =$$
$$\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{Q}_L \not{D} Q_L + \bar{Q}_R \not{D} Q_R + \frac{1}{2} \text{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] +$$
$$\frac{\mu_0^2}{2} \text{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda_0}{4} \left(\text{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 +$$
$$\eta \left(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L\right) +$$
$$\frac{b^2}{2} \rho \left(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \bar{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu Q_L\right) \qquad \text{“pseudo-Wilson term”}$$

UV cutoff: $\Lambda_{\text{UV}} \sim b^{-1} \to \infty$

Toy model Lagrangian symmetries

- **Exact $\chi_L \times \chi_R$ acting on fermions and scalars**

χ_L: $\tilde\chi_L \otimes \chi_L^\Phi$

$$\tilde\chi_L\colon\ Q_L \to \Omega_L Q_L\,, \qquad \bar Q_L \to \bar Q_L \Omega_L^\dagger$$
$$\chi_L^\Phi\colon\ \Phi \to \Omega_L \Phi \qquad \Omega_L \in \mathrm{SU}(2)_L$$

χ_R: $\tilde\chi_R \otimes \chi_R^\Phi$

$$\tilde\chi_R\colon\ Q_R \to \Omega_R Q_R\,, \qquad \bar Q_R \to \bar Q_R \Omega_R^\dagger$$
$$\chi_R^\Phi\colon\ \Phi \to \Phi \Omega_R^\dagger \qquad \Omega_R \in \mathrm{SU}(2)_R$$

- **Poincaré, T, P, C, SU(3) gauge, $\chi_L \times \chi_R$**

$\Rightarrow$ **Renormalisability**

$\Rightarrow$ **no divergent $\Lambda_{\mathrm{UV}}\bar Q Q$ mass terms, only b^2 cutoff effects on the lattice**

- **Fermionic chiral transformations $\tilde\chi_L \times \tilde\chi_R$ are explicitly broken by $\mathcal{L}_{\mathrm{Yuk}}$ and $\mathcal{L}_{\mathrm{Wil}}$**

$$\mathcal{L}_{\mathrm{kin}}(Q,A,\Phi) = \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar Q_L \not{\mathcal{D}} Q_L + \bar Q_R \not{\mathcal{D}} Q_R + \tfrac{1}{2}\,\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right]$$

$$\mathcal{V}(\Phi) = \tfrac{\mu_0^2}{2}\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \tfrac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 \qquad \mathcal{L}_{\mathrm{Yuk}}(Q,\Phi) = \eta\left(\bar Q_L \Phi Q_R + \bar Q_R \Phi^\dagger Q_L\right)$$

$$\mathcal{L}_{\mathrm{Wil}}(Q,A,\Phi) = \tfrac{b^2}{2}\rho\left(\bar Q_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \bar Q_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu Q_L\right)$$

Toy model $\tilde{\chi}$-symmetry restoration

- Toy model *not* invariant under $\tilde{\chi} = \tilde{\chi}_L \times \tilde{\chi}_R \Rightarrow$ non-cons. currents $\Rightarrow$ SDEs

$\tilde{\chi}$-variation currents

$$\tilde{J}_\mu^{[L,R]i} = \bar{Q}_{[L,R]}\gamma_\mu \frac{\tau^i}{2} Q_{[L,R]} - \frac{b^2}{2}\rho \left(\bar{Q}_{[L,R]} \frac{\tau^i}{2} \Phi \mathcal{D}_\mu Q_{[R,L]} - \bar{Q}_{[R,L]} \overleftarrow{\mathcal{D}} \Phi^\dagger \frac{\tau^i}{2} Q_{[L,R]} \right)$$

Renormalised $\tilde{\chi}$ Schwinger-Dyson-equations (SDEs)

$$\partial_\mu \left\langle Z_{\tilde{J}} \tilde{J}_\mu^{[L,R]i}(x)\hat{O}(0) \right\rangle = \left\langle \tilde{\Delta}^i_{[L,R]} \hat{O}(0) \right\rangle \delta(x) \qquad (\text{ variation of } \hat{O}, \text{ if any })$$

$$+ (\bar{\eta} - \eta) \left\langle \left(\bar{Q}_{[L,R]} \frac{\tau^i}{2} \Phi Q_{[R,L]} - \bar{Q}_{[R,L]} \Phi^\dagger \frac{\tau^i}{2} Q_{[L,R]} \right)(x) \hat{O}(0) \right\rangle$$

$$+ \mathcal{O}(b^2)$$

- $\bar{\eta}$ term due to mixing of $d = 6$ pseudo-Wilson operator under renormalisation, just like $m_{\rm cr}$ counterterm for Wilson LQCD
- **can tune $\eta \to \eta_{\rm cr}$ to restore $\tilde{\chi}$ up to $\mathcal{O}(b^2)$**
- **eliminate Yukawa term!**

Central conjecture: The non-perturbative anomaly

Glossed over essential detail discussing $\tilde{\chi}$-restoration: the scalar v.e.v. $\langle\Phi\rangle$!

In the Wigner phase, $\langle\Phi\rangle = 0$, argument holds

- $\eta \to \eta_{\mathrm{cr}}$: scalars and fermions decoupled
- massless quarks (up to $\mathcal{O}(b^2)$)

In the Nambu-Goldstone phase, $\langle\Phi\rangle \neq 0$

- even $\chi_L \times \chi_R$ spontaneously broken
- $\eta \to \eta_{\mathrm{cr}}$: but, still no Yukawa or quark mass term
- **Massive quarks due to "NP anomaly" in $\tilde{\chi}$-restoration**
  - $\langle\Phi\rangle \neq 0$ provides seed for SSB
  - $\tilde{\chi}$-breaking at UV scale by higher-dimensional operator
  - even at η_{cr}, strong interactions and residual $\tilde{\chi}$-breaking induce $\tilde{\chi}$-SSB
  - **1 PI effective Lagrangian at "SM scale" has a quark mass term $\propto \Lambda_s$!**
    - **"irrelevant" effects at high energy affect low-energy behaviour *fundamentally***
    - **independent of actual value of $\langle\Phi\rangle$ in a realistic model including weak interactions**

The NP anomaly 1/2

Renormalised $\tilde{\chi}$-SDEs in NG phase

$$\partial_\mu \left\langle Z_{\tilde{J}} \tilde{J}_\mu^{[L,R]i}(x)\hat{O}(0)\right\rangle\Big|_{\eta_{cr}} = \left\langle \tilde{\Delta}^i_{[L,R]}\hat{O}(0)\right\rangle\Big|_{\eta_{cr}} \delta(x) \qquad (\text{ variation of } \hat{O}, \text{ if any })$$

$$+ \cancelto{0}{(\bar{\eta}-\eta)} \left\langle \left(\bar{Q}_{[L,R]}\frac{\tau^i}{2}\Phi Q_{[R,L]} - \bar{Q}_{[R,L]}\Phi^\dagger \frac{\tau^i}{2} Q_{[L,R]} \right)(x)\hat{O}(0) \right\rangle$$

$$+ c_1 \Lambda_s \left\langle \left(Q_L \frac{\tau^i}{2} U Q_R - \text{H.c.} \right) \hat{O}(0) \right\rangle\Big|_{\eta_{cr}}$$

$$+ \mathcal{O}(b^2)$$

- L.H.S. of SDE is RGI $\Rightarrow$ all terms on R.H.S. must be RGI!
- effective scalar degrees of freedom
  - Massive ζ_0
  - Goldstone $\zeta_{1,2,3}$

$$\Phi = v_\Phi + \sigma + i\vec{\tau}\vec{\pi} = RU \,, \quad R = (v_\Phi + \zeta_0) \,, \quad U = \exp\left[i v_\Phi^{-1} \tau^k \zeta_k \right]$$

The NP anomaly 2/2

1 PI effective Lagrangian in the NG phase

$$
\begin{aligned}
\Gamma_4^{\rm NG} =& \frac{1}{4}(F \cdot F) + \bar{Q}\not{D}Q + \frac{1}{2}\,\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \frac{\hat{\mu}_\Phi}{2}\,\mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\hat{\rho}}{4}\left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 \\
&+ (\eta - \overline{\eta}(\eta, \rho, \ldots))\left[\bar{Q}_L \Phi Q_R + \mathrm{H.c.}\right] \\
&+ c_1 \Lambda_s \left(\bar{Q}_L U Q_R + \mathrm{H.c.}\right) \\
&+ \left(c_2 \Lambda_s^2 + \tilde{c}\Lambda_s R\right) \frac{1}{2}\,\mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\mu U\right)
\end{aligned}
$$

$$
\Phi = v_\Phi + \sigma + i\vec{\tau}\vec{\pi} = RU\,, \quad R = (v_\Phi + \zeta_0)\,, \quad U = \exp\left[i v_\Phi^{-1} \tau^k \zeta_k\right]
$$

- **expand** U **around** $1 \Rightarrow \Gamma^{\rm NG} \supset c_1 \Lambda_s \bar{Q}_L Q_R + \ldots$
- In realistic model including weak interactions
  - ρ is no longer free parameter
  - $\tilde{c}$ term vanishes when $\tilde{\chi}$ is maximally enhanced
  - c_2 term leads to weak gauge boson masses

Lattice study targets 1/2

- For a given ρ, need to determine $\eta_{\rm cr}$, conveniently done in the Wigner phase

Enforce restoration of $\tilde{\chi}$ up to lattice artefacts based on $\tilde{J}_\mu^{[L,R]i}$ SDEs:

$$\frac{\partial_\mu \left\langle \tilde{A}^i_\mu(x)\tilde{D}^i_P(0)\right\rangle}{\left\langle \tilde{D}^i_P(x)\tilde{D}^i_P(0)\right\rangle} = (\eta-\bar{\eta}) + \mathcal{O}(b^2)$$

$$\tilde{A}^i_\mu = \tilde{J}^{Li}_\mu - \tilde{J}^{Ri}_\mu \qquad \tilde{D}^i_P = \bar{Q}_L\left\{\Phi, \frac{\tau^i}{2}\right\}Q_R - \bar{Q}_R\left\{\frac{\tau^i}{2}, \Phi^\dagger\right\}Q_L$$

Practical correlator ratio with better signal/noise

$$r_{\rm AWI} = \frac{\sum_{\vec{x},\vec{y}}\left\langle P^1(0)\partial_0^{\rm FW}\tilde{A}_0^{1,{\rm BW}}(x)\varphi_0(y)\right\rangle}{\sum_{\vec{x},\vec{y}}\left\langle P^1(0)\tilde{D}^1_P(x)\varphi_0(y)\right\rangle}, \qquad P^i = \bar{Q}\gamma_5\frac{\tau^i}{2}Q\,, \qquad \varphi_0 = \frac{{\rm Tr}[\Phi]}{2}$$

- $\partial_0^{\rm FW}\tilde{A}_0^{1,{\rm BW}}$: fwd lattice derivative of bwd point-split $\bar{Q}\gamma_0\gamma_5(\tau^1/2)Q$
- $y_0 - x_0$ appropriately fixed in physical units to 0.6 fm

Lattice study targets 2/2

- In the NG phase, demonstrate that massive quarks observed @ $\eta_{\rm cr}$

Pseudoscalar meson correlation function

$$C^i_{\rm PS}(x_0) = \sum_{\vec{x}} \left\langle [P^i(x)]\,[P^i(0)]^\dagger \right\rangle \qquad\qquad P^i = \bar{Q}\gamma_5 \frac{\tau^i}{2} Q$$

PCAC quark mass

$$\frac{Z_{\tilde{A}}}{Z_P} m_{\rm AWI} = \left. \frac{Z_{\tilde{A}} \sum_{\vec{x}} \partial_0 \, \langle \tilde{A}^i_0(x) P^i(0) \rangle}{2 Z_P \sum_{\vec{x}} \langle P^i(x) P^i(0) \rangle} \right|_{\eta_{\rm cr}}$$

- Both $M_{\rm PS}$ and $(Z_{\tilde{A}}/Z_P) m_{\rm AWI}$ should remain non-zero as $b \sim \Lambda^{-1}_{\rm UV} \to 0$

Lattice study setup

- Need setup with exact $\chi_L \times \chi_R$ symmetry
- NP anomaly also expected in quenched approximation $\Rightarrow$ modified naïve lattice fermions

$$S_L = S_g[U] + S_s[\Phi] + b^4 \sum_x \left\{ \bar{Q}(x) D_L[U,\Phi] Q(x) + i\mu \bar{Q}(x)\gamma_5 \tau^3 Q(x) \right\}$$

- $S_g[U]$: Wilson plaquette gauge action ($\beta \in [5.75, 5.85, 5.95]$)
- $S_s[\Phi]$: $b^4 \sum_x \left\{ \frac{1}{2} \mathrm{Tr}\left[\Phi^\dagger(-\partial^*_\mu \partial_\mu)\Phi\right](x) + \frac{m_\Phi^2}{2} \mathrm{Tr}\left[\Phi^\dagger \Phi\right](x) \frac{\lambda_0}{4} \left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 (x) \right\}$
- $D_L[U,\Phi]Q(x) = \gamma_\nu \tilde{\nabla}_\nu Q(x) + \eta F(x) Q(x) - b^2 \frac{\rho}{2} F(x) \tilde{\nabla}_\nu \tilde{\nabla}_\nu Q(x)$ $-b^2 \frac{\rho}{4} \left[(\partial_\nu F)(x) U_\nu(x) \tilde{\nabla}_\nu Q(x+\hat{\nu}) + (\partial^*_\nu F)(x) U^\dagger(x-\hat{\nu}) \tilde{\nabla}_\nu Q(x-\hat{\nu}) \right]$
- twisted mass to avoid spurious zero modes and keep $M_{\mathrm{PS}}L$ large

$$\Phi \equiv \left[\varphi_0 \mathbf{1} + i\varphi_j \tau^j\right] \qquad F \equiv \left[\varphi_0 \mathbf{1} + i\gamma_5 \tau^j \varphi_j\right]$$

$$\tilde{\nabla}_\mu f(x) = \frac{1}{2}\left(\nabla^*_\mu + \nabla_\mu\right) f(x)$$

$$b\nabla_\mu f(x) = U_\mu(x) f(x+\hat{\mu}) - f(x) \qquad b\nabla^*_\mu = f(x) - U^\dagger_\mu(x-\hat{\mu}) f(x-\hat{\mu})$$

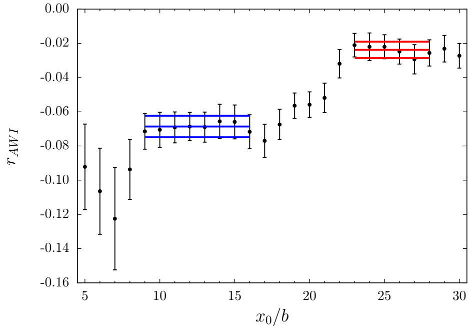
Lattice, finite volume systematics and renormalisation

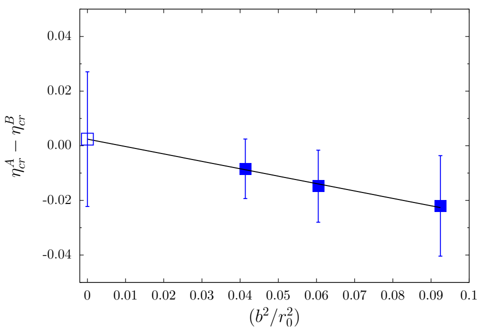
- many doublers (32 flavours!) but symmetric derivatives ensure ρ and $(\eta - \bar{\eta})$ common to all
- quenched setup $\rightarrow$ doublers irrelevant for demonstration of mechanism
- gauge and scalar sectors independently renormalised

- $\beta \in [5.75, 5.85, 5.95] \Rightarrow b^2/r_0^2$ varies by ~ 2.2
- r_0/b values taken from [ALPHA, Nucl.Phys. B535 (1998) 389-402]
- $M_{\zeta_0}^2 r_0^2 = 1.284(6)\,, \qquad \lambda_R = \frac{M_{\zeta_0}^2}{2v_R^2} = 0.441(4)\,, \qquad v_\Phi^2 r_0^2 = 1.458(2)$
- parameters s.t. $M_{\mathrm{PS}} L \geq 4.7$
- many μ and η values for inter-/extrapolations
- $\rho \in [0, 1.96, 2.94]$

Lattice study results 1/8

r_{AWI} plateaus

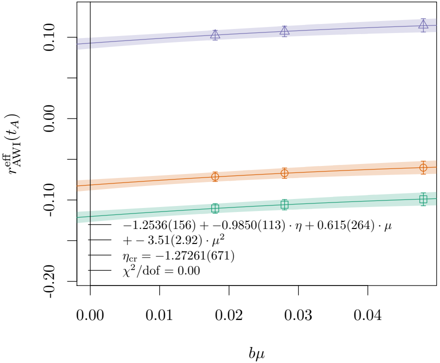


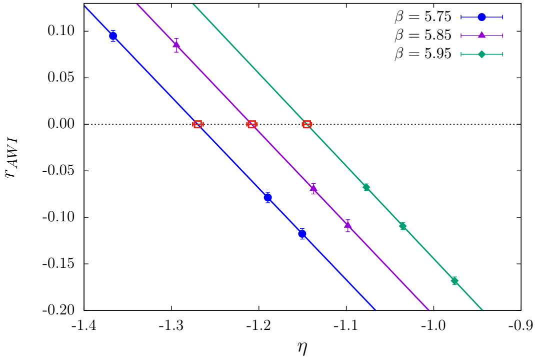


- r_{AWI} has two plateau regions
- choose fit interval (in each region) in physical units
- difference is a lattice artefact due to different states dominating

Lattice study results 2/8

η_{cr} determination



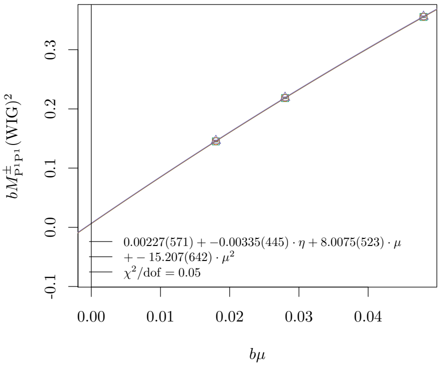


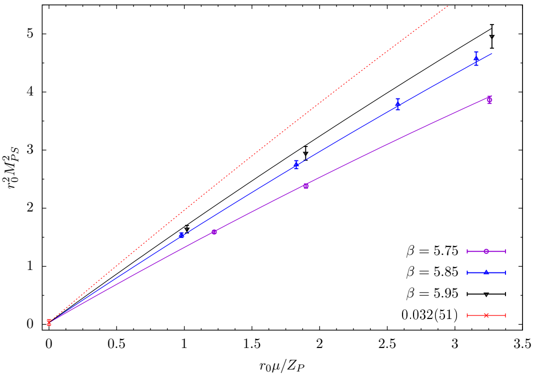
- at each β, parametrise r_{AWI} in various ways $\Rightarrow$ account for small systematic parametrisation errors

- @ $b\mu = 0$, r_{AWI} well linear in η
- slightly better than 1% precision in η_{cr} with final statistics & systematics

Lattice study results 3/8

Wigner phase sanity checks

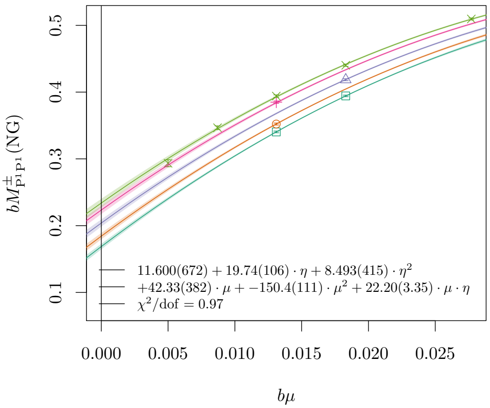


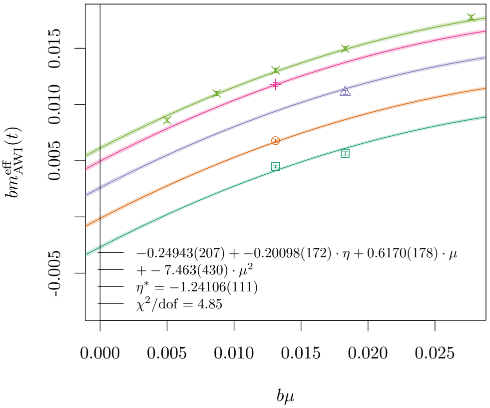


- Wigner phase: three values of μ, three values of η
- As expected no η-dependence of M^2_{PS}
- Must be able to extrapolate to $M^2_{\mathrm{PS}} = 0$ in chiral limit @ η_{cr}
- Result comparable to best quenched LQCD studies

Lattice study results 4/8

NG phase parametrisations





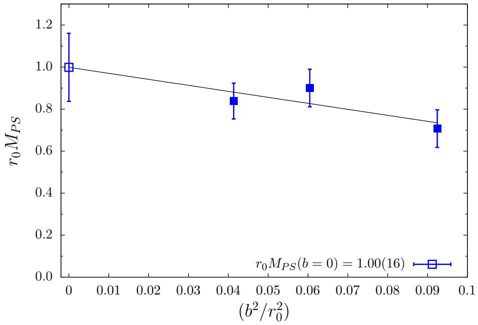
- NG phase parameter dependence of M_{PS} requires "complicated" polynomial
- Also m_{AWI} must be appropriately parametrised

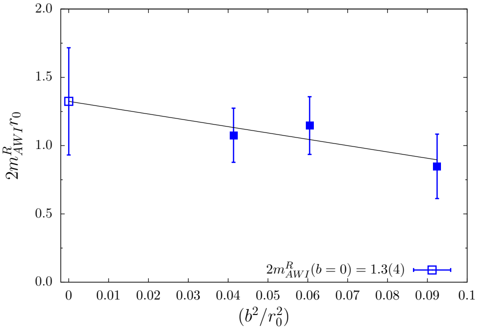
$\Rightarrow$ ready to interpolate to η_{cr} and take continuum limit!

Lattice study results 5/8

Main result: renormalising & taking the continuum limit

- Expect b^2 scaling
- significant evidence for NP'vely generated quark mass!





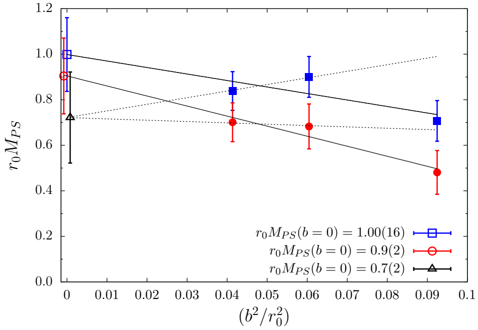
- Median over all analyses
- $r_0 M_{\mathrm{PS}} = 0.93(0.09)_{\mathrm{stat}}(0.10)_{\mathrm{sys}}$

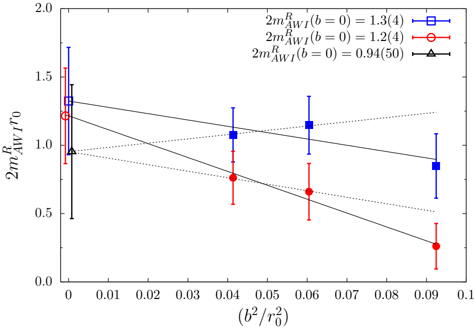
- Median over all analyses
- $2r_0 m^R_{\mathrm{AWI}} = 1.20(0.39)_{\mathrm{stat}}(0.19)_{\mathrm{sys}}$

Lattice study results 6/8

Scaling tests

- Different definitions of η_{cr} should be consistent up to $\mathcal{O}(b^2)$
  - Compare continuum limit with different definitions
- At $\beta = 5.75$, perhaps worry about scaling?
  - Compare separate 3-β cont.exp. and common 2-β cont.exp.



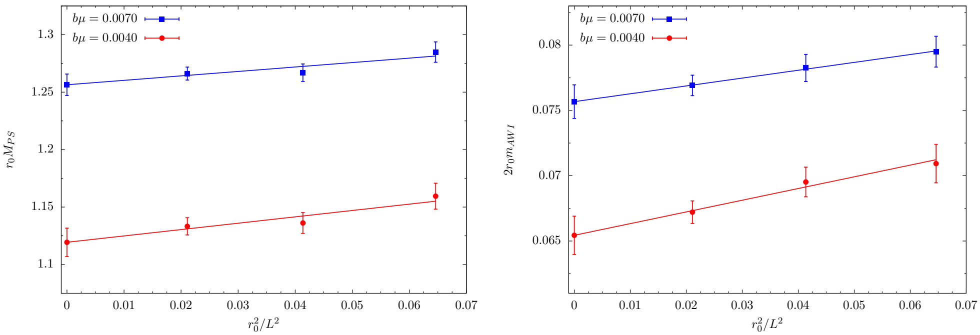


■ η_{cr} from region A ● η_{cr} from region B

Lattice study 7/8

NG phase finite volume effects

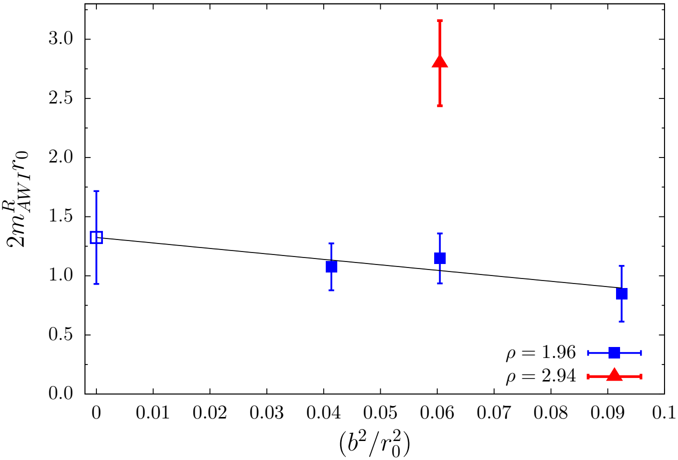
- M_{PS} always ≥ 4.7
- But: NG phase has massless elementary NG bosons (as $L \to \infty$)



- Here at $\beta = 5.85$, no way for effect to disappear in $L \to \infty$ limit

Lattice study 8/8

NG phase ρ-dependence



- Expect $m^R_{\mathrm{AWI}} \propto \rho^2$ from theory
- $\rho_1 = 1.96\,, \rho_2 = 2.94$
- Expect $\frac{m^R_{\mathrm{AWI}}(\rho_2)}{m^R_{\mathrm{AWI}}(\rho_2)} \sim 2.25$
- find ~ 2.4

- Note: in Wigner phase find change in η_{cr} linear in ρ, also as expected

Outlook: Construction of a realistic starting point for BSM physics

- All masses are parametrically $\propto \mathcal{O}(\Lambda_{\rm RGI})$
  - Implies $\Lambda_{\rm RGI} = \Lambda_T \gg \Lambda_s$ to explain m_t and $M_{[Z,W]}$
  - New gauge interaction that gets strong at scale Λ_T
  - "Terafermions" $Q \in (3_T, 3_s)$ and fermions $q \in (1_T, 3_s)$
  - Auxiliary scalar field coupled to both $\leftrightarrow$ "remnant" of complicated interactions at UV scale

$$
\begin{aligned}
\Gamma_4^{\rm NG} =& \frac{1}{4}(F\cdot F)^{G,A,W} + \bar{Q}\not{D}Q + \bar{q}\not{D}q + \frac{1}{2}\,{\rm Tr}\left[\partial_\mu\Phi^\dagger\partial_\mu\Phi\right] + \frac{\hat{\mu}_\Phi}{2}\,{\rm Tr}\left[\Phi^\dagger\Phi\right] + \frac{\hat{\rho}}{4}\left({\rm Tr}\left[\Phi^\dagger\Phi\right]\right)^2 \\
& + (\eta - \overline{\eta}(\eta,\rho,\ldots))\left[\bar{Q}_L\Phi Q_R + \bar{q}_L\Phi q_R + {\rm H.c.}\right] \\
& + \Lambda_T\left(C_{1,Q}\bar{Q}_L U Q_R + C_{1,q}\bar{q}_L U q_R + {\rm H.c.}\right) \\
& + \left(C_2\Lambda_T^2 + \tilde{C}\Lambda_T R\right)\frac{1}{2}\,{\rm Tr}\left(\mathcal{D}_\mu^W U^\dagger \mathcal{D}_\mu^W U\right)
\end{aligned}
$$

- Criticality: $\eta \to \eta_{\rm cr}$, $\rho \to \rho_{\rm cr}$, no $\mathcal{O}(v_\Phi)$ mass terms, ζ_0 decouples
- ζ_i provide longtitudinal d.o.f. for massive weak gauge bosons
- Ignoring many details: $m_Q^{\rm eff} \propto \alpha_T^2\rho_{\rm cr}^2\Lambda_T$ $\quad m_q^{\rm eff} \propto \alpha_s^2\rho_{\rm cr}^2\Lambda_T$ $\quad M_W^{\rm eff} \propto g_W\rho_{\rm cr}^2\Lambda_T$

Outlook: testable predictions

It's nice to be able to do the most important things first

weak gauge boson to T-meson mass ratio

- Expect $M_W^{\text{eff}}/M_{\text{T-meson}} \sim g_W \sqrt{C_2} \sim 10^{-2}$
- Mass ratio amenable to lattice calculation with controlled $\mathcal{O}(20\%)$ errors
- M_W^{eff} shift from double pole of $\sum_y e^{i(p\cdot y)} \langle J^{\text{weak}}_{\mu,Q}(y) J^{\text{weak}}_{\nu,Q}(0)\rangle$ at $g_W = g_Y = 0$

eventually perhaps: Higgs as a bound state in $WW \to WW$ channel

- T-strong force $\Rightarrow$ scalar T-meson exchange in t-channel
  - WW attractive and strong over distance $\sim \Lambda_T^{-1} \lesssim M_W^{-1}$
- May be possible to determine $WW - WW$ coupling in quenched lattice calculation

- Complete model complicated and not yet unique. However: many experimental constraints $\to$ predictivity?