Status report on the $\rho \to \pi\pi$ scattering at the physical point

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Motivation

- The first part of the talk is based on Werner et al.(arXiv:1907.01237)
- $\pi\pi$ $I = 1$ $\ell = 1$ channel
- Experiments tells us that a single vector meson resonance dominatas the scattering amplitude
- We have an incoming $\pi\pi$ state that transforms to ρ and then decays to $\pi\pi$
- The parameters of the ρ are encoded in the difference in the phase of the in and outgoing wave packet.
- Determine the phase shift directly is not possible from lattice simulations

Scattering on the lattice

- Interaction energy between two hadrons in finite volume with periodic boundary conditions Lüscher(1986)

Introduction

Recent LQCD studies

- Luescher method X.Feng, K. Jansen and D.B. Renner (2011)
- Anisotropic lattices, distillation, more *CM* momenta Dudek et al. (2013)
- Coupled channel analysis Wilson et al.(2015)
- Staggered fermions Z. Fu, L. Wang (2016)
- Elongated boxes Guo, A . Alexandru, R, Molina and M, Döring (2016)
- Different fit models C. Alexandrou et al.(2017)
- *CLS* ensembles M_π downto 200 MeV C.Andersen, J. Bulava, B. Hörz, C. Morningstar.(2019)

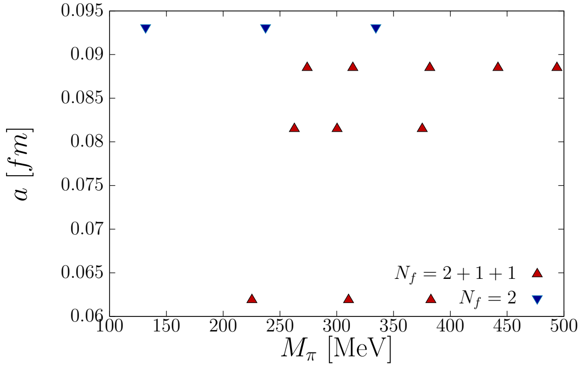
Account for all systematics

- Finite volume (exponential finite volume dependence)
- Finite lattice spacing
- Chiral extrapolations (simulations at the physical point)

Simulation details

Dirac operator

- Light sector: $D_\ell^{tm,clover} = D_W + i\mu_\ell\gamma_5\tau^3 - \frac{1}{4}c_{SW}\sigma_{\mu\nu}\mathscr{F}^{\mu\nu}$



- Stochastic distillation methods Morningstar et al. (2011)

The Lüscher's finite volume method

Two particle quantization condition

$$\det\left(\mathcal{M}_{\ell m,\ell' m'}(k) - \delta_{\ell,\ell'}\delta_{mm'}\cot\left(\delta_\ell(k)\right)\right) = 0,$$

where

- ℓ, m are indices of the irreducible representations of continuous rotation group $SU(2)$
- k scattering momenta

$$k^2 = \frac{E_{\pi\pi,CM}}{4} - M_\pi^2$$

- δ_ℓ phase shift for the ℓ-th partial wave
- The determinant is taken in angular momentum space

Reduction of the Lüscher matrix

- Finite volume $\mathscr{M}$ is not diagonal in ℓ
- To partially diagonalize the Lüscher matrix we do a "subduction"

Subduction

- Contribution of (infinite volume) SU(2) irrep states $|\ell, m\rangle$ to a certain lattice irrep $|\Gamma, \alpha, n\rangle$
  - Γ irrep
  - α row of the irrep
  - n multiplicity of the irrep
- $|\Gamma, \alpha, n\rangle = c_{LM}^{\Gamma,\alpha,n} |\ell, m\rangle$

Reduction of the Lüscher matrix

- The Lüscher matrix can be expressed in this new basis

$$M\left(\ell,\ell'\right)^{\Gamma,\alpha;\Gamma',\alpha'} = \langle \Gamma,\alpha,n|M|\Gamma',\alpha',n'\rangle$$
$$= \mathscr{M}_{\ell m,\ell' m'}\left(k\right) c^{\Gamma,\alpha\star}_{\ell,m} c^{\Gamma',\alpha'}_{\ell,m}$$

- Γ,α remain good quantum numbers in finite volume
- The quantization condition becomes

$$\det\left(\mathscr{M}\left(\ell,\ell'\right)^{\Gamma,\alpha} - \delta_{\ell,\ell'}\cot\left(\delta_\ell\right)\right) = 0$$

- In practice we consider several total momentum for each ensemble ($\vec{p}_{cm} = \frac{2\pi}{L}\vec{d}$
- To obtain the CM energy we perform a boost to the CM

$$E_{CM} = \frac{1}{\gamma}E_{lat} = \sqrt{E_{lat}^2 - \vec{p}_{CM}^2}$$

Extraction of energy levels

- We construct a gevp using the following interpolating operators

$$\mathcal{O}_{\pi^+} = \bar{d} i\gamma_5 u(x)$$
$$\mathcal{O}_{\pi^-} = \bar{u} i\gamma_5 d(x)$$

$$\mathcal{O}_{\pi\pi}(t,\vec{x}_1,\vec{x}_2) = \frac{1}{2}\left(\mathcal{O}_{\pi^+}(t,\vec{x}_1)\mathcal{O}_{\pi^-}(t,\vec{x}_2) - \mathcal{O}_{\pi^-}(t,\vec{x}_1)\mathcal{O}_{\pi^+}(t,\vec{x}_2)\right)$$

$$\mathcal{O}_{\rho}(x) = \frac{1}{\sqrt{2}}\left(\bar{u}(x)\Gamma^{\rho} u(x) - \bar{d}\Gamma^{\rho} d(x)\right),$$

where $\Gamma^{\rho} \in \{i\gamma_i, \gamma_0\gamma_i\}$.

- To compute the correlation function we project to
  - definite total momentum
  - an irreducible representation Γ
  - to a basis vector α

Thermal states

- around the world effect: reflextion at the temporal boundary
- finite lattice time extent
- Functional form can be predicted: Spectral decomposition

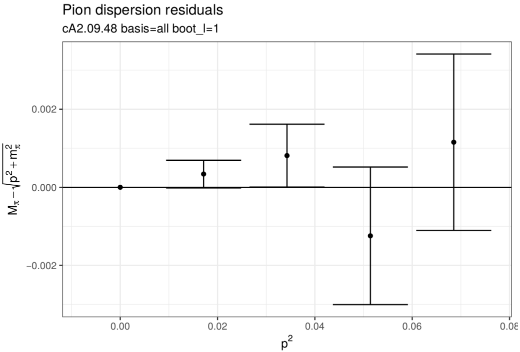
$$e^{-E_\pi(\vec{p}_1)T}e^{-(E_\pi(\vec{p}_2)-E_\pi(\vec{p}_1))t}+e^{-E_\pi(\vec{p}_2)T}e^{-(E_\pi(\vec{p}_1)-E_\pi(\vec{p}_2))t}$$

- $E_\pi\left(\vec{p}_1\right), E_\pi\left(\vec{p}_2\right)$ can be determined from the two point functions

Disperpion relation for the pion

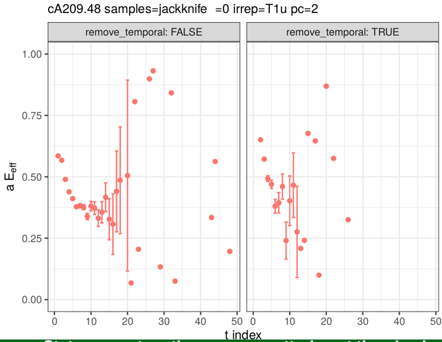
- To obtain a better estimate for $E_\pi(\vec{p})$ we use the continuum dispersion relation

$$E_\pi(\vec{p})^2 = m_\pi^2 + \vec{p}^2$$



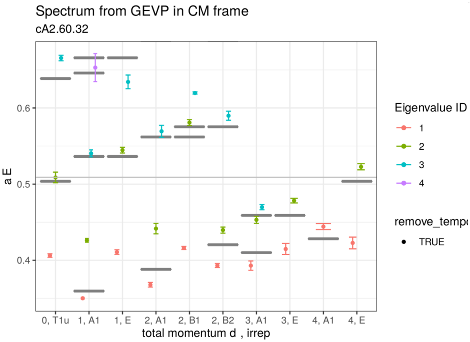
$\pi\pi$ correlators with and without thermal states removal

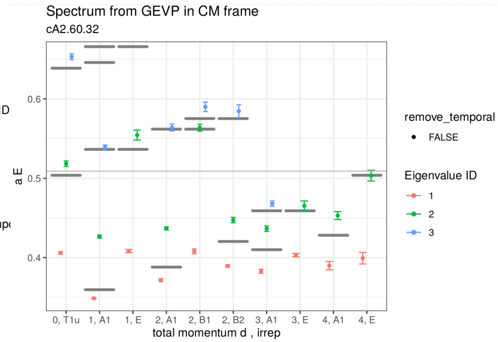
- To remove the thermal states we the weighting and shifting technique
- The correlation between the time-slice reduced drastically
- Hard to identify plato in the effective mass
- Bayesian fit methods could help



aE_{CM} for $N_f = 2, M_\pi = 334\mathrm{MeV}$

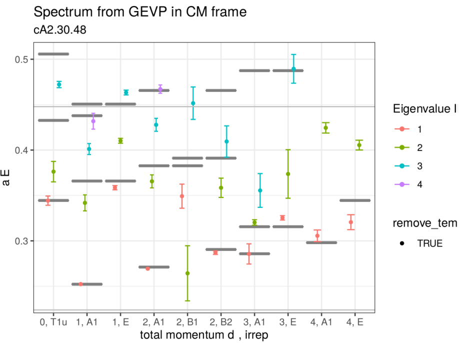
The center of mass frame energy spectrum for all irreps(Γ) and boost ($\vec{d}$)

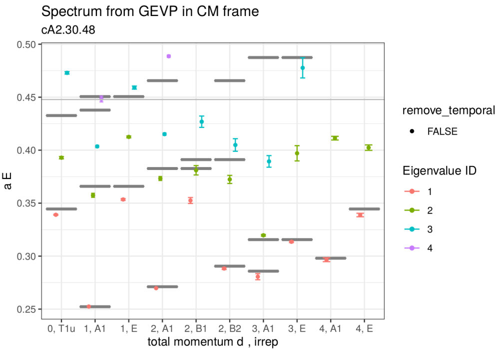




aE_{CM} for $N_f = 2, M_\pi = 235\mathrm{MeV}$

The center of mass frame energy spectrum for all irreps(Γ) and boost ($\vec{d}$)





aE_{CM} for $N_f = 2, M_\pi = 132\mathrm{MeV}$

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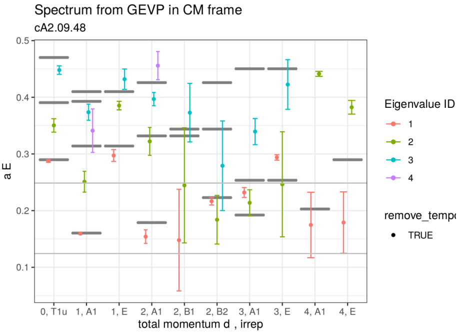
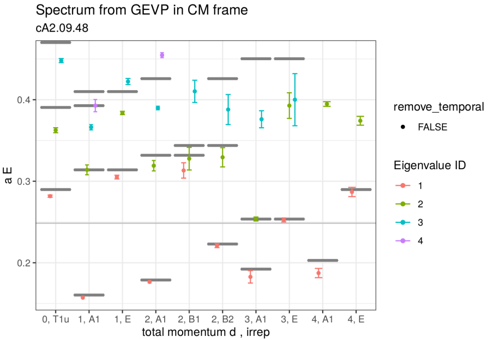
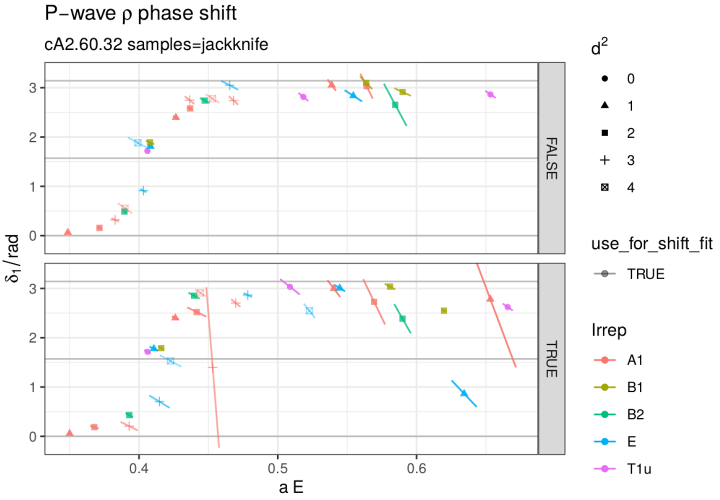
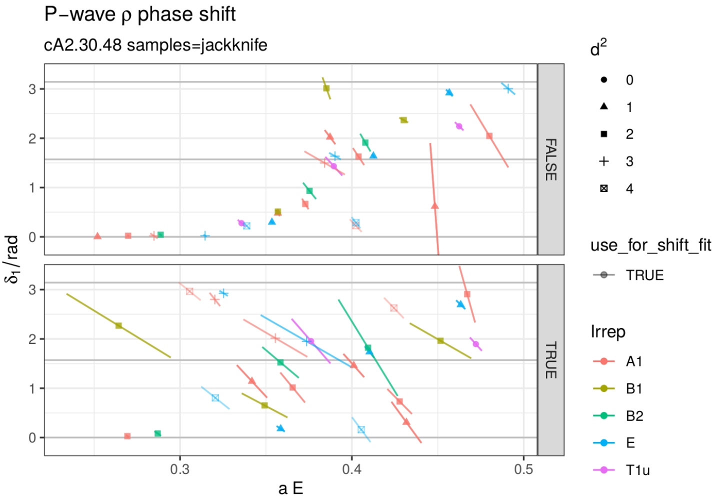




Phaseshift curve for $N_f = 2, M_\pi = 334\mathrm{MeV}$



Phaseshift curve for $N_f = 2, M_\pi = 235\text{MeV}$



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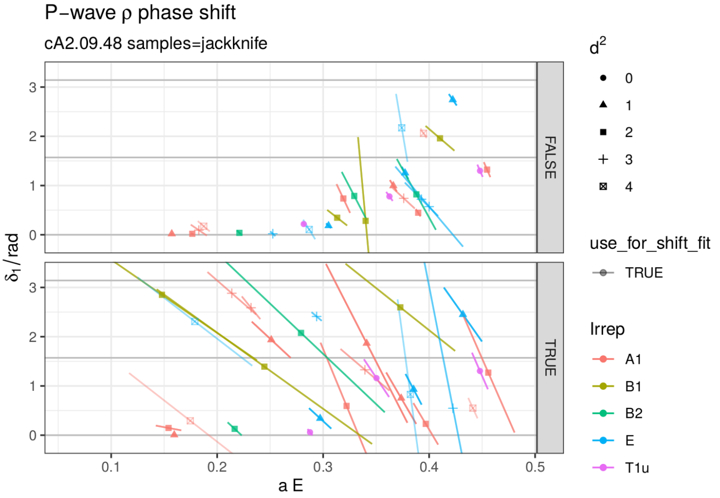

Outlook, Conclusion

- We determined the phase shift curve for the physical point ensemble
- 4π threshold is very low, very few points in the interesting region
- The signal with thermal state removal gets worse as the pion mass is lowered
- The Breit-Wigner analysis for extracting M_ρ, Γ_ρ and its chiral extrapolation is on-going