TMDPDFs in twisted mass lattice QCD

Aniket Sen

HISKP, University of Bonn

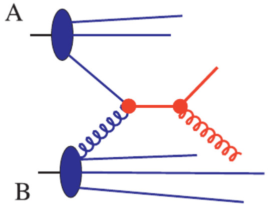
in collaboration with
C. Alexandrou, S. Bacchio, K.Cichy, M. Constantinou, G. Spanoudes, F. Steffens and J. Tarello

[Alexandrou et al., Phys. Rev. D 108, 114503 (2023)]

HISKP Theory Seminar
15.01.2024

Parton Distribution Functions (PDFs)

PDFs give the distribution of fraction (x) of hadron momentum carried by the constituent partons (quarks and gluons).



[Soper, Nucl.Phys.Proc.Suppl. 53, 69-80 (1997)]

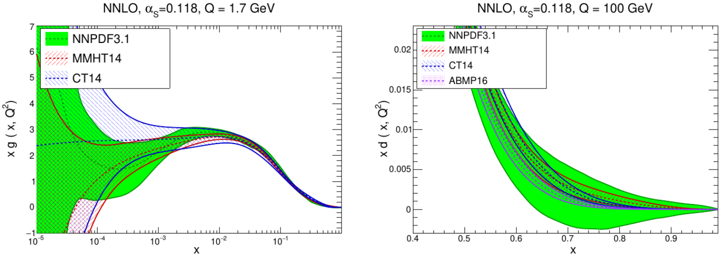
$$\frac{d\sigma}{dP_T} \sim \sum_{a,b} \int dx_A q_{a/A}(x_A,\mu) \int dx_B q_{a/B}(x_B,\mu) \frac{d\hat{\sigma}}{dP_T}$$

The PDF can be derived from the OPE

$$q(x) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} d\xi^- e^{-ixP^+\xi^-} \langle N| \bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0) |N\rangle$$

PDFs from global QCD analysis

Historically, theoretical calculation of PDFs have been limited to global fits to experimental scatttering data aided by phenomenologically motivated ansätze.

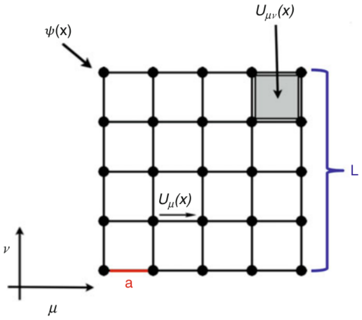


[Gao et al., Phys.Rept. 742, 1 (2018)]

Due to the non-perturbative nature of PDFs, lattice QCD is an ideal method for direct calculation.

Lattice QCD

- Continuum space time is discretized into a hypercubic Euclidean lattice with lattice spacing *a*.
- Quarks, ψ, are Grassmann variables associated with the lattice sites.
- The gluon fields are represented by SU(3) matrices located at the links connecting 2 lattice sites.



$$U_\mu(x) = e^{iaT^a A^a_\mu(x)}$$

PDFs from Euclidean lattice

Calculation of light-cone/time-separated correlation functions is not possible on the lattice.

Solution: Large Momentum Effective Theory (LaMET)

[Ji, Phys. Rev. Lett. 110, 262002 (2013); Ji, Zhang and Zhao, Phys. Rev. Lett. 111, 112002 (2013)]

- Quark distribution can be recovered by boosting an equal-time correlator to a large momentum.
- Defined the basis for the quasi-PDF approach.

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{P_z}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_z\right) + O\left(\frac{m_N^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2}\right)$$

Quasi-PDF

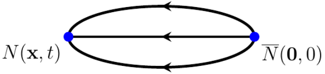
Quasi-PDFs can be calculated on the lattice,

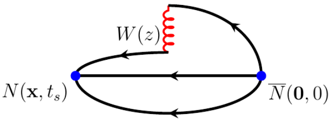
$$\tilde{q}(x, P_z) = \int_{-z_{max}}^{+z_{max}} \frac{dz}{4\pi} e^{-ixP_z z} \langle N | \bar{\psi}(0,z)\Gamma W(z,0)\psi(0,0) | N \rangle .$$

The hadronic matrix element can be obtained from the ratio of a 3-point and a 2-point function.

$$C^{2pt}(\vec{P}, t, 0) = \mathcal{P}_{\alpha,\beta} \sum_{\vec{x}} e^{-i\vec{P}\cdot\vec{x}} \langle 0 | N_\alpha(\vec{x}, t) \bar{N}_\beta(\vec{0}, 0) | 0 \rangle ,$$

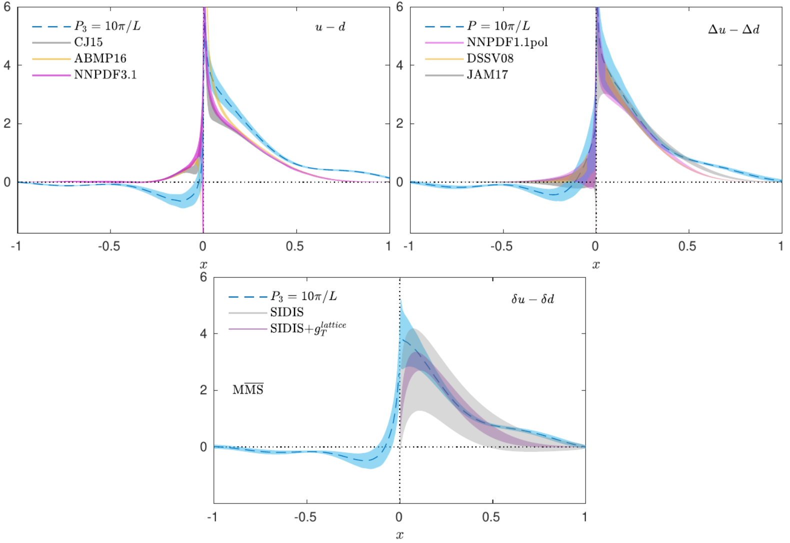
$$C^{3pt}(\vec{P}, t_s, \tau, 0) = \tilde{\mathcal{P}}_{\alpha,\beta} \sum_{\vec{x},\vec{y}} e^{-i\vec{P}\cdot\vec{x}} \langle 0 | N_\alpha(\vec{x}, t_s) \mathcal{O}(\vec{y}, \tau, z) \bar{N}_\beta(\vec{0}, 0) | 0 \rangle ,$$





PDFs from lattice QCD

ab initio calculation of PDFs in lattice QCD has been very successful over the last decade.



[Alexandrou et al., Phys. Rev. D 99, 114504 (2019)]

TMDPDFs

Transverse momentum dependent PDFs (TMDPDFs) give the distribution of parton momenta in the transverse plane.

- To understand the 3-dimensional structure of proton, we need to measure and compute TMDPDFs and GPDs (generalized parton distributions).

- Global fits for TMDPDFs lack in accuracy.

- With the upcoming Electron-Ion-Collider (EiC), the calculation of TMDPDFs from first principle, is of great importance.

TMDPDFs from LaMET

Using LaMET, the TMDPDF can be written as

$$f^{TMD}(x, b, \mu, \zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln\left(\frac{\zeta_z}{\zeta}\right) K(b,\mu)} \tilde{f}(x, b, \mu, \zeta_z)\, S_r^{\frac{1}{2}}(b, \mu) + \dots$$

- $\tilde{f}(x, b, \mu, \zeta_z)$ is the quasi-TMDPDF.
- $S_r(b, \mu)$ is the reduced soft function.
- $\zeta_z = (2xP^z)^2$ is the Collins-Soper scale of the quasi-TMDPDF.
- $H\left(\frac{\zeta_z}{\mu^2}\right)$ is the perturbative matching kernel.
- $K(b, \mu)$ is the Collins-Soper kernel.

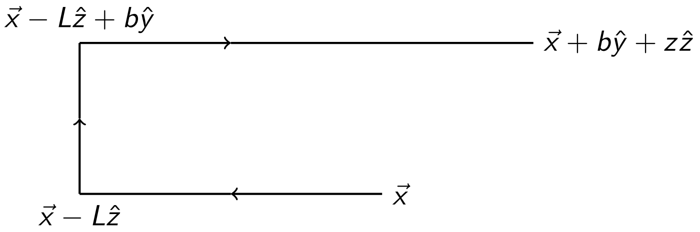
Quasi-TMDPDF

$$\tilde{f}(x, b, \mu, \zeta_z) = \lim_{L\to\infty} \int \frac{P^z\, dz}{2\pi} e^{-ix(zP^z)} B(z, b, L, P^z, \mu).$$

With the quasi-beam function

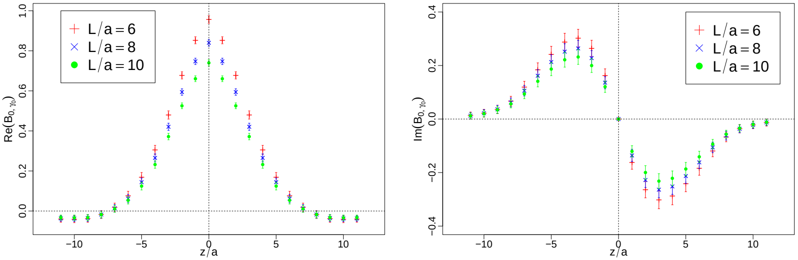
$$\begin{aligned} B_{0,\Gamma}(z, b, L, P^z) =& \langle N(P^z)|\mathcal{O}^\Gamma(z, b, L)|N(P^z)\rangle \\ =& \langle N(P^z)|\bar{\psi}(b+z)\Gamma W(b+z; L)\psi(0)|N(P^z)\rangle. \end{aligned}$$

Here W is an asymmetric staple shaped Wilson link.



Bare quasi-beam function

$24^3 \times 48$, $N_f = 2+1+1$ twisted mass lattice at a pion mass of 350 MeV. $P^z \sim 1.7$ GeV.



Results symmetrized using

$$B_{0,\Gamma}(z, b, L, P^z) = B^{\dagger}_{0,\Gamma}(-z, -b, -L, P^z)$$

Renormalization

The staple-shaped gauge link has three types of divergences.

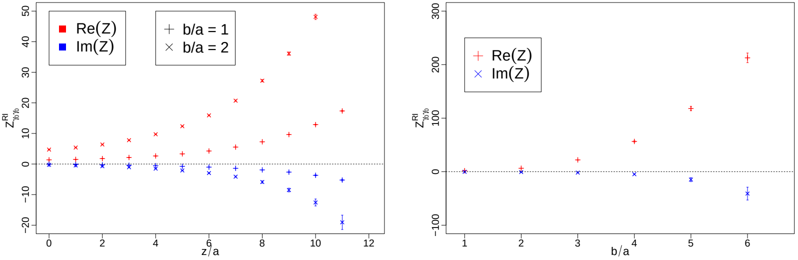
I Linear divergence coming from the Wilson line, which connects the quark fields and which depends on the length of the staple-shaped link.

II Logarithmic divergences coming from the endpoints of the staple link.

III Logarithmic divergences coming from the presence of cusps in the staple.

RI/MOM

$$\frac{Z^{\mathrm{RI}}_{\Gamma\Gamma'}(z,b,L,\mu_0;1/a)}{Z^{\mathrm{RI}}_q(\mu_0;1/a)}\frac{1}{12}\mathrm{Tr}\left[\frac{\Lambda^{\Gamma}_0(z,b,L,p;1/a)\Gamma'}{e^{ip^z z+ip_\perp b}}\right]\Bigg|_{p^2=\mu_0^2}=1.$$



RI/MOM introduces unwanted non-perturbative effects at large z and large b regions. The renormalization becomes unreliable at large distances.

Alternative methods

- If the divergences associated with the staple-shaped Wilson line can be eliminated, then the only remaining divergence is the multiplicative UV divergence.

- This can then be eliminated using an appropriate multiplicative factor.

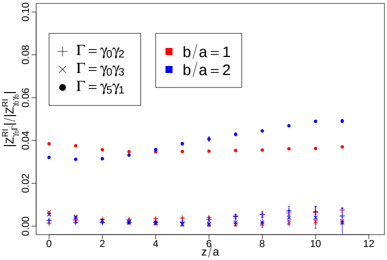
- Can only be performed when there is no mixing.

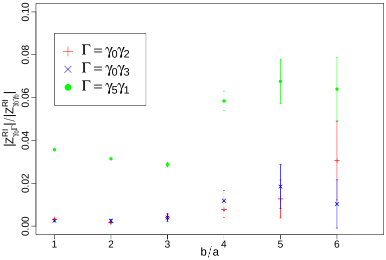
Study of mixing

Using symmetry arguments, we showed that any operator Γ is only allowed to mix with

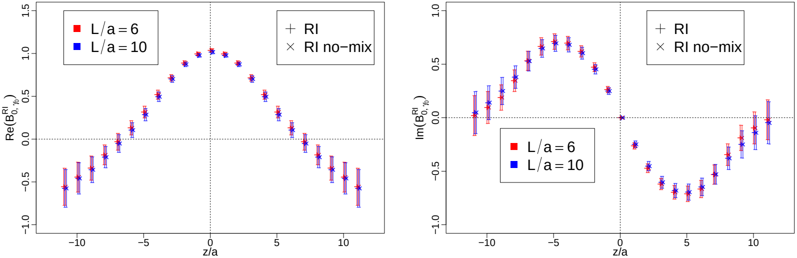
- $\Gamma\gamma_2$, $\Gamma\gamma_3$ and $\Gamma\gamma_2\gamma_3$.

In case of γ_0, we need to consider the mixing of the operators $\{\gamma_0, \gamma_0\gamma_2, \gamma_0\gamma_3, \gamma_5\gamma_1\}$.



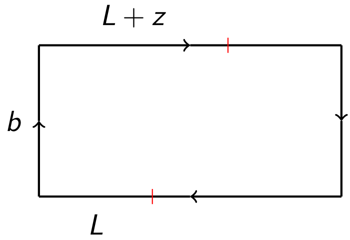


Mixing contribution



Mixing is negligible and can be ignored for the renormalization procedure.

Wilson loop subtraction

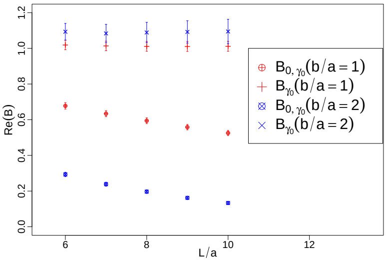


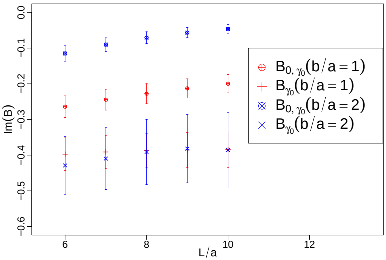
The Wilson loop, Z_E of sides $2L + z$ and b, is by construction the product of the staple-shaped Wilson line and its reflection.

$\Longrightarrow$ $\sqrt{Z_E}$ has the same divergences as that of the staple-shaped gauge link.

$$B_\Gamma(z, b, P^z; 1/a) = \lim_{L\to\infty} \frac{B_{0,\Gamma}(z, b, L, P^z; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}.$$

Effect of Wilson loop subtraction





This ratio takes care of the divergences associated with the length L and width b of the staple-shaped operator.

Multiplicative renormalization factor

Short distance ratio (SDR) [LPC, Phys. Rev. Lett. 129, 082002 (2022)]

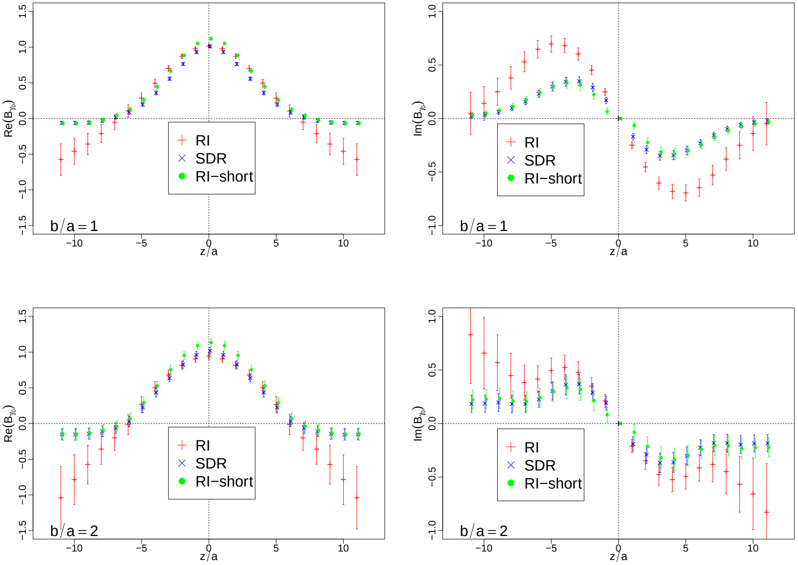
$$Z^{SDR}(z_0, b_0; 1/a) = \frac{1}{B_\Gamma(z = z_0, b = b_0, P^z = 0; 1/a)}.$$

Short distance RI/MOM (RI-short) [Ji et al., Phys. Rev. D 104, 094510 (2021)]

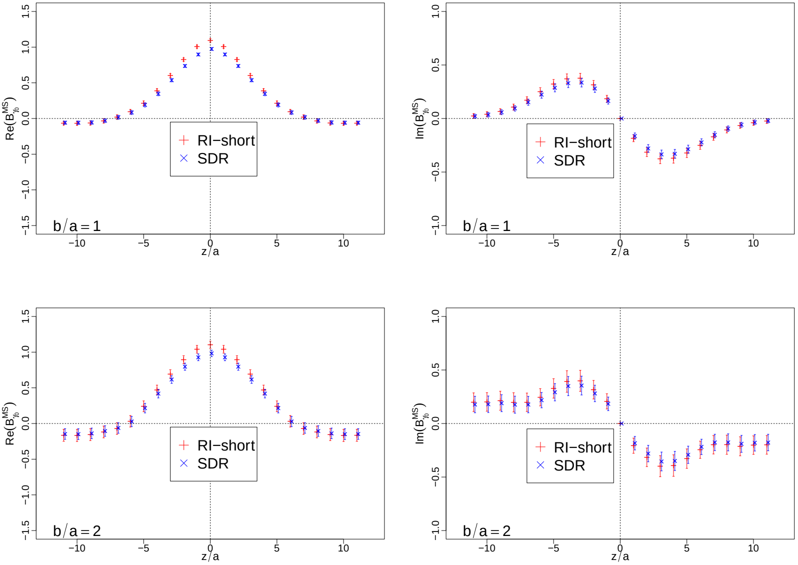
$$\Lambda^\Gamma(z, b, p; 1/a) = \frac{\Lambda_0^\Gamma(z, b, p; 1/a)}{\sqrt{Z_E(b, 2L + z; 1/a)}}.$$

$$\frac{Z_{\Gamma\Gamma'}^{\mathrm{RI-short}}(z_0, b_0, \mu_0; 1/a)}{Z_q^{\mathrm{RI}}(\mu_0; 1/a)} \frac{1}{12} \mathrm{Tr}\left[\frac{\Lambda^\Gamma(z, b, p; 1/a)\Gamma'}{e^{ip^z z + ibp_\perp}}\right]\Bigg|_{p^2=\mu_0^2, z=z_0, b=b_0} = 1.$$

Quasi-TMDPDF



Quasi-TMDPDF ($\overline{\text{MS}}$)



Soft function

$$S_r(b,\mu) = \frac{F(b,P^z,\mu)}{\int dx\, dx'\, H(x,x')\tilde{\psi}^\dagger(x',b)\tilde{\psi}(x,b)}.$$

$F(b,P^z,\mu)$ is the meson form factor.

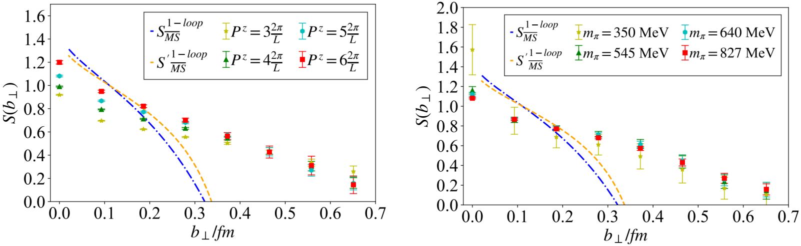
$$F_\Gamma(b,P^z) = \langle \pi(-P^z)|\bar{u}\Gamma u(b)\bar{d}\Gamma d(0)|\pi(P^z)\rangle.$$

$\tilde{\psi}(x,b)$ is the quasi-TMDWF.

$$\begin{aligned}\psi_{0,\Gamma}(z,b,L,P^z) =& \langle 0|\mathcal{O}^\Gamma(z,b,L)|\pi(P^z)\rangle \\ =& \langle 0|\bar{q}(b+z)\Gamma\mathcal{W}(b+z;L)q(0)|\pi(P^z)\rangle.\end{aligned}$$

$H(x,x')$ is a perturbative matching kernel.

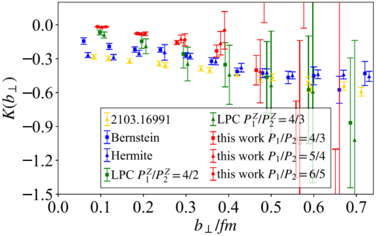
Soft function



[Li et al., Phys. Rev. Lett. 128, 062002 (2022)]

Collins-Soper kernel

The Collins-Soper kernel $K(b, \mu)$ governs the rapidity evolution of the TMDPDFs.



[Li et al., Phys. Rev. Lett. 128, 062002 (2022)]

$K(b, \mu)$ can be extracted by taking ratios of either quasi-TMDPDFs or quasi-TMDWFs at different values of P^z.

Outlook

- Computation of TMDPDFs is important for future accelerators that will study 3-D structure of protons.

- We have a better understanding of the renormalization procedure for the asymmetric staple-shaped Wilson line operator.

- We have a systematic setup for calculating all the different observables necessary for constructing the full TMDPDF.

- Computations at different lattices are ongoing for studying discretization effects and finite volume effects.

- Calculation of quasi-TMDPDF at the physical point is also underway.