Hexagons in the $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM and in the fishnet theory

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Works in progress with S. Komatsu, G. Lefundes, and D. Serban, and with E. Olivucci

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Motivation

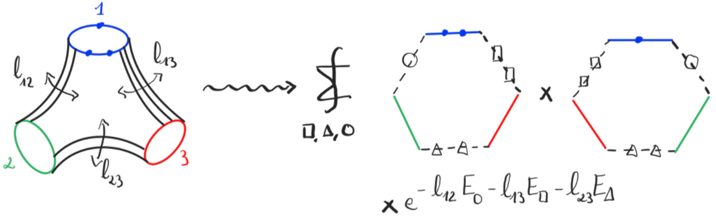
A non-perturbative solution of planar $\mathcal{N} = 4$ SYM should be reachable thanks to integrability. It also allows to explore an instance of the AdS/CFT correspondence.

This has been achieved for the spectrum of conformal dimensions (QSC).

For three- and higher-point correlation functions, there is still work to be done. Various approaches: hexagons, T-functions, separation of variables (SoV).

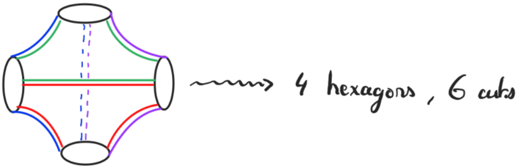
In this talk, we will test the hexagons in the $\mathbb{Z}_2$ orbifold and the fishnet theory.

Hexagonalisation



Hexagon form factors = building blocks for n-point correlators.
Gluing along a seam = sum over a complete basis of mirror magnons.
[Basso, Komatsu, and Vieira (2015)] [Fleury and Komatsu (2016-2017)]
[Eden and Sfondrini (2016)]

The hexagon expansion is the analogue of the Lüscher expansion for the spectrum.



The Octagon

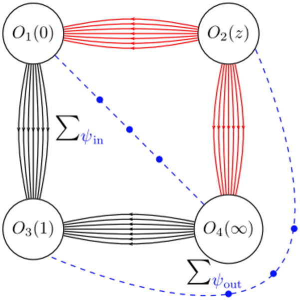
Take $\mathcal{O}_1 = \mathrm{Tr}\big(Z^K (X^\dagger)^K\big) + \ldots$, $\mathcal{O}_2 = \mathrm{Tr}\big((Z^\dagger)^{2K}\big)$, $\mathcal{O}_3 = \mathrm{Tr}\big(X^{2K}\big)$.
Then, when $K \to +\infty$, one has

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_1(x_4)\rangle \sim \frac{\mathbb{O}_0^2(z,\bar{z})}{(x_{12}^2 x_{24}^2 x_{13}^2 x_{34}^2)^K}\,.$$

Conformal ross-ratios:

$$z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad \frac{z}{\bar{z}} = e^{2\,\mathrm{i}\,\phi}\,,$$

$$(1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\,.$$



[Coronado (2018)]

Generalisation with arbitrary bridge length ℓ and R-symmetry polarisation vectors: [Coronado (2018)]

$$\mathbb{O}_\ell(z,\bar{z},\alpha,\bar{\alpha}) = 1 + \sum_{n=1}^{+\infty} \frac{\lambda_+^n + \lambda_-^n}{2\, n!} \sum_{a_1,\ldots,a_n=1}^{+\infty} \prod_{k=1}^{n} \frac{\sin(a_k\phi)}{\sin(\phi)}$$

$$\times \int \prod_{i<j} H_{a_i,a_j}(u_i,u_j) \prod_{k=1}^{n} (z\bar{z})^{-\mathrm{i}\, p_{a_k}(u_k)}\, \mathrm{e}^{-\ell E_{a_k}(u_k)}\, \mu_{a_k}(u_k) \frac{\mathrm{d}u_k}{2\pi},$$

where

$$p_a = \frac{g}{2}\left(x^{[+a]} + x^{[-a]} - \frac{1}{x^{[+a]}} - \frac{1}{x^{[-a]}}\right), \quad \mathrm{e}^{E_a} = x^{[+a]}x^{[-a]},$$

$$\mu_a = \frac{\mathrm{i}\left(x^{[+a]} - x^{[-a]}\right) x^{[+a]}x^{[-a]}}{g\left((x^{[+a]})^2 - 1\right)\left((x^{[-a]})^2 - 1\right)\left(1 - x^{[+a]}x^{[-a]}\right)},$$

$$H_{a,b}(u,v) = \prod_{\delta,\epsilon=\pm} \frac{x^{[\delta a]}(u) - x^{[\epsilon b]}(v)}{1 - x^{[\delta a]}(u)x^{[\epsilon b]}(v)},$$

$$x(u) + \frac{1}{x(u)} = \frac{u}{g}, \quad x^{[a]}(u) = x\left(u + \mathrm{i}\,\frac{a}{2}\right).$$

$$\mathbb{O}_\ell(z,\bar{z},\alpha,\bar{\alpha}) = \frac{\det(1-\lambda_+ K_{\ell+1}) + \det(1-\lambda_- K_{\ell+1})}{2},$$

where K_ℓ is a semi-infinite matrix: for $m, n \geqslant 0$,

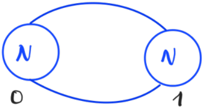
$$(K_\ell)_{mn} = \sqrt{(\ell+2m)(\ell+2n)} \int_0^{+\infty} \frac{J_{\ell+2m}(2gt) J_{\ell+2n}(2gt)}{\cos\phi - \operatorname{ch}\sqrt{t^2+z\bar{z}}} \frac{\mathrm{d}t}{t}.$$

[Kostov, Petkova, and Serban (2019)] [Belitsky and Kortchemsky (2019)]

This object occurs in several other situations.

[Beisert, Eden, and Staudacher (2006)] [Basso, Dixon, and Papathanasiou (2020)]
[Sever, Tumanov, and Wilhelm (2020-2021)] [Basso and Tumanov (2024)]

$\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM



- $\mathcal{N} = 2$ SCFT with gauge group $SU(N)_0 \times SU(N)_1$, 2 vector multiplets and 2 bifundamental hypermultiplets.
- In $\mathcal{N} = 4$ language, fields are $2N \times 2N$ matrices that satisfy

$$[A_\mu, \tau] = [Z, \tau] = \{X, \tau\} = \{Y, \tau\} = 0\,,$$

where the $\mathbb{Z}_2$ twist is $\tau = \mathrm{Diag}(I_N, -I_N)$.

- At the orbifold point $g_0 = g_1$, theory expected to be integrable in the planar limit.
- The twist breaks $PSU(2,2|4)$ down to $SU(2,2|2) \times SU(2)$.

We focus on correlation functions of the following BPS operators:

$$U_\ell = \frac{\mathrm{Tr}\big(Z^\ell\big)}{\sqrt{2}} = \frac{\mathrm{Tr}\big(\phi_0^\ell\big) + \mathrm{Tr}\big(\phi_1^\ell\big)}{\sqrt{2}} \quad \text{(untwisted)}$$

$$T_\ell = \frac{\mathrm{Tr}\big(\tau Z^\ell\big)}{\sqrt{2}} = \frac{\mathrm{Tr}\big(\phi_0^\ell\big) - \mathrm{Tr}\big(\phi_1^\ell\big)}{\sqrt{2}} \quad \text{(twisted)}$$

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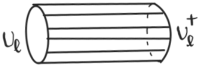


$$\langle U_\ell^\dagger(x) U_\ell(0)\rangle = \frac{\ell}{x^{2\ell}}\,, \qquad \langle T_\ell^\dagger(x) T_\ell(0)\rangle = \frac{C_\ell}{x^{2\ell}}\,,$$

where the normalisation is expressed in terms of the octagon for $z = \bar{z} = 1$ and $\alpha = \bar{\alpha} = -1$:

$$C_\ell = \ell\,\frac{\det(1 - 4K_{\ell+2})}{\det(1 - 4K_\ell)}\,.$$

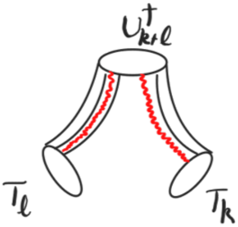
[Galvagno and Preti (2020)] [Billò, Frau, Galvagno, Lerda, and Pini (2021)]

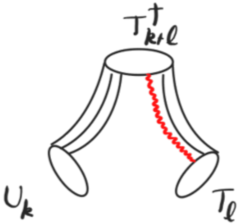
Three-point functions

$$\langle U_k(x) U_\ell(y) U^\dagger_{k+\ell}(z) \rangle = \frac{k\ell(k+\ell)}{\sqrt{2}N|x-z|^k|y-z|^\ell} \,,$$

$$\langle T_k(x) T_\ell(y) U^\dagger_{k+\ell}(z) \rangle = \frac{G_{k,\ell}}{|x-z|^k|y-z|^\ell} \,,$$

$$\langle U_k(x) T_\ell(y) T^\dagger_{k+\ell}(z) \rangle = \frac{G'_{k,\ell}}{|x-z|^k|y-z|^\ell} \,.$$





Normalised structure constants:

$$C_{k,\ell} = \frac{G_{k,\ell}}{\sqrt{(k+\ell)C_k C_\ell}} = \frac{\sqrt{k\ell(k+\ell)}}{\sqrt{2}N}\sqrt{1+\frac{g}{2k}\partial_g \ln C_k}\sqrt{1+\frac{g}{2\ell}\partial_g \ln C_\ell}\,,$$

$$C'_{k,\ell} = \frac{G'_{k,\ell}}{\sqrt{kC_\ell C_{k+\ell}}} = \frac{\sqrt{k\ell(k+\ell)}}{\sqrt{2}N}\sqrt{1+\frac{g}{2\ell}\partial_g \ln C_\ell}\sqrt{1+\frac{g}{2(k+\ell)}\partial_g \ln C_{k+\ell}}$$

[Billò, Frau, Lerda, Pini, and Vallarino (2022)]

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[Billò, Frau, Lerda, Pini, and Vallarino (2022)]

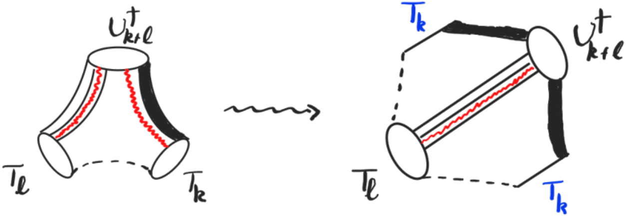
$$\sqrt{1+\frac{g}{2\ell}\partial_g \ln C_\ell} = \frac{\det(1-4K_{\ell+1})}{\sqrt{\det(1-4K_\ell)\det(1-4K_{\ell+2})}}$$

[Ferrando, Komatsu, Lefundes, and Serban (unpublished)] [Korchemsky (unpublished)]

How can we recover this result in the hexagon framework?

- Numerator = bridge magnons

$$\det(1-4K_{\ell+1}) = \mathbb{O}_\ell(z=\bar{z}=1, \alpha=\bar{\alpha}=-1)$$



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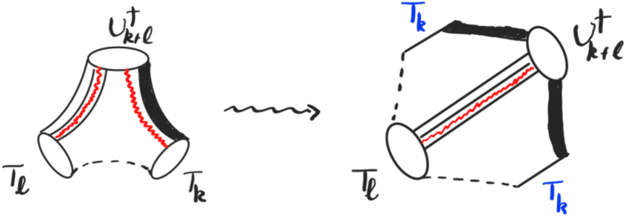
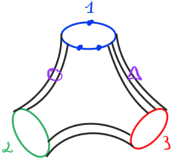
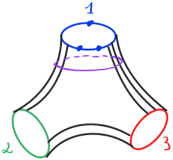


- Denominator = wrapping magnons?

One wrapping magnon





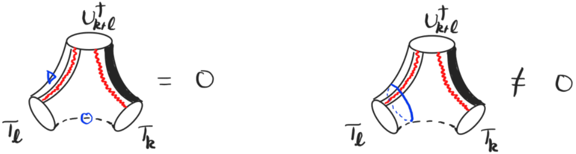
Naïve hexagon prediction:

$$\sum_{a,b=1}^{+\infty} \iint \frac{T_a(u)T_b(v)}{H_{a,b}(u,v)}\, \mathrm{e}^{-\ell_{\mathbf{12}} E_a(u) - \ell_{\mathbf{13}} E_b(v)}\, \mu_a(u)\mu_b(v) \left(\sum_{\text{partitions}} \cdots \right) \frac{\mathrm{d}u\,\mathrm{d}v}{(2\pi)^2}\,.$$

[Basso, Gonçalves, Komatsu, and Vieira (2015)]

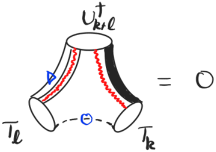
However $H_{a,b}(u,v) \sim \mu_a^2(u)(u-v)^2$ when $a=b$ and $v \to u$.

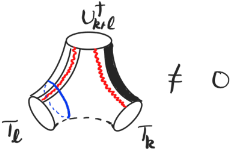
Regularisation prescription $=$ $H_{a,b}(u,v) \to H_{a,b}(u + \mathrm{i}\,\epsilon, v - \mathrm{i}\,\epsilon)$ and add some contact terms. [Basso, Gonçalves, and Komatsu (2017)]



$$T_a(u) = \mathrm{STr}_a(\mathcal{S}_{a1}(u^\gamma, u_1)\ldots\mathcal{S}_{a1}(u^\gamma, u_K))$$
$$\longrightarrow \mathrm{STr}_a(1) = 0 \quad \text{or} \quad \mathrm{STr}_a(\tau_a) = 4a\,,$$

where $\mathcal{S}_{ab}(u, v)$ is Beisert's $\mathfrak{su}(2|2)$-invariant S-matrix.





$$T_a(u) = \mathrm{STr}_a(\mathcal{S}_{a1}(u^\gamma, u_1)\dots\mathcal{S}_{a1}(u^\gamma, u_K)) \longrightarrow \mathrm{STr}_a(1) = 0 \quad \text{or} \quad \mathrm{STr}_a(\tau_a) = 4a\,,$$

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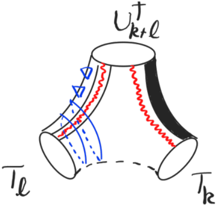
Consequence: only terms with wrapping magnons in the untwisted channel can be present. And only terms with derivatives of the S-matrix.



The one-wrapping-magnong term exactly reproduces the first contribution to the denominator:

$$\frac{1}{\sqrt{\det(1-4K_\ell)\det(1-4K_{\ell+2})}} = 1 + 2\,\mathrm{Tr}(K_\ell + K_{\ell+2}) + \dots .$$

We indeed observe that

$$2\,\mathrm{Tr}(K_\ell + K_{\ell+2}) = \sum_{a=1}^{+\infty}\int_{-\infty}^{+\infty} \mathcal{K}_{aa}(u,u)\,\mathrm{e}^{-\ell E_a(u)}\,\frac{\mathrm{d}u}{2\pi}\,,$$

for

$$\mathcal{K}_{ab}(u,v) = \mathrm{i}\,\mathrm{STr}_{a\otimes b}(\mathcal{S}_{ab}^{-1}\tau_b\partial_1\mathcal{S}_{ab})(u^\gamma, v^\gamma)\,.$$

Similarly, one can reproduce the factorised terms

$$2\,\mathrm{Tr}(K_\ell + K_{\ell+2}) \times \det(1-4K_{\ell+1})\,.$$

Several wrapping magnons

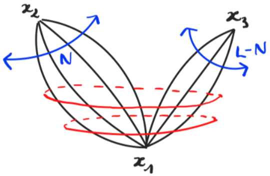
How can we systematically generate the wrapping corrections?

One needs to introduce a regulator. For instance, a cross-ratio in a four point-function: [Basso (IGST 2021)]

$$\langle U_k(x_4)\, T_\ell(x_1)\, T_m(x_2)\, U^\dagger_{k+\ell+m}(x_3)\rangle$$

$$\underset{x_4 \to x_1}{\sim} \frac{\langle U_k(0)\, T_\ell(1)\, T^\dagger_{k+\ell}(\infty)\rangle \langle T_{k+\ell}(0)\, T_m(1)\, U^\dagger_{k+\ell+m}(\infty)\rangle}{\langle T_{k+\ell}(0)\, T^\dagger_{k+\ell}(\infty)\rangle |x_2 - x_3|^m |x_1 - x_3|^{k+\ell}}$$

Example in the Fishnet Theory



$$\left\langle \mathrm{Tr}\left[Z^L(x_1)\right] \mathrm{Tr}\left[(Z^\dagger)^N(x_2)(Z^\dagger)^{L-N}(x_3)\right]\right\rangle = \frac{\sqrt{\mathcal{N}_{\mathrm{Tr}[Z^L]}}\, C_{L,N}\, |x_{23}|^{\gamma_L}}{|x_{12}|^{\gamma_L+2N}|x_{13}|^{\gamma_L+2L-2N}}\,,$$

where $\Delta_{\mathrm{Tr}(Z^L)} = L + \gamma_L$.

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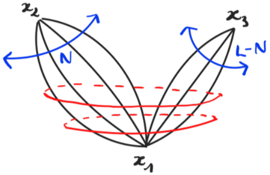

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where $\Delta_{\mathrm{Tr}(Z^L)} = L + \gamma_L$.The regularised version

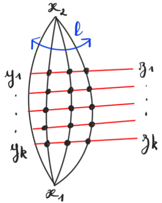
$$\mathcal{G}(\{x_i\}) = \left\langle \mathrm{Tr}\left[Z^M(x_4)Z^{L-M}(x_1)\right] \mathrm{Tr}\left[(Z^\dagger)^N(x_2)(Z^\dagger)^{L-N}(x_3)\right]\right\rangle$$

behaves as

$$\mathcal{G}(\{x_i\}) \underset{x_4 \to x_1}{\sim} \left(\frac{|x_{14}||x_{23}|}{|x_{12}||x_{13}|}\right)^{\gamma_L} \frac{C^*_{L,M}C_{L,N}}{|x_{12}|^{2N}|x_{13}|^{2L-2N}}\,.$$

$$x_{12}^{-2\ell} \sum_{\vec{a}} \int \langle \vec{y} | \vec{a}, \vec{s} \rangle_{x_1}^{x_2} \, {}_{x_1}^{x_2}\langle \vec{a}, \vec{s} | \vec{z} \rangle \, \mathrm{e}^{-\ell \sum_{i=1}^{k} E_{a_i}(s_i)} \, \mu_{\vec{a}}(\vec{s}) \mathrm{d}^k \vec{s},$$

$$\mathrm{e}^{E_a(s)} = s^2 + \frac{a^2}{4}$$



[Derkachov and Olivucci (2019-2020)]

$$x_{12}^{-2\ell} \sum_{\vec{a}} \int \langle \vec{y} | \vec{a}, \vec{s} \rangle^{x_2}_{x_1} \, {}^{x_2}_{x_1}\langle \vec{a}, \vec{s} | \vec{z} \rangle \, \mathrm{e}^{-\ell \sum_{i=1}^{k} E_{a_i}(s_i)} \, \mu_{\vec{a}}(\vec{s}) \mathrm{d}^k \vec{s} \,,$$

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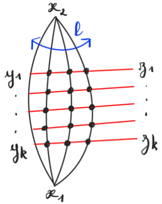

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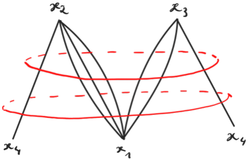

We only need to compute ${}^{x_2}_{x_4}\left\langle \vec{b}, \vec{t} \middle| \vec{a}, \vec{s} \right\rangle^{x_2}_{x_1}$.
Naïvely divergent when $a_i = b_j$ and $t_j \to s_i$.
But there is a natural regularisation:

$$s_i \to u_i + \mathrm{i}\,\delta_i \,, \quad t_k \to t_k - \mathrm{i}\,\epsilon_k \,,$$

with $0 < \epsilon_i, \delta_k \ll 1$.

$$\mathcal{G}_k = \sum_{\vec{a},\vec{b},\vec{c},\vec{d}} \int_{\mathcal{D}} \frac{|1-1/z|^{2\,\mathrm{i}\sum_{j=1}^{k}(t_j+v_j)}\,|z|^{2\,\mathrm{i}\sum_{j=1}^{k}(s_j+u_j)}}{h_{\vec{b},\vec{a}}(\vec{t},\vec{s})h_{\vec{b},\vec{c}}(\vec{t},\vec{u})h_{\vec{d},\vec{a}}(\vec{v},\vec{s})h_{\vec{d},\vec{c}}(\vec{v},\vec{u})}$$

$$\times \mathcal{T}_{\vec{a},\vec{b},\vec{c},\vec{d}}(\vec{s},\vec{t},\vec{u},\vec{v};z,\bar{z})\,\mathrm{e}^{-\sum_{i=1}^{k}(\ell_1 E_{a_i}(s_i)+\ell_2 E_{b_i}(t_i)+\ell_3 E_{c_i}(u_i)+\ell_4 E_{d_i}(v_i))}$$

$$\times \mu_{\vec{a}}(\vec{s})\mu_{\vec{b}}(\vec{t})\mu_{\vec{c}}(\vec{u})\mu_{\vec{d}}(\vec{v})\,\mathrm{d}^k\vec{s}\,\mathrm{d}^k\vec{t}\,\mathrm{d}^k\vec{u}\,\mathrm{d}^k\vec{v}\,,$$

where $\mathrm{Im}(s_i), \mathrm{Im}(u_i) > \mathrm{Im}(t_j), \mathrm{Im}(v_j)$ and

$$h_{\vec{b},\vec{a}}(\vec{t},\vec{s}) = \prod_{i,j=1}^{k}\left(\mathrm{i}(t_j - s_i) + \frac{|a_i - b_j|}{2}\right)\left(\mathrm{i}(s_i - t_j) + \frac{a_i + b_j}{2}\right).$$

If we only care about terms up to order $O(|z|^0)$, then we can replace

$$|1-1/z|^{2\,\mathrm{i}\sum_{j=1}^{k}(t_j+v_j)} \longrightarrow |z|^{-2\,\mathrm{i}\sum_{j=1}^{k}(t_j+v_j)}.$$

Relevant contributions come from residues at decoupling poles of the form $(a_i, s_i) = (b_j, t_j)$ or $(a_i, s_i) = (d_j, v_j)$, etc.

Recall that $1 + \sum_{k=1}^{+\infty} \xi^{2kL} \mathcal{G}_k \propto_{z,\bar{z}\to 0} |z|^{\gamma_L} C^*_{L,M} C_{L,N}$. Consistency check:

$$\mathcal{G}_1 = \gamma_{L,1} \ln |z| + B_{L,N} + B_{L,M} + 2A_L + o(1)\,,$$

where $\gamma_{L,1} = -2\zeta_{2L-3}\binom{2L-2}{L-1}$ and

$$B_{L,N} = \sum_{a,c=1}^{+\infty} a^2 c^2 \iint_{\substack{\mathrm{Im}(s)=\epsilon \\ \mathrm{Im}(u)=-\epsilon}} \frac{\mathrm{e}^{-NE_a(s)-(L-N)E_c(u)}}{h_{a,c}(s,u)h_{c,a}(u,s)} \frac{\mathrm{d}s\,\mathrm{d}u}{(2\pi)^2}\,,$$

$$A_L = -\sum_{a=1}^{+\infty} a^2 \int \left[\psi\left(\mathrm{i}\,s + \frac{a}{2}\right) + \psi\left(-\mathrm{i}\,s + \frac{a}{2}\right) - \psi(1) - \psi(2) + \frac{a/2}{s^2 + \frac{a^2}{4}}\right] \frac{\mathrm{d}s}{2\pi}\,.$$

[Basso, Caetano, and Fleury (2018)]

$$\mathcal{G}_2 = \frac{(\gamma_{L,1} \ln|z|)^2}{2} + [(B_{L,N} + B_{L,M} + 2A_L)\gamma_{L,1} + \gamma_{L,2}] \ln|z| \\ + (B_{L,M} + A_L)(B_{L,N} + A_L) + D_{L,M} + D_{L,N} + o(1)\,,$$

We have explicit (sum + integral) formulae for $\gamma_{L,2}$, i.e. second order Lüscher corrections, and for $D_{L,M}$.

From $D_{L,M}$, there is a natural guess for the 2-wrapping-magnon contact term in the $\mathbb{Z}_2$ orbifold. It reproduces the second order term of

$$\frac{1}{\sqrt{\det(1-4K_\ell)\det(1-4K_{\ell+2})}} = 1 + 2\,\mathrm{Tr}(K_\ell + K_{\ell+2}) \\ + 4\,\mathrm{Tr}\big(K_\ell^2 + K_{\ell+2}^2\big) + 2\,[\mathrm{Tr}(K_\ell + K_{\ell+2})]^2 + \ldots\,.$$

Future Directions

- Natural all order guess for the hexagons in the $\mathbb{Z}_2$ orbifold setting, can we show that it resums to the localisation result?
- Can we interpret the result in the language of T-functions? of Q-functions? [Basso, Georgoudis, and Klemenchuk Sueiro (2022)] [Bercini, Homrich, and Vieira (2022)]
- Higher-point correlation functions of BPS operators in the $\mathbb{Z}_2$ orbifold.
- In the fishnet theory, what additional information can we extract from the SoV representation of the four-point function? What about other correlation functions ?

Thank you!