Thermodynamic limits of fishnet graphs with various boundary conditions

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LPENS

Fishnets: Conformal Field Theories and Feynman Graphs

Bethe Center for Theoretical Physics, Bonn 2024

Based on works
with L.Dixon, G.Ferrando, V.Kazakov,
D.Kosower, A.Krajenbrink, D.I.Zhong

Introduction

Understand planar diagrams and their relation to sigma models

['t Hooft]
[Polyakov]
[Maldacena]

Main goal: N=4 SYM in 4d

1. String dual is (conjecturally) known
2. Theory is (conjecturally) integrable (exact methods at large N)

Huge progress at "bootstrapping" solution (ABA,TBA,QSC,HEX,SOV)

But hard to be rigorous and derive solution from 1st principles

Introduction

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[Zamolodchikov'80]
[Gurdogan,Kazakov'15]
[many more studies]

Simpler theories: fishnet theories

Large family of planar QFTs without SUSY
but with conformal symmetry and integrability

Simple "graphic" content: **fishnet graphs** with regular lattice structures

Interesting connections to conformal spin chains, sigma models,
deformations of SUSY theories

Aim to be "more" rigorous and draw lessons for more sophisticated theories

4d fishnet theory

Matrix scalar fields with quartic coupling [Gurdogan,Kazakov'15]

$$\mathcal{L}_{\text{fishnet}} = \text{tr}\ \partial_\mu \phi_1 \partial_\mu \phi_1^\dagger + \partial_\mu \phi_2 \partial_\mu \phi_2^\dagger - (4\pi g)^2 \phi_1 \phi_2 \phi_1^\dagger \phi_2^\dagger$$

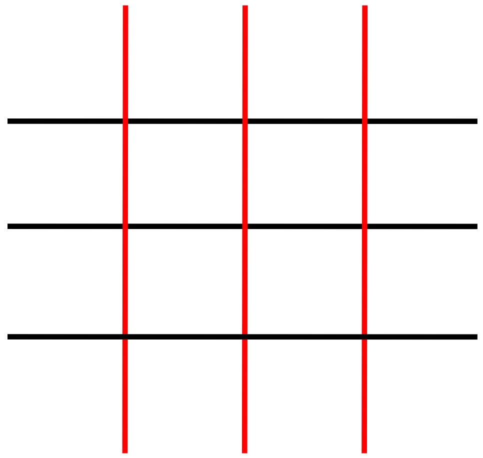
Related to N=4 SYM it by 'extreme' deformation (twist)

[Caetano,Gurdogan,Kazakov'16]

Planar graphs have a rigid structure

Edge: massless propagator in position space $\dfrac{1}{(x-y)^2}$

Vertex: integration point $\displaystyle\int \frac{d^4x}{\pi^2}$



bulk of fishnet graph

Different observables correspond to different BC for external legs [Zamolodchikov'80] [Isaev'03] [Gromov,Kazakov,Korchemsky,Negro,Sizov'17]

Integrability follows from quartic vertex in d=4 [Chicherin,Kazakov,Loebbert,Muller,Zhong'16]

Thermodynamic limit

Continuum limit and relation to string world-sheet?

- Hard to follow AdS/CFT all the way from N=4 theory to fishnet theory
- Deformation procedure sends the YM coupling to zero (highly curved AdS background for string)

Yet fishnet graphs have been known for a long to exhibit interesting behaviour in the thermodynamic limit

[Zamolodchikov'80]

Volume 97B, number 1 PHYSICS LETTERS 17 November 1980

"FISHING-NET" DIAGRAMS AS A COMPLETELY INTEGRABLE SYSTEM

A.B. ZAMOLODCHIKOV
The Academy of Sciences of the USSR, L.D. Landau Institute for Theoretical Physics, Chernogolovka, USSR

Received 29 July 1980

The "fishing-net" planar Feynman diagrams with massless scalar propagators are shown to be equivalent to some completely integrable lattice statistical system. The infinite-volume partition function for this system is computed exactly.

Thermodynamic limit

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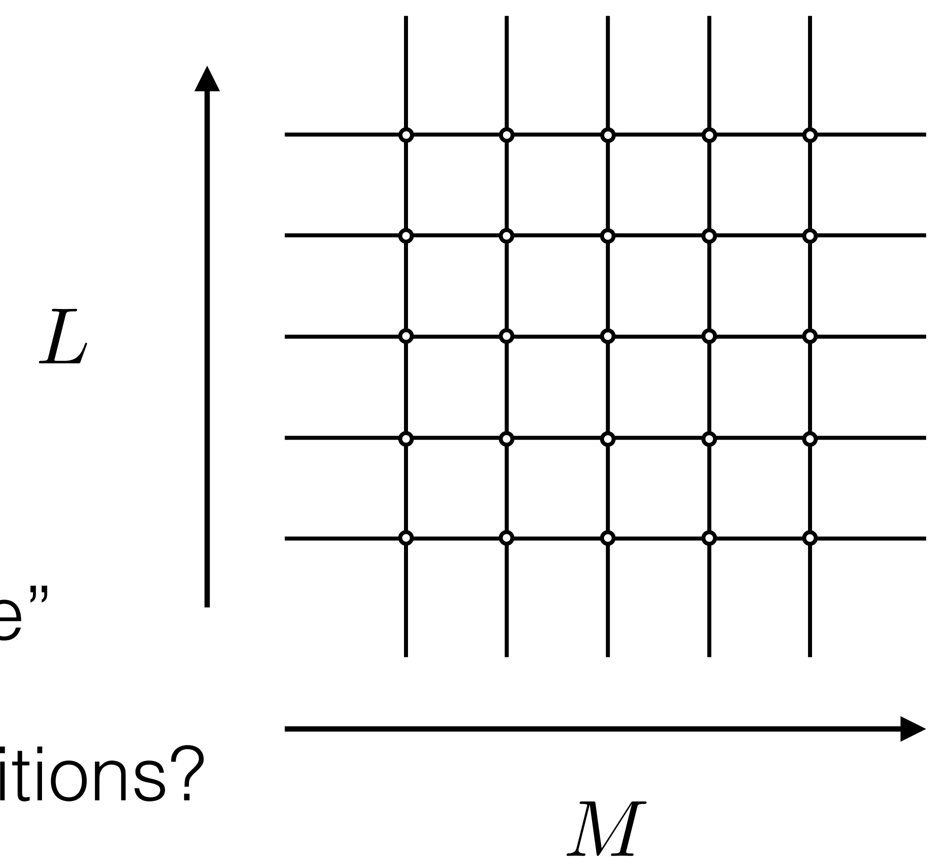
Yet fishnet graphs have been known for a long to exhibit interesting behaviour in the thermodynamic limit [Zamolodchikov'80]

Thermodynamic scaling $L, M \to \infty$

$$\ln Z_{L\times M} = -LM \ln g_c^2$$

$$g_c = \frac{\Gamma(3/4)}{\sqrt{\pi}\Gamma(5/4)} = 0.7...$$



Critical coupling: where graphs become "dense"

Q1: How does that depend on Boundary Conditions?

Q2: 2d Effective Field Theory of large fishnet graphs?

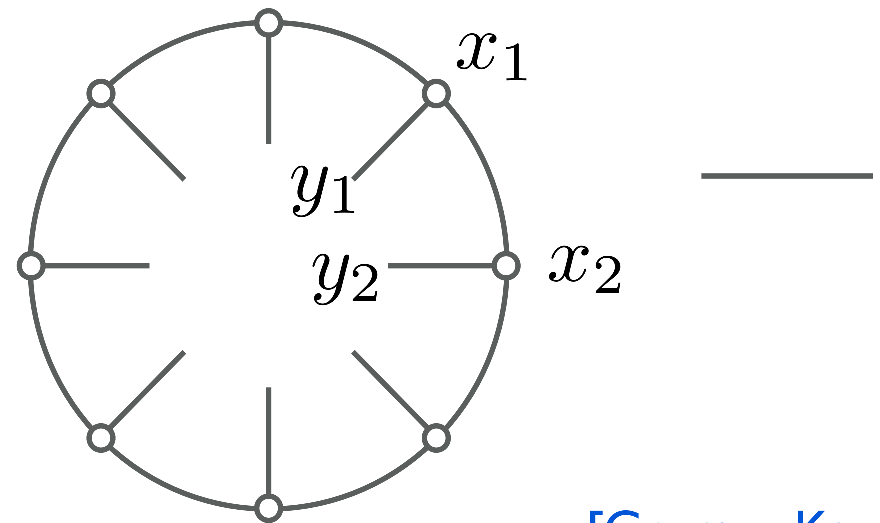
Periodic Fishnets

PBC

Closed-string channel

Graph building operator B_L

Integral operator acting on functions of L points subject to PBC



[Gromov,Kazakov,Korchemsky,Negro,Sizov'17]

$$B_L \cdot \psi(x_1, \ldots, x_L) = \int \prod_{i=1}^{L} \frac{d^4 y_i}{\pi^2 (x_i - x_{i+1})^2 (x_i - y_i)^2} \psi(y_1, \ldots, y_L)$$

with $x_{L+1} = x_1$

Commute with conformal symmetry generators $\in SO(1,5)$

Action on a free scalar field

Lorentz $M_{\mu\nu} = x_u \partial_\nu - x_\nu \partial_\mu$ Translation $P_\mu = \partial_\mu$

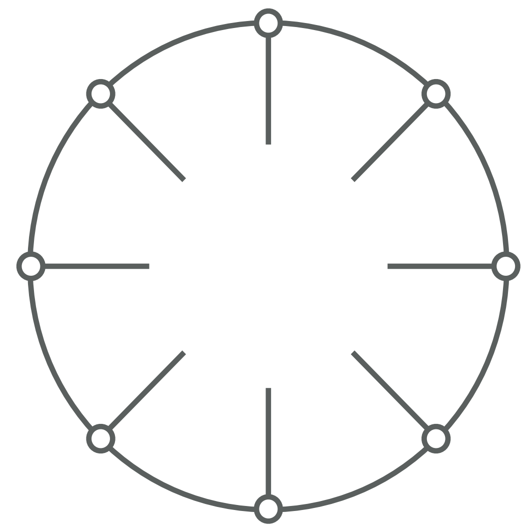
Dilatation $D = (x\partial) + 1$ Conformal boost $K_\mu = 2x_\mu (x\partial) - x^2 \partial_\mu + 2x_\mu$

PBC

Closed-string channel

Graph building operator B_L

Integral operator acting on functions of L points subject to PBC



[Gromov,Kazakov,Korchemsky,Negro,Sizov'17]

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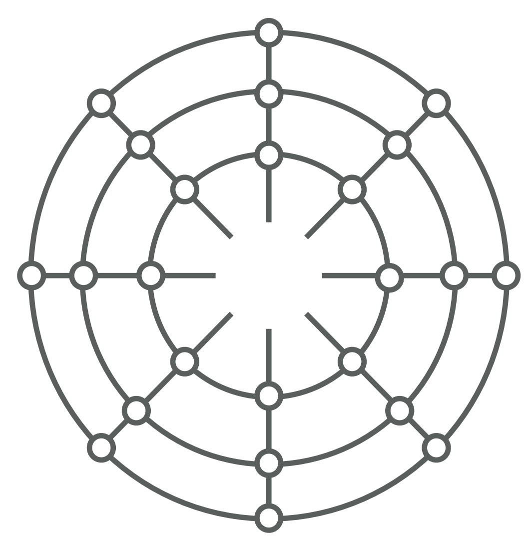
with $x_{L+1} = x_1$

Fishnet graphs have iterative structure and can be generated by iterating B

E.g. Graph with M wheels

$(B_L)^M$

$M \times L$ fishnet



Thermodynamic limit
Study large L and large M

Spectrum

[Gromov,Kazakov,Korchemsky,Negro,Sizov'17]
[Grabner,Gromov,Kazakov,Korchemsky'17]
[Kazakov,Olivucci'18],[Gromov,Sever'19'20]

Eigenvalue problem

Diagonalize B_L & conformal generators simultaneously

Eigenvalues labelled with scaling dimension Δ and spins

Conformal group enough to diagonalise L = 2 - Higher lengths require **integrability**

Key observation: operator B falls inside a family of commuting conserved charges

Difficulty: Unusual spin chain with spins in "wild" infinite dimensional representations of the conformal group (complementary or principal series)

No Bethe ansatz, non magnons - Need more sophisticated methods
(e.g. SoV for higher-rank models)

Spectrum of local single-trace operators

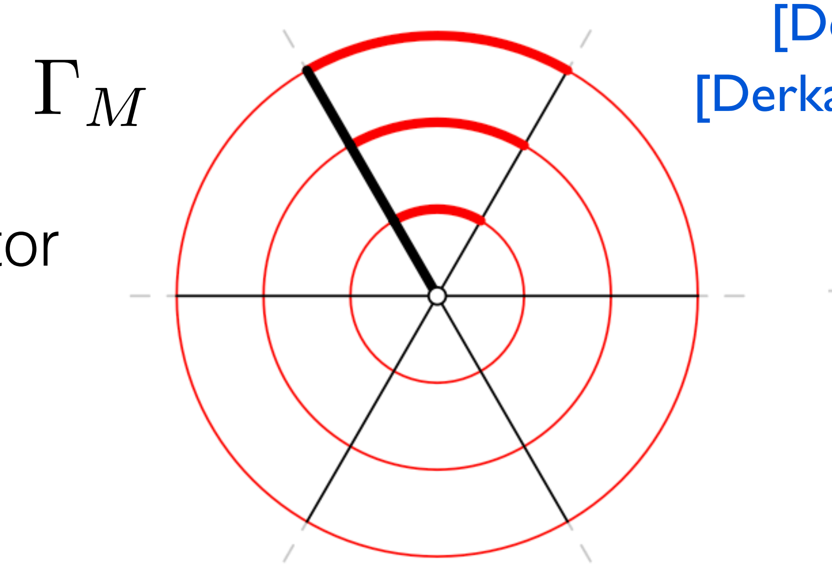
Solve $1 = g^{2L} B_L(\Delta)$

Logic is to solve for arbitrary dimension and then impose quantization: $\Delta = \Delta_L(g^2)$

TBA strategy

[Derkachov,Kazakov,Olivucci'18]
[BB,Ferrando,Kazakov,Zhong'19]
[Derkachov,Olivucci'19'20'21]
[Derkachov,Ferrando,Olivucci'21]

Open-string channel



Insight from N=4 SYM mirror TBA
Switch to 'pizza slice' graph building operator

Still an integral operator but with open BC

Finite-time evolution operator in 'angular direction'

Symmetries: fix two points (origin and infinity), left with dilatation and Lorentz

(Same residual conformal symmetries as for N=4 SYM mirror magnons)

Excitations (magnons) classified accordingly

Magnon = 3-sphere Harmonics carrying momentum

Energy is sum of individual energies of these magnons
Wave functions give us access to their factorised S-matrix

$$\Gamma_M = \prod_{i=1}^{M} e^{-\epsilon_i}$$

Enough to formulate TBA equations for the scaling dimensions

TBA strategy

Goal: calculate grand canonical partition function

$$Z_L(g^2) = \sum_{M=0}^{\infty} g^{2LM} (\Gamma_M)^L$$

Free energy density = scaling dimension $\Delta_L(g^2)$ of local operator $\mathcal{O}_L(x) \sim \mathrm{tr}\, \phi_1(x)^L$

TBA equations

[Yang,Yang'60s]
[Zamolodchikov'90s]

$$\log Y_a(u) = Lh - L\epsilon_a(u) + \sum_b \mathcal{K}_{ab} * \log\left(1 + Y_b(u)\right) + \ldots$$

1) **Coupling constant** enters as chemical potential $h = \log g^2$

2) Length **L** of chain acts as inverse temperature

Solution to TBA determines the free energy = scaling dimension

$$\Delta = L - 2 \sum_a \int \frac{du}{2\pi} \log\left(1 + Y_a(u)\right)$$

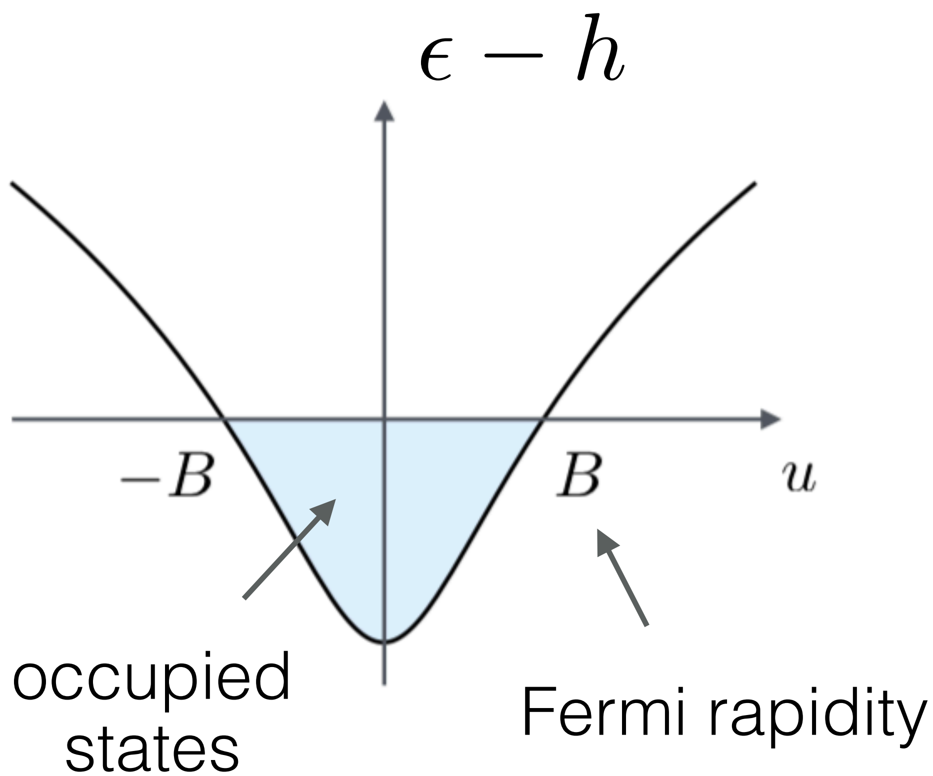
Thermodynamic limit

Thermodynamic limit $L \to \infty$

[BB,Zhong'18]
[BB,Ferrando,Kazakov,Zhong'19]

Lightest modes (s-wave, a=1) dominate in the thermodynamic limit

A **Fermi sea** forms: all states below the Fermi rapidity B are filled



Increasing coupling amounts to increasing B

Dense fishnet limit corresponds to Fermi sea extending across all rapidity axis

The value of the coupling for which it is realized is Zamolodchikov critical coupling g_c

Analysis

General: solve linear integral equation for the distribution of energy levels

$$\chi(u) = C - \epsilon(u) + \int_{-B}^{B} \frac{du}{2\pi} \mathcal{K}(u-v)\chi(v)$$

With $\chi(u = \pm B) = 0$

$$C = \log g^2 - \int_{-B}^{B} \frac{du}{2\pi} k(u)\chi(u)$$ and $$\Delta/L = 1 - \int_{-B}^{B} \frac{du}{\pi} \chi(u)$$

Critical regime: (maximal density of wheels)

(i) Scaling function vanishes $f = \Delta/L \to 0$

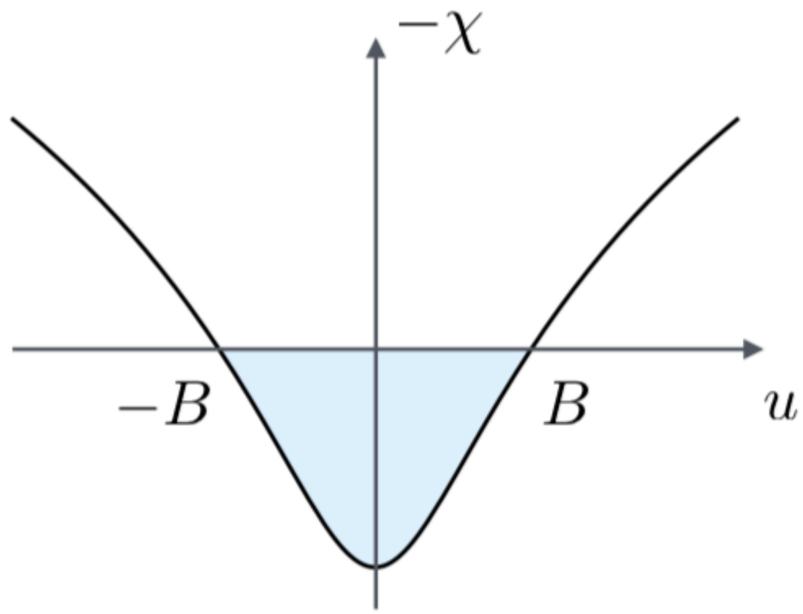
(ii) Chemical potential (coupling) approaches predicted value

$$\chi_{cr} \sim e^{-|\theta|} \quad \Rightarrow \quad C_{cr} = 0 \quad \Rightarrow \quad g_c = \Gamma(3/4)/\sqrt{\pi}\Gamma(5/4)$$

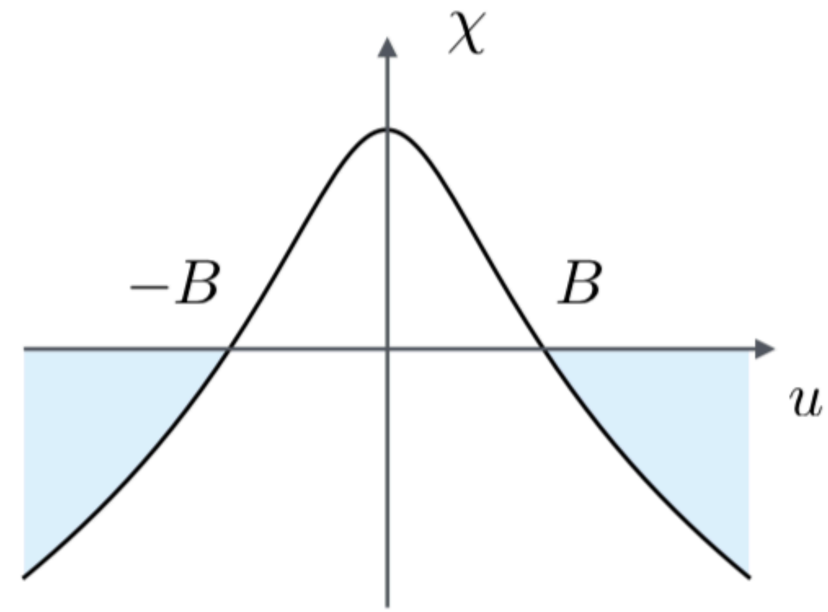
(Same as Zamolodchikov result for 4d regular square lattice)

Duality transformation

Particle-hole transformation (similar to map between ferro and anti-ferro)



Fermi sea of magnons



Dual sea

Dual equation

1) dualize kernel

$$K = -\frac{\mathcal{K}}{1-\mathcal{K}*} = -\mathcal{K} - \mathcal{K}*\mathcal{K} - \ldots$$

2) act on both sides of the equation with $1 - K*$

Sigma model picture

Dual equation: $\chi(\theta) = E(\theta) + \int\limits_{\theta^2 > B^2} \frac{d\theta'}{2\pi} K(\theta - \theta')\chi(\theta')$

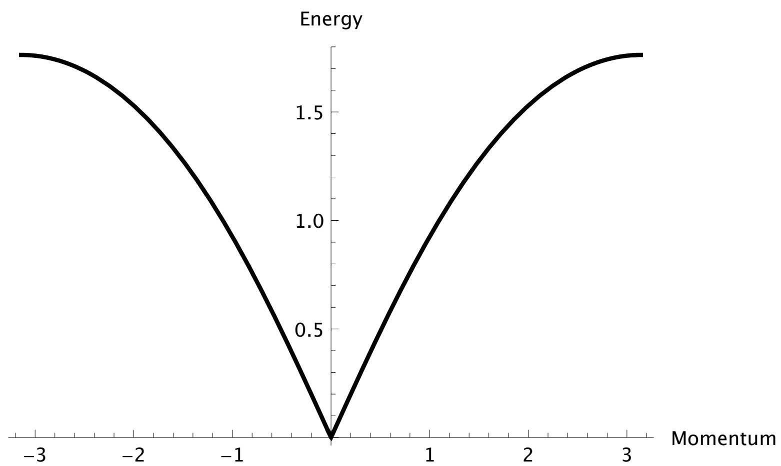
Dual energy: $\log g^2 = \log g_c^2 + \int\limits_{\theta^2 > B^2} \frac{d\theta}{2\pi} P'(\theta)\chi(\theta)$

No chemical potential $\chi(\theta) \sim -2\rho \log\theta$ $\rho = \Delta/L =$ charge density

1) **Kernel**: $K = -i\partial_\theta \log S_{O(6)}$ [Zamolodchikov&Zamolodchikov'78]

Particles scatter as in 2d O(6) non-linear sigma model

2) **Dispersion relation**: $\sinh^2(\frac{1}{2}E) = \sin^2(\frac{1}{2}P)$



But they are gapless (unlike in O(6) model)

Interpretation: sigma model on pseudo-sphere

Dual = some integrable lattice version of AdS_5 sigma model

Dual theory: hyperbolic sigma model

2d sigma model with curved target space

$$\mathcal{L} = -\frac{1}{2e^2} G^{AB} \partial_a Y_A \partial^a Y_B$$

Weak coupling (large radius of curvature) : $e^2 \ll 1$

Beta function related to Ricci scalar

$$AdS_{d+1} \;:\; -Y_0^2 + Y_\perp^2 - Y_{d+1}^2 = -1$$

Negative curvature ⟶ a **positive** beta function $\mu \frac{\partial}{\partial \mu} e^2(\mu) = \frac{d}{2\pi} e^4(\mu) + \ldots$

Alternatively, the coupling **grows** with the energy $\frac{1}{e^2(\mu)} = \frac{d}{2\pi} \log\left(\Lambda/\mu\right)$

1. Theory is weakly coupled in IR
2. There is no mass gap
3. Integrable but gapless and no good particle picture

akin to massless factorized scattering theories

[Zamolodchikov&Zamoldchikov'92]
[Fendley,Saleur,Zamolodchikov'93]
[Fateev,Onofri,Zamolodchikov'93]

Dual state: tachyon

Consider sigma model on cylinder of radius L

Interested in 2d "ground state" energy: tachyon

(best candidate for extremum of energy at given charge = global time energy)

$$V_\Delta \sim e^{-i\Delta t}$$

Classically, it corresponds to solution $Y^0 \pm iY^{d+1} = e^{\pm iH\tau} \quad Y_\perp = 0$

charge density $\Delta/L = \frac{1}{e^2}(Y^0\dot{Y}_{d+1} - Y^{d+1}\dot{Y}_0) = -H/e^2$

energy density $E/L = \frac{1}{2e^2}(\dot{Y}^0\dot{Y}_0 + \dot{Y}^{d+1}\dot{Y}_{d+1}) = -H^2/(2e^2)$

Classical result :
(c-o-m energy)

$$E = -\frac{e^2\Delta^2}{2L}$$

Same as in O(d+2) model if not for the sign of the coupling $e^2 \leftrightarrow -e^2$

Graph building operator

Duality exchanges energy and chemical potential

Dual energy $= L \log g^2$

It describes eigenvalues of the closed string operator B_L

$$-\log B_L = L \log g_c^2 + H_{AdS}$$

Hamiltonian of 2d sigma model up to vacuum energy density $\log g_c^2$

Dual TBA formula calculates energy levels of H in finite volume L

$$\log Y_1 = LE - K_{O(6)} * \log\left(1 + 1/Y_1\right) + \dots$$

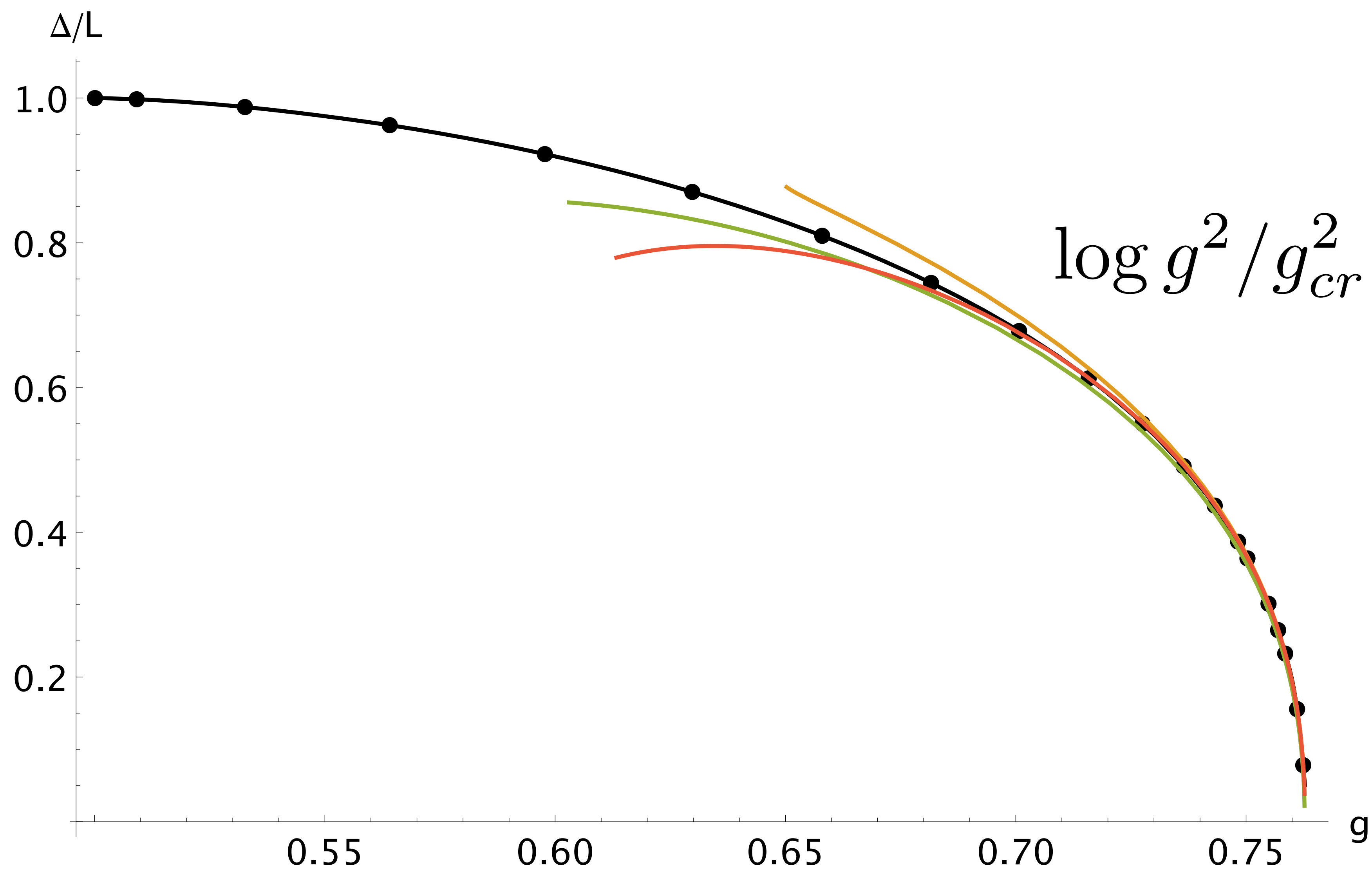
$$H = -\int \frac{d\theta}{2\pi} P'(\theta) \log\left(1 + 1/Y_1\right)$$

Good description (sigma model is weakly coupled) when length is large and energy is small

With Δ entering in the asymptotics of Y function

Numerics

Quadratic scaling near critical point controlled by continuum sigma model



$$\log g^2/g_{cr}^2 = -\frac{e^2\Delta^2}{2L^2}$$

WS PT
at 1st, 2nd
and 3rd
order

$$g_{\rm thr} = 0.5 \qquad g_{\rm cr} = 0.76276$$

thermodynamical window

Fishchain model

Duality with quantum mechanical system in AdS for any L [Gromov,Sever'19]

Not a smooth sigma model, closer to a string-bit model

$$S = g \int dt \sum_{i=1}^{L} L_i$$

$$L_i = -\frac{\dot{X}_i^2}{2} - \prod_{k=1}^{L} (-X_k \cdot X_{k+1})^{1/L} - \eta_i (X_i^2 + R^2) + R^2$$

Chain of L point particles moving inside AdS with nearest neighbour interactions

Should be equivalent to original fishnets for any L and at any coupling

Complicated interactions, hard to take continuum limit directly

Open Fishnets

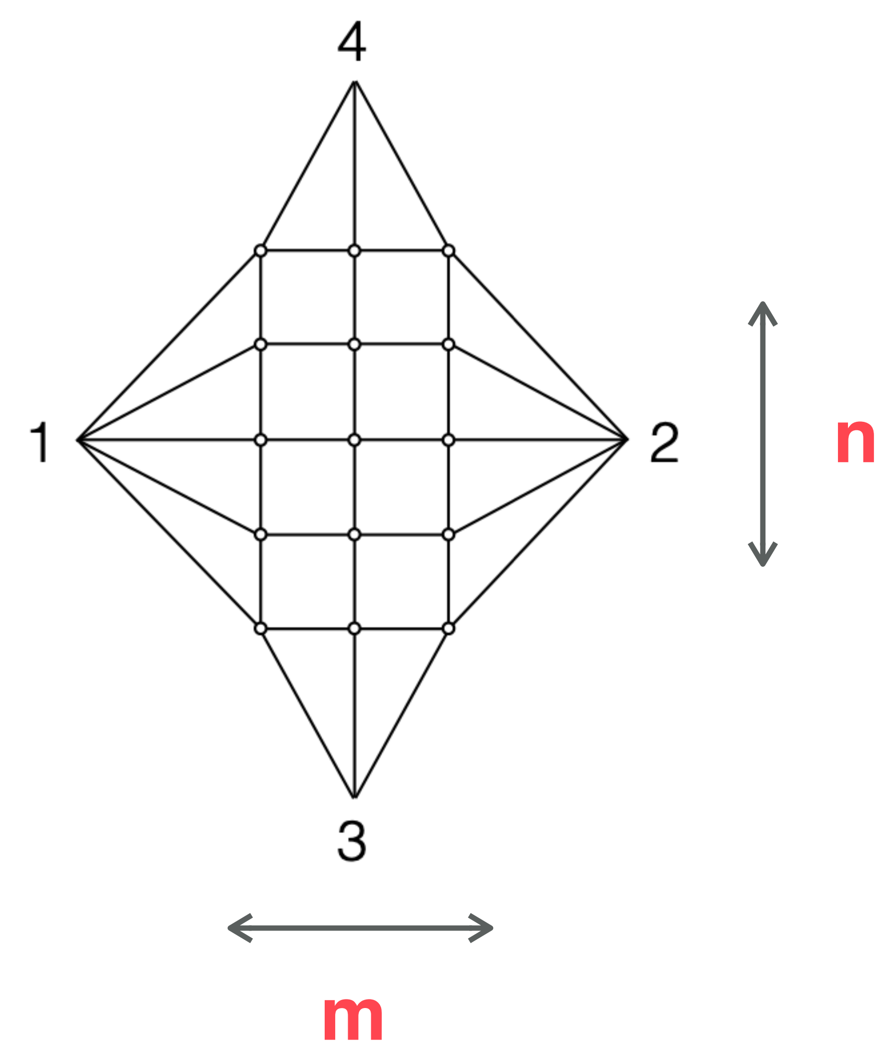
Open Boundary Conditions

4pt function

Attach **n** horizontal lines at spacetime point 1 and 2

Attach **m** vertical lines at spacetime point 3 and 4

*Function of 2 integers (**m**&**n**)
*Function of 2 cross ratios



$$u = \frac{x_{14}^2 x_{23}^2}{x_{12}^2 x_{34}^2} = \frac{z\bar{z}}{(1-z)(1-\bar{z})}$$

$$v = \frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2} = \frac{1}{(1-z)(1-\bar{z})}$$

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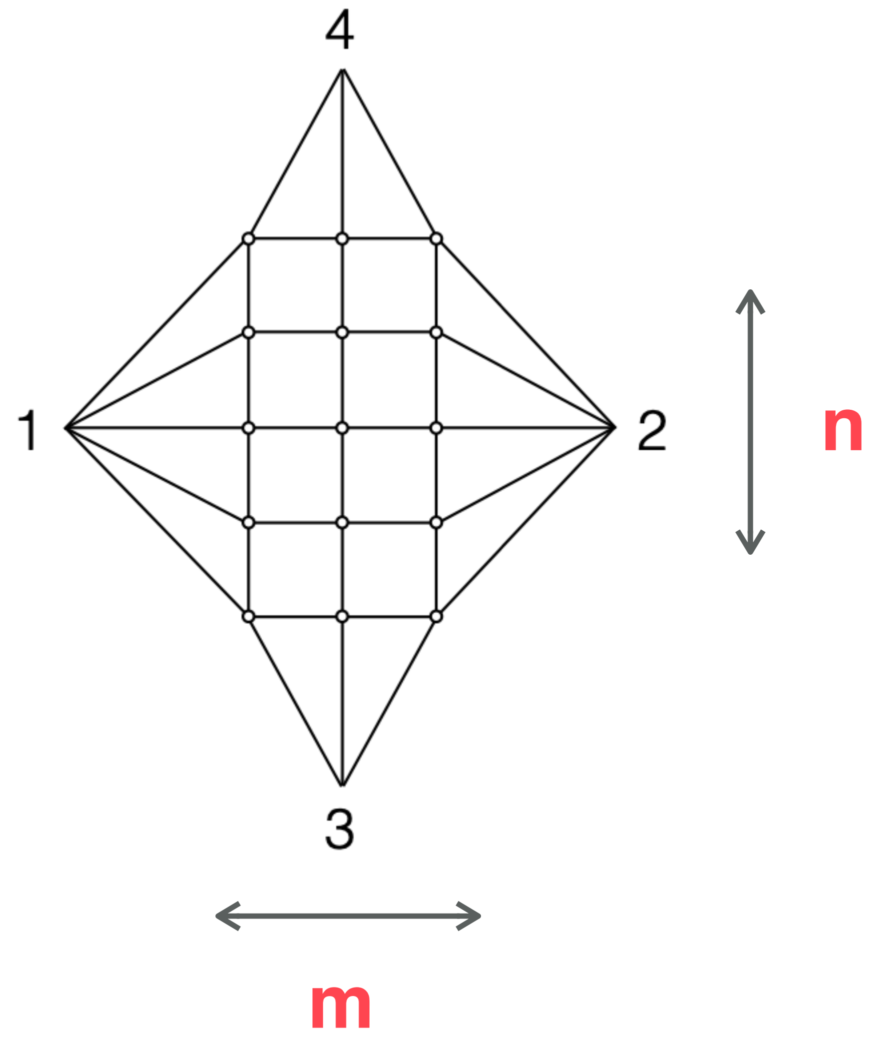


Integral is both UV and IR finite

$$G_{m,n} = \frac{g^{2mn}}{(x_{12}^2)^n (x_{34}^2)^m} \times \Phi_{m,n}(u,v)$$

Parametrization: $$\Phi_{m,n}(u,v) = \left[\frac{(1-z)(1-\bar{z})}{z-\bar{z}}\right]^m \times I_{m,n}(z,\bar{z})$$

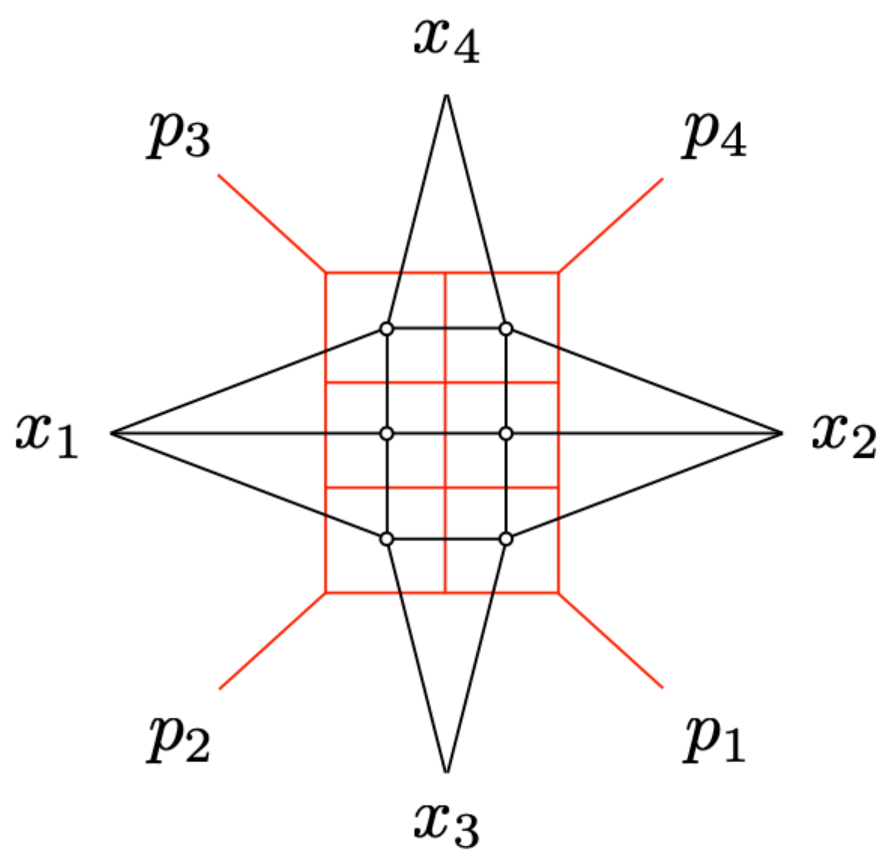
with $I_{m,n}$ a pure function (i.e. iterated integral) of weight $2mn$

Shortcut

1) *Analyticity*

[BB,Dixon'17]

Dual graph = Amplitude (subject to stringent analytical constraints, such as Steinmann relations for double discontinuity)



Dual momenta

$$p_1 = x_2 - x_3$$

$$p_2 = x_3 - x_1$$

$$\ldots$$

2) *Integrability (several reps)*

Determinant of Hankel matrix

$$I_{m,n} = \frac{1}{\mathcal{N}} \det_{1 \leqslant i,j \leqslant m} M_{i+j+n-m-1}$$

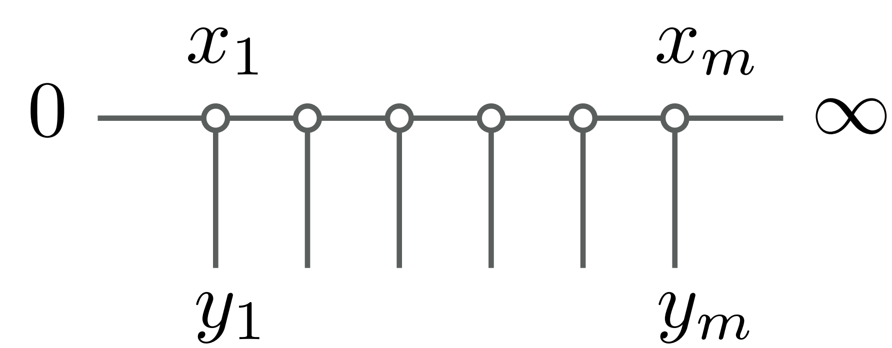
With entries given by Ladders (m=1, n=p) $\quad M_p = p!(p-1)!\; L_p(z, \bar{z})$

with $\quad L_p(z,\bar{z}) = \sum_{j=p}^{2p} \frac{j![-\ln(z\bar{z})]^{2p-j}}{p!(j-p)!(2p-j)!} [\mathrm{Li}_j(z) - \mathrm{Li}_j(\bar{z})]$

[Usyukina,Davydychev'93]

Rigorous approach

Hexagonalization
SoV factorization using open string graph building operator



$$\Gamma_m \cdot \psi(x_1, \ldots, x_m) = \int \prod_{i=1}^{m} \frac{dy_i}{\pi^2 (x_{i-1} - x_i)^2 (x_i - y_i)^2} \psi(y_1, \ldots, y_m)$$

with $x_0 = 0$

[Derkachov,Kazakov,Olivucci'18]
[BB,Ferrando,Kazakov,Zhong'19]
[Derkachov,Olivucci'19'20'21]
[Olivucci'21'23]
[BB,Caetano,Fleury'18]
[Fleury,Komatsu'16]
[BB,Dixon'17]

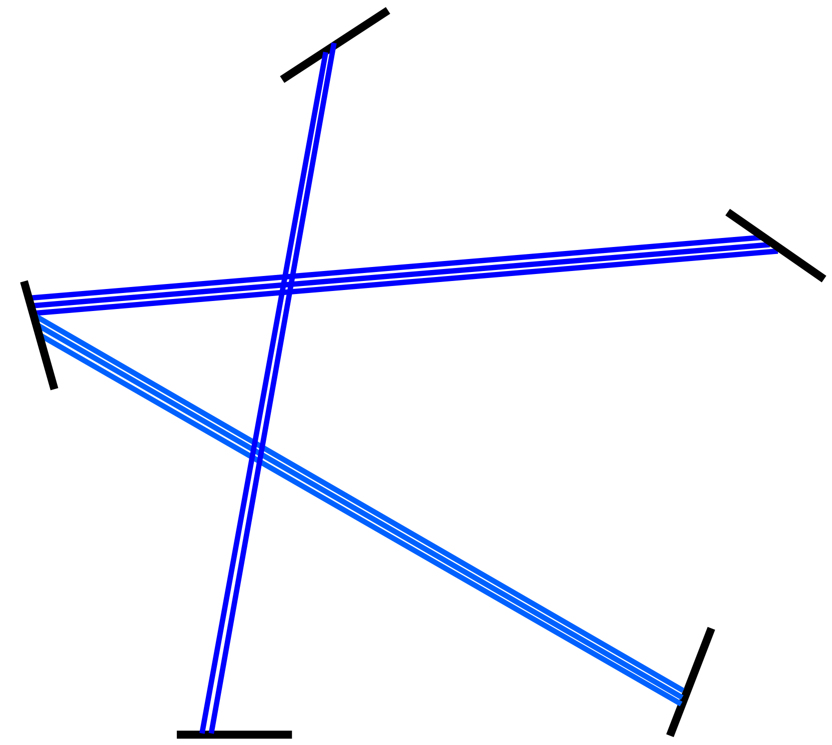
Wave functions ψ_m can be constructed exactly:
(states labelled by rapidities and Harmonics on 3-sphere)

Completeness -> concise matrix-model like representation for correlator

$$I_{m,n} = \langle \text{out} | (\Gamma_m)^n | \text{in} \rangle = \int_{\mathcal{H}_m} d\mu(\psi_m) \lambda(\psi_m)^n$$

Generalizations

HEX/SoV for higher-point correlators



Provide Fourier-Mellin-type representations

[Komatsu,Fleury'16]
[BB,Caetano,Fleury'18]
[Derkachov,Olivucci'19'20'21]
[Olivucci'21'23] [Aprile,Olivucci'23]

Can we integrate them? Space of functions? Bootstrap?

Contact with methods based on Yangian symmetry (in d dimensions)?

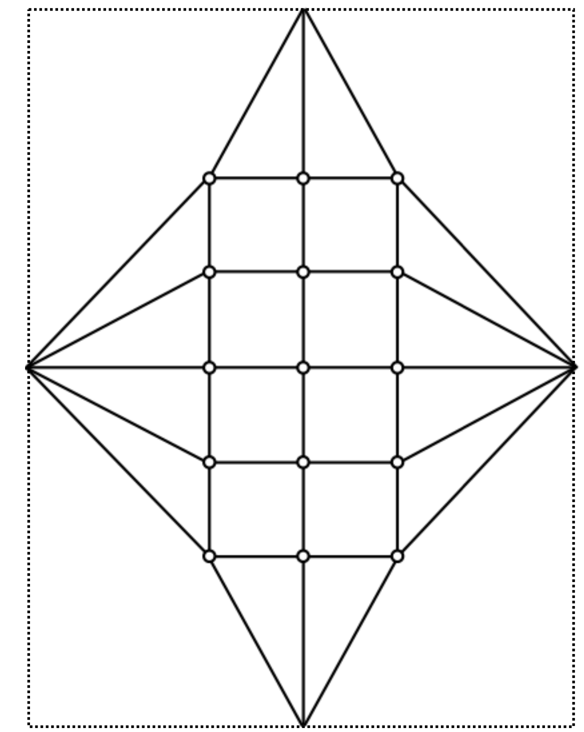
[Chicherin,Kazakov,Loebbert,Muller,Zhong'16]
[Loebbert,Muller,Munkler'19]
[Duhr,Klemm,Loebbert,Nega,Porkert'22]

Null Wilson loop approach

Insert graph in a null WL frame

(Easy step since gauge field decouples in fishnet limit)

Scalar lines map to excitations of the flux tube sourced by the WL



Flux-tube factorization:

$$\Phi_{m,n}(z,\bar{z}) = \int e^{2iu\sigma_1 + 2iv\sigma_2} \mu(\mathbf{u}) S_\star(\mathbf{u},\mathbf{v}) \mu(\mathbf{v})$$

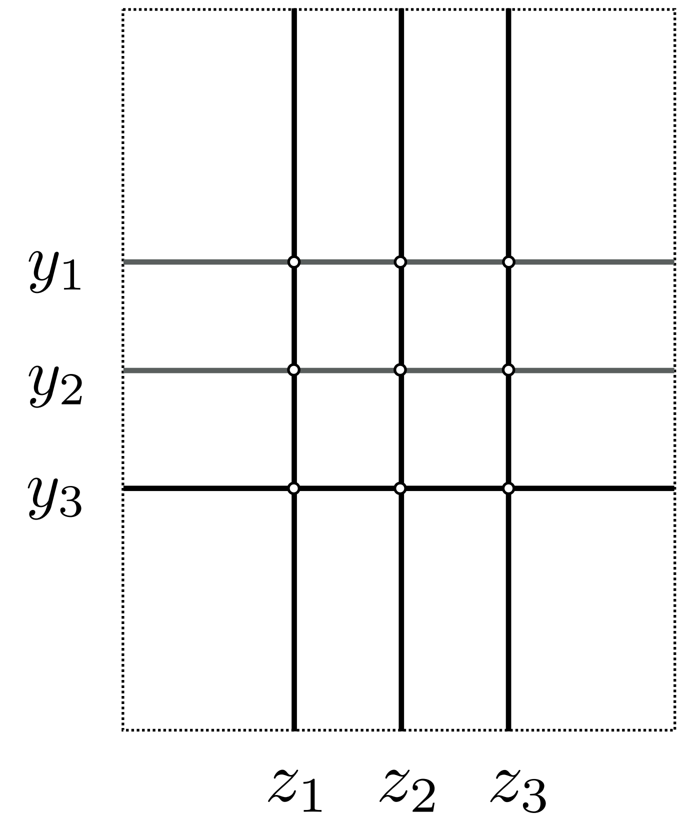
Interaction point yields an S-matrix $S_\star(u,v) = \dfrac{\sinh(\pi(u-v))}{\pi(u-v)}$

Wave functions for productions of excitations $\mu(\mathbf{u}) \propto \prod_{i\neq j} \dfrac{1}{|P(u_i|u_j)|^2}$

Not same representation as from hexagons but fully equivalent

Generalizations

Higher-point amplitudes by point splitting along null ray



Same bulk interaction as before

Different boundary conditions (wave functions)

$$\text{Int} = \langle \Psi(\mathbf{u},\mathbf{z}), \Psi(\mathbf{u},\mathbf{z}')\rangle \times S_\star(\mathbf{u},\mathbf{v}) \times \langle \Psi(\mathbf{v},\mathbf{y}), \Psi(\mathbf{v},\mathbf{y}')\rangle$$

Wave functions can be constructed using SL(2) SoV method

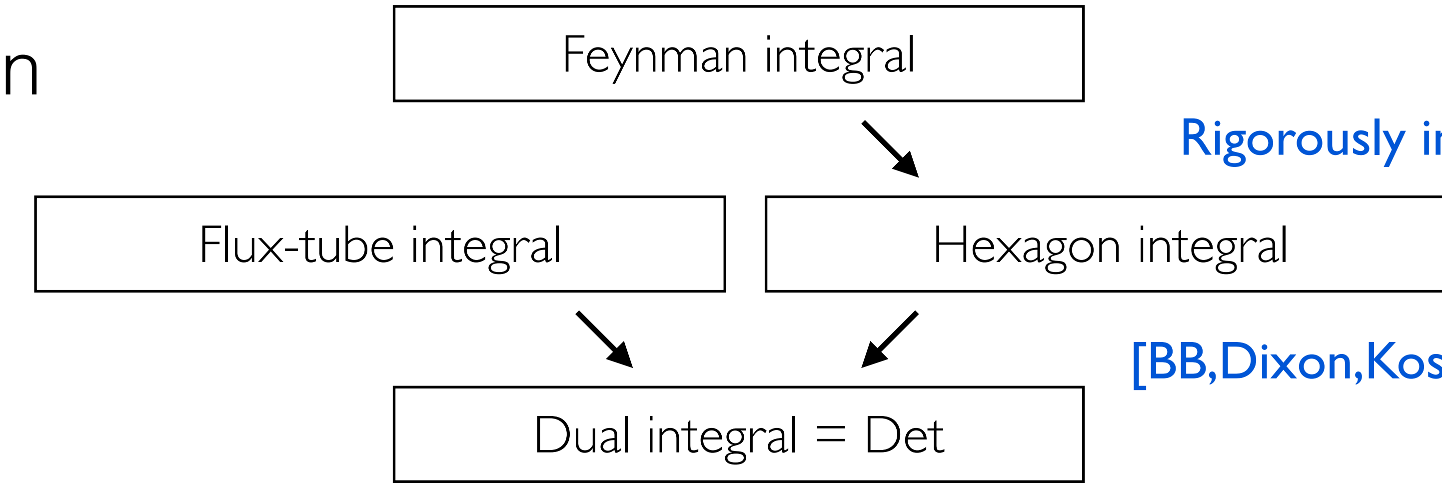
[BB,Sever,Vieira'13]
[Belitsky,Derkachov,Manashov'14]

Not obviously simpler than hexagon representation

But certainly better suited to study the light-ray kinematics

Integral representation

Back to 4pt function



Rigorously in [Derkachov,Olivucci'19'20]

[BB,Dixon,Kosower,Krajenbrink,Zhong,'21]

Dual integral representation

$$I_{m,n} = \frac{1}{2^m m! \mathcal{N}} \prod_{i=1}^{m} \int_{|\sigma|}^{\infty} \frac{dx_i x_i (x_i^2 - \sigma^2)^{n-m}}{\cosh \frac{1}{2}(x_i + \varphi) \cosh \frac{1}{2}(x_i - \varphi)} \prod_{i<j}^{m} (x_i^2 - x_j^2)^2$$

With cross ratios parametrized as $z = -e^{\sigma+\varphi}$ $\quad \bar{z} = -e^{\sigma-\varphi}$

Determinant follows from standard manipulation of Vandermonde interaction

Nice form for thermodynamic (large mn) limit

Thermodynamic limit

Large mn : Electrostatic equilibrium in potential $V \approx \frac{m}{n-m} \log x^2 + \frac{|x|}{m}$

Roots (x's) scale large, cross ratios (z,zb) go away

log (repulsive) linear (confining)

Upon rescaling, get well-known singular equation for density

$$0 = \frac{1}{x} - 2\pi + \int_a^b \frac{4x dy \rho(y)}{x^2 - y^2}$$

Same as for O(-2) matrix model, classical limit of Bethe equations for SL(2) spin chain, classical string in AdS, …

[Kostov'97]
[Beisert,Minahan,Staudacher,Zarembo'03]

Solution given in terms of elliptic integrals

Calculate free energy density and compare with PBC?

Tedious but doable - must use several tricks (differential equations)

Solution I

Free energy density $F = \lim_{m,n\to\infty} \frac{\ln I_{m,n}}{mn}$ [BB,Dixon,Kosower,Krajenbrink,Zhong'21]

Surprisingly (or not) F depends on the ratio of lengths of the fishnet

$$k = n/m \quad \in (1,\infty)$$

Parametric solution in terms of elliptic integrals E&K

$$F = \ln\pi^2 + k\ln\frac{1+\sqrt{1-q}}{2} + \frac{(k-1)^2}{2k}\ln K(q)$$
$$+\frac{1}{k}\ln\frac{1-\sqrt{1-q}}{2} - \frac{(k+1)^2}{2k}\ln E(q)$$

where $$k = \frac{E(q)+\sqrt{1-q}K(q)}{E(q)-\sqrt{1-q}K(q)}$$

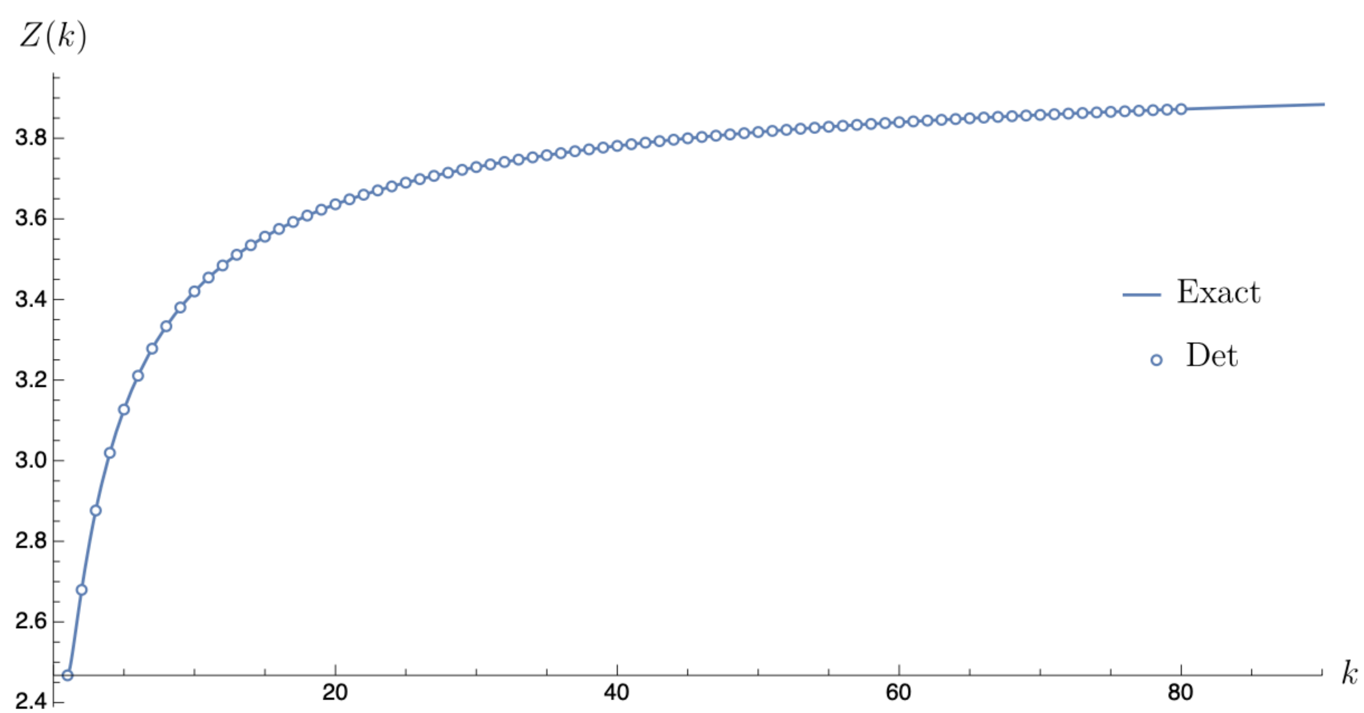
However, no dependence on cross ratios

Solution II

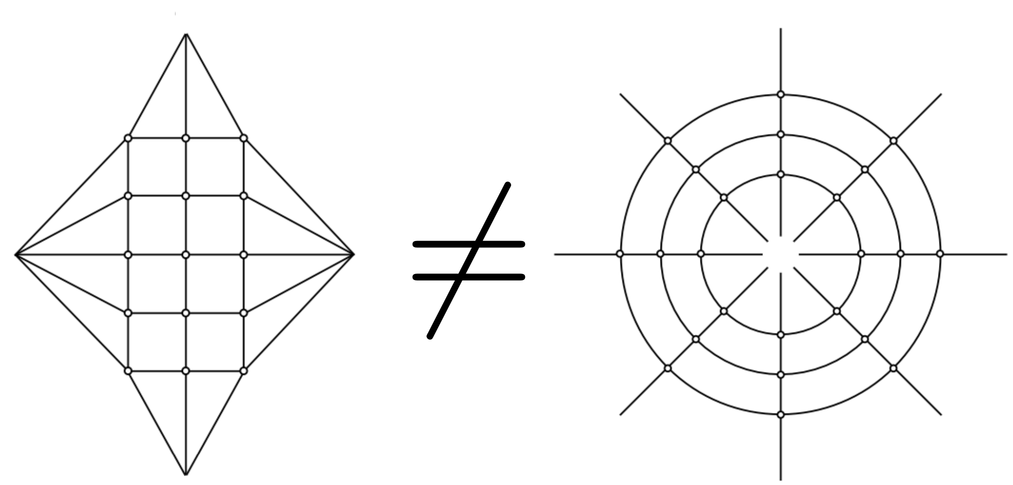
Explicit expressions may be found in particular limits $k \to 1 \text{ or } \infty$

Numerical plot for other values
(Compare well to direct extrapolation of the determinant)

$$Z = e^F$$



Result appears everywhere different from the free energy for PBC



Thermodynamic puzzle

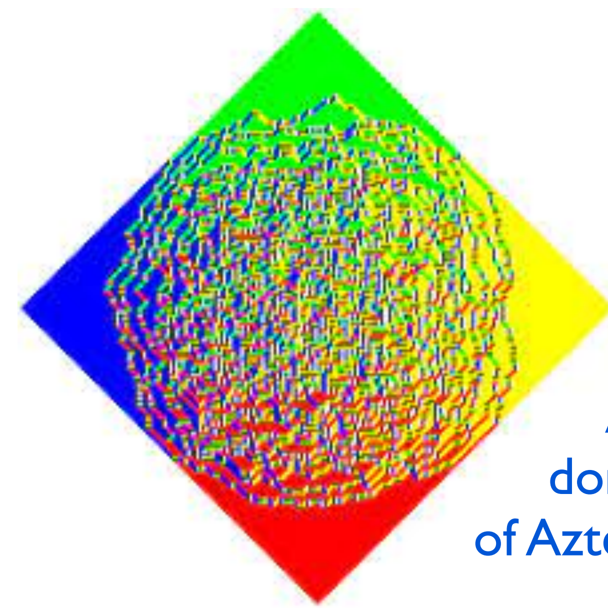
Should the thermodynamic limit not be universal (same for all BCs)?
Probably yes for local enough systems and regular enough BCs

Strong sensitivity to BCs have been seen in 2d lattice models

See e.g.
[Zinn-Justin'19]
[Korepin,Zinn-Justin'00]
[Colomo,Pronko'10]

Example: 6-vertex model shows very different thermodynamic behaviours for PBC or DWBC (Domain Wall BC)

Interpretation: two macroscopic phases coexist,
frozen near corners and *melted* in bulk



Artic curve domino tilings of Aztec diamond (particular case of DWBC 6-vertex model)

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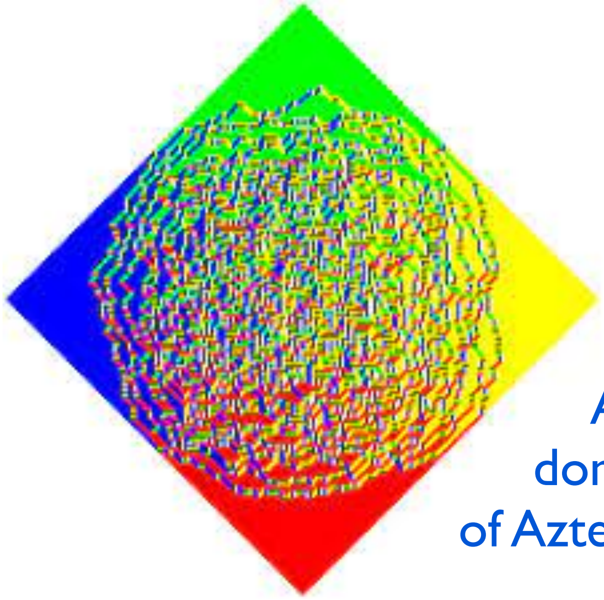

Artic curve domino tilings of Aztec diamond (particular case of DWBC 6-vertex model)

Arctic curve for fishnets?

Analogy mathematically convincing
PBC -> TBA vs DWBC -> Determinant

Physically, operators (corners) carry large dimensions, which invalidate the low energy (sigma model) description there

Can we probe with arctic curve for fishnets?

A stringy limit

Unexpected connection to string theory appears certain scaling limit

Scaling with dependence on cross ratios in the thermodynamic limit

Very **short-distance** limit $|\sigma|, m, n \to \infty$

$$z = -e^{\sigma+\varphi}$$
$$\bar{z} = -e^{\sigma-\varphi}$$

$$0 = \frac{x}{x^2-\xi^2} - 2\pi + \int_a^b \frac{4x dy \rho(y)}{x^2-y^2} \qquad \xi = |\sigma|/m$$

See Ivan's talk

Deformed potential (back to spin chain eq. when xi ->0)

Same eq. as for a folded string spinning in $AdS_3 \times S^1$

[Kazakov,Zarembo'04]
[Casteill,Kristjansen'07]

Mathematical coincidence?

Hint at a world-sheet description in the "frozen phase"?

Conclusion

Planar graphs are a nice arena for exploring connections among integrable systems, from spin chains to sigma models

Fishnet graphs particular nice, direct connection between continuum limit and sigma models through TBA equations

Limit not universal though, strong sensitivity on boundary conditions

What is the general behaviour for general graphs?

Classes of boundary conditions sharing same large order behaviours?

What does it imply for validity of AdS sigma model description?

Conclusion

Can other fishnets be described using TBA?

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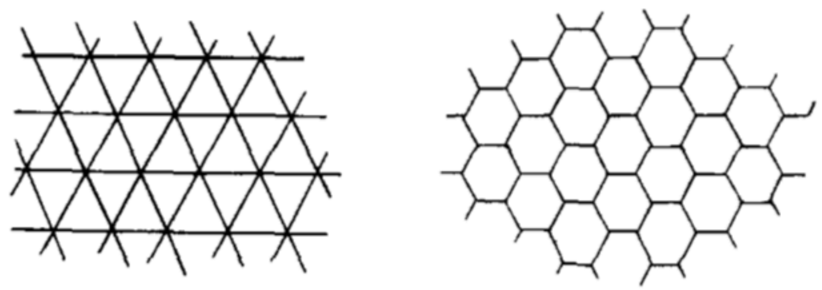

(a) (b)

Fig. 6.

shown in fig. 6a (at $\mathcal{D} = 3$) and in fig. 6b (at $\mathcal{D} = 6$)

means that the diagram with a large number P of vertices behaves as $(gc)^P$ (we mean the main factor of the asymptotics), the constant c being independent of the external momenta. The value of c can be obtained from eq. (18) by substituting the appropriate values of α and $\mathcal{D}$. Thus, one obtains

$$c_4 = \left[\frac{1}{16\sqrt{\pi}} \frac{\Gamma(1/4)}{\Gamma(3/4)}\right]^2 , \quad c_3 = \left[\frac{1}{12\sqrt{\pi}} \frac{\Gamma(1/6)}{\Gamma(2/3)}\right]^3 ,$$

$$c_6 = \left[\frac{1}{24\sqrt{\pi}} \frac{\Gamma(1/3)}{\Gamma(5/6)}\right]^{3/2} , \qquad (19)$$

Enlarged family of fishnets with Yukawa interactions,
Brick Walls, Dynamical fishnets,
Checkerboards, etc.

[Kazakov,Olivucci,Preti'19]
[Pittelli,Preti'19],[Kazakov,Olivucci'22]
[Alfimov,Ferrando,Kazakov,Olivucci'23]
[Kade,Staudacher'23'24]

Correspondence with non-compact sigma models?