UNIVERSITÄT BONN

The 7th Meeing of the Low Energy Nuclear Physics International Collaboration (LENPIC2024)

LENPIC

SRG evolution of NCSM wave functions

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In collaboration with

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- SRG-independent NCSM wave functions
- Benchmark: Deutron wave functions
- Two-body density, SRC, and rms radius of ^{4}He
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Similarity renormalization group (SRG)

Unitary transformations on the Hamiltonian decouple high- and low-momentum parts of the interaction

$$H_\lambda = U_\lambda H_0 U_\lambda^\dagger$$

$$\frac{dV_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda], \quad \frac{dU_\lambda}{d\lambda} = \eta_\lambda U_\lambda,$$

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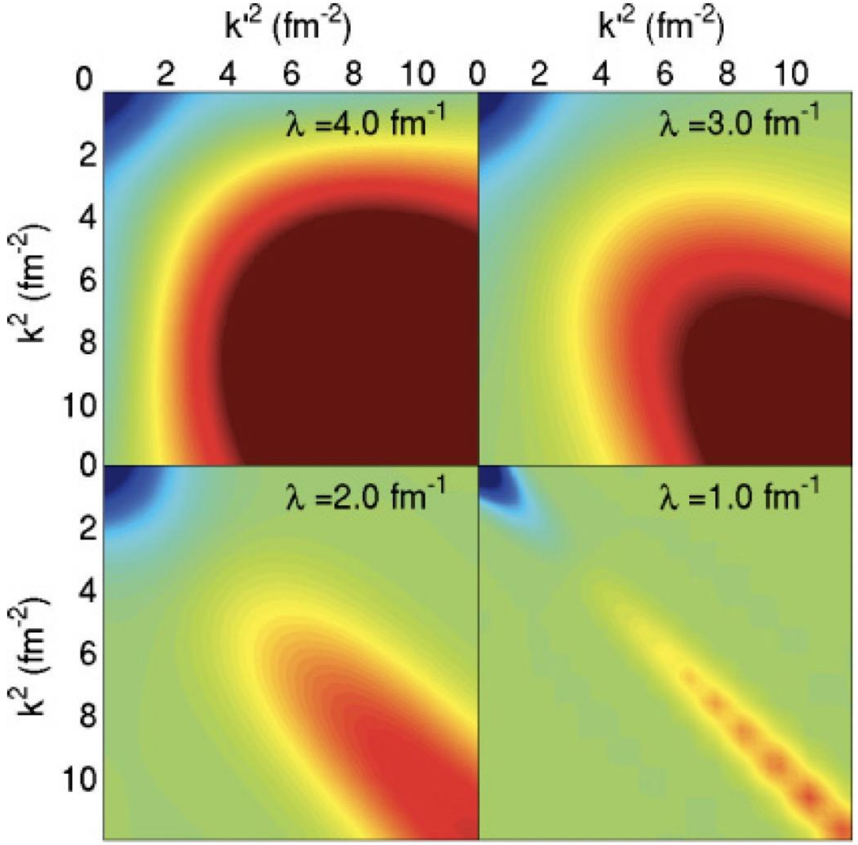
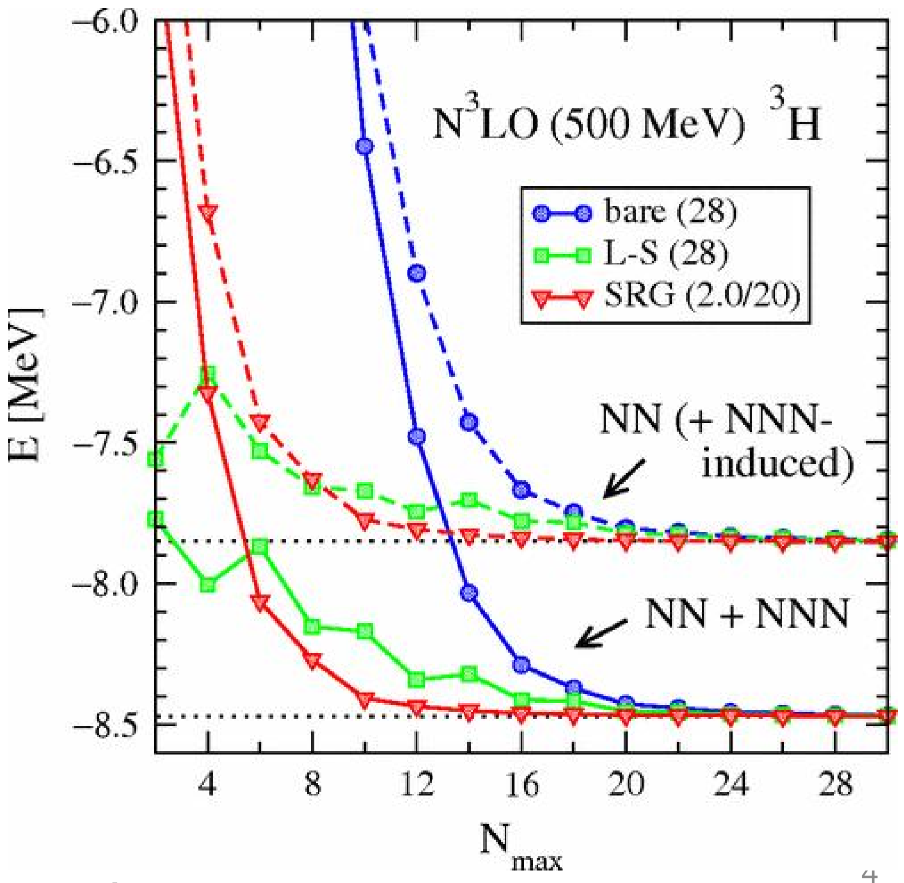




Bogner, Furnstahl, Perry, Phys. Rev. C75, 061001(R) (2007)

Jurgenson, Navrátil, Furnstahl, Phys. Rev. Lett. 103, 082501(2009)

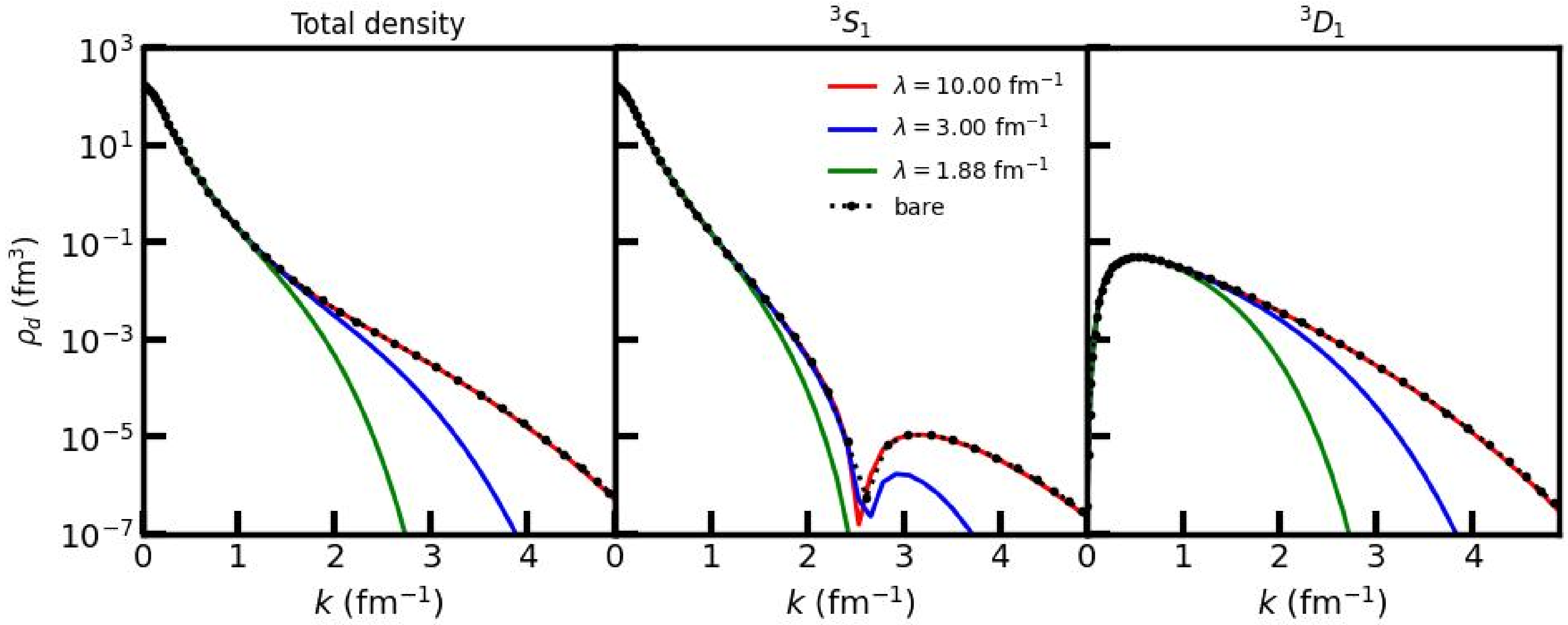
NCSM wave functions

High-resolution wfs: $H_0|\Psi_0\rangle = E|\Psi_0\rangle$ bare-interaction solutions

- ✓ A=2, 3, and 4 systems using FH method with the bare interaction
- ✓ light nuclei with quantum Monte Carlo

Low-resolution wfs: $H_\lambda|\Psi_\lambda\rangle = E|\Psi_\lambda\rangle$ softened-interaction solutions

- ✓ all the *ab initio* calculations (NCSM, IMSRG) with SRG softened interaction



Observables in SRG

$$\langle O \rangle = \langle \Psi_\lambda | O | \Psi_\lambda \rangle$$

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- construct the unitary transfer matrix by energy eigenstates

$$U_\lambda = \sum_\alpha |\Psi_\lambda^\alpha\rangle\langle\Psi_{\lambda=0}^\alpha|$$

- get the SRG evolved operator

$$\frac{dO_\lambda}{d\lambda} = [\eta_\lambda, O_\lambda], \qquad \frac{dU_\lambda}{d\lambda} = \eta_\lambda U_\lambda$$

Anderson, et al Phys. Rev. C 82, 054001 (2010)
Schuster, *et al* Phys. Rev. C 90, 011301R (2014)
Schuster, *et al* Phys. Rev. C 92, 014320 (2015)
Neff, Feldmeier, Horiuchi, Phys. Rev. C 92, 024003 (2015)
Tropiano, Bogner, Furnstahl, Phys. Rev. C 102, 034005 (2020)
Miyagi, *et al* Phys. Rev. C 100, 034310 (2019)

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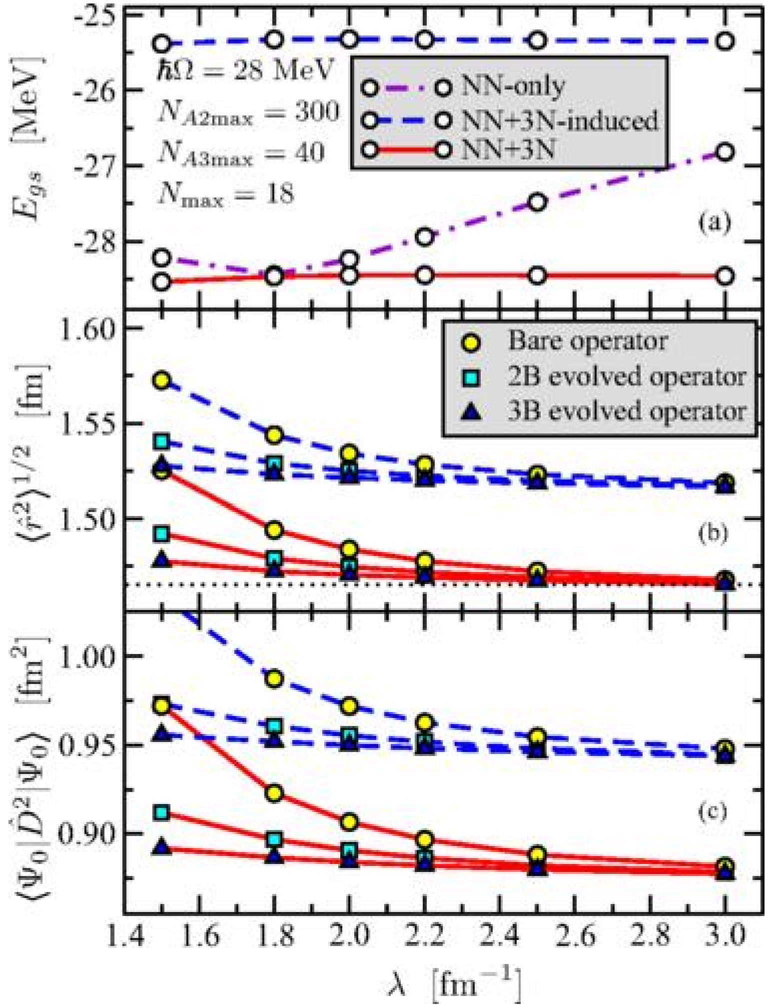

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High-resolution wave functions

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$$U_\lambda = I + \sum \delta U_\lambda^{(2)} a^\dagger a^\dagger a a + \sum \delta U_\lambda^{(3)} a^\dagger a^\dagger a^\dagger a a a + [\text{four body}] + \ldots$$

Tropiano, Bogner, Furnstahl, Phys. Rev. C 104, 034311 (2021)

Low-resolution wfs: $H_\lambda | \Psi_\lambda \rangle = E | \Psi_\lambda \rangle$

$$|\Psi_\lambda : NJTM_T\rangle = \sum_{\gamma_{A-2},\alpha} F_\lambda^{\alpha,\gamma_{A-2}} C^{TM_T}_{t_{12}T_{A-2};m_t M_T - m_{t12}} C^{JM}_{j_{12}I,m_{12}M-m_{12}}$$
$$|n_{12}(l_{12}s_{12})j_{12}m_{j\,12}; t_{12}m_{t12}\rangle$$
$$|\gamma_{A-2} : N_{A-2}IM - m_{j\,12}; T_{A-2}M_T - m_{t12}\rangle,$$

High-resolution wave functions

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$$U_\lambda^\dagger | n_{12} (l_{12} s_{12}) j_{12} m_{j12}; t_{12} m_{t12} \rangle$$
$$| \gamma_{A-2} : N_{A-2} I M - m_{j12}; T_{A-2} M_T - m_{t12} \rangle,$$

High-resolution wave functions can be constructed by low-resoltion solution combined with the SRG evolved two-body Jacobi basis

SRG evolved two-body Jacobi basis

Since we have already known the interaction in the momrntum space

$$V_{ll'}^{j_{12}s_{12}}(p,p';\lambda) = \langle p(ls_{12})j_{12}|V_\lambda|p'(ls_{12})j_{12}\rangle$$

$$\frac{dU_\lambda}{d\lambda} = \eta_\lambda U_\lambda, \frac{dU_\lambda^\dagger}{d\lambda} = -U_\lambda^\dagger \eta_\lambda$$

Let's define $\phi_{n_{12},l_{12}l'_{12}}^{j_{12}s_{12}}(p;\lambda) \equiv \langle n_{12}(l_{12}s_{12})j_{12}|U_\lambda^\dagger|p(l'_{12}s_{12})j_{12}\rangle$

$$\frac{d}{d\lambda}\phi_{n_{12},l_{12}l'_{12}}^{j_{12}s_{12}}(p;\lambda) = -\sum_{\tilde{l}}\int dp' p^2 \phi_{n_{12},l_{12}\tilde{l}}^{j_{12}s_{12}}(p';\lambda) \times (\frac{p'^2}{2\mu} - \frac{p^2}{2\mu})V_{\tilde{l}l'_{12}}^{j_{12}s_{12}}(p',p;\lambda)$$

SRG evolved two-body Jacobi basis

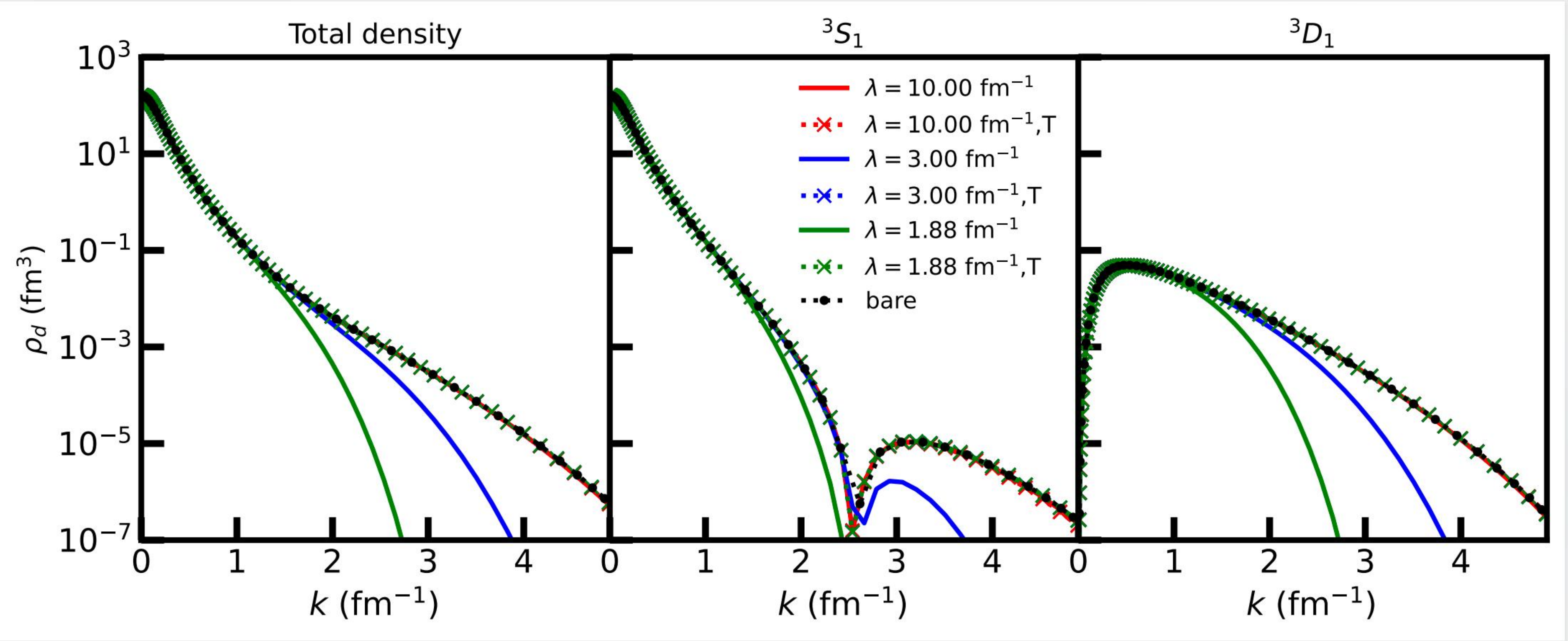
$$\tilde{R}_{n_{12},ll_{12}}^{j_{12}s_{12}}(p;\lambda) = \langle p(ls_{12})j_{12}|U_\lambda^\dagger|n_{12}(l_{12}s_{12})j_{12}\rangle = \sum_{n'} R_{n'l}(p)\int dp' p'^2 \phi_{n',ll_{12}}^{j_{12}s_{12}}(p';\lambda)R_{n_{12}l_{12}}(p')$$

$R_{nl}(p) = \langle p(ls)j|n(ls)j\rangle$ HO basis function in the momentum space

Such procedures can also be implemented in the HO basis

Deutron wave functinos in momentum space

the chiral semilocal momentum-spaced regularized (SMS) N^4LO+ (450 MeV), $\omega = 16$ MeV, $N_{max} = 50$

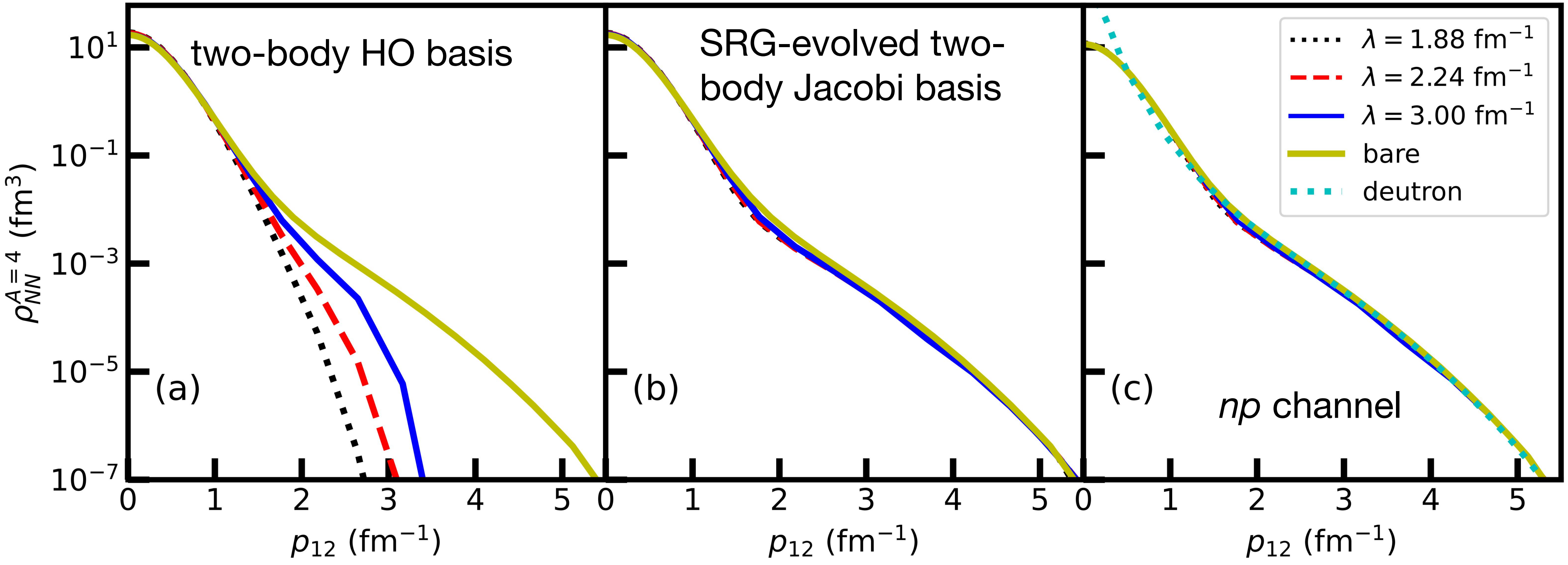


Transfer low-resolution wave function to high-resolution ones accurately for two-body system.

Two-body density of ^{4}He in momentum space

$$\rho_{NN}(p) = \frac{A(A-1)}{2(2J+1)} \langle \Psi^{JM} | \delta(\vec{p} - \vec{p}_{12}) | \Psi^{JM} \rangle$$

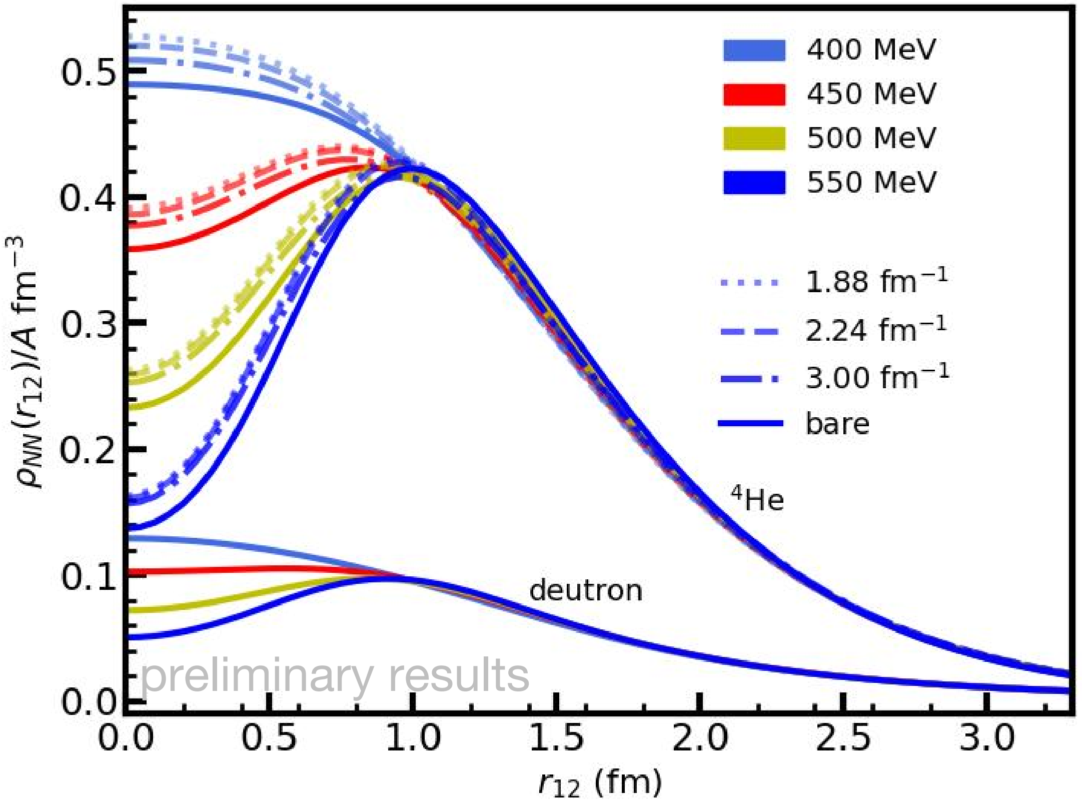
SMS N^{4}LO(450 MeV) + N^{2}LO,
ω = 16 MeV, N_{max}= 20 -- 28



- Expending the NCSM in the SRG-evolved two-body basis accurately reproduces the densities of bare interaction calculations
- In medium momentum part, there are some small discrepancies due to the absence of the three- and four-body part of unitary transformation operator
- The np-channel behavior agrees with the deuteron density, important to nuclear SRC

Two-body density in coordinate space

SMS N^{4}LO + N^{2}LO, ω = 16 MeV, N_{max}= 20 -- 28, J-NCSM

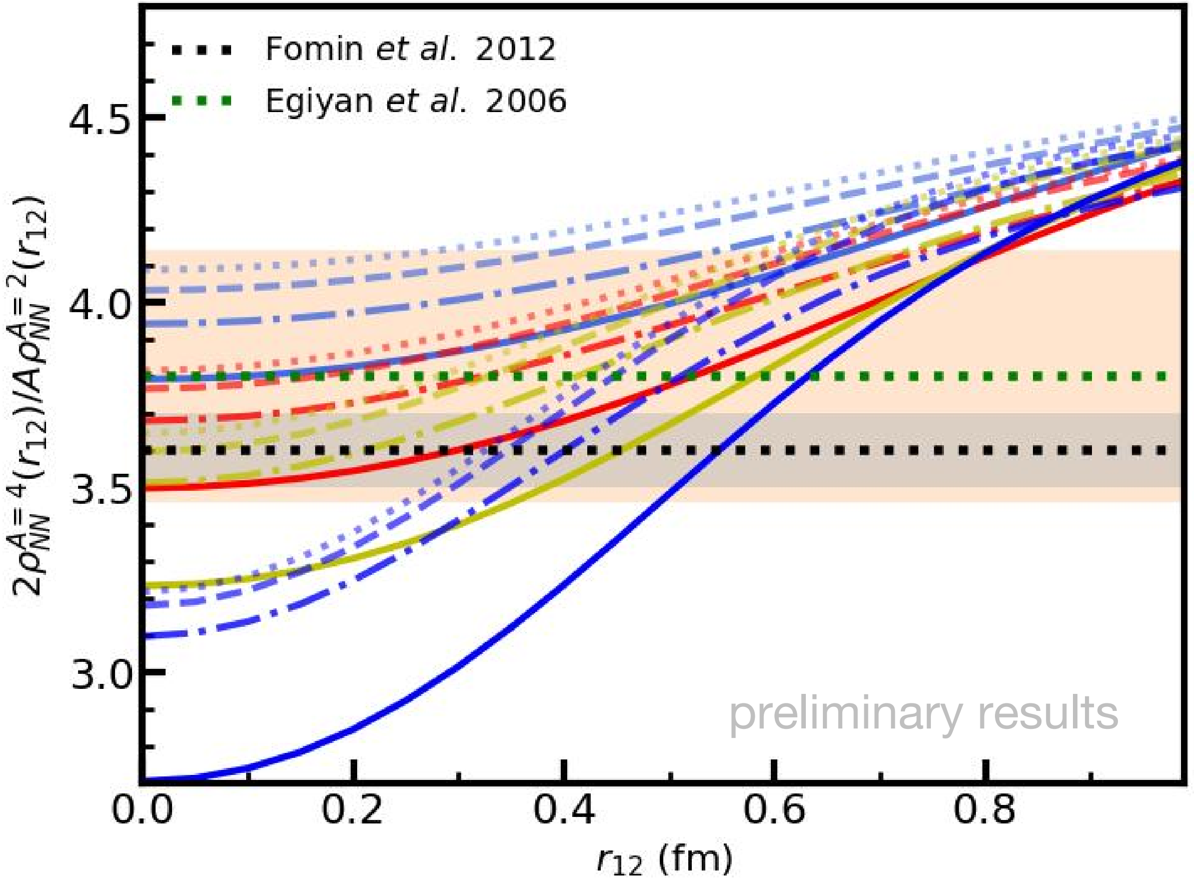


- Short-distance repulsive core depends on the interactions
- Densities with different SRG cutoffs are almost consistent with each other
- At $r_{12} \approx 1$ fm, all densities have similar values, scale and scheme independence of SRC

The scaling factor for nuclear SRC

$$a_2 \approx \lim_{r_{12}\to 0} \frac{2\rho_{NN}(A, r_{12})}{A\rho_{NN}(2, r_{12})}$$

Chen, Detmold, Lynn, Schwenk,
Phys. Rev. Lett. 119, 262502 (2017)



$$\frac{|a_2(\lambda = 1.88) - a_2(\text{bare})|}{a_2(\text{bare})}$$

- < 10% for $\Lambda = 400, 450$ MeV

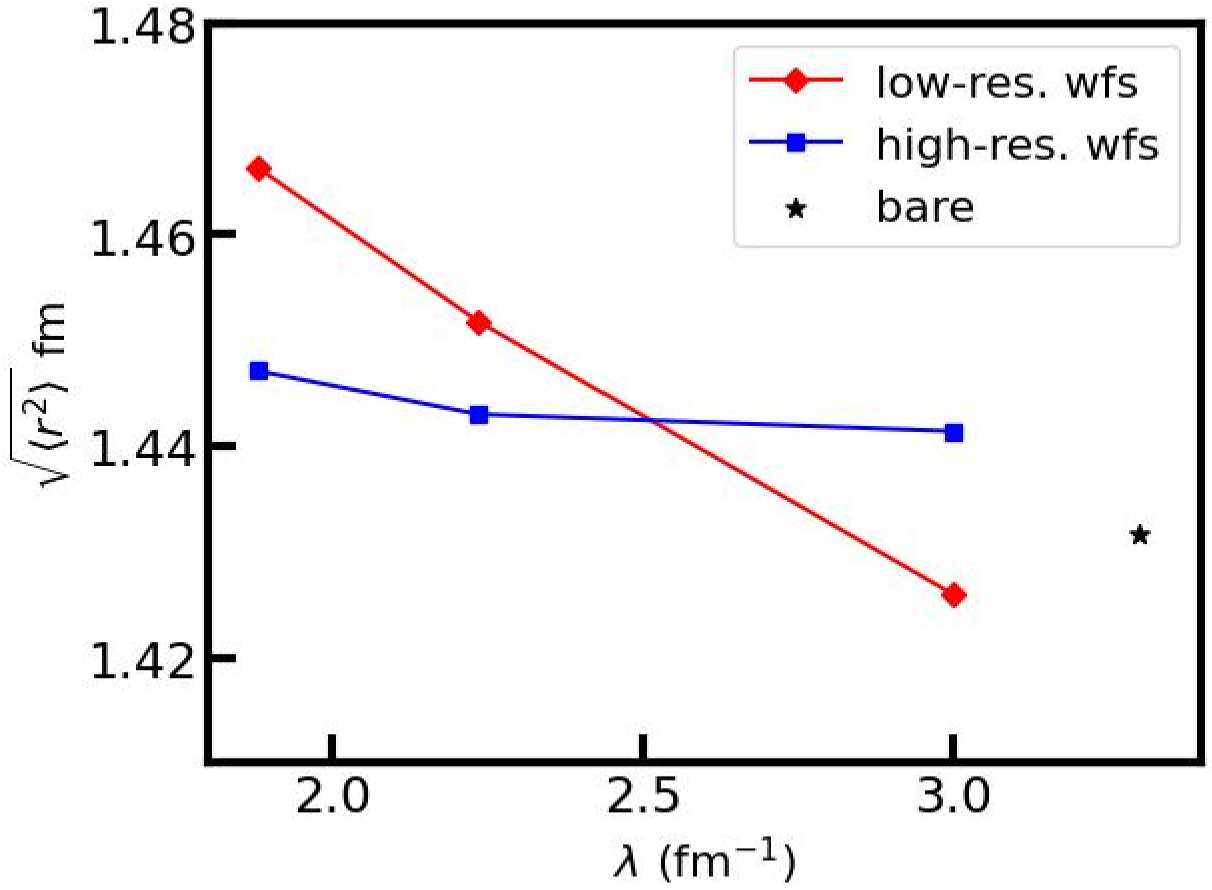
- SMS N[4]LO+ (450 MeV) is suitable for SRC of [4]He
- a_2 depends on flow parameters slightly
- Λ-dependence larger than λ-dependence

Nuclear SRC can be studied by the wavefunctions corresponding to softened interaction via our method !

rms matter radius of ^{4}He

SMS N^{4}LO + N^{2}LO, ω = 16 MeV, N_{max}= 20 -- 28, J-NCSM

$$\langle r^2 \rangle = \frac{1}{2A^2} \sum_{ij} r_{ij}^2$$



- The dependence of rms radius on SRG cutoffs almost disappears (~0.005 fm)
- Comparing with the bare results, still overestimated (~0.01 fm)
- The three-body part also influence the radius very slightly

Summary

- A new method can construct SRG-independent wfs (high-res.) based on low-resolution solution with a SRG evolved Jacobi two-body basis
- This method can is exact for the deutron wfs
- Well reproduce the two-body density of He4
- Sofened interaction solutions can also describe short-range correlations in He4
- The dependence of rms radius on SRG cutoff almost disappears

Perspective

- SRC in light nuclei with Chiral potentials
- Gobal study on the matter and charge radii of light nuclei with NCSM
- Influence of two-body currents on nuclear electromagnetic observables
- Include the three-body part of the unitary transformations
- ……