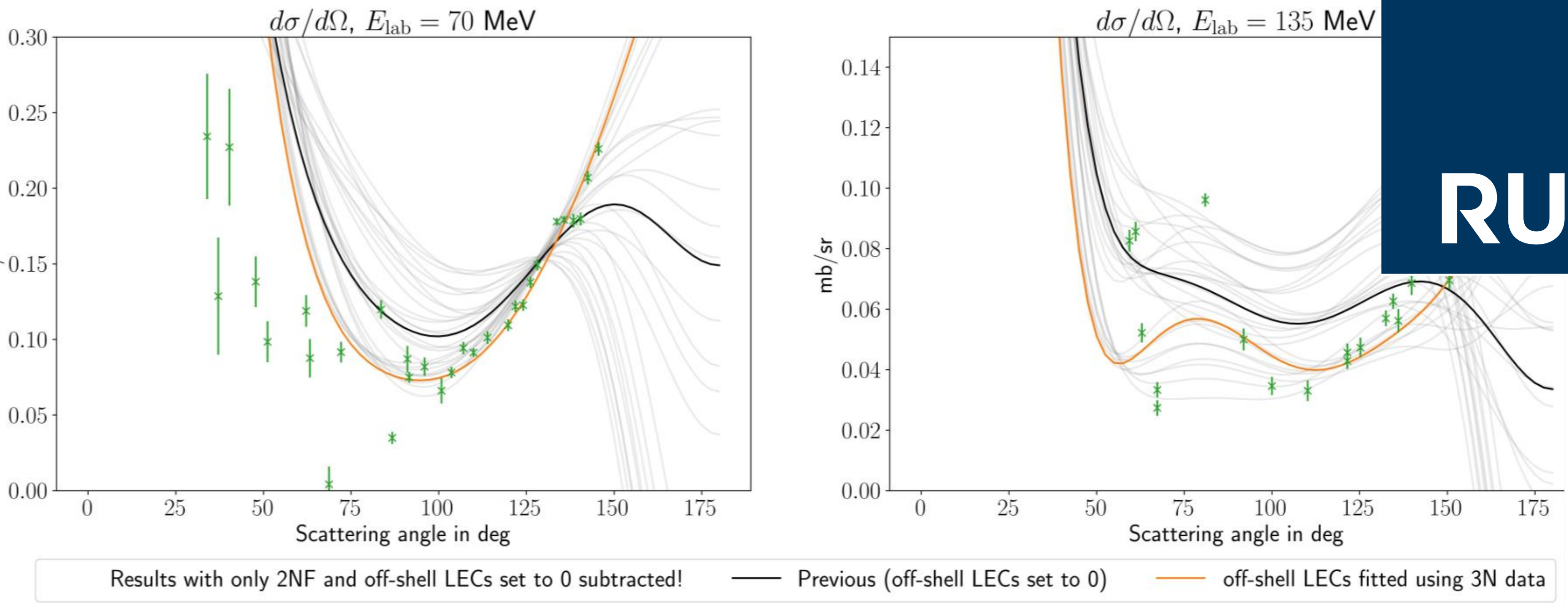

RUB

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CONSTRAINING THE TWO-NUCLEON FORCE IN CHIRAL EFT FROM THREE-NUCLEON DATA

Observing the unobservable off-shell behavior?

And: Smooth evolution from pion-full to pion-less theory

Sven Heihoff, Evgeny Epelbaum

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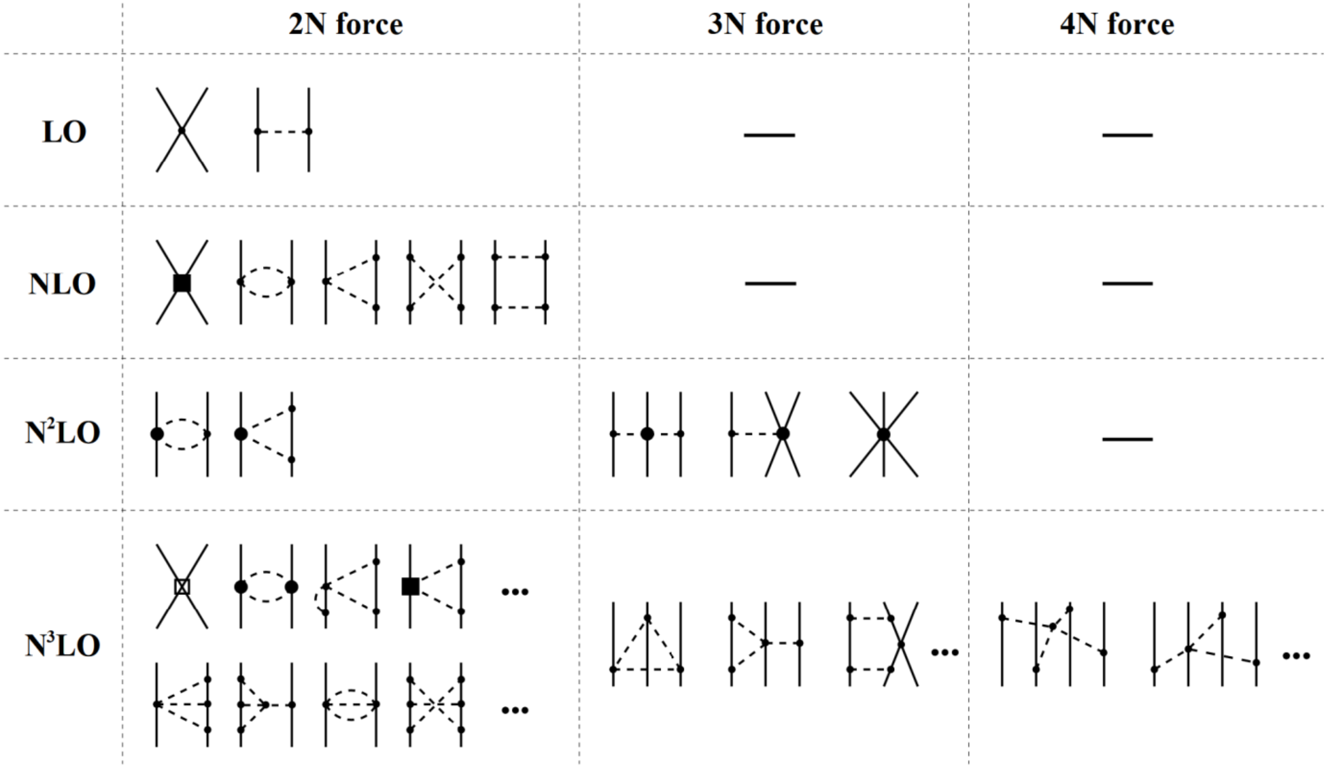
Introduction

- Chiral EFT is a low-energy effective theory of QCD
- Degrees of freedom: Pions and Nucleons
- Uses momenta and pion mass divided by the breakdown scale as expansion parameter



E. Epelbaum, Nuclear Forces from Chiral Effective Field Theory: A Primer (2010)

Introduction

- Chiral EFT potential comes with a-priori unknown Low-Energy-Constants (LECs)
  - Usually determined by fits to experimental data
  - At N^3LO three redundant LECs (so called off-shell LECs) appear in the 2N potential

- 2N potential in chiral EFT leads to high-precision description of 2N data
  (P. Reinert, et al., Eur. Phys. J. A 54, 86 (2018))

- 3N observables are not described precisely (large χ^2 for some observables and/or energies)
  - 2 Nucleon force makes up most of the 3N scattering amplitude
  - Need to push chiral expansion of three nucleon forces (3NF) to higher orders

- **In this talk:** Calculating 3NFs without calculating 3NFs and determining redundant LECs.

Off-shell Low-Energy Constants (off-shell LECs)

- 2N potential in chiral EFT in the order N^{3}LO in the $^1S_0$ partial wave:

$$\langle\, ^1S_0, p' | V^{(4)} |\, ^1S_0, p\rangle = D_{1S0}\, p^2\, p'^2 + D_{1S0}^{\text{off}}\, \left(p^2 - p'^2\right)^2 + \,\ldots$$

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- We can only observe $p = p'$ → D_{1S0}^{off} can not be determined by using 2N experimental data
- Off-shell LECs are somewhat similar to transformation angles γ_i of a unitary transformation

$$\widehat{U} = \exp\left(\gamma_1 \widehat{T}_1 + \gamma_2 \widehat{T}_2 + \gamma_3 \widehat{T}_3\right), \qquad \langle \vec{p}' | \widehat{T}_1 | \vec{p} \rangle = \frac{m_\text{N}}{2\Lambda_\text{b}^4} \left(\vec{p}'^2 - \vec{p}^2\right)/2$$

$$\langle \vec{p}' | \delta \widehat{H} | \vec{p} \rangle = \langle \vec{p}' | \widehat{U}^\dagger \widehat{H}^{(0)} \widehat{U} - \widehat{H}^{(0)} | \vec{p} \rangle = \Sigma_i\, \gamma_i \left\langle \vec{p}' \left| \left[\widehat{H}_\text{kin}^{(0)}, \widehat{T}_i \right] \right| \vec{p} \right\rangle + \cdots = \gamma_1 \frac{1}{\Lambda_\text{b}^4} \left(\left(\vec{p}'^2 - \vec{p}^2\right)/2 \right)^2 + \cdots$$

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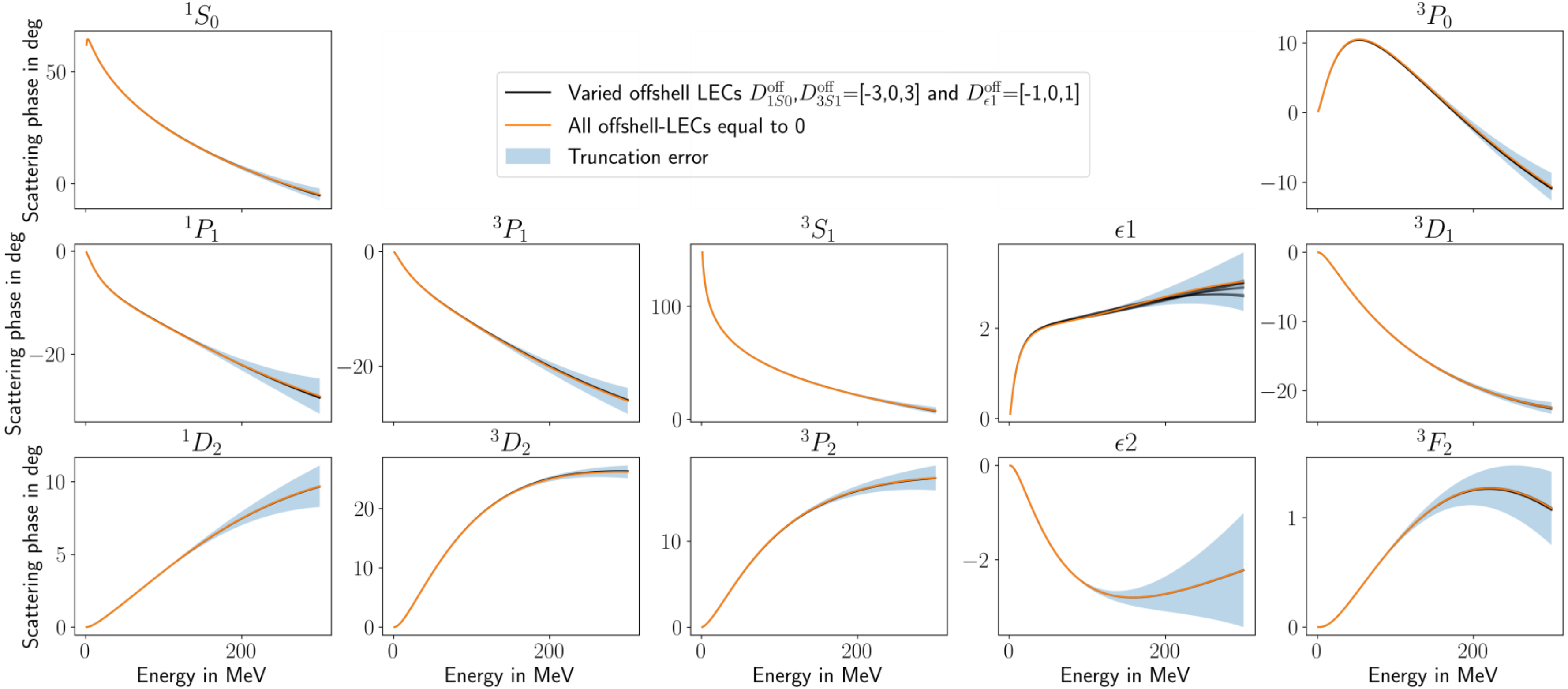
- **Conclusion:** Fixing off-shell LECs is equivalent to fixing arbitrary transformation angles!

Off-shell LECs do not affect 2N observables



Unitary Transformation in 3N Systems

- 3N forces are induced by the unitary transformation $\hat{U} = \exp\left(\gamma_1 \hat{T}_1 + \gamma_2 \hat{T}_2 + \gamma_3 \hat{T}_3\right)$

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$$= \gamma_1 \frac{\mathrm{m_N}}{4\Lambda_{\mathrm{b}}^4} |\vec{p}_3' - \vec{p}_3|^2 \left(C_S + C_T \vec{\sigma}_1 \vec{\sigma}_2\right) + \text{permutations} + \cdots$$

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- This is observable → γ_{i} can be determined → off-shell LECs can be determined!

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- This is observable → γ_i can be determined → off-shell LECs can be determined!
- Induced 3N forces appear at $\mathrm{N^3LO}$ and there are similar terms in $\mathrm{N^4LO}$
  - Determination necessary for complete $\mathrm{N^3LO}$ calculation
  - Going to infinite chiral order → all off-shell effects stay unobservable
- More generally: “Different off-shell extensions of two-body forces can be equivalently realized as three-body interactions.” (W. N. Polyzou and W. Glöckle, Few-Body Syst. 9, 97 (1990))

Emulator for the 3N Scattering Amplitude

- To obtain 3N scattering observables, Faddeev equation must be iterated

  → Problem: takes a lot of time

Emulator for the 3N Scattering Amplitude

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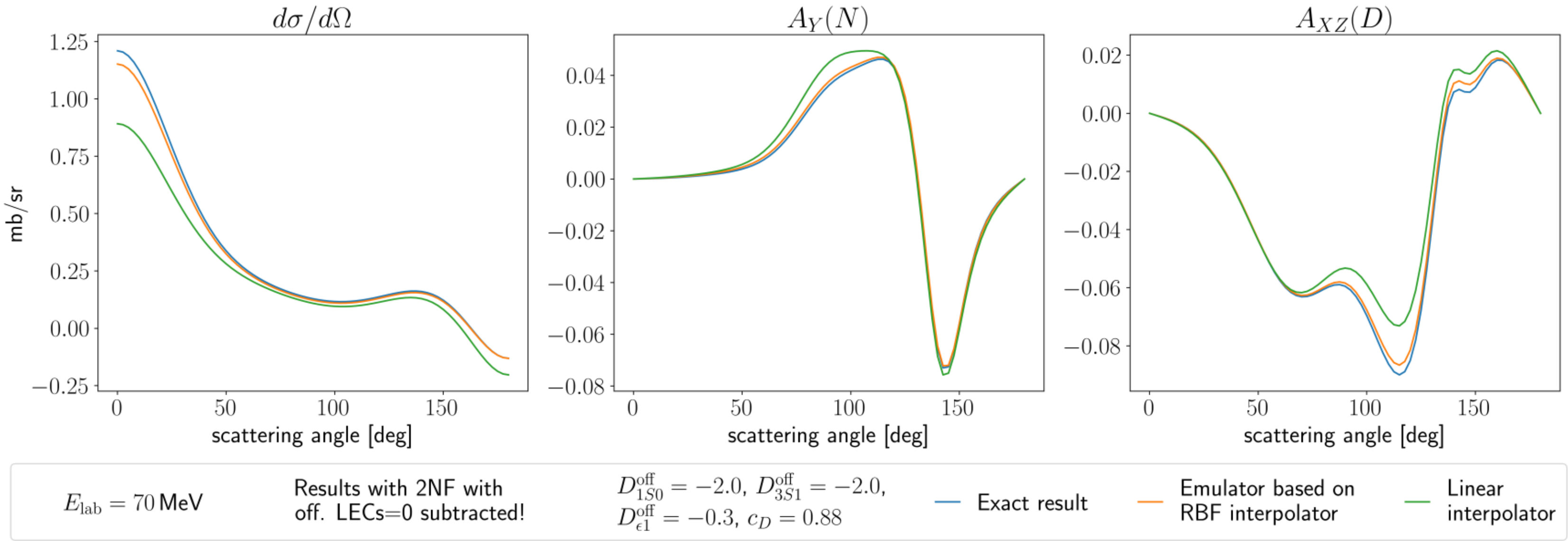
- Solution: use an emulator
  - Fitting off-shell LECs takes ~ 1 min instead of ~ 1 week
  - Cost: on average 3% error
  - Algorithm: radial basis function interpolation (RBF)
  - LECs sampled at 135 different combinations used as the basis mesh for interpolation
  - $c_D \in \{-5, -3, -1, 2, 5\}$, $D_{1S0}^{\text{off}}, D_{3S1}^{\text{off}} \in \{-3, 0, 3\}$ and $D_{\epsilon 1}^{\text{off}} \in \{-1, 0, 1\}$
  - Same procedure at four different energies (10, 70, 135 and 200 MeV)

Precision of the Emulator

- Emulator tested for seven different sets of randomly chosen LEC combinations
  → Averaged error for the differential cross-section is ~ 2%
- For comparison: linear interpolator ~ 15% error



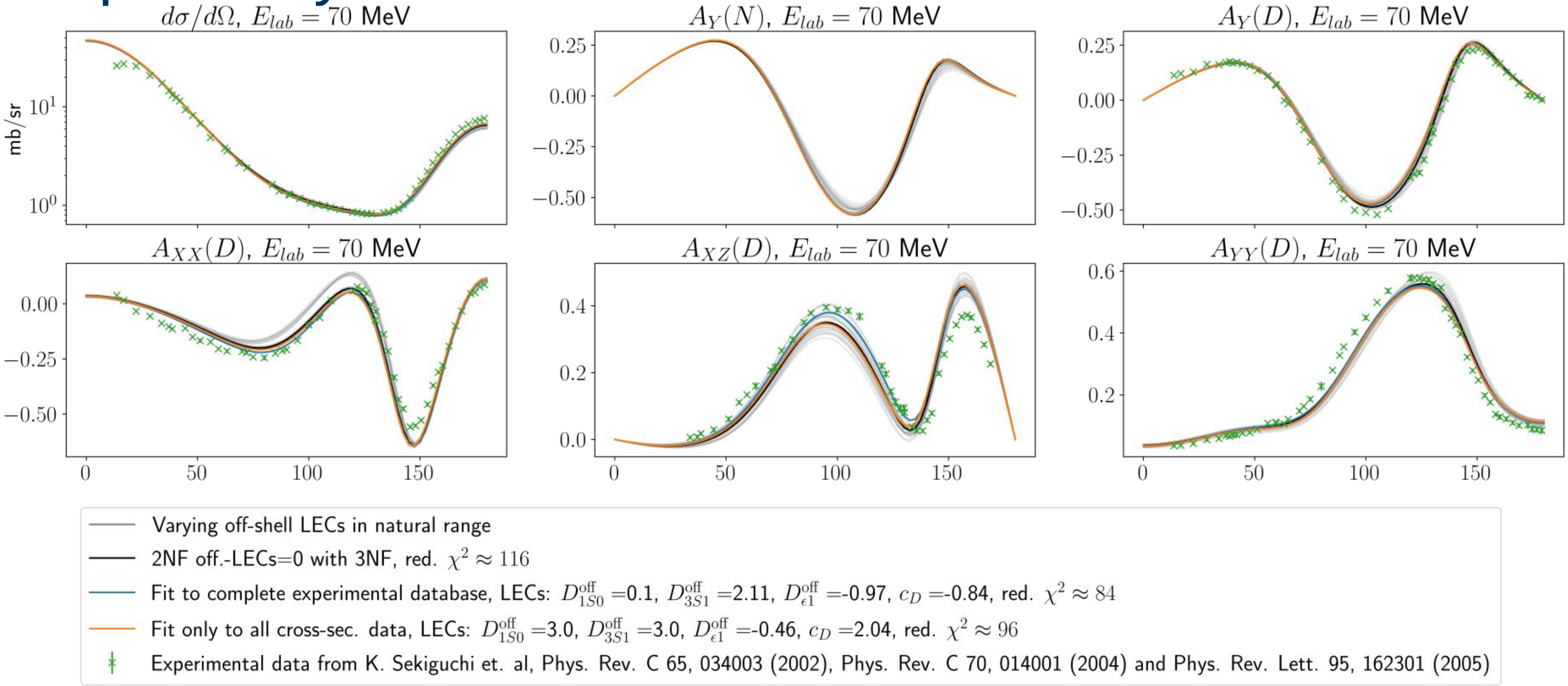
Fitting Procedure

- 2N potential at order N^4LO+ and 3N potential at order N^2LO

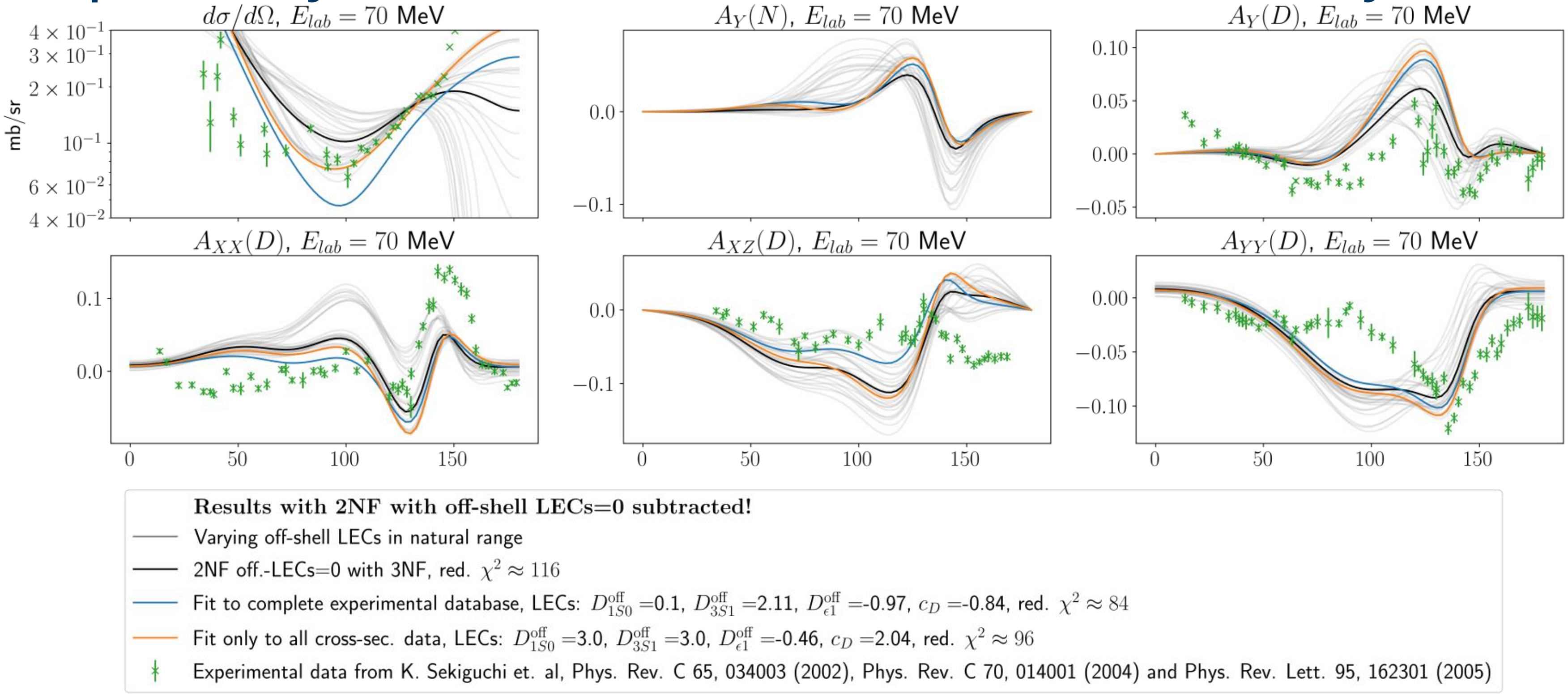
- 4 LECs to be fitted in total
  - 3 off-shell LECs from the 2N potential
  - 2 LECs (c_D and c_E) from the 3N potential, c_E is determined indirectly from Triton binding energy

- Experimental data from 3N scattering experiments at 10, 70, 135 and 200 MeV
  - Scattering angle > 40° to keep Coulomb effects small

Exploratory fits of off-shell LECs to 3N data



Exploratory fits – subtract 2NF for better visibility



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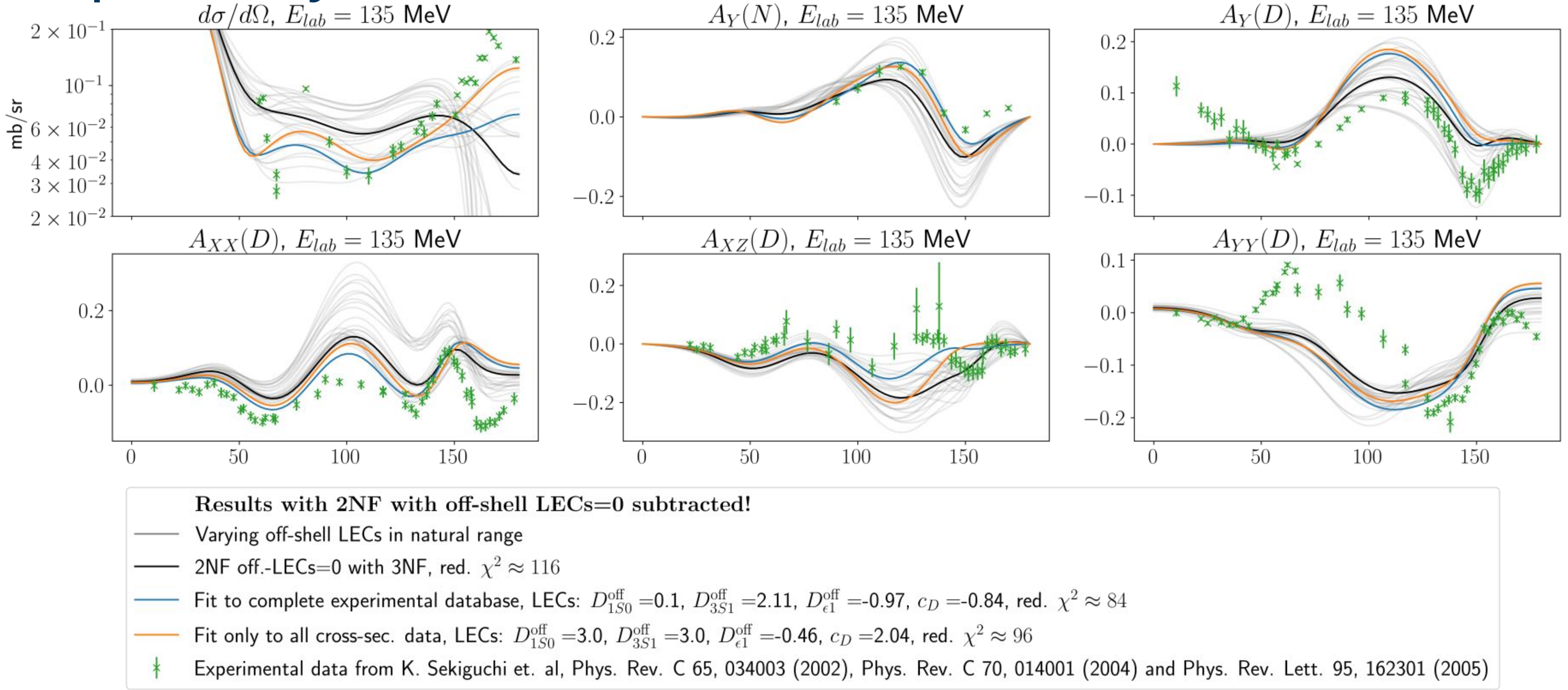
Exploratory fits 135MeV



Conclusion and Outlook

- Radial basis function interpolation is an efficient way to compute 3N scattering observables for LEC-fitting

- Tuning the off-shell LECs of the 2N potential improves description of 3N data
  - 3N data is not yet fully described → need to increase chiral order of 3NF

- Looking forward to do full N^3LO calculation of 3N observables → LENPIC

- Fits to 3N data can be extended
  - Including other data than scattering data (e.g. Triton beta decay) or theoretical uncertainties

- There are further generators of the unitary transformation, which vanish in 2N c.o.m frame
  (Girlanda et al. Phys. Rev. C 102, 064003(2020))

Evolving ChEFT into $\not\pi$−EFT

How much theory is in a model with 30 tunable parameters?

Too many fit parameters in 2N potential?

- Chiral EFT consists of
  - Long-range potential based on pion-exchange
  - Short-range potential based on contact interactions of unknown strength, so-called LECs, in total: 30

- "With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.“
  - John von Neumann

- 30 fit parameters (LECs) are needed for a sufficient description of 2N data

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- 30 fit parameters (LECs) are needed for a sufficient description of 2N data
  → Can we describe a herd of elephants having a tea party on the moon?

- LECs are not completely arbitrary, constrained by naturalness assumption and hierarchy scheme

Fits without 2π exchange

- Excluding highest order long-range potential

- Fit result for 450MeV cutoff
  - np and pp scattering data, [0, 280] MeV lab energy
  - Red. χ^2 with 2π exchange: 1.006
  - Red. χ^2 without 2π exchange: 1.9 deviates ~ 45σ from 1!

  → 30 LECs are not enough to describe the data

- We expect no sufficient description because of:
  - Branching cut in scattering amplitude according to 2π exchange

  → To find a good description of 2N data, i.e. red. $\chi^2 \leq 1 + \sigma$, energy range of database must be limited:
  Cutoff: 450MeV, upper lab energy limit: 95MeV, Cutoff 400MeV, upper lab energy limit 135MeV

Evolving the cutoff – evolving the LECs?

- Wilsonian point of view, Renormalization group evolution of LECs

  → Rescaling of LECs

- Evolving the cutoff – going from 450MeV down to ~130MeV
  - "Smoothly" integrating out the pions from our theory

    → Going from a pion-full theory to a pion-less theory

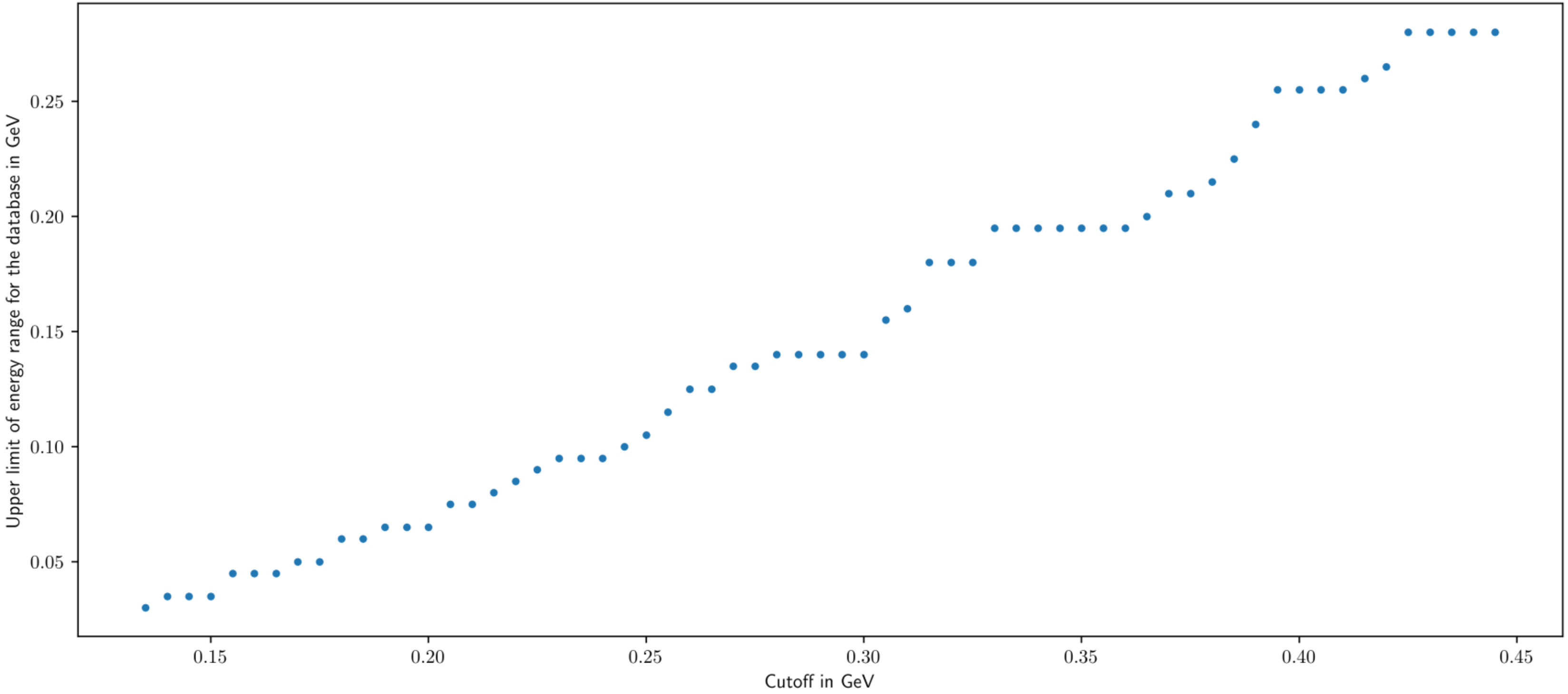
Evolving the cutoff – method

- Derive potential for full database and different cutoffs, varied in steps of 5MeV

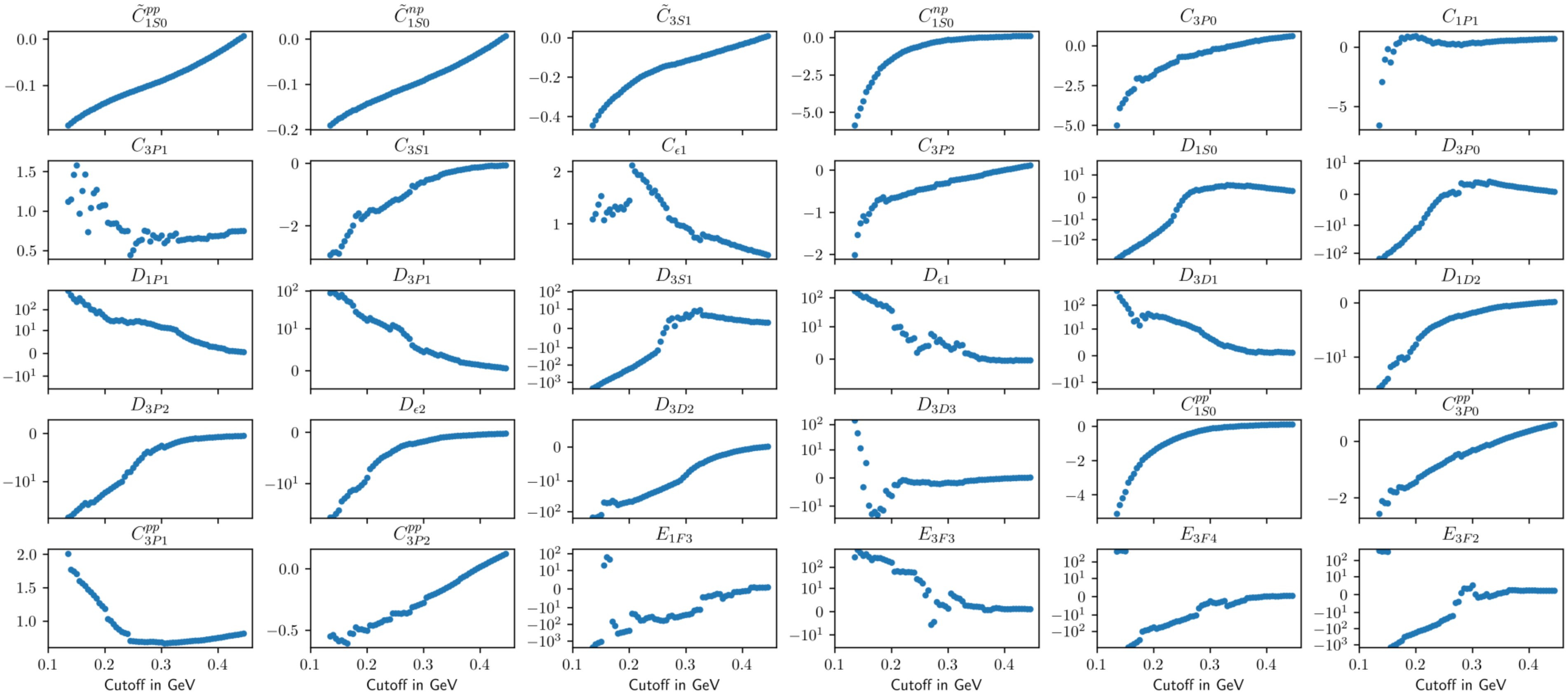
- Start with cutoff = 450MeV
  - Fit the 30 LECs to a database, containing experiments with lab energy between 0 and x MeV
  - Check if red. $\chi^2 \leq 1 + \sigma = 1 + \sqrt{\frac{2}{N_{dat}}}$
  - If yes: continue to the next lower cutoff
  - If no: lower the upper energy range limit by 5MeV and repeat

- Fit:
  - Starting point for LECs: solution from next-higher cutoff value
  - Two constraints: Deuteron binding energy and coherent scattering length

Which cutoff can describe which energy range?

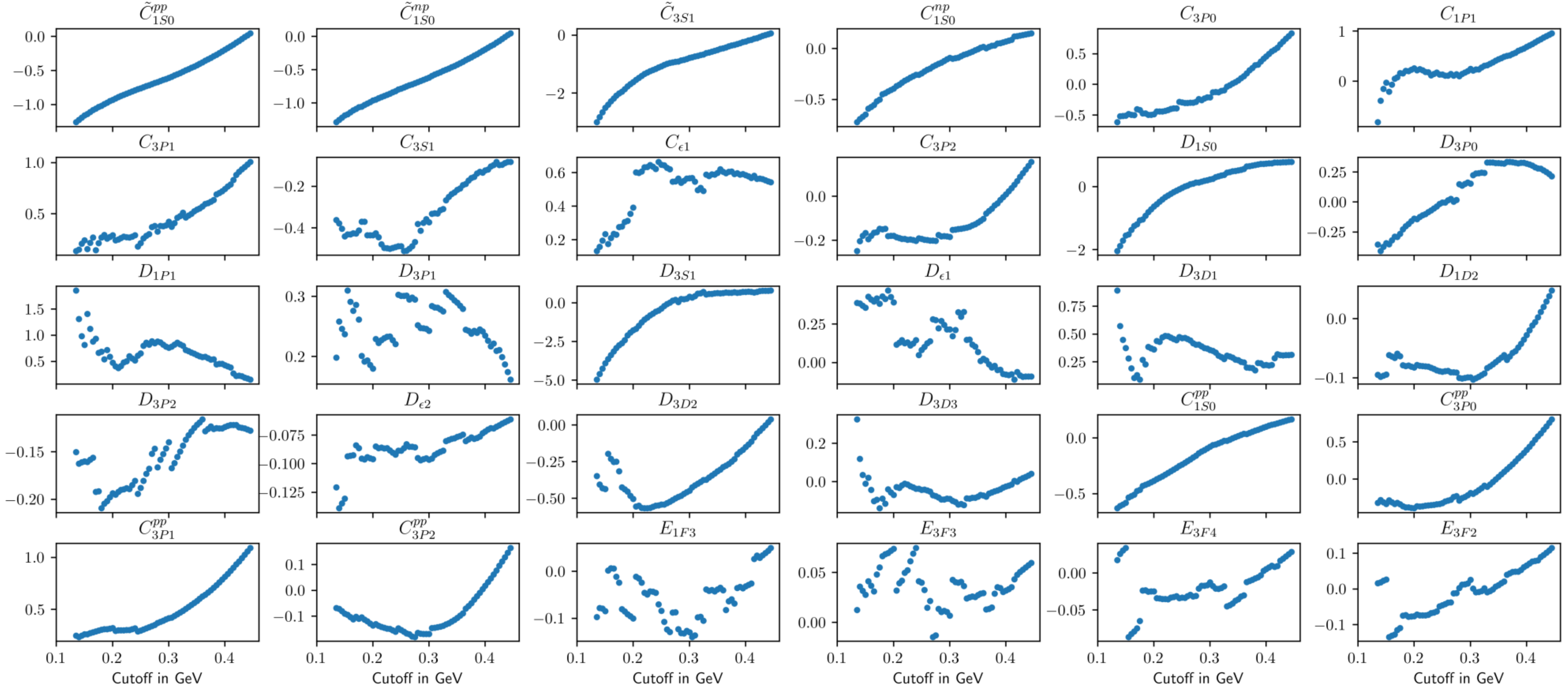


Evolution of LECs



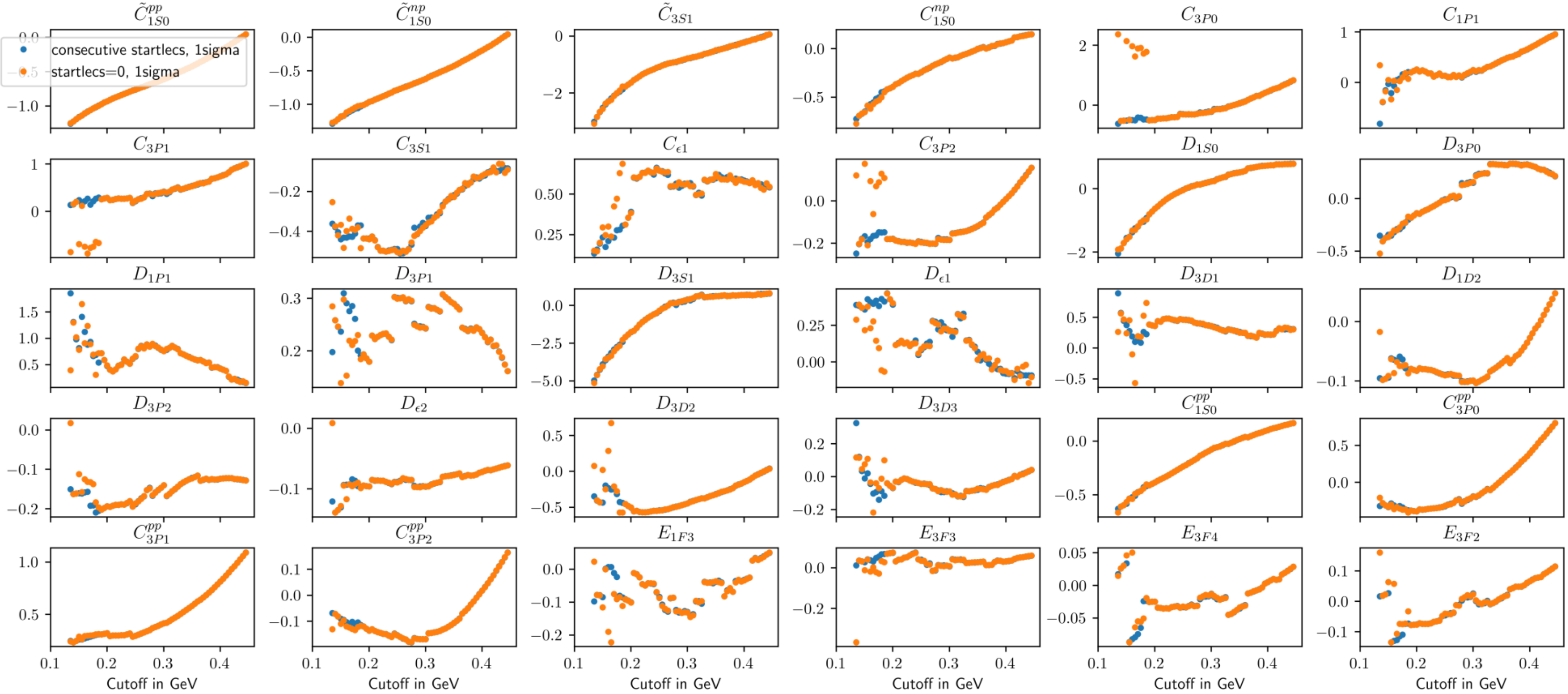
Evolution of LECs divided by their natural size

- Natural size of LECs: $\sim 4\pi/(F_\pi^2\ \Lambda_b^n)$, where $\Lambda_b = 600\mathrm{MeV}$ is the breakdown scale, here replaced by cutoff



Evolution of LECs – starting value always 0

- Are we chasing an arbitrary minimum of χ^2 so far?



Conclusion and Outlook

- The pure contact potential is not enough for a good description of 2N data for lab energies up to 280MeV
  - 30 parameters are not enough for an elephant tea party at the moon

- Most LECs evolve “smoothly and continuously” with the cutoff
  - Especially true for LECs appearing at leading order in chiral expansion and low partial wave numbers
  - Not so true for LECs appearing at the higher orders

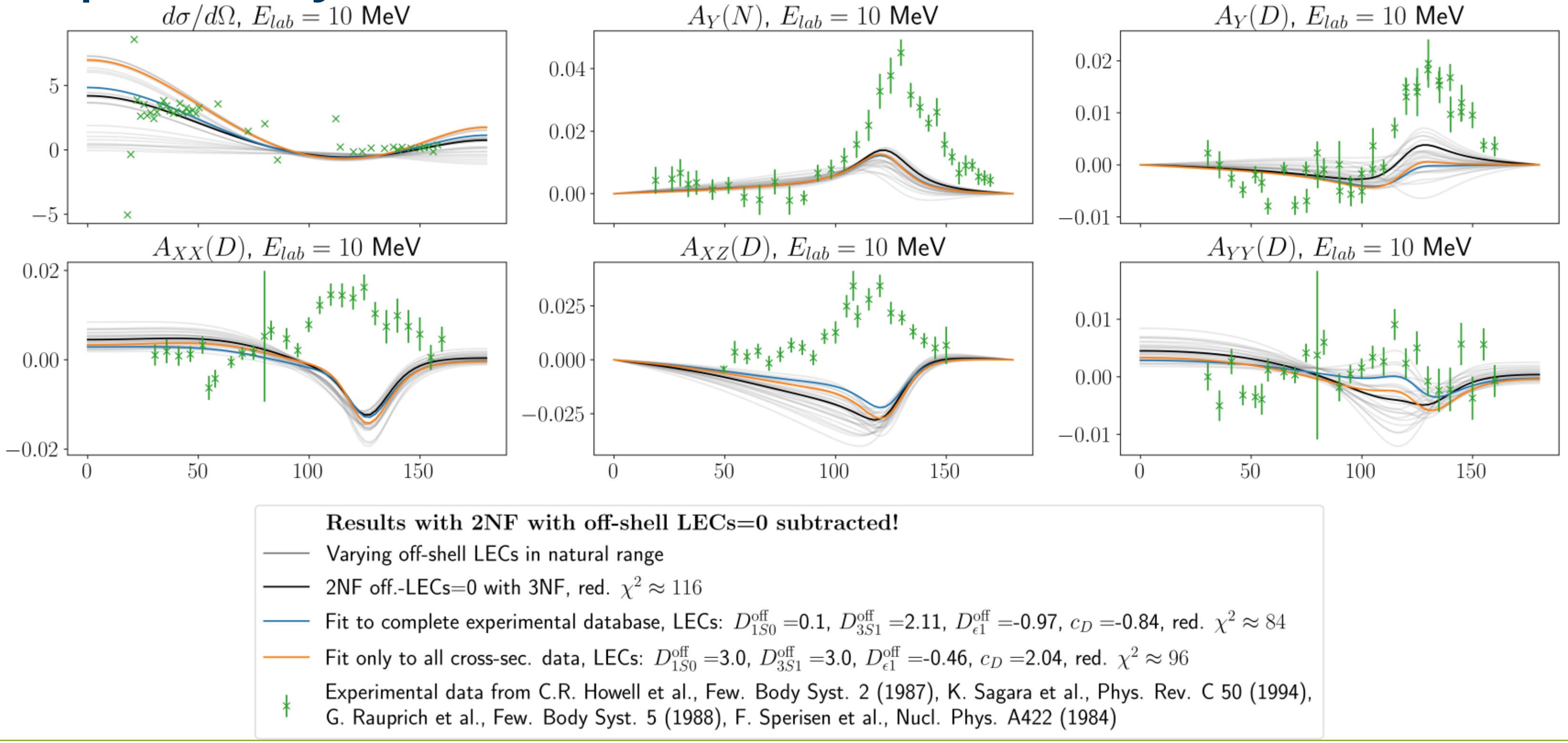
  → Not enough data for small energy ranges to constrain these

- Smooth evolution into a pionless theory → LECs are not just random and arbitrary fit parameters

- Investigate uncertainty at low energies

Backup slides
More results for the 3N fits

Exploratory fits 10MeV



Exploratory fits 200MeV