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Tritium β-decay with LENPIC interactions

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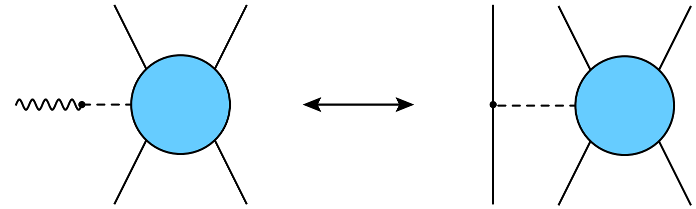
- Motivation and introduction
- Axial currents in χEFT: The status
- Regularized axial currents at N^2LO
- Calculations and benchmarks
- Summary and outlook

erc European Research Council
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen
ERATO Exploratory Research for Advanced Technology

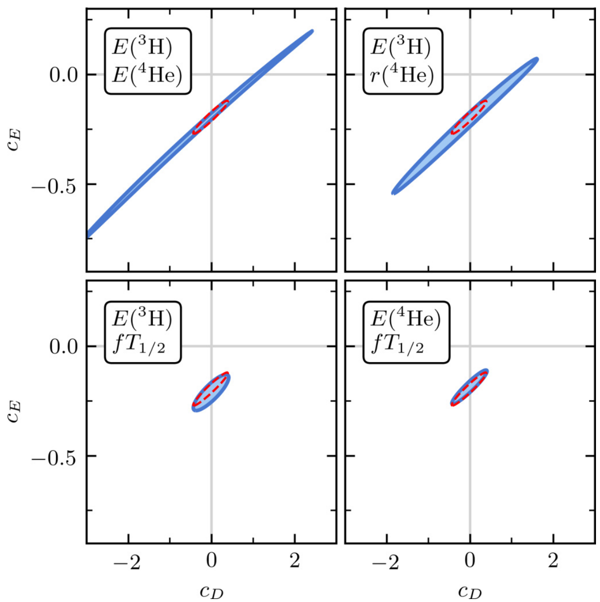
Motivation

- Well-known relationship between axial currents and 3NF
- The GT ME is uncorrelated with the ^{3}H BE

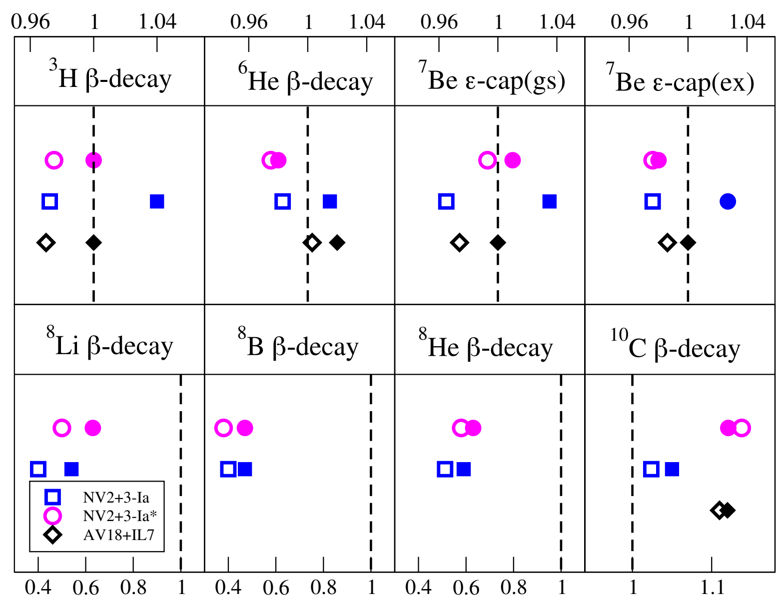


⇒ ^{3}H β-decay can be used to fix c_D

- 2-body axial currents are claimed to improve the description of the GT ME in nuclei



Wesolowski et al., 2021



King et al., 2020

⇒ Study ^{3}H β-decay using LENPIC interactions: Convergence? A reliable determination of c_D?

Phenomenology of ^{3}H β-decay

Tritium half-life $\underbrace{(1+\delta_R)tf_V}_{\substack{\text{radiative corrections:}\\ \delta_R = 1.9\,\%}} = \dfrac{K/G_V^2}{\langle \boldsymbol{F}\rangle^2 + 3\underbrace{f_A/f_V}_{\substack{\text{Fermi functions}\\ \text{(known numerically)}}} g_A^2 \langle \boldsymbol{GT}\rangle^2}$, with $K = 2\pi^3 \ln 2/m_e^5$ and $G_V = G_F V_{ud}$ [see e.g., Hardy, Towner, PRL 94 (2005) 092502]

Is calculable from the GT and F MEs that read (in the 1-body limit):

$$\langle \boldsymbol{F}\rangle = \left\langle {}^3\text{He} \middle\| \sum_i \tau_{i,+} \middle\| {}^3\text{H} \right\rangle$$

← would be 1 in the isospin limit. For 2NF@N^{4}LO$^+$ combined with 3NF@N^{2}LO: $\langle \boldsymbol{F}\rangle = 1.0001 \ldots 1.0013$

$$\langle \boldsymbol{GT}\rangle = \frac{1}{\sqrt{3}} \left\langle {}^3\text{He} \middle\| \sum_i \vec{\sigma}_i \tau_{i,+} \middle\| {}^3\text{H} \right\rangle$$

← controls the ^{3}H life time

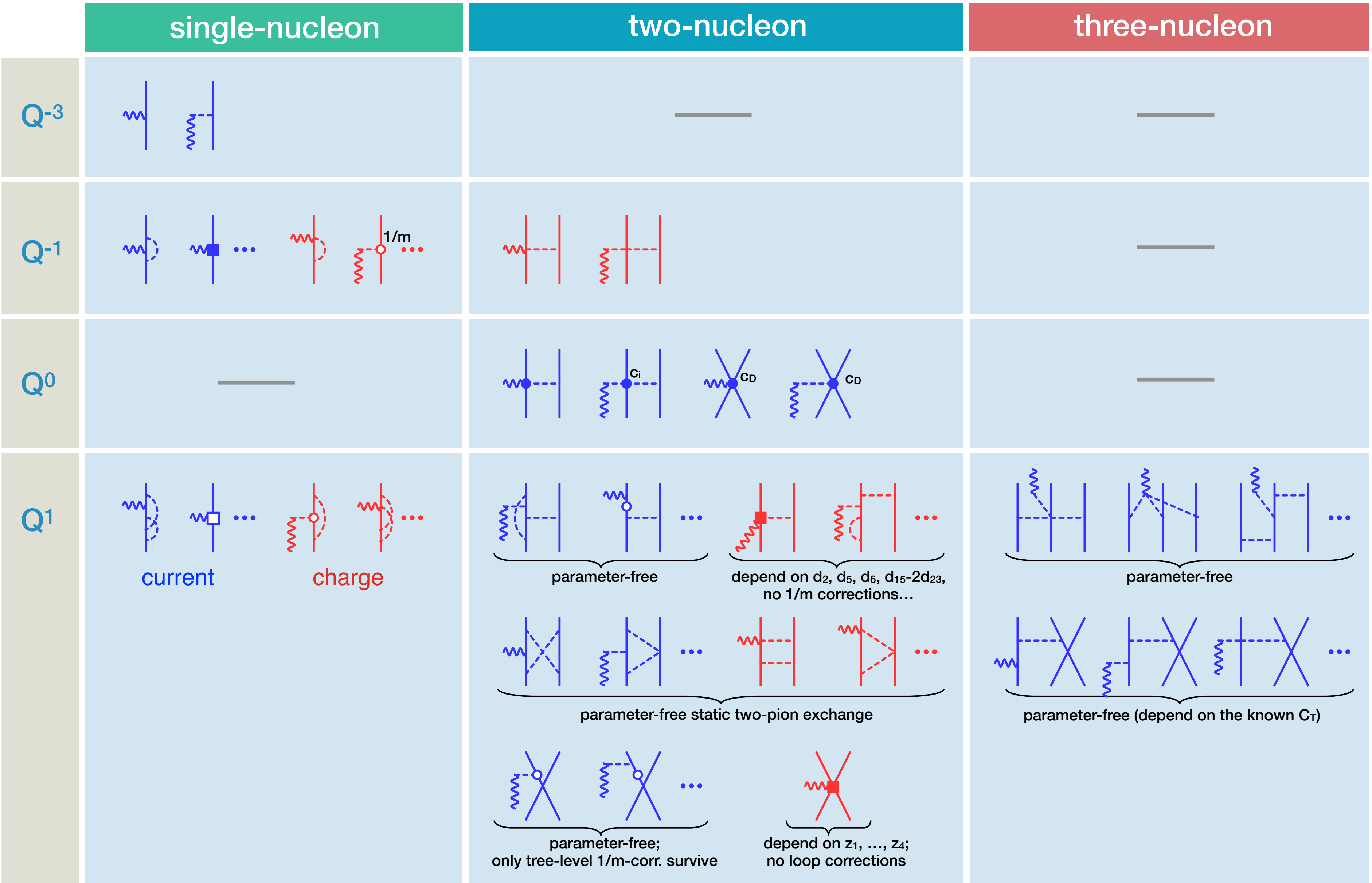
Using the experimental values $(1+\delta_R)tf_V = 1134.6 \pm 3.1$ s [Simpson, PRC 35 (1987) 752] and $g_A = 1.2756 \pm 0.0013$ [PDG20], one finds the empirical value for the GT ME:

$$\langle \boldsymbol{GT}\rangle_{\text{emp}} = 0.9484 \pm 0.0019$$

This ME is what we are going to calculate in chiral EFT.

Chiral expansion of axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317



Chiral expansion of axial currents

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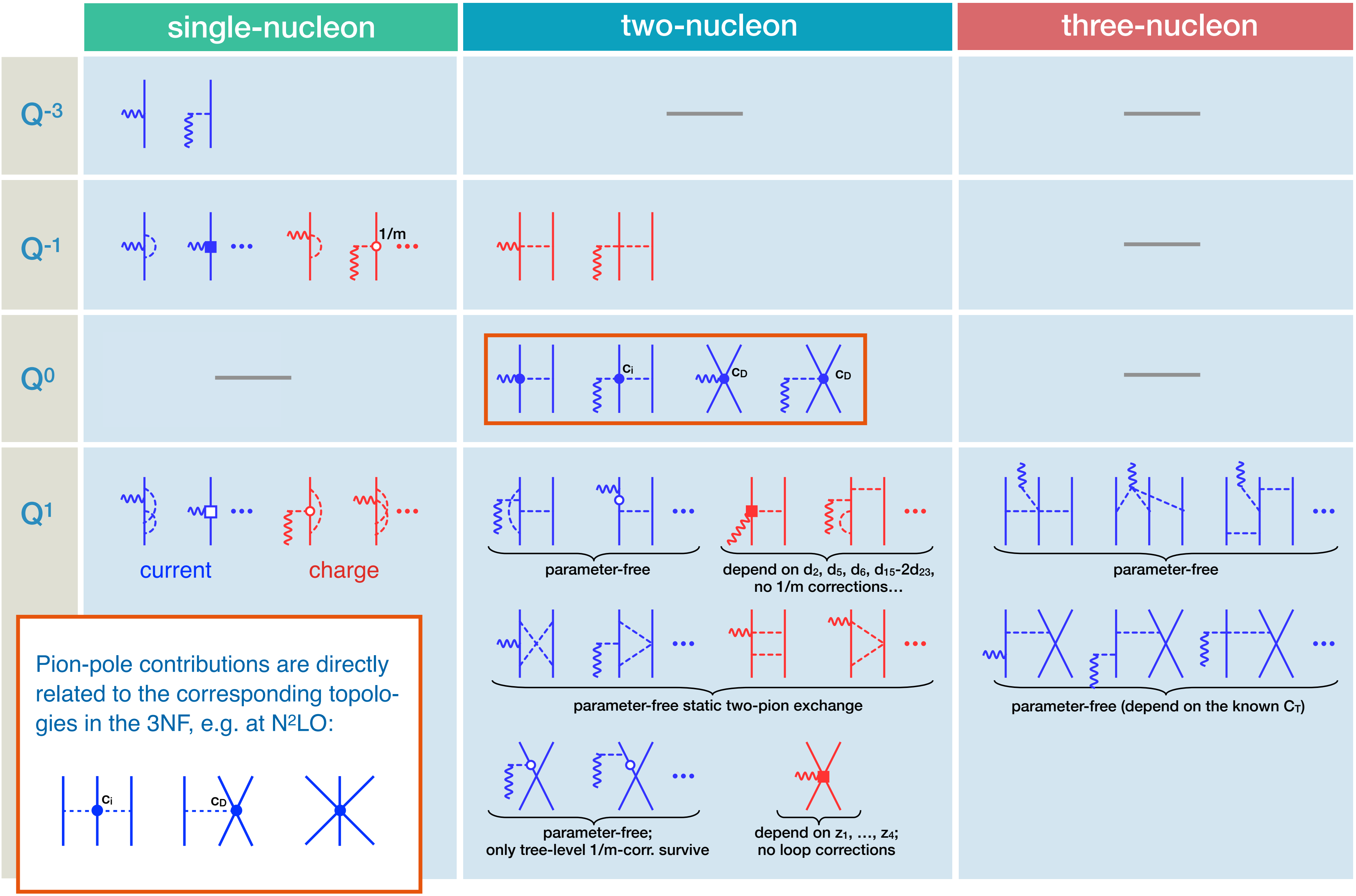

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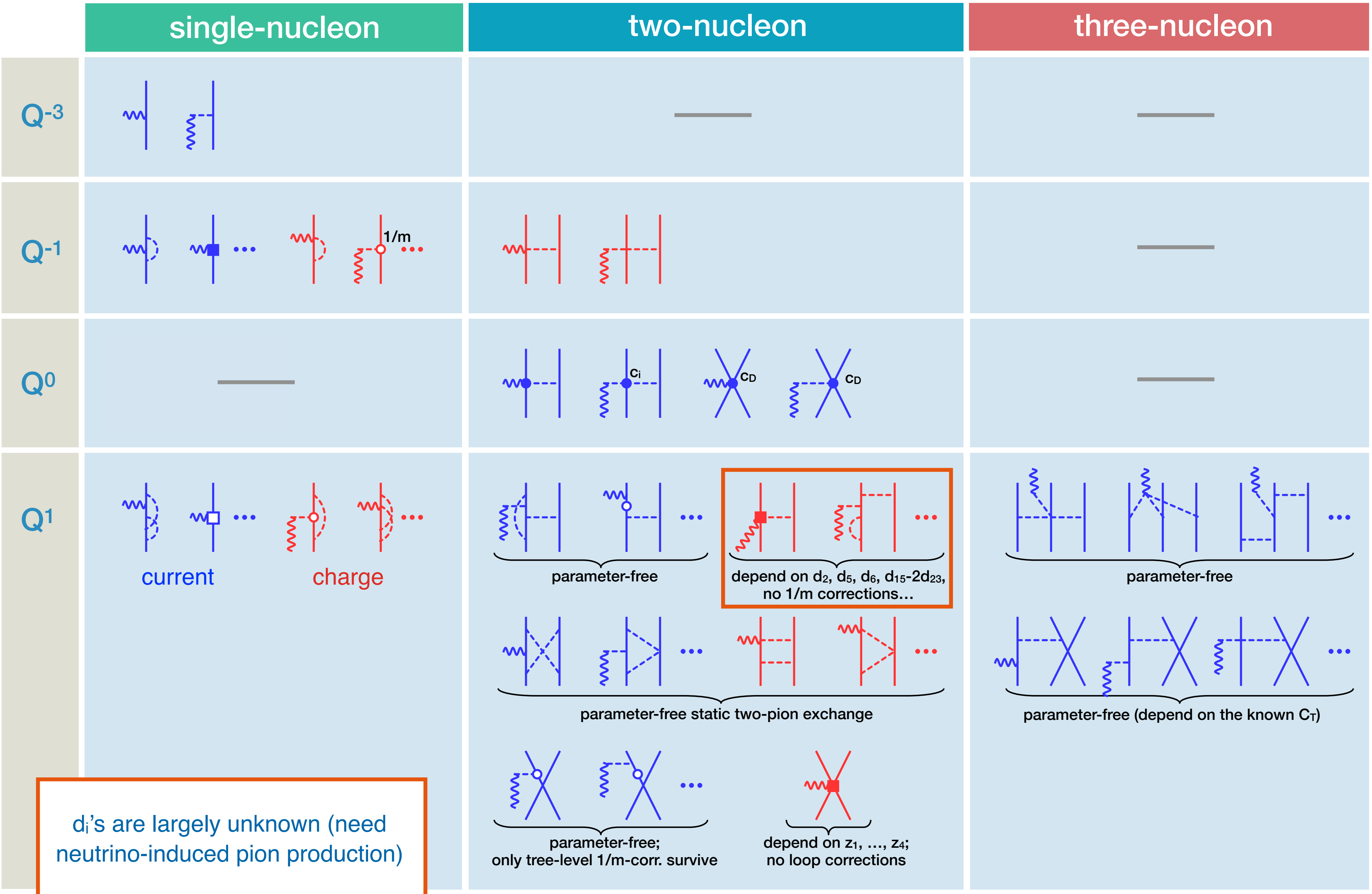
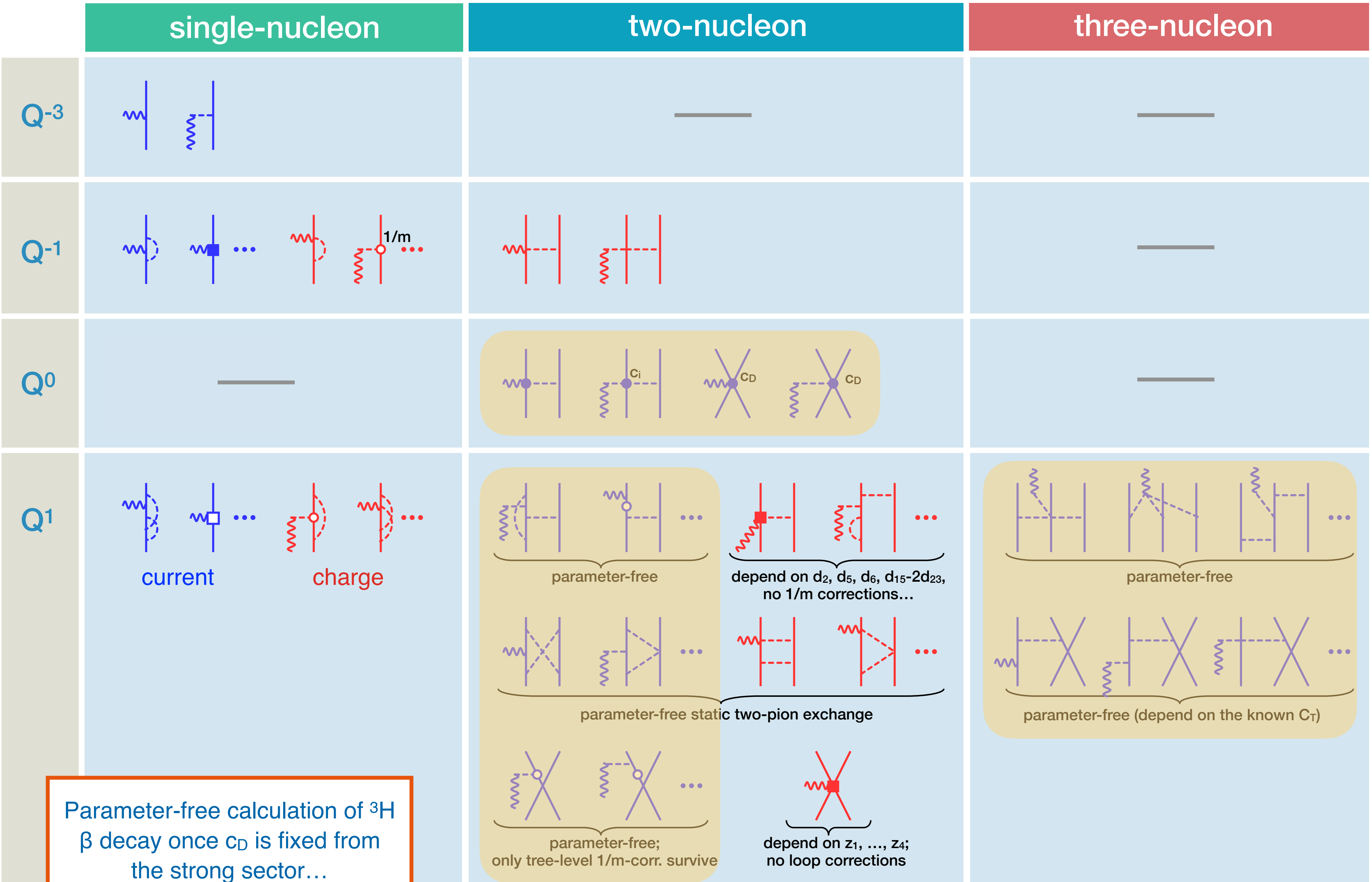


Chiral expansion of axial currents

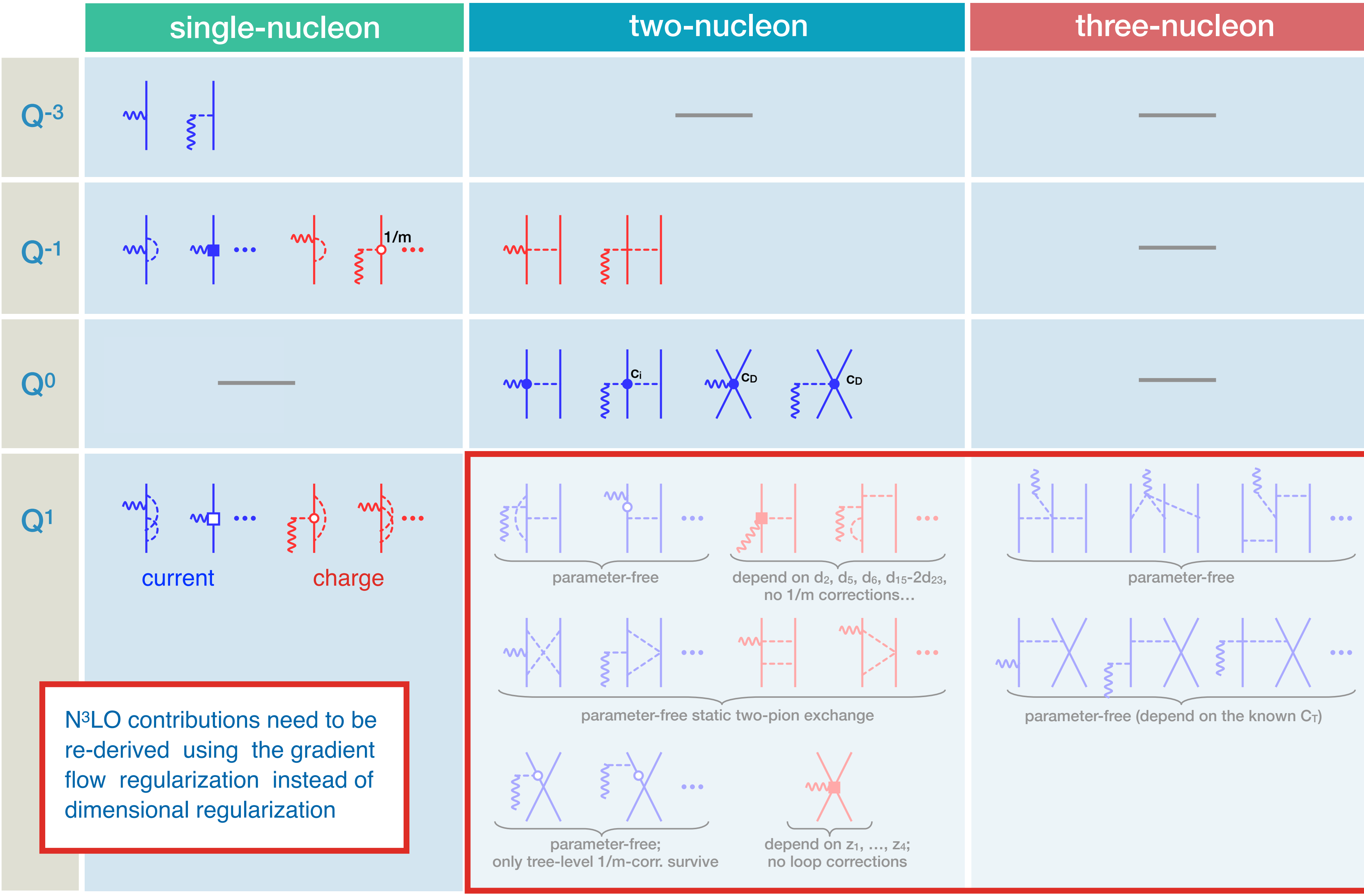
Krebs, EE, Meißner, Annals Phys. 378 (2017) 317



Parameter-free calculation of ^3H β decay once c_D is fixed from the strong sector…

Chiral expansion of axial currents

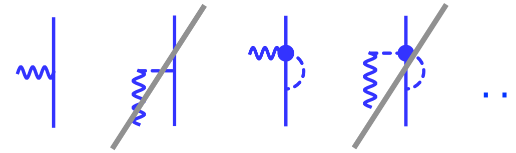
Krebs, EE, Meißner, Annals Phys. 378 (2017) 317



N³LO contributions need to be re-derived using the gradient flow regularization instead of dimensional regularization

One-body contributions to the GT ME

- Because of the kinematics of the ^{3}H β-decay, no pion-pole contributions need to be considered



- Similarly to the e.m. currents, 1-body axial currents are expressible in terms of FFs:

$$\boldsymbol{A}^0_{\rm 1N} = -\frac{G_A(-\vec{k}^2)}{2m}\boldsymbol{\tau}_i \vec{k}_i\cdot\vec{\sigma}_i + \frac{G_P(-\vec{k}^2)}{8m^2}\boldsymbol{\tau}_i k_0\,\vec{k}\cdot\vec{\sigma}_i = -\frac{g_A}{2m}\boldsymbol{\tau}_i\vec{k}_i\cdot\vec{\sigma}_i + \ldots$$

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

$$\vec{\boldsymbol{A}}_{\rm 1N} = -\frac{G_A(-\vec{k}^2)}{2}\boldsymbol{\tau}_i\vec{\sigma}_i + \frac{G_P(-\vec{k}^2)}{8m^2}\boldsymbol{\tau}_i\vec{k}\,\vec{k}\cdot\vec{\sigma}_i + \underbrace{\vec{\boldsymbol{A}}^{(Q)}_{{\rm 1N:}\,1/m,{\rm UT}'}}_{\text{pion-pole contribution}} + \vec{\boldsymbol{A}}^{(Q)}_{{\rm 1N:}\,1/{\rm m}^2} = -\frac{g_A}{2}\boldsymbol{\tau}_i\vec{\sigma}_i + \underbrace{\ldots}_{1/m^2 \text{ corrections (N}^3\text{LO)}}$$

- The GT matrix element $\times\, 10^2$ calculated without MECs using the SMS 2NFs

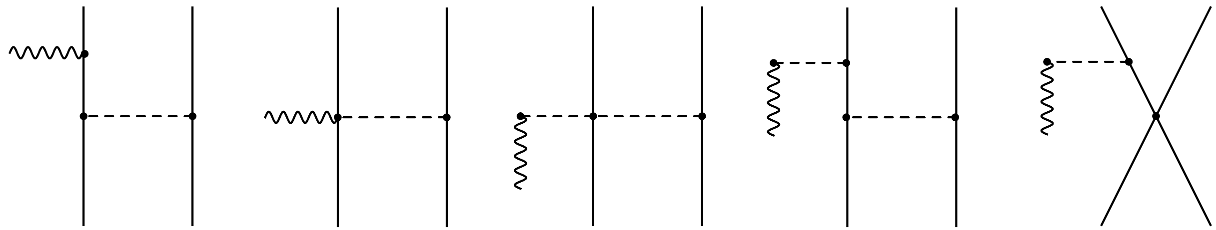
	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
2NF at LO	96.73	96.15	95.45	94.64
2NF at NLO	94.52	93.99	93.52	93.04
2NF at N^{2}LO	93.88	93.08	92.28	91.44
2NF at N^{3}LO	93.63	93.12	92.64	92.23
2NF at N^{4}LO	93.83	93.32	92.83	92.44
2NF at N^{4}LO$^+$	93.78	93.23	92.73	92.31

To compare, one finds 92.24 for AV18 and 93.63 (93.22) for EM N^{3}LO 500 (600)

To recall: $\langle \boldsymbol{GT}\rangle_{\rm emp} = 0.9484 \pm 0.0019$

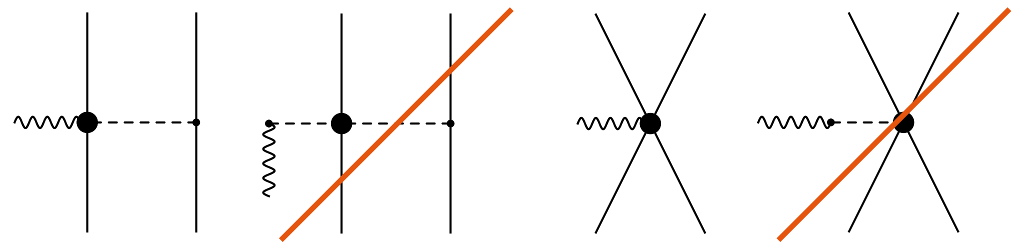
Two-body currents

NLO (Q^{-1}):



lead to a non-vanishing axial charge density

N^{2}LO (Q^0):



generate the first contribution to the current density

Un-regularized expressions for the current density:

$$\vec{\boldsymbol{A}}^{(Q^0)}_{2\mathrm{N}:\,1\pi} = \frac{g_A}{2F_\pi^2}\frac{\vec\sigma_1\cdot\vec q_1}{q_1^2+M_\pi^2}\bigg\{\boldsymbol{\tau}_1\bigg[-4c_1M_\pi^2\frac{\vec k}{k^2+M_\pi^2}+2c_3\bigg(\vec q_1-\frac{\vec k\,\vec k\cdot\vec q_1}{k^2+M_\pi^2}\bigg)\bigg]+c_4\,\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\bigg(\vec q_1\times\vec\sigma_2-\frac{\vec k\,\vec k\cdot\vec q_1\times\vec\sigma_2}{k^2+M_\pi^2}\bigg)$$
$$-\frac{\kappa_v}{4m}\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\,\vec k\times\vec\sigma_2\bigg\}-\frac{1}{4}D\,\boldsymbol{\tau}_1\bigg(\vec\sigma_1-\frac{\vec k\,\vec\sigma_1\cdot\vec k}{k^2+M_\pi^2}\bigg)+1\leftrightarrow 2\,,$$

does not contribute for $k=0$ too

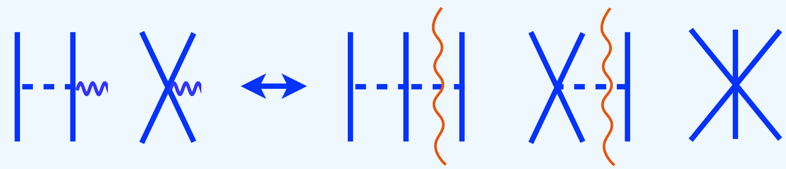
pion poles do not contribute…

Two-body currents: Regularization

SMS regularization of the current (only terms that survive for $k=0$ are shown):

$$\vec{\boldsymbol{A}}^{(Q^0)}_{\text{2N, reg.}} = \frac{g_A}{2F_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\, e^{-\frac{q_1^2+M_\pi^2}{\Lambda^2}}\Big(2c_3\,\boldsymbol{\tau}_1\, q_1 + c_4\,\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\,\vec{q}_1\times\vec{\sigma}_2\Big) - \frac{1}{4}D\,\boldsymbol{\tau}_1\,\vec{\sigma}_1\, e^{-\frac{p^2+p'^2}{\Lambda^2}}$$
$$+ \underbrace{\frac{g_A}{2F_\pi^2}\,C\, e^{-\frac{q_1^2+M_\pi^2}{\Lambda^2}}\Big(2c_3\boldsymbol{\tau}_1\,\vec{\sigma}_1 + c_4\,\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\,\vec{\sigma}_1\times\vec{\sigma}_2\Big)}_{\text{subtraction}\quad C\,=\,-\frac{\Lambda(\Lambda^2-2M_\pi^2)+2\sqrt{\pi}M_\pi^3 e^{\frac{M_\pi^2}{\Lambda^2}}\operatorname{erfc}\left(\frac{M_\pi}{\Lambda}\right)}{3\Lambda^3}} + 1\leftrightarrow 2\,.$$

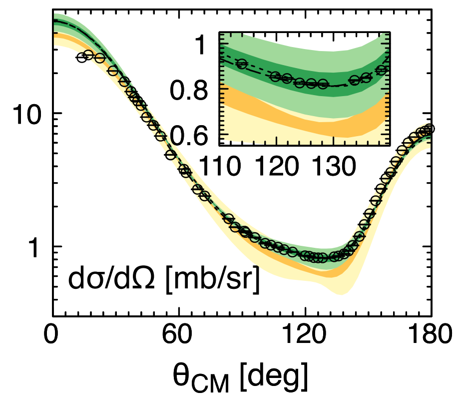
Consistent with the 3NF from LENPIC, PRC 103 (2021) 054001



$$V^{3N}_\Lambda = \frac{g_A^2}{8F_\pi^4}\, e^{-\frac{\vec{q}_1^{\,2}+M_\pi^2}{\Lambda^2}}\, e^{-\frac{\vec{q}_3^{\,2}+M_\pi^2}{\Lambda^2}} \bigg\{ \frac{\vec{\sigma}_1\cdot\vec{q}_1\;\vec{\sigma}_3\cdot\vec{q}_3}{(\vec{q}_1^{\,2}+M_\pi^2)\,(\vec{q}_3^{\,2}+M_\pi^2)}\Big[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\big(2c_3\,\vec{q}_1\cdot\vec{q}_3 - 4c_1M_\pi^2\big) + c_4\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\,\vec{q}_1\times\vec{q}_3\cdot\vec{\sigma}_2\Big]$$
$$+ C\,\frac{\vec{\sigma}_1\cdot\vec{q}_1}{\vec{q}_1^{\,2}+M_\pi^2}\Big(2c_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\vec{\sigma}_3\cdot\vec{q}_1 + c_4\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\,\vec{q}_1\times\vec{\sigma}_3\cdot\vec{\sigma}_2\Big)$$
$$+ C\,\frac{\vec{\sigma}_3\cdot\vec{q}_3}{\vec{q}_3^{\,2}+M_\pi^2}\Big(2c_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\vec{\sigma}_1\cdot\vec{q}_3 + c_4\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\,\vec{\sigma}_1\times\vec{q}_3\cdot\vec{\sigma}_2\Big)$$
$$+ C^2\Big(2c_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\vec{\sigma}_1\cdot\vec{\sigma}_3 + c_4\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\,\vec{\sigma}_1\times\vec{\sigma}_3\cdot\vec{\sigma}_2\Big)\bigg\}$$
$$- \frac{g_A D}{8F_\pi^2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\, e^{-\frac{\vec{p}_{12}^{\,2}+\vec{p}_{12}^{\,\prime 2}}{\Lambda^2}}\, e^{-\frac{\vec{q}_3^{\,2}+M_\pi^2}{\Lambda^2}}\bigg[\frac{\vec{\sigma}_3\cdot\vec{q}_3}{\vec{q}_3^{\,2}+M_\pi^2}\,\vec{\sigma}_1\cdot\vec{q}_3 + C\,\vec{\sigma}_1\cdot\vec{\sigma}_3\bigg] + \frac{1}{2}E\,T_{12}\, e^{-\frac{\vec{p}_{12}^{\,2}+\vec{p}_{12}^{\,\prime 2}}{\Lambda^2}}\, e^{-\frac{3\vec{k}_3^{\,2}+3\vec{k}_3^{\,\prime 2}}{4\Lambda^2}} + 5\text{ perm.}$$

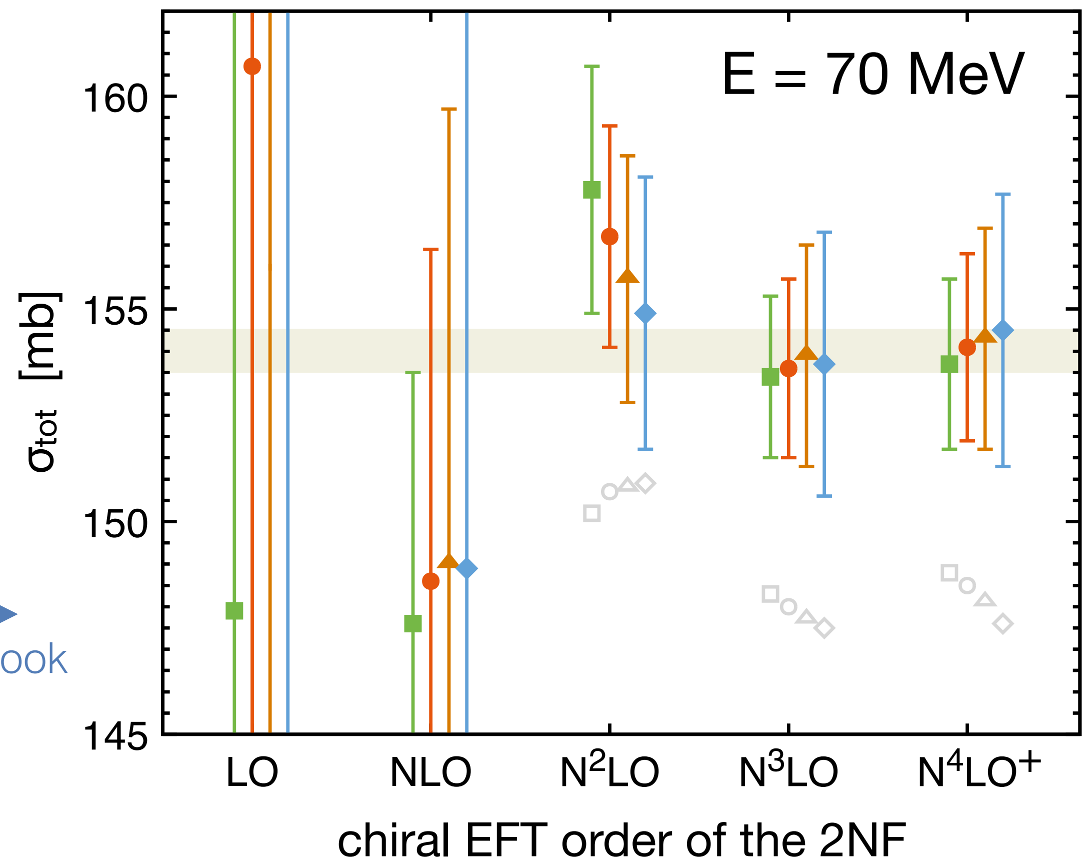
Two-body currents: LECs

^{3}H beta decay: An interesting interplay of the LECs c_3, c_4 and c_D (and c_E through the 3NF)



LENPIC fitting protocol: Use ^{3}H BE + the cross section minimum at 70 MeV to fit c_D and c_E

few-N predictions look reasonable



⇒ no free parameters for ^{3}H β decay

Technical performance and results

Technical performance:

— use 2NF@N^{4}LO$^+$ together with the 3NF@N^{2}LO

— use the (shifted) N^{4}LO values of c_i's (in GeV^{-1}): $c_1 = -1.23, \quad c_3 = -4.65, \quad c_4 = 3.28$

— tune c_D and c_E to the ^{3}H BE and the nucleon-deuteron differential cross section at 70 MeV

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
c_D (c_E)	3.328 (−0.454)	0.892 (−0.386)	−1.279 (−0.382)	−3.626 (−0.410)

— the ^{3}H and ^{3}He wave functions provided by the Bochum group

— partial-wave decomposition of the 2N current carried out by the Cracow group

— the GT ME calculated by the Cracow group

Calculated values of the GT matrix element

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
2NF at N^{4}LO$^+$	93.78	93.23	92.73	92.31
2NF at N^{4}LO$^+$ + 3NF	93.87	93.33	92.86	92.50
2NF at N^{4}LO$^+$ + 3NF + MEC	103.41	101.19	99.63	98.37

— adding the 3NF has negligible effect on the GT ME

— strong overestimation of the GT ME (remember: $\langle \boldsymbol{GT} \rangle_{\text{emp}} = 0.9484 \pm 0.0019$)

Benchmarking

What can possibly go wrong?

Could be bugs in the implementation of MECs (factors of 2, units) $\Rightarrow$ compare with Baroni et al.

Baroni et al. PRC 94 (2016) + 2 errata

local cutoff for the 3-body current (no subtractions)

Λ	500 MeV	600 MeV	
LO	0.9363(0.9224)	0.9322 (0.9224)	
N2LO	$-0.569(-0.844)\times10^{-2}$	$-0.457(-0.844)\times10^{-2}$	
N3LO(OPE)	$0.825(1.304)\times10^{-2}$	$0.043(7.517)\times10^{-2}$	$\leftarrow$ $\delta\langle GT\rangle$ for $c_3 = -3.20,\ c_4 = 5.40$
N3LO*(OPE)	$0.579(0.812)\times10^{-1}$	$0.652(1.413)\times10^{-1}$	$\leftarrow$ $\delta\langle GT\rangle$ for $c_3 = -5.61,\ c_4 = 4.26$
	EM N³LO + 3NF	AV18 + UIX	

Assuming that (i) the 3NF has no impact and (ii) the relativistic corrections they include in the OPE MEC are small, one can extract their individual contributions of c_3, c_4:

Using EM N³LO + 3NF:

$$10^2 \times \langle GT\rangle^{\text{Baroni}}_{c_3=1} = -1.68\,/\,-2.12 \quad \text{for } \Lambda = 500\,/\,600 \text{ MeV}$$

$$10^2 \times \langle GT\rangle^{\text{Baroni}}_{c_4=1} = -0.80\,/\,-1.19 \quad \text{for } \Lambda = 500\,/\,600 \text{ MeV}$$

Using AV18 + UIX:

$$10^2 \times \langle GT\rangle^{\text{Baroni}}_{c_3=1} = -2.32\,/\,-2.66 \quad \text{for } \Lambda = 500\,/\,600 \text{ MeV}$$

$$10^2 \times \langle GT\rangle^{\text{Baroni}}_{c_4=1} = -1.08\,/\,-0.18 \quad \text{for } \Lambda = 500\,/\,600 \text{ MeV}$$

Without subtractions ($C = 0$), we obtain:

$$10^2 \times \langle GT\rangle_{c_3=1} = -1.53\,/\,-2.06 \quad \text{for } \Lambda = 400\ldots550 \text{ MeV}$$

$$10^2 \times \langle GT\rangle_{c_4=1} = -1.50\,/\,-0.51 \quad \text{for } \Lambda = 400\ldots550 \text{ MeV}$$

Benchmarking

To compare the cD-contribution, look at their more recent paper Baroni et al, PRC 98 (2018)

Here, one needs to be careful since in this calculation, the quoted cD values include the admixture of the short-range part of the c3, c4 terms $\delta c_D = 4.1313$ (similar to our subtractions).

Further, they use $\Lambda_\chi = 1$ GeV instead of $\Lambda_\chi = 700$ MeV in $D = c_D/(F_\pi^2 \Lambda_\chi)$

different Norfolk N3LO interaction models

	Ia	Ib	IIa	IIb
c_D	3.666	−2.061	1.278	−4.480
c_E	−1.638	−0.982	−1.029	−0.412
LO	0.9248	0.9237	0.9249	0.9259
N2LO(Δ)	0.0401	0.0586	0.0406	0.0589
N2LO(RC)	−0.0055	−0.0063	−0.0059	−0.0077
N3LO(OPE)	0.0327	0.0457	0.0330	0.0462
N3LO(CT)	−0.0036	−0.0487	−0.0249	−0.0668

$\Lambda = 493$ MeV

$\Lambda = 564$ MeV

Bringing their results to our convention yields:

$$10^2 \times \langle GT \rangle^{\text{Baroni}}_{c_D=1} = 1.26\,/\,1.27$$ for their models Ia, IIa based on $\Lambda = 493$ MeV

$$10^2 \times \langle GT \rangle^{\text{Baroni}}_{c_D=1} = 1.12\,/\,1.13$$ for their models IIb, Ib based on $\Lambda = 564$ MeV

This is to be compared with our results:

$$10^2 \times \langle GT \rangle_{c_D=1} = 1.43 \ldots 1.25$$ for $\Lambda = 400 \ldots 550$ MeV

Consistency check with subtractions

Another nontrivial consistency check is provided by switching off subtractions in the 3NF and MECs

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
c_D (c_E)	3.328 (−0.454)	0.892 (−0.386)	−1.279(−0.382)	−3.626 (−0.410)
c_D (c_E) using unsubtracted 3NF ($C = 0$)	5.208 (0.723)	2.756 (0.369)	0.520 (−0.014)	−2.025 (−0.503)

With subtractions:

$$\Lambda = 400 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = \underbrace{0.80}_{c_3} + \underbrace{3.92}_{c_4} + \underbrace{4.75}_{c_D} = \boxed{9.47}$$

$$\Lambda = 450 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 1.57 + 4.95 + 1.28 = 7.80$$

$$\Lambda = 500 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 2.47 + 5.98 - 1.74 = 6.71$$

$$\Lambda = 550 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 3.40 + 6.95 - 4.54 = 5.81$$

Without subtractions:

$$\Lambda = 400 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 7.12 - 4.92 + 7.44 = 9.64$$

$$\Lambda = 450 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 8.15 - 4.24 + 3.94 = 7.85$$

$$\Lambda = 500 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 8.96 - 3.07 + 0.71 = 6.60$$

$$\Lambda = 550 \text{ MeV}: \quad 10^2 \times \langle \boldsymbol{GT} \rangle = 9.59 - 1.67 - 2.53 = 5.39$$

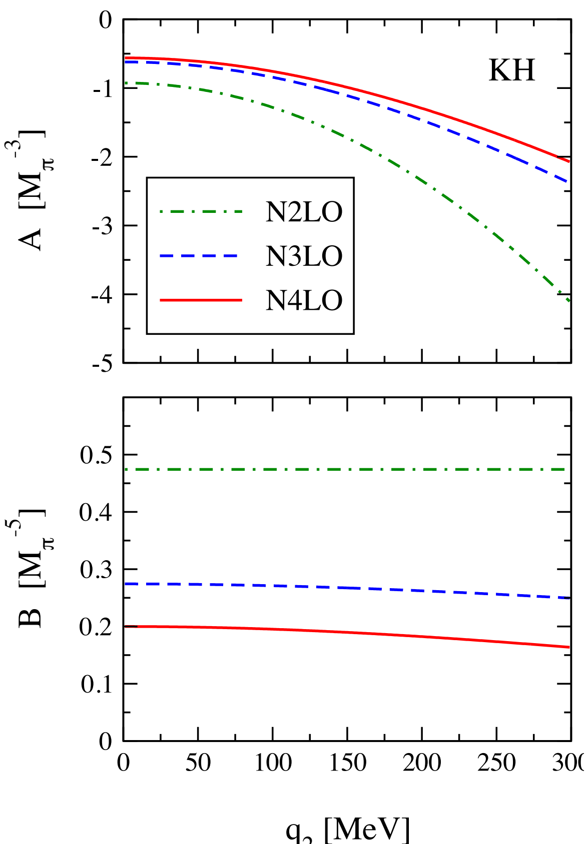
differences consistent with higher-order effects

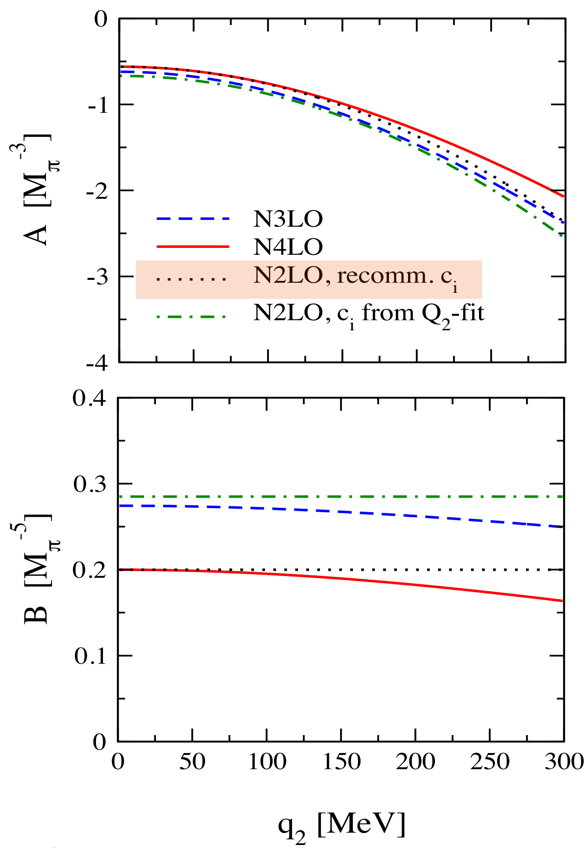
Effective c_i's?

Can the problem be related to inadequate values of the c_i's?

The 2π-exchange 3NF: $$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

$$\mathcal{A}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big[(2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big] + \ldots \qquad \mathcal{B}(q_2) = \frac{g_A^2}{8F_\pi^4} c_4 + \ldots$$





Krebs, Gasparyan, EE, PRC 85 (2012) 054006

Effective c_i's?

The N⁴LO result for the 2π-exchange 3NF can be approximately reproduced using effective values:

$c_1 = -0.37,\ c_3 = -2.71,\ c_4 = 1.44$ (tbc with the original ones: $c_1 = -1.23,\ c_3 = -4.65,\ c_4 = 3.28$)

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
c_D (c_E)	3.328 (−0.454)	0.892 (−0.386)	−1.279(−0.382)	−3.626 (−0.410)
c_D (c_E) using effective values $c_i^{\rm eff}$ in the 3NF	5.479 (−0.538)	3.643 (−0.498)	2.346 (−0.547)	1.208 (−0.670)

Effective c_i's:

$$\begin{aligned}
\Lambda &= 400\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= \underbrace{0.45}_{c_3} + \underbrace{1.69}_{c_4} + \underbrace{7.91}_{c_D} = 10.05 \\
\Lambda &= 450\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 0.90 + 2.14 + 5.29 = 8.33 \\
\Lambda &= 500\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 1.43 + 2.59 + 3.26 = 7.27 \\
\Lambda &= 550\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 1.99 + 3.01 + 1.55 = 6.55
\end{aligned}$$

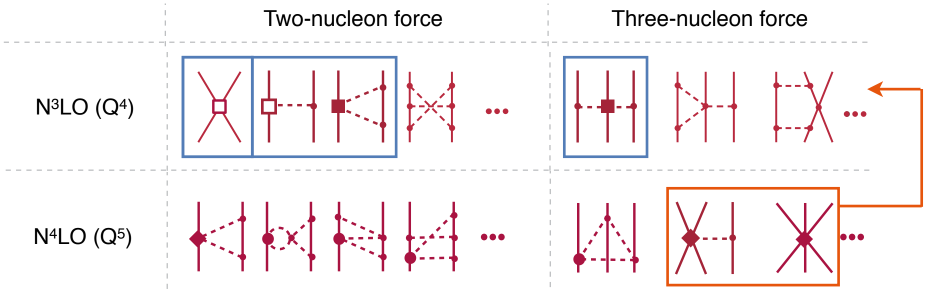
Original c_i's:

$$\begin{aligned}
\Lambda &= 400\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 0.80 + 3.92 + 4.75 = 9.47 \\
\Lambda &= 450\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 1.57 + 4.95 + 1.28 = 7.80 \\
\Lambda &= 500\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 2.47 + 5.98 - 1.74 = 6.71 \\
\Lambda &= 550\ \text{MeV}: & 10^2 \times \langle \boldsymbol{GT} \rangle &= 3.40 + 6.95 - 4.54 = 5.81
\end{aligned}$$

Remarkably, the total result is almost unchanged (naturally explained within π-less EFT…)

Off-shell NN contact interactions at N³LO

The over-prediction of the GT ME at N²LO seems robust ⇒ large N³LO corrections?



One particular type of N³LO „corrections" emerges from 3 off-shell NN contact interactions ($^1S_0$, $^3S_1$, $^3S_1$-$^3D_1$)

E.g.: $\langle p', {}^1S_0 | V_{\rm cont} | p, {}^1S_0 \rangle = \underbrace{\tilde{C}_{1S0}}_{\text{tuned to the scatt. length}} + \underbrace{C_{1S0}(p'^2 + p^2)}_{\text{tuned to the effective range}} + \underbrace{D_{1S0}p^2p'^2 + D^{\rm off}_{1S0}(p'^2 - p^2)^2}_{\text{tuned to the first shape parameter}}$

— can be eliminated using a suitable UT (SMS choice), at the cost of an enhancement of some of the (linear combinations of) short range 3NF & currents from N⁴LO to N³LO

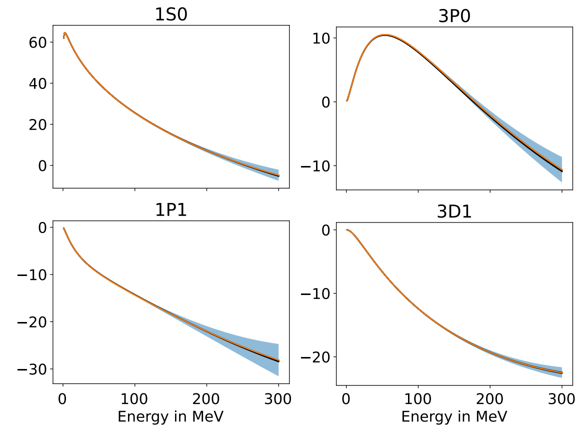
— alternatively, fix the 3 NN off-shell LECs from data other than NN scattering (these LECs become completely redundant at N⁴LO) ← talk by Sven Heihoff

⇒ Extended the SMS N⁴LO⁺ potential ($D_i^{\rm off} = 0$) with 26 potentials with $D_S^{\rm off} = \{-3,0,3\}$, $D_{\epsilon 1}^{\rm off} = \{-1,0,1\}$:

— on-shift equivalent: $\chi^2_{\rm datum} = 1.010 \ldots 1.014$

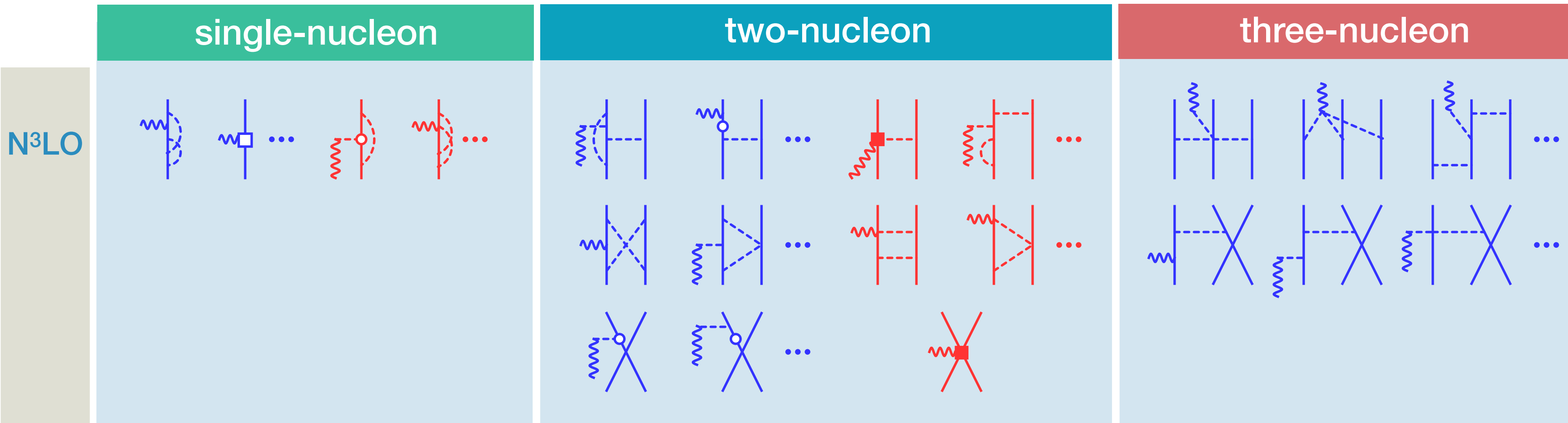
— but the ³H BE varies by ~ 1.5 MeV (without 3NFs)

repeating the calcs., we find: $\langle \mathbf{GT} \rangle_{1N} = 87.40 \ldots 94.19$ (!)



What else?

It seems one needs a *large and negative* contribution to the GT ME at N^{3}LO in order to bring the predictions for ^{3}H β-decay in agreement with the experimental datum.



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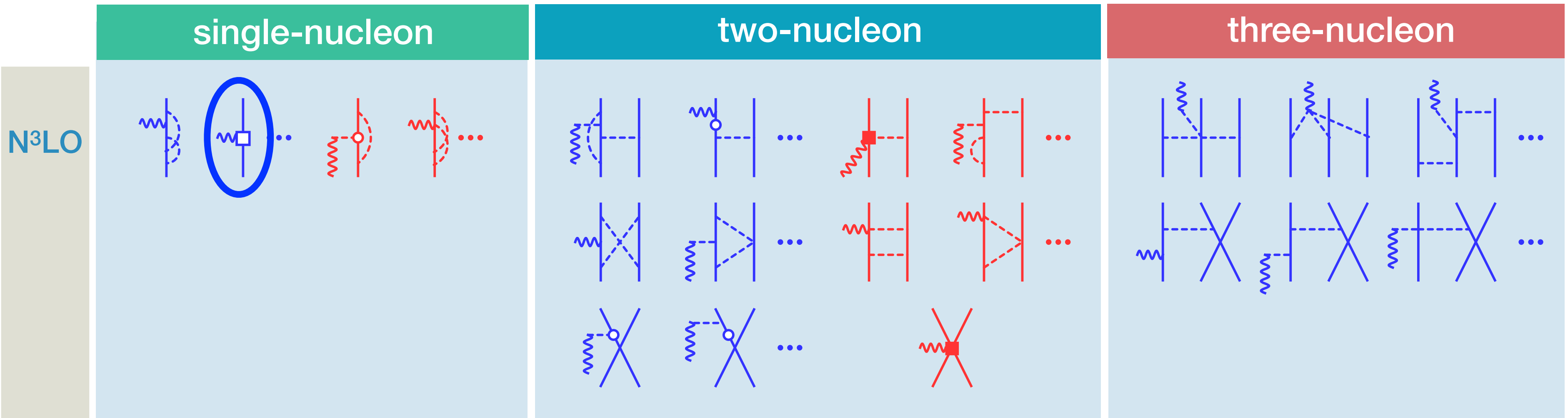

— 1N relativistic corrections help a bit, but not much (i.e., $\delta\langle \mathbf{GT}\rangle_{\text{relat.}} \sim -0.5\ldots-1$)

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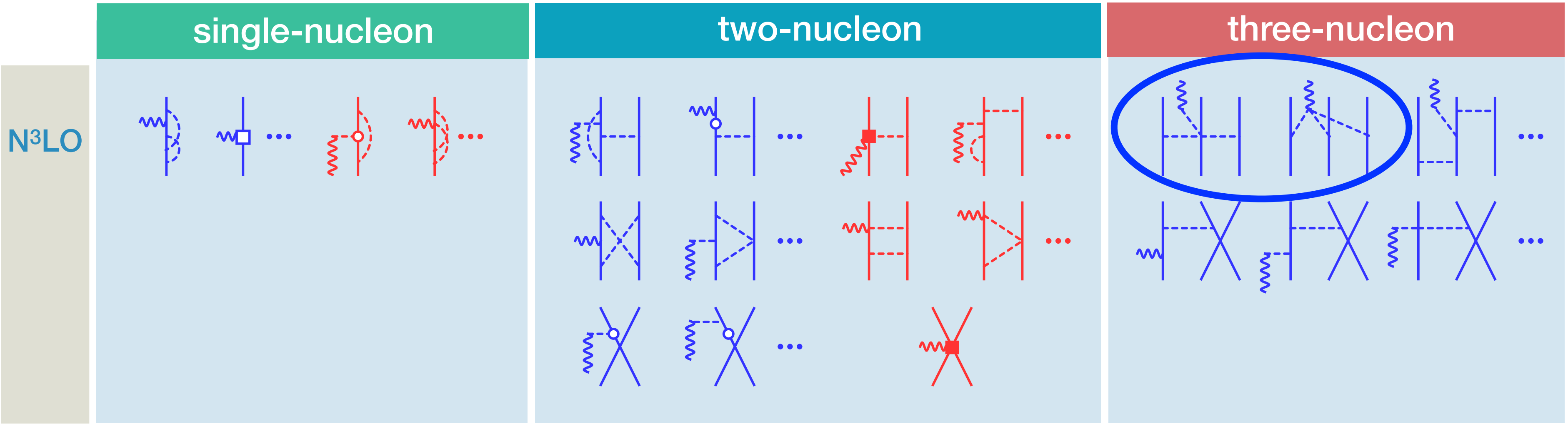

- 1N relativistic corrections help a bit, but not much (i.e., $\delta\langle \mathbf{GT}\rangle_{\text{relat.}} \sim -0.5\ldots-1$)
- (some) 3N currents were considered by Baroni et al. to be tiny

Λ	500 MeV	600 MeV
LO	0.9363(0.9224)	0.9322 (0.9224)
N2LO	$-0.569(-0.844)\times10^{-2}$	$-0.457(-0.844)\times10^{-2}$
N3LO(OPE)	$0.825(1.304)\times10^{-2}$	$0.043(7.517)\times10^{-2}$
N3LO*(OPE)	$0.579(0.812)\times10^{-1}$	$0.652(1.413)\times10^{-1}$
N3LO(CT)	$-0.586(-0.721)\times10^{-3}$	$-0.717(-0.644)\times10^{-3}$
N4LO(OPE)	$-0.697(-0.964)\times10^{-2}$	$-0.867(-1.216)\times10^{-2}$
N4LO(MPE)	$-0.430(-0.565)\times10^{-1}$	$-0.532(-0.775)\times10^{-1}$
N4LO(3Ba)	$-0.143(-0.183)\times10^{-2}$	$-0.153(-0.205)\times10^{-2}$

Baroni et al. PRC 94 (2016)

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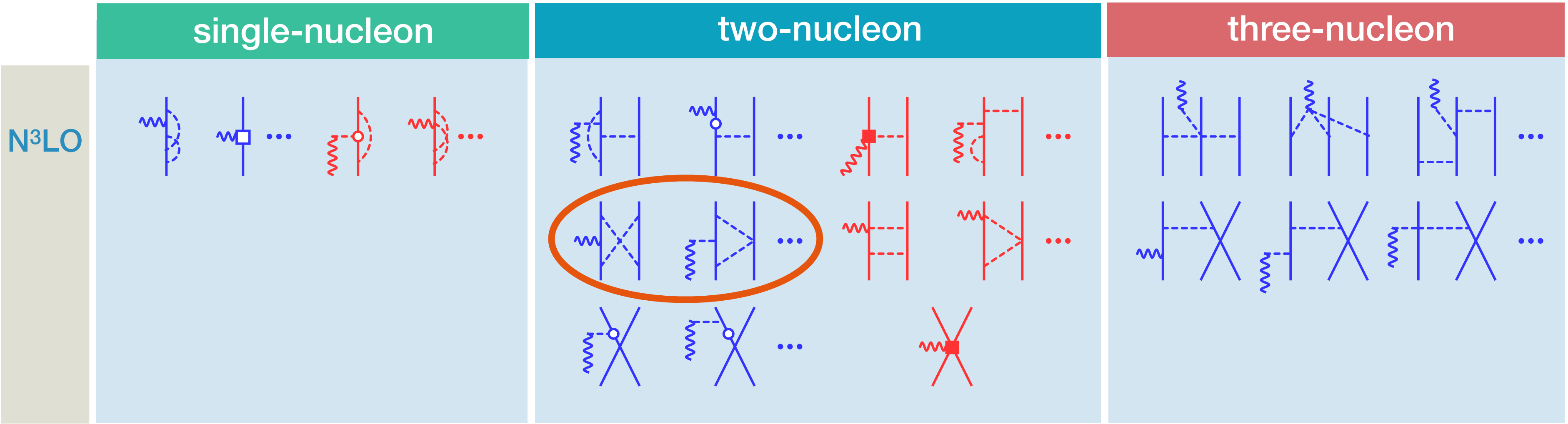

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- Baroni et al. claim to find a large and negative N³LO contribution stemming from TPE!

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Summary and conclusions

- The 1N contribution to the axial current gives $\sim 98\,\%$ of $\langle \mathbf{GT} \rangle_{\text{emp}}$; the remaining $\sim 2\,\%$ have to be generated by MECs
- MECs first appear at N^{2}LO ($\propto c_{3,4,D}$), but their individual contributions $\sim 5 \ldots 10\,\%$ suggest that no precise description should be expected at N^{2}LO
- Our N^{2}LO parameter-free predictions for $\langle \mathbf{GT} \rangle$ are too large by $\sim 4 \ldots 9\,\%$ $\Rightarrow$ expect large and negative N^{3}LO contribution from MECs. Indeed, large and negative contributions were claimed by Baroni et al.
- Our findings put into question some of the recent calculations…
- With the new regularization scheme, a complete and consistent N^{3}LO analysis is in reach!

Thank you for your attention