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Ab initio calculation of hyper-matter and the hyperon puzzle

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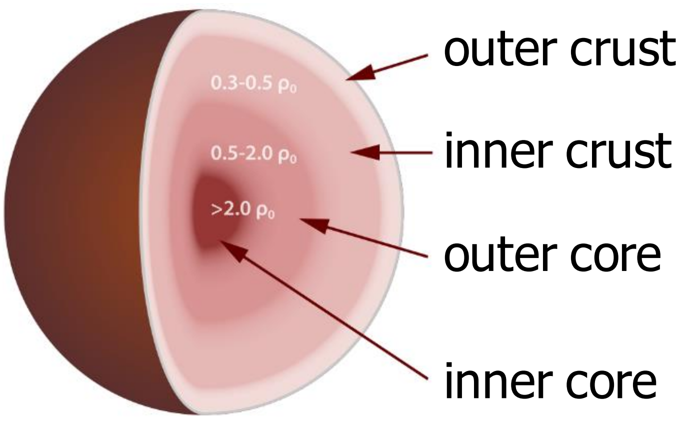
Contents

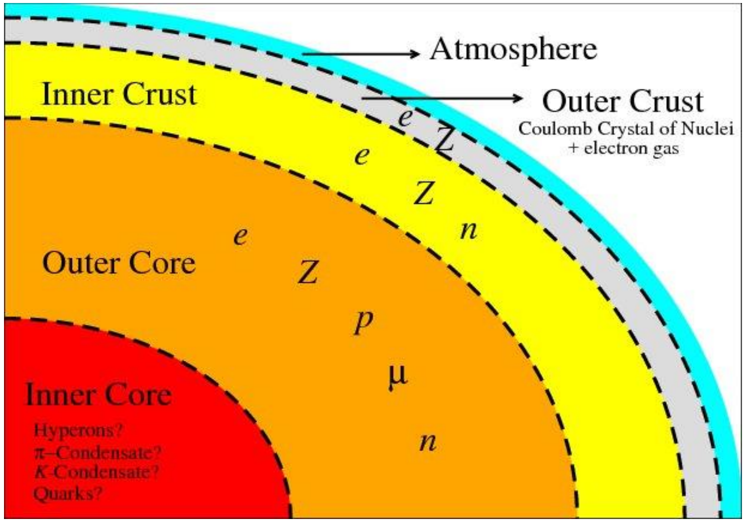
Neutron star

- Neutron stars are one of the densest massive objects in the universe.

✲ *J. M. Lattimer, et al., Science 304, 536 (2004)*



✲ *Figure from www.Universe.com*



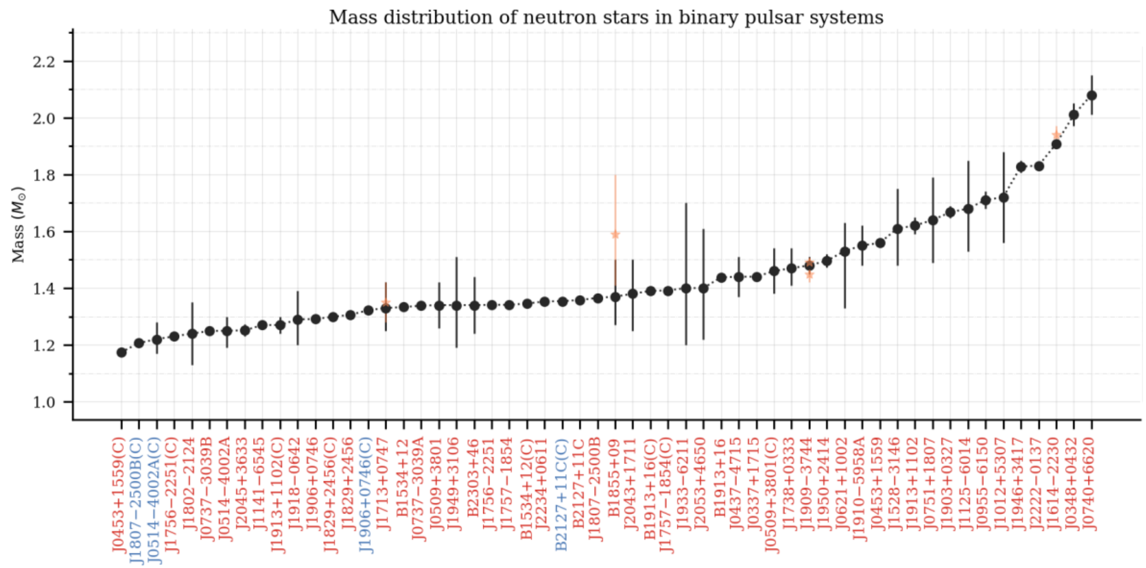
✲ *Figure from J. Piekarewicz*

① Usually refer to a star with a mass *M* on the order of 1-2 solar masses, a radius *R* of 10-12 km.

② The central density can reach several times the empirical nuclear matter saturation density ($\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$).

Observations of Neutron stars

- *Mass* measurements



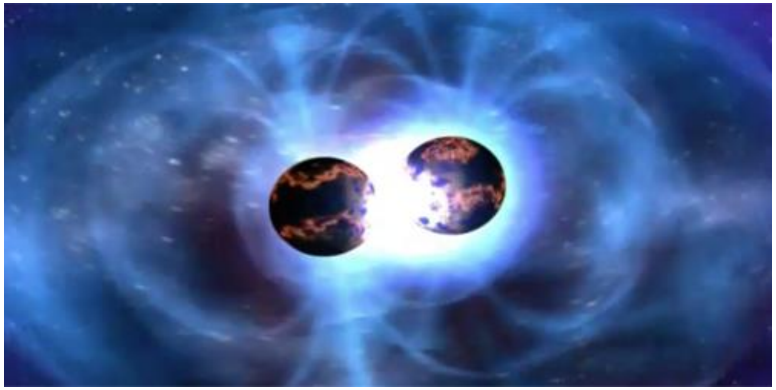
✡ *Figure from Vivek V. Krishnan*

- The mass and *radius* of PSR J0030+0451 were simultaneously measured by the NICER collaboration.

✡ *Raaijmakers, et al., ApJ 887, L22 (2019)*

- The gravitational wave (GW) signal provides the astrophysical measurements of masses, *tidal deformabilities*, etc.

✡ *B. P. Abbott, et al., PRL 119, 161101 (2017)*

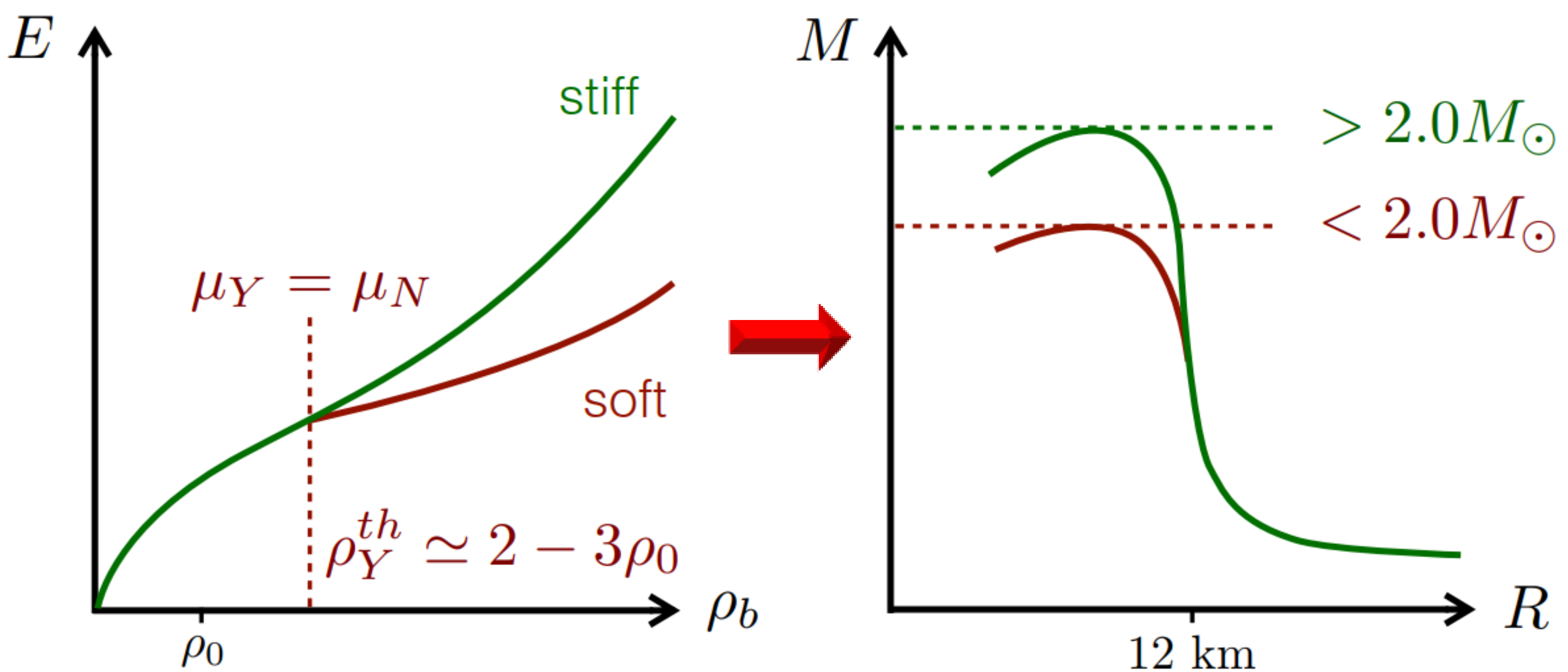


✡ *Figure from NASA/Goddard Space Flight Center*

Hyperon puzzle

- Some of the nuclear many-body approaches, such as Hartree-Fock and Brueckner-Hartree-Fock, predict the appearance of hyperons at a density of $(2-3)\rho_0$, and a softening of the EoS, implying a reduction of the maximum mass.

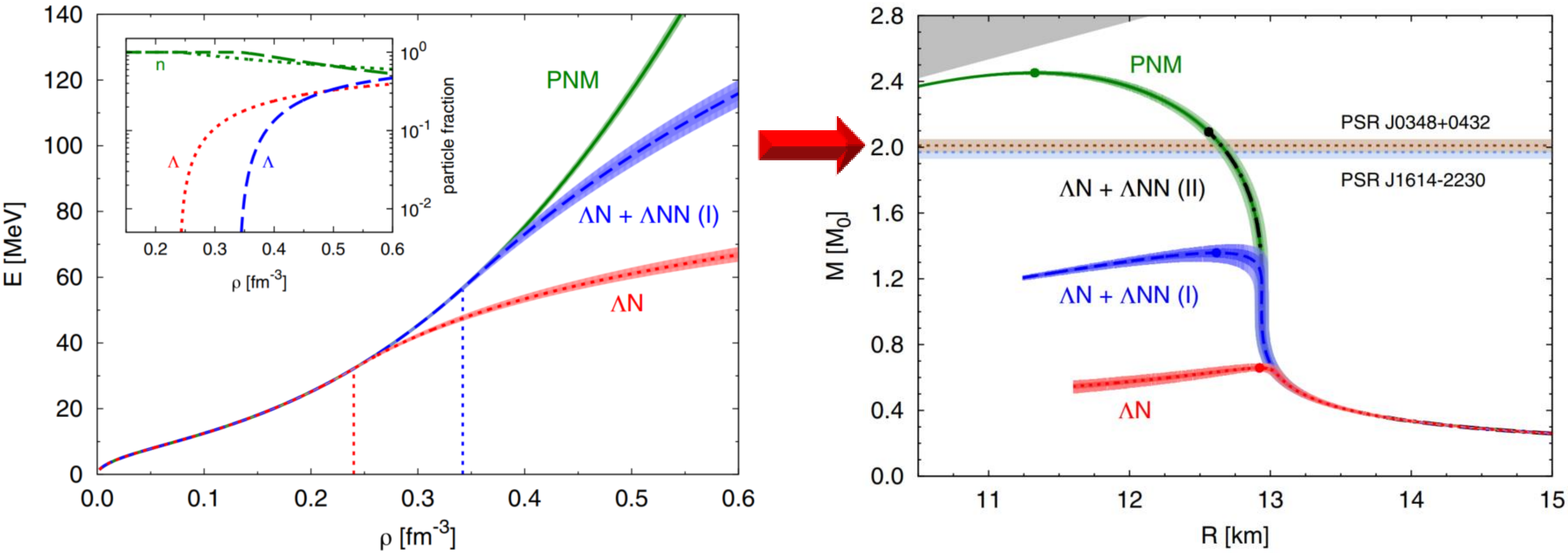
✡ H. Đapo, et al., PRC 81, 035803 (2010)
✡ H.-J. Schulze and T. Rijken, PRC 84, 035801 (2011)



✡ Figure from D. Lonardoni

EoS with Hyperons from AFDMC

- ΛNN(II) can support $2M_{\odot}$, but the onset of Λ is above the maximum density ($0.56\ \mathrm{fm}^{-3}$). No Λ present in Neutron Star?



D. Lonardoni, et al., PRL 114, 092301 (2015)

① Phenomenological $\Lambda N + \Lambda NN$ potential + Auxiliary field diffusion Monte Carlo, no $\Lambda\Lambda + \Lambda\Lambda N$ potential

② Only some fixed number of neutrons (N_n=66, 54, 38) and hyperons (N_Λ=1, 2, 14) in the simulation box used from AFDMC, the EoS of hyper-neutron matter needs to be parametrized

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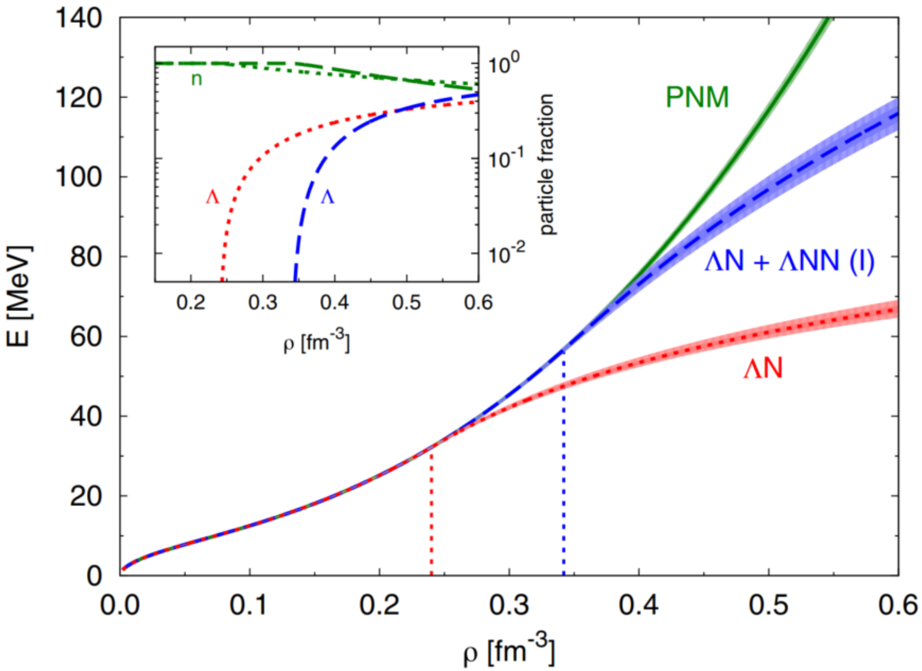

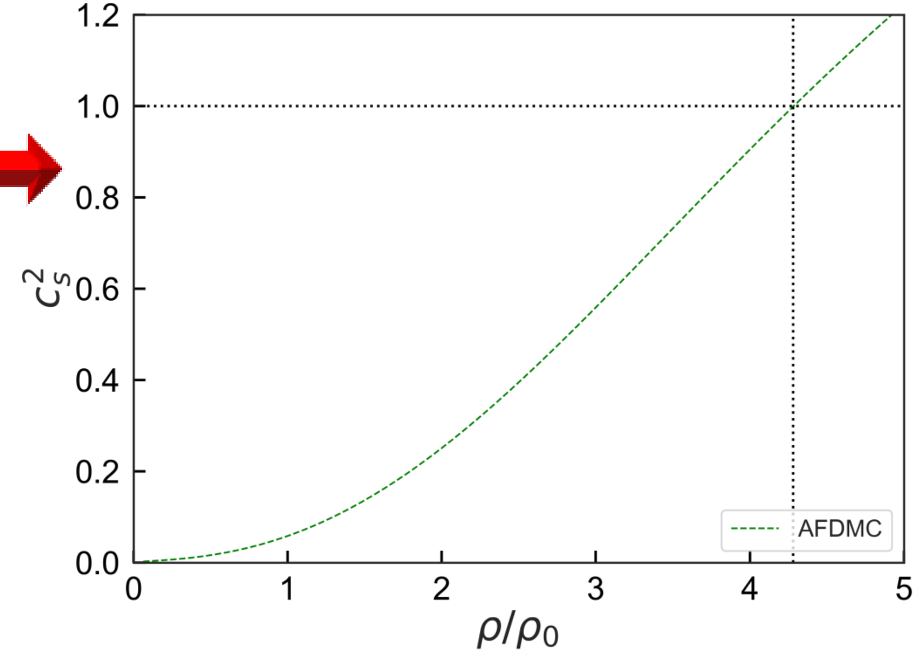

✡ *D. Lonardoni, et al., PRL 114, 092301 (2015)*

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- ❑ Introduction
- ❑ Hyper-Neutron matter
- ❑ Summary and Outlook

SU(4)-invariant leading order EFT

- Hamiltonian

$$H_{\mathrm{SU}(4)} = H_{\mathrm{free}} + \frac{1}{2!}C_{NN}\sum_{\boldsymbol{n}}\tilde{\rho}_N(\boldsymbol{n})^2 + \frac{1}{3!}C_{3N}\sum_{\boldsymbol{n}}\tilde{\rho}(\boldsymbol{n})^3,$$

the density operator $\tilde{\rho}_N(\boldsymbol{n})$ is defined as

$$\tilde{\rho}_N(\boldsymbol{n}) = \sum_{i,j}\tilde{a}^\dagger_{i,j}(\boldsymbol{n})\tilde{a}_{i,j}(\boldsymbol{n}) + s_L^{NN}\sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1}\sum_{i,j}\tilde{a}^\dagger_{i,j}(\boldsymbol{n}')\tilde{a}_{i,j}(\boldsymbol{n}'),$$

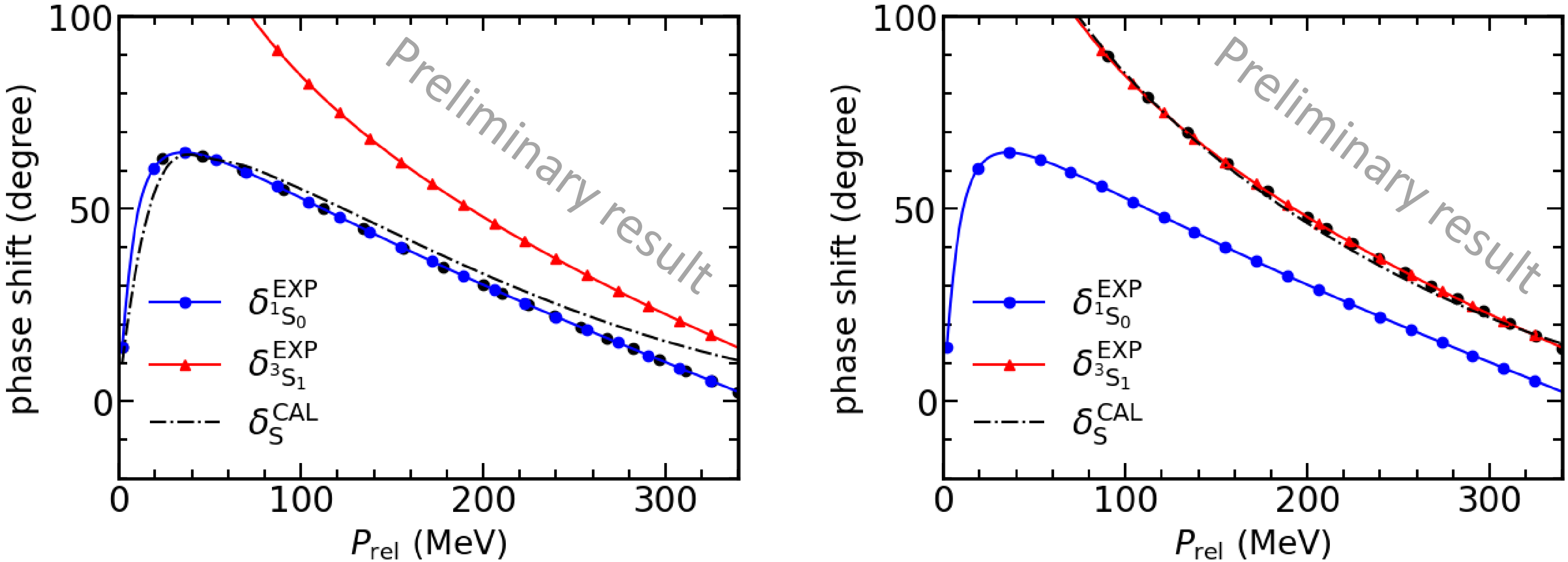
where i is the spin index, j is the isospin index. The smeared annihilation and creation operators are defined as

$$\tilde{a}_{i,j}(\boldsymbol{n}) = a_{i,j}(\boldsymbol{n}) + s_{NL}^{NN}\sum_{|\boldsymbol{n}'-\boldsymbol{n}|=1} a_{i,j}(\boldsymbol{n}').$$

The parameter s_L is a local smearing parameter, s_{NL} is a nonlocal smearing parameter, C_{NN} and C_{3N} gives the strength of the two-body and three-body interaction.

Phase shift

- The C_{NN} couplings are determined by fitting the phase shift.



- The C_{3N} are determined by the empirical value for nuclear matter.

- The Galilean invariance restoration for each channel are obtained by tuning $C_{{\rm GIR},i}\,(i = 0,1,2)$ with the constraint

$$C_{\rm GIR,0} + 6C_{\rm GIR,1} + 12C_{\rm GIR,2} = 0$$

EoS with hyperons

- Hamiltonian

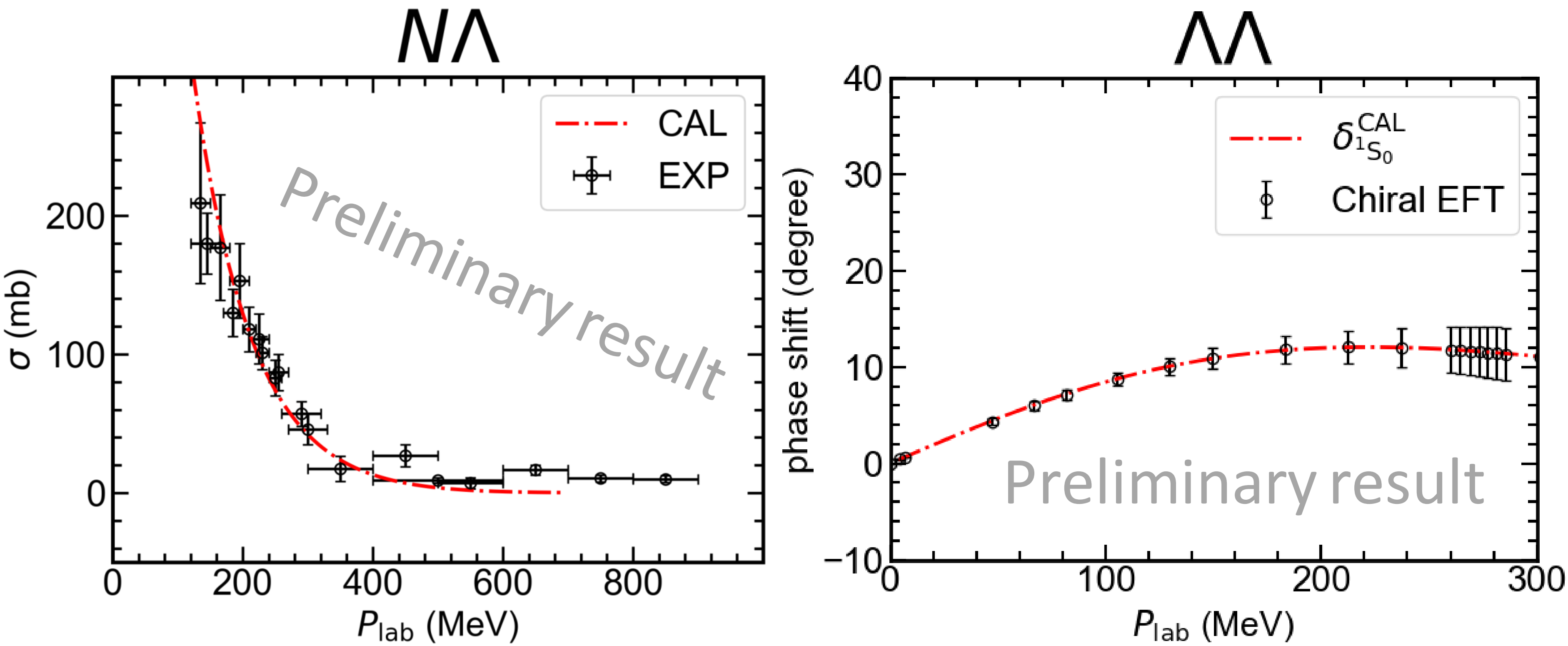
$$
\begin{aligned}
H =& H^N_{\text{free}} + H^\Lambda_{\text{free}} \\
&+ \frac{1}{2!} C_{NN} \sum_{\boldsymbol{n}} \left[\tilde{\rho}_N(\boldsymbol{n}) + \frac{C_{N\Lambda}}{C_{NN}} \tilde{\rho}_\Lambda(\boldsymbol{n}) \right]^2 \\
&+ \frac{1}{2!} \left(C_{\Lambda\Lambda} - \frac{C^2_{N\Lambda}}{C_{NN}} \right) \sum_{\boldsymbol{n}} \tilde{\rho}_\Lambda(\boldsymbol{n}) \tilde{\rho}_\Lambda(\boldsymbol{n}), \\
&+ \frac{1}{3!} C_{3N} \sum_{\boldsymbol{n}} \tilde{\rho}_N(\boldsymbol{n})^3 + \frac{1}{2!} C_{NN\Lambda} \sum_{\boldsymbol{n}} \tilde{\rho}_N(\boldsymbol{n})^2 \tilde{\rho}_\Lambda(\boldsymbol{n}) + \frac{1}{2!} C_{N\Lambda\Lambda} \sum_{\boldsymbol{n}} \tilde{\rho}_N(\boldsymbol{n}) \tilde{\rho}^2_\Lambda(\boldsymbol{n})
\end{aligned}
$$

$C_{N\Lambda}, C_{\Lambda\Lambda}, C_{N\Lambda\Lambda}, C_{NN\Lambda}$ give the strength of the two-body and three-body interactions. The simulation of systems with both neutrons and Λ hyperons can be achieved by using a single auxiliary field.

Phase shift with hyperons

- $C_{N\Lambda}$ is determined by fitting the cross section, and $C_{\Lambda\Lambda}$ by fitting the phase shift $({}^1S_0)$.

✲ *J. Haidenbauer, Ulf-G. Meißner, and S. Petschauer, Nucl. Phys. A 954, 273 (2016).*



- The $C_{NN\Lambda}$ and $C_{N\Lambda\Lambda}$ are determined by the separation energy for single Λ and double Λ hyper-nuclei.

Neutron Star EoS

- The energy density can be obtained as

$$\varepsilon_{\rm HNM} = \rho[E_{\rm HNM}(\rho_n, \rho_\Lambda)/N + \frac{\rho_n}{\rho} m_n + \frac{\rho_\Lambda}{\rho} m_\Lambda],$$

the chemical potentials for neutrons and lambdas are evaluated via

$$\mu_n(\rho, x) = \frac{\partial \varepsilon_{\rm HNM}}{\partial \rho_n}, \mu_\Lambda(\rho, x) = \frac{\partial \varepsilon_{\rm HNM}}{\partial \rho_\Lambda},$$

the Λ threshold density ρ_Λ^{th} is determined by imposing $\mu_\Lambda = \mu_n$, and the pressure is defined as

$$P(\rho) = \rho^2 \frac{d}{d\rho} \frac{\varepsilon_{\rm HNM}}{\rho} = \sum_{i=n,\Lambda} \rho_i \mu_i - \varepsilon_{\rm HNM}.$$

Neutron Star properties

- Tolman-Oppenheimer-Volkoff (TOV) equations

✲ *R. C. Tolman, Phys. Rev. 55, 364 (1939)* ✲ *J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939)*

$$\frac{dP(r)}{dr} = -\frac{[P(r)+\varepsilon(r)][M(r)+4\pi r^3 P(r)]}{r[r-2M(r)]},$$
$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r),$$

where $P(r)$ is the pressure, and $M(r)$ is the total star mass.

- Neutron star tidal deformability Λ

✲ *E. E. Flanagan and T. Hinderer, PRD 77, 021502 (2008)* ✲ *T. Hinderer, ApJ 677, 1216 (2008)*

$$\Lambda = \frac{2}{3} k_2 \left(\frac{R}{M}\right)^5,$$

where k_2 is the second Love number

$$k_2 = \frac{1}{20}\left(\frac{2M}{R}\right)^5 \left(1-\frac{2M}{R}\right)^2 \left[2 - y_R + (y_R - 1)\frac{2M}{R}\right] \times \left\{\frac{2M}{R}\left(6 - 3y_R + \frac{3M}{R}(5y_R - 8)\right.\right.$$
$$\left.+\frac{1}{4}\left(\frac{2M}{R}\right)^2 \left[26 - 22y_R + \frac{2M}{R}(3y_R - 2) + \left(\frac{2M}{R}\right)^2 (1 + y_R)\right]\right) + 3\left(1-\frac{2M}{R}\right)^2$$
$$\left.\times\left[2 - y_R + (y_R - 1)\frac{2M}{R}\right] \log\left(1 - \frac{2M}{R}\right)\right\}^{-1}.$$

The moment of inertia

- In the slow-rotation approximation, the moment of inertia is given by

✲ *F. J. Fattoyev and J. Piekarewicz, Phys. Rev. C 82, 025810 (2010)*

$$I = \frac{8\pi}{3}\int_0^R r^4 e^{-\nu(r)} \frac{\bar{\omega}(r)}{\Omega} \frac{\varepsilon(r)+P(r)}{\sqrt{1-2M(r)/r}} dr$$

The quantity $\nu(r)$ is a radially dependent metric function is defined as

$$\nu(r) = \frac{1}{2}\ln\left(1-\frac{2M}{R}\right) - \int_r^R \frac{M(x)+4\pi x^3 P(x)}{x^2[1-2M(x)/x]} dx$$

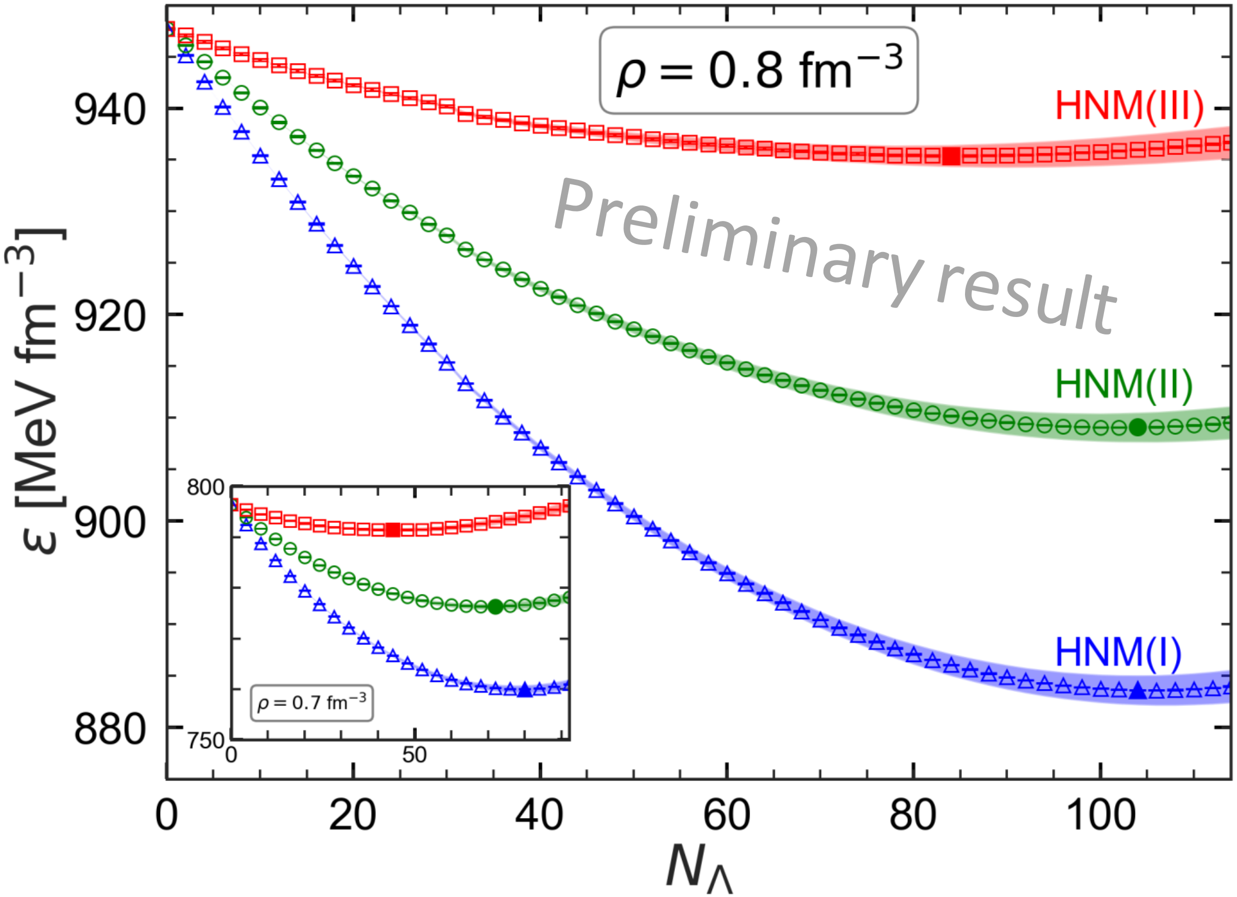
The frame-dragging angular velocity $\bar{\omega}$ is usually obtained by the dimensionless relative frequency $\tilde{\omega} \equiv \bar{\omega}/\Omega$, which satisfies

$$\frac{d}{dr}\left[r^4 j(r) \frac{d\tilde{\omega}(r)}{dr}\right] + 4r^3 \frac{dj(r)}{dr}\tilde{\omega}(r) = 0$$

where

$$j(r) = e^{-\nu(r)}\sqrt{1-2M(r)/r}$$

Energy density for different number of hyperons

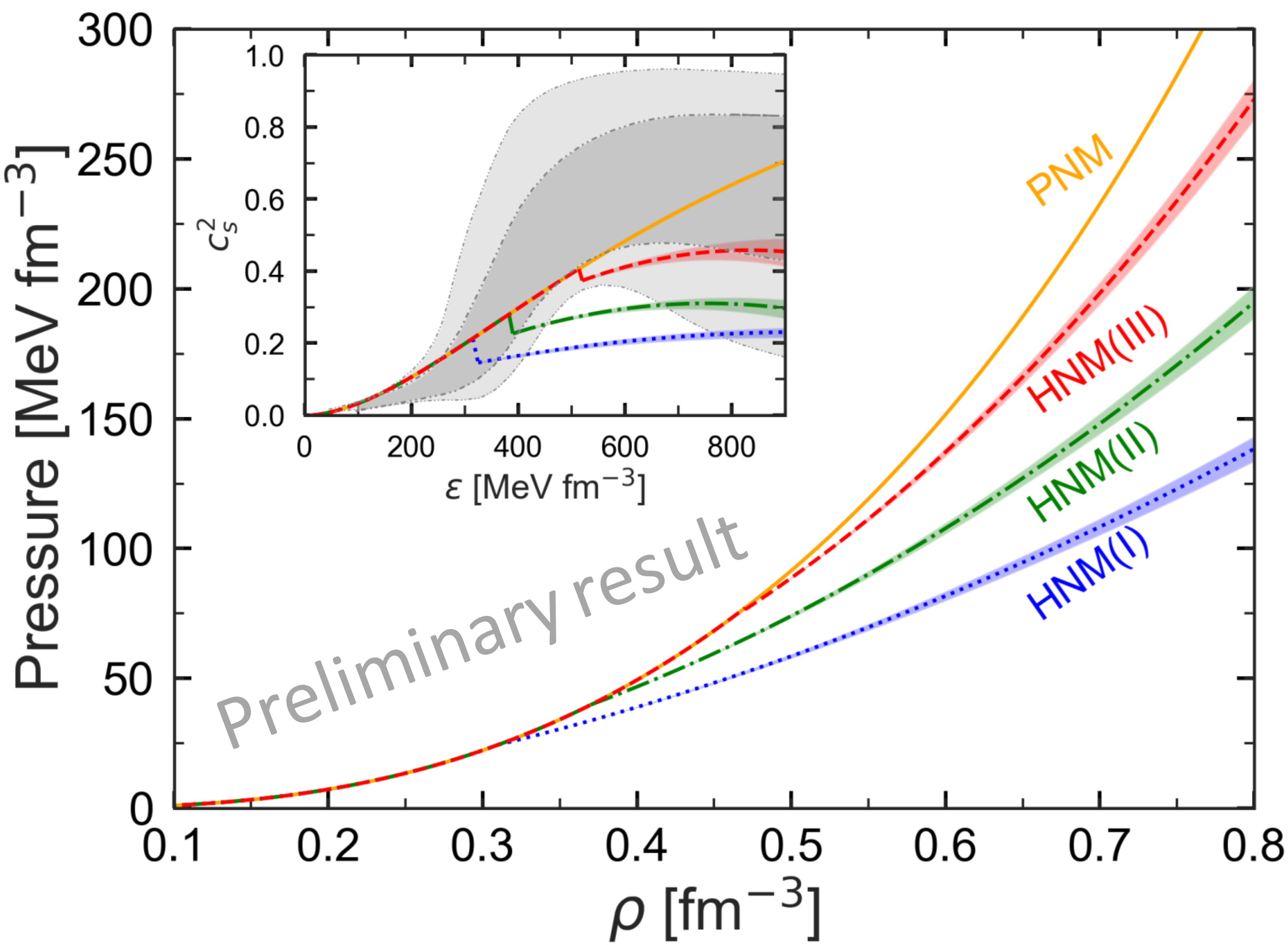


① ***Different number of hyperons can be simulated in our calculations.***

② Only N_n=66,54,38 and N_Λ=1,2,14 are used in AFDMC.

③ HNM(I,II,III) have different couplings for $NN\Lambda$ and $N\Lambda\Lambda$ interactions.

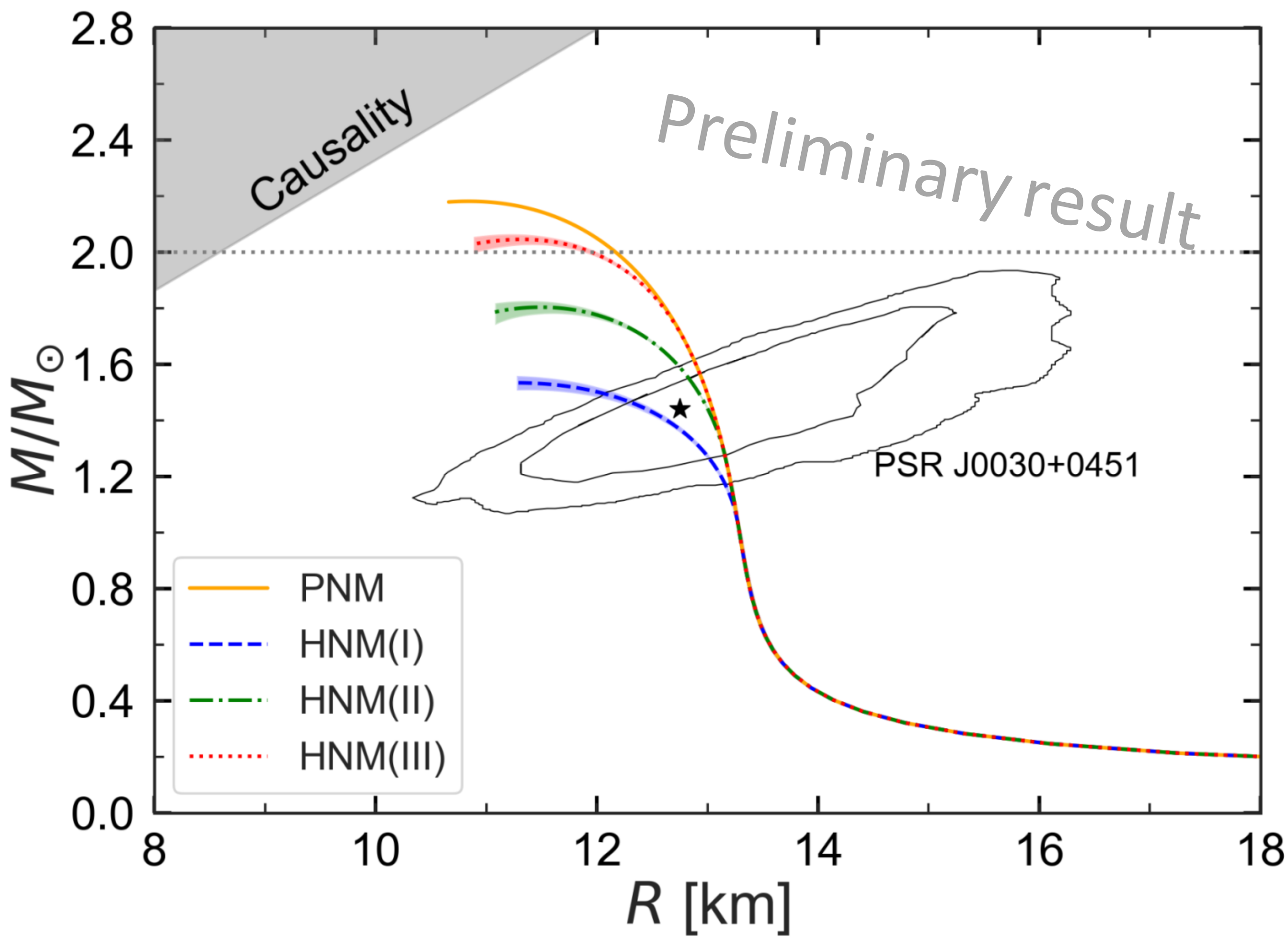
Equation of State for hyper-neutron matter



① HNM(I): ρ_Λ^{th}=0.33(1) fm^{-3}, HNM(II): ρ_Λ^{th}=0.38(1) fm^{-3}, HNM(III): ρ_Λ^{th}=0.48(1) fm^{-3}

② AFDMC : AV8'+3N interaction inspired by the Urbana IX and the Illinois models

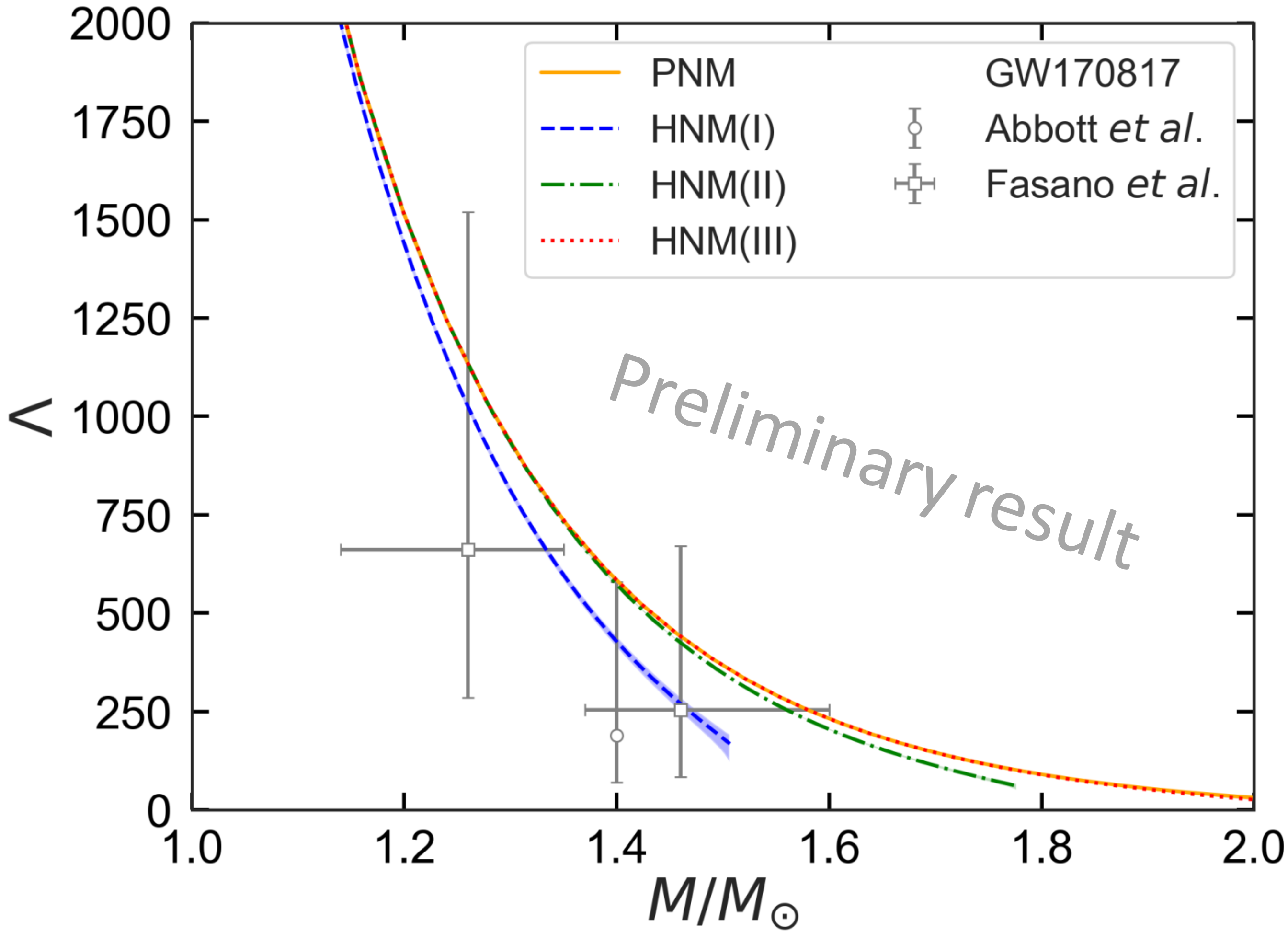
Neutron star Mass and radius



① HNM(I): $M_{\rm max}$=1.53(3)$M_\odot$, HNM(II): $M_{\rm max}$=1.80(3)$M_\odot$, HNM(III): $M_{\rm max}$=2.05(3)$M_\odot$

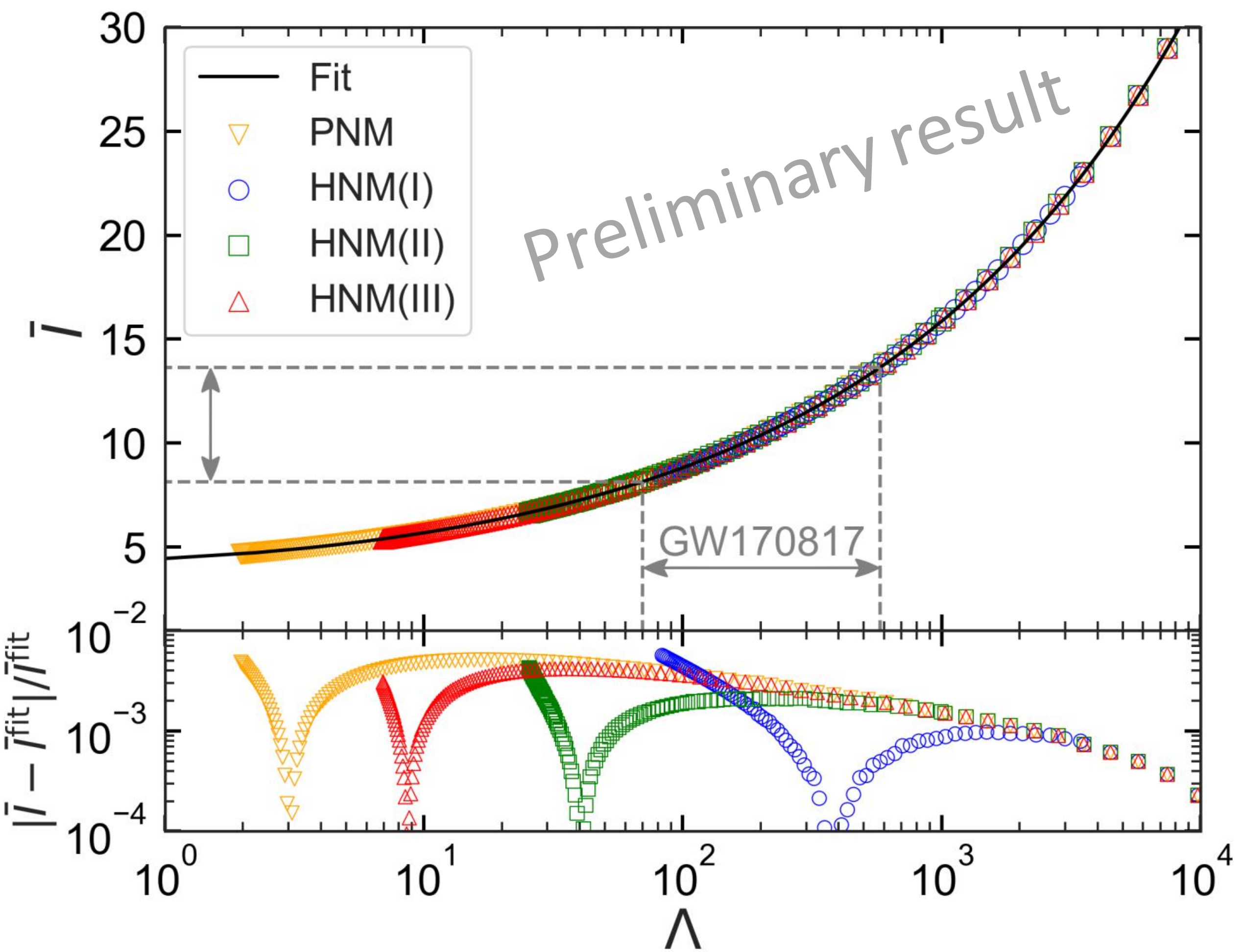
② PSR J0030+0451 : pulsar observed by the Neutron Star Interior Composition Explorer (NICER).

Neutron star tidal deformability



① The tidal deformability Λ are consistent with astrophysical observations GW170817

Universal relations I-Love-Q



① $\bar{I}$ is the dimensionless quantities for the moment of inertia, 8.2<$\bar{I}$<13.7

② Fitting function

✡ K. Yagi and N. Yunes, Science 341, 365 (2013)

$$\ln y_i = a_i + b_i \ln x_i + c_i(\ln x_i)^2 + d_i(\ln x_i)^3 + e_i(\ln x_i)^4$$

Contents

- Introduction
- Hyper-Neutron matter
- Summary and Outlook

Summary and Outlook

① Simulations for different number of hyperons and neutrons can be achieved from our *ab initio* calculations, and hence the neutron star EoS are calculated precisely.
② The three-body hyperon-nucleon interaction plays an important role for the maximum mass. HNM(III) is the best solution for hyperon puzzle in our simulation.
③ The universal relation is found for hyper-neutron matter in ab initio calculations.

① Include protons in our hyper-neutron system

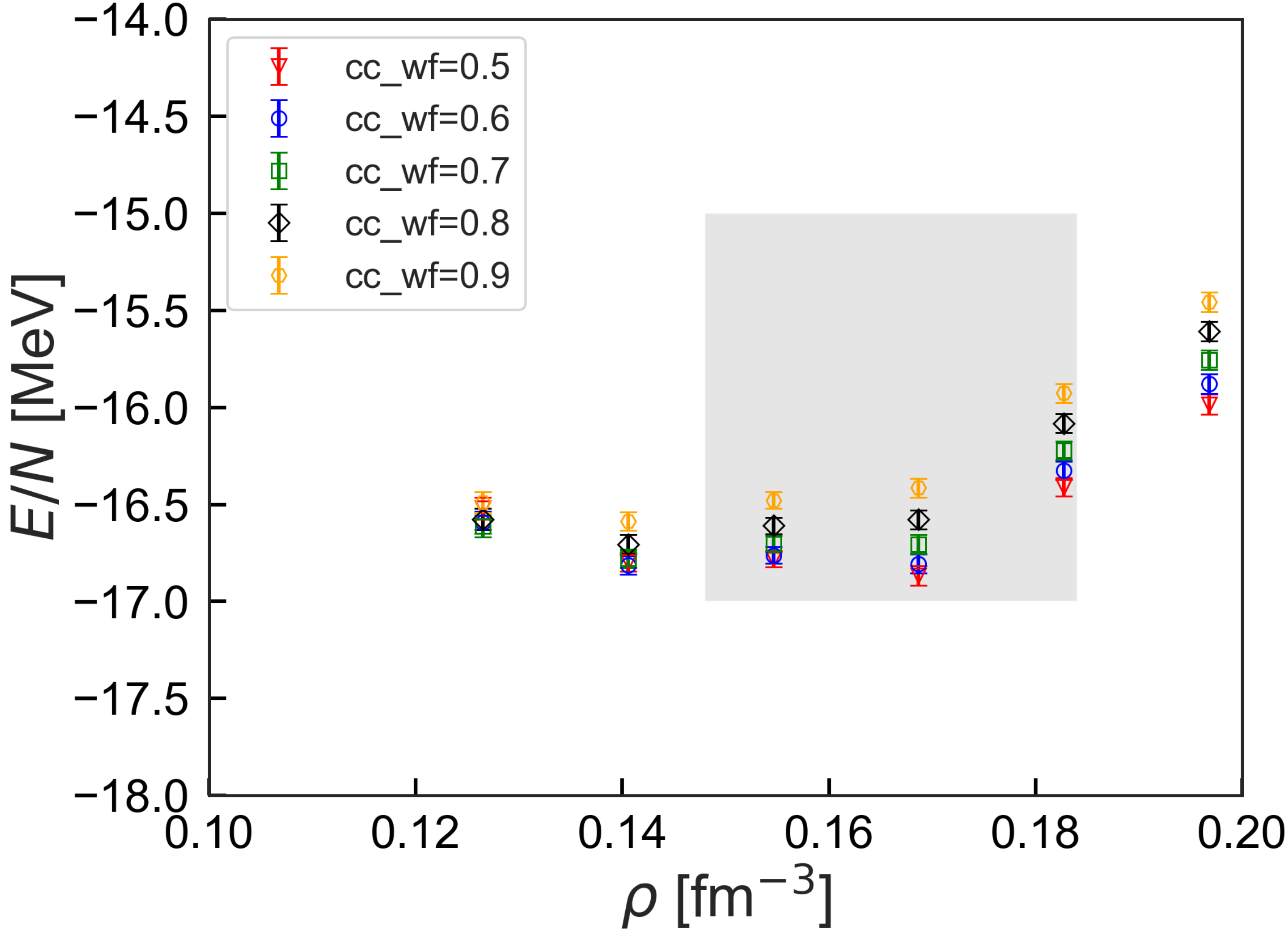
Acknowledgement

Prof. Ulf-G. Meißner, Prof. Serdar Elhatisari, Dr. Zhengxue Ren

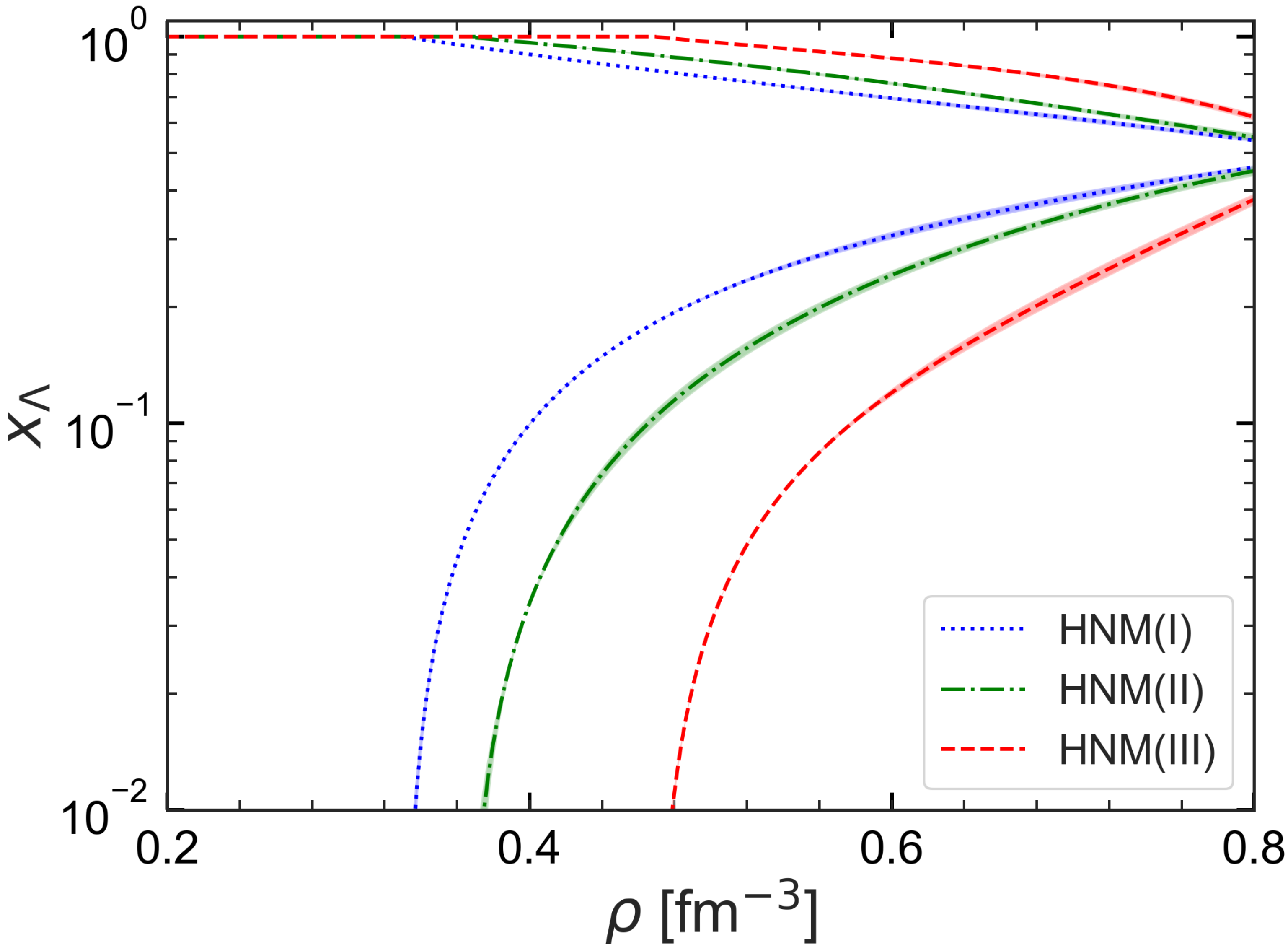
Thanks for your attention !

Appendix

Symmetric nuclear matter



Fraction

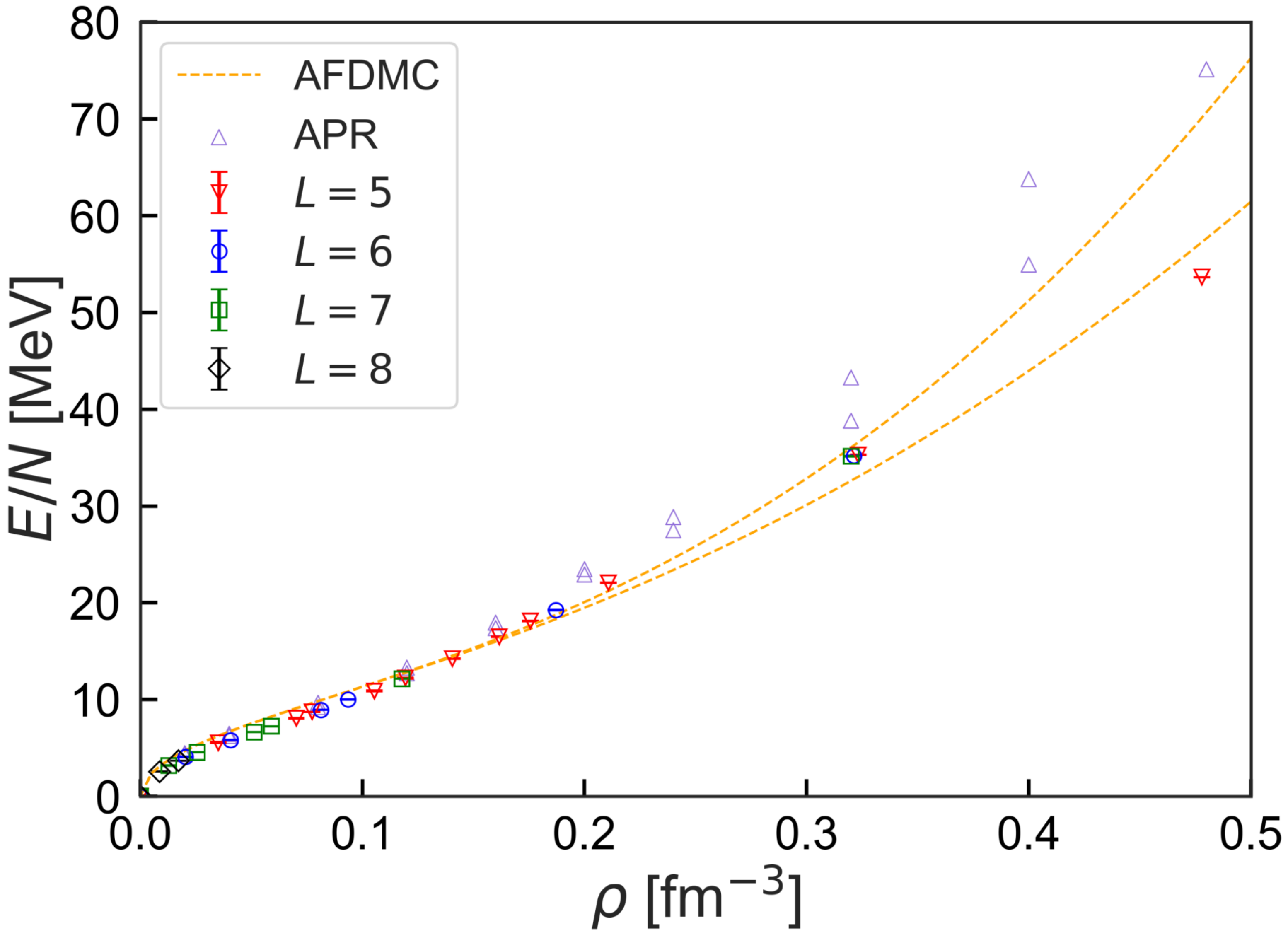


Hyper-Nuclei

Separation Energy B_Λ			
	${}^{5}_{\Lambda}\mathrm{He}$	${}^{9}_{\Lambda}\mathrm{Be}$	${}^{13}_{\Lambda}\mathrm{C}$
NLEFT	3.104 ± 0.086	6.641 ± 0.127	11.712 ± 0.144
EXP.	3.102±0.030	6.614±0.072	11.797±0.157

Separation Energy $B_{\Lambda\Lambda}$		
	${}^{6}_{\Lambda\Lambda}\mathrm{He}$	${}^{10}_{\Lambda\Lambda}\mathrm{Be}$
NLEFT	6.961 ± 0.087	14.349 ± 0.128
EXP.	6.910±0.160	14.700±0.400

Finite volume



2B and 3B interaction in AFDMC

For the hyperon sector, we adopted the phenomenological hyperon-nucleon potential that was first introduced by Bodmer, Usmani, and Carlson in a similar fashion to the Argonne and Urbana interactions [44]. It has been employed in several calculations of light hypernuclei [45–51] and, more recently, to study the structure of light and medium mass Λ hypernuclei [34,35]. The two-body ΛN interaction, $v_{\lambda i}$, includes central and spin-spin components and it has been fitted on the available hyperon-nucleon scattering data. A charge symmetry breaking term was introduced in order to describe the energy splitting in the mirror Λ hypernuclei for $A = 4$ [34,47]. The three-body ΛNN force, $v_{\lambda ij}$, includes contributions coming from P- and S-wave 2π exchange plus a phenomenological repulsive term. In this work we have considered two different parametrizations of the ΛNN force.

The authors of Ref. [49] reported a parametrization, hereafter referred to as parametrization (I), that simultaneously reproduces the hyperon separation energy of ${}^{5}_{\Lambda}$He and ${}^{17}_{\Lambda}$O obtained using variational Monte Carlo techniques. In Ref. [34], a diffusion Monte Carlo study of a wide range of Λ hypernuclei up to $A = 91$ has been performed. Within that framework, additional repulsion has been included in order to satisfactorily reproduce the experimental hyperon separation energies. We refer to this model of ΛNN interaction as parametrization (II).

No $\Lambda\Lambda$ potential has been included in the calculation. Its determination is limited by the fact that $\Lambda\Lambda$ scattering data are not available and experimental information about double Λ hypernuclei is scarce. The most advanced theoretical works discussing $\Lambda\Lambda$ force [52,53], show that it is indeed rather weak. Hence, its effect is believed to be negligible for the purpose of this work. Self-bound multi-strange systems have been investigated within the relativistic mean field framework [54–56]. However, hyperons other than Λ have not been taken into account in the present study due to the lack of potential models suitable for quantum Monte Carlo calculations.

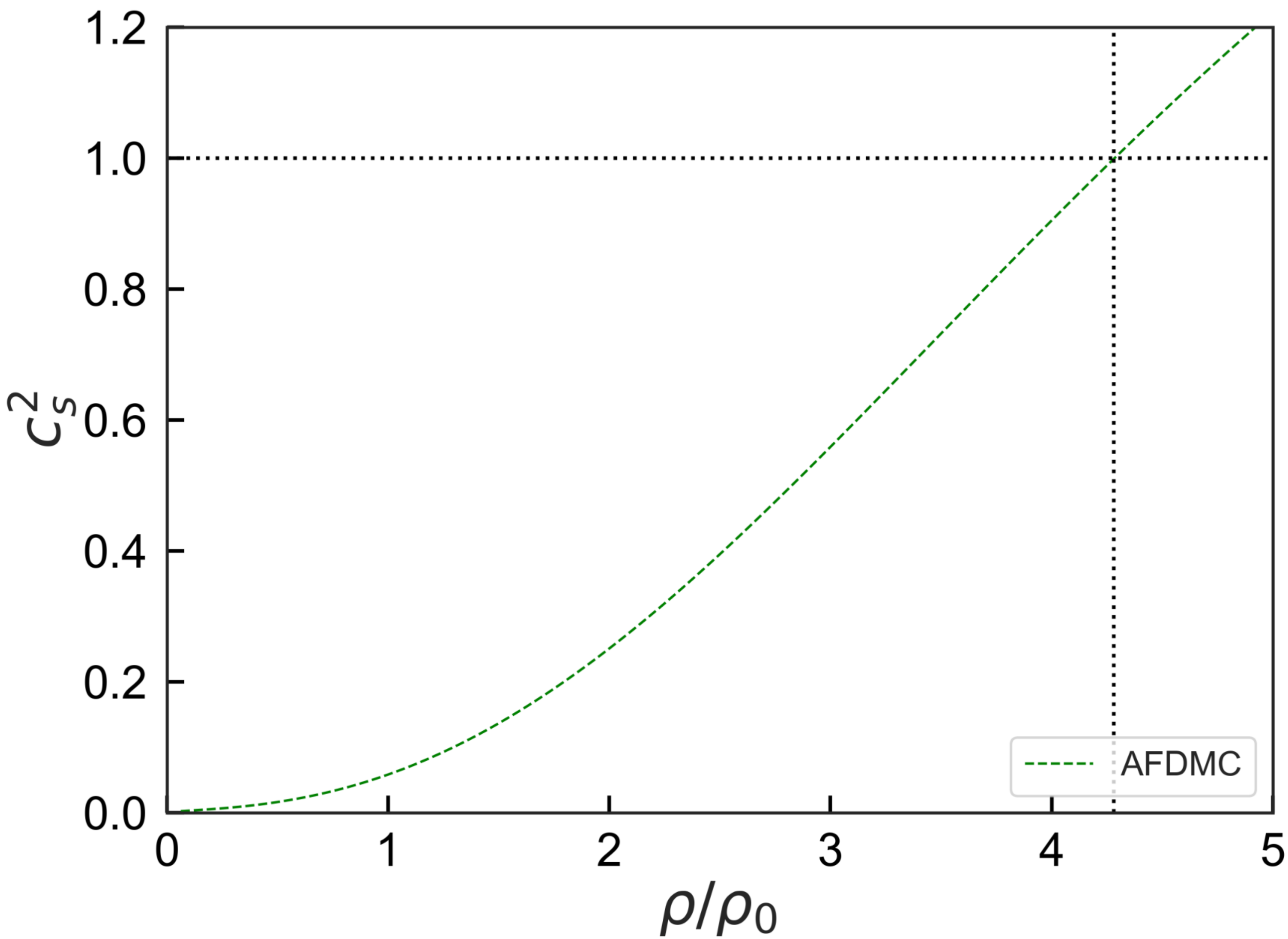
Parameterization in AFDMC

$\rho_\Lambda = x\rho$ are the neutron and hyperon densities, respectively. The energy per particle can be written as

$$E_{\mathrm{HNM}}(\rho, x) = [E_{\mathrm{PNM}}((1-x)\rho) + m_n](1-x) + [E_{\mathrm{P\Lambda M}}(x\rho) + m_\Lambda]x + f(\rho, x). \quad (2)$$

We parametrized the energy of pure lambda matter $E_{\mathrm{P\Lambda M}}$ with the Fermi gas energy of noninteracting Λ particles. Such a formulation is suggested by the fact that in the Hamiltonian of Eq. (1) there is no $\Lambda\Lambda$ potential. The reason for parametrizing the energy per particle of hyperneutron matter as in Eq. (2) lies in the fact that, within AFDMC calculations, $E_{\mathrm{HNM}}(\rho, x)$ can be easily evaluated only for a discrete set of x values. They correspond to a different number of neutrons ($N_n = 66, 54, 38$) and hyperons ($N_\Lambda = 1, 2, 14$) in the simulation box giving momentum closed shells. Hence, the function $f(\rho, x)$ provides an analytical parametrization for the difference between Monte Carlo energies of hyperneutron matter and pure neutron matter in the (ρ, x) domain that we have considered. Corrections for the finite-size effects due to the

Speed of Sound



Many body calculations and NN Interactions

① Density functional theories (DFTs) with effective nucleon–nucleon (NN) interactions

② *ab initio* methods with realistic ones

Realistic Interaction

AV18, CD-Bonn,

χEFT, LQCD…

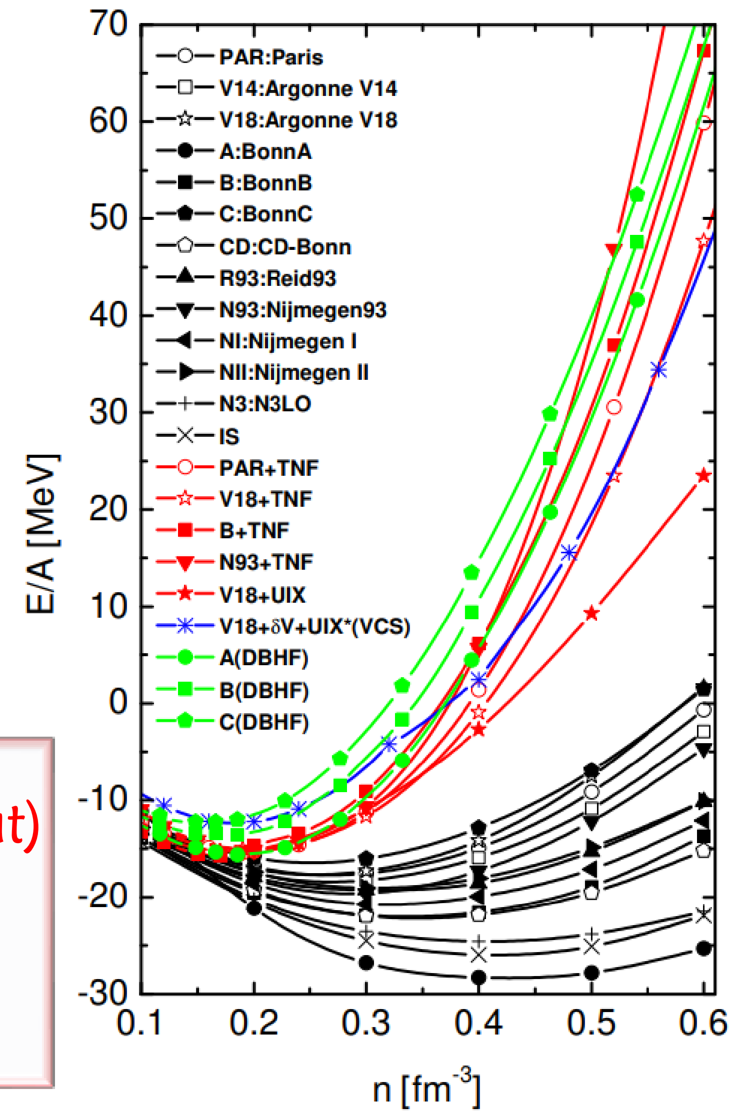
Many-body method

MBPT, NCSM, CC,

QMC, IM-SRG, BBG,

NLEFT…

✓ EoS with (without) hyperons from NLEFT



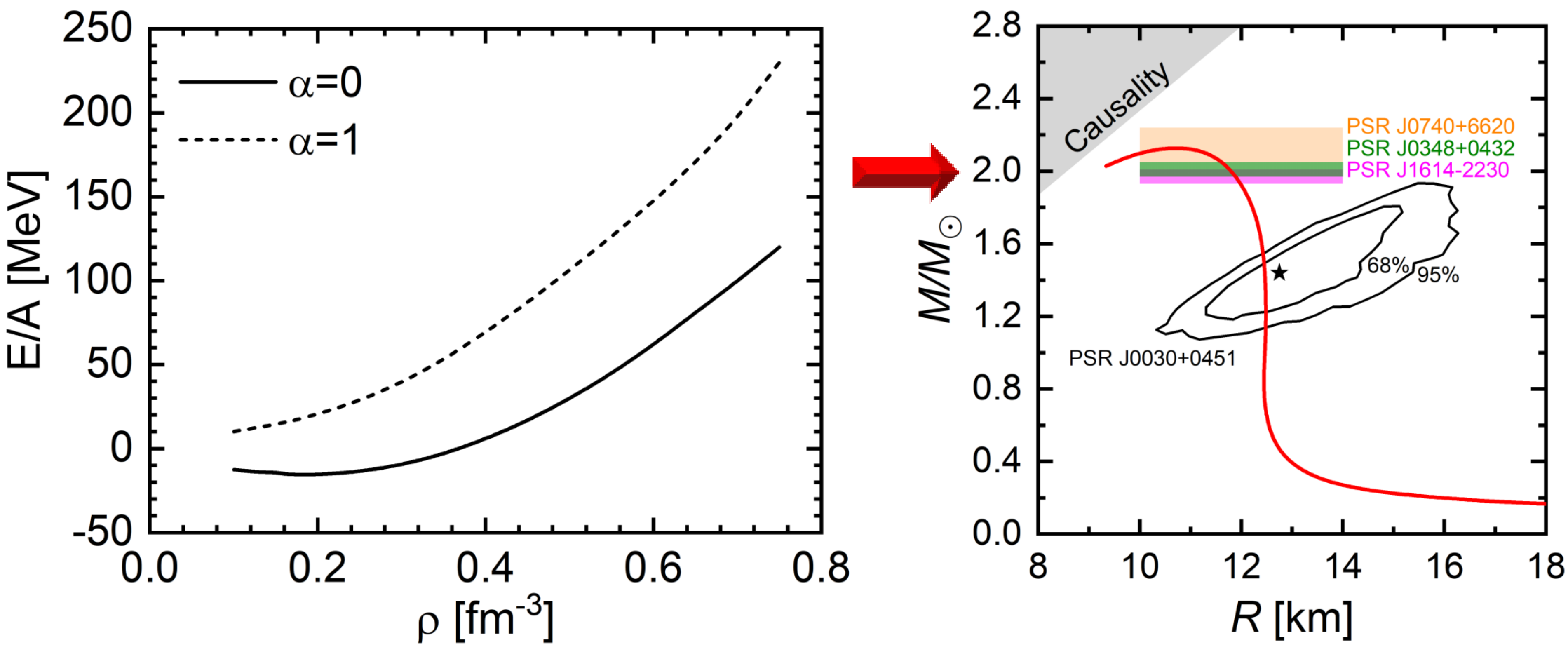
✡ *G. F. Burgio, et al., arXiv 1804, 03020 (2018)*

From EoS to Neutron Star

- The equation of state (EoS) for dense nuclear matter constitutes the basic input quantity for the theoretical reconstruction of a neutron star.

✲ R. C. Tolman, Phys. Rev. 55, 364 (1939)

✲ J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939)



✲ C. H. Lee, et al., Phys. Rev. C 57, 3488 (1998)