Renormalizability in Effective Field Theories: Recent Developments

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Outline

Explicit renormalizationin chiral EFT: motivation

Ingredients of Renormalizability

Examples and counterexamples

Summary

Renormalization in chiral EFT

Expansion parameter: (soft scale)/(hard scale) $Q = \frac{q}{\Lambda_b}$

$$q \in \{ |\vec{p}| \, , M_\pi \} \, , \qquad \Lambda_b \sim M_\rho$$

“Perturbative” calculation of observables

Renormalization in chiral EFT

Expansion parameter: (soft scale)/(hard scale) $Q = \frac{q}{\Lambda_b}$

$$q \in \{|\vec{p}|\,, M_\pi\}\,, \qquad \Lambda_b \sim M_\rho$$

“Perturbative” calculation of observables

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Contains bare parameters → Renormalization: power counting for renormalized quantities

Implicit renormalization

$T = T_0$ fit bare $C_i^{(0)}$

$T = T_0 + T_2$ (re)fit bare $C_i^{(0)}, C_i^{(2)}$

$T = T_0 + T_2 + T_4$ (re)fit bare $C_i^{(0)}, C_i^{(2)}, C_i^{(4)}$

$\ldots$

Implicit renormalization

$T = T_0$ fit bare $C_i^{(0)}$

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$\ldots$

Explicit renormalization

$$T = \mathbb{R}(T_0) + \mathbb{R}(T_2) + \mathbb{R}(T_4) + \ldots$$

$C_i = C_i^r + \delta C_i$ bare =renormalized + counter term

Identify each term individually, or at least prove this is possible

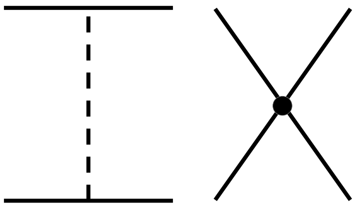
Justifies theoretical error estimation!

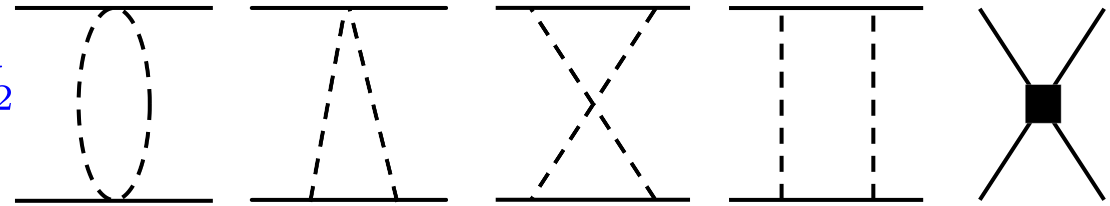
Power counting for NN chiral EFT

Weinberg, S., **NPB363**, 3 (1991)

$\mathcal{O}(Q^0)$

$\mathcal{O}(Q^2)$

$V_{\rm LO} = V_0$ 

$V_{\rm NLO} = V_2$ 

$$|V_0(p',p)| \leq V_{\rm max} \times 1$$

$$|V_2(p',p)| \leq V_{\rm max} \frac{p^2 + p'^2}{\Lambda_b^2} \log \frac{p^2 + p'^2}{M_\pi^2}$$

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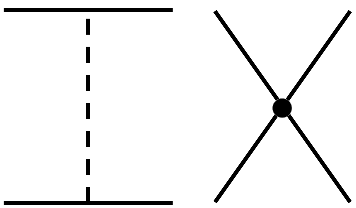

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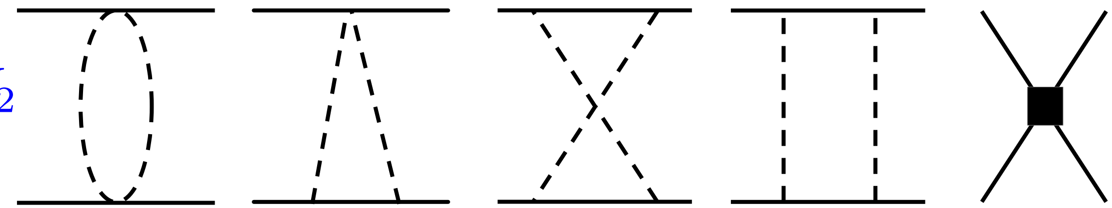

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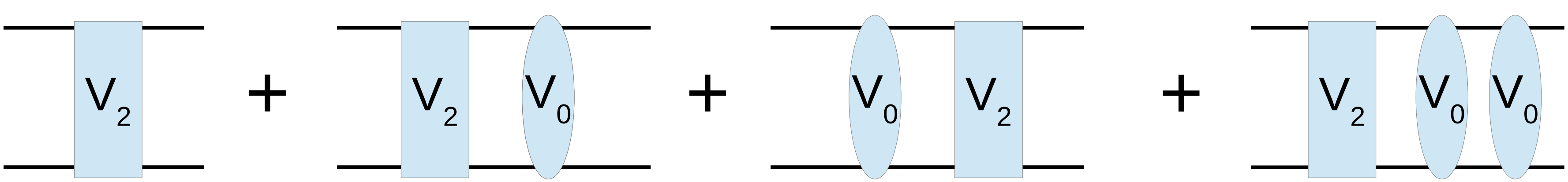
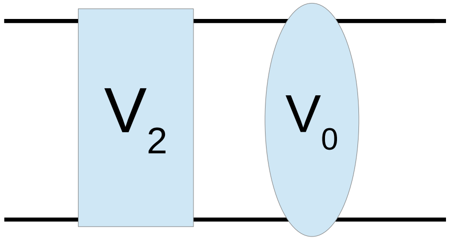
Iterating (resummig) LO potential

$$T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$



Estimating integrals using bounds on potentials



$$T_2^{[0,1]} = \int_0^\infty \frac{p''^2 dp''}{(2\pi)^3} V_2(p', p'') G(p''; p_{\rm on}) V_0(p'', p)$$

LO potential: $V_0 \sim 1$

NLO potential: $V_2 \sim \frac{p^2}{\Lambda_b^2} \log \frac{p^2}{M_\pi^2}$

2-nucleon Green's function: $G \sim \frac{1}{p^2}$

Integral converges at $p \sim \Lambda$ (regulator) $\longrightarrow$ $T_2^{[0,1]} \sim \frac{\Lambda^3}{\Lambda_b^3} \log \frac{\Lambda}{M_\pi} \neq \mathcal{O}(Q^2)$

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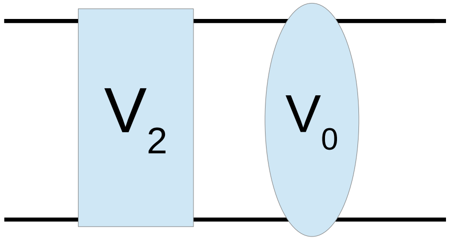

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Renormalization   Subtraction   Counter term δC_0

Structure of the interaction in chiral EFT

Interaction obtained from chiral EFT: $V(\vec{p}\,', \vec{p}) = V_{\rm short}(\vec{p}\,', \vec{p}) + V_{\rm long}(\vec{p}\,', \vec{p})$

$$V_{\rm short}(\vec{p}\,', \vec{p}) = {\rm Polynomial}(\vec{p}\,', \vec{p})\, F_\Lambda(\vec{p}\,', \vec{p})$$

$$V_{\rm long}(\vec{p}\,', \vec{p}) = V_L(\vec{q} = \vec{p}\,' - \vec{p})\, \tilde{F}_\Lambda(\vec{p}\,', \vec{p}), \qquad V_L = V_{1\pi} + V_{2\pi} + \ldots$$

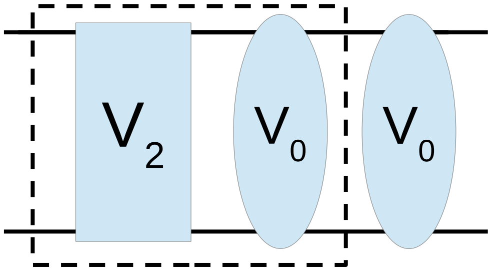
Subtractions: $\left| V(p\,', p) - V(p\,', 0) \right| \le \left| \frac{p}{p'} \right| \times (\ldots) \text{ if } |p'| > |p|$

$$\left| V(p\,', p) - \sum_{i=0}^{n} \frac{\partial^i V(p', p)}{i! (\partial p)^i} \bigg|_{p=0} p^i \right| \le \left| \frac{p}{p'} \right|^{n+1} \times (\ldots) \text{ if } |p'| > |p|$$

AG, E.Epelbaum, **PRC 105**, 024001 (2022)

- Large loop momenta are suppressed
- Renormalizability

More iterations of V_0



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Renormalized NLO amplitude: $\mathbb{R}\left(T_2^{[0,1]}\right) \sim \frac{p^2}{\Lambda_b^2}\frac{\Lambda}{\Lambda_b}\log\frac{\Lambda}{M_\pi}$

2-nucleon Green's function: $G \sim \frac{1}{p^2}$

Integral converges at $p \sim \Lambda$ $\longrightarrow$ $T_2^{[0,2]} \sim \frac{\Lambda^4}{\Lambda_b^4}\log\frac{\Lambda}{M_\pi} \neq \mathcal{O}(Q^2)$

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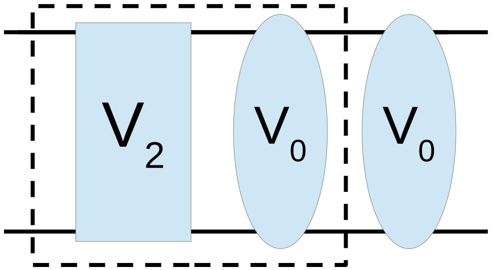

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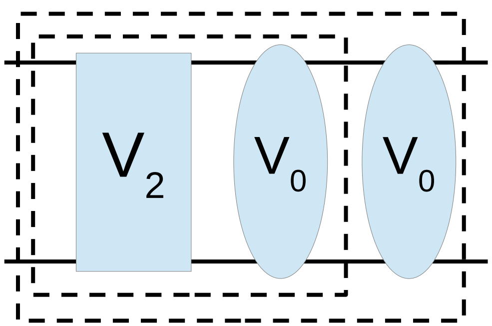
One more subtraction

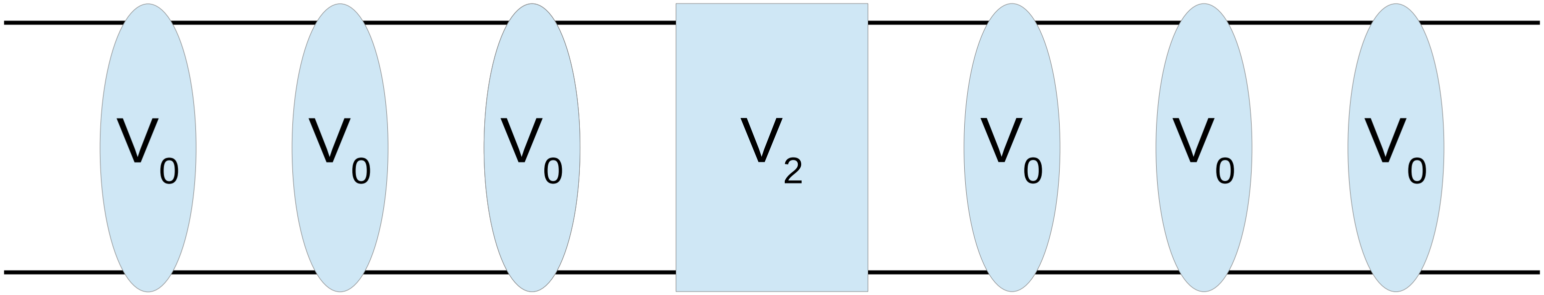
The same form of a counter term δC_0



$$\mathbb{R}\left(T_2^{[0,2]}\right) \sim \frac{p^2}{\Lambda_b^2}\frac{\Lambda^2}{\Lambda_b^2}\log\frac{\Lambda}{M_\pi}$$

General case, BPHZ

N. N. Bogoliubov, O. S. Parasiuk, **AM97**, 227 (1957); K. Hepp, **CMP2**, 301 (1966); W. Zimmermann, **CMP15**, 208 (1969)



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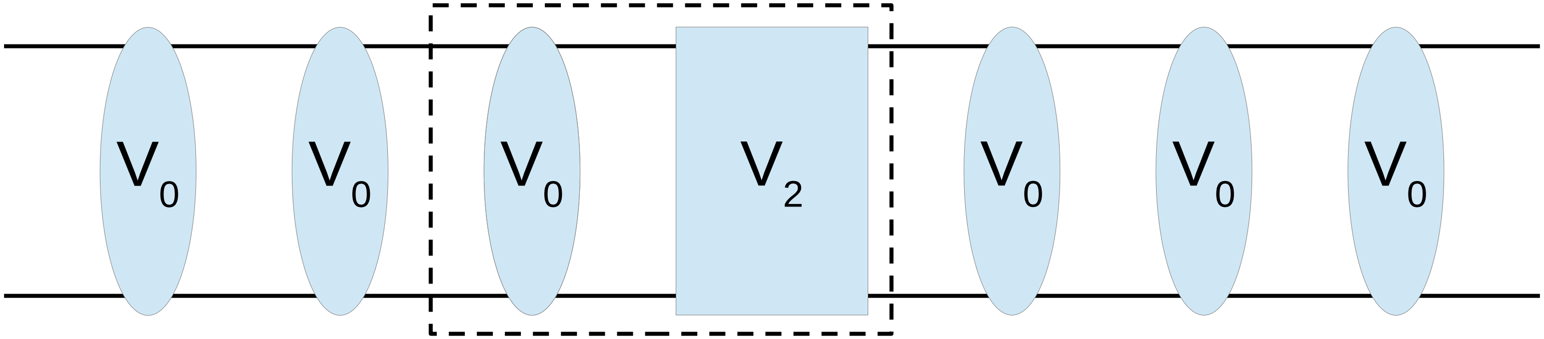

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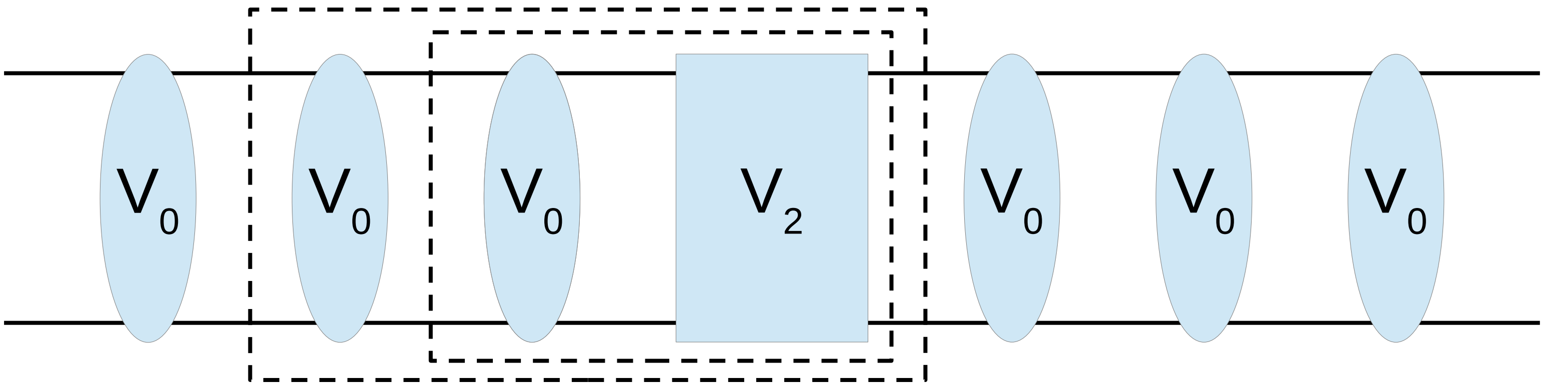

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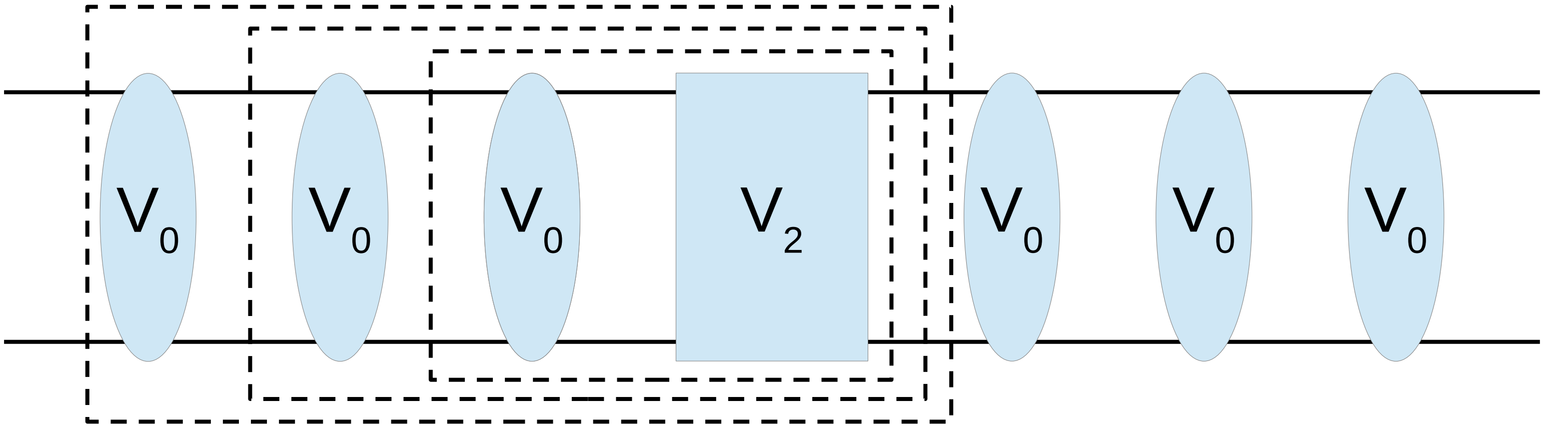

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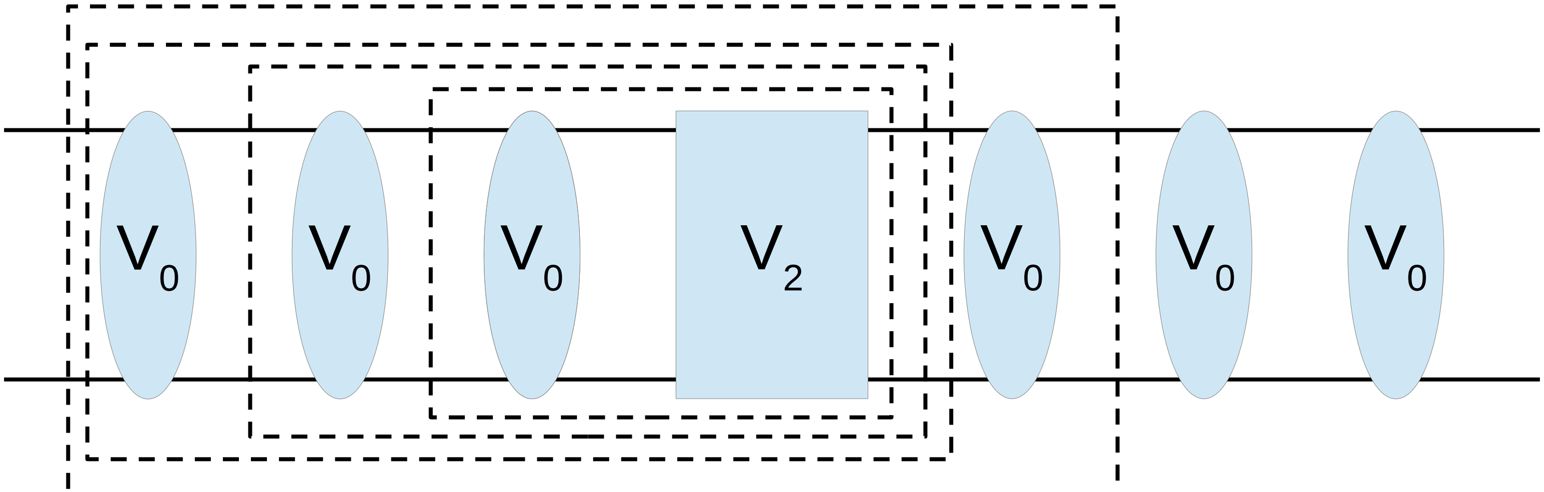

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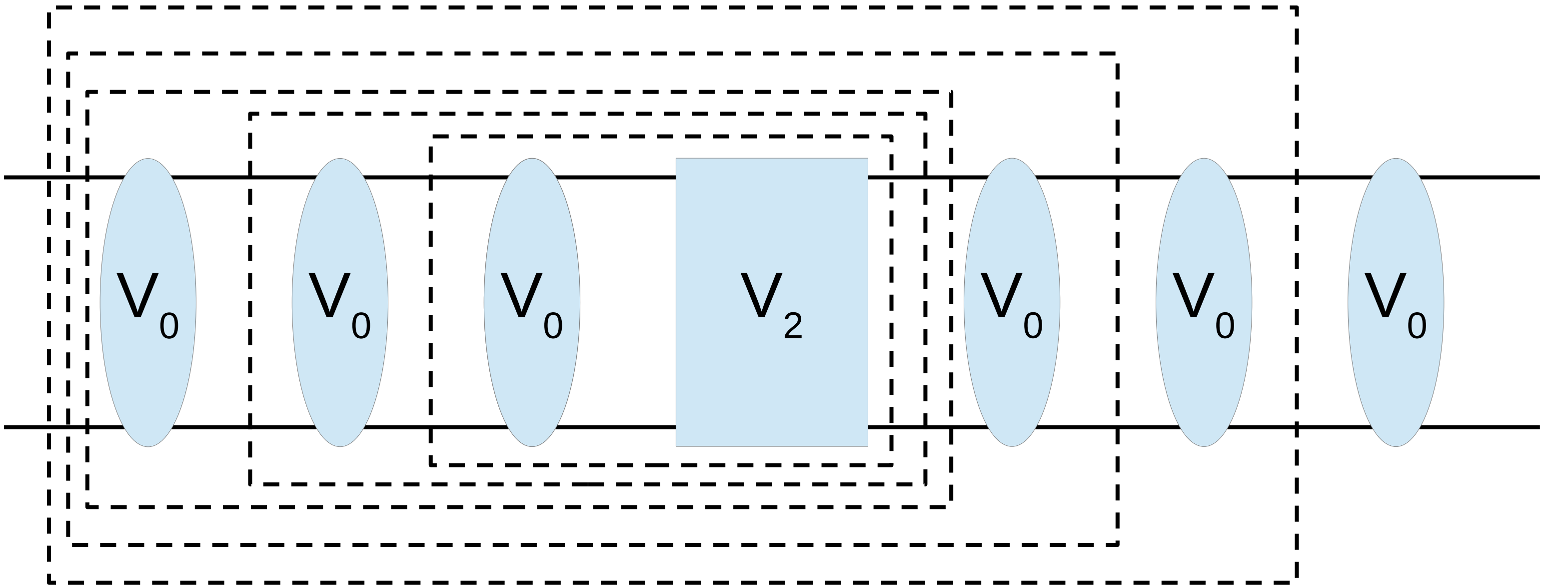

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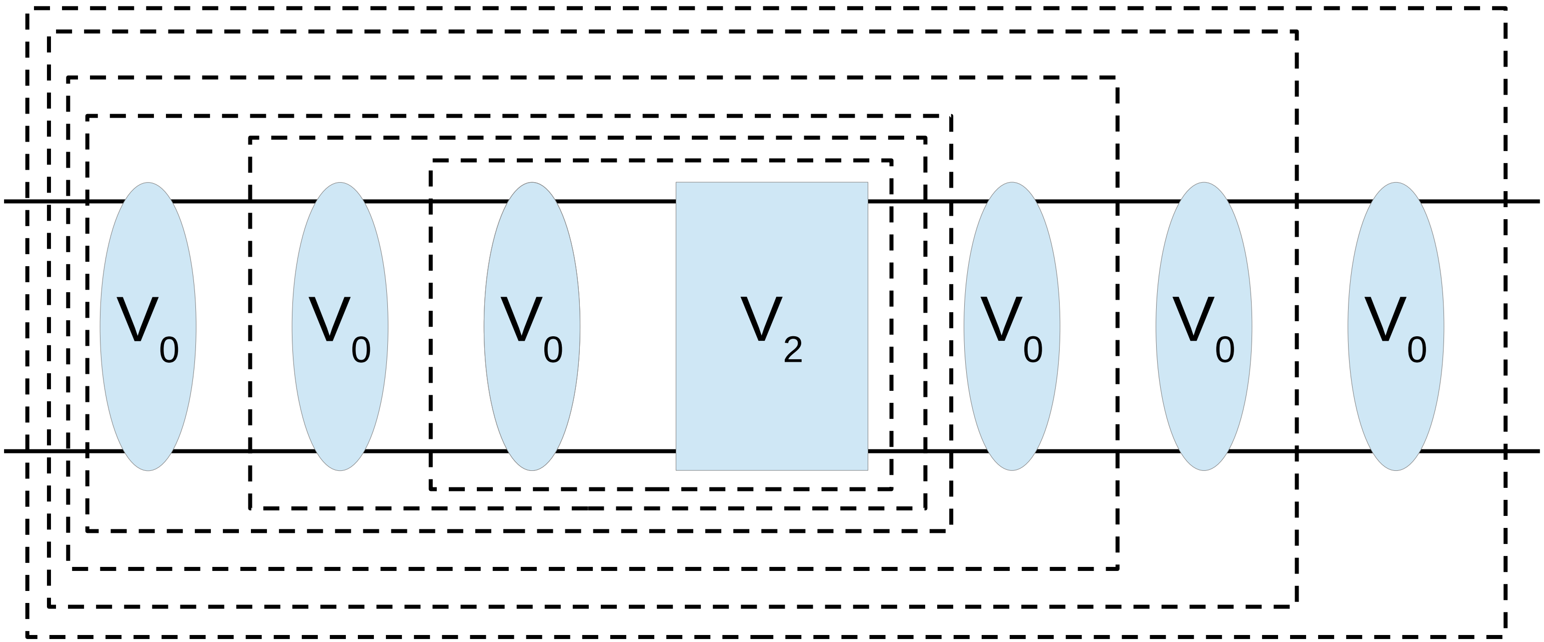

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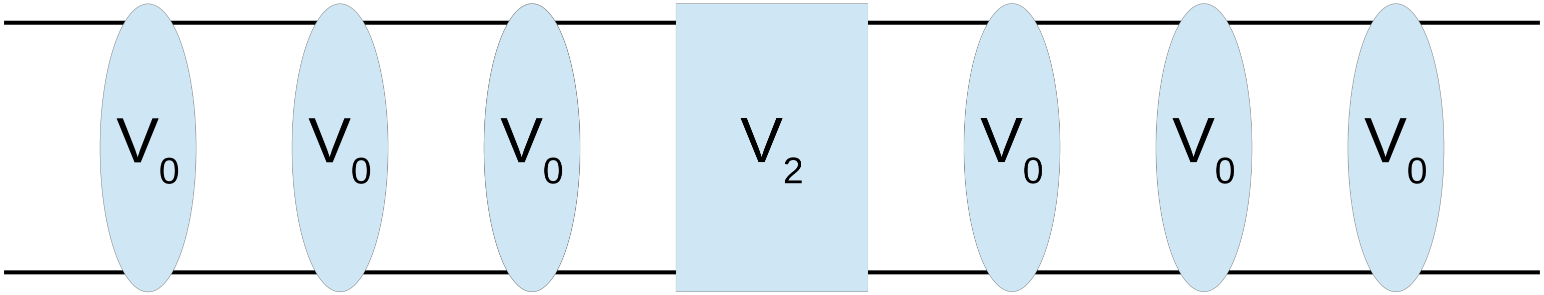

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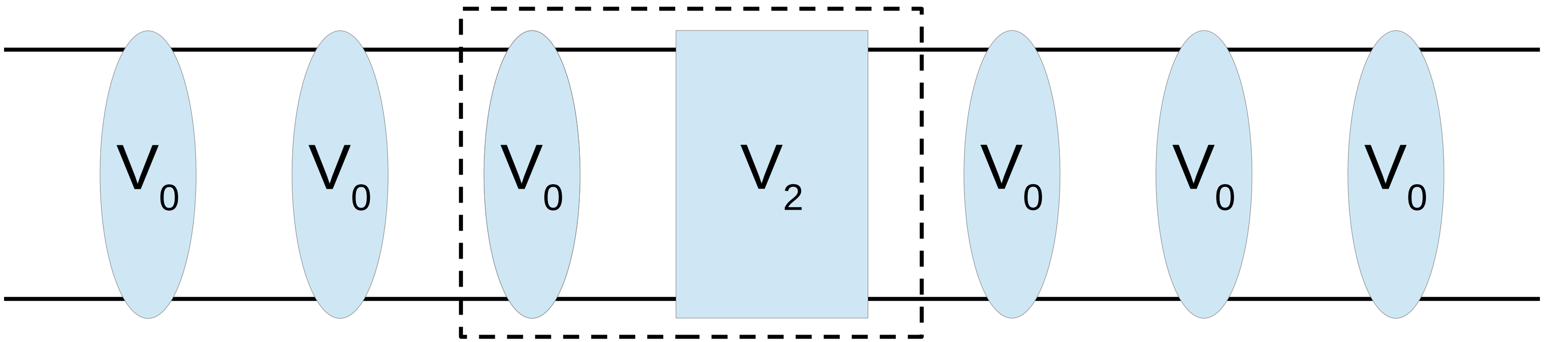

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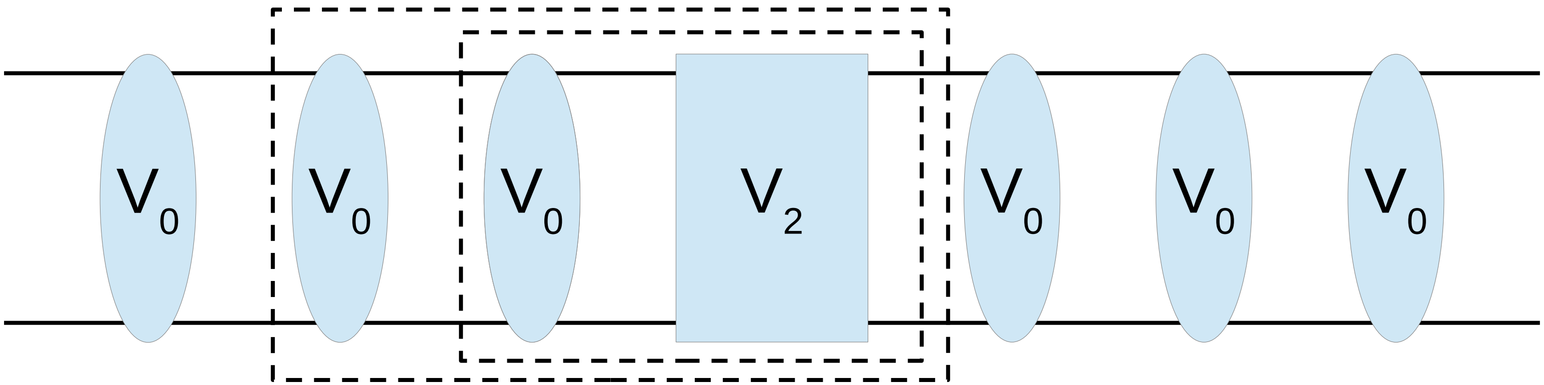

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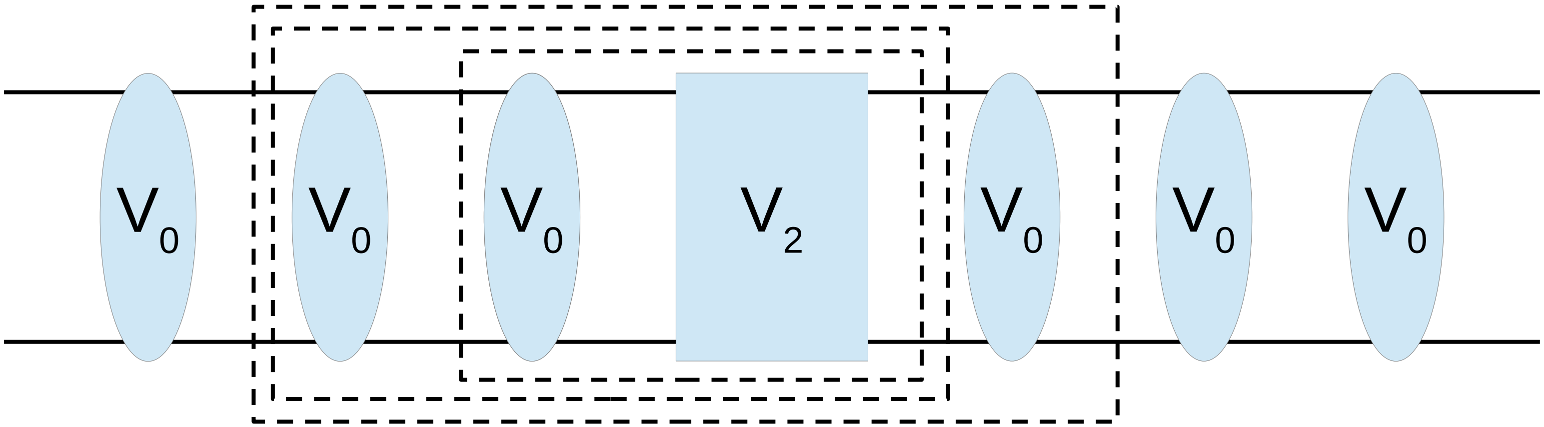

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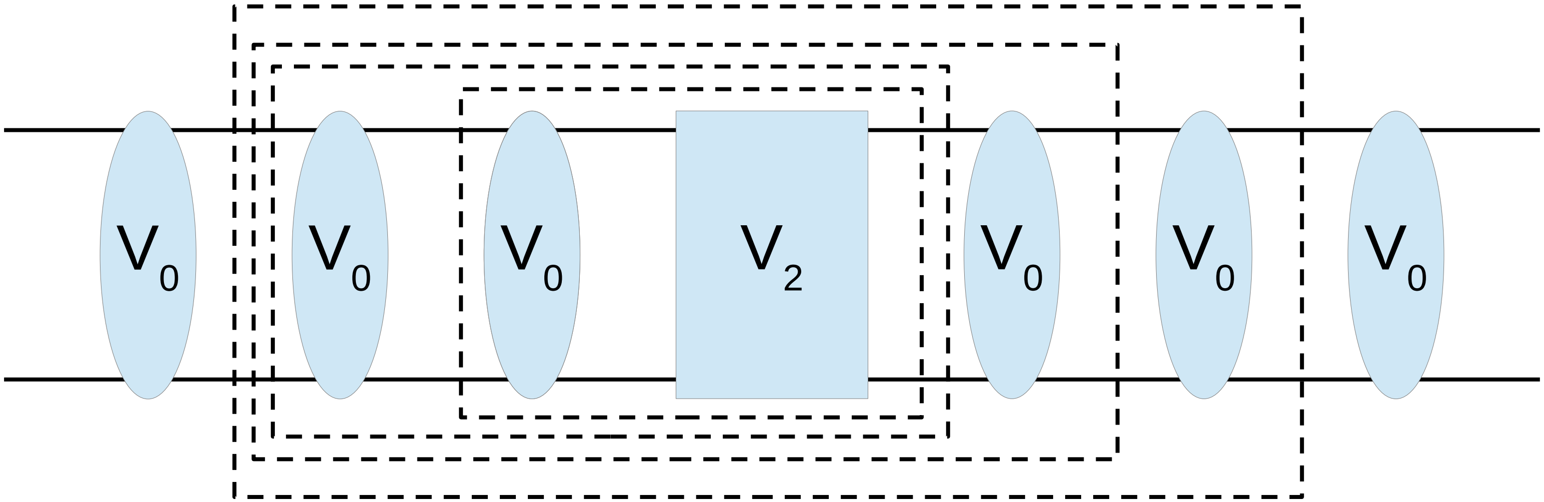

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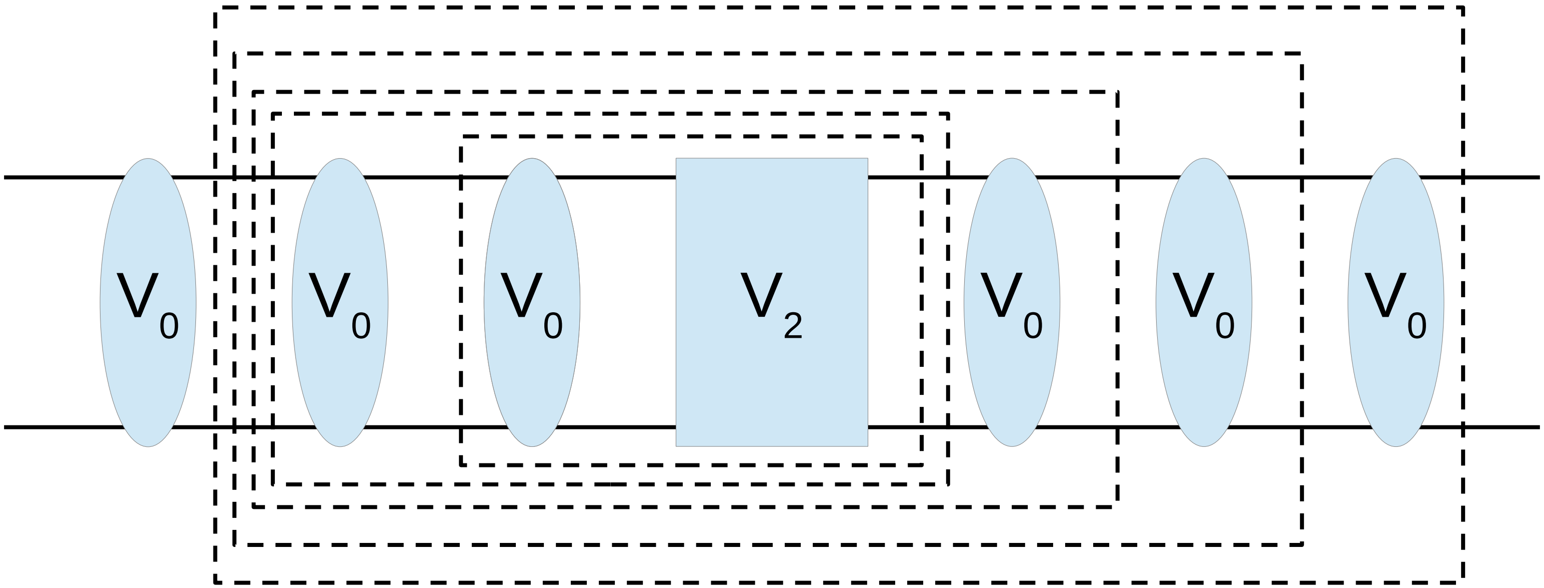

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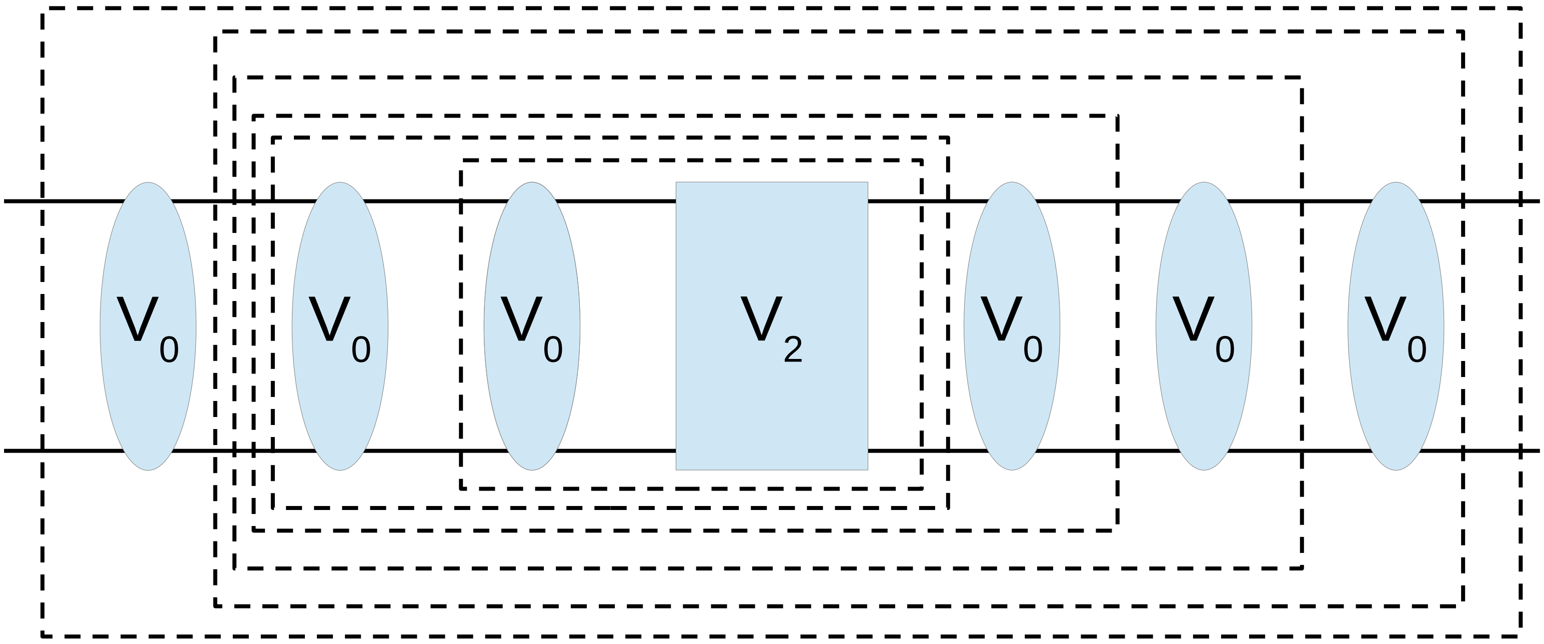

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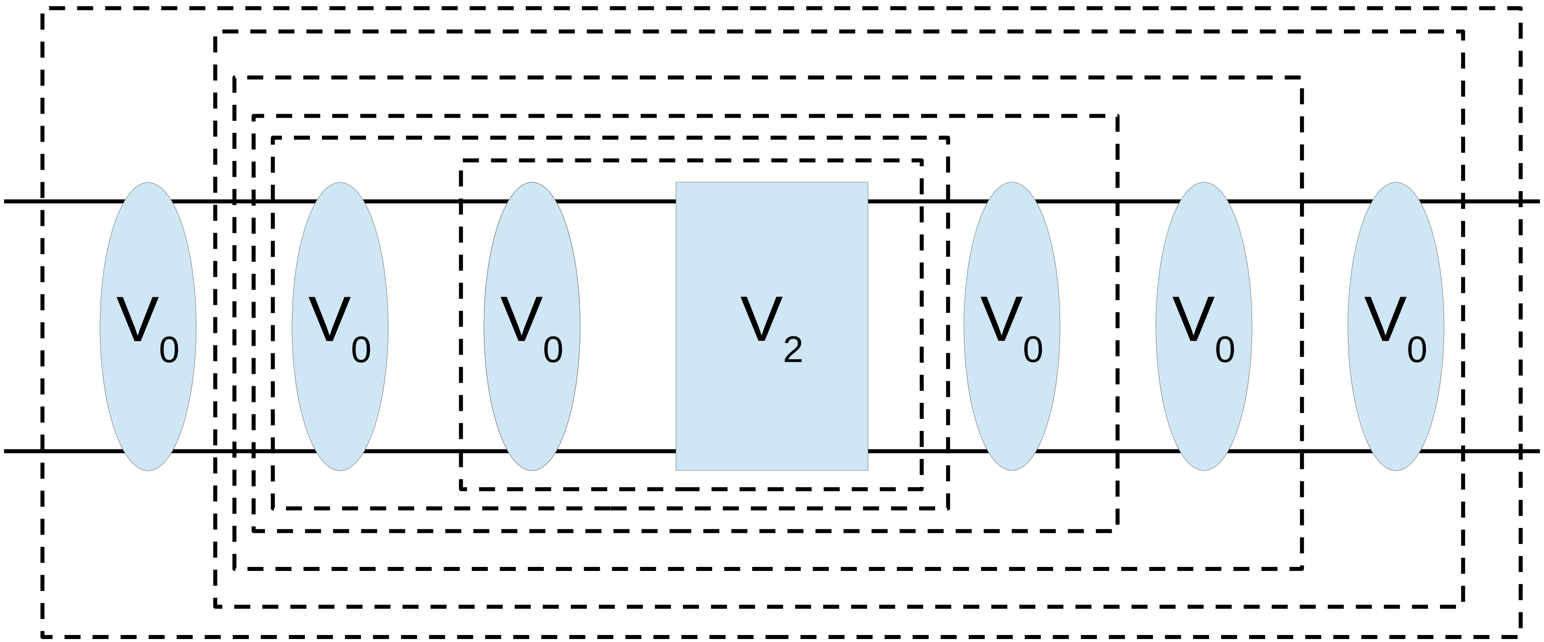

Partition integrations over p_i into sectors to avoid double counting

All counter terms add up to a single counter term δC_0

Power counting in the finite-cutoff scheme, NLO

AG, E.Epelbaum, PRC 105, 024001 (2022)

$$\mathbb{R}\left(T_2\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)$$

$$\mathbb{R}(T_2^{[m,n]})(p) \sim \frac{p^2}{\Lambda_b^2}\left(\frac{\Lambda}{\Lambda_b}\right)^{m+n} \log \Lambda/M_\pi$$

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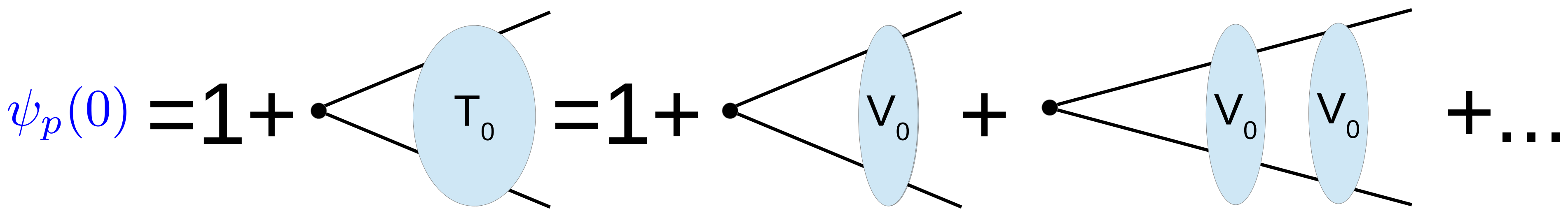
$$\Lambda \approx \Lambda_b$$

$$\mathbb{R}(T_2^{[m,n]})(p) = \mathcal{O}(Q^2)$$

Renormalization in the non-perturbative regime

The series for ***R($T_2^{[m,n]}$)*** can be summed explicitly

$$\mathbb{R}\left(T_2\right)(p) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)(p) = T_2(p) + \delta C_0\, \psi_p(0)^2, \qquad \delta C_0 = -\frac{T_2(0)}{\psi_0(0)^2}$$



$$\mathbb{R}\left(T_2\right)(p=0) = 0$$

Using Fredholm formula to match to the perturbative regime

$$T_2(p) = (1 + T_0 G) V_2 (1 + G T_0) = \frac{N_2(p)}{D(p)^2}$$

$D(p)$ -Fredholm determinant

Convergent series in V_0: $$N_2 = \sum_{i=0}^{\infty} N_2^{[i]}\,, \;\; D = \sum_{i=0}^{\infty} D^{[i]}$$

The same for the counter terms:

$$\delta T_2 = (1 + T_0 G)\delta V_0^{ct}(1 + G T_0) = \delta C_0 \left[\psi_p(0)\right]^2$$

$$\psi_p(0) = \frac{\nu(p)}{D(p)} \qquad \nu(p) = \sum_{i=0}^{\infty} \nu^{[i]}(p)$$

Renormalizability constraints

$$\mathbb{R}\left(T_2\right)(p) = \frac{1}{D(p)^2}\left[N_2(p) + \delta C_0\, \nu(p)^2\right], \qquad \delta C_0 = -\frac{N_2(0)}{\nu_0^2}$$

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Potentially problematic factor

Renormalizability constraints

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Potentially problematic factor

Renormalizability constraints on (the short-range part of) the LO potential.
The simplest formulation: LECs must be of natural size (If $\Lambda \sim \Lambda_b$).

Key ingredients of renormalizability

(1) Locality of the long-range forces

(2) Cutoff of the order of the hard scale $\Lambda \approx \Lambda_b$

(3) Naturalness of the counter terms

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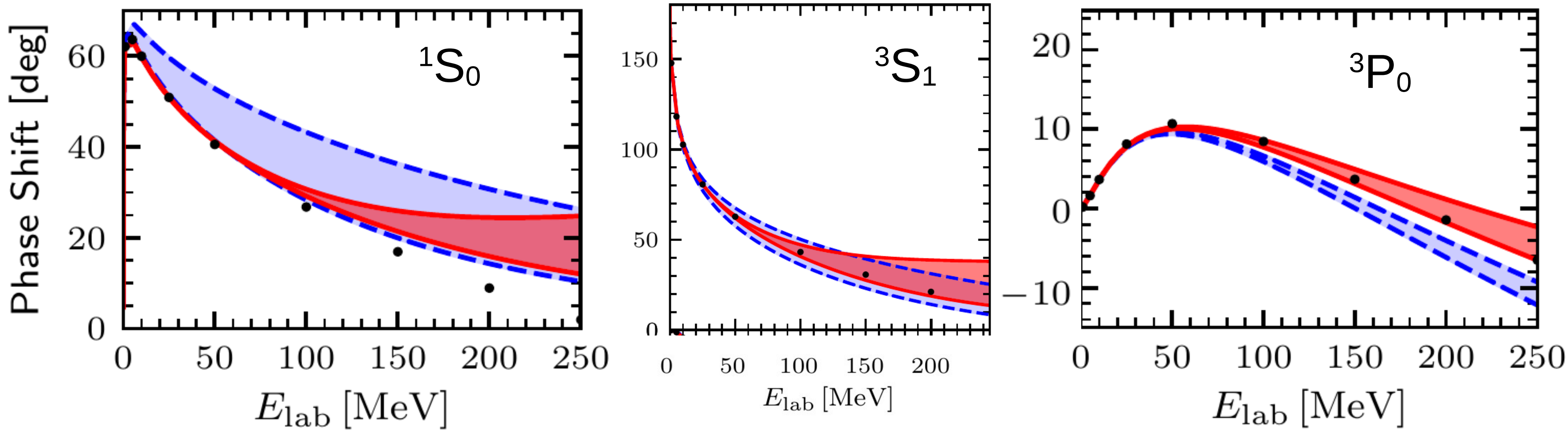
(1)+(2) in most cases imply (3): (1)+(2) $\rightarrow$ (3)

Examples and counterexamples:
1. Finite cutoff scheme (e.g. Bochum group)

AG, E.Epelbaum, **PRC107**, 044002 (2023)

NN at NLO: “Nonperturbative partial waves” $450\text{MeV} \leq \Lambda \leq 800\text{MeV}$



LECs are natural for cutoffs below 600 MeV
and start to rapidly increase when the cutoff approaches 800 MeV and higher

Examples and counterexamples:
2. Fully local interaction

AG, E.Epelbaum, Y.Komissarova, in progress

All potentials (short- and long-range) are local

A.Gezerlis et al., **PRC 90**, 054323 (2014)

$$V(\vec{p}\,',\vec{p}) = V(\vec{q} = \vec{p}\,' - \vec{p}), \quad V(\vec{r}\,',\vec{r}) = V(r)\delta(\vec{r} - \vec{r}\,').$$

Analysis of renormalizability is much easier

$$T_2(p) = \int r^2 dr V_2(r) \psi_p^{(+)}(r)^2 = \frac{(4\pi)^2}{f(p)^2} \int dr V_2(r) \phi_p(r)^2$$

$f(p)$ -Jost function (Fredholm determinant) containts nonperturbative physics

$\phi_p(r)$-regular solution is represented by the rapidly converging series (1/n!):

$$\phi_p = \sum_{n=0}^{\infty} \phi_p^{(n)}, \qquad \phi_p^{(n+1)}(r) = \frac{m_N}{p} \int_0^r dr' \sin[p(r - r')] V_0(r') \phi_p^{(n)}(r')$$

Naturalness condition can be satisfied automatically

$\psi_p(0) \equiv D^{-1}(p)$ ⟶ dangerous numerator $\nu(p) \equiv 1$

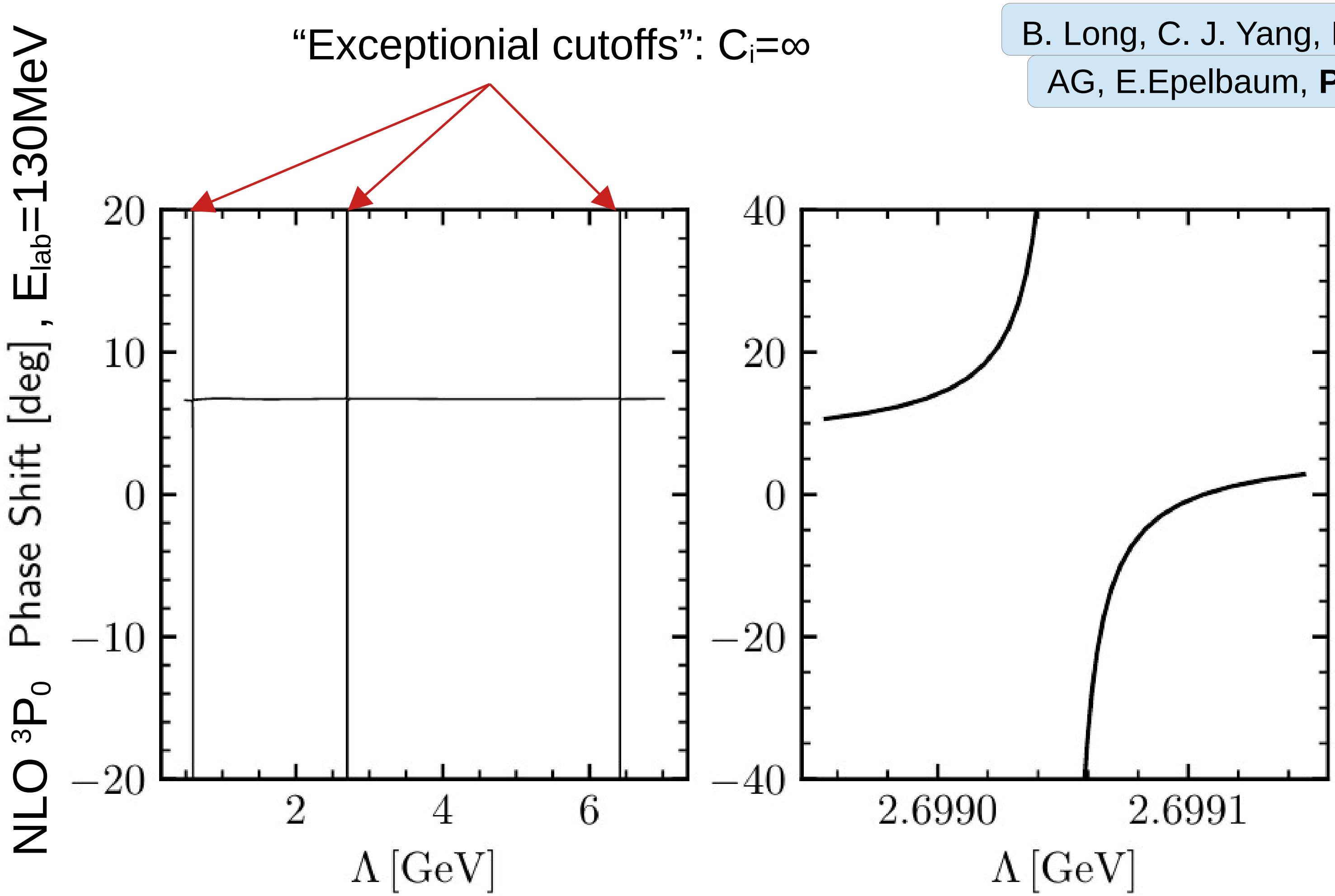
Examples and counterexamples:
3. Increasing the cutoff above the hard scale.

Naturalness condition does not make sense

→ numerical verification

B. Long, C. J. Yang, **PRC84**, 057001 (2011)

AG, E.Epelbaum, **PRC107**, 034001 (2023)



Non-local separable long-range interaction

AG, E.Epelbaum, N.Jacobi, in progress

$$V_0 = C_0 F_\Lambda(p') F_\Lambda(p) + \ldots ,$$

$$V_2 = C_2 \frac{p'^2 + p^2}{\Lambda_b^2} \frac{p'^2 p^2}{(M_\pi^2 + p'^2)(M_\pi^2 + p^2)} F_\Lambda(p') F_\Lambda(p).$$ two-pion exchange

$$F_\Lambda(p) = \frac{\Lambda^2}{(\Lambda^2 + p^2)}$$

$$V_0 G V_2 \sim \frac{\Lambda^2}{\Lambda_b^2} \frac{p^2}{(M_\pi^2 + p^2)} \sim O(Q^0)$$

$$\int dp', \qquad p' \sim \Lambda$$

- Long-range power-counting-breaking terms
- Nonrenormalizability (in terms of local counter terms)

Similar result when V_0 is nonperturbative

Summary

- ✔ Explicit renormalization of an EFT provides a justified systematic expansion of observables and theoretical error estimate

- ✔ Three ingredients of renormalizability

- ✔ Examples and counterexampes