RUB

LENPIC 2024 Meeting, Bonn, 12.03.2024

Precision theory for charge radii of A=2,3,4 nuclei

Preliminary results

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in collaboration with

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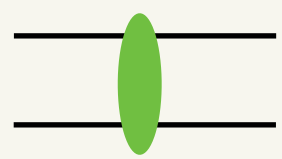
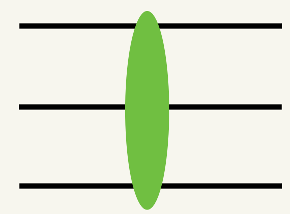
Nuclear charge radii from chiral effective field theory

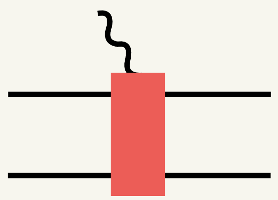
Low-energy **chiral effective field theory** of the standard model



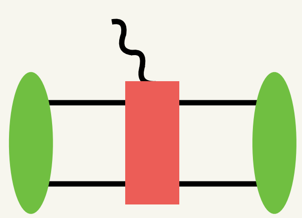
Chiral two-nucleon forces



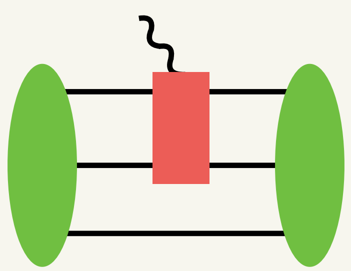
Chiral three-nucleon forces



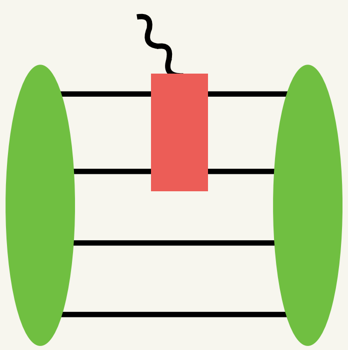
Chiral electromagnetic currents



^{2}**H**

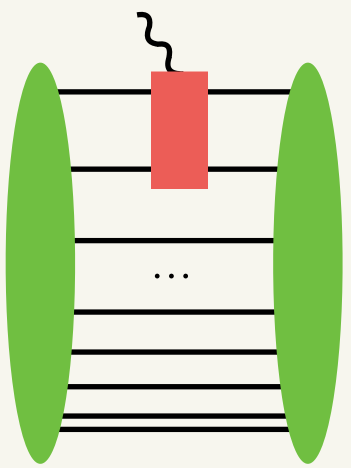


^{3}**He** ^{3}**H**



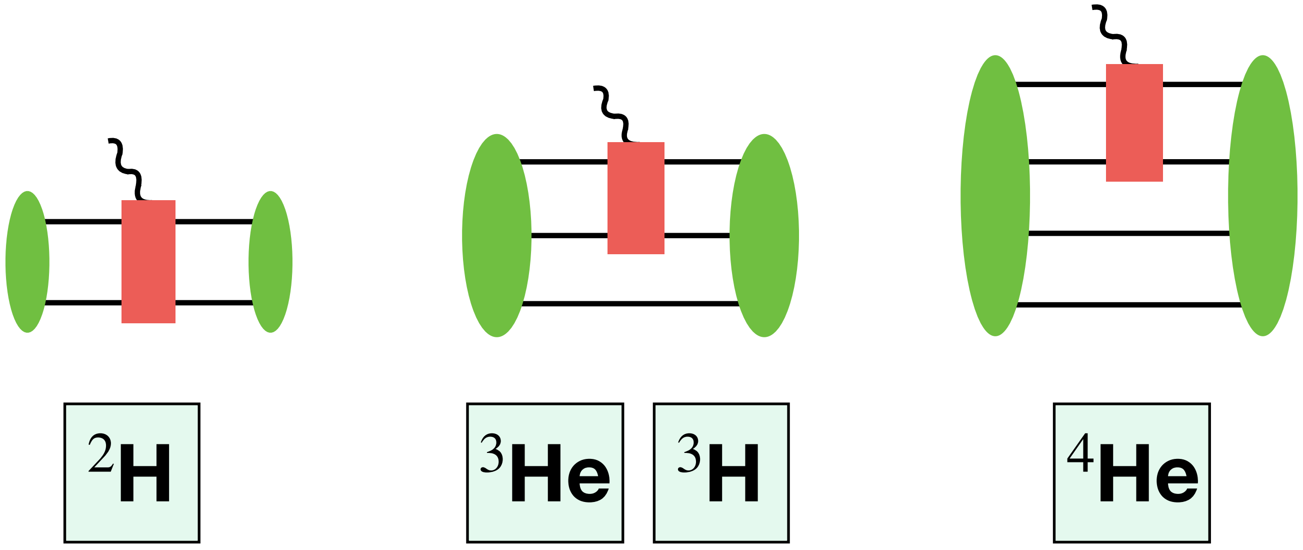
^{4}**He**

Charge form factors and radii of light nuclei



Charge FF and radii of medium-mass nuclei

Precise calculations of charge radii of super-light nuclei



Motivation:

- Precision **tests of nuclear chiral effective field theory (Chiral EFT)**

- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**

- **Extract the neutron and the proton charge radii** from few-nucleon data

Charge radii of A ≤ 4 nuclei — experimental data

Number of protons Z

Number of neutrons N

^{3}He

$r = 1.97007(94)\,\mathrm{fm}$

0.05 %

CREMA 2023
arXiv:2305.11679

^{4}He

$r = 1.6786(12)\,\mathrm{fm}$

0.07 %

Krauth et al. 2021
Pachucki:2022tgl

Pohl:2013yb,
CREMA:2016idx,
Pohl:2016glp,
Schmidt:2018kjc,
Krauth:2021foz
...

^{1}H

$r_p = 0.8406(4)\,\mathrm{fm}$

0.05 %

Antognini et al. 2013
Pachucki:2022tgl

^{2}H

$r_d = 2.12758(78)\,\mathrm{fm}$

0.04 %

CREMA:2016idx
Pachucki:2022tgl

^{3}H

$r = 1.7xxx(\quad 2)\,\mathrm{fm}$

0.01 %

TREX Mainz
[in progress]

Data from isotope shift

$(r_d^2 - r_p^2) = 3.82070(31) fm^2$

0.01 %

Pachucki et al. 2018
Jentschura et al. 2011

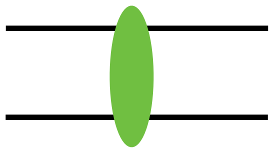
n

$r_n^2 = -0.1155(17)\,\mathrm{fm}^2$

1.5 %

PDG 2022

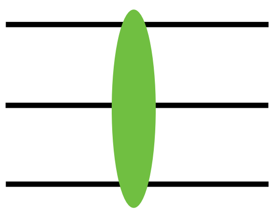
Chiral effective field theory - **precise, accurate** and **consistent**



N^4LO^+

New **high-precision chiral NN forces** (N^4LO^+) Reinert et al. PRL 126, 092501 (2021)

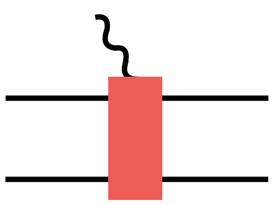
- Nearly perfect description of pp and pn scattering data up to pion production threshold



N^2LO

Chiral 3N forces (general N^2LO; selected terms at N^4LO) Epelbaum:2019kcf

- LECs cD and cE (N^2LO) are fitted to RIKEN Nd DCS data and ^{3}He binding energy
- **Consistent** regularisation of N^3LO is in progress (talk by Hermann)
- Inclusion of the N^3LO off-shell effects are in progress (talk by Sven)



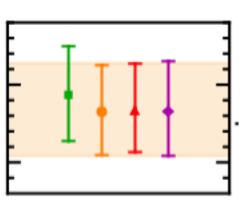
N^2LO

Isoscalar: N^4LO

2N Chiral electromagnetic currents (general N^2LO; isoscalar N^4LO)

- N^2LO (**isoscalar N^4LO**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^3LO (isovector) is in progress

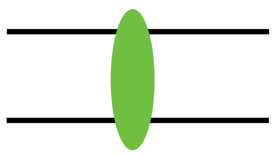
Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)



Reliable methods to quantify truncation uncertainty of the EFT expansion

Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017), Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

Chiral effective field theory - **precise, accurate** and **consistent**



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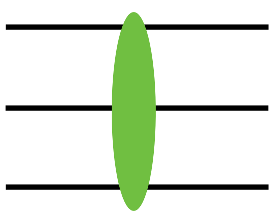

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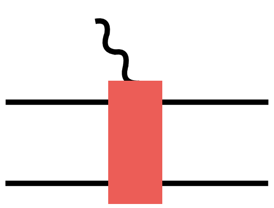

N^2LO
Isoscalar: N^4LO

2N Chiral electromagnetic currents (general N^2LO; isoscalar N^4LO)

Kolling:2009iq
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- N^2LO (**isoscalar N^4LO**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^3LO (isovector) is in progress

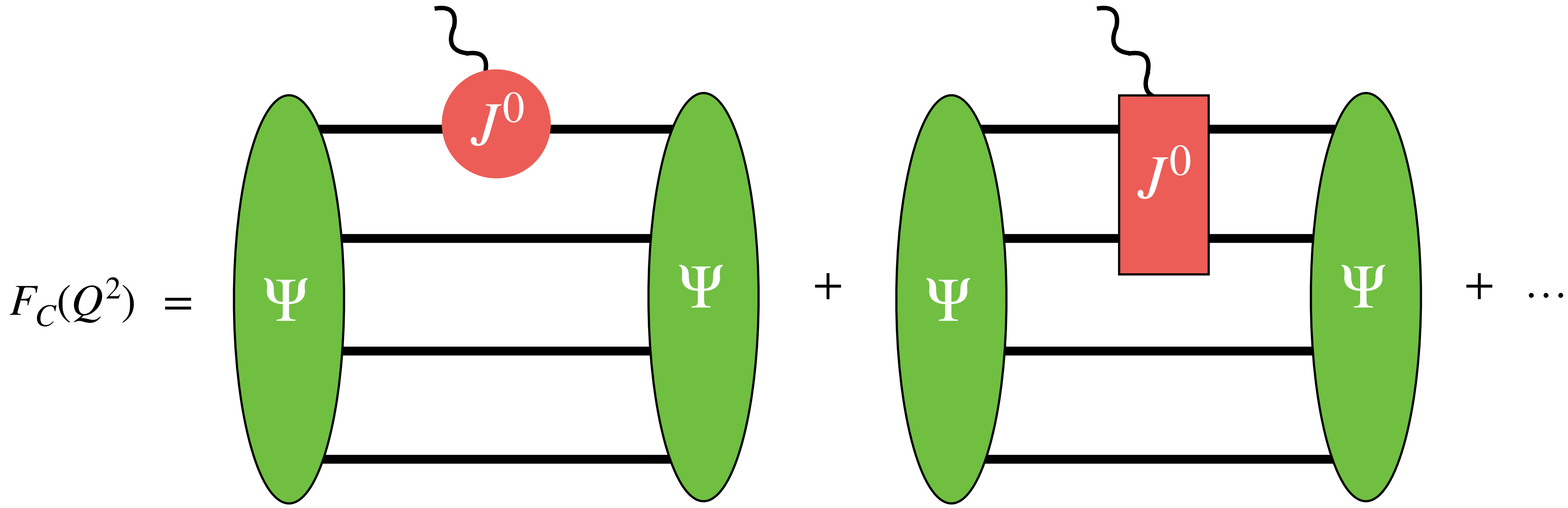
Goals of this study:

- consistent χEFT calculation of isoscalar structure radii of A = 2, 3, 4 nuclei
- aim at N^4LO level of accuracy even in the incomplete calculation
- careful estimation of uncertainties (truncation, statistical, numerical and other)

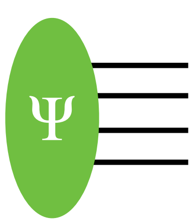
Chiral EFT calculation of the nuclear charge radius

Charge radius r_C is related to the charge form factor $F_C(Q)$

$$r_C^2 = (-6)\frac{\partial}{\partial Q^2} F_C(Q^2)\Big|_{Q=0}$$

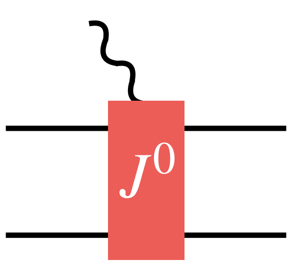


The matrix element is a convolution of nuclear wave function and charge density operator



Nuclear wave function based on high-precision chiral EFT interactions

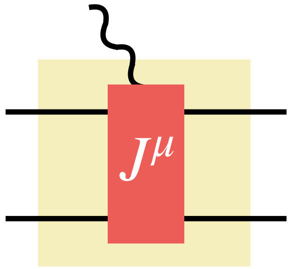
- based on LENPIC 2N (N4LO+) & 3N (N2LO+selected N4LO) interactions
- N4LO 3NFs are used to exactly reproduce physical binding energy of 4He
- Additional relativistic effects: relativistic kinetic energy & boost of 2N interaction

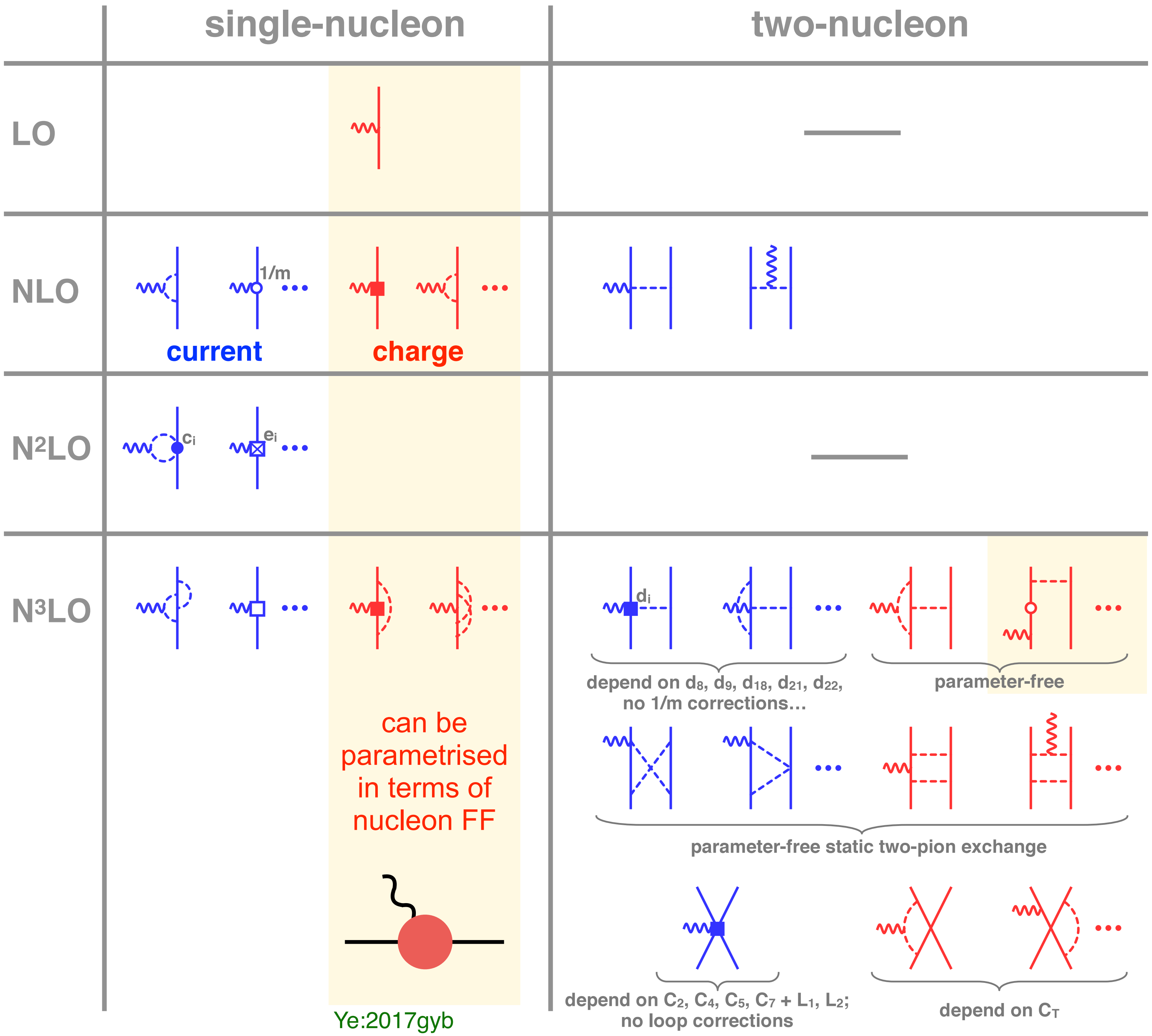


Charge density operator - consistent with chiral nuclear forces

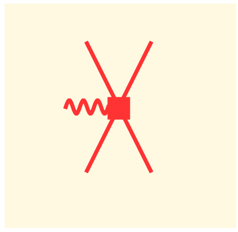
Nuclear electromagnetic currents

Kolling:2009iq, Kolling:2012cs, Krebs:2019aka
Review: H. Krebs, EPJA 56 (2020) 240





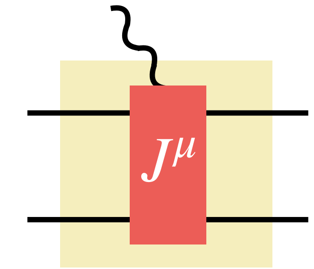
Ye:2017gyb
Belushkin:2006qa
Lin:2021xrc



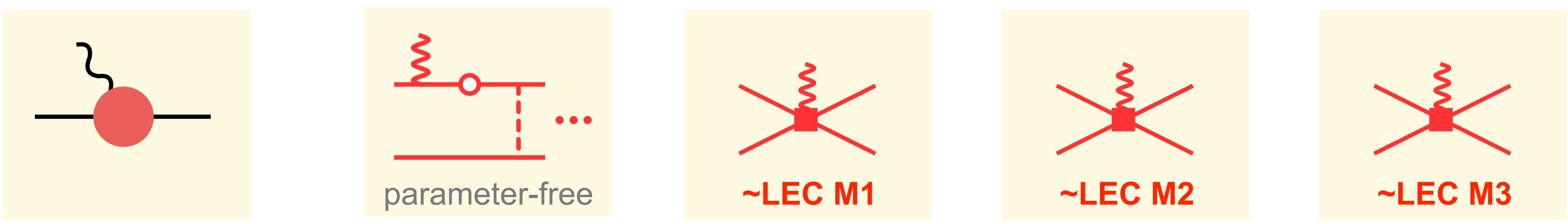
depend on **3 parameters (LECs)**
$^3S_1$-$^3S_1$ - fitted to deuteron FF data
$^3S_1$-$^3D_1$ - fitted to deuteron FF data too
$^1S_0$-$^1S_0$ - fitted to ^{4}He FF data

Chen, Rupak, Savage '99;
Phillips '07
AF et al. '20

Charge density operators



1N charge density | one-pion exchange 2N charge density | contact 2N charge density



parametrised in terms of nucleon FF

Contact density depends on **3** parameters **(LECs): M1, M2, M3**

M1 $^3S_1$-$^3S_1$ - fitted to deuteron FF data

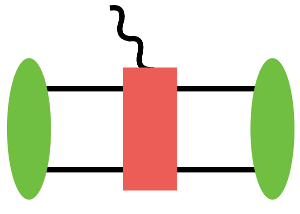
M2 $^3S_1$-$^3D_1$ - fitted to deuteron FF data too

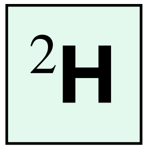
M3 $^1S_0$-$^1S_0$ - fitted to ^{4}He FF data

Three-body charge density does not contribute to isoscalar N4LO

Deuteron structure radius

& extraction of the neutron charge radius

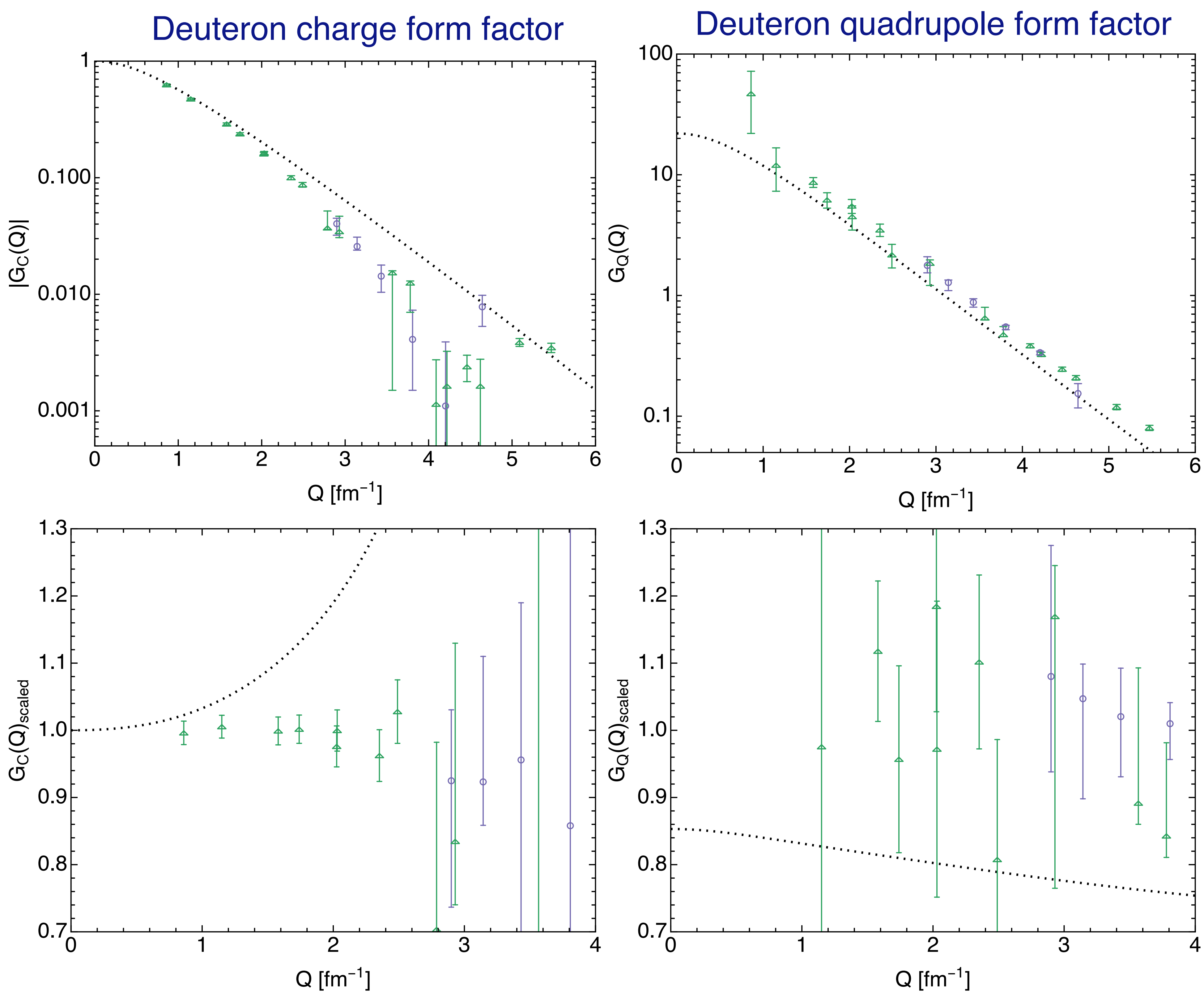




PRL124 082501 (2020)
Phys.Rev.C 103, 024313 (2021)

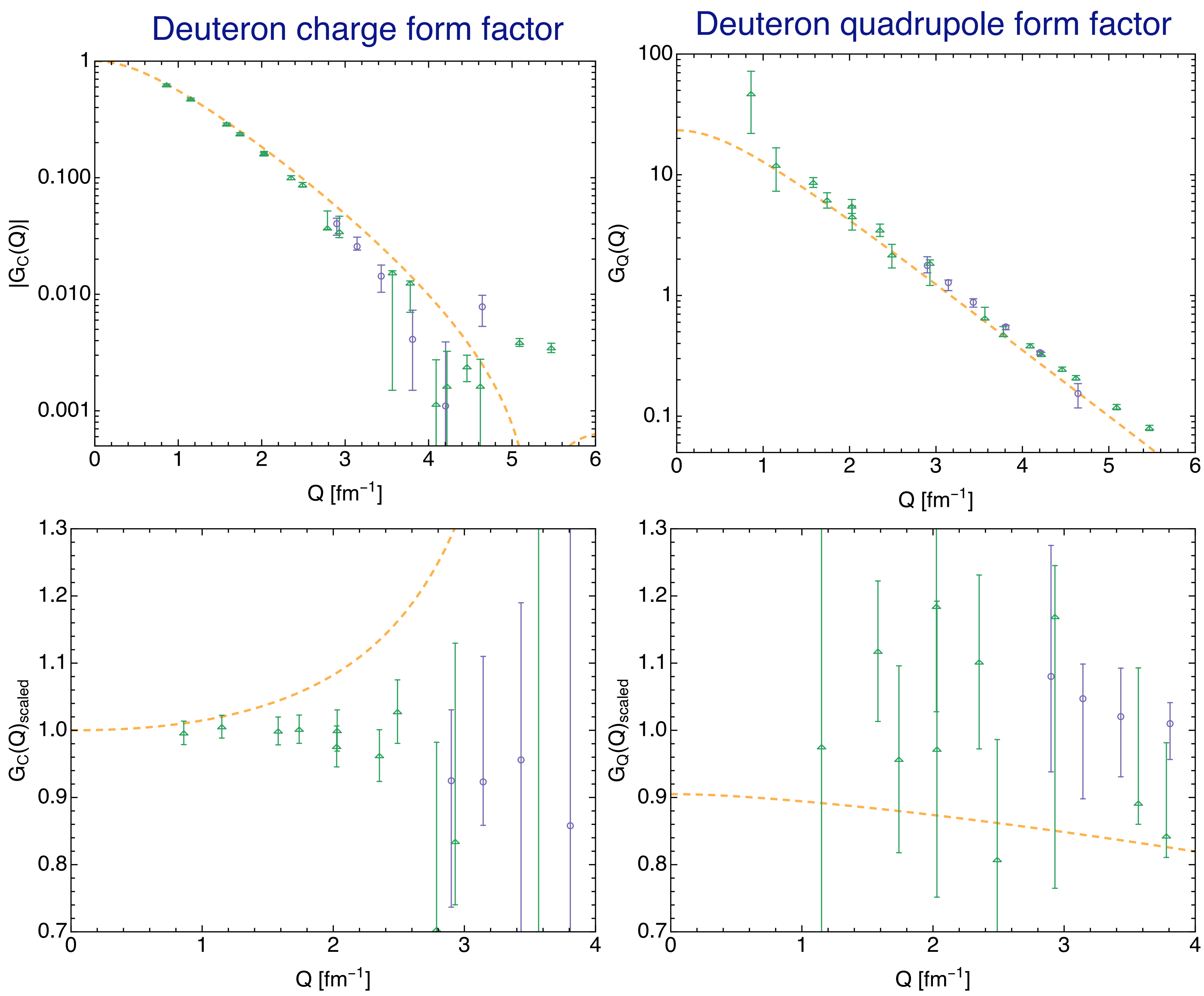
Deuteron charge and quadrupole form factors

LO



Deuteron charge and quadrupole form factors

NLO

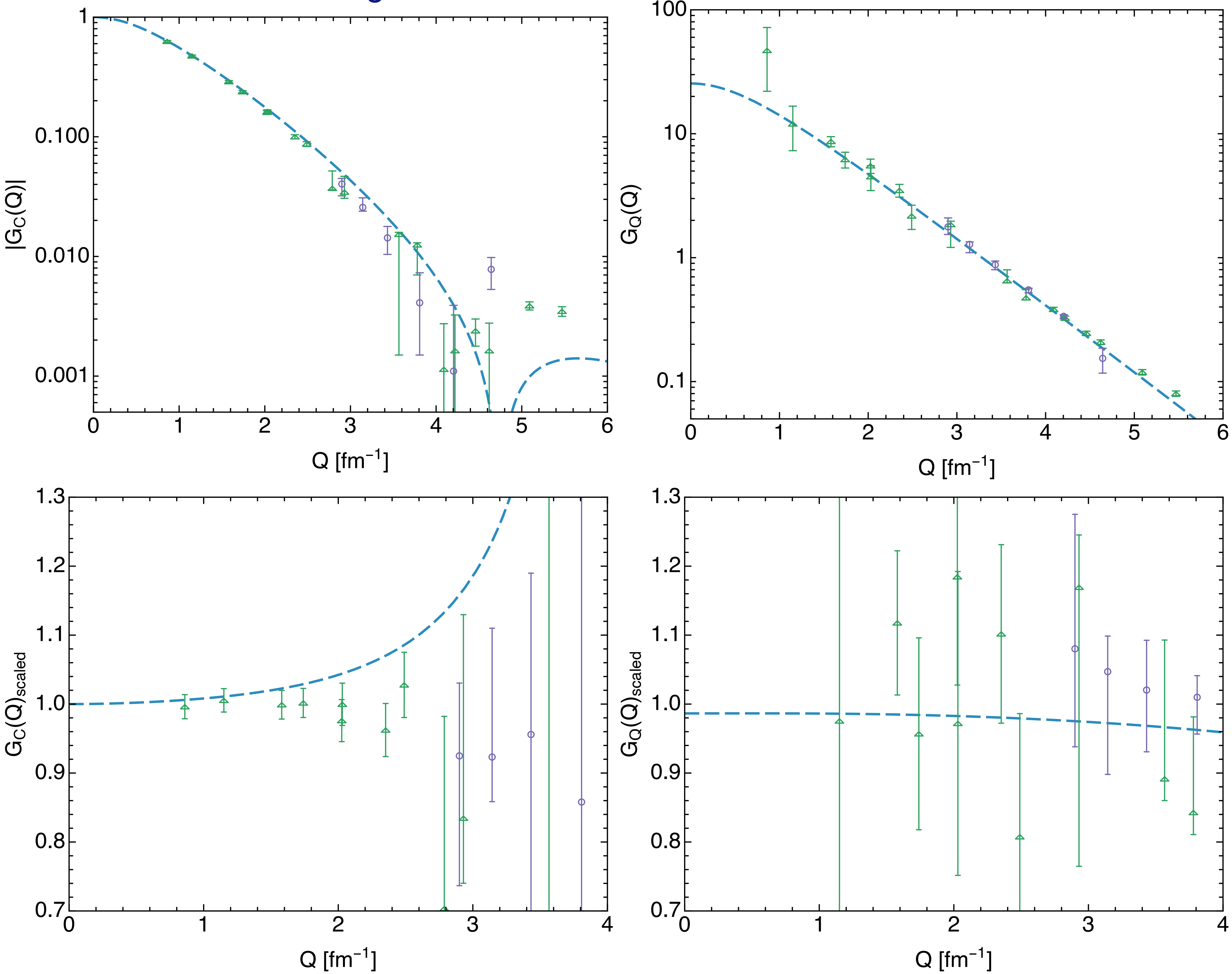


Deuteron charge and quadrupole form factors

N2LO

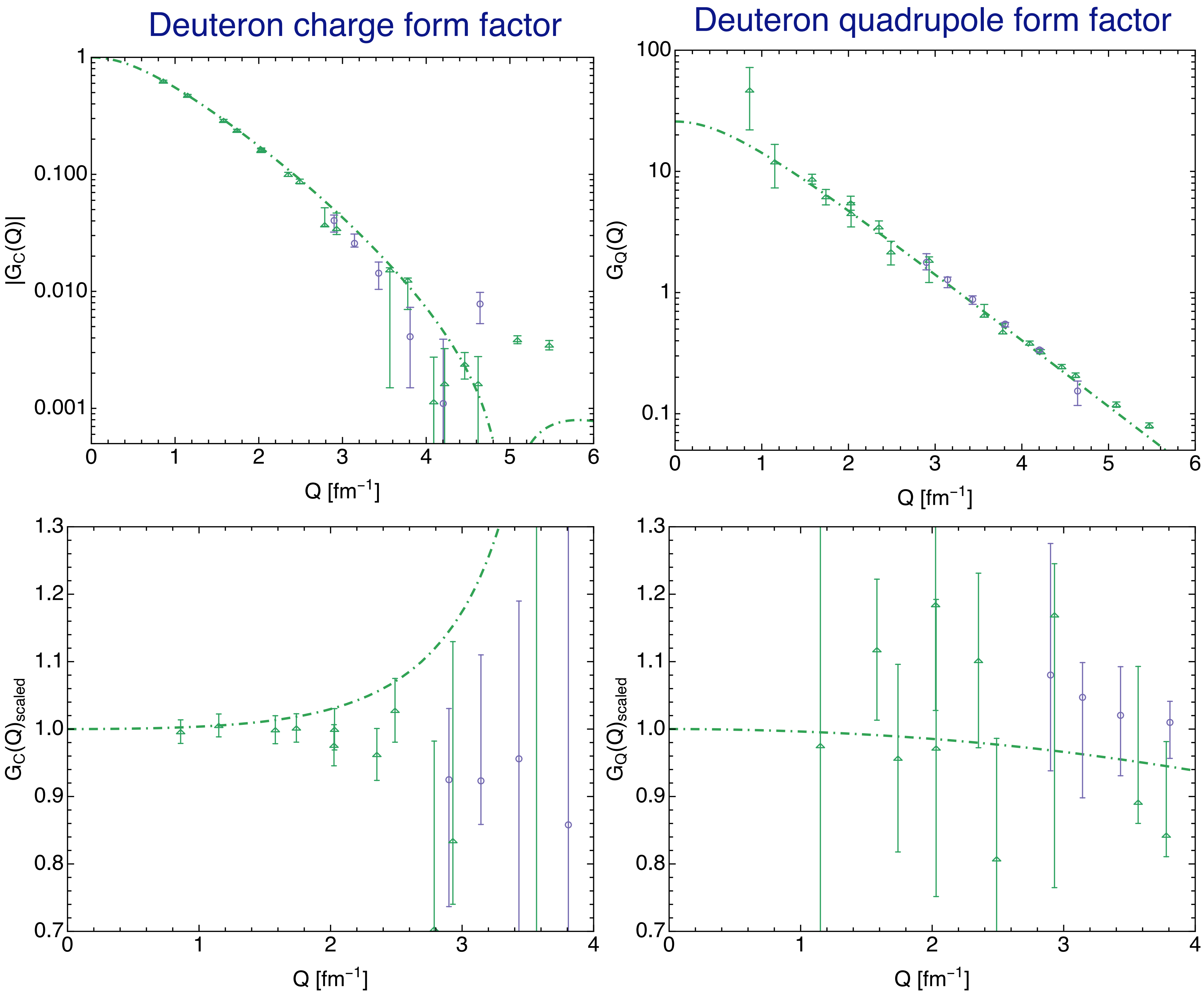
Deuteron charge form factor

Deuteron quadrupole form factor

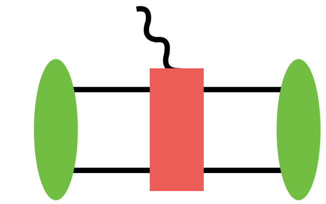


Deuteron charge and quadrupole form factors

N3LO
without
2N charge
density

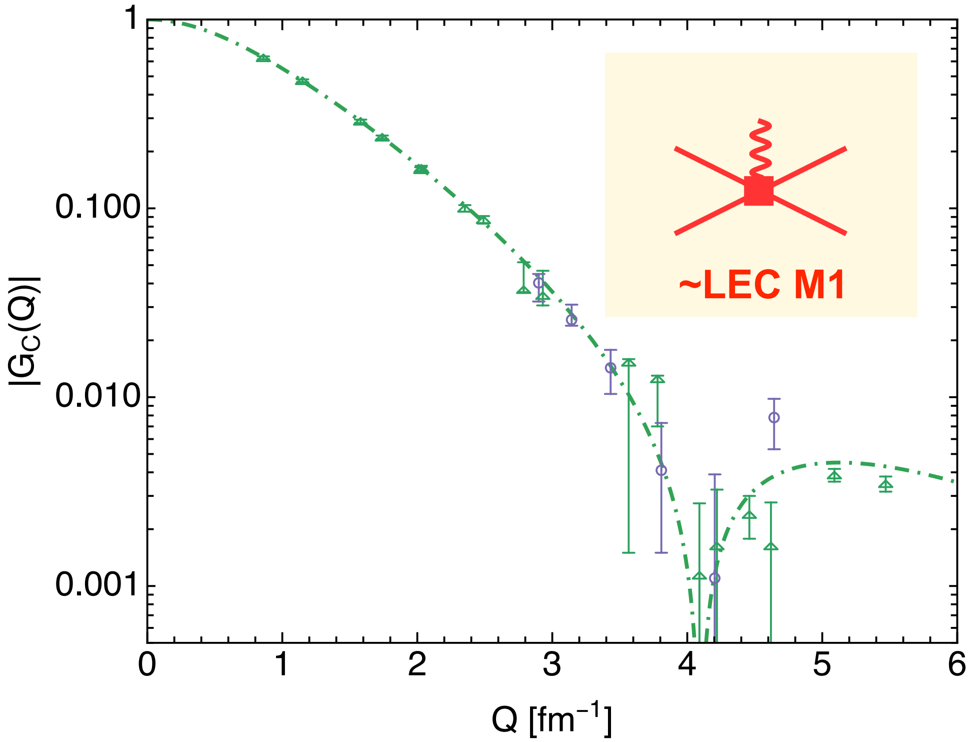


Deuteron charge and quadrupole form factors

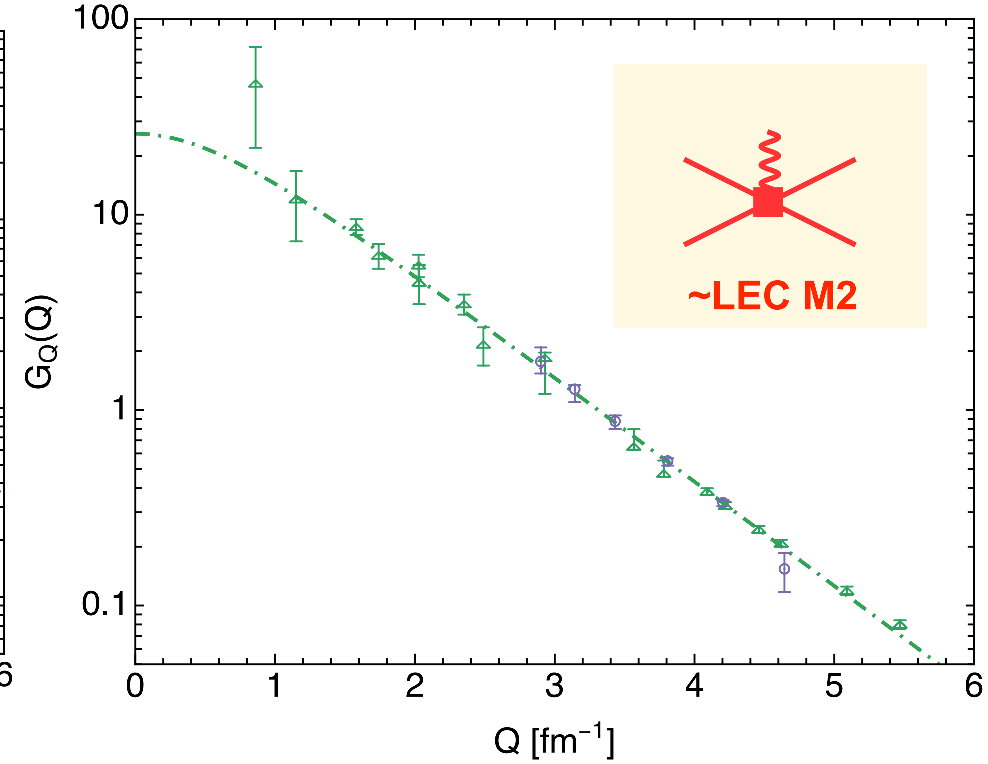


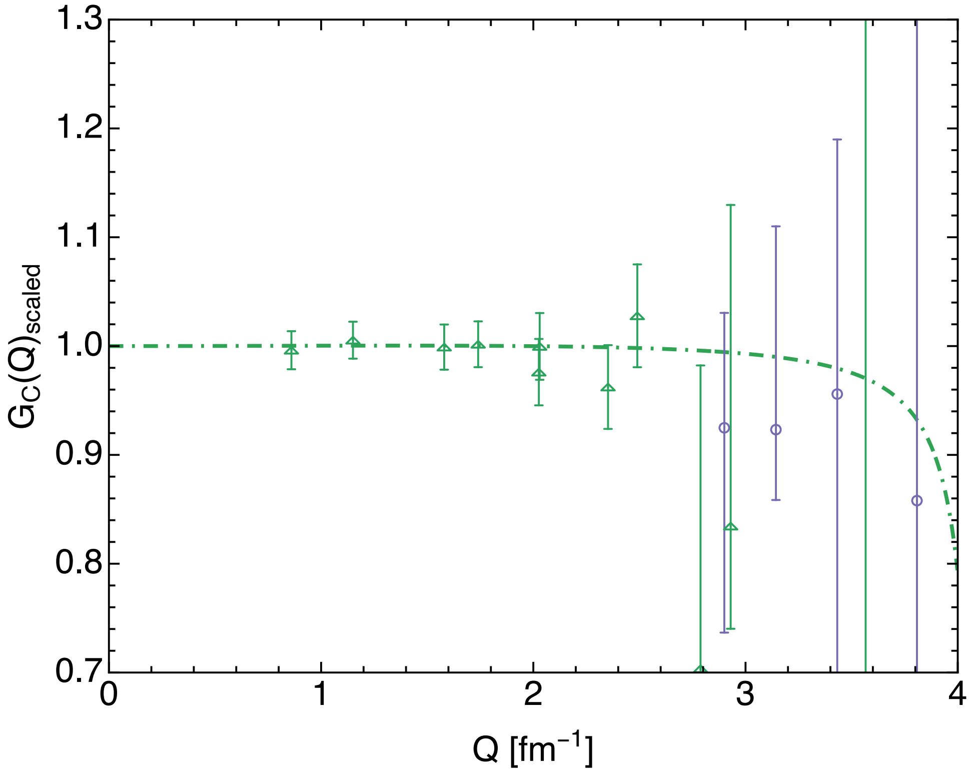
N3LO
with
2N charge
density

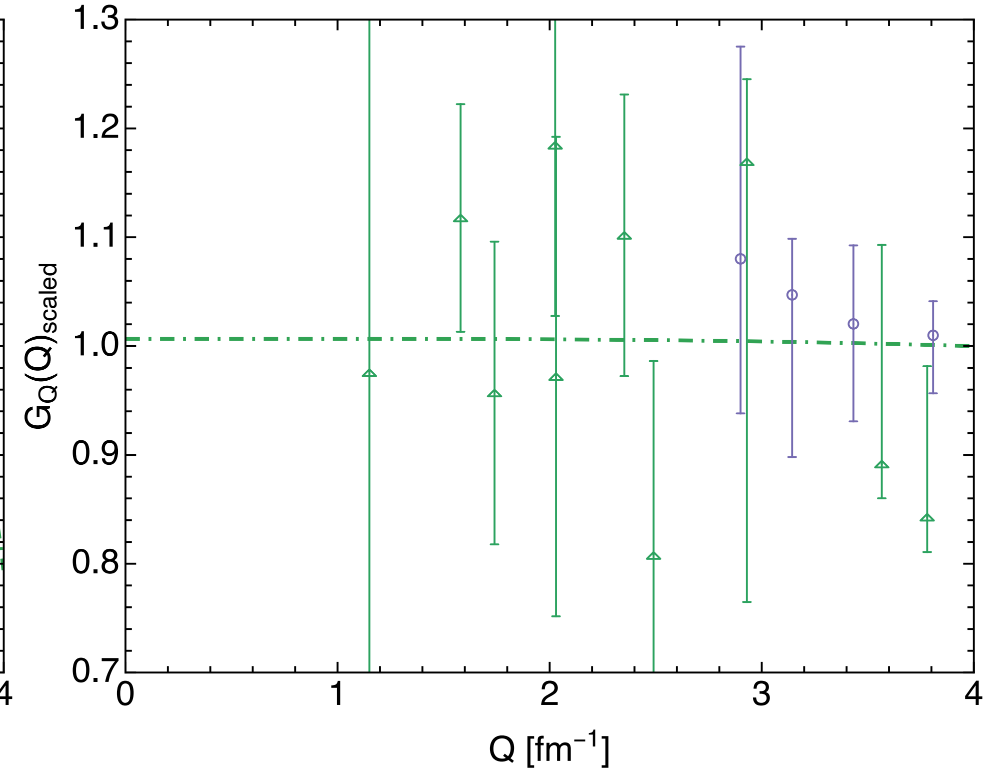
Deuteron charge form factor



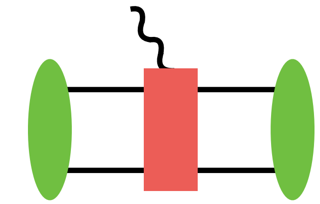
Deuteron quadrupole form factor





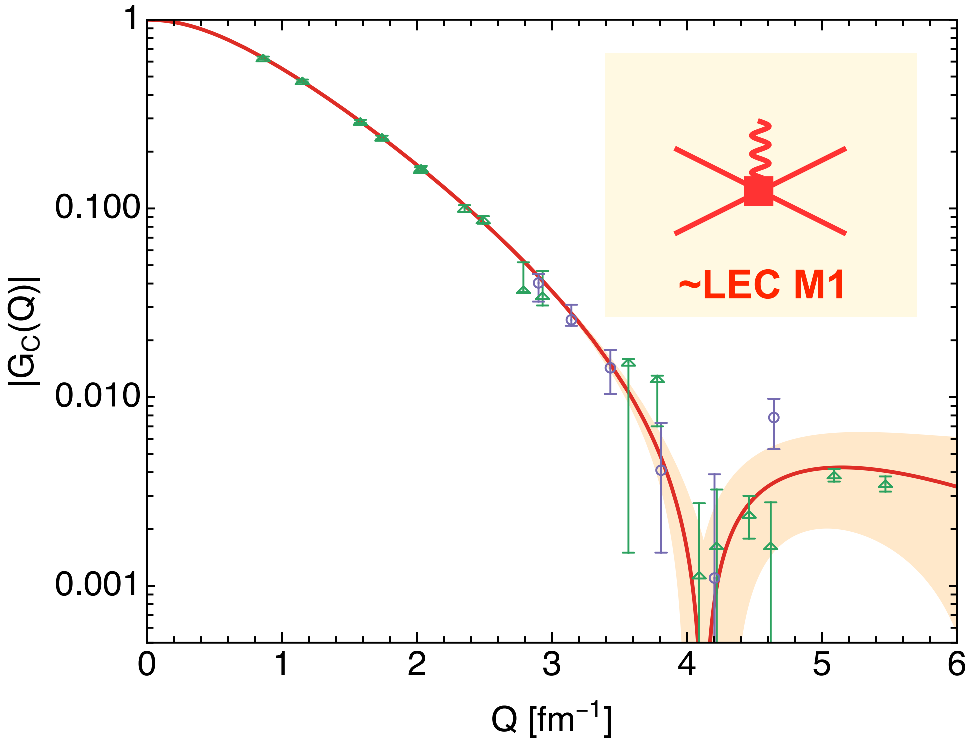


Deuteron charge and quadrupole form factors

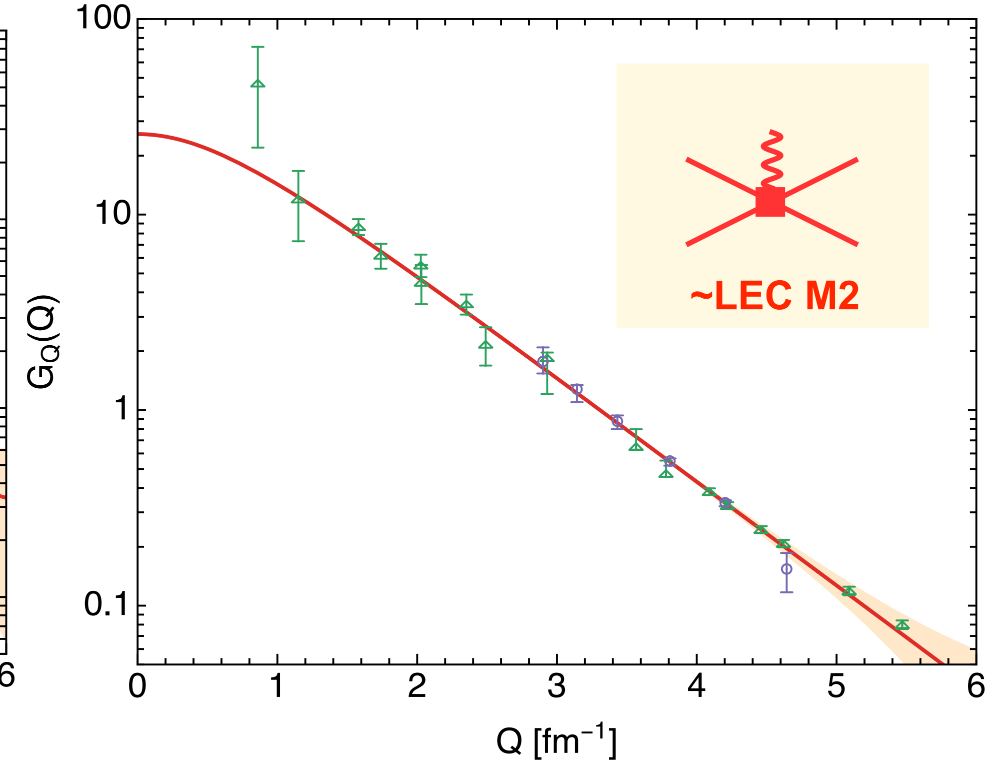


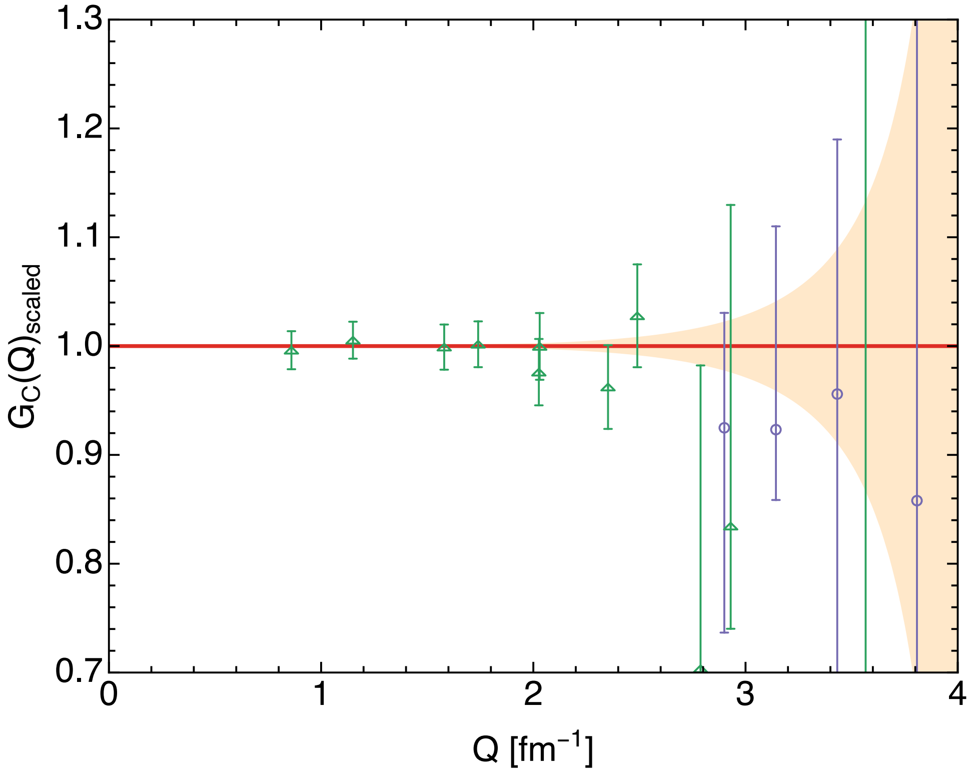
N4LO
with
2N charge
density

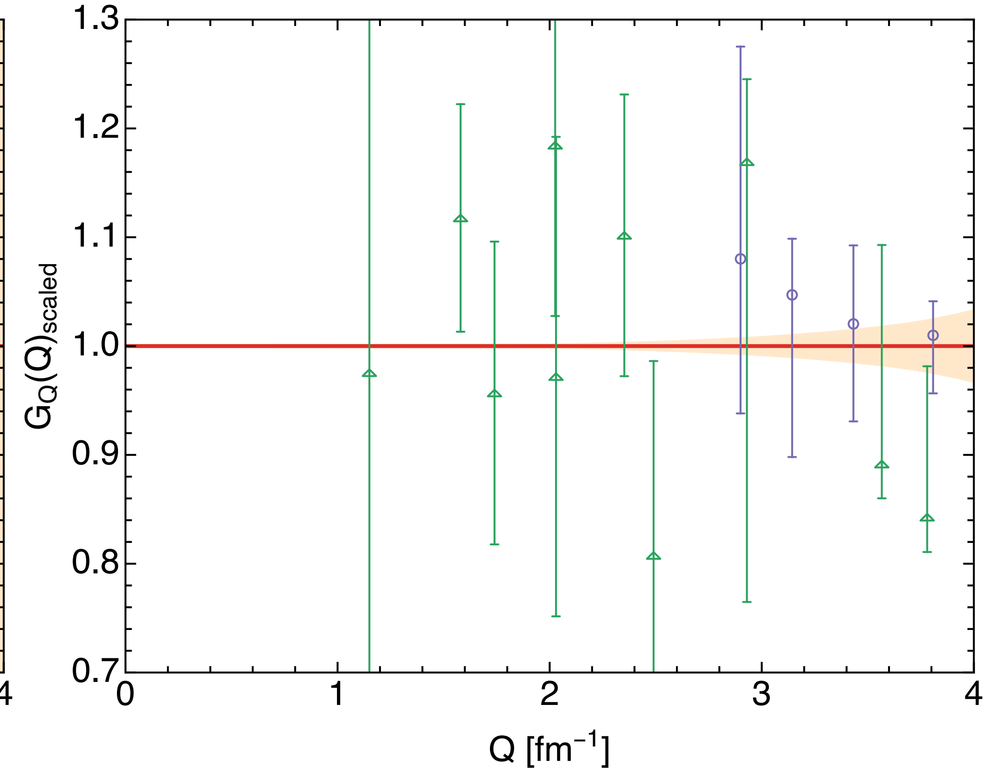
Deuteron charge form factor



Deuteron quadrupole form factor



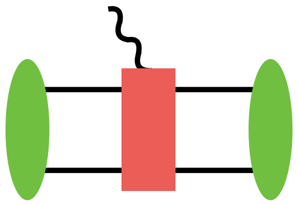


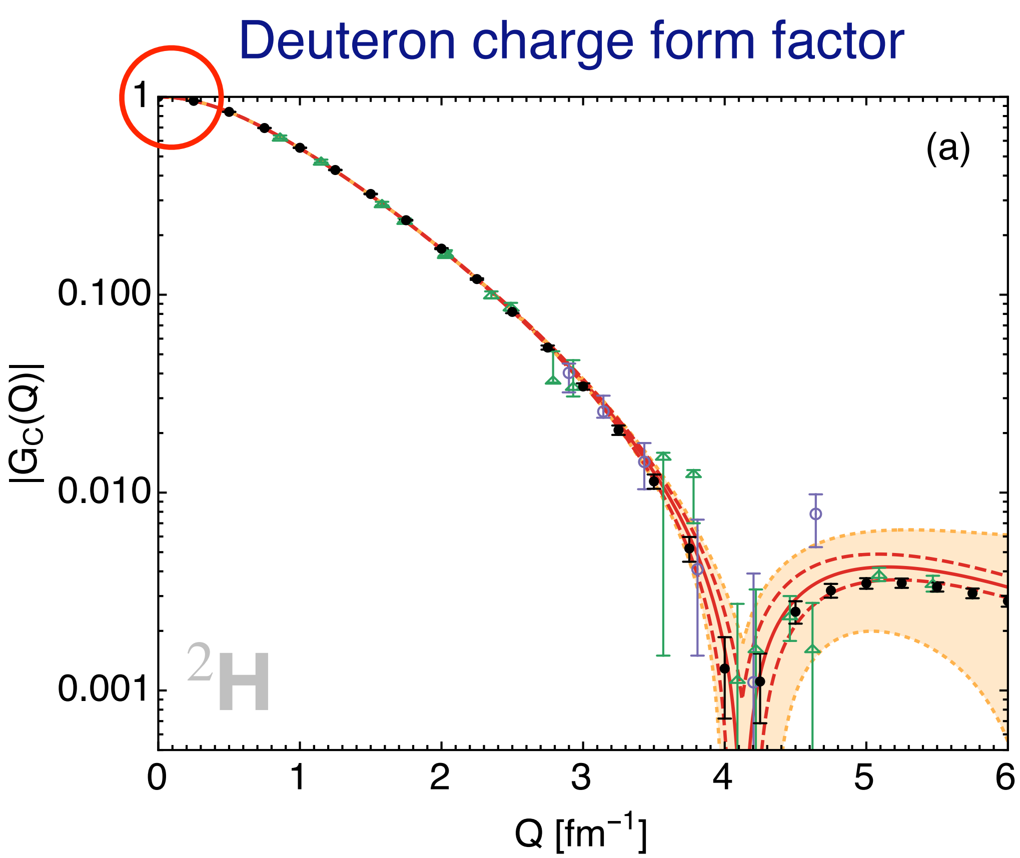


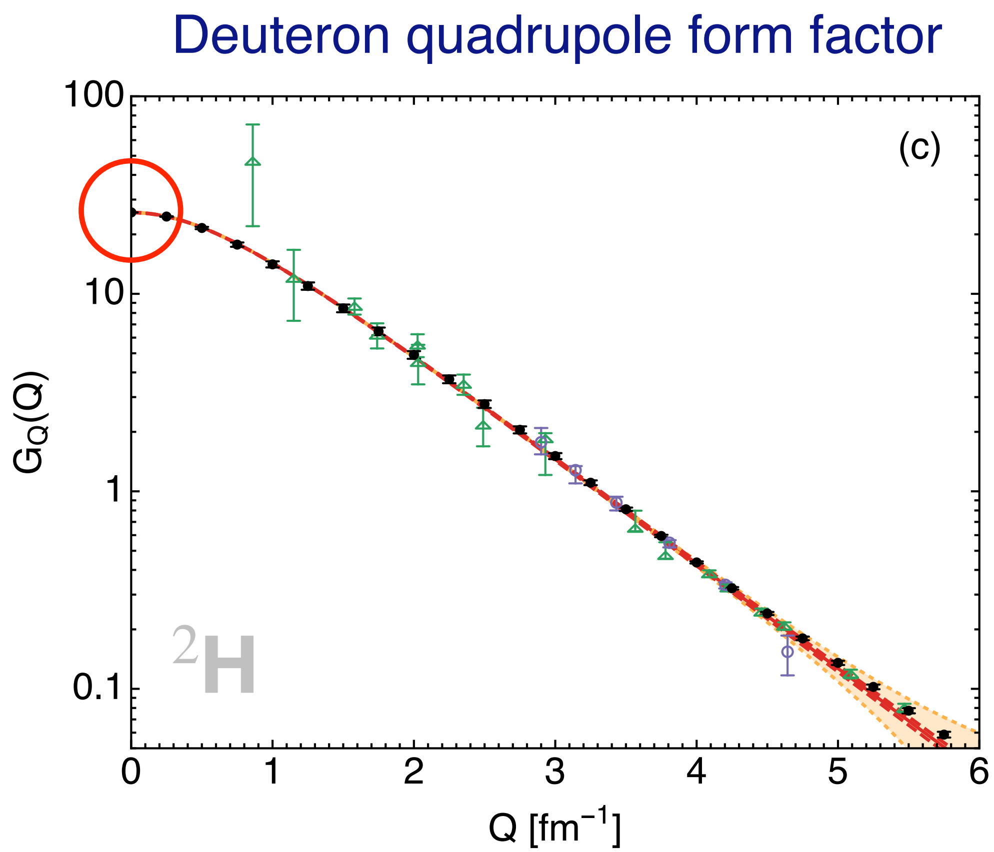
LECs M1 and M2 are fitted to GC and GQ data

Good description of low-Q behaviour and also node position

Deuteron charge and quadrupole form factors







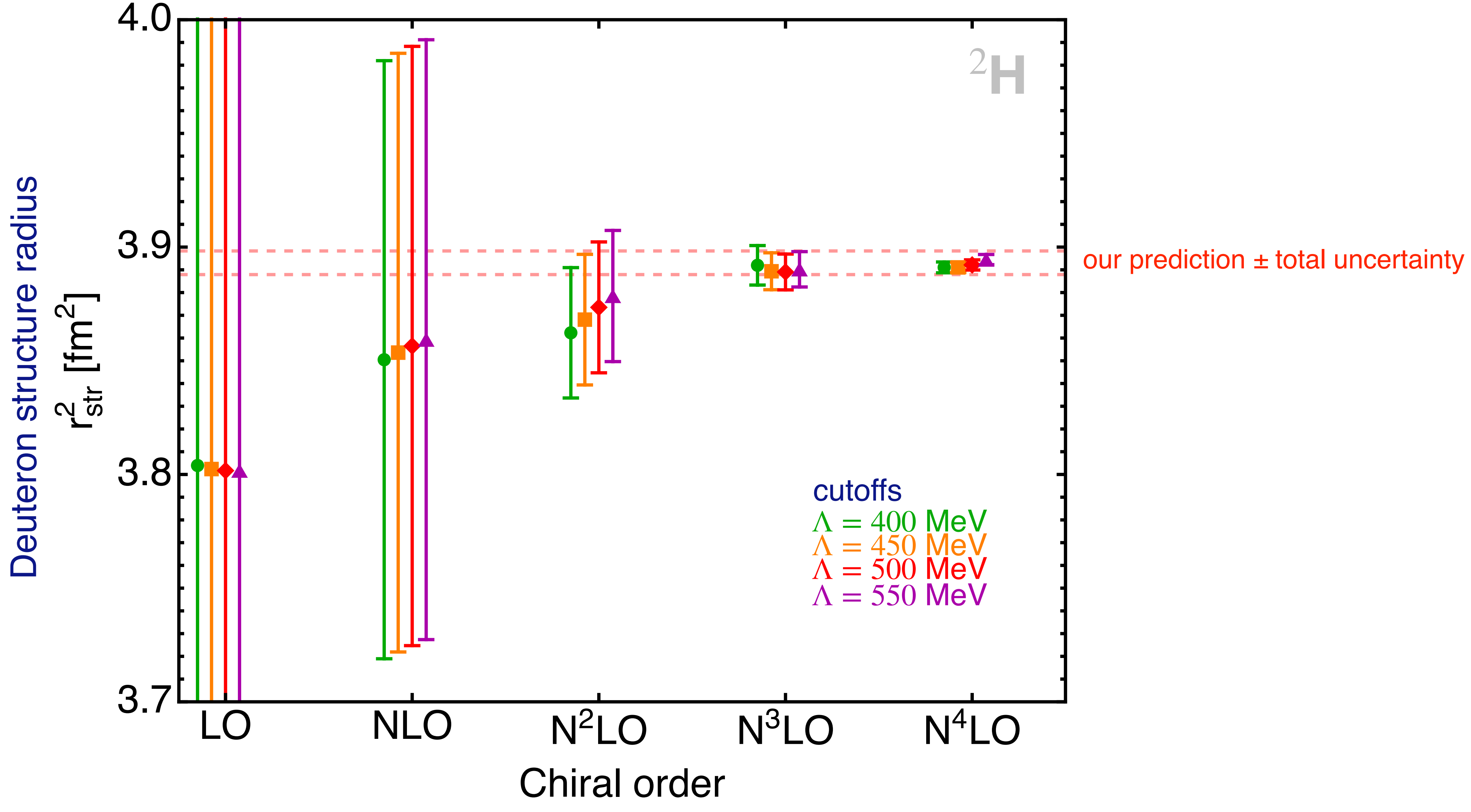
Extraction of deuteron structure radius and quadrupole moment

$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm \qquad Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Truncation uncertainty of ^2H structure radius

Using Bayesian model to estimate truncation uncertainty at each order Epelbaum et al. EPJA 56, 92 (2020)

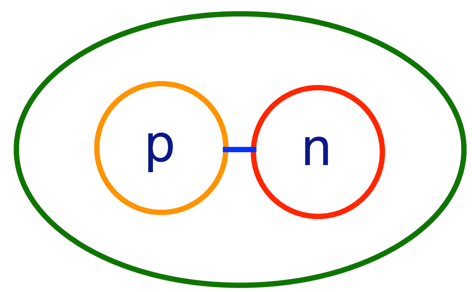


Chiral EFT expansion converges very well

Cutoff dependence is smaller than the truncation uncertainty

Neutron charge radius from high-accuracy χEFT calculation of deuteron structure radius

^{2}H



$$r_d^2 = r_{str}^2(^2\text{H}) + \left(r_p^2 + \frac{3}{4m_p^2}\right) + r_n^2$$

Deuteron-proton radius difference

$$(r_d^2 - r_p^2) = 3.82070(31) fm^2$$

Atomic spectroscopy

Hydrogen-deuterium 1S-2S isotope shift

+ QED corrections

Pachucki et al., PRA 97, 062511 (2018)
Jentschura et al. PRA 83 (2011)

Our accurate **χEFT calculation**

of the **deuteron structure** radius

New method of determination

of the neutron charge radius

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2(^2\text{H})$$

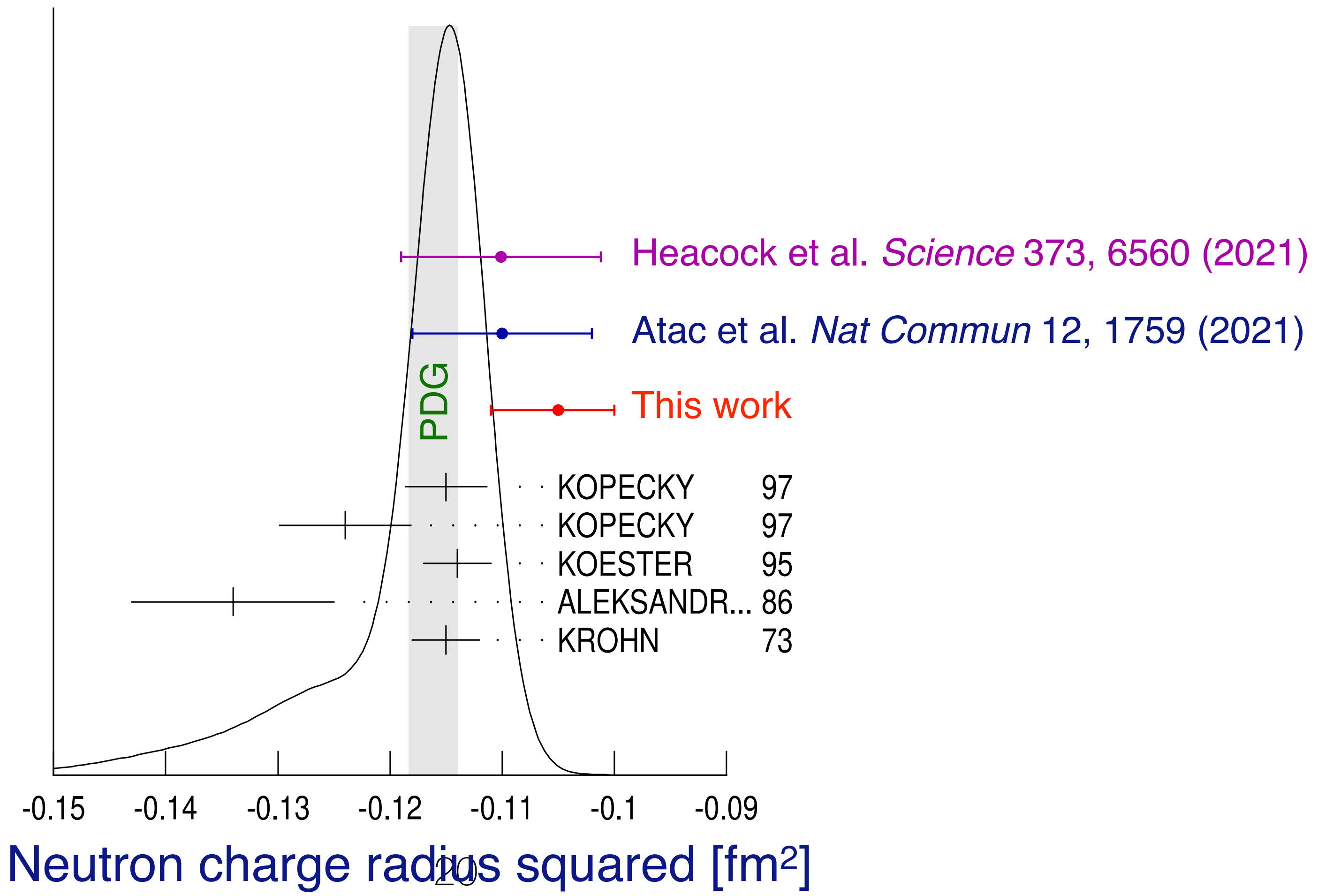
$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

Our extraction of the neutron charge radius

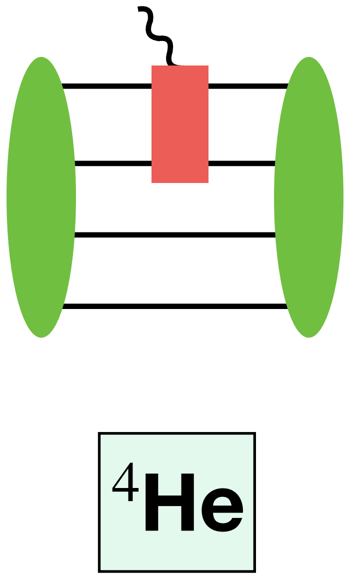
$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

~2σ deviation from the PDG (2022) weighted average $r_n^2 = -0.1155(17) fm^2$



^{4}He charge radius

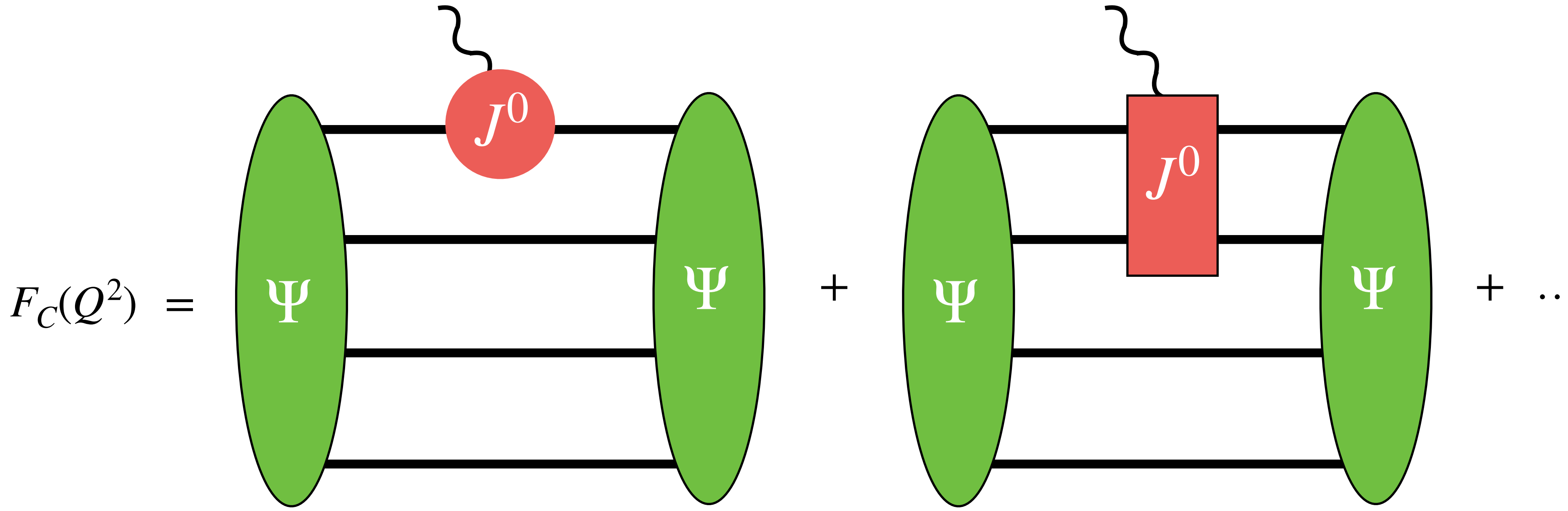
Precision test of the chiral EFT for ^{4}He



Preliminary results

4He FF precision calculation challenge

Conceptually, calculation is simple: convolution of 4He WF with 1N and 2N charge density



In practice, **many complications, when one aims at sub-percent accuracy:**

Incompleteness of 3NFs: complete N2LO; missing N3LO; selected contacts at N4LO

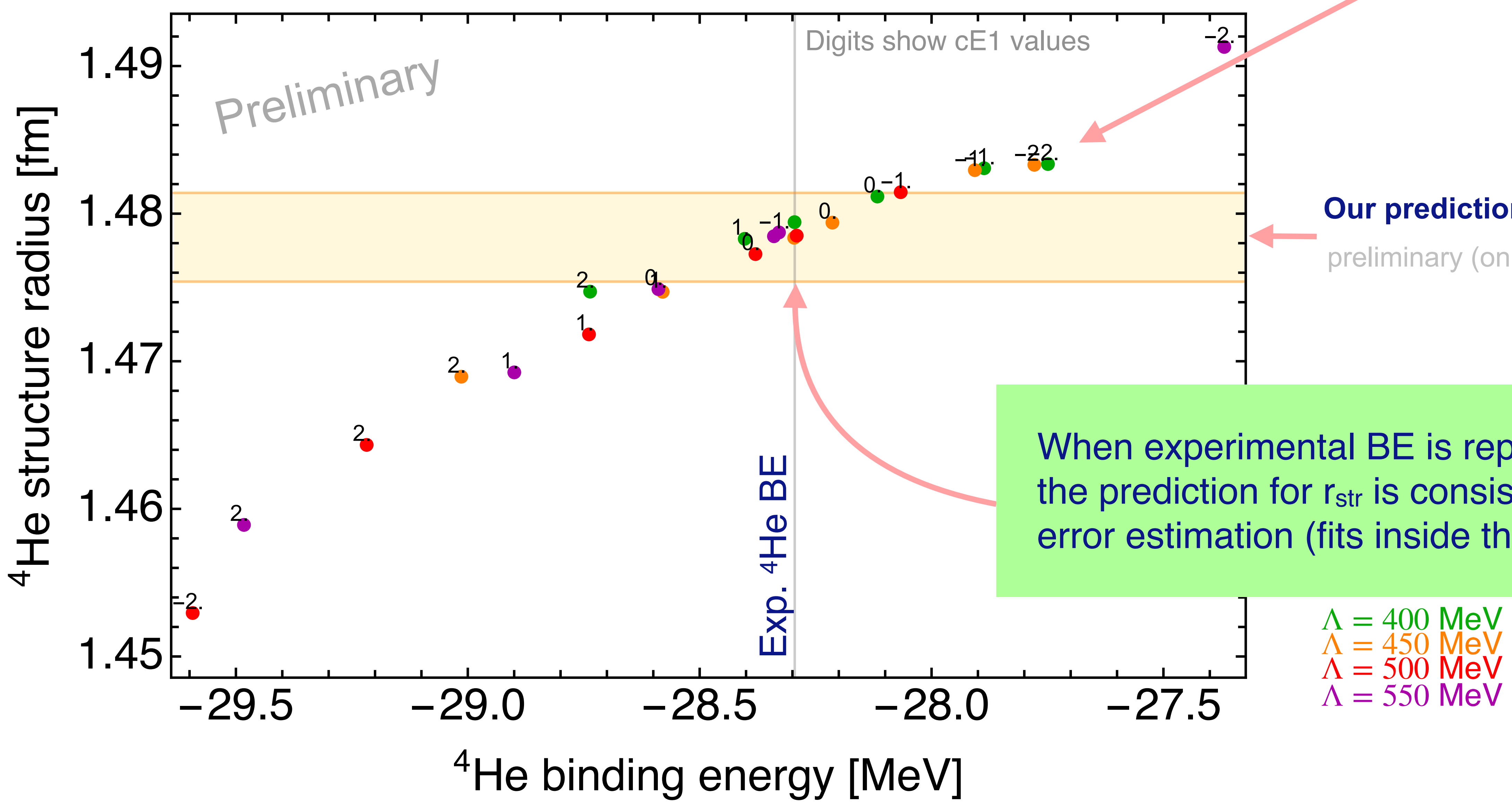
Additional relativistic effects: relativistic kinetic energy in SE, boost of 2N operators

Sensitivity of LEC M3 fit **to nucleon FF parametrisation and 4He FF data**

Correlation between ^{4}He structure radius and binding energy

using variation of **cE1** (N^{4}LO contact 3NF LEC)

^{4}He **binding energy**
and **r_{str}** are **strongly correlated**!



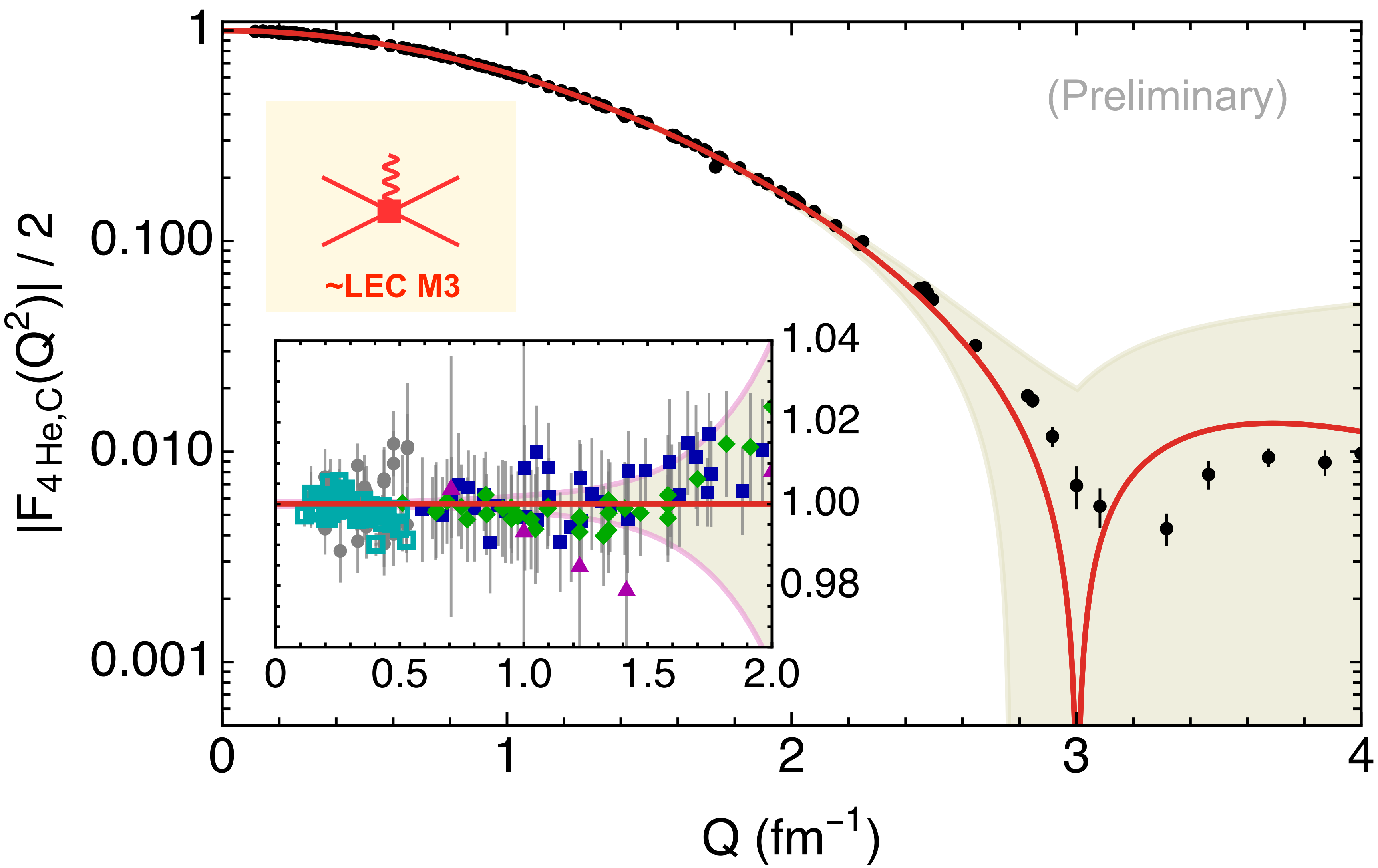
Our prediction for ^{4}He structure radius:
preliminary (only truncation error is shown)

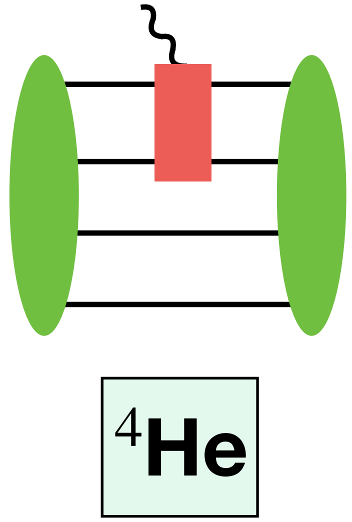
When experimental BE is reproduced the prediction for r_{str} is consistent with our error estimation (fits inside the orange band)

Many effects (higher-order 3NFs, some relativistic corrections) change BE and radius simultaneously.
Once BE is fitted back to physical value the residual effect is usually ver small.

4He charge form factor

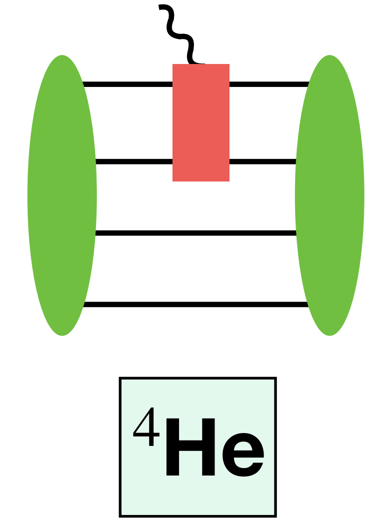
LEC M3 is fitted to the shape of 4He charge form factor





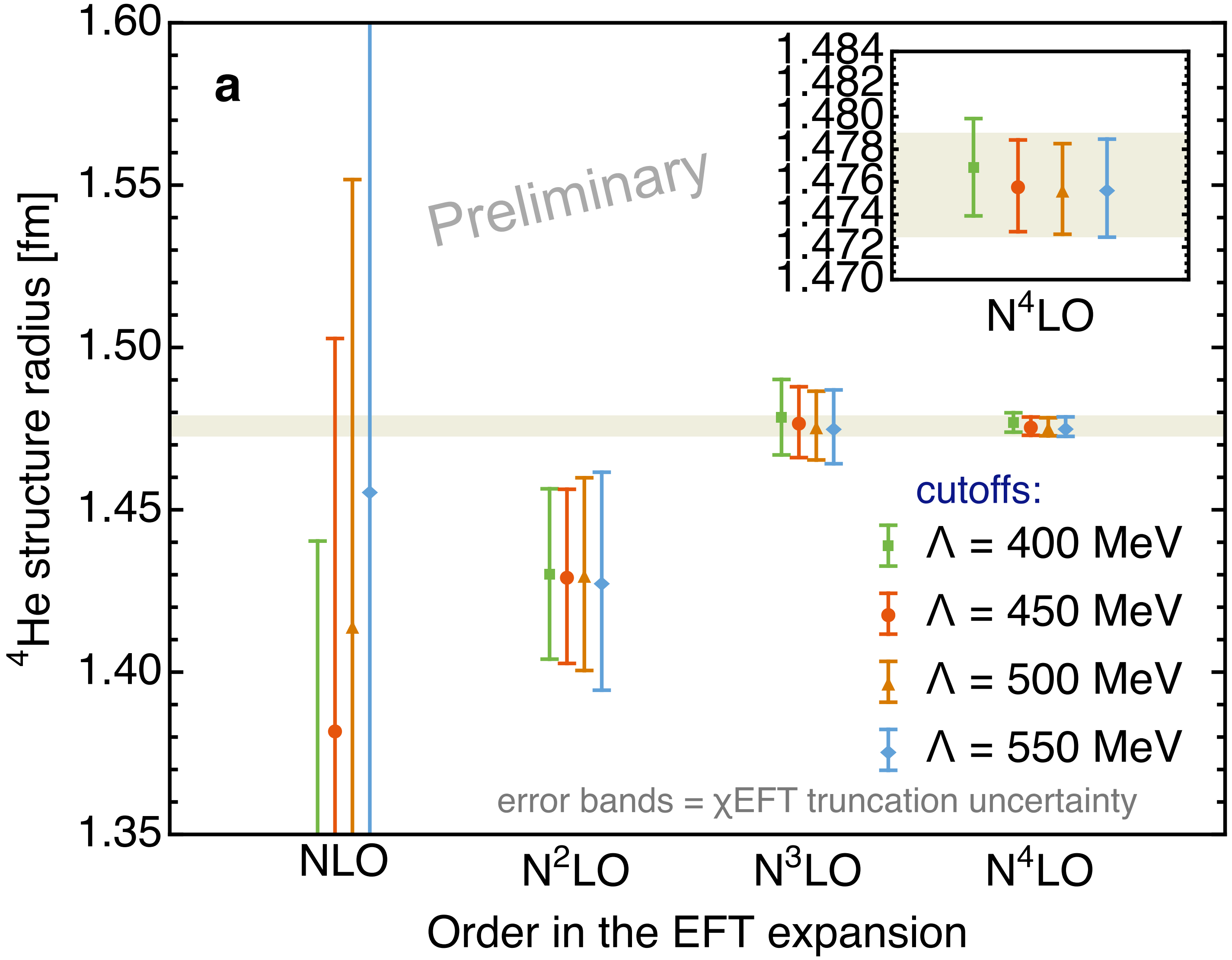
Prediction of 4He structure radius

Our preliminary prediction for 4He structure radius:



$$r_{str}(^4\mathbf{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}}\ fm \quad \text{(Preliminary)}$$

Estimation of truncation error:



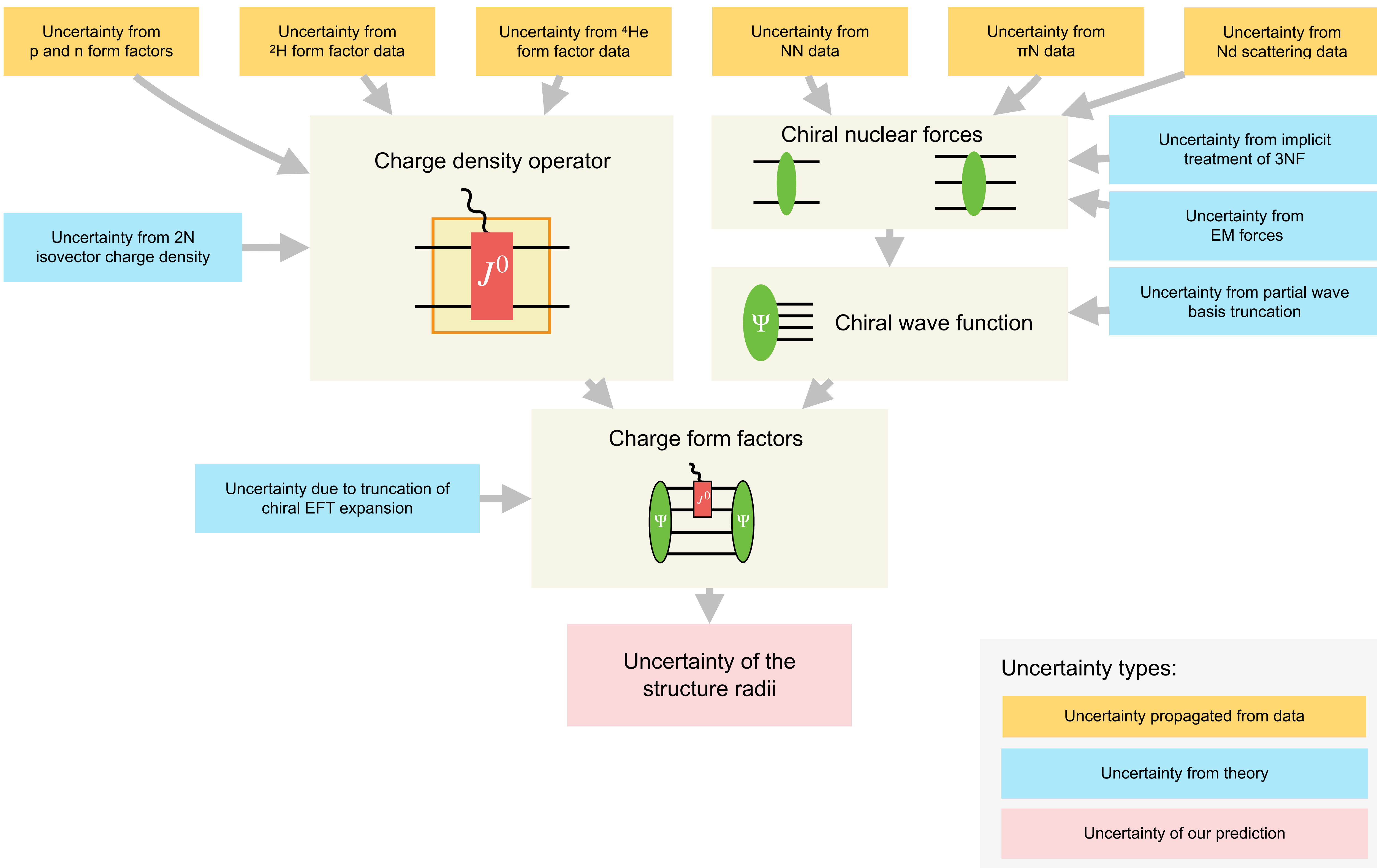
band = our prediction ± total uncertainty

Cutoff dependence is smaller than the truncation uncertainty

Chiral EFT expansion converges well

Extensive uncertainty analysis

Propagation of uncertainties from data and theory



Prediction for ^{4}He charge radius

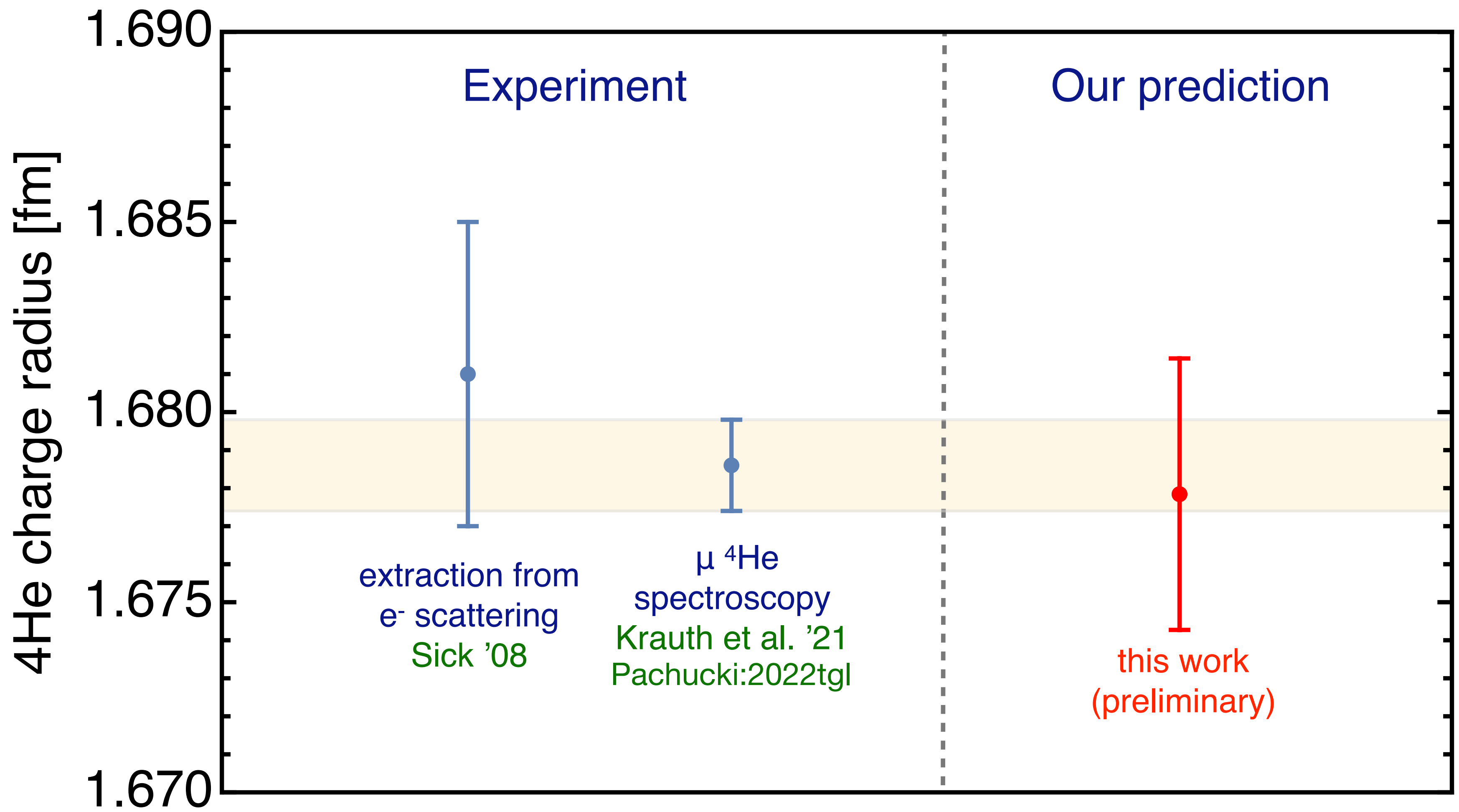
$r_{str}(^4\mathbf{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}}\ fm$ (Preliminary)

Our prediction for ^{4}He **charge** radius

$$r_C(^4\mathbf{He}) = (1.6775 \pm 0.0035)\, fm$$

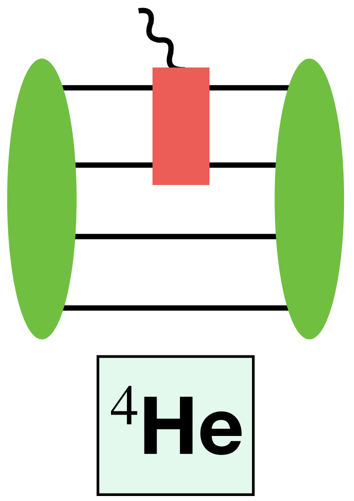
preliminary, using CODATA 2018 r_p and own determination of r_n

$$r_C(^4\mathbf{He}) = r_{str}^2(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$



Our prediction for ^{4}He charge radius is fully consistent with the muonic-atom spectroscopy

Isoscalar nucleon charge radius from data on ⁴He



Our prediction for ⁴He **structure** radius

Experimental ⁴He charge radius
Krauth et al., Nature 589 (2021) 7843, 527-531
+ theory update from Pachucki et al. (2022)

$$r_C(^4\mathbf{He}) = r^2_{str}(^4\mathbf{He}) + \left(r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Determination

of the isoscalar nucleon charge radius

$$(r_n^2 + r_p^2) = (0.607 \pm 0.010)\, fm$$

preliminary

Proton charge radius from isoscalar nucleon radius

Our determination of the

isoscalar nucleon charge radius from ^{4}He

$(r_n^2 + r_p^2) = (0.607 \pm 0.010)\, fm$ preliminary

Our determination of the

neutron charge radius from ^{2}H

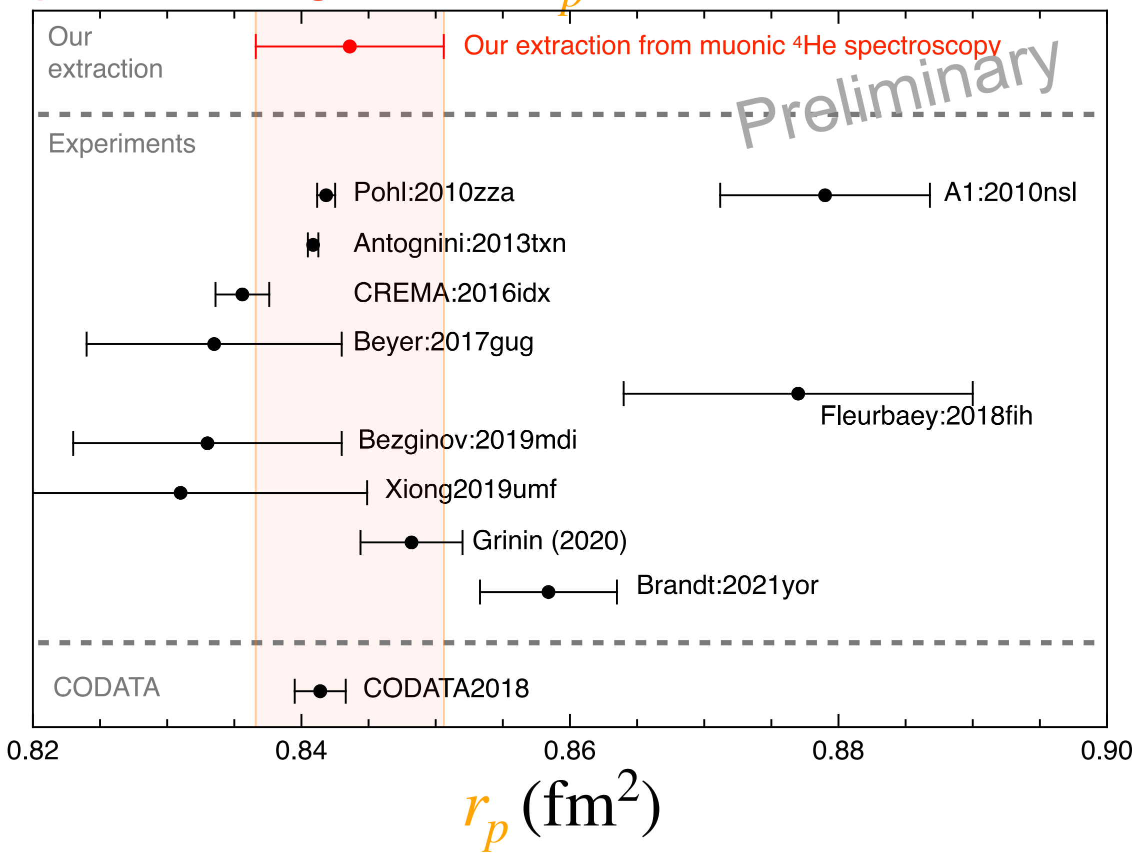
$r_n^2 = -0.105^{+0.005}_{-0.006}\, fm^2$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

New determination of the proton charge radius: $r_p = (0.844 \pm 0.007)\, fm$

preliminary

proton charge radius r_p



Our extraction supports the „small“ proton radius

Summary

General results:

Precise and accurate calculation of A = 2, 3, 4 isoscalar charge radii + extensive uncertainty analysis

Extraction of neutron and proton charge radii from few-nucleon system data.

Derivation of consistent 2N isoscalar N3LO charge density operators and determination of LECs.

Main updates:

2N charge density are important for A=2,3,4 radii, but **seems to be less important for larger systems**

Relativistic kinetic energy and boost of 2N interaction can significantly affect A=3,4 BE.

However, once the physical binding energy of 4N or 3N system is reproduced the **„residual“ effect of all these effects on charge radii is small** (smaller than our N4LO truncation uncertainty).

Predictions for 3He charge FF and charge radius (computed without N3LO isovector charge density) are very close to experimental data.

Outlook

- **Consistent** inclusion of **isovector 2N currents at N^{3}LO** and **N^{4}LO**

- **Consistent** inclusion of **N^{3}LO** and **N^{4}LO three-nucleon forces**

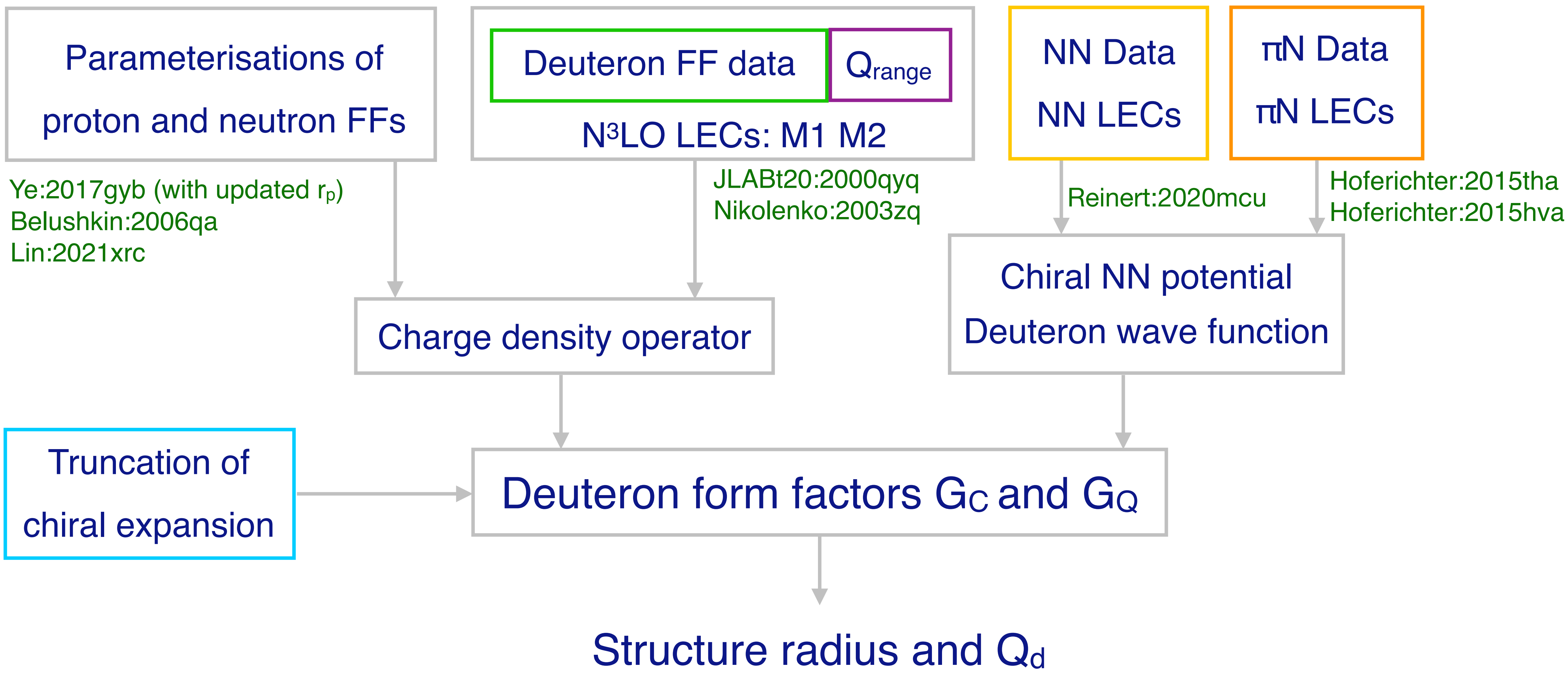
- Analysis of **magnetic form factors** (PhD thesis of D. Möller)

- Application to processes with two photons (**polarizabilities**, ...)

Spares

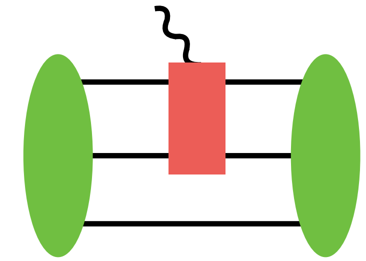
Uncertainty analysis of deuteron structure radius

We propagate uncertainties from multiple sources



	Central	Truncation	$\rho^{\text{reg}}_{\text{Cont}}$	πN LECs RSA	2N LECs and f_i^2	Q range	Total
r^2_{str} (fm^2)	3.8925	±0.0030	±0.0024	±0.0003	±0.0025	$^{+0.0035}_{-0.0005}$	$^{+0.0058}_{-0.0046}$
Q_d (fm^2)	0.2854	±0.0005	±0.0007	±0.0003	±0.0016	$^{+0.0035}_{-0.0005}$	$^{+0.0038}_{-0.0017}$

Prediction for isoscalar 3N charge radius



With all LECs being fixed, we can predict the isoscalar 3N charge radius: $r_C^{isoscalar3N} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{isoscalar3N} = (1.9061 \pm 0.0026)\, fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Our result is 10x more precise than current experimental data:

the ^{3}H charge radius from e^- scattering experiments:	$r_C^{3H} = (1.7550 \pm 0.0860)\, fm$ (5 %)	Amroun et al. '94 (world average)
the ^{3}He charge radius from muonic ^{3}He:	$r_C^{3He} = (1.9701 \pm 0.0009)\, fm$ (0.05 %)	CREMA 2023 arXiv:2305.11679

Exp. **3N isoscalar charge radius**: (using muonic ^{3}He and old ^{3}H) $r_{C,exp.}^{isoscalar3N} = (1.9010 \pm 0.0260)\, fm$ (1.4 %)

T-REX experiment in Mainz [Pohl et al.] aims at measuring r_C^{3H} within ±0.0002 fm (400x more precise)

The isoscalar 3N radius will be then known within ±0.0009 fm

⇒ **precision tests of nuclear chiral EFT!**

Structure radius

Nuclear **charge radius** can be decomposed into structure, proton and neutron radii

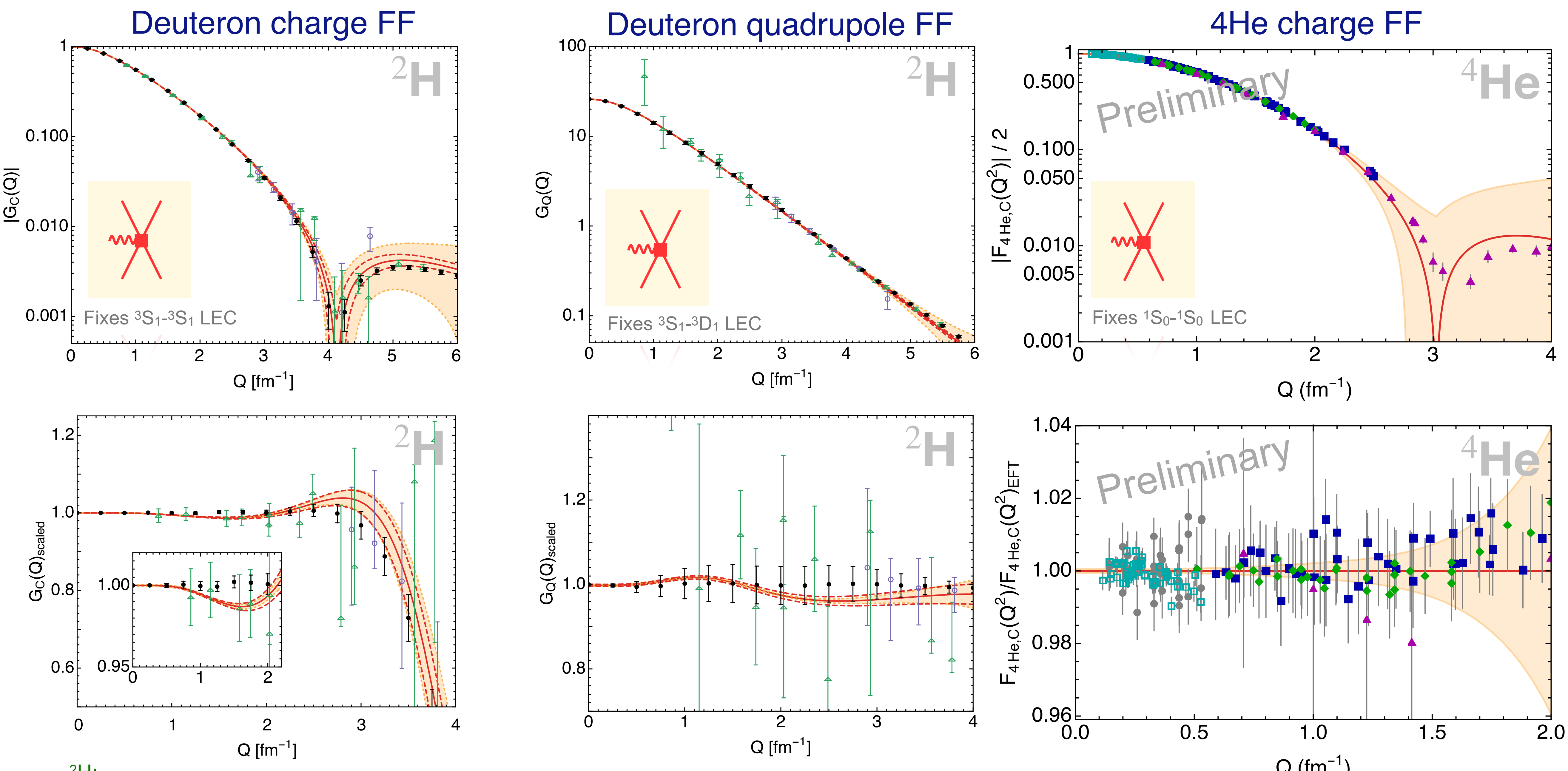
$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2} \right) + \frac{N}{Z} r_n^2$$

Nuclear structure radius

- depends on distribution of matter (proton and neutrons) inside the nuclei
- depends on many-body electromagnetic currents (meson-exchange currents)
- **can be accurately calculated using chiral nuclear forces and EM currents**

Structure radius = charge radius if protons and neutrons have point-like charge distributions

Low-energy constants from a fit to charge and quadrupole form factors



^{2}H:
AF, Möller, Baru, Epelbaum, Krebs, Reinert,
PRL 124 (2020) 082501; PRC 103 (2021) 024313

^{4}He: Preliminary

best fit + N^{4}LO truncation uncertainty

3 parameters (LECs) in 2N charge density J^0 are fixed from the form factor data of deuteron and ^{4}He

Neutron charge radius in PDG 2022

R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022) and 2023 update

n MEAN-SQUARE CHARGE RADIUS

VALUE (fm^2)			*DOCUMENT ID*		*COMMENT*
−0.1155±0.0017 OUR AVERAGE					
−0.115	±0.002	±0.003	KOPECKY	97	*n e* scattering (Pb)
−0.124	±0.003	±0.005	KOPECKY	97	*n e* scattering (Bi)
−0.114	±0.003		KOESTER	95	*n e* scattering (Pb, Bi)
−0.115	±0.003		[1] KROHN	73	*n e* scattering (Ne, Ar, Kr, Xe)
• • • We do not use the following data for averages, fits, limits, etc. • • •					
−0.1101	±0.0089		[2] HEACOCK	21	*n* interferometry
−0.106	$^{+0.007}_{-0.005}$		[3] FILIN	20	chiral EFT analysis
−0.117	$^{+0.007}_{-0.011}$		BELUSHKIN	07	Dispersion analysis
−0.113	±0.003	±0.004	KOPECKY	95	*n e* scattering (Pb)
−0.134	±0.009		ALEKSANDR...	86	*n e* scattering (Bi)
−0.114	±0.003		KOESTER	86	*n e* scattering (Pb, Bi)
−0.118	±0.002		KOESTER	76	*n e* scattering (Pb)
−0.120	±0.002		KOESTER	76	*n e* scattering (Bi)
−0.116	±0.003		KROHN	66	*n e* scattering (Ne, Ar, Kr, Xe)

[1] KROHN 73 measured -0.112 ± 0.003 fm^2. This value is as corrected by KOESTER 76.

[2] HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

[3] FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.