Scattering processes on the lattice

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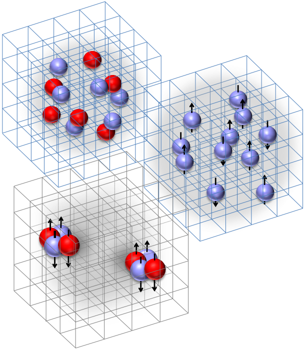
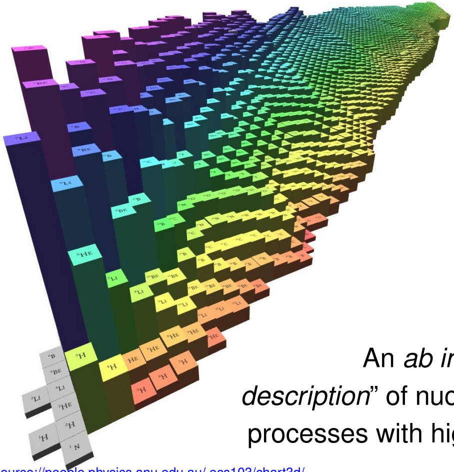
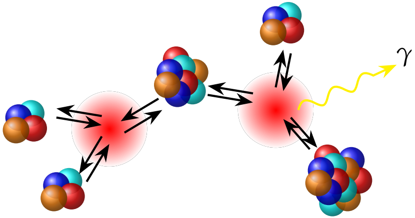


Ab initio nuclear theory

The aim is to predict the properties of nuclear systems from microscopic nuclear forces



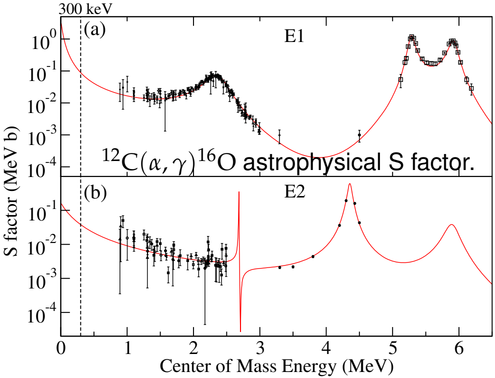


An *ab initio* nuclear theory that has a “*unified description*” of nuclear structure and scattering/reaction processes with high predictive power.

source://people.physics.anu.edu.au/ ecs103/chart3d/

Why *ab-initio* nuclear reactions: nucleosynthesis processes

- ^{4}He: fuels the nucleosynthesis of the heavier elements.
- The reaction cross section must be determined within the energy range of $0.15 - 3.4$ MeV, and obtaining the reaction rate accurately is essential for stellar evolution models.
- Direct measurements at the 300 keV, corresponding to helium-burning temperatures, are impossible due to the presence of the Coulomb barrier between nuclei.
- Therefore, the inaccessible reaction rate depends on extrapolating experimental data obtained at higher energies, leading to significant uncertainties in stellar evolution models.



^{12}C$(\alpha,\gamma)^{16}$O astrophysical S factor.

deBoer *et al.*, *Rev. Mod. Phys.* 89, 035007

Progresses and challenges in *ab initio* scattering and reactions

- ☐ QMC calculations of n-^{4}He scattering.
  Nollett, Pieper, Wiringa, Carlson, & Hale, PRL 99, 022502 (2007).
- ☐ *Ab initio* many-body calculations of n-^{3}H, n-^{4}He, p-^{3,4}He,and n-^{10}Be scattering.
  Quaglioni & Navratil, PRL 101, 092501 (2008).
- ☐ Ab initio many-body calculations of the ^{3}H(d,n)^{4}He, ^{3}He(d,p)^{4}He fusion.
  Navratil & Quaglioni, PRL 108, 042503 (2012).
- ☐ Elastic proton scattering of medium mass nuclei from CC theory.
  Hagen & Michel PRC 86, 021602 (2012).
- ☐ Coupling the lorentz integral transform (LIT) and the CC Methods.
  Orlandini, G. et al. , Few Body Syst. 55, 907â911 (2014).
- ☐ Ab Initio Prediction of the ^{4}He(d,γ)^{6}Li Big Bang Radiative Capture.
  Hebborn, Hupin, Kravvaris, Quaglioni, Navratil & Gysbers, PRL 129, 042503 (2022).
- ☐ *Ab initio* investigations of A=8 nuclei.
  Navratil, Kravvaris et al., J.Phys.Conf.Ser. 2586 (2023) 1, 012062
  Kravvaris and Volya, PRC 100, 034321 (2019)

Progresses and challenges in *ab initio* scattering and reactions

Ab initio calculations of scattering and reactions are limited by the computational scaling with the number of nucleons in the target and projectile (clusters).

In general, for most of the many-body approaches it remains a challenge to address important processes relevant for stellar astrophysics.

☐ Scattering of alpha particles: $^4\mathrm{He} + {}^4\mathrm{He} \to {}^4\mathrm{He} + {}^4\mathrm{He}$

☐ Triple- alpha reaction: $^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} \to {}^{12}\mathrm{C} + \gamma$

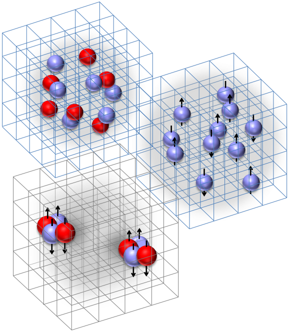
☐ Alpha capture: $^4\mathrm{He} + {}^{12}\mathrm{C} \to {}^{16}\mathrm{O} + \gamma$

$^4\mathrm{He} + {}^{16}\mathrm{O} \to {}^{20}\mathrm{Ne} + \gamma$

$\vdots$

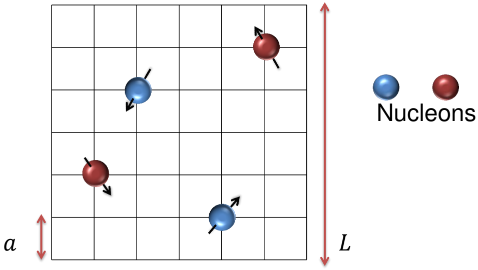
Outline

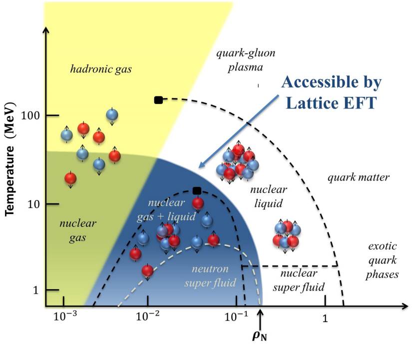
- Introduction
- Lattice effective field theory
- Scattering on the lattice
- Adiabatic projection method
- Recent progress in LEFT
- Summary



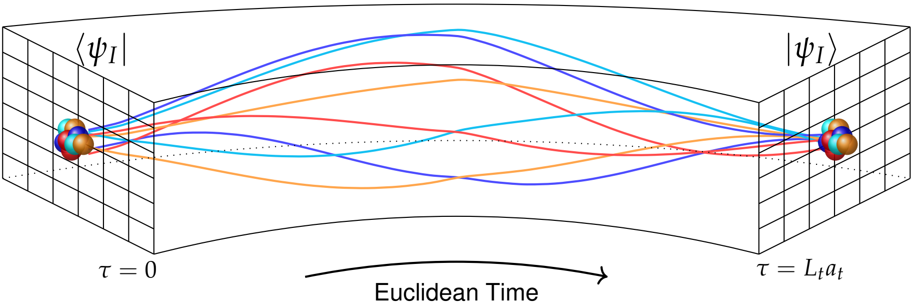
Lattice effective field theory

- Lattice effective field theory is a powerful numerical method formulated in the framework of chiral effective field theory.

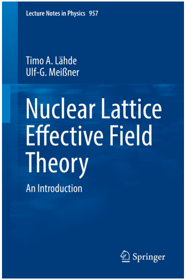




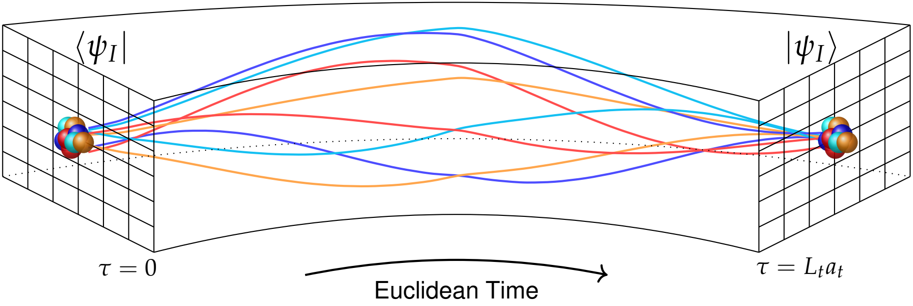
Lattice effective field theory



- ☐ construct an initial/final state of nucleons, $|\psi_I\rangle$, as a Slater determinant of free-particle standing waves on the lattice.
- ☐ evolve nucleons forward in Euclidean time, $e^{-H_{\rm LO}\,\tau}\,|\psi_I\rangle$, where $\tau = L_t a_t$.
- ☐ The evolution in Euclidean time automatically incorporates the induced deformation, polarization and clustering.



Auxiliary field Monte Carlo



Use a Gaussian integral identity

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}} \int ds \, \exp\left[-\frac{s^2}{2} + \sqrt{-C}\, s \left(N^\dagger N\right)\right]$$

s is an auxiliary field coupled to the particle density. Each nucleon evolves as if a single particle in a fluctuating background of pion fields and auxiliary fields.

Lattice Monte Carlo calculations

Projection Monte Carlo uses a given initial state, $|\psi_I\rangle$, to evaluate a product of a string of transfer matrices $\hat{M}$.

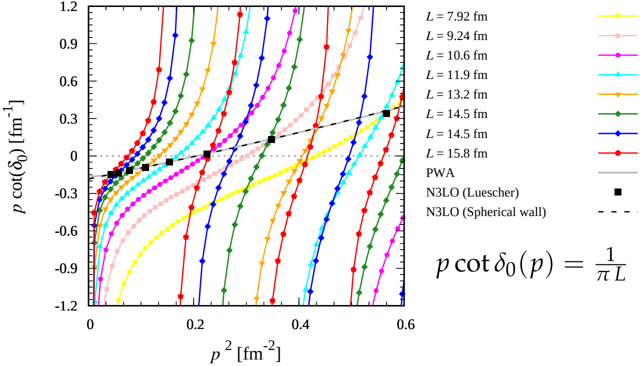
$$Z(L_t) = \langle\psi_I\,|\hat{M}(L_t-1)\,\hat{M}(L_t-2)\,\ldots\hat{M}(1)\,\hat{M}(0)\,|\,\psi_I\rangle$$

In the limit of large Euclidean time the evolution operator $e^{-H_{\mathrm{LO}}\,\tau}$ suppress the signal beyond the low-lying states, and the ground state energy can be extracted by

$$\boxed{\lim_{L_t\to\infty}\frac{Z(L_t+1)}{Z(L_t)} = e^{-E_0\,a_t}}$$

$$\boxed{\lim_{L_t\to\infty}\frac{\langle\psi_I|\,\hat{M}^{L_t/2}\,H_{\mathrm{LO}}\,\hat{M}^{L_t/2}\,|\psi_I\rangle}{\langle\psi_I|\,\hat{M}^{L_t}\,|\psi_I\rangle} = E_0}$$

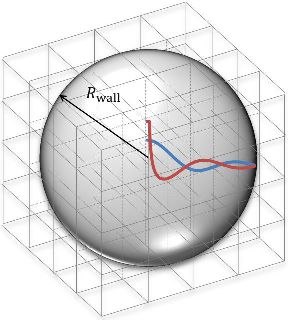
Scattering on the lattice



$$p\cot\delta_0(p) = \frac{1}{\pi L}\left[\sum_{\vec{n}}^{\Lambda} \frac{\theta\left(\Lambda^2-\vec{n}^2\right)}{\vec{n}^2-(Lp/2\pi)^2} - 4\pi\,\Lambda\right]$$

Lüscher’s finite volume method:

Lüscher, Comm. Math. Phys. 105 (1986) 153; NPB 354 (1991) 531



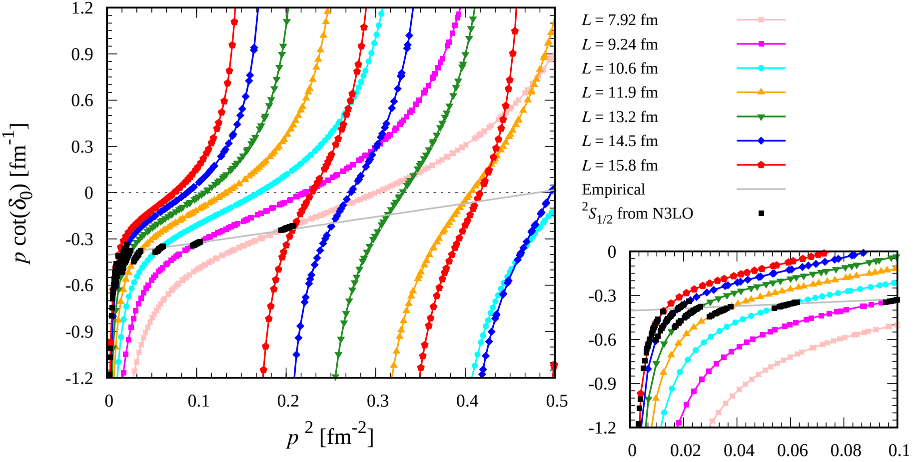
Spherical wall method:

$$R_\ell^{(p)}(r) = N_\ell(p) \times \begin{cases} \cot\delta_\ell(p)\, j_\ell(p\,r) - n_\ell(p\,r) \\ \cot\delta_\ell(p)\, F_\ell(p\,r) + G_\ell(p\,r) \end{cases}$$

Nucl. Phys. A 424, 47-59 (1984), Eur. Phys. J. A 34, 185-196 (2007).

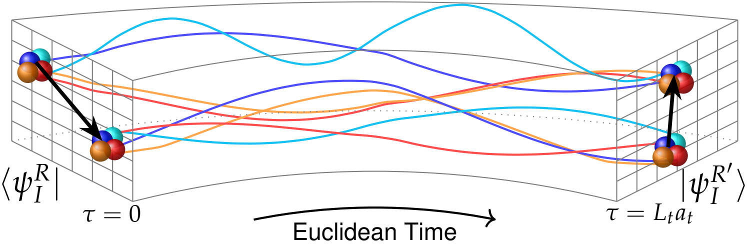
Scattering on the lattice

Neutron-alpha scattering at N3LO



SE and Meißner, [*in progress*].

Adiabatic projection method



The method constructs a low energy effective theory for the clusters by using initial states, $|\psi_I^R\rangle$ and $|\psi_I^{R'}\rangle$, parameterized by the relative spatial separation between clusters, and project them in Euclidean time to get dressed cluster states, $|\psi_I^R\rangle_\tau = e^{-H\tau}\, |\psi_I^R\rangle$.

Hamiltonian matrix

$$[H_\tau]^{J,J_z}_{R,R'} = {}^{J,J_z}_{\tau}\langle \psi_I^R | H | \psi_I^{R'} \rangle^{J,J_z}_{\tau}$$

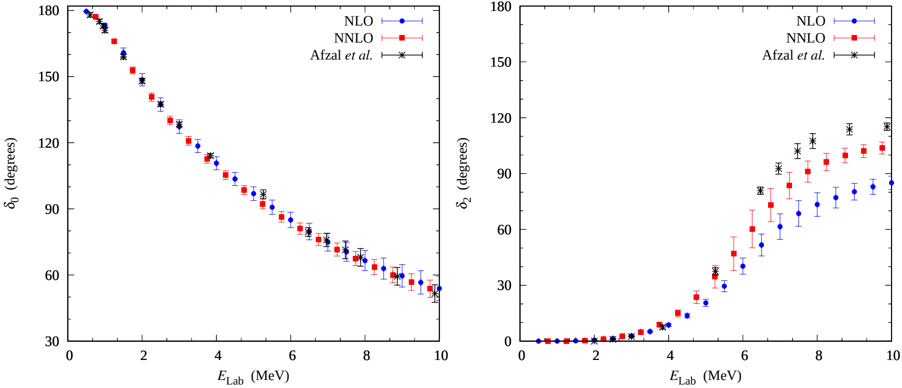
Norm matrix

$$[N_\tau]^{J,J_z}_{R,R'} = {}^{J,J_z}_{\tau}\langle \psi_I^R | \psi_I^{R'} \rangle^{J,J_z}_{\tau}$$

$$[H^a_\tau]^{J,J_z}_{\vec{R},\vec{R}'} = \left[N_\tau^{-1/2}\; H_\tau\; N_\tau^{-1/2} \right]^{J,J_z}_{\vec{R}\,\vec{R}'}$$

Eur.Phys.J.A 52 (2016) 6, 174.
Eur.Phys.J.A 55 (2019) 8, 144.

Ab-initio alpha-alpha scattering N2LO



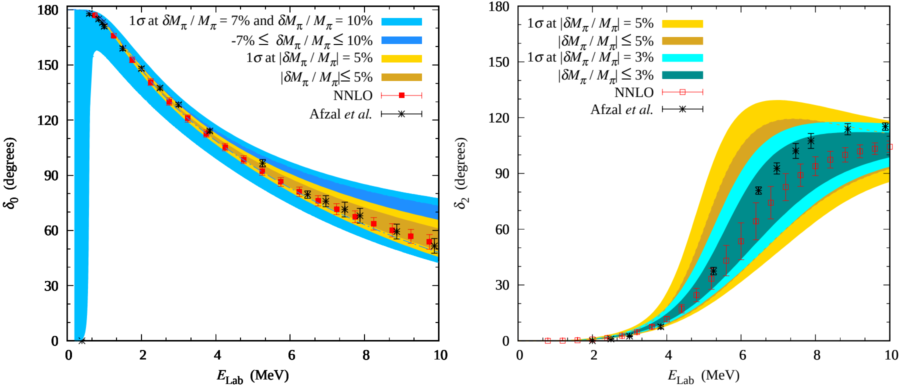
Afzal, Ahmad, Ali, *Rev. Mod. Phys.* 41, 247, (1969).

SE, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, & Meißner. *Nature* 528, 111-114 (2015).

Ab-initio alpha-alpha scattering in the Multiverse

Alpha-alpha scattering phase shifts under variations of the fundamental parameters of the Standard Model.

$$\left.\frac{\partial E_{\alpha\alpha}}{\partial M_\pi}\right|_{M_\pi^{\rm ph}} = \left.\frac{\partial E_{\alpha\alpha}(\tilde{M}_\pi, m_N(M_\pi), \tilde{g}_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi))}{\partial M_\pi}\right|_{M_\pi^{\rm ph}}$$



Afzal, Ahmad, Ali, *Rev. Mod. Phys.* 41, 247, (1969).

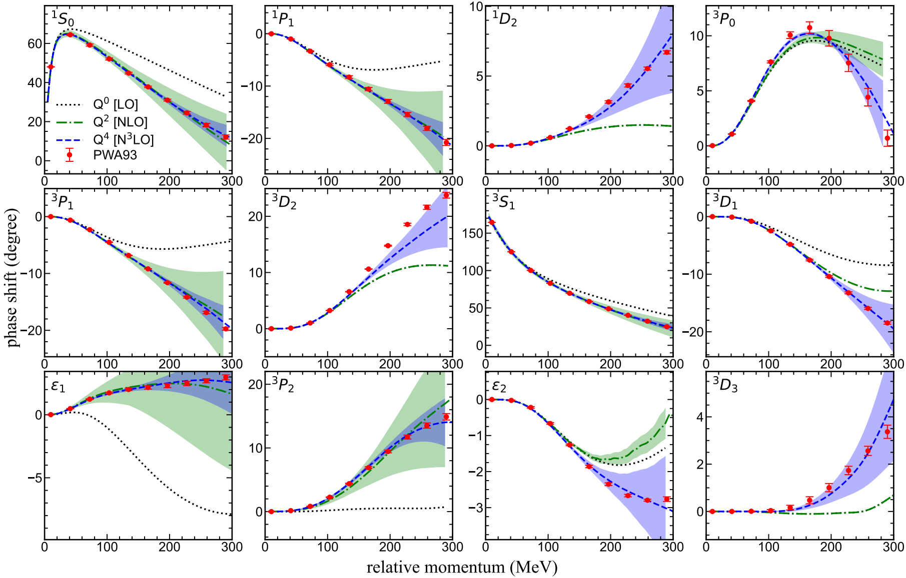
SE, Lähde, Lee, Meißner, Vonk. *JHEP* 02 (2022) 001

Chiral interactions at N3LO – 2NFs + 3NFs

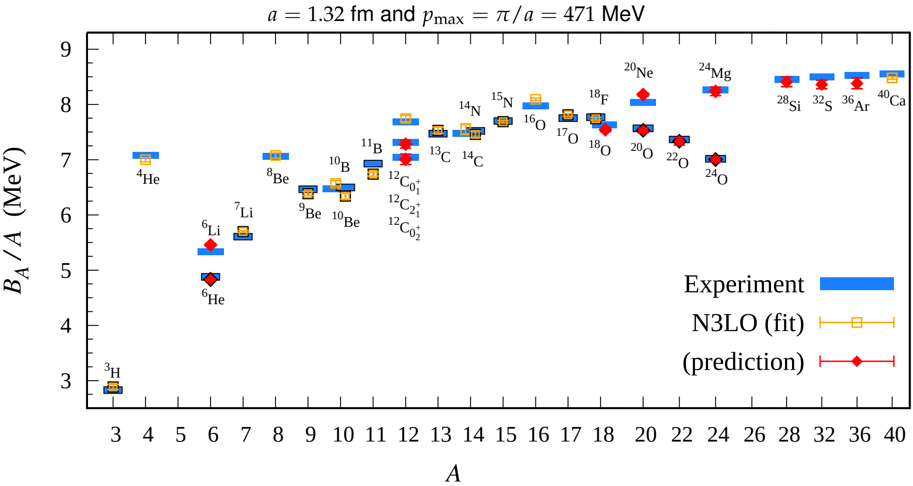
Work	Constraints	Predictions
NCSM, Barrett *et al.*, Nogga *et al.*	BE of ^{3}H and ^{4}He	Spectrum of ^{6}Li and ^{7}Li
NCSM, Navratil *et al.*	^{3}H, ^{6}Li, ^{10}B, ^{12}C	^{4}He, ^{6}Li, ^{10,11}B, ^{12,13}C
NCSM, Maris *et al.*, Roth *et al.*	BE of ^{3}H and ^{3}H β decay	Structures of $A = 7, 8$. ^{4}He, ^{6}Li, ^{12}C and ^{16}O
CC, Hagen *et al.*	BE of ^{3}H and ^{3}H β decay	EoS of nucleonic matter
BMBPT, Tichai *et al.*	BE of ^{3}H and ^{3}H β decay	BE of $^{16-26}$O, $^{36-60}$Ca and $^{50-78}$Ni
IT-NCSM, Roth *et al.*	BE of ^{3}H and ^{4}He, and ^{3}H β decay	BE of ^{4}He, ^{16}O, ^{40}Ca
CC, Roth *et al.*	BE of ^{3}H and ^{4}He, and ^{3}H β decay	BE of ^{16,24}O, ^{40,48}Ca
SCGF, Cipollone *et al.*	BE of ^{3}H and ^{4}He, and ^{3}H β decay	BE of ^{13,27}N, ^{14,28}O and ^{15,29}F
AFDMC, Lynn *et al.*	BE of ^{3}H and n-^{4}He P-wave phase shifts	EoS of nucleonic matter
MBPT, Bogner *et al.*, Hebeler *et al.*, Drischler *et al.*, Wienholtz *et al.*, Simonis *et al.*	BE ^{3}H and R_c of ^{4}He	symmetric and asymmetric NM, BE of $^{48-58}$Ca, spectrum of *sd*-shell nuclei with $8 \leq Z, N \leq 20$, BE and R_c of open- and closed-shell nuclei up to $A = 78$
NCCI, Epelbaum *et al.*, Maris *et al.*	BE of ^{3}H, nd spin-doublet scattering length and the pd differential cross section	the spectrum of light nuclei with $A = 3$–16, elastic nd scattering and in the deuteron breakup reactions, properties of the $A = 3, 4$ nuclei, and for spectra of p-shell nuclei up to $A = 16$, BE and R_c of the oxygen and calcium isotope chains
CC, Carlsson *et al.*, Ekström *et al.*, Hagen *et al.*	BE of ^{3}H, ^{3,4}He, ^{14}Li and ^{16,22,24,25}O	R_c and BE of nuclei up to ^{40}Ca, symmetric nuclear matter, neutron skin of ^{48}Ca, structure of ^{78}Ni
NCSM, IM-SRC, IM-NCSM, Hüther *et al.*	BE of ^{3}H and ^{16}O	R_c and BE of ^{4}He, $^{14-26}$O, $^{36-52}$Ca and $^{48-78}$Ni, the spectrum of ^{7}Li, ^{8}Be, ^{9}Be and ^{10}B
CC, Jiang *et al.*	properties of $A \leq 4$	properties of nuclei from $A = 16 - 132$

Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm and $p_{\max} = \pi/a = 471$ MeV



Ab initio nuclear theory: recent progress in NLEFT



SE et al. [NLEFT collaboration] arXiv:2210.17488

Ab initio nuclear theory: recent progress in NLEFT

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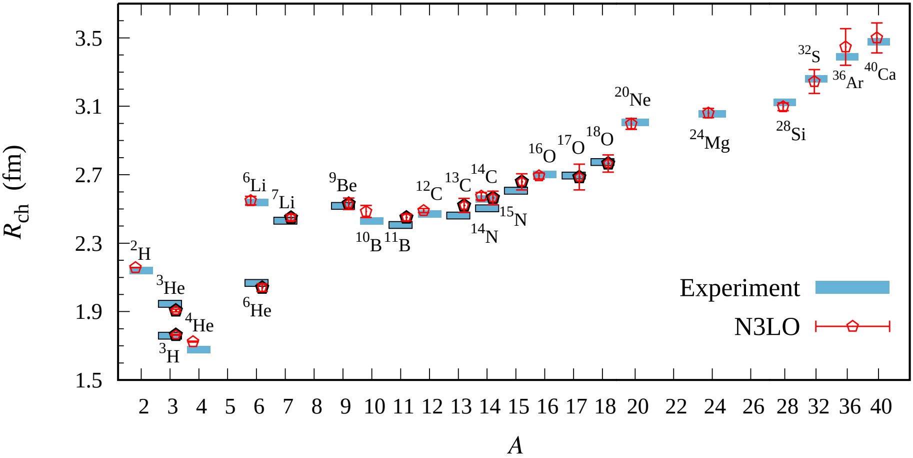

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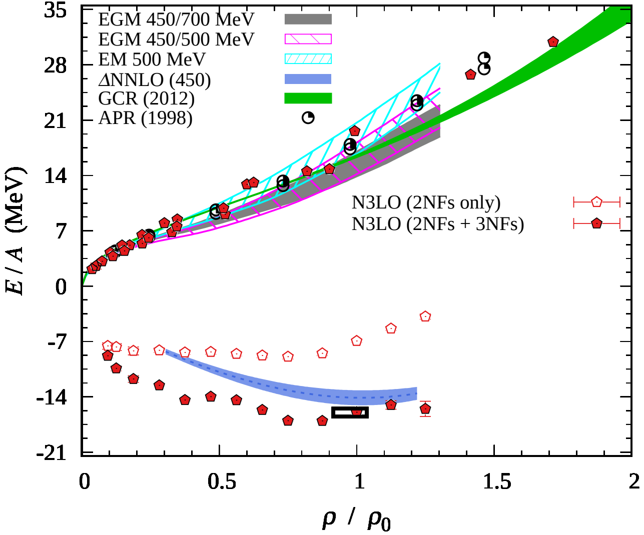

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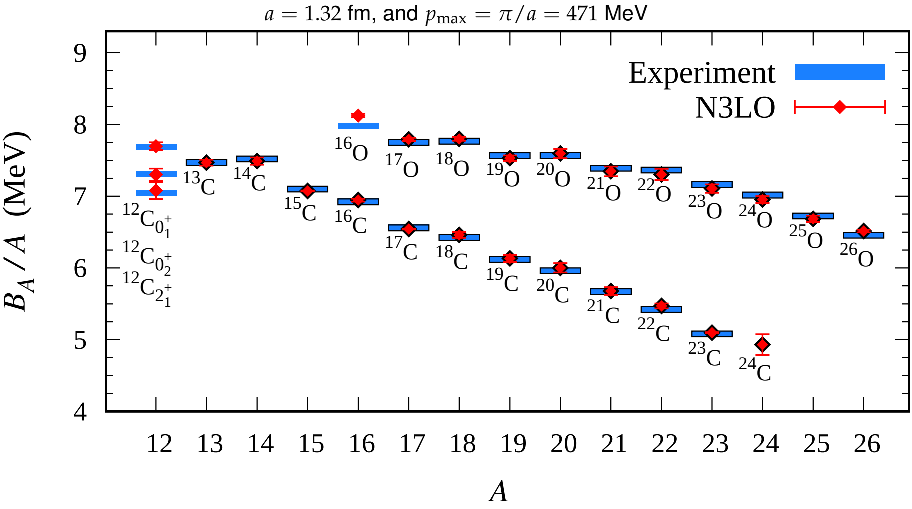
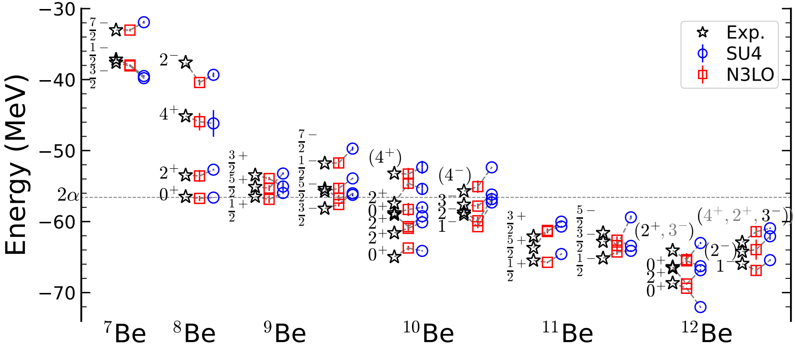


[NLEFT collaboration] in progress

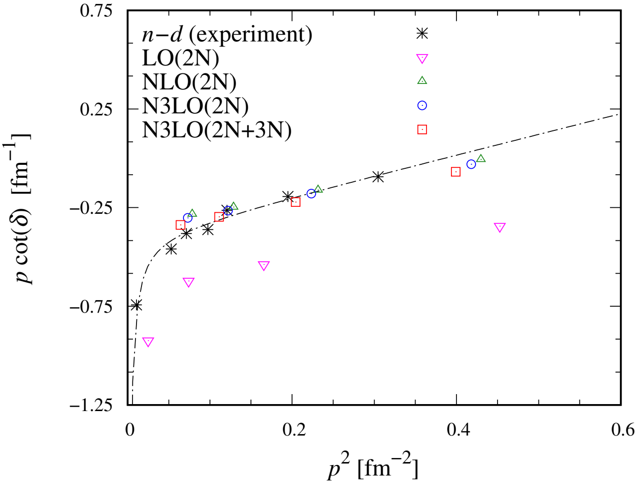
Ab initio nuclear theory: recent progress in NLEFT

$a = 1.32$ fm, and $p_{\max} = \pi/a = 471$ MeV



[NLEFT collaboration] in progress

Spin doublet S-wave neutron-deuteron scattering at N3LO



SE, Hildenbrand and Meißner, [*in progress*].

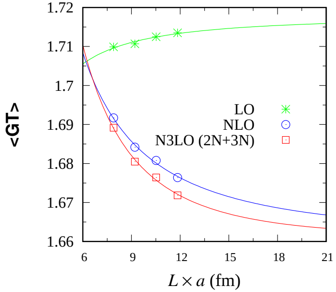
Triton-β decay at N3LO

$$(1+\delta_R)\, t_{1/2}\, f_V = \frac{K/G_V^2}{\langle \mathbf{F}\rangle^2 + \frac{f_A}{f_V}\, g_A^2\, \langle \mathbf{GT}\rangle^2}$$

$$\langle \mathbf{F}\rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He}\| \tau_{n,+} \| {}^3\mathrm{H}\rangle = 0.9998 \qquad \langle \mathbf{GT}\rangle = \sum_{n=1}^{3} \langle {}^3\mathrm{He}\| \sigma_n \tau_{n,+} \| {}^3\mathrm{H}\rangle = 1.6474(23)\,.$$

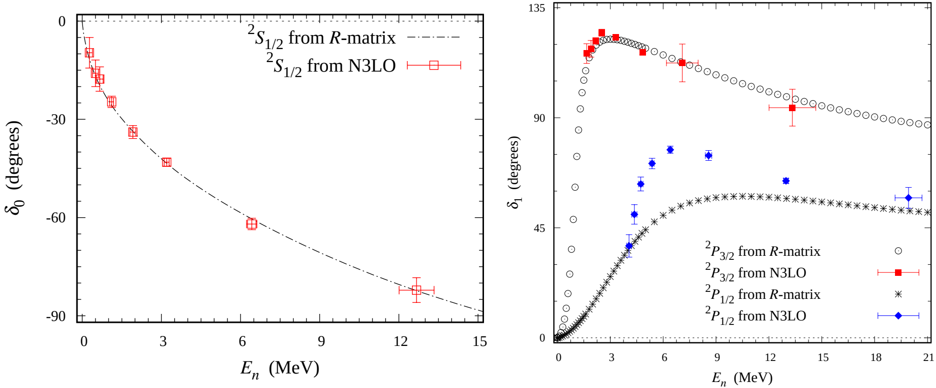
$$\langle \mathbf{GT}\rangle_{\mathrm{N3LO}} = 1.661(35)\,.$$

L (fm)	$\langle \mathbf{F}\rangle$ LO (2N)	NLO (2N)	N3LO (2N+3N)	$\langle \mathbf{GT}\rangle$ LO (2N)	NLO (2N)	N3LO (2N+3N)
6.60	0.99984	0.99997	0.99997	1.7115	1.6937	1.6927
7.92	0.99969	0.99989	0.99991	1.7099	1.6917	1.6891
9.24	0.99967	0.99977	0.99980	1.7107	1.6842	1.6805
10.6	0.99973	0.99956	0.99961	1.7125	1.6808	1.6764
11.9	0.99980	0.99940	0.99947	1.7135	1.6764	1.6718



SE, Hildenbrand and Meißner, [*in progress*].

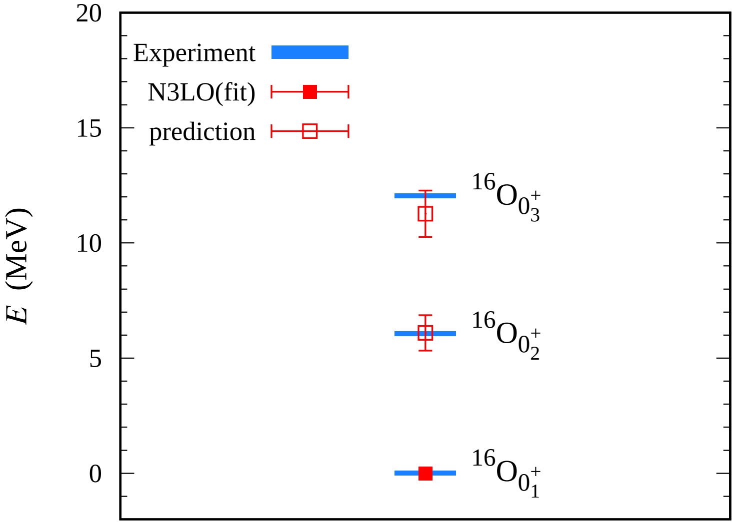
Neutron-alpha scattering at N3LO



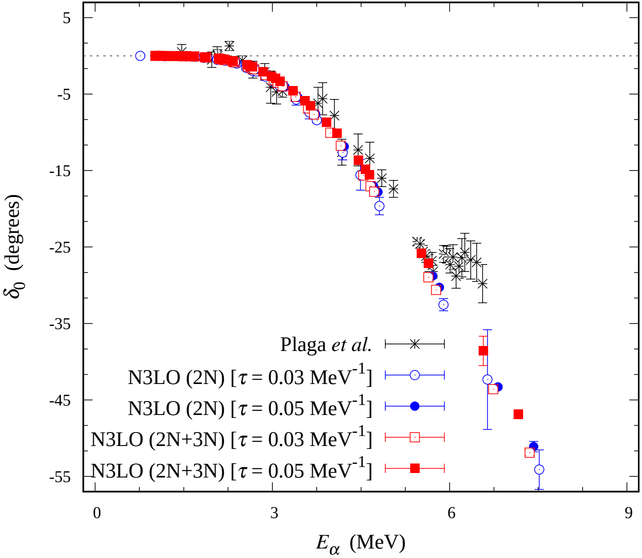
SE, Hildenbrand and Meißner, [*in progress*].

G. M. Hale, Private Communication, [*R-matrix*].

Alpha-carbon scattering at N3LO

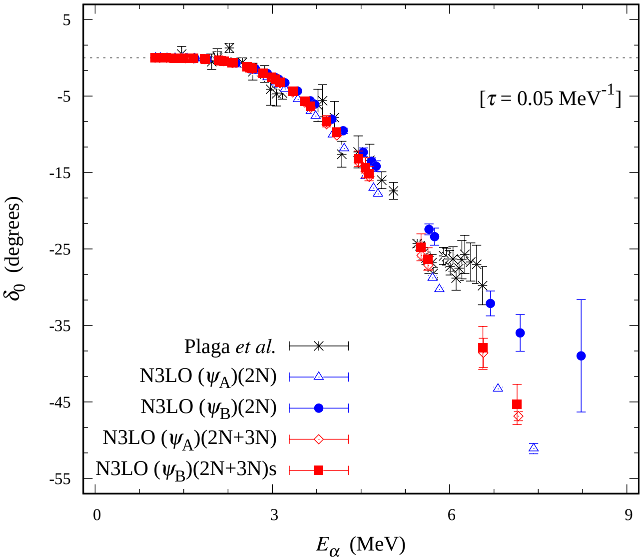


Ab initio alpha-carbon scattering at N3LO



SE, Hildenbrand, Meißner, ... NLEFT [*in progress*].

Ab initio alpha-carbon scattering at N3LO



SE, Hildenbrand, Meißner, ... NLEFT [*in progress*].

Summary

☐ Nuclear forces in the framework of chiral effective field theory are well-established, and it is very important time for *ab initio* methods to make predictions in many-nucleon system using these forces.

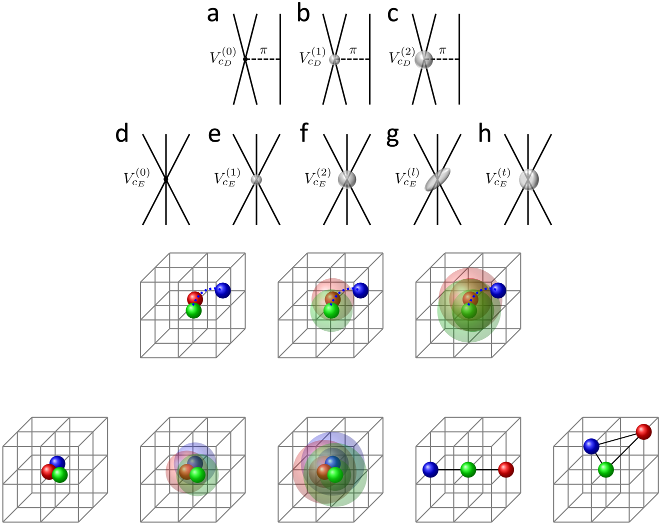
☐ A recently developed method so called the wave function matching provides a rapid convergence in perturbation theory for many-body nuclear physics. Using this new method now we are able to calculate the nuclear binding energies, neutron matter, symmetric nuclear matter and charge radii of nuclei simultaneously in very good agreements with the experimental results.

☐ With the recently developed N3LO lattice action and powerful numerical methods, we are ready to perform the first *ab initio* calculation of alpha-carbon scattering, “holy grail” of nuclear astrophysics.

Thanks!

Three-nucleon forces

$a = 1.32$ fm and $p_{\max} = \pi/a = 471$ MeV



SE et al. [NLEFT collaboration] arXiv:2210.17488

Lattice EFT: (Euclidean time) projection Monte Carlo

Transfer matrix operator formalism $\hat{M} =: \exp(-H_{\mathrm{LO}}\, a_t) :$

Microscopic Hamiltonian $H_{\mathrm{LO}} = H_{\mathrm{free}} + V_{\mathrm{LO}}$

$$Z(L_t) = \mathrm{Tr}(\hat{M}^{L_t}) = \int Dc\, Dc^* \,\exp[-S(c, c^*)]$$

Creutz, Found. Phys. 30 (2000) 487.

The exact equivalence of several different lattice formulations.

Lee, PRC 78:024001, (2008); Prog.Part.Nucl.Phys., 63:117-154 (2009)