(Puzzling) QCD effects in semileptonic $\bar{B} \to D^{(*)}\ell\bar{\nu}$ decays

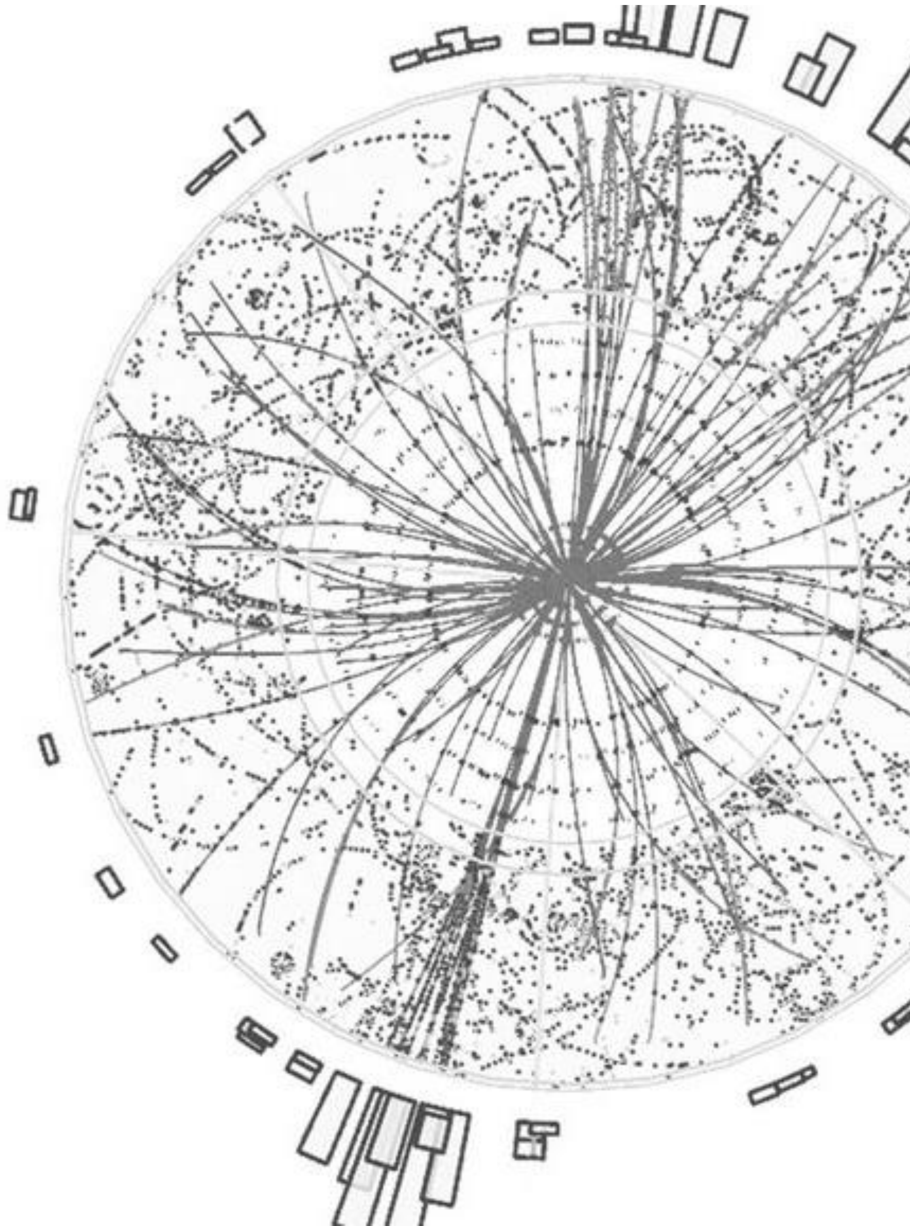
---

Nico Gubernari

Based on
arXiv: 1908.09398
arXiv: 1912.09335
arXiv: 24xx.xxxxx
in collaboration with
Marzia Bordone, Martin Jung, and Danny van Dyk

Particle Physics Seminar
University of Bonn
25-Jan-2024

Talk outline



Introduction

Theoretical framework

- hadronic matrix elements
- form factors

Form factor calculations

- lattice QCD (LQCD)
- light-cone sum rules (LCSRs)

Form factor parametrizations

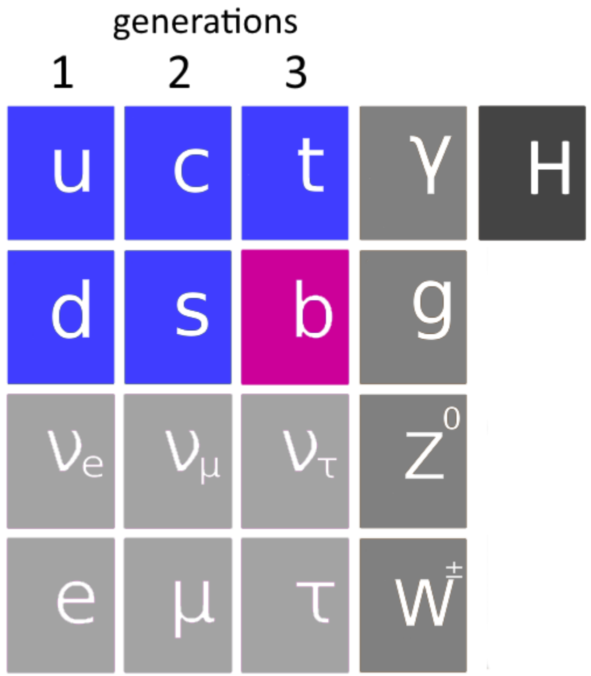
- z expansion
- unitarity bounds and BGL
- HQET parametrization

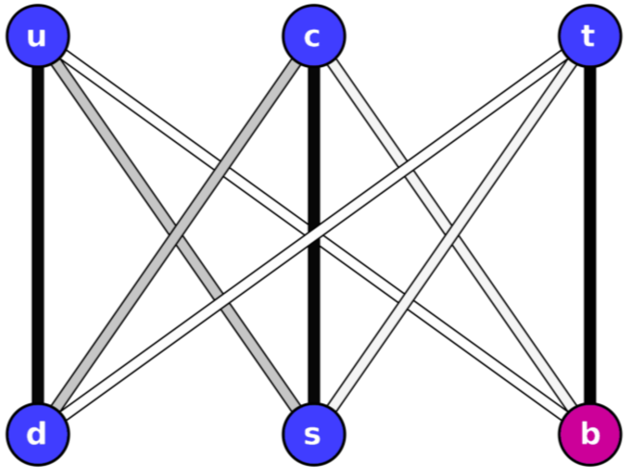
($\boldsymbol{B} \to \boldsymbol{D}^{**}$ form factors)

Summary and outlook

Introduction

The beauty in flavour physics





In the SM 6 quark flavours and 6 lepton flavours

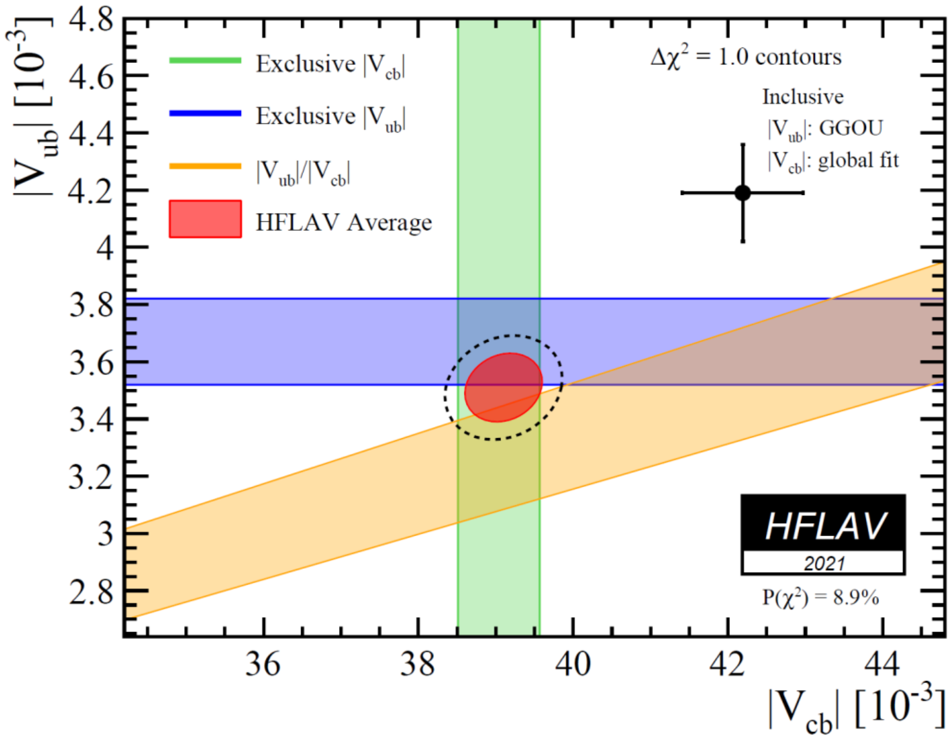
flavour physics: investigate the properties, the transitions, and the spectrum of the different quark and lepton flavours

transitions between different (flavours) mediated by $W^\pm$

why is the *b* quark interesting?

- third generation quark
- heaviest fermion that forms bound states ($m_b \gg \Lambda_{\mathrm{QCD}}$)
- lighter than the t quark
  $\Longrightarrow$ decays in quarks of another generation
  $\Longrightarrow$ CKM suppressed decay

$b \to c\ell\nu$ decays



why study $b \to c\ell\nu$ transitions?

1. extract $|V_{cb}|$ - fundamental parameter of the SM
   - using inclusive $B \to X_c\ell\nu$ decays
   - using exclusive $B \to D\ell\nu$ or $B \to D^*\ell\nu$ decays

   $\Longrightarrow |V_{ub}| - |V_{cb}|$ puzzle

2. probe the SM and constrain new physics
   - do the SM predictions agree with the corresponding measurements?
   - possible deviations form a coherent pattern?

focus of this talk: theory predictions for $\boldsymbol{B \to D\ell\nu}$ and $\boldsymbol{B \to D^*\ell\nu}$ (and $B \to D^{**}\ell\nu$) decays

Optimised observables and LFU

test the lepton flavour universality to test the SM

lepton flavour universality (LFU) = the 3 lepton generations have the same couplings to the gauge bosons

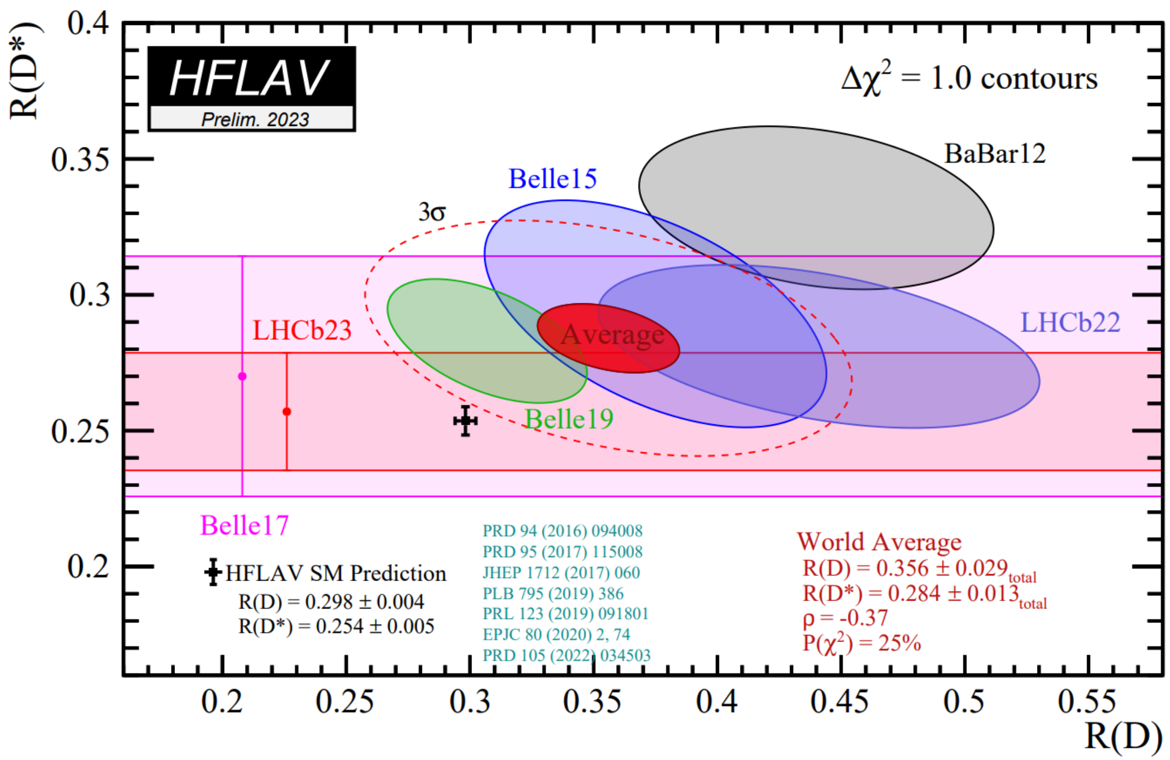
violations of LFU $\Longrightarrow$ new physics

define observables smartly to reduce theory uncertainties and cancel V_{cb}

observables to test LFU

$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)} \tau \nu)}{\Gamma(B \to D^{(*)} \ell \nu)}$$

3.3 σ tension between the SM and data



Theoretical framework

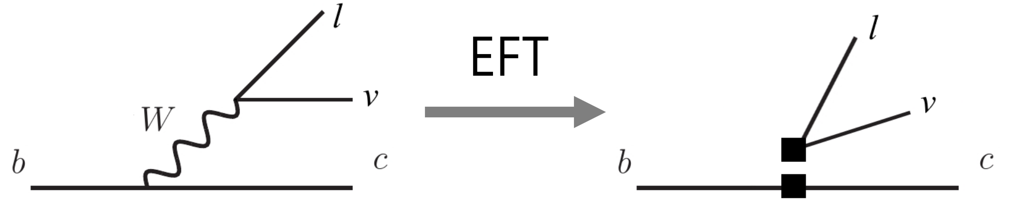
Flavour changing currents

flavour changing charged currents (FCCC) occur at tree level (mediated by $W^{\pm}$) in the SM

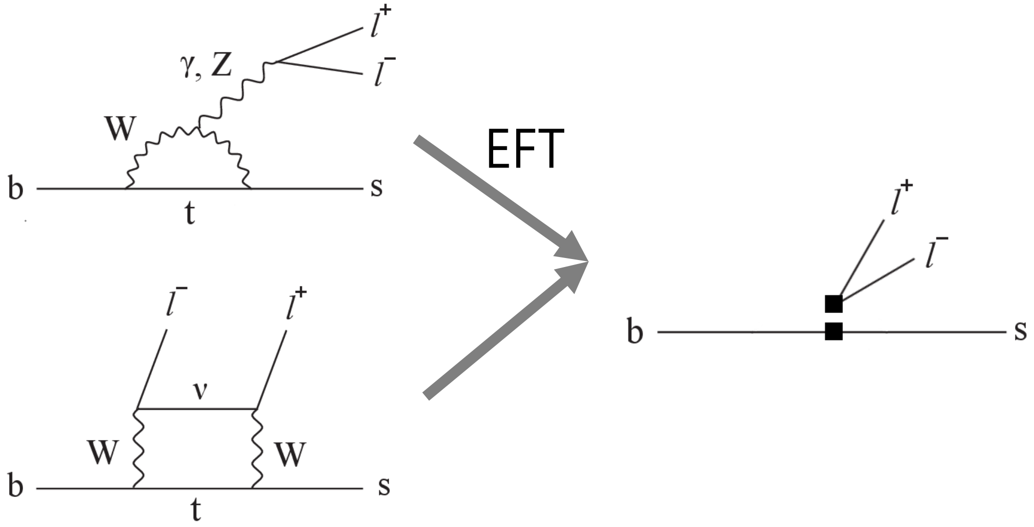
$b \to c\ell\nu$ very frequent transitions ($\Gamma(B \to X_c \ell\nu \simeq 11\%$)

flavour changing neutral currents (FCNC) absent at tree level in the SM
FCNC are loop, GIM and CKM **suppressed in the SM**

integrate out DOF heavier than the b
⇓
weak effective field theory



FCCC



FCNC

Hadronic matrix elements

study $\boldsymbol{B}$-meson decays to test the $b \to c\ell\nu$ transitions
factorise decay amplitude (neglecting QED corrections)

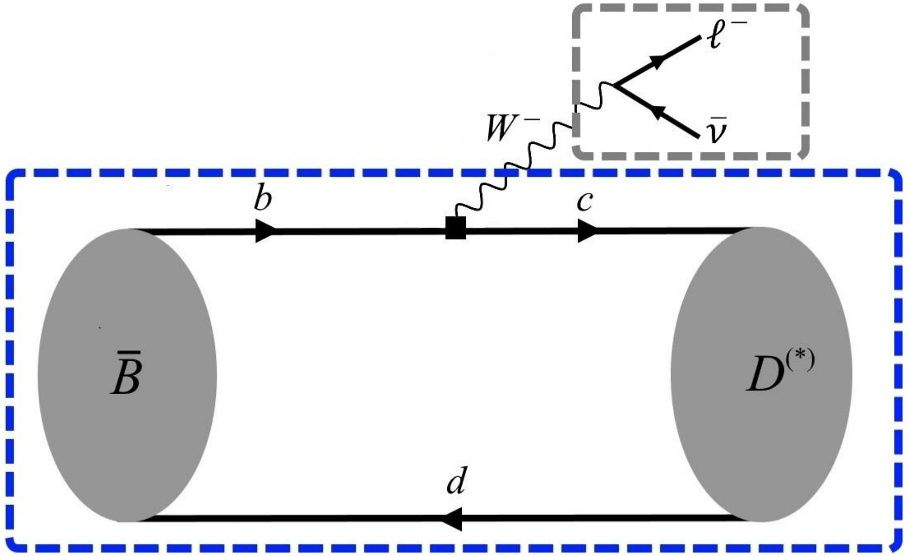
charged currents: $\langle \bar{D}^{(*)}\ell\nu_\ell | \mathcal{O}_{eff} | B\rangle = \langle \ell\nu_\ell | \mathcal{O}_{lep} | 0\rangle \langle D^{(*)} | \mathcal{O}_{had} | B\rangle$

neutral currents: $\langle K^{(*)}\ell^+\ell^- | \mathcal{O}_{eff} | B\rangle = \langle \ell\ell | \mathcal{O}_{lep} | 0\rangle \langle K^{(*)} | \mathcal{O}_{had} | B\rangle$ + non−fact.

leptonic matrix elements: perturbative objects, high accuracy
QED corrections mostly unknown but small (~1%)

hadronic matrix elements: non-perturbative QCD effects,
usually large uncertainties (~10%)

(local) hadronic matrix elements are crucial
to obtain precise predictions for $\boldsymbol{b \to c\ell\nu}$ decays



Definition of the form factors

form factors (FFs) parametrize exclusive hadronic matrix elements

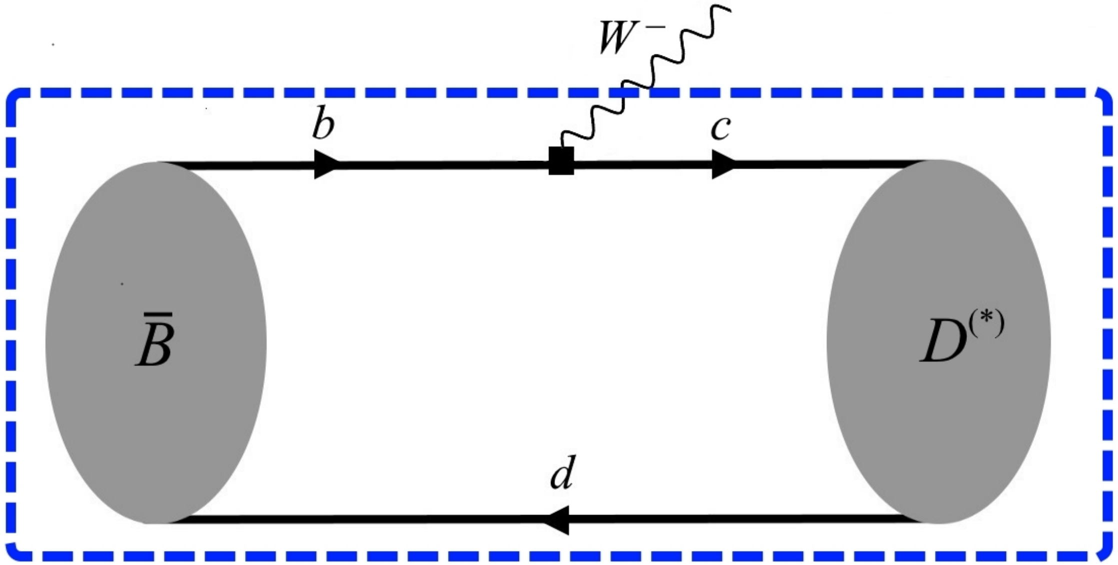
$$\langle D(k)|\bar{c}\,\gamma_\mu b|B(q+k)\rangle = 2\,k_\mu f_+(q^2) + q_\mu\big(f_+(q^2) + f_-(q^2)\big)$$

$$\langle D(k)|\bar{c}\,\sigma_{\mu\nu}q^\nu b|B(q+k)\rangle = \frac{i f_T(q^2)}{m_B + m_P}\left(q^2(2k+q)_\mu - (m_B^2 - m_P^2)q_\mu\right)$$

decomposition follows from Lorentz invariance

FFs are functions of the momentum transferred q^2
(q^2 is the dilepton mass squared)

2(+1) independent $B \to D$ FFs
4(+3) independent $B \to D^*$ FFs



Form factors calculations

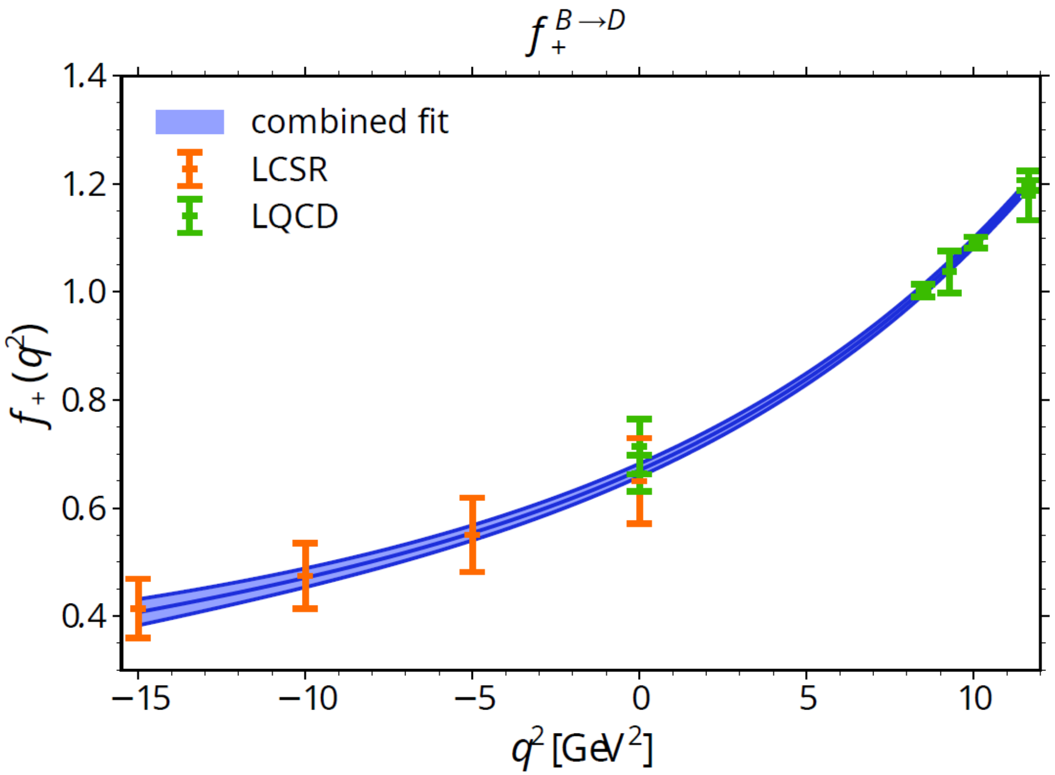
Methods to compute FFs

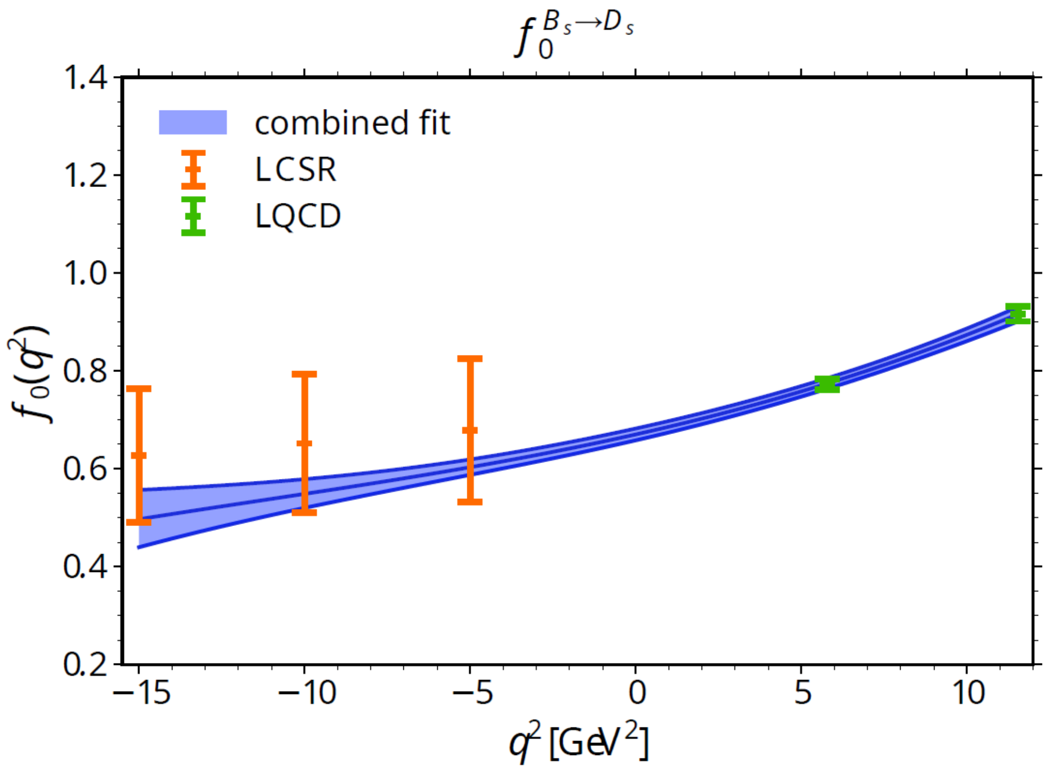
non-perturbative techniques are needed to compute FFs

1. Lattice QCD (LQCD)
   more efficient usually at **high q^2**

2. Light-cone sum rules (LCSRs)
   only applicable at **low q^2**

complementary approaches to calculate FFs





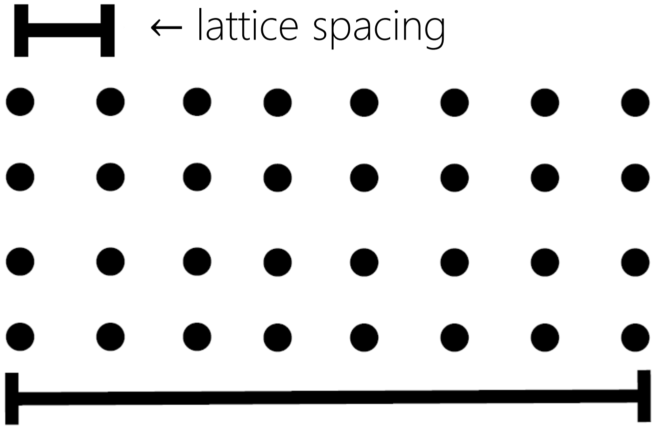
Lattice QCD in a nutshell

LQCD = evaluating path integrals numerically

$$\textbf{observable} = \int \prod_i \mathrm{d}\phi_i\ (\textbf{correlator})$$

to perform the calculation approximations are needed

1. **nonzero** lattice **spacing**
2. **finite volume**
3. Euclidian space time



Pros

can be used potentially for any q^2

first principles calculations

reducible systematic uncertainties

Cons

nonlocal matrix elements,
unstable states,
are still work in progress

computationally very expensive

Light-cone sum rules in a nutshell

LCSRs are a method to calculate hadronic matrix elements

method based on:

- dispersion relation
- quark-hadron duality assumption
- light-cone OPE

Pros

compute hadronic matrix elements
not accessible yet with LQCD

effective at small q^2
(complementary to LQCD)

Cons

need universal non-perturbative inputs
(B-meson distribution amplitudes)

non-reducible systematic uncertainties

in the long run LQCD will dominate the theoretical predictions (smaller and reducible syst unc.)

State of the art $B_{(s)} \to D_{(s)}^{(*)}$ FFs

- $B \to D$
  LQCD calculations available at **high** q^2
  [FNAL/MILC 2015] [HPQCD 2015]

- $B \to D^*$
  LQCD calculations available at **high** q^2
  [FNAL/MILC 2021] [JLQCD 2023]
  in the **whole** semileptonic region of q^2
  [HPQCD 2023]

- $B_s \to D_s$
  LQCD calculations available
  in the **whole** semileptonic region of q^2
  [HPQCD 2019]

- $B_s \to D_s^*$
  LQCD calculations available
  in the **whole** semileptonic region of q^2
  [HPQCD 2021] [HPQCD 2023]

LCSRs available for the four processes at **low** q^2

how to **combine** different calculations for the same channel?
how to obtain result in the **whole** semileptonic region if not available from LQCD?

Form factor parametrization

Parametrization for FFs

when LQCD data are available only at high q^2
obtain FFs in the whole semileptonic region by either

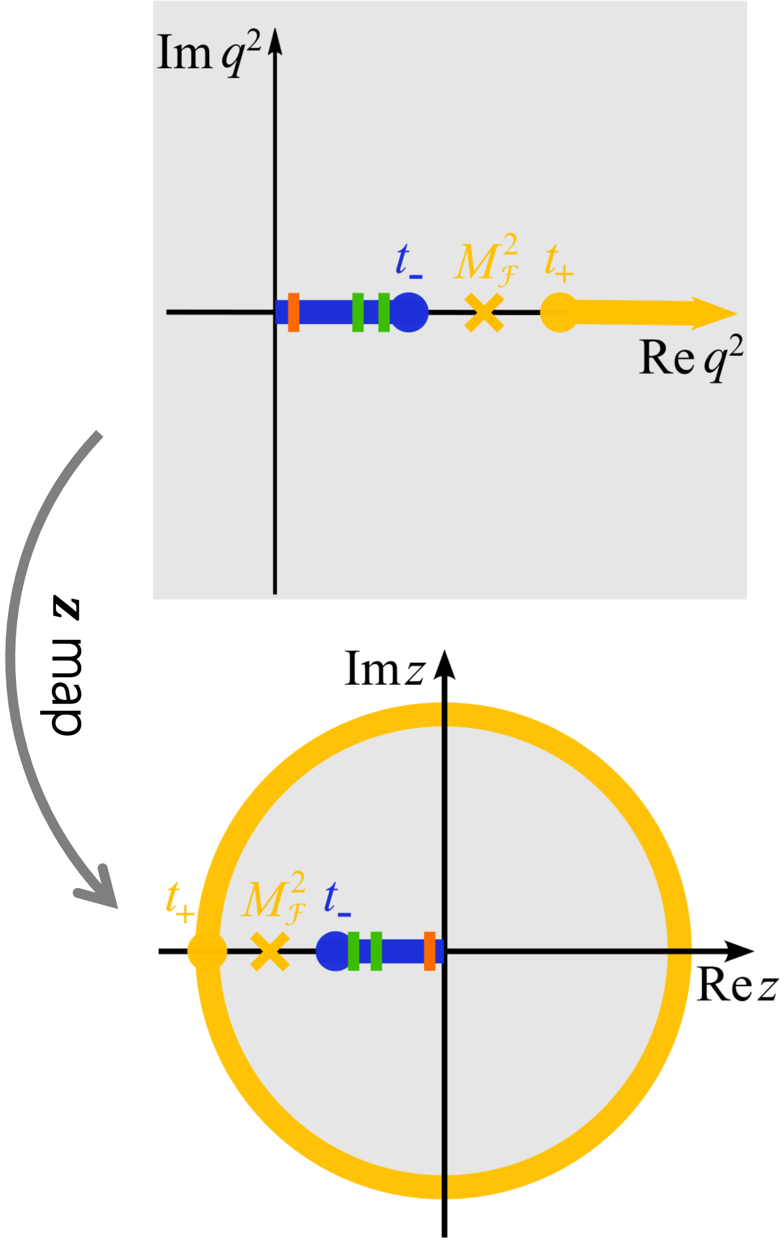
- extrapolating LQCD calculations to low q^2
- or combining LQCD and LCSRs

FFs are analytic functions of q^2 except for
branch cut for $q^2 > t_+ = \left(M_B + M_{D^{(*)}}\right)^2$

fit results to a **z parametrization** = Taylor series (standard approach)
[Boyd/Grinstein/Lebed 1997] [Bourrely/Caprini/Lellouch 2008]
[Bharucha/Straub/Zwicky 2015] [...]

$$\mathrm{FF} \propto \sum_{n=0}^{\infty} \alpha_n^{\mathrm{FF}} \, z^k$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$



Combine LQCD and LCSRs with naïve z param.

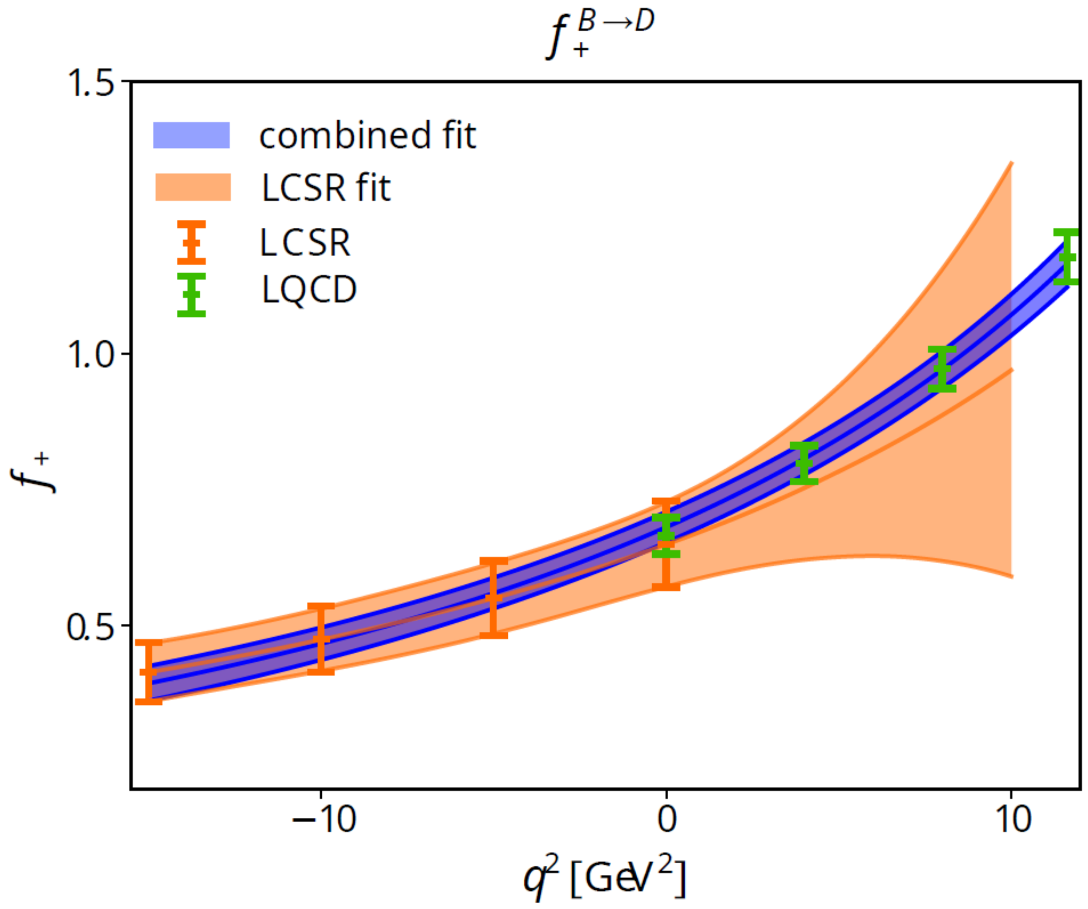
combine LQCD and LCSRs to obtain the FF values
in the whole semileptonic region

good agreement between LQCD and LCSRs calculations

use only first 3 terms in the **z parametrization**

issues of the naïve z parametrization

- **errors blow up** when extrapolating
- what is the “right” **truncation order?**
- what is the **truncation error?**



[NG/Kokulu/van Dyk 2018]

Unitarity bounds

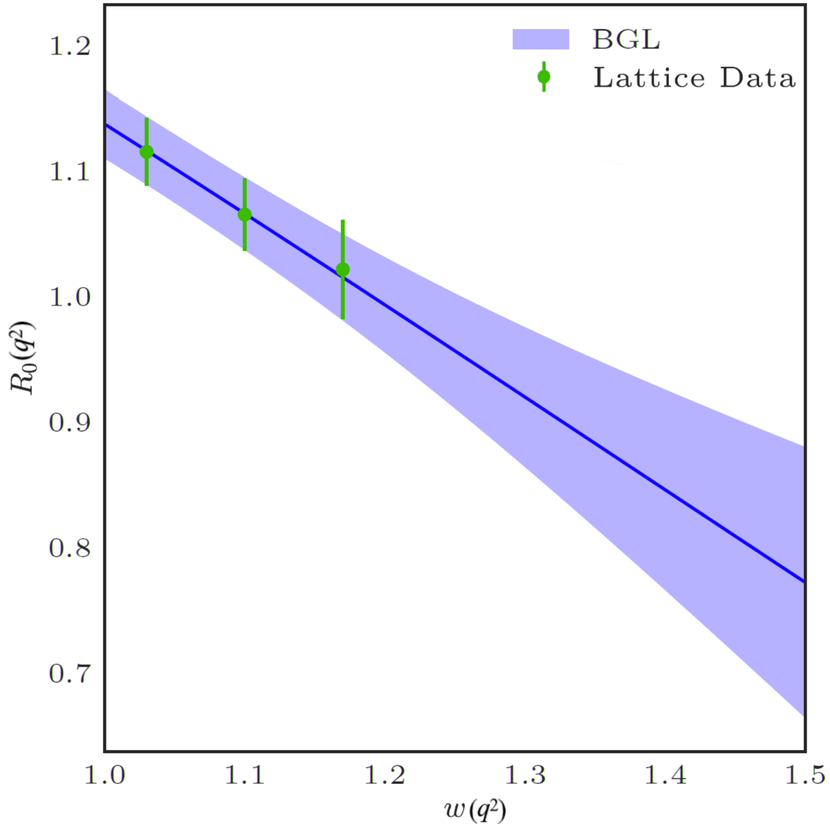
obtain constraints analyticity and unitarity
$\Longrightarrow$ **unitarity bounds**

BGL parametrization: [Boyd/Grinstein/Lebed 1994]

$$\mathrm{FF}(z) = \frac{1}{\mathcal{B}(z)\phi(z)} \sum_{n=0}^{\infty} a_n^{\mathrm{FF}}\, z^k$$

$$\boxed{\sum_{FF} \sum_{n=0}^{\infty} \left|a_n^{\mathrm{FF}}\right|^2 < 1}$$

determine the truncation error



[FNAL/MILC 2021]

two different ways to apply use the bounds:

1. "standard" BGL fit
2. dispersive matrix method

two methods substantially equivalent

$$w(q^2) = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

Combined fit with unitarity bounds

combine all $\boldsymbol{B \to D^*}$ FFs results using unitarity bound
[FNAL/MILC 2021] [JLQCD 2023][HPQCD 2023]

good fit can be obtained [Martinelli/Simula/Vittorio 2023]

calculate

$$R(D^*) = 0.266 \pm 0.009$$

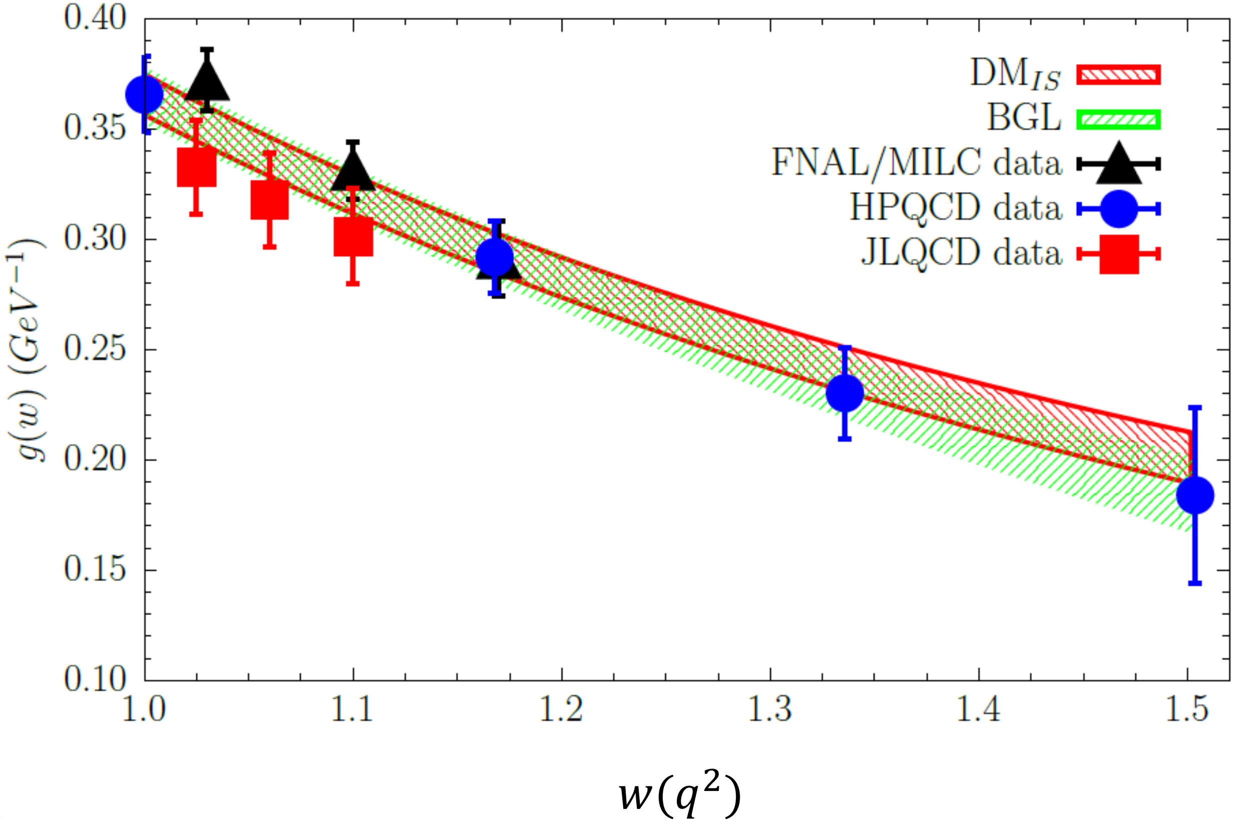
tension with $R(D^*)$ measurement reduced from $2.2\,\sigma$ to 1.2σ

extract $|V_{cb}|$ by comparing with recent Belle (II) data

$$|V_{cb}| \cdot 10^3 = 40.16 \pm 0.53$$

reduced tension with inclusive determination to $2.5\,\sigma$

$\boldsymbol{B \to D^*}$ FF



[Martinelli/Simula/Vittorio 2023]

and they lived happily ever after…

didn’t they?

A closer look to lattice QCD for $B \to D^*$

compute the differential width
using $B \to D^*$ FF from LQCD

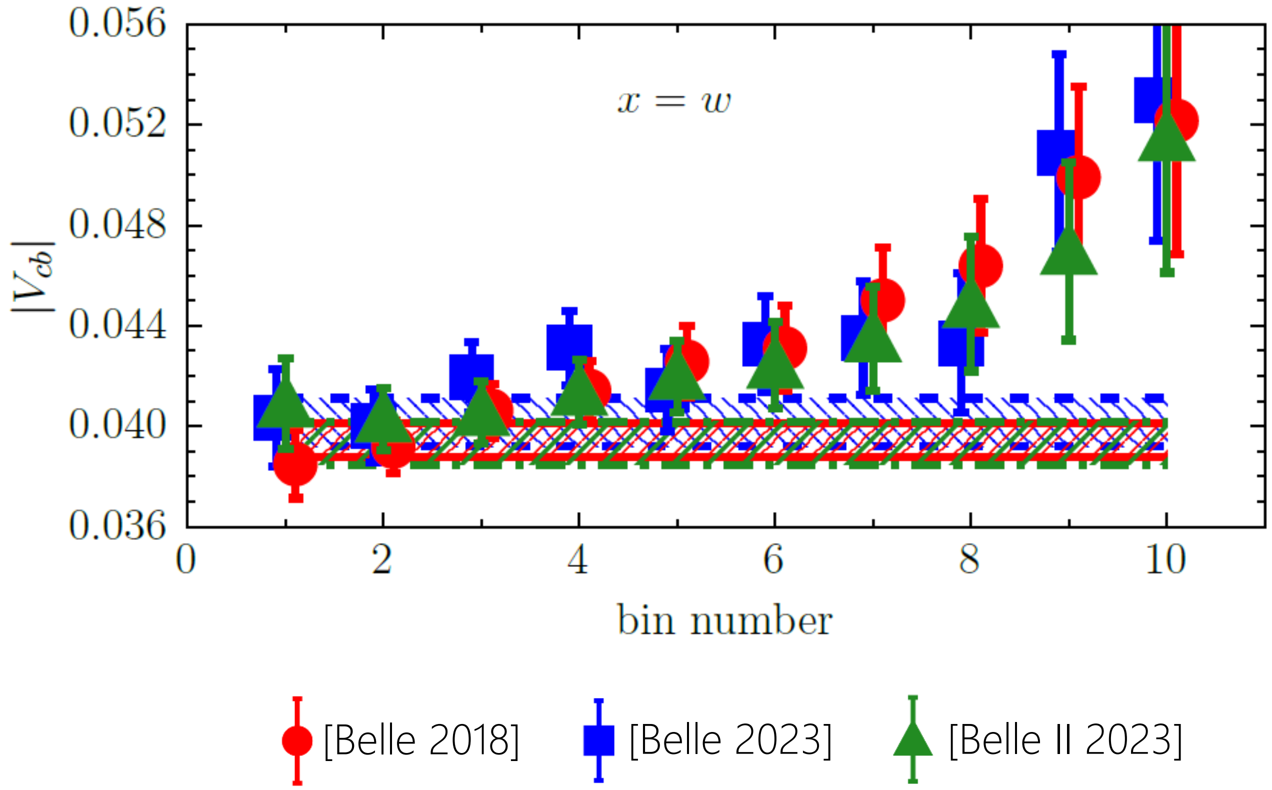
extract $|V_{cb}|$ bin by bin by comparing
with available experimental data
[Belle (II) 2018 and 2023]

shape (q^2 dependence) **discrepancy** between
data and lattice QCD

- systematic issue in (some) LQCD results
- New Physics in $B \to D^*\{e,\mu\}\nu$

no issues in $B \to D\ell\nu$ decays

$|V_{cb}|$ extraction with FNAL/MILC FFs



[Martinelli/Simula/Vittorio 2023]

How to proceed?

? can the theory predictions from the current LQCD results be **trusted**?

? does it make sense to **extract** $|V_{cb}|$ if theory and experimental shapes disagree?

? are the different $B \to D^{(*)}$ LQCD calculations **consistent** among each other?

? is there a way to check whether the LQCD results have **issues**?

Heavy quark expansion for form factors

HQE for the $B \to D^{(*)}$ FFs

use heavy-quark expansion (**HQE**), i.e. that $m_b, m_c \gg \Lambda_{\text{QCD}}$, to relate $B \to D^{(*)}$ FFs

$$FF^{B\to D^{(*)}}(q^2) = \xi(q^2)\left(c_0 + c_1\frac{\alpha_s}{\pi}\right) + c_2\frac{1}{m_b}L_i(q^2) + c_3\frac{1}{m_c}L_i(q^2) + c_4\frac{1}{m_c^2}l_i(q^2)$$

essential to include $\Lambda_{\text{QCD}}^2/m_c^2$ corrections (CLN not sufficient) [Bordone/Jung/van Dyk 2019]

all the $B \to D^{(*)}$ FFs can be expressed in terms of **10 Isgur-Wise functions**
(1 leading, 3 subleading, 6 subsubleading)

$\Longrightarrow$ **relations between** $B \to D^{(*)}$ FFs

LQCD calculations must fulfil these relations (within errors)

alternative method to include $1/m_c^2$ corrections proposed in Bernlochner F. et al. (2022)

Our HQE parametrization

expand in z the **Isgur-Wise functions**

- leading power $\xi(q^2) = \sum_{n=0}^{N} \xi^{(n)} z^n\,(q^2)$
- subleading $L_i(q^2) = \sum_{n=0}^{M} L_i^{(m)} z^m\,(q^2)$
- subsubleading $l_i(q^2) = \sum_{n=0}^{K} l_i^{(k)} z^k\,(q^2)$

$N/M/K$ parametrization

3/2/1 parametrization is the minimal order to achieve a good description [Bordone/Jung/van Dyk 2019]

rewrite the BGL unitarity bounds in terms of **Isgur-Wise parameters**

$$\boxed{\sum_{FF}\sum_{n=0}^{\infty}\left|\alpha_n^{\mathrm{FF}}\right|^2 \equiv \sum_{FF}\sum_{n=0}^{\infty}\left|\alpha_n^{\mathrm{FF}}(\xi^{(n)}, L_i^{(m)}, l_i^{(k)})\right|^2 < 1}$$

weak bound $\Longrightarrow$ **strong bound**

Some (concerning) comparison

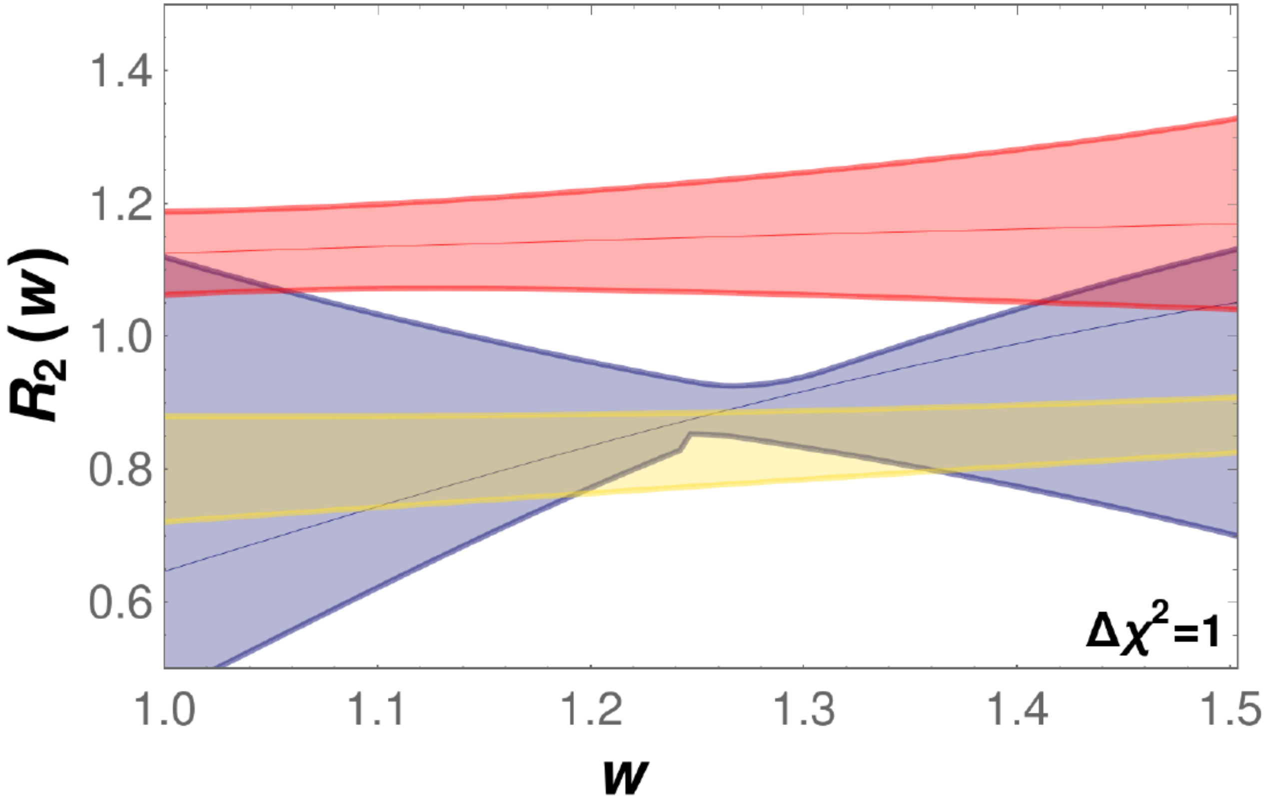
tension between experimental measurements (BGL) and FNAL/MILC 2021 (HPQCD 2023)

tension between HQE fit 2019 ($1/m_c^2$) and FNAL/MILC 2021 (HPQCD 2023)

solid pheno analyses need stable inputs

discussion about different approaches (parametrizations) is useless if inputs are faulty

until LQCD results are well understood theory predictions ($\boldsymbol{R(D^{(*)})}$) and $|\boldsymbol{V_{cb}}|$ extractions cannot be trusted



[credit: Martin Jung – LHCb impl. 2022]

Preliminary HQE fit (FF results)

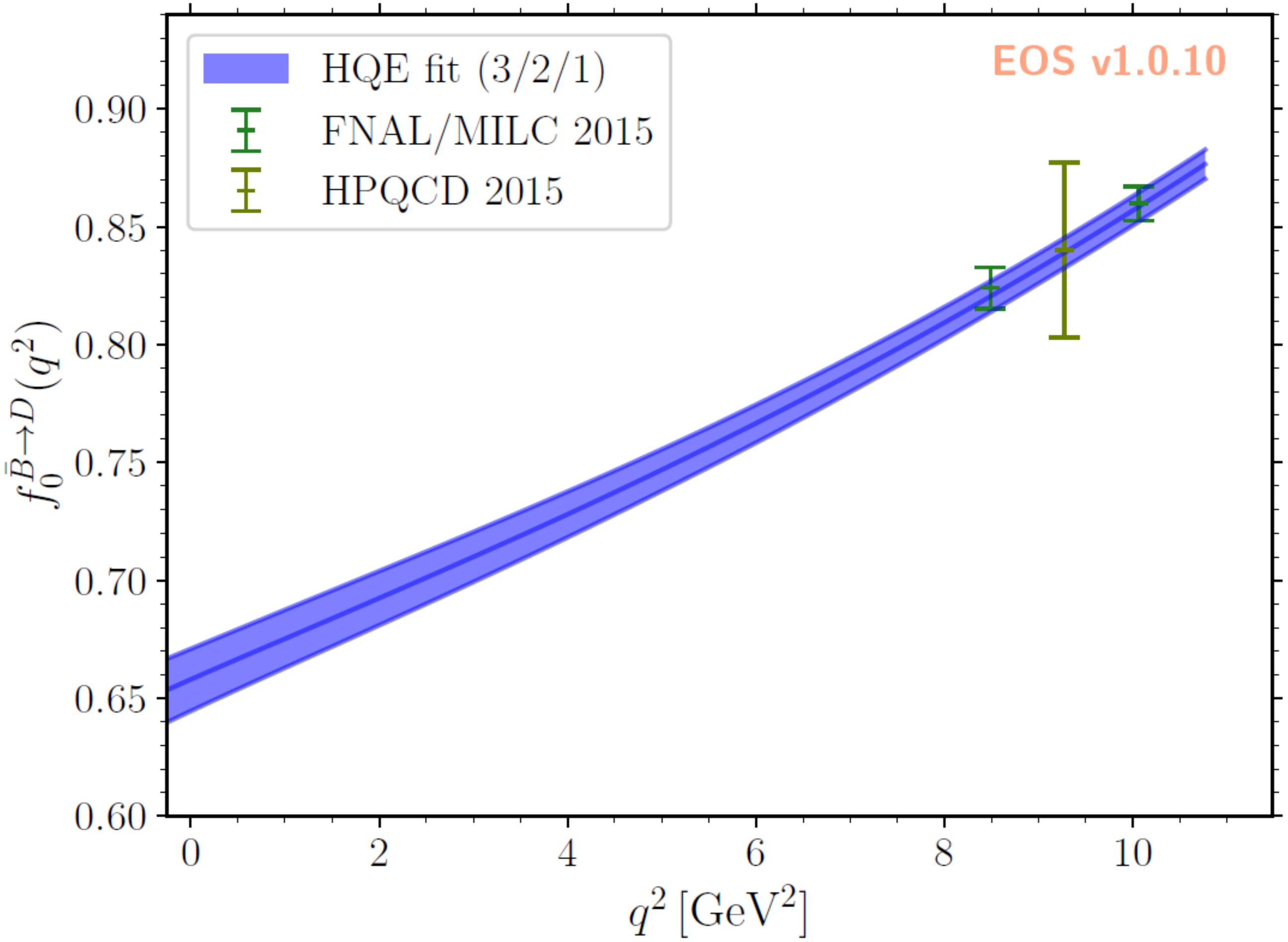
use HQE up to $1/m_c^2$ corrections

fit all available $B \to D^{(*)}$ FFs from LQCD
$B \to D$: [FNAL/MILC 2015] [HPQCD 2015]
$B \to D^*$: [FNAL/MILC 2021] [JLQCD 2023] [HPQCD 2023]

use (strong) dispersive bounds

obtain a good fit p-value ~ 50%
HPQCD 2023 has still to be published

obtain $B \to D^{(*)}$ FFs
predict physical observable

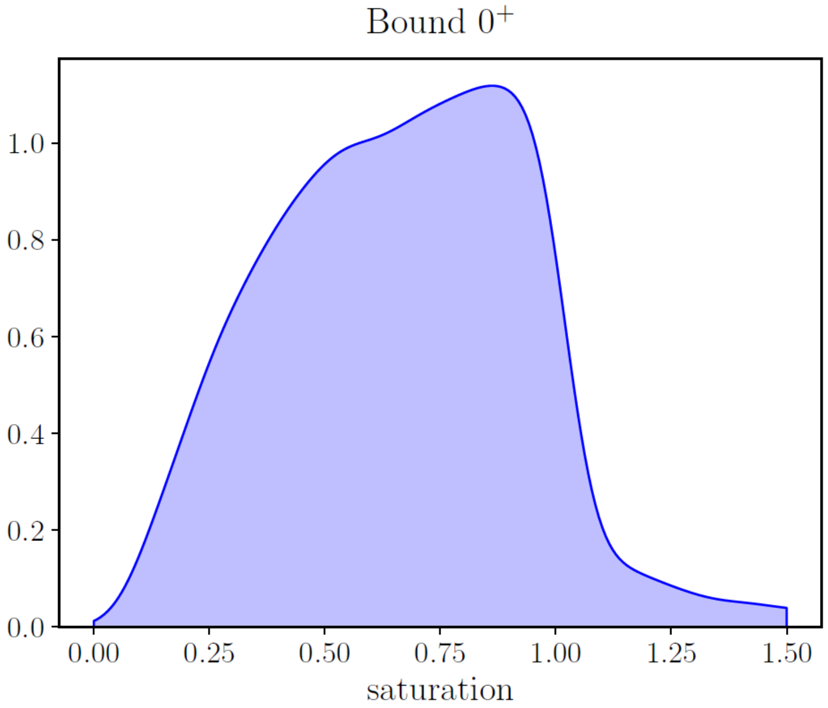


Preliminary HQE fit (bounds)

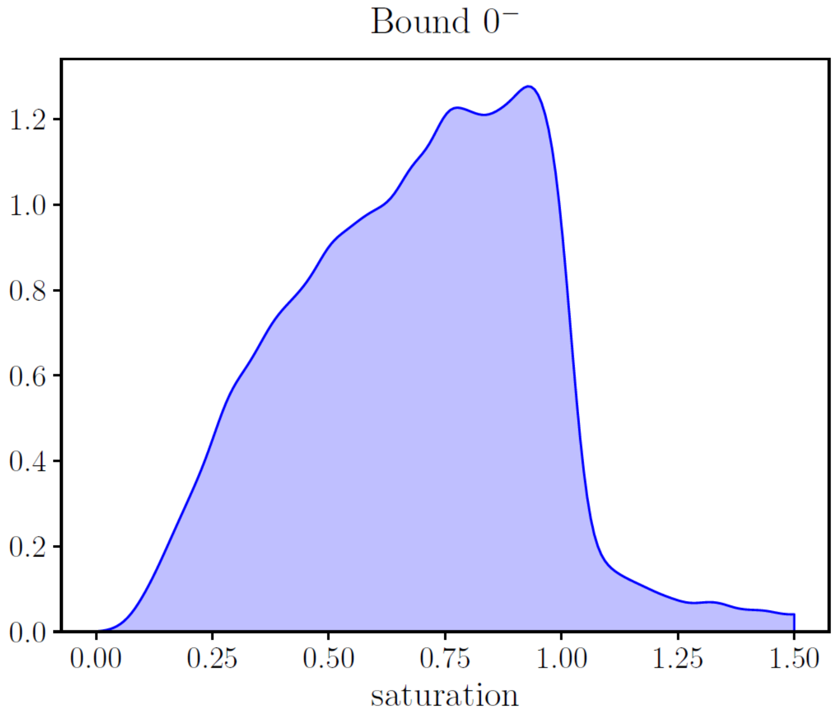
unitarity bound is (suspiciously) almost saturated by LQCD results. ⟹ essential for the analysis

also observed by Martinelli et al.

unitarity bounds not only control the truncation error but also **check the consistency of LQCD results**



Bound 0^+



Bound 0^-

A few words on $B_s \to D_s^{(*)}$

different Isgur-Wise functions

$$FF^{B\to D^{(*)}}(q^2) = \xi^{s}(q^2)\left(c_0 + c_1\frac{\alpha_s}{\pi}\right) + c_2\frac{1}{m_b}L_i^{s}(q^2) + c_3\frac{1}{m_c}L_i^{s}(q^2) + c_4\frac{1}{m_c^2}l_i^{s}(q^2)$$

contribute to the same unitarity bound ⟹ extract more precisely $B \to D^{(*)}$ FFs

$$\sum_{FF}\sum_{n=0}^{\infty}\left|\alpha_n^{\mathrm{FF}(B\to D^{(*)})}\right|^2 + \sum_{FF}\sum_{n=0}^{\infty}\left|\alpha_n^{\mathrm{FF}(B_s\to D_s^{(*)})}\right|^2 < 1$$

$B_s \to {D_s}^{(*)}$ FFs easier to compute than $B \to D^{(*)}$ for LQCD

fewer experimental results (only LHCb), Belle II does produce B_s mesons (for the moment being)

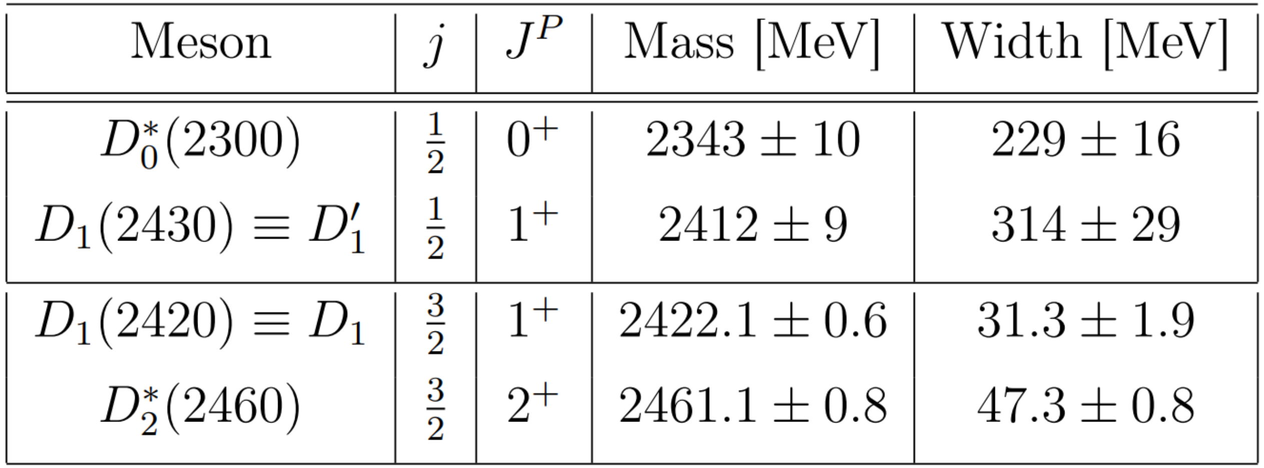
Next steps

1. combine analysis of $B_s \to D_s^{(*)}$ and $B_s \to D_s^{(*)}$FFs
2. predict observables (branching ratios, angular observables, $R(D^{(*)})$)
3. compare with experimental data (Belle and Belle II 2023)
4. extract $|V_{cb}|$?

WORK IN PROGRESS

$B \to D^{**}$ form factors

D^{**} mesons



Meson	j	J^P	Mass [MeV]	Width [MeV]
$D_0^*(2300)$	$\frac{1}{2}$	0^+	2343 ± 10	229 ± 16
$D_1(2430) \equiv D_1'$	$\frac{1}{2}$	1^+	2412 ± 9	314 ± 29
$D_1(2420) \equiv D_1$	$\frac{3}{2}$	1^+	2422.1 ± 0.6	31.3 ± 1.9
$D_2^*(2460)$	$\frac{3}{2}$	2^+	2461.1 ± 0.8	47.3 ± 0.8

why study $B \to D^{**}\ell\nu$ decays?

- **alternative** way to study $b \to c\ell\nu$ transitions ($R(D^{**})$ ratios, $|V_{cb}|$ etc.)
- **background** in $B \to D^*\ell\nu$ measurements
- understand the **gap** inclusive vs. sum of exclusive $B \to X_c\ell\nu$

State of the art $B \to D^{**}$ FFs

calcultions of $B \to D^{**}$ FFs using **sum rules and HQET** for both the b and c quarks [Colangelo/De Fazio/…]

data driven determination of the $B \to D^{**}$ **Isgur-Wise functions** [Bernlochner/Ligeti/…]

no LQCD calculation available

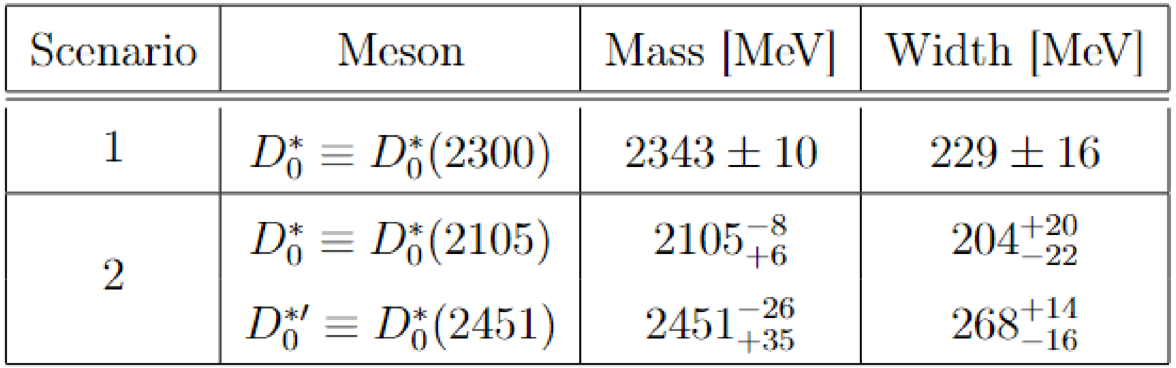
LCSRs

- $B \to D_2^*$ FFs [Aliev et al 2019]
- $\boldsymbol{B \to D_1}$ **and** $\boldsymbol{B \to D_1'}$ **FFs** (first calculation with finite m_c) [NG/Khodjamirian/Mandal/Mannel 2022]
- $\boldsymbol{B \to D_0^*}$ **FFs** [NG/Khodjamirian/Mandal/Mannel 2023]

$B \to D_0^*$ FFs

two scenarios [Du et al. 2017]:

1. single broad resonance $D_0^*(2300)$
2. two scalar resonances $D_0^*(2105)$ and $D_0^*(2451)$



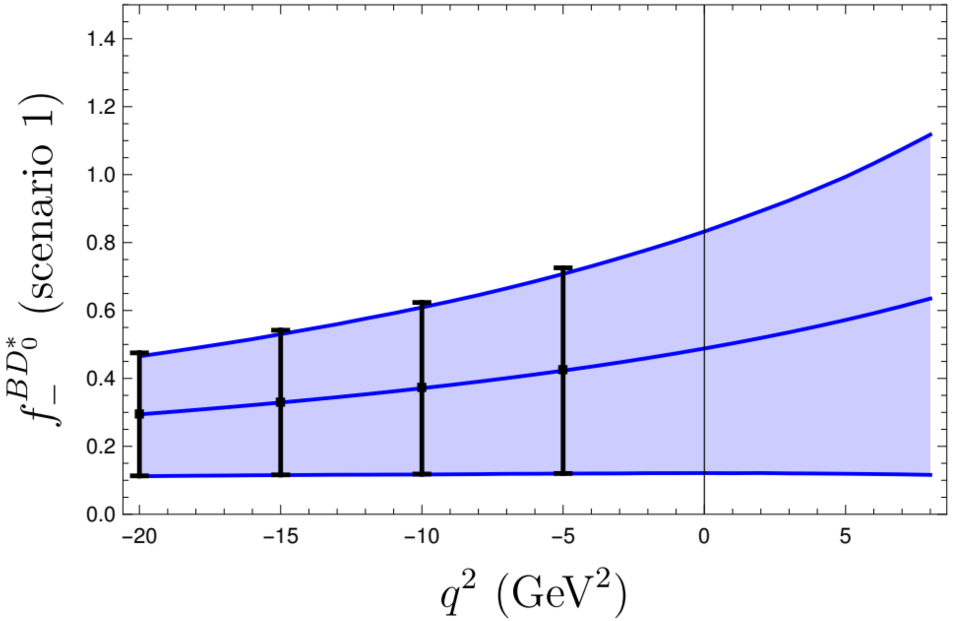
Scenario	Meson	Mass [MeV]	Width [MeV]
1	$D_0^* \equiv D_0^*(2300)$	2343 ± 10	229 ± 16
2	$D_0^* \equiv D_0^*(2105)$	2105^{-8}_{+6}	204^{+20}_{-22}
	$D_0^{*\prime} \equiv D_0^*(2451)$	2451^{-26}_{+35}	268^{+14}_{-16}

calculate all $B \to D_0^*$ FFs in both scenarios using standard LCSR approach

calculate also branching ratio and LFU ratio

scenario 1: $R(D_0^*) = 0.11^{+0.03}_{-0.01}$
scenario 2: $R(D_0^*) = 0.16^{+0.04}_{-0.01}$

calculate also $B_s \to D_{0s}^*(2317)$ FFs



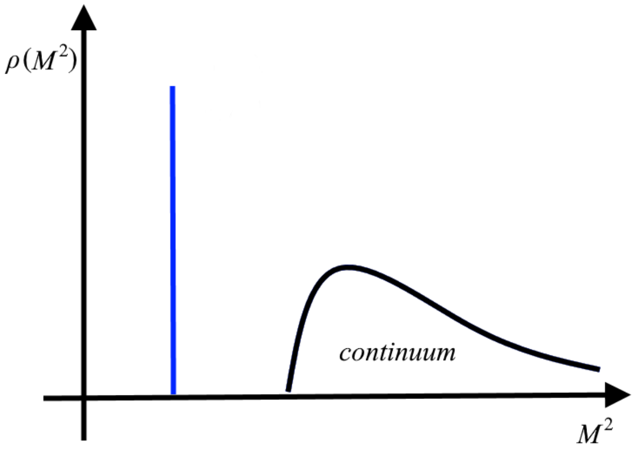
[NG/Khodjamirian/Mandal/Mannel 2023]

New LCSRs for $B \to D_1^{(\prime)}$ FFs

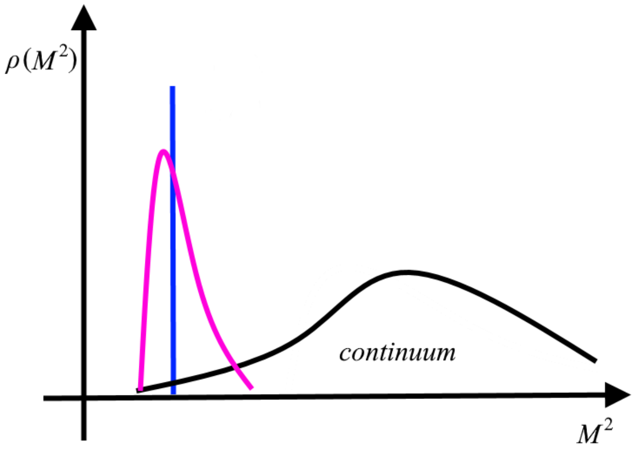
define a correlator and study spectral density

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{J_{int}(x), J_{weak}(0)\}|B(k+q)\rangle$$

usual LCSRs
(e.g. $B \to D$) **one** ground **state**



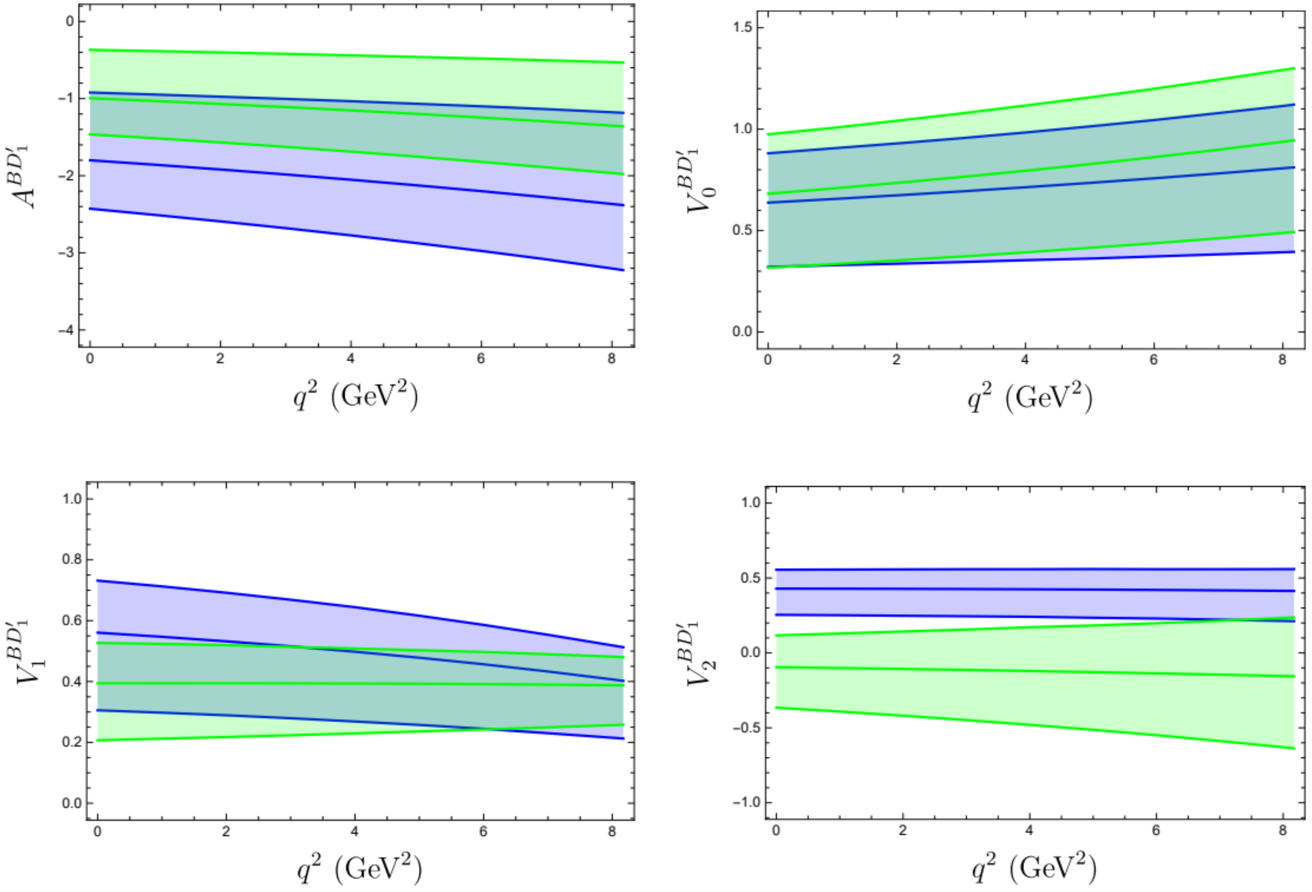
two states (**D_1** and **D_1'**) with similar masses and $J^P = 1^+$ (cannot be disentangled using a standard LCSRs)



define new type of LCSRs to deal with states with similar masses

Numerical results for $B \to D_1^{(\prime)}$ FFs

new method yields a **twofold ambiguity** (could be resolved with more experimental data or LQCD results)



[NG/Khodjamirian/Mandal/Mannel 2022]

both solutions give

$R(D_1) = 0.10 \pm 0.02$

$R(D_1') = 0.10 \pm 0.03$

in agreement with Bernlochner, Ligeti et al.

Summary and conclusion

Summary and conclusion

- **amazing progress by recent LQCD** calculations (but a few concerns for $B \to D^*$ FFs)

- **combine** theory inputs using $\boldsymbol{z}$ **parametrization**
  $\Longrightarrow$ control the truncation error using **unitarity bounds**

- **HQET** gives additional and precious constraints

- puzzle in the non-zero recoil $B \to D^*$ FFs from LQCD
  ([FNAL/MILC 2021] [HPQCD 2023])
  $\Longrightarrow$ **understand these results otherwise theory predictions ($\boldsymbol{R(D^{(*)})}$)
  and $|\boldsymbol{V_{cb}}|$ extractions cannot be trusted**

- $B \to D^{**}\ell\nu$ decays interesting and promising alternative channel to test $\boldsymbol{b \to c\ell\nu}$ transitions

Thank you!