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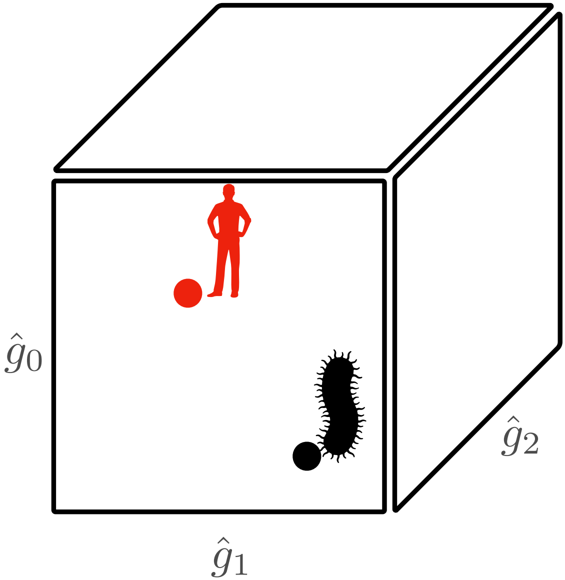
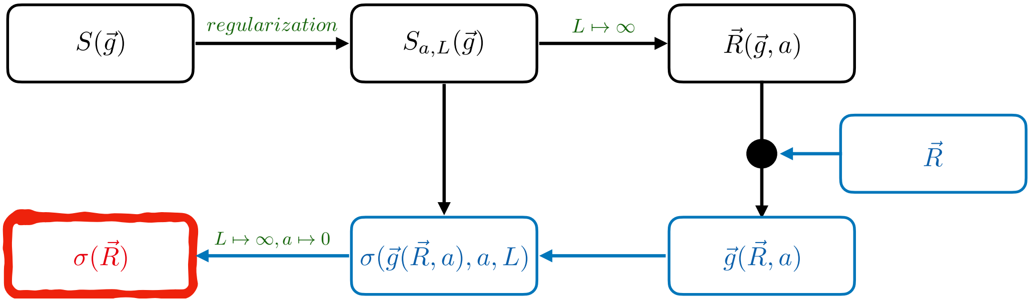
Lattice 2022, Bonn

Matching QC+ED to Nature

- the first step of any QFT calculation, aiming at phenomenological predictions, is the matching of the theory to Nature

- a renormalizable theory depending upon N parameters can describe $\mathbb{R}^N$ Universes
- and we can choose the one that we want!



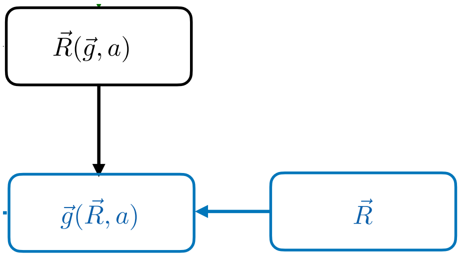


- $\vec{g}$ are the N bare couplings
- $\vec{R}$ are N experimental inputs
- σ is the prediction

- this algorithm, a.k.a. renormalization, can be implemented on the lattice as it is

$$\vec{R}(\vec{g}, a) = \vec{R} \quad \mapsto \quad \vec{g}(\vec{R}, a)$$

$$\lim_{a \mapsto 0} \sigma(\vec{g}(\vec{R}, a), a) = \sigma(\vec{R})$$



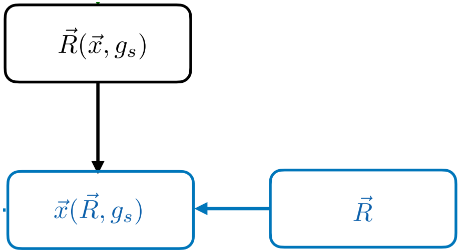
- in practice we usually prefer to fix g_s and solve for a

$$\vec{g} = (g_s, e, a\vec{m}) \ ,$$

$$\vec{x} = (a, e, a\vec{m})$$

$$\vec{R}(\vec{x}, g_s) = \vec{R} \quad \mapsto \quad \vec{x}(\vec{R}, g_s)$$

$$\lim_{g_s \mapsto 0} \sigma(\vec{x}(\vec{R}, g_s), g_s) = \sigma(\vec{R})$$



- at the (sub)percent level of precision the hadronic universe is described by QCD+QED

$$\vec{g} = (g_s, e, a\vec{m})$$

- we need to fix n_f masses and 2 gauge couplings in terms of an equal number of experimental inputs

- these must include a dimensional quantity!!

$$\sigma\left(\hat{g}_s(\mu), \hat{\alpha}(\mu), \hat{m}_f(\mu); \mu\right) = \sigma \underbrace{\left(\Lambda, \frac{M^i}{\Lambda}\right)}_{\vec{R}}, \qquad i = 1, \cdots, n_f + 1$$

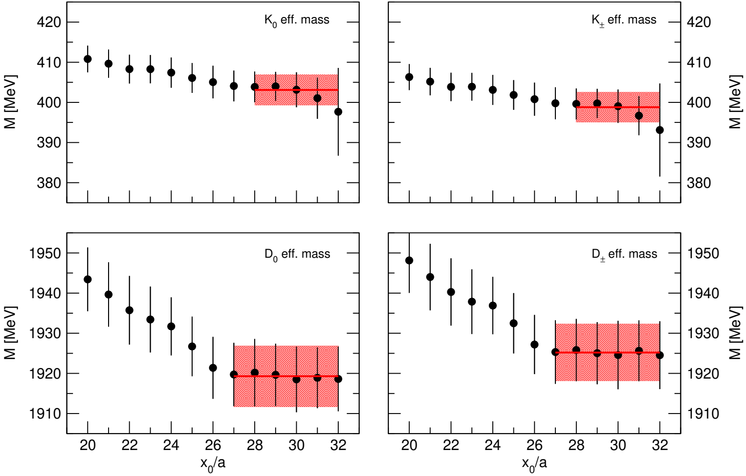
- the fact that the bare strong coupling g_s vanishes in the continuum doesn't mean that we don't need an input to tune it or to fix the lattice spacing
- this is the well known mechanism of dimensional transmutation: $g_s \to \Lambda_{QCD}$

pseudoscalar meson masses?

hi simplicio!

$$n_f = 4\,, \quad \vec{R} = \left(\frac{M^2_{\pi^+}}{M^2_{K^0}}\,,\ \frac{M^2_{D^+}}{M^2_{K^0}}\,,\ \frac{M^2_{D_s}}{M^2_{K^0}}\,,\ M_{K^0}\,,\ \underbrace{\frac{M^2_{K^+} - M^2_{K^0}}{M^2_{K^0}}\,,\ \frac{M^2_{D^+} - M^2_{D^0}}{M^2_{K^0}}}_{\vec{R}_{IB} \quad \mapsto \quad e,\ m_u - m_d} \right)$$

RC* in preparation, j.lücke talk



- full QCD+QED$_C$, $\hat{\alpha} \simeq 1/137$
- no gauge–fixing
- unphysical quark masses $m_d = m_s$

- we could discuss forever on the best choice for the input parameters. . .

- i'm pretty sure we all agree that the inputs should be
  - precisely measured experimental observables
  - precisely computable on the lattice
  - under theoretical control w.r.t. quark–mass, volume dependence, excited–states contaminations, induced cutoff effects

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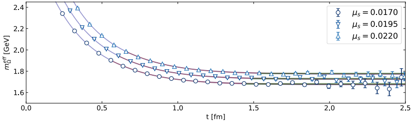
$$n_f = 4\,, \quad \vec{R} = \left(\frac{M^2_{\pi^+}}{M^2_{K^0}}\,, \frac{M^2_{D^+}}{M^2_{K^0}}\,, \frac{M^2_{D_s}}{M^2_{K^0}}\,, M_{K^0}\,, \frac{M^2_{K^+} - M^2_{K^0}}{M^2_{K^0}}\,, \frac{M^2_{D^+} - M^2_{D^0}}{M^2_{K^0}} \right)$$

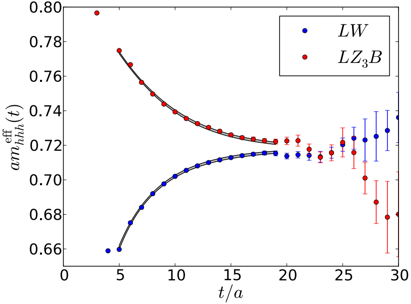
so Momega?

$$n_f = 4\,, \qquad \vec{R} = \left(\frac{M_{\pi^+}^2}{M_\Omega^2}\,,\ \frac{M_{K^0}^2}{M_\Omega^2}\,,\ \frac{M_{D_s}^2}{M_\Omega^2}\,,\ M_\Omega\,,\ \frac{M_{K^+}^2 - M_{K^0}^2}{M_\Omega^2}\,,\ \frac{M_{D^+}^2 - M_{D^0}^2}{M_\Omega^2} \right)$$

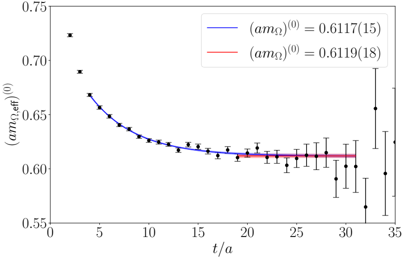
ETMC PRD.104.2021



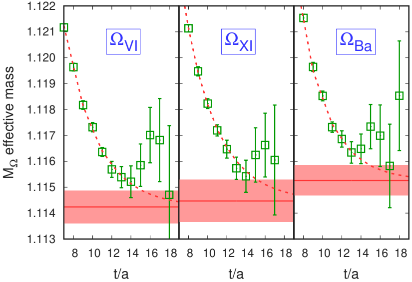
RBC/UKQCD PRD.93.2016 r.mawhinney talk



MAINZ a.segner talk



BMW Nature.593.2021

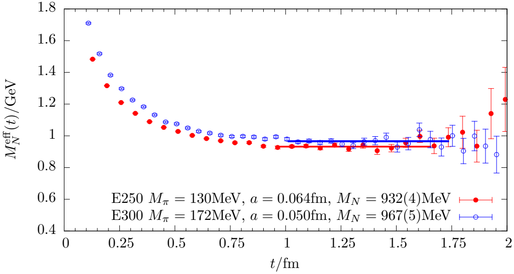


- excited states contamination is (will become) a serious issue

$$m_\pi^{eff}(t) = m_\pi + c\, e^{-2m_\pi t} + \cdots$$

$$\mapsto m_\pi + c\, e^{-\frac{2\pi}{L} t} + \cdots$$

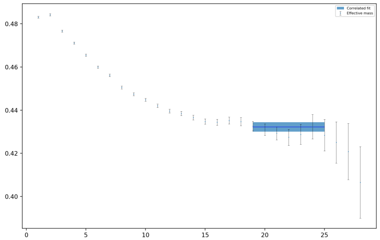
k.ottnad talk

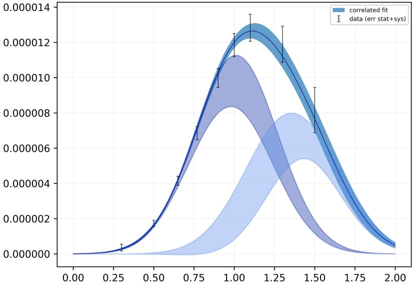


- spectral densities techniques might become a useful tool. . .

$$C(t) = \int_{e_0}^{\infty} dE\, \rho(E)\, e^{-Et}$$

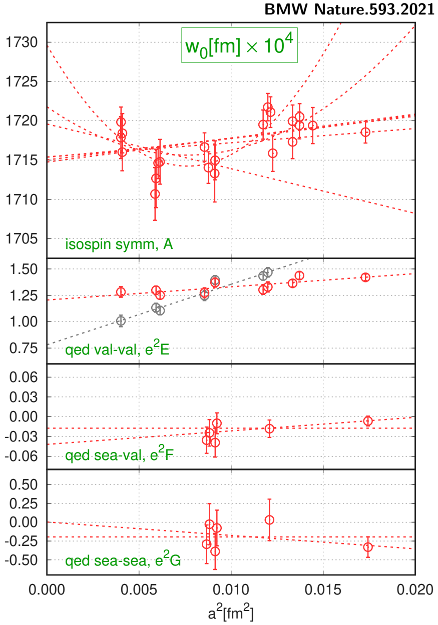
$$\mapsto \int_{e_0}^{\infty} dE\, \rho(E)\, \delta_\sigma(E - E_\star)$$





what about theory scales?

once you know their "physical" values ...



- the so–called theory scales (w_0, $\sqrt{t_0}$, M_{qq}, ...) are quantities that cannot be directly measured in experiments but can be computed very precisely on the lattice
- **renormalized couplings** can also be viewed as **theory scales** and/or viceversa
- having matched the theory to Nature, by using experimental inputs, these quantities can be computed as any other observable

$$\vec{x} = (a, e, a\vec{m})$$

$$\vec{R}^{exp} \quad \mapsto \quad \vec{x}(\vec{R}^{exp}, g_s) \quad \mapsto \quad \sigma,\ w_0,\ M_{ss},\ \hat{\alpha}(\mu),\ \hat{m}_c(\mu)\cdots$$

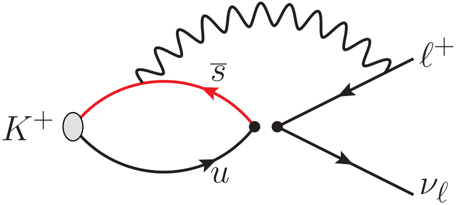
- this has to be done, once in history at least, then

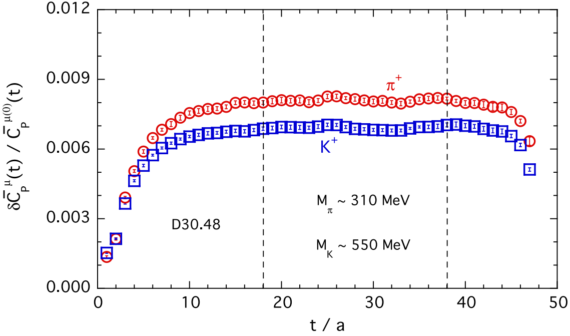
$$w_0,\ M_{ss},\ \cdots \quad \mapsto \quad \vec{x}(\vec{R}^{exp}, g_s) \quad \mapsto \quad \sigma$$

- i'll come back to theory scales and to the BMW plot later...

why not FK?

mhm...





- leptonic decay rates are cumbersome on the lattice because of the $\log(L)$ divergences appearing at intermediate stages of the calculations

- the infrared–safe measurable quantity is the sum of virtual and real photons contributions

- moreover, we have to give up V_{ud} or V_{us}

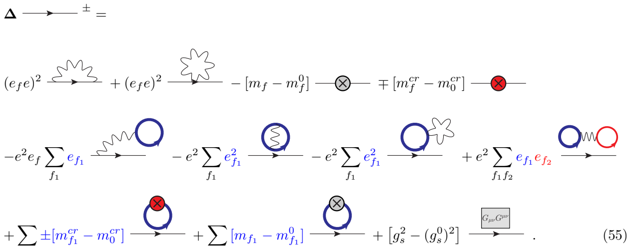
$$\mathcal{F}_\pi(E_\gamma) = \sqrt{\frac{\Gamma\left[\pi^+ \mapsto \mu^+\nu_\mu(\gamma), E_\gamma\right]}{\frac{G_F^2}{8\pi}|V_{ud}|^2 M_{\pi^+}^{exp}(M_\mu^{exp})^2\left[1-\left(\frac{M_\mu^{exp}}{M_{\pi^+}^{exp}}\right)^2\right]^2}}$$

look, QED corrections are small!!

RM123 PRD.87.2013

look, QED corrections are small!!

:)
let's talk about QCD and corrections



$$\alpha = \frac{1}{137.035999084(21)}$$

- even if we neglect $O(\alpha^2)$, QCD and QCD+QED are two different theories

$$\vec{x} = (a, a\vec{m})$$

$$\vec{R}(g_s, \vec{x}, \alpha) = \vec{R} \quad \mapsto \quad \vec{x}^R \equiv \vec{x}(g_s, \vec{R}, \alpha)$$

$$\vec{R}(g_s, \vec{x}, 0) = \vec{R} \quad \mapsto \quad \vec{x}_0^R \equiv \vec{x}(g_s, \vec{R}, 0)$$

- a (trivial?) statement: QCD and QCD+QED are two different theories

$(e_f e)^2$  $\longrightarrow$ $[m_f - m_f^0]$ 

$$J^\mu(x) J_\mu(0) \longrightarrow c_1(x)1 + \sum_f \left[c_m^f(x) m_f + c_{cr}^f(x) \right] \bar{\psi}_f \psi_f + c_{g_s}(x) G_{\mu\nu} G^{\mu\nu} + \cdots$$

electromagnetic currents generate divergent contributions that redefine the vacuum energy, c_1, the quark masses, c_m^f, the quark critical masses (if chirality is broken), c_{cr}^f, and the strong coupling constant (the lattice spacing), c_g

$$\vec{x} = (a, a\vec{m})$$

$$\vec{R}(g_s, \vec{x}, \alpha) = \vec{R} \quad \mapsto \quad \vec{x}^R \equiv \vec{x}(g_s, \vec{R}, \alpha)$$

$$\vec{R}(g_s, \vec{x}, 0) = \vec{R} \quad \mapsto \quad \vec{x}_0^R \equiv \vec{x}(g_s, \vec{R}, 0)$$

- in QCD+QED, in order to match the theory and to do predictions, we only need $\vec{x}^R$

$$\vec{R}(g_s, \vec{x}^R, \alpha) = \vec{R}(g_s, \vec{x}^R, 0) + \alpha \frac{\partial \vec{R}}{\partial \alpha}(g_s, \vec{x}^R, 0) = \vec{R}$$

$$\sigma(\vec{R}, \alpha) = \lim_{g_s \mapsto 0} \left\{ \sigma(g_s, \vec{x}^R, 0) + \alpha \frac{\partial \sigma}{\partial \alpha}(g_s, \vec{x}^R, 0) \right\}$$

- if we want to take the continuum limit of the two terms separately, i.e. match/define QCD, we need $\vec{x}_0^R$

$$\sigma(\vec{R},\alpha) = \lim_{g_s\mapsto 0}\left\{\sigma(g_s,\vec{x}^R,0) + \alpha\frac{\partial\sigma}{\partial\alpha}(g_s,\vec{x}^R,0)\right\}\ , \qquad \lim_{g_s\mapsto 0}\sigma(g_s,\vec{x}^R,0) = \infty$$

$$\vec{x}^R = \vec{x}_0^R + \Delta\vec{x}^R$$

$$\sigma(\vec{R},0) = \lim_{g_s\mapsto 0}\sigma(g_s,\vec{x}_0^R,0)\ , \qquad \Delta\sigma(\vec{R},\alpha) = \lim_{g_s\mapsto 0}\left\{\Delta x_i^R\frac{\partial}{\partial x_i} + \alpha\frac{\partial}{\partial\alpha}\right\}\sigma(g_s,\vec{x}_0^R,0)$$

$$\Delta\sigma(\vec{R},\alpha) = \sigma(\vec{R},\alpha) - \sigma(\vec{R},0) = \alpha\left.\frac{\partial\sigma(\vec{R},\alpha)}{\partial\alpha}\right|_{\vec{R},\alpha=0}$$

- we don't need to use the same inputs to match QCD+QED and to define QCD

$$\vec{R}(g_s, \vec{x}, \alpha) = \vec{R} \;\mapsto\; \vec{x}^R \;, \qquad\qquad \vec{S}(g_s, \vec{x}, 0) = \vec{S} \;\mapsto\; \vec{x}_0^S$$

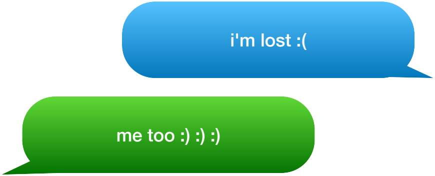
$$\left\{ \Delta x_i^S \frac{\partial}{\partial x_i} + \alpha \frac{\partial}{\partial \alpha} \right\} \vec{R}(g_s, \vec{x}_0^S, 0) = R - \vec{R}(g_s, \vec{x}_0^S, 0) \qquad \mapsto \qquad \Delta \vec{x}^S = \vec{x}^R - \vec{x}_0^S$$

$$\sigma(\vec{R}, 0) + \Delta\sigma(\vec{R}, \alpha) = \sigma(\vec{S}, 0) + \Delta\sigma(\vec{S}, \alpha) = \sigma$$

$$\sigma(\vec{R}, 0) - \sigma(\vec{S}, 0) = O(\alpha)$$

i'm lost :(

me too :) :) :)



- if $O(\alpha)$ matters QCD is unphysical
- QCD can be defined by matching it directly to Nature
- or by using a convenient prescription
- QCD results coming from different prescriptions differ

$$\sigma(\vec{R}+\vec{\varepsilon},0)-\sigma(\vec{R},0)=\varepsilon^i\frac{\partial}{\partial R^i}\sigma(\vec{R},0)$$

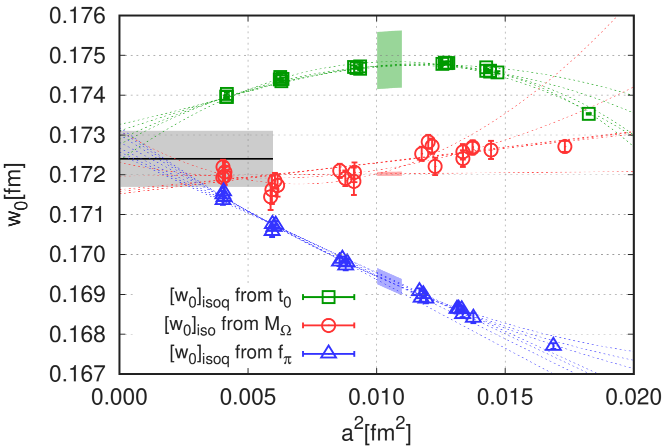
at least *in principle* ...

- *in practice* . . .

BMW Nature.593.2021

$$\vec{R} = \left(\frac{M_{\pi^0}^2}{M_\Omega^2},\ \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{2M_\Omega^2},\ \frac{\hat{m}_c(\mu)}{\hat{m}_s(\mu)} = 11.85,\ M_\Omega,\ \frac{M_{K^+}^2 - M_{K^0}^2}{M_\Omega^2} \right), \quad \alpha = \frac{1}{137.035999084(21)}$$

BMW Nature.593.2021



$M_\pi = M_{\pi^0}$

$M_{ss} = 689.89(28)(40)[49] MeV$

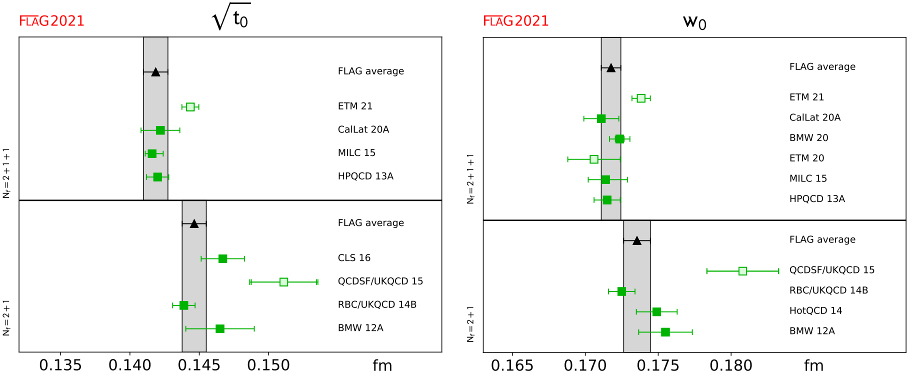
M_Ω

$M_\pi = 134.8(3)\ MeV$

$M_K = 494.2(3)\ MeV$

$t_0 = 0.1416(^{+8}_{-5})\ fm$

$f_\pi = 130.50(14)\ MeV$



FLAG2021
√t0
FLAG average
ETM 21
CalLat 20A
MILC 15
HPQCD 13A
FLAG average
CLS 16
QCDSF/UKQCD 15
RBC/UKQCD 14B
BMW 12A
Nf=2+1+1
Nf=2+1
0.135
0.140
0.145
0.150
fm
FLAG2021
w0
FLAG average
ETM 21
CalLat 20A
BMW 20
ETM 20
MILC 15
HPQCD 13A
FLAG average
QCDSF/UKQCD 15
RBC/UKQCD 14B
HotQCD 14
BMW 12A
Nf=2+1+1
Nf=2+1
0.165
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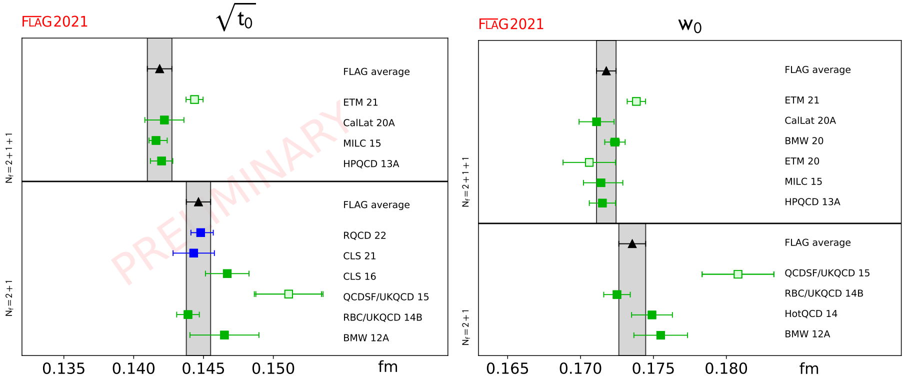
fm



- CLS21 $f_{K+\pi}$, **arXiv:2112.06696**, RQCD **s.collins talk**

- different inputs, e.g. CalLat+RBC/UKQCD M_Ω, CLS $f_{K+\pi}$, ETMC+HPQCD+MILC f_π, RQCD M_Ξ,

$$M_\pi^{iso} = M_{\pi^0} \ , \qquad M_\Omega^{iso} = M_\Omega \ , \qquad f_\pi^{iso} = 130.4(3)\ MeV \ , \qquad M_K^{iso} \in [494.2, 497.6]\ MeV$$

- maybe, for these quantities, the charm–quenching effect is presently more relevant than QED scheme ambiguities

see **TUMQCD j.weber talk**

- the matching of the $n_f = 2 + 1$ and $n_f = 2 + 1 + 1$ QCDs is a problem that can be addressed by using the same strategy we have been discussing so far

- although many (not very) different prescriptions have been used

- and some dedicated investigations have been done

RM123+SOTON PRD.100.2019

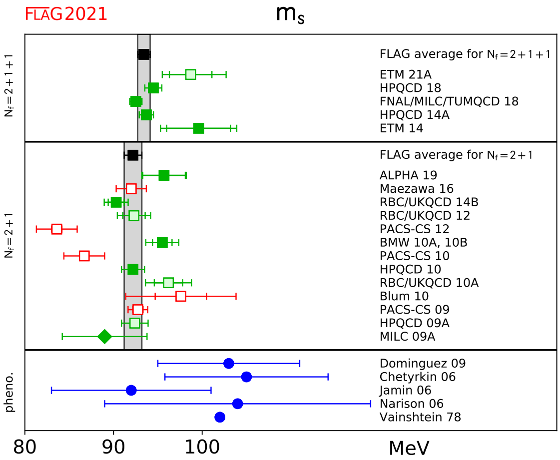
BMW Nature.593.2021

RBC-UKQCD c.lehner talk

- no significant differences have been observed yet wait wait, some prescriptions can have a big impact on quark masses

hi antonin!

see **a.portelli talk**



the GRS schemes

gasser et al. EPJ.C32.2003, RM123 PRD.87.2013

wait wait, some prescriptions can have a big impact on quark masses

hi antonin!

see a.portelli talk

$$\hat{m}_{ud}(\mu)|_{QCD} = \left.\frac{\hat{m}_u(\mu) + \hat{m}_d(\mu)}{2}\right|_{QCD+QED}$$

$$\hat{m}_s(\mu)|_{QCD} = \hat{m}_s(\mu)|_{QCD+QED}$$

$$\hat{m}_c(\mu)|_{QCD} = \hat{m}_c(\mu)|_{QCD+QED}$$

$$\hat{g}_s(\mu)|_{QCD} = \hat{g}_s(\mu)|_{QCD+QED}$$

FLAG $<$ 2021

RM123+SOTON PRD.100.2019

wait wait, some prescriptions can have a big impact on quark masses

hi antonin!

see **a.portelli talk**

$$M_{\pi}^{GRS} = M_{\pi^0}\,(1+\varepsilon_{\pi^0}) = 135.0(2)\ MeV$$

$$M_{K}^{GRS} = M_{K^0}\,(1+\varepsilon_{K^0}) = 494.6(1)\ MeV$$

$$M_{D_s}^{GRS} = M_{D_s}\,(1+\varepsilon_{D_s}) = 1966.7(1.5)\ MeV$$

$$f_{\pi}^{GRS} = \mathcal{F}_{\pi}(E_{\gamma}^{max})\,(1+\varepsilon_{f_\pi}) = 130.65(12)\ MeV$$

- the past is the past. . .
- in the future, can we agree on this?

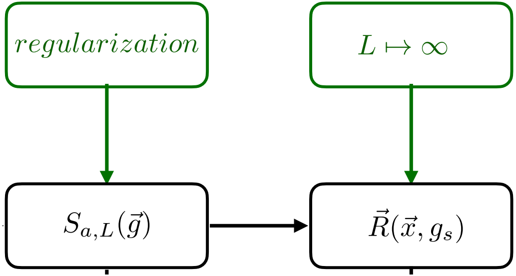
$$QCD^{2+1}\ , \qquad \vec{R} = \left(\frac{M^2_{\pi^0}}{M^2_\Omega},\ \frac{M^2_{K^0}}{M^2_\Omega},\ M_\Omega \right)$$

- maybe you prefer this?

$$QCD^{2+1}\ , \qquad \vec{R} = \left(\frac{M^2_{\pi^0}}{f^2_\pi},\ \frac{M^2_{K^0}}{f^2_\pi},\ f_\pi \right)\ , \qquad f_\pi = 130.56\ MeV$$

- GRS?

backup



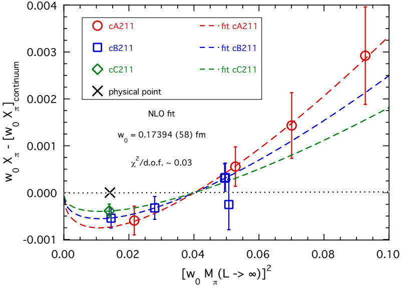
- in QCD+QED hadron masses are affected by large finite volume effects
- the leading ones are universal and can be removed

davoudi-savage PRD.90.2014, BMW Science.347.2015
lucini et al. JHEP.02.2016

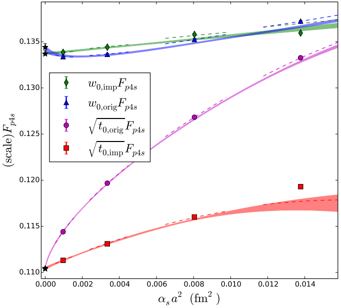
$$\frac{M(\vec{x}, g_s, L)}{M(\vec{x}, g_s, \infty)} = 1 + \alpha \left(\frac{q^2 \xi(1)}{LM(\vec{x}, g_s, \infty)} + \frac{q^2 \xi(2)}{[LM(\vec{x}, g_s, \infty)]^2} + O(L^{-n}, \alpha^2) \right)$$

concerning cutoff effects in gradient–flow scales see e.g. a.ramos talk

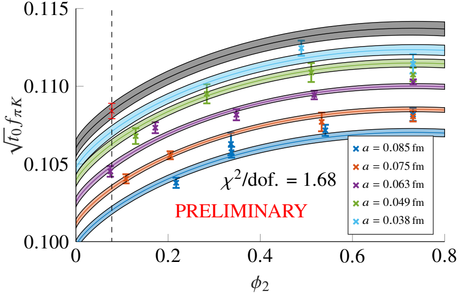
ETMC PRD.104.2021



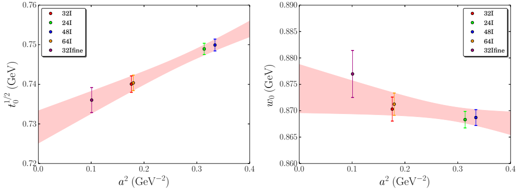
MILC PRD.93.2016



CLS arXiv:2112.06696



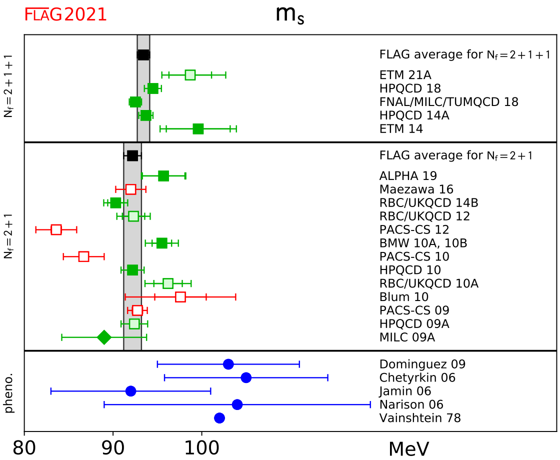
RBC/UKQCD PRD.93.2016



Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume	physical scale	$\sqrt{t_0}$ [fm]	w_0 [fm]
ETM 21	[53]	2+1+1	P	★	★	★	f_π	0.14436(61)	0.17383(63)
CalLat 20A	[31]	2+1+1	A	★	★	★	m_Ω	0.1422(14)	0.1709(11)
BMW 20	[3]	1+1+1+1	A	★	★	★	m_Ω		0.17236(29)(63)[70]
ETM 20	[1037]	2+1+1	C	★	★	★	f_π		0.1706(18)
MILC 15	[67]	2+1+1	A	★	★	★	$F_{p4s}(f_\pi)^{\#}$	0.1416(+8/-5)	0.1714(+15/-12)
HPQCD 13A	[68]	2+1+1	A	★	○	★	f_π	0.1420(8)	0.1715(9)
CLS 16	[69]	2+1	A	○	★	★	f_π, f_K	0.1467(14)(7)	
QCDSF/UKQCD 15B	[70]	2+1	P	○	○	○	$m_P^{SU(3)}$	0.1511(22)(6)(5)(3)	0.1808(23)(5)(6)(4)
RBC/UKQCD 14B	[32]	2+1	A	★	★	★	m_Ω	0.14389(81)	0.17250(91)
HotQCD 14	[71]	2+1	A	★	★	★	$r_1(f_\pi)^{\#}$		0.1749(14)
BMW 12A	[39]	2+1	A	★	★	★	m_Ω	0.1465(21)(13)	0.1755(18)(4)

$$M_\pi^{isoQCD} = M_{\pi^0} \ , \qquad M_\Omega^{isoQCD} = M_\Omega \ , \qquad f_\pi^{isoQCD} = 130.4(3) \ MeV \ ,$$

$$M_K^{isoQCD} \in [494.2, 497.6] \ MeV \ , \qquad M_{ss}, \qquad \cdots$$



ETM21A $M_K = 494.2\ MeV$

ALPHA19 $M_K = 494.2\ MeV$

HPQCD18 $M_{\eta_s} = 0.6885(22)\ GeV$

FNAL/MILC/TUMQCD18 $M_K = 494.5\ MeV$

RBCUKQCD14B $M_K = 495.7\ MeV$

BMW10 $494.2\ MeV$

the GRS schemes. . . by implementing it once in history

wait wait, some prescriptions can have a big impact on quark masses

hi antonin!

see **a.portelli talk**

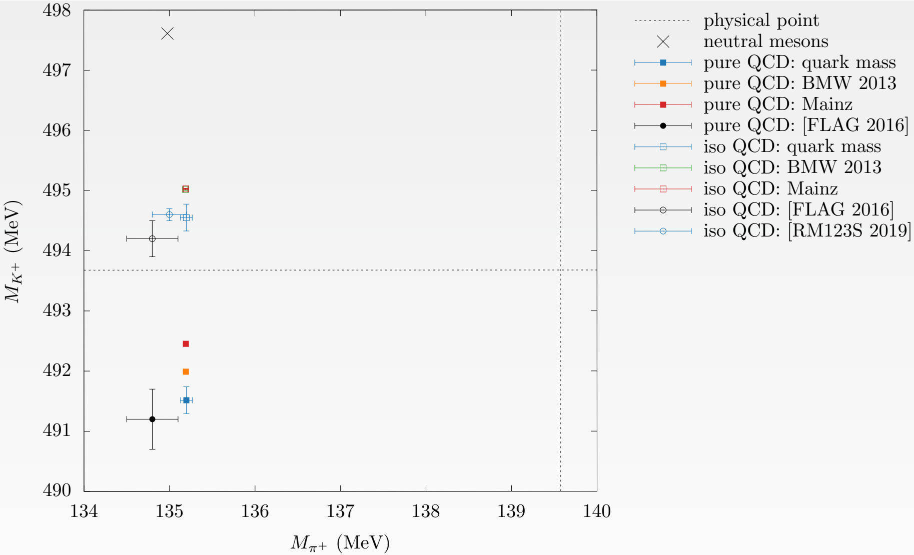
$$\mu = 2\ GeV$$

$$\mu = \left\{ \left(\frac{\sqrt{\tau}}{a}\right)_{GF},\ \left(\frac{2\pi|\vec{n}|}{L}\right)_{RI-MOM},\ \left(\frac{1}{L}\right)_{SF},\ \cdots \right\}$$

$$\hat{g}_s^{QCD}(a\mu, g_s)$$

$$a \mapsto \hat{g}_s^{QCD}(a\mu, g_s^\star) = \hat{g}^\star\ , \qquad g_s^\star(a\mu, \hat{g}^\star)$$

$$g_s \mapsto \hat{g}_s^{QCD}([a\mu]^\star, g_s) = \hat{g}^\star\ , \qquad a = \frac{[a\mu]^\star(g_s, \hat{g}_s^\star)}{\mu}$$



physical point
neutral mesons
pure QCD: quark mass
pure QCD: BMW 2013
pure QCD: Mainz
pure QCD: [FLAG 2016]
iso QCD: quark mass
iso QCD: BMW 2013
iso QCD: Mainz
iso QCD: [FLAG 2016]
iso QCD: [RM123S 2019]
M_{K^+} (MeV)
M_{π^+} (MeV)
498
497
496
495
494
493
492
491
490
134
135
136
137
138
139
140